1. Prove that $\|Bx\| \geq \|x\|/\|B^{-1}\|$, for any non singular matrix B.

2. Given a square nonsingular matrix A. Consider its rank-one update $\bar{A} = A + ab^T$, where $a, b \in \mathcal{R}^n$

   (a) Verify that when $\bar{A}$ is nonsingular, we have

   $$\bar{A}^{-1} = A^{-1} - \frac{A^{-1}ab^TA^{-1}}{1 + b^TA^{-1}a}$$

   (b) Using the above formula to show that

   $$B_{k+1} = B_k + \frac{(y_k - B_ks_k)(y_k - B_ks_k)^T}{(y_k - B_ks_k)^Ts_k}$$

   is the inverse of

   $$H_{k+1} = H_k + \frac{(s_k - H_ky_k)(s_k - H_ky_k)^T}{(s_k - H_ky_k)^Ty_k}$$

   where $H_k^{-1} = B_k$ is symmetric, $s_k = x_{k+1} - x_k$ and $y_{k+1} = \nabla f(x_{k+1}) - \nabla f(x_k)$.

3. Minimize the Rosenbrock function $f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$ and Beale function $f(x) = (1.5 - x_1 + x_1x_2)^2 + (2.25 - x_1 + x_1x_2^2)^2 + (2.625 - x_1 + x_1x_2^3)^2$ by the steepest descent method and Newtons method respectively, where $x^{(0)} = (-1.2, 1)^T$.

4. Let $f(x) = \frac{1}{2}x^Tx + \frac{1}{4}\sigma(x^TAx)^2$, where

   $$A = \begin{bmatrix} 5 & 1 & 0 & \frac{1}{2} \\ 1 & 4 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 3 & 0 \\ \frac{1}{2} & 0 & 0 & 2 \end{bmatrix}$$

   Let (1) $x^{(0)} = (\cos 70°, \sin 70°, \cos 70°, \sin 70°)^T$;
   (2) $x^{(0)} = (\cos 50°, \sin 50°, \cos 50°, \sin 50°)^T$.
   In the case of $\sigma = 1$ and $\sigma = 10^4$, discuss the numerical results and behavior of convergence rate of pure Newtons method and Newtons method with line search respectively.

5. Please solving the modified LASSO problem in $x \in \mathbb{R}^n$,

   $$\min f(x) = \frac{1}{2}\|Ax - b\|^2 + \mu L_\delta(x)$$

   with $L_\delta(x) = \sum_{i=1}^n \ell_\delta(x_i)$, and

   $$\ell_\delta(x_i) = \begin{cases} \frac{1}{2\delta}x_i^2, & |x_i| < \delta \\ |x_i| - \frac{\delta}{2}, & otherwise. \end{cases}$$

Parameter Setting: $A \in R^{m \times n}, m = 512;\ n = 1024;$ A is a random matrix, which can be generated by $randn(m, n); x$ is a random sparse matrix, which can be generated by $sprandn(n, 1, r)$ with sparticity rate $r = 0.1, \mu = 10^{-2}$ or 10^{-3}, $\delta = 10^{-3}\mu$.

Utilizing steepest decent method and Barzilar-Borwein method to solve this problem respectively.