

Type-II Antiferroelectricity

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Antiferroelectricity (AFE) is a fundamental concept in physics and materials science. Conventional AFEs have the picture of alternating local electric dipoles defined in real space. Here, we discover a new class of AFEs, termed type-II AFEs, which possess opposite polarizations defined in momentum space across a pair of symmetry decoupled subspaces. Unlike conventional AFEs, the order parameter of type-II AFEs is rigorously formulated through Berry-phase theory and can be quantitatively extracted from the electronic band structure. Focusing on a subclass of type-II AFEs that preserve spin-rotation symmetry, we establish the relevant symmetry constraints and identify all compatible spin point groups. Remarkably, we find that type-II AFE order intrinsically coexists with antiferromagnetism, revealing a robust form of magnetoelectric coupling. We construct an altermagnetic model and identify several concrete antiferromagnetic/altermagnetic materials, such as FeS, Cr₂O₃, MgMnO₃, monolayer MoCl₂ and bilayer CrI₃, that exhibit this novel ordering. Furthermore, we uncover unique physical phenomena associated with type-II spin-AFE systems, including spin current generation upon AFE switching and localized spin polarization at boundaries and domain walls. Our findings reveal a previously hidden class of quantum materials with intertwined ferroic orders, offering exciting opportunities for both fundamental exploration and technological applications.

The concept of antiferroelectricity (AFE) was initially proposed in the 1950s [1–3]. It refers to dielectric materials having spontaneously ordered electric dipoles arranged in an antiparallel manner, such that the net electric polarization vanishes. Till now, AFEs have been discovered in several families of materials [4–6], and have attracted wide-spread interest due to their promising applications in high-performance capacitors, solid-state cooling, actuators, transducers, and memory devices [7–11].

The physical picture of these *conventional* AFEs is a *real-space* one, i.e., one needs to identify the *local* electric dipoles as real-space quantities in the crystal lattice as well as the antiparallel alignment of these dipoles [see Fig. 1(a)]. It is hence not fully compatible with band theory, which relies on Bloch states and a momentum-space description. This situation differs from ferroelectricity, where its order parameter, the electric polarization, has a purely band theoretic formulation in terms of Berry phases of Bloch states [12–15]. In contrast, the AFE order parameter does not have a band formulation and can only be approximately estimated in practice [16–19].

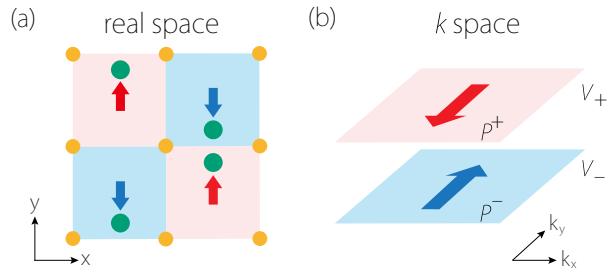


FIG. 1. (a) Illustration of conventional AFE. The green and yellow dots represent two kinds of ions. The AFE order is associated with opposite local electric dipoles defined in real space, as indicated by the red and blue arrows. (b) Illustration of type-II AFE. The AFE order is connected to opposite polarization P^\pm defined in k space for two decoupled sectors V_\pm of the total Hilbert space.

In this work, we propose a new class of AFE, which is termed as the type-II AFE. It refers to dielectrics which possess a symmetry allowing the decoupling of valence bands into two sectors. Each sector is polar and has a nonzero electric polarization, while polarizations of the two sectors are opposite, making a zero net polarization. For such type-II AFEs, the order parameter has a well-defined Berry phase theory and can be quantitatively calculated from the band structure. Focusing on the subclass of type-II AFEs with spin rotational symmetry, named as spin-AFE, we clarify its symmetry con-

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straint and discover that AFE order must coexist with an antiferromagnetic (AFM) order, i.e., they are intrinsically multiferroic. Particularly, we show spin-AFE can be realized in spin groups including several for altermagnets. We construct lattice models to demonstrate the essential physics and identified several concrete spin-AFE materials, including FeS, Cr₂O₃, MgMnO₃, monolayer MoICl₂ and bilayer CrI₃. These type-II AFEs may exhibit unique properties, such as generation of pure spin current upon flip of AFE order, spin polarization at boundaries and domain walls, and strong magnetoelectric coupling. Another subclass, i.e., type-II AFE with mirror symmetry, is also discussed. Our work uncovers a new type of AFE materials, which host distinct characters from conventional AFEs. Their multiferroic nature and unique properties are of great interest for fundamental research and potential device applications.

Concept of type-II AFE. Consider a band insulator with certain symmetry W that decouples the Hilbert space V of electronic states into two sectors (subspaces): $V = V_+ \oplus V_-$. For example, if W is the spin rotation symmetry, then V_{\pm} would be the subspaces for the two spin species. Accordingly, the Bloch Hamiltonian $\mathcal{H}(\mathbf{k})$ can also be decoupled into two blocks:

$$\mathcal{H}(\mathbf{k}) = h_+(\mathbf{k}) \oplus h_-(\mathbf{k}). \quad (1)$$

If the two subsystems defined by h_{\pm} are polar, they each have a nonzero electric polarization, which can be evaluated by the Berry-phase theory [15], with

$$\mathbf{P}^{\pm} = -e \sum_n^{\text{occ}} \int_{\text{BZ}} [\mathbf{dk}] \mathcal{A}_n^{\pm}(\mathbf{k}), \quad (2)$$

where $-e$ is the electron charge, the sum of band index n is over valence bands, $[\mathbf{dk}]$ is a shorthand notation for $d^D k / (2\pi)^D$ with D the dimension of the system, and $\mathcal{A}_n^{\pm}(\mathbf{k}) = i \langle u_n^{\pm}(\mathbf{k}) | \nabla_{\mathbf{k}} | u_n^{\pm}(\mathbf{k}) \rangle$ is the Berry connection with $|u_n^{\pm}(\mathbf{k})\rangle$ the Bloch eigenstates of subsystem h_{\pm} . It should be noted that the polarization is well defined modulo $e\mathbf{R}/V_{\text{cell}}$ [15], where \mathbf{R} is a lattice vector and V_{cell} is the unit cell volume.

Assume that the whole system \mathcal{H} is nonpolar and there exists certain symmetry X connecting h_+ and h_- , which ensures the net polarization vanishes: $\mathbf{P} = \mathbf{P}^+ + \mathbf{P}^- = 0$. Then, the system would be AFE, with a well-defined AFE order parameter from band theory:

$$\mathbf{Q} = \frac{1}{2} (\mathbf{P}^+ - \mathbf{P}^-). \quad (3)$$

We have a few remarks here. First, as discussed above, for conventional AFEs, the evaluation of AFE order parameter relies on the identification of local dipoles in real space, which cannot be directly connected to band structure. In comparison, for type-II AFEs, the order parameter \mathbf{Q} permits a momentum-space formulation, and is

determined as a band structure property. This makes \mathbf{Q} well defined and readily evaluated from first-principles computations.

