1. Form the KKT conditions for

$$\begin{aligned} \max \ & (x+1)^2 + (y+1)^2 \\ s.t. \ & x^2 + y^2 \leq 2, \ 1 - y \geq 0. \end{aligned}$$

and then determine the solution.

2. Consider

$$\begin{aligned} \min \ & f(x,y) = (x-1)^2 + (y-2)^2 \\ s.t. \ & (x-1)^2 = 5y. \end{aligned}$$

(a) find all the KKT points for this problem. Is the LICQ satisfied?

(b) Which of these points are solutions?

(c) By directly substituting the constraint into the objective function and eliminating the variable x, we obtain an unconstrained optimization problem. Show that the solutions of this problem cannot be solutions of the original problem.

3. Consider the following linear program:

$$\begin{aligned} \max \ & -5x_1 - x_2 \\ s.t. \ & x_1 + x_2 \leq 5, \\ & 2x_1 + \frac{1}{2}x_2 \leq 8, \qquad\qquad x \geq 0. \end{aligned}$$

(a) Add slack variables x_3 and x_4 to convert this problem to standard form.

(b) Solve this problem using the simplex method.

4. Consider the quadratic program

$$\begin{aligned} \max \ & 6x_1 + 4x_2 - 13 - x_1^2 - x_2^2, \\ s.t. \ & x_1 + x_2 \leq 3, \ x_1 \geq 0, \ x_2 \geq 0. \end{aligned}$$

Utilizing active set method to solve it.