Home work 0327

14.

Setting $B = \frac{1}{2}(A + A^T)$

$B^T = \frac{1}{2}(A^T + A) = \frac{1}{2}(A + A^T) = B$

$\Rightarrow$ B is symmetric

Setting $C = \frac{1}{2}(A - A^T)$

$C^T = \frac{1}{2}(A^T - A) = -\frac{1}{2}(A - A^T) = -C$

$\Rightarrow$ C is skew-symmetric

$A = B + C$

Suppose $\exists\ B_1, C_2$: $\begin{cases} B_1 \text{ is symmetric} \\ C_1 \text{ is skew-symmetric} \end{cases}$

s.t. $A = B_1 + C_1$

$A^T = B_1^T + C_1^T = B_1 - C_1$

$B = \frac{1}{2}(A + A^T) = B_1$

$C = \frac{1}{2}(A - A^T) = C_1$

15.

(a) for all $A, B \in M$, and $a \in K$

1) $P(A+B) = \dfrac{A+B+(A+B)^T}{2} = \dfrac{A+A^T}{2} + \dfrac{B+B^T}{2} = P(A) + P(B)$

2) $P(aA) = \dfrac{aA + (aA)^T}{2} = \dfrac{aA + aA^T}{2} = a\dfrac{A+A^T}{2} = aP(A)$

(b) for all skew-symmetric matrices $A \in M_{skew}$

1) $P(A) = \frac{A+A^T}{2} = \frac{-A^T+A^T}{2} = 0$

2) suppose $P(B) = 0$

$\Rightarrow \frac{B+B^T}{2} = 0 \Rightarrow B+B^T = 0 \Rightarrow B^T = -B$

i.e. B is skew-symmetric

$$\begin{cases} M_{skew} \subseteq \ker P \\ M_{skew} \supseteq \ker P \end{cases} \Rightarrow M_{skew} = \ker P$$

(C) $\dim \ker P = \dim M_{skew} = \frac{n(n-1)}{2}$ over $\mathbb{C}$