Second, the polarizations discussed above are from electronic contributions. Usually, there are also ionic contributions [15]. However, as one can see below, by choosing the origin located at the center of X symmetry operation, the ionic contribution can always be made zero.

Third, a key ingredient for type-II AFE is the decoupling of a nonpolar system into two polar subsystems under some symmetry W . As mentioned, a natural case is with decoupling into two spin sectors under spin rotation symmetry, which is named as the spin-AFE. There also exist other possibilities. For example, two dimensional (2D) systems may have W being horizontal mirror symmetry, and the decoupling is into the two subspaces with even and odd mirror eigenvalues. For 1D, one may also consider W being twofold rotational symmetry along the axis of the system. In the following, we shall mainly focus on the spin-AFE subclass, which is the most interesting, and other cases shall be mentioned at the end.

Symmetry condition for spin-AFE. Spin-AFE occurs in systems with spin rotation symmetry, i.e., systems with negligible spin-orbit coupling (SOC). It must be noted that the decoupling into two spin sectors does not require the full $SU(2)$ spin rotation group; a $U(1)$ subgroup suffices. This extends scope of material platforms to also include the *collinear* magnets, where the spin components s_m in the magnetic moment direction are conserved and serve as the label of the two sectors.

First of all, one can easily see that spin-AFE cannot exist in nonmagnetic systems. For a nonmagnetic system with spin rotation symmetry, the two spin sectors must have the same polarization, hence cannot realize AFE. With a similar reasoning, one can see that spin-AFE also cannot exist in ferromagnetic systems, since the spin sectors are inequivalent and cannot have $\mathbf{P}^+ = -\mathbf{P}^-$. It follows that the only possibility of spin-AFE is in AFM systems.

Consider a collinear AFM system with negligible SOC. Let the local moments be in the direction \hat{m} , then the two spin sectors are labeled with $s_m = \pm$. The AFE order is constrained by the spin point group \mathcal{G} of the system. It can always be decomposed as:

$$\mathcal{G} = \mathcal{G}_0 + X\mathcal{G}_0, \quad (4)$$

where \mathcal{G}_0 is the set of symmetries preserving each spin sector, and $X\mathcal{G}_0$ are those switching the two sectors.

The elements of a spin point group is usually expressed in the form of $[S|R]$, where R is an ordinary point group element acting on real space, and S acts independently on spin space [20]. For elements $[S|R] \in \mathcal{G}_0$, we have $S \in \{E, C_2T\} \times SO(2)$ [20], where E is the identity, C_2 is the spin-flip operation, and T is time reversal; meanwhile, the collection of all the R 's form a point group, which we

TABLE I. Collinear spin point groups for spin-AFE. The fifth column shows the type of magnetic order, and the last column indicates the direction of the AFE order parameter \mathbf{Q} . AM denotes altermagnetism.

Lattice	\mathcal{G}	Generators of \mathcal{G}_0	Symmetry X	Magnetic order	\mathbf{Q}
Triclinic	$\bar{1}\bar{1}^{\infty m}1$	$[C_2T E]$	$[C_2 P]$	PT-AFM	$(\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3)$
Monoclinic	$\bar{1}2/\bar{1}m^{\infty m}1$	$[C_2T E], [E 2010]$	$[C_2 P]$	PT-AFM	\mathcal{Q}_2
	$\bar{1}2/\bar{1}m^{\infty m}1$	$[C_2T E], [E m_{010}]$	$[C_2 P]$	PT-AFM	$(\mathcal{Q}_1, 0, \mathcal{Q}_3)$
Orthorhombic	$\bar{1}2\bar{1}2\bar{1}2^{\infty m}1$	$[C_2T E], [E 2001]$	$[C_2 2100]$	AM	\mathcal{Q}_3
	$1m^1\bar{m}^1m^{\infty m}1$	$[C_2T E], [E m_{001}], [E m_{010}]$	$[C_2 P]$	PT-AFM	\mathcal{Q}_1
	$\bar{1}\bar{4}^{\infty m}1$	$[C_2T E]$	$[C_2 \bar{4}_{001}^+]$	AM	\mathcal{Q}_3
Tetragonal	$14/\bar{1}m^{\infty m}1$	$[C_2T E], [E 4_{001}^+]$	$[C_2 P]$	PT-AFM	\mathcal{Q}_3
	$14\bar{1}2\bar{1}2^{\infty m}1$	$[C_2T E], [E 4_{001}^+]$	$[C_2 2100]$	AM	\mathcal{Q}_3
	$\bar{1}\bar{4}\bar{1}2^1m^{\infty m}1$	$[C_2T E], [E m_{110}], [E 2001]$	$[C_2 \bar{4}_{001}^+]$	AM	\mathcal{Q}_3
	$14/\bar{1}m^1m^1m^{\infty m}1$	$[C_2T E], [E 4_{001}^+], [E m_{100}], [E m_{010}]$	$[C_2 P]$	PT-AFM	\mathcal{Q}_3
Trigonal	$\bar{1}\bar{3}^{\infty m}1$	$[C_2T E]$	$[C_2 \bar{3}_{001}^+]$	PT-AFM	\mathcal{Q}_3
	$1\bar{3}\bar{1}2^{\infty m}1$	$[C_2T E], [E 3_{001}^+]$	$[C_2 2100]$	AM	\mathcal{Q}_3
	$\bar{1}\bar{3}^1m^{\infty m}1$	$[C_2T E], [E m_{100}]$	$[C_2 \bar{3}_{001}^+]$	PT-AFM	\mathcal{Q}_3
Hexagonal	$\bar{1}\bar{6}^{\infty m}1$	$[C_2T E]$	$[C_2 \bar{6}_{001}^+]$	AM	\mathcal{Q}_3
	$1\bar{6}\bar{1}2\bar{1}2^{\infty m}1$	$[C_2T E], [E 6_{001}^+]$	$[C_2 2100]$	AM	\mathcal{Q}_3
	$16/\bar{1}m^{\infty m}1$	$[C_2T E], [E 6_{001}^+]$	$[C_2 P]$	PT-AFM	\mathcal{Q}_3
	$\bar{1}\bar{6}^1m^1\bar{2}^{\infty m}1$	$[C_2T E], [E 3_{001}^+], [E m_{100}]$	$[C_2 \bar{6}_{001}^+]$	AM	\mathcal{Q}_3
$16/\bar{1}m^1m^1m^{\infty m}1$	$[C_2T E], [E 6_{001}^+], [E m_{100}], [E m_{1\bar{1}0}]$	$[C_2 P]$	PT-AFM	\mathcal{Q}_3	

denote as \mathcal{G}_R . For each spin sector to have a nonzero polarization, we must have the condition (i) \mathcal{G}_R must be a polar point group.

