Assignment Four (Due 23/04/2024)

1. Assume that 100 observations from an AR(2) model:

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t,$$

gave the following sample ACF: $\hat{\rho}_1 = 0.8$, $\hat{\rho}_3 = 0.5$ and $\hat{\rho}_3 = 0.4$. Estimate ϕ_1 and ϕ_2.

2. Given the set of observations 2.2, 4.5, 2.5, 2.3, 1.1, 3.0, 2.1, and 1.0, calculate the conditional sum of squares $S(\theta_1, \theta_2)$ for the MA(2) model with $\theta_1 = -0.5$ and $\theta_2 = 0.2$.

3. Consider the AR(1) model

$$(1 - \phi B)(Z_t - \mu) = a_t.$$

(a) Find the maximum likelihood estimators for ϕ and μ.

(b) Discuss the relationship between the LSE and the MLE for ϕ in the above model.