Signal flow graph

→ What is signal flow graph?
– Def related to signal flow graph (SFG)
→ How to find T·F from given SFG
→ How to convert block dia representation to SFG

$$TF = \frac{LT\ q\ o/P}{LTq\ i/P}$$

- → electrical system
- → Block dia representation
- → Signal flow graph.
- → Mechanic system

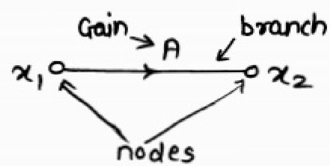
★ Signal Flow Graphs ★

S·F·G :- pictorial representation of a system which displays transmission of signals in the system graphically.

★ **Features :-**

- applies to time-invariant linear systems only.
- signal flow along the direction of arrows.
- linear algebraic equations are used to draw S·F·G.
- contains dependent & independent variables. (nodes)
- relationship bet^{n} nodes (branches)
- Gain of S·F·G. is given by Mason's eq^{n}.
- every block diagram can be represented by SFG but the converse is not true.

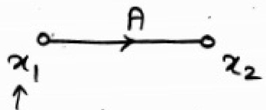
eg:-



$$x_2 = A \cdot x_1$$

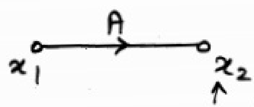
• Terms used in S.F.G.

1] Input or source node :-



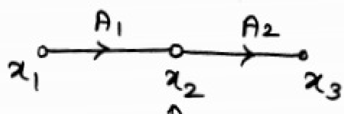
source / i/p node
(has only outgoing branches)

2] Output or sink node :-



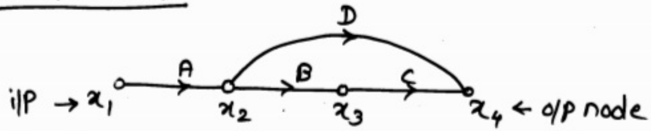
o/p / sink node
(has only incoming branches)

3] Chain node :-



chain node
(has both incoming and outgoing branches)

4] Forward Path :-

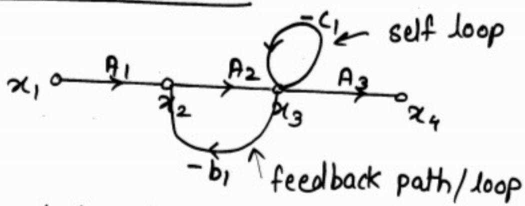


Path from i/p node to output node is forward path.

$$x_1 - x_2 - x_3 - x_4$$

$$x_1 - x_2 - x_4$$

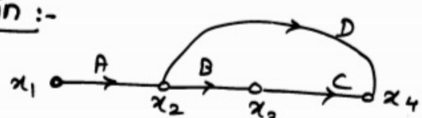
5] Feedback path / loop :-



Path which originates & terminates on same node.

a loop (feedback) which consists of only one node is self loop.

6] Path Gain :-



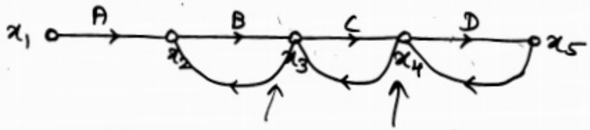
Product of branch gains in a forward path is called as path gain.

for $x_1 - x_2 - x_3 - x_4 \Rightarrow A \cdot B \cdot C$.

for $x_1 - x_2 - x_4 \Rightarrow A \cdot D$

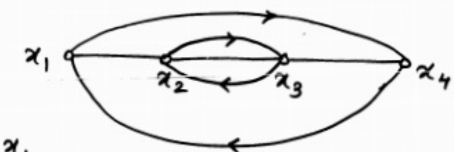
7] **Path :-** Traversal of connected branches in the direction of branch arrows such that no node is traversed more than once.

8] **Touching path :-**



Loops with one or more nodes in common are called as touching loops.