Furthermore, to have $\mathbf{P}^+ = -\mathbf{P}^-$ exerts a requirement on symmetry X . Writing $X = [S_X||R_X]$, this requirement translates into the condition (ii): The action of operation R_X must reverse \mathbf{P}^+ .

Based on these two conditions, we search through the 90 collinear spin point groups and find 18 groups that can accommodate spin-AFE. These candidate groups are listed in Table I. We observe that first, as expected, all these groups that can host spin-AFE correspond to systems also with AFM ordering. In other words, spin-AFE materials are intrinsically multiferroic, with both electric and magnetic orderings. Second, from our analysis above, magnetic ordering is the primary driving force for the emergence of spin AFE, which indicates a strong coupling between the two orderings and also justifies naming such systems as type-II AFEs. In addition, out of the 18 candidate groups in Table I, we find 10 groups are with PT -symmetric AFM, whereas the other 8 are with altermagnetic ordering which is a currently an active research topic [21–26]. From Table I, such spin-AFE altermagnets can exist in a variety of lattice types, opening the possibility to explore novel AFE altermagnets and interplay between AFE and altermagnetism.

Lattice model. We construct a simple lattice model

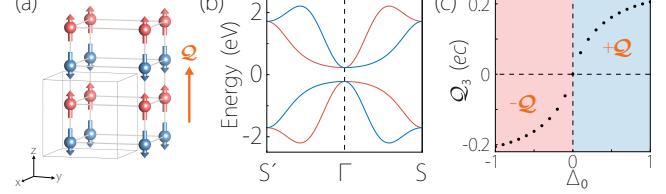


FIG. 2. (a) Illustration of the spin-AFE lattice model (5). The red and blue arrows denote the magnetic moments on the sites. (b) Band structure of the model, showing altermagnetic spin splitting. The Brillouin zone is presented in Supplemental Material (SM) [27]. The resulting AFE order \mathbf{Q} is along z axis, as shown in (a). (c) The AFE order Q_3 vs the AFM order Δ_0 . Here, we set $t_1 = -0.9$ eV, $t_2 = 0.8$ eV, and $\chi = -0.8$ in (b-c) and $\Delta_0 = -1$ eV in (b).

for spin-AFE with altermagnetism. Let's choose an orthorhombic lattice belonging to spin point group $\bar{1}2\bar{1}2\bar{1}2^{\infty m}1$ (see Table I). Each unit of the lattice contains two sites at positions $(0, 0, \frac{1}{4})$ and $(0, 0, \frac{3}{4})$. The two sites each has two spin orbitals, and feature an AFM order, as illustrated in Fig. 2(a). Then, a lattice Hamiltonian allowed by symmetry can be constructed as:

$$\mathcal{H}(\mathbf{k}) = \Delta_0 \sigma_3 \tau_3 + t_+ \cos \frac{k_z}{2} \tau_1 + t_- \sin \frac{k_z}{2} \tau_2 + \chi \Delta_0 (\sin k_x \sin k_y \sigma_3 - \cos k_x \cos k_y \sigma_3 \tau_3) \quad (5)$$

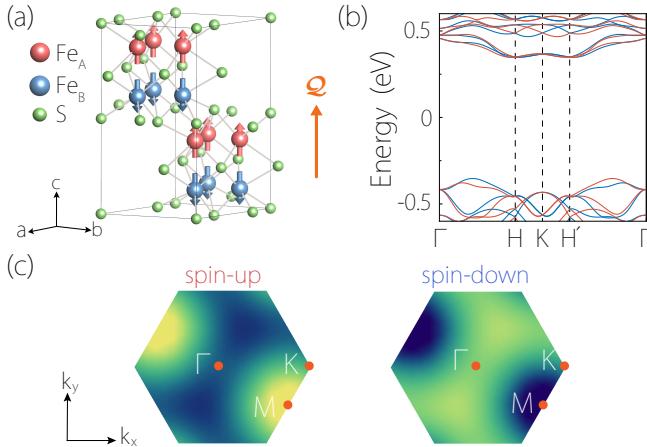


FIG. 3. (a) Crystalline structure of FeS, where the red and blue spheres represent the Fe atoms with opposite magnetic moments. It is an altermagnetic type-II AFE. (b) Band structure of FeS without SOC. Red and blue lines denote the spin-up and spin-down bands, showing altermagnetic spin splitting. (c) The k resolved polarization $P_z = \int_0^{2\pi} A_z(k_x, k_y) dk_z$ in the k_x - k_y plane. It takes opposite values for the two spin subspaces, leading to AFE.

with $t_{\pm}(k_x) = \pm t_1 + t_2 \cos k_x$, and τ 's and σ 's are Pauli matrices acting on site and spin spaces respectively, Δ_0 is exchange term associated with AFM order, and t 's and χ are hopping parameters.

The band structure of model (5) is shown in Fig. 2(b), showing the characteristic spin splitting for altermagnetism. After computing the polarization of each spin subspace, we find that the spin-AFE order \mathcal{Q} is along the z axis, with $\mathcal{Q}_3 = -0.206$ ec (c is the lattice constant). Importantly, nonzero Δ_0 is essential for finite spin-AFE order [see Fig. 2(c)], demonstrating that spin-AFE results from the AFM order. Moreover, by reversing the Néel vector, the spin-AFE order \mathcal{Q} is also reversed [see Fig. 2(c)], showing that spin-AFE materials are intrinsically multiferroic with strong magnetoelectric coupling.

Material realization. Guided by the symmetry conditions in Table I, we find several material candidates for spin-AFE. Here, we present two examples FeS and monolayer MoICl₂. Other examples including Cr₂O₃, MgMnO₃ and bilayer CrI₃ are given in SM [27].

The first example FeS is a bulk collinear AFM, with a Morin transition temperature ~ 220 K [28–30]. Experimentally, it was reported that FeS has a hexagonal crystal structure with lattice parameters $a = 5.966$ Å and $c = 11.761$ Å below 220 K [28–30]. The local moments are mainly on the Fe sites, with an easy axis in the c direction forming a collinear AFM configuration, as illustrated in Fig. 3(a). In the AFM state, the spin layer-point group is $\bar{1}\bar{6}1m\bar{1}2^{\infty m}1$ [27]. According to Table I, the system is an altermagnet, which allows AFE order \mathcal{Q} in the z direction (c axis).

