MATH 4824C - HW 2

Due on March 15th, 2024

Please (1) show the work to the questions and (2) upload your answers through Canvas.

1. Assume the covariates $X_i \in \{1, \ldots, K\}$, and for $z = 0, 1$ and $k = 1, \ldots, K$ define

$$\bar{Y}_{k,z} = \frac{\sum_{X_i = k \wedge Z_i = z} Y_i}{\sum_{i=1}^n \mathbf{1}\{X_i = k \wedge Z_i = z\}}, \quad \bar{Y}_z = \frac{\sum_{Z_i = z} Y_i}{\sum_{i=1}^n \mathbf{1}\{Z_i = z\}}$$

and $n_k = \sum_{i=1}^n \mathbf{1}\{X_i = k\}$ and $n_{k,z} = \sum_{i=1}^n \mathbf{1}\{X_i = k \wedge Z_i = z\}$. Identify the condition that the stratified estimator

$$\widehat{ACE}_{\text{block}} = \sum_{k=1}^{k=K} \frac{n_k}{n} \widehat{ACE}_k$$

is equal to $\widehat{ACE} = \bar{Y}_1 - \bar{Y}_0$ where $\widehat{ACE}_k = \bar{Y}_{k,1} - \bar{Y}_{k,0}$;

2. Consider the setting in page 16 in Chapter 2 of lecture notes, show that $\arg\min_\gamma Var(\widehat{ACE}(\gamma, \gamma))$ is equivalent to $\arg\min_{\alpha,\gamma} E(Y - \alpha - \gamma X)^2$

3. In observational studies without unmeasured confounders; i.e. $(Y_i(1), Y_i(0)) \perp Z_i \mid X_i$, if $Y = \alpha + \beta Z + \gamma X + \epsilon$, write down $E(Y(1)|X)$ and ACE;

4. In observational studies without unmeasured confounders; i.e. $(Y_i(1), Y_i(0)) \perp Z_i \mid X_i$, if $0 < e(X) = P(Z = 1|X) < 1$, show that $EY_i(0) = E\dfrac{(1 - Z_i)Y_i}{1 - e(X_i)}$ and $E\dfrac{1 - Z_i}{1 - e(X_i)} = 1$.