HW4

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Q1

$$\begin{cases} E(Z(Y-\tau_{adj}Z-\alpha X))=0 \\ E(X(Y-\tau_{adj}Z-\alpha X))=0 \end{cases}$$

$$\Rightarrow \begin{cases} EZY-\tau_{adj}EZ^2-\alpha EZX=0 \\ EXY-\tau_{adj}EXZ-\alpha EX^2=0 \end{cases}$$

$EXZ=EX(aX+bU)=aEX^2=a$

$EXY=EX(\tau Z+cU)=\tau EXZ=\tau a$

$EX^2=1,\ EZ^2=a^2+b^2+1$

$EZY=EZ(\tau Z+cU)=\tau EZ^2+cE(aX+bU)U=\tau(a^2+b^2+1)+cb$

So we get $\begin{cases} \tau(a^2+b^2+1)+cb-\tau_{adj}(a^2+b^2+1)-\alpha\cdot a=0 \\ \tau a-\tau_{adj}a=\alpha \end{cases}$

Solve this and we get : $\begin{cases} \tau_{adj}=\tau+\dfrac{bc}{b^2+1} \\ \alpha=-\dfrac{abc}{b^2+1} \end{cases}$

Q2 :

$RR_{ZY}^{obs}<1$

$$\text{E-value}=\frac{1}{RR_{ZY}^{obs}}+\sqrt{\frac{1}{RR_{ZY}^{obs}}\left(\frac{1}{RR_{ZY}^{obs}}-1\right)}=\frac{5+\sqrt{5}}{4}$$

which means the maximum of the confounding measures $\frac{1}{RR_{uY}}$, RR_{Zu} need to be as large as the E-value to explain away the observed relative work.

Q3

Theorem 18.3 *With known $\varepsilon_0(X)$, we have*

$$E\{Y(0) \mid Z=1\} = E\{Z\mu_0(X)\varepsilon_0(X)\}/e = E\left\{e(X)\varepsilon_0(X)\frac{1-Z}{1-e(X)}Y\right\}/e,$$

where $e = \mathrm{pr}(Z=1)$

① $\mu_0(X) = E(Y \mid Z=0, X)$

$E(Y(0) \mid Z=1) = E(\mu_0(X) \mid Z=1)$

$= E\{\mu_0(X)\cdot\varepsilon_0(X) \mid Z=1)$

$= E\{\mu_0(X)\cdot\varepsilon_0(X), Z=1\} / E(Z=1)$

$= \dfrac{E(Z\cdot\mu_0(X)\cdot\varepsilon_0(X))}{P(Z=1)} = E(Z\mu_0(X)\varepsilon_0(x)\}/e$

② $E(e(x)\varepsilon_0(x)\dfrac{1-Z}{1-e(x)}Y] /e = E\left(\dfrac{e(x)}{1-e(x)}\varepsilon_0(x)\, E[(1-Z)Y|x]\right] /e$

$= E\left[\dfrac{e(x)}{1-e(x)}\varepsilon_0(x)\cdot E[(1-Z)Y(0)|X]\right) /e$

$= E\left(\dfrac{e(x)}{1-e(x)}\varepsilon_0(x)\cdot E(1-Z)|X\cdot E(Y(0)|X)\right) /e$

$= E(\varepsilon_0(x)\cdot E(Y(0)|X)e(x)) /e$

$= \dfrac{E(e(x))}{e}\cdot E[\varepsilon_0(X)\cdot E(Y(0)|X)] = E[E(Y(0)|Z=1, X)] = E(Y(0)|Z=1)$

Q4

Under Sharp RDD design with cutoff c and continuous assumptions as desired, if $E(Y|Z,X) = \alpha_0 + \alpha_1 X + \alpha_2 Z + \alpha_3 ZX$, please write down $E(Y(1)|Z=1,X)$ and ACE.

$$E(Y(1)|Z=1,X) = E(ZY(1)+(1-Z)Y(0)|Z=1,X)$$

$$= E(Y|Z=1,X) = \alpha_0 + \alpha_1 X + \alpha_2 + \alpha_3 X$$

$$= \alpha_0 + \alpha_2 + (\alpha_1 + \alpha_3)X$$

$$E(Y(0)|Z=0,X) = E(Y|Z=0,X) = \alpha_0 + \alpha_1 X$$

$$ACE(c) = E(Y(1)|X=c) - E(Y(0)|X=c)$$

$$= (\alpha_1+\alpha_3)c + \alpha_0 + \alpha_2 - \alpha_0 - \alpha_1 c$$

$$= \alpha_2 + \alpha_3 c$$