In Fig. 3(b), we plot the calculated band structure of

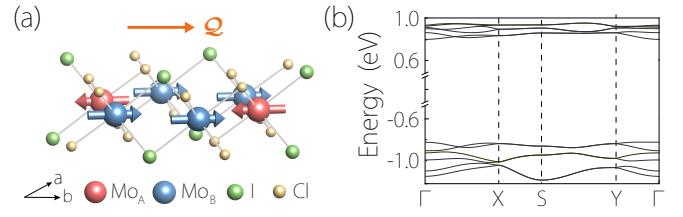


FIG. 4. (a) Crystalline structure of monolayer MoICl₂, where the red and blue spheres represent Mo atoms with opposite magnetic moments. Its AFE order \mathcal{Q} is along b axis. (b) Band structure of monolayer MoICl₂ without SOC, which are spin-degenerate.

FeS in the absence of SOC. One observes it is a magnetic semiconductor with a band gap ~ 0.32 eV. The spin splitting in the band structure can be clearly seen, manifesting its altermagnetic character. We also checked that SOC only has weak effect on the band structure [27]. Using Eqs. (2) and (3), we compute the AFE order parameter \mathcal{Q} . The result confirms that \mathcal{Q} is along z direction, with a value ~ 1.343 $\mu\text{C}/\text{cm}^2$. This value is comparable to the ferroelectric polarization in perovskite oxides [31]. In addition, consistent with observation on model (5), we find that the type-II AFE order vanishes above magnetic transition, and is flipped under the reversal of Néel vector. These results reveal FeS as an altermagnetic spin-AFE with sizable AFE and strong coupling between AFM and AFE orders.

As the second example, we consider a 2D system: monolayer MoICl₂ [32]. It has the CrI₃-type structure, consisting of three atomic layers Cl(I)-Mo-Cl(I), as shown in Fig. 4(a). The lattice constants are $a = 6.63$ Å and $b = 11.48$ Å [27]. The ground state of MoICl₂ is AFM, with the local spin configuration in Fig. 4(a). The magnetic easy axis is along $(1, \bar{1})$ direction. This state has spin point group $1^2/\bar{1}^m\infty^m1$, corresponding to a PT -symmetric AFM. From Table I, its AFE order \mathcal{Q} should be along b axis. Figure 4(b) shows the calculated band structure in the absence of SOC, from which \mathcal{Q}_2 is found to be 11.875 pC/m, consistent with our expectation. This value is also quite sizable compared to other 2D materials such as WTe₂ [33].

Discussion. We have proposed a new class of materials with AFE ordering. Distinct from conventional AFE materials, these type-II AFEs are intrinsically multiferroic: their AFE orders must coexist and are actually resulted from magnetic AFM ordering. This necessarily indicates a strong magnetoelectric coupling. We have demonstrated that flipping the magnetic Néel vector, e.g., by electric Néel torque or optical method [34–36], will reverse the AFE order. One can expect that applied magnetic field, via spin-flop or spin-flip transitions [37–40], can also strongly influence the AFE order. Compared to conventional AFEs, another important difference of type-II AFEs is on the hysteresis behavior un-

der electric field. For conventional AFEs, the electric dipoles are defined in real space, often associated with some local lattice distortions, e.g., displacement of ions in perovskite structure. Hence, usually one can identify a metastable ferroelectric state with all local dipoles aligned in the same direction, and the transitions between AFE and this ferroelectric state lead to a double hysteresis loop often observed in conventional AFEs [4, 5]. In comparison, for type-II AFEs, there is no simple identification of a metastable ferroelectric state (it depends on details of specific material), so a type-II AFE may or may not exhibit double hysteresis loops.

Type-II AFE also manifest other interesting properties. For example, in ferroelectrics, the change of polarization induces a charge current [15]. In analogy, in spin-AFEs, the change of AFE order parameter will generate a pure spin current:

$$j_a^p = \frac{\partial}{\partial t} \mathcal{Q}_a, \quad (6)$$

where polarization p of spin current j_a^p is the Néel vector direction, and a labels the Cartesian component. Consider the material FeS, the reverse of its AFE order will generate a spin current in the c direction. Assuming the reversal process occurs in a time scale of ps (which is the typical time scale for switching antiferromagnets), the generated spin current can reach a magnitude of order 10^{10} A/m². This pure spin current can be detected by inverse spin Hall effect after injecting it into a nearby metallic layer [see Fig. 5(a)], or by its induced magnetization dynamics after injecting it into a nearly ferromagnet [36, 41].

In addition, different from conventional AFEs, spin-AFE could host spin polarization at system boundaries, domain walls, and other topological defects. For example, at a boundary, we should have spin polarization density $s^p = \mathbf{Q} \cdot \hat{n}$ where \hat{n} is the boundary normal vector. Figure 5(c) shows the calculated electronic spectrum of a head-to-head AFE domain wall for model (5) [see Fig. 5(b)]. One observes two bands inside the band gap. They each is localized at the domain wall, corresponding to a polarized spin density [see Fig. 5(d)]. Such local spin density can be probed by magneto-optical measurement [42].

Finally, we mentioned there are other subclasses of type-II AFEs, such as those enabled by mirror symmetry or twofold rotation in reduced dimensions. In SM [27], we give a symmetry analysis for the mirror type-II AFEs. We find such AFEs must also coexist with AFM ordering, and some material candidates are predicted. The search for more type-II AFE systems and the study of their unique physical properties will be interesting directions for future research.

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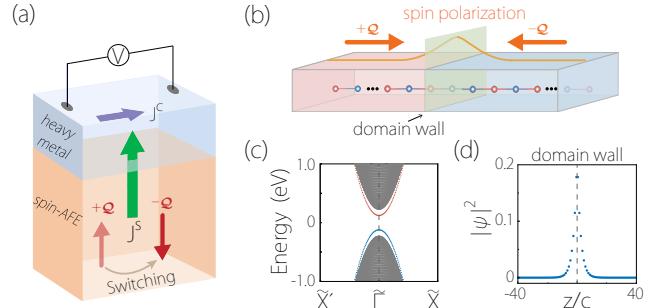


FIG. 5. (a) Schematic of a possible junction geometry for detection of spin-AFE. A pure spin current is generated by reversing the spin-AFE order, and it flows into an adjacent heavy-metal layer and produces a transverse voltage signal via the inverse spin Hall effect. (b) Domain wall between two spin-AFE domains hosts a local spin polarization. (c) Calculated band structure for such a domain wall based on model (5). The red and blue dots are respectively the spin-up and spin-down domain wall modes. (d) The real-space distribution of spin-down domain wall mode at Γ point in (c).

