

hw 0419

$$1. \begin{vmatrix} I_n & B \\ A & I_m \end{vmatrix} = |I_m - AB| = |I_n - BA|$$

$$\begin{vmatrix} I_n & B \\ A & I_m \end{vmatrix} = \begin{vmatrix} I_n - BA & B \\ A - I_m A & I_m \end{vmatrix} = \begin{vmatrix} I_n - BA & B \\ 0 & I_m \end{vmatrix}$$

$$= |I_n - BA| |I_m| = |I_n - BA|$$

$$= \begin{vmatrix} I_n & B - I_n B \\ A & I_m - AB \end{vmatrix}$$

$$= |I_m - AB|$$

$$2. \text{Rank } \tilde{A} \geq \text{Rank}(A\tilde{A}) = \text{Rank}(I) = n$$

Since \tilde{A} is a $n \times n$ matrix

$$\text{Rank } \tilde{A} \leq n$$

$$\text{Hence Rank } \tilde{A} = n$$