1 Fee Calibration

We approach the calibration of minting fees similar to credit pricing. We calibrate the fees that we charge to the minters so that the expected losses to the stakers are paid for by fees. The Liquid Collateral Contract, as outlined in the Frankencoin Manifesto, has the following features.

- The participants are minters, challengers, auction participants, and ZCHF stakers.
- The minter deposits collateral and thereby mints ZCHF. The ZCHF are overcollateralized at the time of minting, that is, the value of collateral deposited exceeds the value of the minted ZCHF.
- Challengers can initiate an auction process for a given position at any time. Afterwards, auction participant bid for the collateral.
    1. If, according to the auction, the value of the collateral falls below a specified threshold, the position is liquidated and the minter loses their collateral. *E.g., the collateral deposited is 1500 LUSD, 1000 ZCHF were minted. Now, the position is challenged and the best bid for 1500 LUSD closes at 1095 ZCHF. Let's assume that the threshold is 10%. Now, because 1095 ZCHF < 1000 (1+10%) ZCHF, the position is closed out.*
    2. If, according to the auction, the collateral value is above the threshold, the position remains in the minter's ownership
    3. If, according to the auction, the collateral value falls below the minted amount of ZCHF, ZCHF in the staking pool have to be burnt, i.e., a loss to ZCHF stakers.

The system outlined above has the following parameters. Let h be the threshold that defines whether the position can be liquidated or not. That is, if, according to the auction, the value of the collateral is below $Z(1+h)$, where Z is the amount of ZCHF minted for a given position, the position is liquidated. Let c be the challenger reward (e.g., 2% of the ZCHF position). Finally, τ is the duration of the auction (e.g., 3 days).

$$h : h > 0, \text{ liquidation threshold, liquidate if highest bid is below Z(1+h)} \tag{1}$$

$$\tau : \text{duration of liquidation process} \tag{2}$$

$$c : 0 < c < h, \text{ challenger reward} \tag{3}$$

By $\tilde{r}$ we denote the random variable that corresponds to the log-return of the collateral using CHF as the quote-currency (or numeraire). Now, we can express the loss to the stakers as the following random variable:

$$\tilde{L} = -\min\left[(1+h)e^{\tilde{r}} - (1+c), 0\right], \tag{4}$$

per unit of ZCHF minted (e.g., if the position consists of Z ZCHF, the loss to the stakers is $Z\tilde{L}$). To see this, first note that the starting value of the position is at least $(1+h)$ per unit of Z. Since $\tilde{L}$ is per ZCHF, we set $Z = 1$. The value of the position at the end of the auction is $Z(1+h)e^{\tilde{r}}$. Stakers need to burn ZCHF for the amount that the end-of-auction value falls short of the minted amount plus the challenger reward, hence we subtract $(1+c)Z$. We need the min[$\cdot$]-function because only losses are charged to the stakers, profits in the liquidation process are not distributed to stakers. Finally, the negative sign is convention to have a positive number for the loss.

Table 1: Minting Fees. We calibrate minting fees for a collateral with 5% daily volatility, assuming a t-distribution with 4 degrees of freedom (left), or a normal distribution (right). The volatility is scaled by $\sqrt{\tau}$, and the mean is assumed to be zero. Further, $h = 0.10$, $c = 0.02$. Fees displayed are in percentages.

τ	**t-distr**	**normal**
1	0.38	0.08
2	0.90	0.38
3	1.38	0.72

If we assume that the auction process will result in prices similar to what we would observe on liquid exchanges, then a remaining risk is that the price falls into a loss region during the auction period. With a known density function for the return distribution over the period τ, $f_\tau(x)$, we can calculate the expected loss as follows

$$\mathbb{E}_\tau\left[\tilde{L}\right] = -\int_{-\infty}^{k} \left((1+h)e^x - (1+c)\right) f_\tau(x)dx \tag{5}$$

$$k = \log\frac{1+c}{1+h}, \tag{6}$$

where the subscript τ emphasizes that the distribution depends on the time-horizon of the auction.

Table 1 shows results from calibrating Eq. (5). BTC-USD has a daily volatility of about 5%. The table compares the result for Eq. (5) when assuming either a t-distribution or a normal distribution for the return.