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Supplemental material for “Type-II Antiferroelectricity”

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I. METHODS

In this work, all first-principles calculations are performed using density functional theory (DFT) within the Vienna Ab initio Simulation Package (VASP) [1], employing the projector augmented wave (PAW) method [2]. The exchange-correlation interaction is calculated by the generalized gradient approximation with the Perdew-Burke-Ernzerhof realization. For calculations except for polarization calculations, the plane-wave basis set is defined by a kinetic energy cutoff of at least 550 eV, while Brillouin zone integration employed a Γ -centered Monkhorst-Pack k -mesh with a space of at least $2\pi \times 0.03$. The energy and force convergence criteria are set to be not lower than 10^{-6} eV and 10^{-2} eV/ \AA , respectively. A vacuum space (20 \AA) for two-dimensional (2D) systems is introduced to avoid interactions between neighboring slabs. To describe the correlation effects in transition-metal elements, the DFT+ U method [3] is employed. Herein, we provide specific U values for transition-metal elements and details for polarization calculation in the following different sections.

II. BRILLOUIN ZONES FOR MODELS AND MATERIALS IN MAIN TEXT

Figure S1 shows the Brillouin zones (BZs) of the models and materials discussed in the main text. Specifically, the BZ of the lattice model (5) in the main text is presented in Fig. S1(a), and the BZ of the junction model [illustrated in Fig. 5(b) in the main text] is presented in Fig. S1(b). The BZs of monolayer MoI₂Cl and FeS are shown as Fig. S1(c) and Fig. S1(d), respectively.

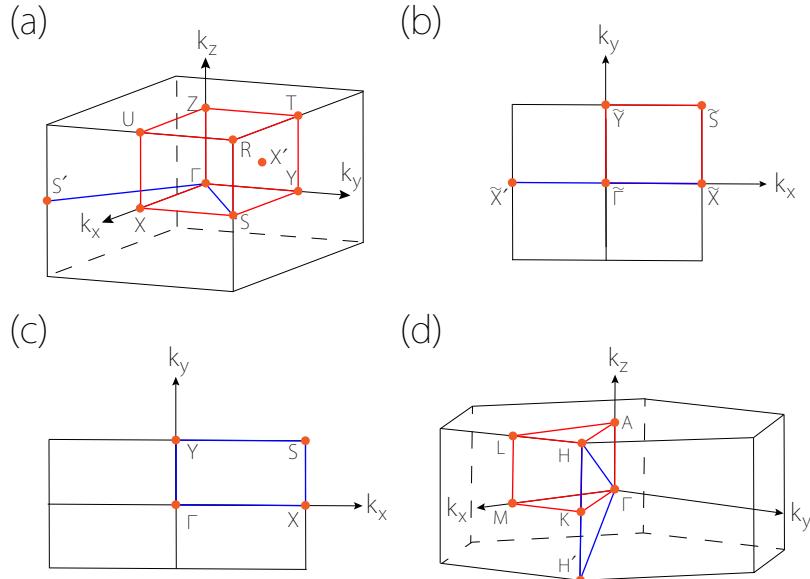


FIG. S1: (a) BZ of lattice model (5) in the main text. (b) BZ of junction model illustrated in Fig. 5(b) in the main text. BZs of (c) monolayer MoI₂Cl and (d) FeS.

III. COLLINEAR SPIN LAYER-POINT GROUPS FOR 2D SPIN-AFE

In the main text, we provide a detailed symmetry analysis for the 3D spin-AFE. The spin-AFE can also exist in 2D systems with negligible SOC. Similarly, we search through the 94 collinear spin layer-point groups (which describe the symmetry of the 2D systems that has a finite thickness along out-of-plane direction) and find 22 groups that can accommodate 2D spin-AFE. These candidate groups are listed in Table S1.

TABLE S1: Collinear spin layer-point groups for 2D spin-AFE. The fifth column shows the type of magnetic order, and the last column indicates the direction of the AFE order parameter \mathbf{Q} . The subscript i in the notation of \mathcal{G} : $[...}_i$ indicates the i -th collinear spin layer-point group obtained from a same 3D collinear spin point group.

Lattice	\mathcal{G}	Generators of \mathcal{G}_0	Symmetry X	Magnetic order	\mathbf{Q}
Triclinic	$\bar{1}\bar{1}^{\infty m}1$	$[C_2T E]$	$[C_2 P]$	$PT\text{-AFM}$	$(\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3)$
	$[1^2/\bar{1}m^{\infty m}1]_1$	$[C_2T E], [E 2_{001}]$	$[C_2 P]$	$PT\text{-AFM}$	\mathcal{Q}_3
Monoclinic	$[1^2/\bar{1}m^{\infty m}1]_2$	$[C_2T E], [E 2_{100}]$	$[C_2 P]$	$PT\text{-AFM}$	\mathcal{Q}_1
	$[\bar{1}2/1^1m^{\infty m}1]_1$	$[C_2T E], [E m_{001}]$	$[C_2 P]$	$PT\text{-AFM}$	$(\mathcal{Q}_1, \mathcal{Q}_2, 0)$
Orthorhombic	$[\bar{1}2/1^1m^{\infty m}1]_2$	$[C_2T E], [E m_{100}]$	$[C_2 P]$	$PT\text{-AFM}$	$(0, \mathcal{Q}_2, \mathcal{Q}_3)$
	$[1^2\bar{2}\bar{2}\bar{1}2^{\infty m}1]_1$	$[C_2T E], [E 2_{001}]$	$[C_2 2_{100}]$	AM	\mathcal{Q}_3
	$[1^2\bar{2}\bar{2}\bar{1}2^{\infty m}1]_2$	$[C_2T E], [E 2_{010}]$	$[C_2 2_{100}]$	AM	\mathcal{Q}_2
	$[1^1m^1m\bar{1}m^{\infty m}1]_1$	$[C_2T E], [E m_{001}], [E m_{010}]$	$[C_2 P]$	$PT\text{-AFM}$	\mathcal{Q}_1
	$[1^1m^1m\bar{1}m^{\infty m}1]_2$	$[C_2T E], [E m_{100}], [E m_{010}]$	$[C_2 P]$	$PT\text{-AFM}$	\mathcal{Q}_3
	$\bar{1}\bar{4}^{\infty m}1$	$[C_2T E]$	$[C_2 \bar{4}_{001}^+]$	AM	\mathcal{Q}_3
Tetragonal	$1^4/\bar{1}m^{\infty m}1$	$[C_2T E], [E 4_{001}^+]$	$[C_2 P]$	$PT\text{-AFM}$	\mathcal{Q}_3
	$1^4\bar{1}2\bar{1}2^{\infty m}1$	$[C_2T E], [E 4_{001}^+]$	$[C_2 2_{100}]$	AM	\mathcal{Q}_3
Trigonal	$\bar{1}\bar{4}\bar{1}2^1m^{\infty m}1$	$[C_2T E], [E m_{110}], [E 2_{001}]$	$[C_2 \bar{4}_{001}^+]$	AM	\mathcal{Q}_3
	$1^4/\bar{1}m^1m^1m^{\infty m}1$	$[C_2T E], [E 4_{001}^+], [E m_{100}], [E m_{010}]$	$[C_2 P]$	$PT\text{-AFM}$	\mathcal{Q}_3
	$\bar{1}\bar{3}^{\infty m}1$	$[C_2T E]$	$[C_2 \bar{3}_{001}^+]$	$PT\text{-AFM}$	\mathcal{Q}_3
	$1^3\bar{1}2^{\infty m}1$	$[C_2T E], [E 3_{001}^+]$	$[C_2 2_{100}]$	AM	\mathcal{Q}_3
	$\bar{1}\bar{3}^1m^{\infty m}1$	$[C_2T E], [E m_{100}]$	$[C_2 \bar{3}_{001}^+]$	$PT\text{-AFM}$	\mathcal{Q}_3
	$\bar{1}\bar{6}^{\infty m}1$	$[C_2T E]$	$[C_2 \bar{6}_{001}^+]$	AM	\mathcal{Q}_3
Hexagonal	$1^6/\bar{1}2^{\infty m}1$	$[C_2T E], [E 6_{001}^+]$	$[C_2 2_{100}]$	AM	\mathcal{Q}_3
	$1^6\bar{1}m\bar{1}2^{\infty m}1$	$[C_2T E], [E 6_{001}^+]$	$[C_2 P]$	$PT\text{-AFM}$	\mathcal{Q}_3
	$\bar{1}\bar{6}^1m\bar{1}2^{\infty m}1$	$[C_2T E], [E 3_{001}^+], [E m_{100}]$	$[C_2 \bar{6}_{001}^+]$	AM	\mathcal{Q}_3
	$1^6/\bar{1}m^1m^1m^{\infty m}1$	$[C_2T E], [E 6_{001}^+], [E m_{100}], [E m_{1\bar{1}0}]$	$[C_2 P]$	$PT\text{-AFM}$	\mathcal{Q}_3

