1.1

1) $\mathcal{L}\{6e^{-0.7t} + 0.5e^{-5t}\}$

$$= 6 \cdot \frac{1}{s+0.7} + 0.5\frac{1}{s+5}$$

2) $\mathcal{L}\{170\sin(60t)\}$

$$= 170\frac{60}{s^2+60^2}$$

3) $\mathcal{L}\{170(60t + \pi/4\}$

Identity

$$= 170 \cdot \frac{s\sin(\frac{\pi}{4}) + 60\cos(\frac{\pi}{4})}{s^2+170^2}$$

4) $\mathcal{L}^{-1}\{(10s^3 - 14s^2 + 4s + 127)X(s)\}$

$$= 10\frac{d^3}{dt^3}x - 14\frac{d^2}{dt^2}x + 4\frac{d}{dt} + 127x$$

1.2

1) Claude.ai Said:

$$H(s) = \frac{2}{s+2} + \frac{48/25 + 4j/25}{s+7+5j} + \frac{48/25 - 4j/25}{s+7-5j}$$

2) By hand

$$\boxed{20} \cdot \left(\frac{s+5}{(s+2)(s+7+5j)(s+7-5j)} \right) = H(s)$$

remember to put this back in

$$\frac{s+5}{(s+2)(s+7+5j)(s+7-5j)} = \frac{A}{s+2} + \frac{B}{s+7+5j} + \frac{C}{s+7-5j}$$

$$A = \frac{(s+5)}{(s+7+5j)(s+7+5j)}\Bigg|_{s=-2} = \frac{3}{(-2+7+5j)(-2+7-5j)}$$

$$= \frac{3}{(5+5j)(5-5j)}$$

$$A = \frac{3}{50} = \boxed{\frac{3}{50}}$$

$$B = \frac{s+5}{(s+2)(s+7-5j)}\Bigg|_{s=-7-5j} = \frac{-7-5j+5}{(-7-5j+2)(-7-5j+7-5j)}$$

$$= \frac{-2-5j}{(-5-5j)(-10j)} = \frac{-2-5j}{50j-50} = \boxed{-0.03+0.07i}$$

$$B$$

$$C = \left.\frac{s+5}{(s+2)(s+7+5j)}\right|_{s=-7+5j} = \frac{-7+5j+5}{(-7+5j+2)(-7+5j+7+5j)}$$

$$= \frac{-2+5j}{(-5+5j)(10j)} = \boxed{-0.03-0.07i} = C$$

$$H(s) = 20\left(\frac{\frac{3}{50}}{s+2} + \frac{-0.03+0.07i}{(s+7+5j)} + \frac{-0.03-0.07i}{(s+7-5j)} \right)$$

3 <u>Python</u>

$n = [20, 100]$

$d =$ numpy polynomial : $[x^3 + 16x^2 + 102x + 148]$

I get:

$$H(s) = \boxed{\frac{1.2}{(s+2)} + \frac{-6.6-1.4j}{(s+7-5j)} + \frac{-6.6+1.4j}{(s+7+5j)}}$$

which is the same as my original by hand answer (with 20 distributed)

The LLM answer is totally different, and I do not trust it

Two out of 3 agree!!! (Python and by hand)

1.3.1

$$f(x) = \frac{1}{4}x^2 + 10x + 25$$

$$f_1(x) = \frac{1}{4}x^2 \text{ and } f_2(x) = 10x + 25$$

$$\hat{f}(x) = \hat{f}_1(x) + f_2(x)$$

$$\hat{f}_1(x) = f_1(x_0) + \frac{d}{dx} f(x) \Big|_{x=x_0} \cdot (x - x_0)$$

$$f_1(x_0) = \frac{1}{4}x_0^2 \qquad \frac{d}{dx} f(x) = \frac{1}{2}x$$

$$\hat{f}_1(x) = \frac{1}{4}x_0^2 + \frac{1}{2}x\Big|_{x_0}(x - x_0)$$

$$\hat{f}_1(x) = \frac{1}{4}x_0^2 + \frac{1}{2}x_0 \cdot x - \frac{1}{2}x_0^2$$

$$\boxed{\hat{f}(x) = \left(10 + \frac{1}{2}x_0\right)x + \left(25 + \frac{1}{4}x_0^2 - \frac{1}{2}x_0^2\right)}$$