9] **Non-Touching loops :-**

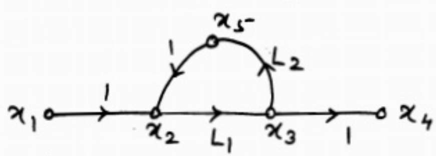


$x_1 - x_4 - x_1$

$x_2 - x_3 - x_2$

loops are non-touching when there is no common node

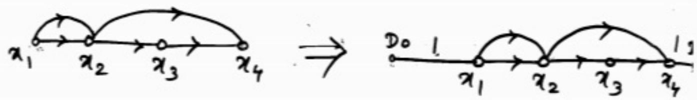
10] **Loop gain :-**



Product of branch gain forming a loop.

$x_2 - x_3 - x_5 - x_2$ · loop gain $= L_1 L_2$

11] Dummy Nodes :-



A branch with gain 1 can be added at i/p as well as o/p node. (Transfer function will not be affected.)

can't be added in between the chain node

★ **Mason's Gain Equation :-** This equation can be used to find the overall transfer funⁿ of system.

Mason's Gain formula :-

$$T.F. = \frac{C(s)}{R(s)} = \frac{1}{\Delta} \sum P_i \Delta_i$$

where i = no. of forward paths

P_i = Gain of i^{th} forward path

Δ_i = Value of Δ for part of graph not touching i^{th} f.w path

Δ = system determinant.

= 1 − (sum of all individual loop gains incl. self loop gain)

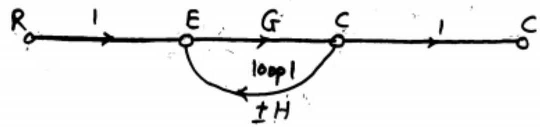
+ (" " " gain products of two non touching loops)

− (sum of all gain products of 3 non touching loops)

$\therefore \Delta = 1 - (P_{11} + P_{21} + \cdots) + (P_{..} + P_{..} + \cdots) - (P_{..} + P_{..} + \ldots)$

Signal Flow Graphs

(1)



1] Find no. of forward paths.

1 forward path R-E-C-C.

$$\underset{\text{Path Gain}}{P_1} = \underline{\underline{G}}$$

2] Find total no. of single loops.

only 1 loop.

$$P_{11} = \pm GH$$

3] There are no two / three non touching loops.

4] Find Δ.

$$\Delta = 1 - (P_{11})$$

$$\Delta = 1 - (\mp GH) = 1 \pm GH$$

5] Δ_1 find loops which are not touching path P_1.

There are no such loops.

$$\Delta_1 = 1 - (0) = 1.$$

$$P_1 \Delta_1 = G \times 1 = G.$$

6] $$\boxed{T.F. = \frac{C}{R} = \frac{P_1 \Delta_1}{\Delta} = \frac{G}{1 \pm GH}}$$

Mason's Gain Formula [MGF]

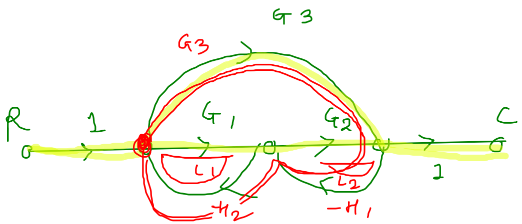
$$T.F. = \sum_{n=1}^{K} \frac{P_n \Delta_n}{\Delta} = \frac{C(s)}{R(s)}$$

K = no of forward path

$$TF = \left[\frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + \cdots P_K \Delta_K}{\Delta}\right]$$

2 → forward paths 3 Forward

$$T.F = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$$



Forward path = 2

$P_1 \rightarrow 1 \cdot G_1 \cdot G_2 \cdot 1 = \underline{G_1 G_2}$

$P_2 \rightarrow 1 \cdot G_3 \cdot 1 = \underline{G_3}$

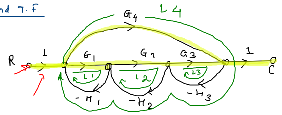
<u>loops</u> → 3 →

$L_1 = -G_1 H_2$

$L_2 = -G_2 H_1$

$L_3 = G_3(-H_1)(-H_2) = G_3 H_1 H_2$

find T.F



$L_1 L_2$ ×
$L_2 L_3$ ×
$L_1 L_3$ ✓
L_4 ×

Step 1 : No of forward path = 2

$P_1 = R - 1 - G_1 - G_2 - G_3 - 1 - C$

$P_2 = R - 1 - G_4 - 1 - C$

Step 2 MGF = $T \cdot F = \frac{C}{R} = \sum_{n=1}^{K} \frac{P_n \Delta_n}{\Delta}$