IV. MATERIAL CANDIDATES FOR SPIN-AFE

Guided by the symmetry conditions in Table I in the main text and Table S1, we find several material candidates for spin-AFE, including FeS, Cr_2O_3 , MgMnO_3 , monolayer MoICl_2 and bilayer CrI_3 . Among them, FeS and monolayer MoICl_2 are presented in the main text. Here, we introduce Cr_2O_3 , MgMnO_3 , bilayer CrI_3 and the calculation details of FeS and monolayer MoICl_2 .

A. 3D systems

1. PT-AFM material Cr_2O_3

The Cr_2O_3 is an experimentally synthesized material [4]. Cr_2O_3 has a trigonal lattice structure with optimized lattice constant $a = b = 4.96 \text{ \AA}$ and $c = 13.59 \text{ \AA}$ [4], as shown in Fig. S2(a). The local moments are mainly on the Cr sites, with an easy axis in the c direction forming a collinear AFM configuration, as illustrated in Fig. S2(a). The ground-state magnetic moment of each Cr atom is about $\pm 2.967 \mu_B$. In the AFM state, the spin point group is $\bar{1}\bar{3}^1m^{\infty m}1$. According to Table I in the main text, the system is a $PT\text{-AFM}$, which allows AFE order \mathbf{Q} in the z direction (c axis).

In Fig. S2(c), we plot the calculated band structure of Cr_2O_3 in the absence of SOC with $U=5.5 \text{ eV}$. One observes

it is a magnetic semiconductor with a indirect band gap ~ 3.61 eV. The spin degeneracy in the band structure can be clearly seen, manifesting its *PT*-AFM character. Using Eqs. (2) and (3) in the main text, we compute the AFE order parameter \mathcal{Q} . The result confirms that \mathcal{Q} is along z direction, with a value $\sim 12.24 \mu\text{C}/\text{cm}^2$ [see Fig. S2(d)]. This value is comparable to the ferroelectric polarization in perovskite oxides [5]. In addition, we find that the type-II AFE order of Cr_2O_3 vanishes above magnetic transition, and is flipped under the reversal of Néel vector. These results reveal Cr_2O_3 as a *PT*-AFM spin-AFE with sizable AFE and strong coupling between AFM and AFE orders.

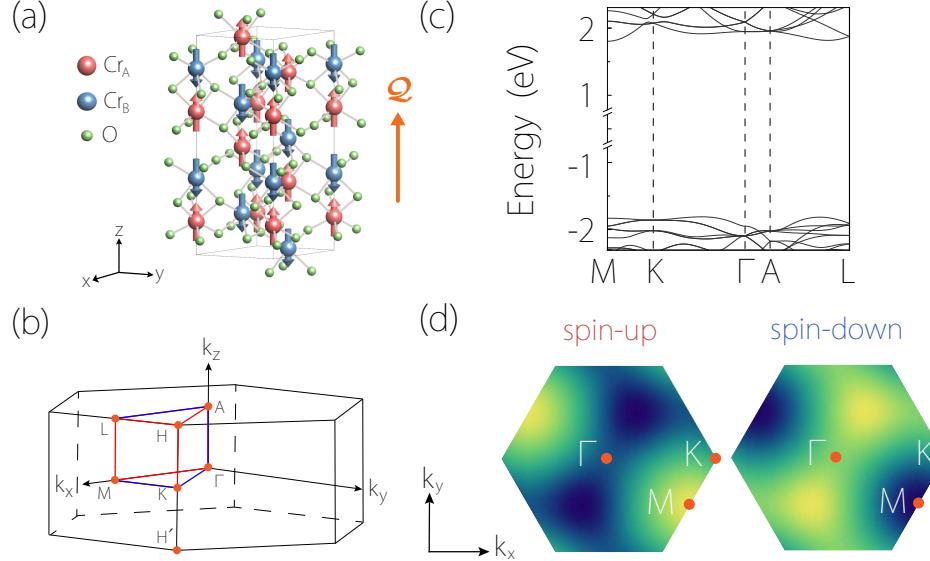


FIG. S2: (a) Crystalline structure of Cr_2O_3 , where the red and blue spheres represent the Cr atoms with opposite magnetic moments. It is an *PT*-AFM type-II AFE. (b) BZ of Cr_2O_3 . (c) Band structure of Cr_2O_3 without SOC, which are spin-degenerate. (d) The k resolved polarization $P_z = \int_0^{2\pi} \mathcal{A}_z(k_x, k_y) dk_z$ in the k_x - k_y plane. It takes opposite values for the two spin subspaces, leading to AFE.