In [ ]:
```
#HW1.2

import scipy
import numpy as np
import matplotlib.pyplot as plt

n = [20, 100]

p1 = np.poly1d([1,2])
p2 = np.poly1d([1, 7+5j])
p3 = np.poly1d([1, 7-5j])

d = p1*p2*p3
print(d)

residues = scipy.signal.residue(n, d)
residues
```

```
   3      2
1 x + 16 x + 102 x + 148
```

Out[ ]:
```
(array([ 1.2-4.69567813e-16j, -0.6-1.40000000e+00j, -0.6+1.40000000e+00j]),
 array([-2.-2.70263575e-16j, -7.+5.00000000e+00j, -7.-5.00000000e+00j]),
 array([], dtype=float64))
```

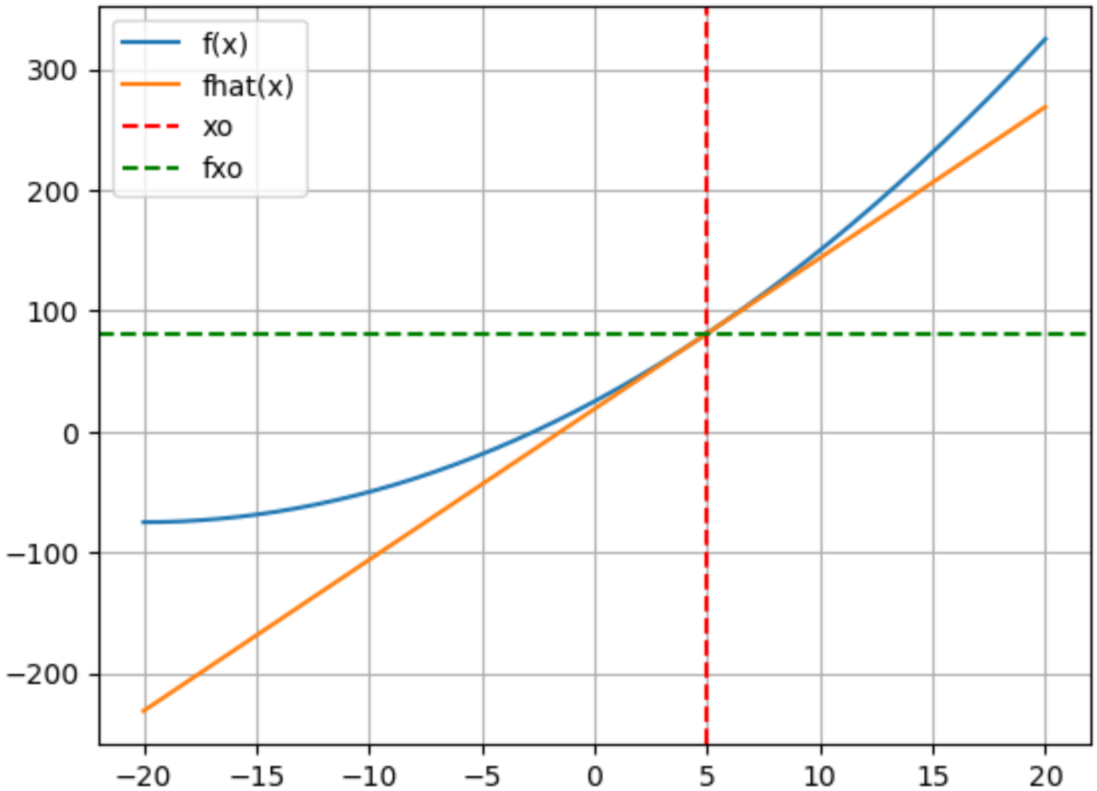
In [25]:
```
#HW1.3
xo = 5
x = xo
fxo = 1/4*(xo**2)+10*xo+25

x = np.linspace(-20, 20, 1000)

f = 1/4*(x**2)+10*x+25
fhat = ((10 + 0.5*xo)*x + (25 + 1/4 * xo**2 - 1/2 * xo**2))


plt.plot(x, f)
plt.plot(x, fhat)
plt.axvline(x=xo, color='red', linestyle='--')
plt.axhline(y=fxo, color='green', linestyle='--')

plt.legend(['f(x)', 'fhat(x)', 'xo', 'fxo'])
plt.grid()
plt.show()
```


f(x)
fhat(x)
xo
fxo
300
200
100
0
−100
−200
−20
−15
−10
−5
0
5
10
15
20