MATH4321 Game Theory (2024 Spring)
Assignment 2

Submission deadline of Assignment 2: <u>**11:59p.m. of 26th Mar, 2023 (Tues)**</u>

Instruction: Please complete all required problems. **Full details (including (i) description of methods used and explanation, (ii) key formula and theorem used and (iii) final answer) must be shown** <u>**clearly**</u> **to receive full credits.** Marks can be deducted for incomplete solution or unclear solution. You may earn extra score by completing some bonus problems. Also, additional score will be given for well-written assignment.

<u>Please submit your completed work via the submission system in canvas</u> before the deadline. Late assignment will not be accepted.

Your submission must (1) be hand-written (typed assignment will not be accepted, you may write on ipad if you wish), (2) in a single pdf. file (other file formats will not be accepted) and (3) contain your full name and student ID on the first page of the assignment.

Problem 1 (Required, 30 marks)

We consider a two person finite games with the following payoff matrix:

		Player 2			
		E	F	G	H
Player 1	A	$(3,5)$	$(7,0)$	$(6,1)$	$(3,21)$
	B	$(5,6)$	$(3,0)$	$(4,20)$	$(2,0)$
	C	$(6,4)$	$(0,21)$	$(8,0)$	$(5,0)$
	D	$(2,8)$	$(6,1)$	$(5,2)$	$(2,5)$

<u>Question</u>

Suppose that the players are allowed to use pure strategies and mixed strategies,

(a) Simplify the games by identifying all dominated strategies.
(☺Hint: Recall that s_i is dominated strategy if it can be dominated by either another pure strategy or mixed strategy. Indeed, the above games (4×4 case) can be reduced to 2×2 case.)

(b) Hence, determine all Nash equilibrium of the games.

Problem 2 (Required, 35 marks)

We consider the following Battle of sexes games: A couple decides a place for dinner. They can choose one of the followings: Chinese food (C), Western food (W) or Japanese food (J). Now the boy can choose whether to let the girl to choose her preference first (Lady first).

- If the boy chooses "lady first", then the girl will choose her preference and let the boy knows her preference. Then the boy then chooses his preference. The payoff matrix is given as follows:

		Girl (Player 2)		
		C	W	J
Boy (Player 1)	C	$(10,6)$	$(-4,-4)$	$(-4,-4)$
	W	$(-4,-4)$	$(7,8)$	$(-4,-4)$
	J	$(-4,-4)$	$(-4,-4)$	$(4,10)$

- If the boy does not want "lady first", then they will choose their preference simultaneously (without knowing the preference of each other). The corresponding payoff matrix is as follows:

		Girl (Player 2)		
		C	W	J
Boy (Player 1)	C	$(9,5)$	$(-3,-3)$	$(-3,-3)$
	W	$(-3,-3)$	$(7,7)$	$(-4,-4)$
	J	$(-3,-3)$	$(-3,-3)$	$(5,9)$

(a) Draw the game tree for this games.
(b) State the strategic profile of two players.
(c) Hence, find all possible subgame perfect equilibrium for this games.

Problem 3 (Required, 35 marks)

Four companies (Firm 1, 2, 3 and 4) are producing a product for the market. Each company will decide the number of products produced. You are given that

- Each firm can choose its q_i.
- Given the quantites produced by four companies (denoted by q_1, q_2, q_3, q_4 respectively), the market price of the product is $P = 400 - q_1 - q_2 - q_3 - q_4$.
- Cost of producing one product is 20 (*Note: So the total cost for producing q_i units of product is $20q_i$.
- Firm 1 and Firm 2 will first decide the quantities produced simultaneously at the beginning. After knowing q_1 and q_2, Firm 3 and Firm 4 will decide the quantities produced simultaneously.

(a) State the strategic profiles of each firm.
(b) Find all possible subgame perfect equilibrium for this games. Provide full justification to your answer.

Bonus Problem (Optional, 40 marks)

Three bidders are bidding an object. The bidders will take turn submitting their bids. The bidder who submits the highest bid will get the object. If there are more than 1 bidders submitting the highest bid, the object will be randomly assigned to one of these bidders with equal chance. The bidder who get the object has to pay for the object immediately. It is given that

- Bidder 1 first submits the bid. After knowing the bidder 1's bid, bidder 2 submits the bid. After knowing the bids submitted by bidders 1 and 2, bidder 3 submits his bid.
- Bidder 1 values the object at \$10.
- Bidder 2 values the object at \$8.
- Bidder 3 values the object at \$5.

(a) Assuming the bid amount submitted must be a non-negative integer, find the sequentially rational Nash equilibrium for this bidding games. Provide full justification to your calculation.
(☺Hint: You may consider the technique that I used in solving the contribution games in the lecture.)

(b) **(Harder)** Suppose that the bidder 1 and bidder 2 will first submit the bid simultaneously and bidder 3 will submit the bid after knowing the bid submitted by bidder 1 and bidder 2. Determine all subgame perfect equilibrium for this games.
(☺Hint: As a starting point, you can first think about the strategy of bidder 3 (last mover) given the bids submitted by bidder 1 and bidder 2.)