hw 0410

1.

SP1. $\langle x, y\rangle = g_A(X, Y) = g_A(Y, X) = \langle y, x\rangle$

SP2. Since $g_A$ is bilinear

$\forall\, x, y, z \in K^n$

$g_A(x, y+z) = g_A(x, y) + g_A(x, z)$

i.e. $\langle x, y+z\rangle = \langle x, y\rangle + \langle x, z\rangle$

SP3. $\forall\, x, y \in K^n$, $\alpha \in K$ Since $g_A$ is bilinear

$g(\alpha x, y) = \alpha g_A(x, y)$

i.e. $\langle \alpha x, y\rangle = \alpha\langle x, y\rangle$

Hence $g_A$ is a scalar product.

3. $\forall\, g_1, g_2 \in Bil(U \times V, W)$

$\forall\, u \in U$, $\forall\, v_1, v_2 \in V$

$(g_1+g_2)(u, v_1+v_2) = g_1(u, v_1+v_2) + g_2(u, v_1+v_2)$

$= g_1(u, v_1) + g_1(u, v_2) + g_2(u, v_1) + g_2(u, v_2)$

$= (g_1+g_2)(u, v_1) + (g_1+g_2)(u, v_2)$

Similarly

$(g_1+g_2)(u_1+u_2, v) = (g_1+g_2)(u_1, v) + (g_1+g_2)(u_2, v)$

$\forall\, c \in K$, $g \in Bil(U \times V, W)$

$(cg)(u, v_1+v_2) = c\, g(u, v_1+v_2) = c(g(u, v_1) + g(u, v_2))$

$= (cg)(u, v_1) + (cg)(u, v_2)$

The same way, respect to the first variable. we can obtain

$(cg)(u_1+u_2, v) = (cg)(u_1, v) + (cg)(u_2, v)$

Therefore.

$cg$ and $g_1+g_2$ are bilinear.

obviously $0 \in Bil(U\times V, W)$

$\{F: U\times V \to W\}$ is a VS

$Bil(U\times V, W) \subseteq \{F: U\times V \to W\}$

Hence $Bil(U\times V, W)$ is a VS.

5 (d) $X^TAY = x_1y_1 + 2x_1y_2 - x_1y_3 - 3x_2y_1 + x_2y_2 + 4x_2y_3 + 2x_3y_1 + 5x_3y_2 - x_3y_3$

6. $C = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$$X^TCY = \sum_{j=1}^{3}\sum_{i=1}^{3} x_i a_{ij} y_j = Y^TC^TX$$

$$Y^TCX - Y^TC^TX \neq 0$$

$$\Leftrightarrow Y^T(C - C^T)X \neq 0$$

$$C - C^T = \begin{pmatrix} 0 & 3 & 2 \\ -3 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix}$$

w.l.o.g Let $X = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$(C - C^T)X = \begin{pmatrix} 0 \\ -3 \\ -2 \end{pmatrix}$$

Let $Y = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

Then $Y^T(C - C^T)X = -1 \neq 0$