Problem1

(a)

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$
$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$
$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$
$$P(x) = f(0)L_0(x) + f(0.3)L_1(x) + f(0.6)L_2(x)$$
$$= L_0(x) + 1.1326L_1(x) - 0.7543L_2(x)$$
$$= -11.22x^2 + 3.81x + 1$$
$$\text{误差}bound\text{：}\ \frac{f^3(\xi)}{3!}(x-x_0)(x-x_1)(x-x_2) \quad \xi \in (0, 0.6)$$
$$f'(\xi) = e^{2\xi}(2cos(3\xi) - 3sin(3\xi))$$
$$f''(\xi) = e^{2\xi}(-5cos(3\xi) - 12sin(3\xi))$$
$$f'''(\xi) = e^{2\xi}(-46cos(3\xi) - 9sin(3\xi))$$
$$\text{故}|\frac{f'''(\xi)}{3!}| \leq 10.94$$
$$\text{当}x \in [0, 0.6]\text{时，}|(x-x_0)(x-x_1)(x-x_2)| \leq 0.0104$$
$$\text{最大误差} = 10.94 * 0.0104 = 0.1137$$

(b)

$$P(x) = f(2)L_0(x) + f(2.4)L_1(x) + f(2.6)L_2(x)$$
$$P(x) = -0.033x^2 + 0.0003x + 1$$
$$\frac{f^3(\xi(x))}{3!}(x-x_0)(x-x_1)(x-x_2) = \frac{3\sin(\ln(\xi(x))) + \cos(\ln(\xi(x)))}{\xi(x)^3}(x-x_0)(x-x_1)(x-x_2)$$
$$|\frac{f'''(\xi)}{3!}| \leq 0.336$$
$$|(x-x_0)(x-x_1)(x-x_2)| \leq 0.0169$$
$$\text{最大误差} = 0,336 * 0.0169 = 0.005675$$

Problem2

$$P(x) = f(0)L_0(x) + f(0.5)L_1(x) + f(1)L_2(x) + f(2)L_3(x)$$
$$= yL_1(x) + 3L_2(x) + 2L_2(x)$$
$$= y\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} + 3\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + 2\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$
$$x^3\text{项的系数：}\quad \frac{y}{0.5*(-0.5)*(-1.5)} + \frac{3}{1*0.5*(-1)} + \frac{2}{2*1.5*1} = \frac{8}{3}y - 6 + \frac{2}{3} = 6$$
$$\Rightarrow y = \frac{17}{4}$$

Problem3

$$P_{2,3}=\frac{(x-x_2)P_3-(x-x_3)P_2}{x_3-x_2}=2.4$$
$$\Rightarrow P_2=2.4$$
$$P_{0,1,2}=\frac{(x-x_2)P_{0,1}-(x-x_1)P_{0,2}}{x_1-x_2}=\frac{9}{4}$$
$$P_{0,1,2,3}=\frac{(x-x_3)P_{0,1,2}-(x-x_0)P_{1,2,3}}{x_0-x_3}=\frac{23}{8}$$

Problem4

$$f[x_1,x_2]=\frac{f[x_2]-f[x_1]}{x_2-x_1}\Rightarrow f[x_1]=3$$
$$f[x_0,x_1,x_2]=\frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}\Rightarrow f[x_0,x_1]=5$$
$$f[x_0,x_1]=\frac{f[x_1]-f[x_0]}{x_1-x_0}\Rightarrow f[x_0]=1$$

Problem5

natural cubic spline

$$\text{在}[0,1]\text{上构造}S_0(x)=a_0+b_0x+c_0x^2+d_0x^3$$
$$\text{在}[1,2]\text{上构造}S_1(x)=a_1+b_1(x-1)+c_1(x-1)^2+d_1(x-1)^3$$
$$\text{由：}f(0)=0=a_0$$
$$f(1)=1=a_0+b_0+c_0+d_0=a_1$$
$$f(2)=2=a_1+b_1+c_1+d_1$$
$$S_0'(1)=S_1'(1)\quad =>\quad b_0+2c_0+3d_0=b_1$$
$$S_0''(1)=S_1''(1)\quad =>\quad 2c_0+6d_0=2c_1$$
$$S_0''(0)=S_1''(2)=0\quad => 6d_1+2c_1=2c_0=0$$
$$\text{解得：}$$
$$a_0=0,b_0=1,c_0=0,d_0=0$$
$$a_1=1,b_1=1,c_1=0,d_1=0$$
$$\text{故在}[0,1]\text{上},S_1(x)=x$$
$$\text{在}[1,2]\text{上},S_2(x)=x$$

clamped cubic spline

$$在[0,1]上构造S_0(x) = a_0 + b_0x + c_0x^2 + d_0x^3$$

$$在[1,2]上构造S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3$$

$$由：f(0) = 0 = a_0$$

$$f(1) = 1 = a_0 + b_0 + c_0 + d_0 = a_1$$

$$f(2) = 2 = a_1 + b_1 + c_1 + d_1$$

$$S_0'(1) = S_1'(1) \quad => \quad b_0 + 2c_0 + 3d_0 = b_1$$

$$S_0''(1) = S_1''(1) \quad => \quad 2c_0 + 6d_0 = 2c_1$$

$$S_0'(0) = S_2'(2) = 1 \quad => \quad b_0 = b_1 + 2c_1 + 3d_1 = 1$$

解得：

$$a_0 = 0, b_0 = 1, c_0 = 0, d_0 = 0$$

$$a_1 = 1, b_1 = 1, c_1 = 0, d_1 = 0$$

$$故在[0,1]上, S_1(x) = x$$

$$在[1,2]上, S_2(x) = x$$