2. *PT*-AFM material MgMnO_3

The MgMnO_3 is an experimentally synthesized material [6]. MgMnO_3 has a trigonal lattice structure with optimized lattice constant $a = b = 4.98 \text{ \AA}$ and $c = 13.82 \text{ \AA}$ [6, 7], as shown in Fig. S3(a). The local moments are mainly on the Mn sites, with an easy axis in the c direction forming a collinear AFM configuration, as illustrated in Fig. S3(a). The ground-state magnetic moment of each Mn atom is about $\pm 3.094 \mu_B$. In the AFM state, the spin point group is $\bar{1}\bar{3}\infty m\bar{1}$. According to Table I in the main text, the system is a *PT*-AFM, which allows AFE order \mathcal{Q} in the z direction (c axis).

In Fig. S3(c), we plot the calculated band structure of MgMnO_3 in the absence of SOC with $U=4$ eV. One observes it is a magnetic semiconductor with a indirect band gap ~ 2.25 eV. The spin degeneracy in the band structure can be clearly seen, manifesting its *PT*-AFM character. Our calculation confirms that \mathcal{Q} of MgMnO_3 is along z direction, with a value $\sim 6.864 \mu\text{C}/\text{cm}^2$ [see Fig. S3(d)], which is comparable to the ferroelectric polarization in perovskite oxides [5]. Similarly, the type-II AFE order of MgMnO_3 vanishes above magnetic transition, and is flipped under the reversal of Néel vector. These results reveal Cr_2O_3 as a *PT*-AFM spin-AFE with sizable AFE and strong coupling between AFM and AFE orders.

3. AM material FeS

The FeS is an experimentally synthesized material [8, 9]. The results of FeS have been presented in the main text. Here, to demonstrate that the effect of SOC on the band structure is negligible, we calculate the band structure of FeS in presence of SOC, as shown in Fig. S4. All calculations employ a Hubbard $U = 1$ eV for Fe atoms and a $15 \times 15 \times 6$ k -mesh for AFE order parameter determination. The energy-minimized configuration yields a magnetic moment of $\pm 3.11 \mu_B$ for Fe atoms.

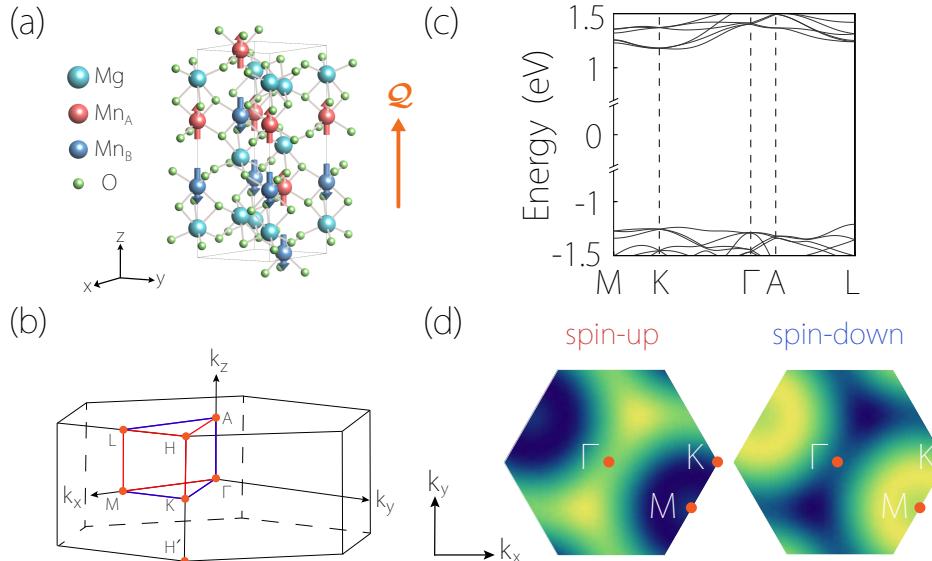


FIG. S3: (a) Crystalline structure of MgMnO_3 , where the red and blue spheres represent the Mn atoms with opposite magnetic moments. It is an PT -AFM type-II AFE. (b) BZ of MgMnO_3 . (b) Band structure of MgMnO_3 without SOC, which are spin-degenerate. (c) The k resolved polarization $P_z = \int_0^{2\pi} \mathcal{A}_z(k_x, k_y) dk_z$ in the k_x - k_y plane. It takes opposite values for the two spin subspaces, leading to AFE.

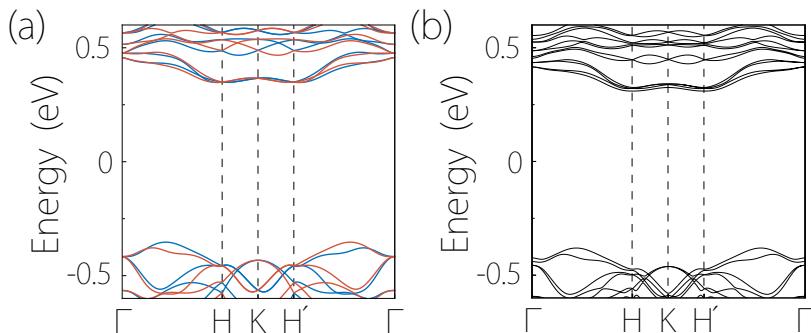


FIG. S4: Band structure of FeS (a) without SOC and (b) with SOC.

B. 2D systems

1. PT -AFM material monolayer MoCl_2

The results of monolayer MoCl_2 have been presented in the main text. All calculations employ a Hubbard $U = 0$ eV for Mo atoms and a $15 \times 9 \times 1$ k -mesh for AFE order parameter determination. The energy-minimized configuration yields a magnetic moment of $\pm 2.353 \mu_B$ for Mo atoms. Here, we further plot the k resolved polarization $P_y = \int_0^{2\pi} \mathcal{A}_y(k_x) dk_y$ of monolayer MoCl_2 in Fig. S5, which directly shows the AFE feature of monolayer MoCl_2 .

2. PT -AFM material bilayer CrI_3

The bilayer CrI_3 is an experimentally synthesized material [10]. Bilayer CrI_3 has an orthorhombic lattice structure with optimized lattice constant $a = 7.01 \text{ \AA}$, and $b = 12.1 \text{ \AA}$ [10, 11], as shown in Fig. S6(a). The local moments are mainly on the Cr sites, with an easy axis in the c direction forming a collinear AFM configuration, as illustrated in Fig. S6(a). The energy-minimized configuration yields a magnetic moment of $\pm 3.042 \mu_B$ for Cr atoms. In the AFM state, the spin layer-point group is $[1/2^1 m^{\infty m} 1]_2$. According to Table I in the main text, the system is a PT -AFM, which allows AFE order \mathbf{Q} in the b - c plane.

