MATH 4824C - Sample Midterm

March 24, 2024

Please (1) show the work to the questions and (2) return the answer sheet on time.

1. Show that in randomized trials; i.e. $(Y_i(1), Y_i(0), X_i) \perp Z_i$, $Var(ZY) = P(Z=1)Var(Y(1))$;

2. Show that in observational studies without unmeasured confounders; i.e. $(Y_i(1), Y_i(0)) \perp Z_i \mid X_i$, $E(Y(1) - X) = E[E(Y - X|Z = 1, X)]$

3. Show that in observational studies without unmeasured confounders; i.e. $(Y_i(1), Y_i(0)) \perp Z_i \mid X_i$, $E(Y(1) - X) = E[Z(Y - X)/e(X)]$ where $e(X) = P(Z = 1|X)$;

4. Show that in observational studies without unmeasured confounders; i.e. $(Y_i(1), Y_i(0)) \perp Z_i \mid X_i$, let $e(\mathrm{x}, \beta)$, be the logistic model of Z_i on X_i, and $\mu_1(\mathrm{x}, \alpha_1)$ be linear or logistic model of $Y_i(1)$ on X_i. Try to show that

$$\tilde{\mu}_{1,\mathrm{DR}} = E\left[\frac{Z_i\left\{Y_i - \mu_1\left(\mathrm{X}_i, \alpha_1\right)\right\}}{e\left(\mathrm{X}_i, \beta\right)} + \mu_1\left(\mathrm{X}_i, \alpha_1\right)\right]$$

   is doubly robust, that is, if either $\mu_1(\mathrm{x}, \alpha_1) = \mu_1(\mathrm{x}) := E[Y_i|Z_i = 1, \mathrm{X}_i = \mathrm{x}]$ or $e(\mathrm{x}, \beta) = e(\mathrm{x}) := P(Z_i|\mathrm{X}_i = \mathrm{x})$, then $\tilde{\mu}_{1,DR} = E\left\{Y_i(1)\right\}$.

5. In observational studies with binary response and binary unmeasured confounders U_i; i.e. $(Y_i(1), Y_i(0)) \perp Z_i \mid (X_i, U_i)$, if $RR^{obs}_{ZY} = 1.2$, calculate the E-value and interpret it.