Items $I,\ i \in I$

Features $F,\ f \in F$

Nodes $N,\ j \in N$

l_j: j is a leaf

v_{ij}: Item i is valid at node j

S_{jf}: Node j discriminates on feature f

t_j: Value for node j

σ_{fi}　The value of feature f in item i

d_{fj}^0　If 1, Node j discriminates on feature f and the feature value is 0 exist at least one item's value of the same feature is 0

d_{fj}^1　If 1, Node j discriminates on feature f and the feature value is 1 exist at least one item's value of the same feature is 1

vd_{ij}　If 1, item i is valid at decision node J and has the same feature value as the feature value at node J

vl_{ij}　If 1, item i is valide at the leaf node J

c_i　The class value of item i

$(1)\ \displaystyle\bigvee_{i \in I} v_{i1} = 1$

$\qquad v_{i1} \wedge v_{21} \wedge v_{31} \cdots \wedge v_{i1}$

(2)

$(2A)\ d_{fj}^0 \leftrightarrow S_{jf} \wedge \neg t_j$　When $\sigma_{fi} = 0$

$(\neg d_{fj}^0 \vee S_{jf}) \wedge (\neg d_{fj}^0 \vee \neg t_j) \wedge (d_{fj}^0 \vee \neg S_{jf} \vee tj)$

$(2B)\ d_{fj}^1 \leftrightarrow S_{jf} \wedge t_j$　When $\sigma_{fi} = 1$

$(\neg d_{fj}^1 \vee S_{jf}) \wedge (\neg d_{fj}^1 \vee t_j) \wedge (d_{fj}^1 \vee \neg S_{jf} \vee \neg tj)$

$(33)\ vd_{ij} \leftrightarrow v_{ij} \wedge \displaystyle\bigvee_{f \in F} d_{fj}^{\sigma_{(f,i)}}$

$A \leftrightarrow B \wedge C$

$(\neg A \vee B) \wedge (\neg B \vee C) \wedge (A \vee \neg B \vee \neg C)$

$(\neg A \vee B) \wedge (\neg B \vee d_{1j} \vee d_{2j} \cdots \vee d_{fj}) \wedge (A \vee \neg B \vee \neg d_{1j}) \wedge (A \vee \neg B \vee \neg d_{2j}) \cdots \wedge (A \vee \neg B \vee \neg d_{fj})$

$(\neg vd_{ij} \vee v_{ij}) \wedge (\neg v_{ij} \vee d_{1j} \vee d_{2j} \cdots \vee d_{fj}) \wedge (vd_{ij} \vee \neg v_{ij} \vee \neg d_{1j}) \wedge (vd_{ij} \vee \neg v_{ij} \vee \neg d_{2j}) \cdots \wedge (vd_{ij} \vee \neg v_{ij} \vee \neg d_{fj})$

$(44)\ \displaystyle\forall_{i,j \in I, 1..N-1} v_{i(j+1)} \leftrightarrow l_j \vee vd_{ij}\ |I||N||F|$

$A \leftrightarrow B \vee C$

$(\neg A \vee B \vee C) \wedge (A \vee \neg B) \wedge (A \vee \neg C)$

$(\neg v_{i(j+1)} \vee l_j \vee vd_{ij}) \wedge (v_{i(j+1)} \vee \neg l_j) \wedge (v_{i(j+1)} \vee \neg vd_{ij})$

$(55)\ \displaystyle\forall_{j \in N} \sum_{f \in F} S_{jf} + l_j = 1\ |N||F|$

$(55a)\ l_j \leftrightarrow \displaystyle\forall_{f \in F} \neg S_{jf}$

$(\neg l_j \vee \neg S_{j1}) \wedge (l_j \vee S_{j1}) \wedge (\neg l_j \vee \neg S_{j2}) \wedge (l_j \vee S_{j2}) \cdots \wedge (\neg l_j \vee \neg S_{jf}) \wedge (l_j \vee S_{jf})$

$(55b)\ \displaystyle\forall_{j \in N} \sum_{f \in F} (S_{jf} \vee l_j) = 1\ |N||F|$

$(6)\ S_{(j+1)f} \rightarrow l_j \vee \displaystyle\mathop{\exists}_{f' < f} S_{jf'}\ \ |N-2||F|$

$\neg A \vee B \vee C$

$$\neg S_{(j+1)f} \vee l_j \vee \underset{f' < f}{\exists} S_{jf'}$$

(7) $vl_{ij} \leftrightarrow l_j \wedge v_{ij}$

$(\neg vl_{ij} \vee l_j) \wedge (\neg vl_{ij} \vee v_{ij}) \wedge (vl_{ij} \vee \neg l_j \vee \neg v_{ij})$

(8) $\underset{i \in I}{\forall} \sum_{j \in N} vl_{ij} = 1 \;\; |N|\,|I|$

(9) $\underset{i \in I}{\forall} \; \underset{j \in J}{\forall} \; vl_{ij} \rightarrow c_i = t_j \;\; |N|\,|I|$

$\neg A \vee B$

$(2XX)$ $S_{jf} \rightarrow \bigvee_{i \in I} (\sigma_{fi} = t_j) \;\; f \in F$

$(3XX)$ $\underset{i,j \in I, 1..N-1}{\forall} v_{i(j+1)} = l_j \vee (v_{ij} \wedge S_{jf}) \;\; |I||N||F|$