

Problem1

(a)

$$\text{取 } h = 0.1 \quad x_1 = x_0 + h \quad x_2 = x_0 + 2h \quad x_3 = x_0 + 3h$$

$$f'(x_0) = \frac{1}{h}[-1.5f(x_0) + 2f(x_1) - 0.5f(x_2)] + \frac{h^2}{3}f^{(3)}(\xi)$$

$$f'(1.1) = 17.769705$$

对于 $x > 1.1$ 时, 用精度更高的三点公式的中间点公式

$$f'(x_0) = \frac{1}{2h}[f(x_0 + h) - f(x_0 - h)] - \frac{h^2}{6}f^{(3)}(\xi)$$

$$\text{对于 } x = 1.4, \text{ 采用 } f'(x_0) = \frac{1}{2h}[f(x_0 - 2h) - 4f(x_0 - h) + 3f(x_0)]$$

计算结果如下

$$f'(1.1) = 17.769705$$

$$f'(1.2) = 22.193635$$

$$f'(1.3) = 27.10735$$

$$f'(1.4) = 21.70365$$

(b)

用与a题类似的方法:

$$f'(8.1) = 3.09205$$

$$f'(8.3) = 3.11615$$

$$f'(8.5) = 3.139975$$

$$f'(8.7) = 3.363525$$

Problem2

$$M = N(h) + K_1h^2 + K_2h^4 + K_3h^6 \dots (1)$$

$$M = N\left(\frac{h}{3}\right) + K_1\frac{h^2}{3^2} + K_2\frac{h^4}{3^4} + K_3\frac{h^6}{3^6} \dots (2)$$

$$M = N\left(\frac{h}{9}\right) + K_1\frac{h^2}{9^2} + K_2\frac{h^4}{9^4} + K_3\frac{h^6}{9^6} \dots (3)$$

设 $(1) + a * (2) + b * (3)$ 可以产生 $O(h^6)$ 的近似

$$\text{则 } 1 + \frac{a}{3^2} + \frac{b}{9^2} = 0$$

$$1 + \frac{a}{3^4} + \frac{b}{9^4} = 0$$

$$\Rightarrow a = -90, b = 729$$

$$\text{于是 } (1) - 90 * (2) + 729 * (3) = 640M$$

$$= N(h) - 90N\left(\frac{h}{3}\right) + 729N\left(\frac{h}{9}\right) + \frac{640}{729}K_3h^6$$

$$\Rightarrow M = \frac{1}{640}(N(h) - 90N\left(\frac{h}{3}\right) + 729N\left(\frac{h}{9}\right)) + \frac{1}{729}K_3h^6$$

Problem3

见文件夹中的Python程序

以下是结果展示

Trapezoidal rule :

$$\int_a^b f(x)dx = \frac{b-a}{2} * [f(a) + f(b)] - \frac{h^3}{12} f''(\xi)$$

Simpon's Rule :

$$h = \frac{b-a}{2} \quad c = \frac{b+a}{2}$$

$$\int_a^b f(x)dx = \frac{h}{3} * [f(a) + 4f(c) + f(b)] - \frac{h^5}{90} f^{(4)}(\xi)$$

题号\方法	Trapezoidal rule	Simpon's Rule
a题	0.46939564047259313	0.4897985468241977
b题	0.08664339756999316	0.05285463856097945
c题	-0.037024252723997224	-0.020271589910295148
d题	0.28638599308922763	0.27271652783901024

Problem4

(a)

$$R_{3,2} = R_{3,1} + \frac{1}{3}(R_{3,1} - R_{2,1})$$

$$R_{2,2} = R_{2,1} + \frac{1}{3}(R_{2,1} - R_{1,1})$$

$$R_{3,3} = R_{3,2} + \frac{1}{15}(R_{3,2} - R_{2,2})$$

$$h = a - b$$

$$\int_a^b f(x)dx = \frac{h}{2^n} [f(a) + f(b) + 2 \sum_{j=1}^{n-1} f(x_j)]$$

$$\Rightarrow R_{3,3} = 1.45281$$

(b)

方法都是一致的

$$\Rightarrow R_{3,3} = 0.327959$$

(c)

$$\Rightarrow R_{3,3} = 1.387063$$

(d)

$$\Rightarrow R_{3,3} = 0.272515$$

Problem5

程序见C++文件。

(a)

$$w_0 = \alpha = 1$$

$$h = 0.1$$

$$N = \frac{2-1}{0.1} = 10$$

$$\Rightarrow y(2) = 1.17065$$

(b)

$$w_0 = \alpha = 0$$

$$h = 0.2$$

$$N = \frac{3-1}{0.2} = 10$$

$$\Rightarrow y(3) = 4.51428$$

Problem6

$$a_0 = \frac{\sum x_i^2 \sum y_i - \sum x_i y_i \sum x_i}{m(\sum x_i^2) - (\sum x_i)^2}$$

$$a_1 = \frac{m \sum x_i y_i - \sum x_i \sum y_i}{m(\sum x_i^2) - (\sum x_i)^2}$$

$$P(x_i) = a_1 x_i + a_0$$

$$E = \sum [y_i - P(x_i)]^2$$

计算得到：

$$P(x_i) = 2.711864407x + 4.542372881$$

$$E = 11.525424$$