Items $I,\ i \in I$

Features $F,\ f \in F$

Nodes $N,\ j \in N$

l_j: j is a leaf

v_{ij}: Item i is valid at node j

S_{jf}: Node j discriminates on feature f

t_j: Value for node j

σ_{fi} The value of feature f in item i

d^0_{fj} If 1, Decision node j discriminates on feature f and the feature value is 0 exist at least one item's value of the same feature is 0

d^1_{fj} If 1, Decision node j discriminates on feature f and the feature value is 1 exist at least one item's value of the same feature is 1

vd_{ij} If 1, item i is valid at decision node j and has the same feature value as the feature value at node j

vl_{ij} If 1, item i is valide at the leaf node j

c_i The class value of item i

$(1)\ \underset{i \in I}{\forall} v_{i1} = 1$ All items are valid at the first node

$v_{i1} \wedge v_{21} \wedge v_{31} \cdots \wedge v_{i1}$

(2)

$(2A)\ \ d^0_{fj} \leftrightarrow S_{jf} \wedge \neg t_j$ When $\sigma_{fi} = 0$

$(\neg d^0_{fj} \vee S_{jf}) \wedge (\neg d^0_{fj} \vee \neg t_j) \wedge (d^0_{fj} \vee \neg S_{jf} \vee t_j)$

$(2B)\ \ d^1_{fj} \leftrightarrow S_{jf} \wedge t_j$ When $\sigma_{fi} = 1$

$(\neg d^1_{fj} \vee S_{jf}) \wedge (\neg d^1_{fj} \vee t_j) \wedge (d^1_{fj} \vee \neg S_{jf} \vee \neg tj)$

$(3)\ \ vd_{ij} \leftrightarrow v_{ij} \wedge \bigvee_{f \in F} d^{\sigma_{(f,i)}}_{fj}$ Iff item i is valid at decision node j and has the same feature value, item i is valid at decision node j and exits one feature and feature value is same as the item i

$(\neg vd_{ij} \vee v_{ij}) \wedge (\neg vd_{ij} \vee d_{1j} \vee d_{2j} \cdots \vee d_{fj}) \wedge (vd_{ij} \vee \neg v_{ij} \vee \neg d_{1j}) \wedge (vd_{ij} \vee \neg v_{ij} \vee \neg d_{2j}) \cdots \wedge (vd_{ij} \vee \neg v_{ij} \vee \neg d_{fj})$

$(4)\ \underset{i,j \in I,1..N-1}{\forall} v_{i(j+1)} \leftrightarrow l_j \vee vd_{ij}\ |I||N||F|$ Iff item i is valid at node $j+1$, node j is a leaf or item i is valid at node j and has the same feature value.

$(\neg v_{i(j+1)} \vee l_j \vee vd_{ij}) \wedge (v_{i(j+1)} \vee \neg l_j) \wedge (v_{i(j+1)} \vee \neg vd_{ij})$

$(55)\ \underset{j \in N}{\forall} \sum_{f \in F} S_{jf} + l_j = 1\ |N||F|$ Node j only discriminates one feature or is a leaf node

$(55a)\ \ l_j \leftrightarrow \underset{f \in F}{\forall} \neg S_{jf}$

$(\neg l_j \vee \neg S_{j1}) \wedge (\neg l_j \vee \neg S_{j2}) \cdots \wedge (\neg l_j \vee \neg S_{jf}) \wedge ((l_j \vee S_{j1} \vee S_{j1} \cdots \vee S_{jf})$

$(55b)\ \ \neg l_j \rightarrow \sum_{f \in F} S_{jf} = 1$

$l_j \vee \sum_{f \in F} S_{jf} = 1$

$(6)\ \ S_{(j+1)f} \rightarrow l_j \vee \underset{f' < f}{\exists} S_{jf'}\ \ |N-2||F|$

$\neg S_{(j+1)f} \vee l_j \vee \underset{f' < f}{\exists} S_{jf'}$

$(7)\ \ vl_{ij} \leftrightarrow l_j \wedge v_{ij}$ Iff item i is valid at leaf node j, node J is a leaf and item i is valid and node j

$(\neg vl_{ij} \vee l_j) \wedge (\neg vl_{ij} \vee v_{ij}) \wedge (vl_{ij} \vee \neg l_j \vee \neg v_{ij})$

$(8)\ \underset{i \in I}{\forall} \bigvee_{j \in N} vl_{ij}\ \ |N|\,|I|$ Each item is valid at at least one leaf node

$(8O)\ \underset{i \in I}{\forall} \bigvee_{j \in N} l_j \wedge v_{ij}\ \ |N|\,|I|$ Each item is valid at at least one leaf node

$(9)\ \underset{i \in I}{\forall}\ \underset{j \in J}{\forall}\ vl_{ij} \rightarrow c_i = t_j\ \ |N|\,|I|$

$(10)\ \neg l_1$