

## Coursework 1: Group Actions

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### Contents

1	Introduction	1
2	Definitions	1
2.1	Lean Code . . . . .	1
3	Main Theorem	2

### 1. Introduction

This is my submission for Coursework 1 of MATH70040-Formalising Mathematics, on the topic of Group Actions. In this project, I formalise the definition of group actions, and prove some basic properties about them using Lean. The Main Theorem is

**Theorem:** Let  $X$  be a  $G$ -Set, for each  $g \in G$ , the map  $\varphi_g : X \rightarrow X$  defined by  $\varphi_g(x) = g \cdot x$  is a permutation of  $X$ . Also the map  $\Phi : G \rightarrow S_X$  defined by  $\Phi(g) = \varphi_g$  is a group homomorphism with the property that for all  $g \in G$  and  $x \in X$ ,  $\Phi(g)(x) = g \cdot x$ .

### 2. Definitions

In this section, I define the basic notions of group actions, including the definition of a group action, and some examples of group actions.

#### 2.1. Lean Code.

```
1  /-! ## Definitions -/
2  -- Group action
3  class GroupAction (G : Type*) [Monoid G] (X : Type*) where
4    act : G → X → X
5    ga1 : (g1 g2 : G) (x : X), act (g1 * g2) x = act g1 (act g2 x)
6    ga2 : (x : X), act (1 : G) x = x
7
8  variable {G : Type*} [Group G] {X : Type*} [GroupAction G X]
9  /-!
```

### **3. Main Theorem**