6.

HP1. $\langle A, B\rangle = \sum_{k=1}^{n} a_k \bar{\beta}_k = \overline{\sum_{k=1}^{n} \beta_k \bar{a}_k} = \overline{\langle B, A\rangle}$

HP2. $\langle A, B+C\rangle = \sum_{k=1}^{n} a_k(\overline{\beta_k + r_k}) = \sum_{k=1}^{n} a_k(\bar{\beta}_k + \bar{r}_k)$

$= \sum_{k=1}^{n} a_k \bar{\beta}_k + a_k \bar{r}_k$

$= \langle A, B\rangle + \langle A, C\rangle$

HP3. $\langle aA, B\rangle = \sum_{k=1}^{n} (a a_k) \cdot \bar{\beta}_k$

$= a \sum_{k=1}^{n} a_k \bar{\beta}_k$

$= a \langle A, B\rangle$

$\langle A, aB\rangle = \sum_{k=1}^{n} a_k (\overline{a\beta_k}) = \sum_{k=1}^{n} a_k \bar{a} \bar{\beta}_k$

$= \bar{a} \sum_{k=1}^{n} a_k \bar{\beta}_k$

$= \bar{a} \langle A, B\rangle$

HP4. Since $A = 0$

$\langle A, A\rangle = \sum_{k=1}^{n} 0 \cdot 0 = 0$

if A is not all zero

$\exists\, a_k \neq 0 \Rightarrow a_k \cdot \bar{a}_k > 0$

$\forall\, a_i \in \mathbb{C} \quad a_i \bar{a}_i \geq 0$

Hence

$\langle A, A\rangle \geq a_k \bar{a}_k > 0$

8. Suppose $f(x) = x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} \cdots a_1x + a_0$

is irreducible polynomial, where $n \geq 3$

Since the complex numbers algebreically closed.

$\exists z \in \mathbb{C}$ s.t. $f(z) = 0$

Case 1. $z \in \mathbb{R}$, a contradiction.

Case 2. $z \in \mathbb{C} \setminus \mathbb{R}$

obviously $\bar{z}^n + a_{n-1}\bar{z}^{n-1} \cdots + a_0 = 0$

i.e. $f(\bar{z}) = 0$

Consider $p(x) = x^2 - (z+\bar{z})x + z\bar{z}$

where $-(z+\bar{z})$, $z\bar{z} \in \mathbb{R}$, and $p(z) = p(\bar{z}) = 0$

~~Hence~~ a contradiction

Hence $\forall n \geq 3$, $f(x)$ is ~~not~~ reducible. over $\mathbb{R}$

x^2+x+1 is irreducible.

all polynomial over $\mathbb{R}$ with degree 1 is irreducib le

Hence every irreducible polynomial over $\mathbb{R}$ have degree 1 or 2.