
Dynamic Programming

Dynamic programming is a useful mathematical technique for making a sequence of interrelated decisions. It provides a systematic procedure for determining the optimal combination of decisions.

In contrast to linear programming, there does not exist a standard mathematical formulation of “the” dynamic programming problem. Rather, dynamic programming is a general type of approach to problem solving, and the particular equations used must be developed to fit each situation. Therefore, a certain degree of ingenuity and insight into the general structure of dynamic programming problems is required to recognize when and how a problem can be solved by dynamic programming procedures. These abilities can best be developed by an exposure to a wide variety of dynamic programming applications and a study of the characteristics that are common to all these situations. A large number of illustrative examples are presented for this purpose.

11.1 A PROTOTYPE EXAMPLE FOR DYNAMIC PROGRAMMING

EXAMPLE 1 The Stagecoach Problem

The STAGECOACH PROBLEM is a problem specially constructed¹ to illustrate the features and to introduce the terminology of dynamic programming. It concerns a mythical fortune seeker in Missouri who decided to go west to join the gold rush in California during the mid-19th century. The journey would require traveling by stagecoach through unsettled country where there was serious danger of attack by marauders. Although his starting point and destination were fixed, he had considerable choice as to which states (or territories that subsequently became states) to travel through en route. The possible routes are shown in Fig. 11.1, where each state is represented by a circled letter and the direction of travel is always from left to right in the diagram. Thus, four stages (stagecoach runs) were required to travel from his point of embarkation in state *A* (Missouri) to his destination in state *J* (California).

This fortune seeker was a prudent man who was quite concerned about his safety. After some thought, he came up with a rather clever way of determining the safest route. Life

¹This problem was developed by Professor Harvey M. Wagner while he was at Stanford University.