MATH4321 Game Theory (2024 Spring)
Assignment 1

<u>Submission Deadline: 11:59p.m., 1st March, 2023 (Friday)</u>
Please complete all required problems. All working steps, including all calculation steps, reasoning, must be shown clearly to receive full credits. Marks can be deducted for any unclear solutions or poor presentation. You may also complete some optional problems for extra credits. The full mark of the assignment is **100**.

Please submit your completed work via the submission system in canvas. Late submission (submitted after the deadline) will not be accepted. Please reserve enough time for online submission. In addition, your submission must

(1) be 100% Handwritten (*Note: You may write your answer using tablet or ipad.);
(2) be in a single pdf. file (Other file formats will NOT be accepted);
(3) including your full name (as shown in student ID card) and student ID number on the first page of your submission.

Problem 1 (30 marks)
We consider the following 2-person games with the following payoff matrix:

		Player 2	
		a	**b**
Player 1	**A**	**(5, 2)**	**(3, 5)**
	B	**(3, 6)**	**(6, 1)**

(a) Find all pure strategy Nash equilibrium of the games, if any.
(b) Find all possible mixed strategy Nash equilibrium of games which at least one player mix between his/her two strategies if any.

Problem 2 (30 marks)
We consider an auction games with 2 bidders. 2 Bidders are bidding for a silver coin.

- Each bidder can submit his/her bid. We let b_i be the bid amount submitted by bidder i (where $i = 1,2$). Here <u>b_i can be *any non-negative integers*</u>.
- Two bidders submit the bids simultaneously.
- The bidder who submitted the highest bid will be the winner and he/she will pay the highest bid for the silver coin.
- If two bidders submit the same bid, each bidder can win the auction with probability 0.5.
- The bidder's payoff will be $v_i - b_i$ if he/she wins, where v_i denotes the bidder's valuation of the silver coin. The bidder's payoff will be 0 if he/she loses.

We take $v_1 = 30$ and $v_2 = 60$.

(a) Determine all Nash equilibrium of the games. Explain your answer.
(☺Hint: Since the bidder will get a negative payoff if he/she submit a bid higher than his/her valuation and win the games. You may assume that each bidder can only submit the bid at or below his/her valuation. To get the Nash equilibrium, we may write down the payoff matrix of the games and follow the procedure as

demonstrated in class. Alternatively, you may first guess all possible equilibrium and verify that those proposed equilibrium is the Nash equilibrium using the definition. However, you will also need to verify that other strategy profile cannot be Nash equilibrium)

(b) Suppose the winner only needs to pay the *second highest bid* for the object, what will be the corresponding Nash equilibrium?

Problem 3 (40 marks)

Consider two competing firms in a declining industry that can support both firms profitably. Each of the firm has 3 possible choices now: (1) Exit the industry immediately, (2) Exit the industry at the end of this quarter, (3) Exit the industry at the end of the next quarter. Two firms need to make the decision simultaneously now. We assume that the firm cannot come back after the firm exits the industry.

- If the firm chooses to exit then its payoff is 0 from that point onwards.
  (For example, if the firm chooses to exit the industry now, then its payoff will be both 0 in coming two quarters. If the firm chooses to exit the industry at the end of this quarter, then its payoff at second quarter will be 0)
- If the firm chooses to stay in the industry during a quarter, the payoff received by the firm in the quarter is calculated as follows:
  - If both firms stay in the quarter, the payoffs to each firm in the quarter is -1.
  - If only one firm stay in the quarter (i.e. another firm quitted), the payoff to this firm is 2 in the quarter.
- The total payoff of each firm is defined as the sum of payoffs received in the coming two quarters. That is,

$$V_i = \{Payoff\ in\ first\ quarter\} + \{Payoff\ in\ second\ quarter\}.$$

(a) Express the games in normal form. Express the payoffs to both firms in matrix form.

(b) Determine if there is any dominated strategy? Is there any weakly dominated strategy?

(c) Determine all pure strategy Nash equilibrium.

(d) Suppose that no firms will choose to quit the industry immediately (i.e. each firm choose either option (2) or option (3)), determine all mixed strategy Nash equilibrium .

Bonus Problem 1 (Optional, 20 marks)

We consider the following n firms Cournot oligopoly model: There are n firms producing a product for a market.

- Each firm can decide the number of products produced by itself. We let $q_i \geq 0$ be the quantity chosen by Firm i, $i = 1,2, \ldots, n$.
- Given the quantities chosen by n firms, the market price of the product is assumed to be $p = a - q_1 - q_2 - \cdots - q_n$.
- The cost of producing q_i units of good is cq_i, where $c < a$.
- The payoff function of firm i is defined as

$$V_i(q_i; q_{-i}) = pq_i - cq_i = (a - q_1 - q_2 - \cdots - q_n)q_i - cq_i.$$

(a) Determine all Nash equilibrium of the games.

(b) Given the Nash equilibrium obtained in (a), what is the market price of the product under Nash equilibrium? Examine the monotonicity of the market price with respect to n? What is the market price when $n \to \infty$?

Bonus Problem 2 (Optional, 15 marks)

We consider a n-person games where $n \geq 2$. Suppose that $s_i^* \in S_i$ is the *unique best response* to any combinations of opponents' strategies $s_{-i} \in S_{-i}$, show that the player i must adopt s_i^* in any Nash equilibrium including pure strategy Nash equilibrium and mixed strategy equilibrium.