$TF = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$ → (1)

Step 3 Path gain

$P_1 = 1 \cdot G_1 \cdot G_2 \cdot G_3 \cdot 1 = G_1 G_2 G_3$

$P_2 = 1 \cdot G_4 \cdot 1 = G_4$

Step 4 Loop with gain

$L_1 = -G_1 H_1$

$L_2 = -G_2 H_2$

$L_3 = -G_3 H_3$

$L_4 = -G_4 H_1 H_2 H_3$

Step 5 two non touching loops gain

$L_1 L_3 = G_1 G_3 H_1 H_3$

three non touching loop ×

×

Step 6 find Δ

$\Delta = 1 -$ (sum of individual loop gain)
+ (sum of two non touching loop gain)
− (sum of 3 non touching loop gain)
+
−

$= 1 - (-G_1 H_1 - G_2 H_2 - G_3 H_3 - G_4 H_1 H_2 H_3)$
$+ (G_1 G_3 H_1 H_3)$

$\Delta = 1 + G_1 H_1 + G_2 H_2 + G_3 H_3 + G_4 H_1 H_2 H_3 + G_1 G_3 H_1 H_3$

Step 7 = $\Delta_1 = 1 -$ sum of non touching loop for 1st Forward path

$= 1 - 0$

$\Delta_1 = 1$

Step 8 → $\Delta_2 = 1 -$ sum of non touching loop for 2nd forward path

$\Delta_2 = 1 + G_2 H_2$

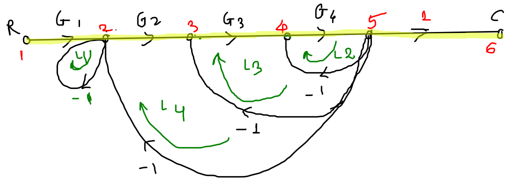
Step 9

R

$$TF = \boxed{\frac{[G_1 G_2 G_3] \cdot (1) + G_4 [1 + G_2 H_2]}{1 + G_1 H_1 + G_2 H_2 + G_3 H_3 + G_4 H_1 H_2 H_3 + G_1 G_3 H_1 H_3}}$$

C

Ex Find T.F of given SFG



Step 1 : No of forward path

$R - G_1 - G_2 - G_3 - G_4 - 1 - C$

$P_1 = G_1 G_2 G_3 G_4$ — path gain

Step 2 → Loop with gain

$L_1 = -1$ $\quad L_3 = -G_3 G_4$

$L_2 = -G_4$ $\quad L_4 = -G_2 G_3 G_4$

Step 3 $\quad MGF = \sum_{n=1}^{K} \frac{P_n \Delta_n}{\Delta} = \frac{P_1 \Delta_1}{\Delta}$

Step 4 : 2 non touching loop

$L_1 \,\&\, L_2 = L_1 L_2 = G_4$

$L_1 \,\&\, L_3 = L_1 L_3 = G_3 G_4$

3 non touching loop ✗

Step 5 $\quad \Delta = 1 - (-1 - G_4 - G_3 G_4 - G_2 G_3 G_4) + (G_4 + G_3 G_4) - 0$

$\Delta = 2 + 2G_4 + 2G_3 G_4 + G_2 G_3 G_4$

Step 6 $\quad \Delta_1 = 1 - (0) = 1$

Step 7 $\quad \boxed{T.F = \frac{(G_1 G_2 G_3 G_4) \cdot 1}{2 + 2G_4 + 2G_3 G_4 + G_2 G_3 G_4}}$