

IMPERIAL

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Coursework 1: Group Actions

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Due date: *January 20, 2026*

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1. Introduction

This is my submission for Coursework 1 of MATH70040-Formalising Mathematics, on the topic of Group Actions. In this project, I formalise the definition of group actions, and prove some basic properties about them using Lean. The Main Theorem is

Theorem: Let X be a G -Set, for each $g \in G$, the map $\varphi_g : X \rightarrow X$ defined by $\varphi_g(x) = g \cdot x$ is a permutation of X . Also the map $\Phi : G \rightarrow S_X$ defined by $\Phi(g) = \varphi_g$ is a group homomorphism with the property that for all $g \in G$ and $x \in X$, $\Phi(g)(x) = g \cdot x$.

2. Definitions

In this section, I define the basic notions of group actions, including the definition of a group action, and some examples of group actions.

2.1. Lean Code.

```
1 /-/ ## Definitions -/
2 -- Group action
3 class GroupAction (G : Type*) [Monoid G] (X : Type*) where
4   act : G → X → X
5   ga1 : (g1 g2 : G) (x : X), act (g1 * g2) x = act g1 (act g2 x)
6   ga2 : (x : X), act (1 : G) x = x
7
8 variable {G : Type*} [Group G] {X : Type*} [GroupAction G X]
9 /-!
```

3. Main Theorem