hw 0412

1(b) $A = (1, -1, 4) \quad B = (-1, 1, 3)$

$$X \cdot Y = (x_1, x_2, x_3) \begin{pmatrix} 1 & 0 & 1 \\ 0 & -3 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$\langle A, A \rangle = 1 - 3 + 2 \times 1 \times 4 + 2 \times -1 \times -4$$

$$= 14$$

$$\langle B, B \rangle = 1 - 3 - 6 - 6 = -14$$

$$\langle A, B \rangle = -1 + 3 + 3 - 4 - 3 + 4 = 2$$

$$(a, b) \begin{pmatrix} 14 & 2 \\ 2 & -14 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = 0$$

$$\Leftrightarrow \langle aA + bB, cA + dB \rangle = 0$$

Seting $a = 1, b = 0, c = 1$

$$\Rightarrow (14, 2) \begin{pmatrix} 1 \\ d \end{pmatrix} = 0$$

$$\Rightarrow 14 + 2d = 0 \Rightarrow d = -7$$

i.e. $\{A, A - 7B\}$ is othogonal basis

where $A - 7B = (8, -8, -17)$

2. $X \cdot Y = (x_1, x_2)\begin{pmatrix} 1 & -i \\ -i & -2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

w.l.o.g Setting $x_1 = 1$ $x_2 = 0$
$y_1 = 1$

$\Rightarrow (1, -i)\begin{pmatrix} 1 \\ y_2 \end{pmatrix} = 0 \Rightarrow 1 - iy_2 = 0$

$\Rightarrow y_2 = -i$

Since $(1,0), (1,-i) \in \mathbb{C}^2$

$\operatorname{span}\{(1,0), (1,-i)\} \subseteq \mathbb{C}^2$

$\forall (\alpha, \beta) \in \mathbb{C}^2$

$(\alpha, \beta) = (\alpha - i\beta)(1,0) + i\beta(1, -i)$

i.e. $\mathbb{C}^2 \subseteq \operatorname{span}\{(1,0), (1,-i)\}$

$\Rightarrow \operatorname{span}\{(1,0), (1,-i)\} = \mathbb{C}^2$

Consider $(1,0) \cdot (1,-i) = 0$

$\Rightarrow \{(1,0), (1,-i)\}$ is an orthogonal basis