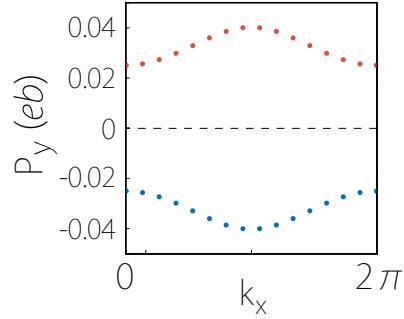


FIG. S5: The k resolved polarization $P_y = \int_0^{2\pi} \mathcal{A}_y(k_x) dk_y$ for different k_x . It takes opposite values for the two spin subspaces, leading to AFE. The red and blue dots denote P_y for spin-up and spin-down, respectively.

In Fig. S6(c), we plot the calculated band structure of bilayer CrI₃ in the absence of SOC with $U=0$ eV. One observes it is a magnetic semiconductor with a band gap ~ 0.67 eV. The spin degeneracy in the band structure can be clearly seen, manifesting its PT -AFM character. Our calculation gives $\mathcal{Q}_2 \sim 1.2$ pC/m and $\mathcal{Q}_3 \sim 1615.31$ pC/m, respectively (see Fig. S7). Notice that since bilayer CrI₃ is confined in z -direction, \mathcal{Q}_3 can be calculated via the distribution of the spin-resolved charge density along z direction, as shown in Fig. S7(b).

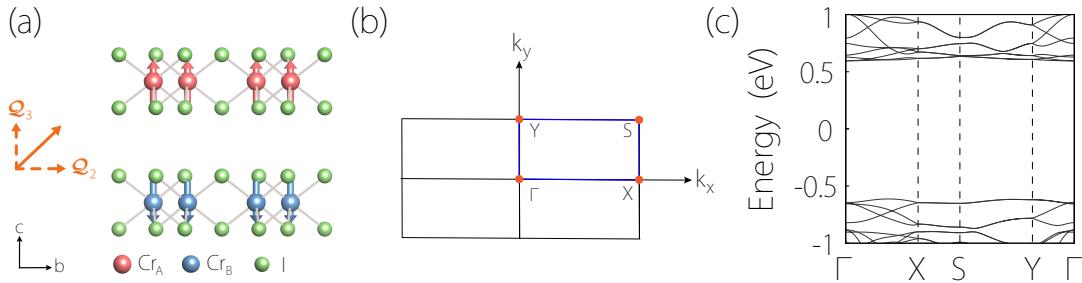


FIG. S6: (a) Crystalline structure of bilayer CrI₃, where the red and blue spheres represent the Cr atoms with opposite magnetic moments. It is an PT -AFM type-II AFE. (b) BZ of bilayer CrI₃. (c) Band structure of bilayer CrI₃ without SOC, which are spin-degenerate.

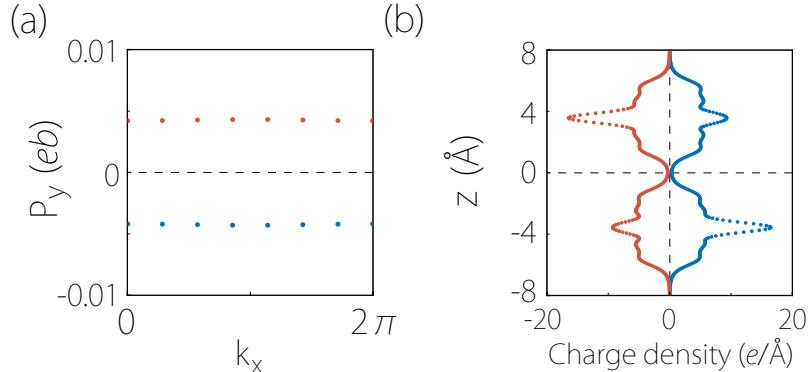


FIG. S7: (a) The k resolved polarization $P_y = \int_0^{2\pi} \mathcal{A}_y(k_x) dk_y$ for different k_x . (b) The distribution of the spin-resolved charge density along z (c) direction. Both (a) and (b) show that \mathcal{Q} take opposite values for the two spin subspaces, leading to AFE. The red and blue dots denote the polarization for spin-up and spin-down, respectively.

V. MIRROR-AFE IN TWO DIMENSIONS

A. Symmetry condition for mirror-AFE

As discussed in the main text, the type-II AFE can be also realized in 2D systems with horizontal mirror symmetry, where W is the horizontal mirror m_{001} , and the decoupling is into the two subspaces with even and odd mirror eigenvalues.

Similarly, to achieve a mirror-AFE, the group of the 2D system \mathcal{G} must be decomposed as

$$\mathcal{G} = \mathcal{G}_0 + X\mathcal{G}_0, \quad (\text{S1})$$

where \mathcal{G}_0 is the set of symmetries preserving each mirror sector, and $X\mathcal{G}_0$ are those switching the two sectors. Specifically, we denote the state with even (odd) mirror eigenvalues as $|m_z = m\rangle$ ($|m_z = -m\rangle$). Here, $m = 1$ for spinless systems and $m = i$ for spinful systems. Since X connect the two mirror subspaces, it must satisfy the following condition

$$X|\pm m\rangle \propto |\mp m\rangle. \quad (\text{S2})$$

This condition indicates that if X is a spatial operator, it must anticommute with m_{001} , and if X is an operator containing T , it must anticommute (commute) with m_{001} for spinless (spinful) systems, as T connects the two states whose eigenvalues are complex conjugates of each other. Besides, the X should ensure the net polarization vanishes: $\mathbf{P} = \mathbf{P}^+ + \mathbf{P}^- = 0$.

Based on these two conditions, it follows that mirror-AFE cannot exist in nonmagnetic systems, which have T symmetry. For nonmagnetic systems without SOC, i.e. spinless systems, T symmetry and all point-group operations fail to switch the two mirror sectors. Moreover, for nonmagnetic systems with SOC, i.e. spinful systems, although T symmetry now enables switching the two mirror sectors, it simultaneously enforces same polarization in both sectors, thereby forbidding the appearance of mirror-AFE.

Then, we consider the magnetic systems both with and without SOC. For magnetic systems without SOC, we search through the 94 collinear spin layer-point groups (see Sec. VIA). Interestingly, due to the decoupling of spin and lattice, there are four spin-group operators that can be considered as mirror, i.e.

$$[E||m_{001}], \quad [C_2||m_{001}], \quad [C_{\parallel,\infty}||m_{001}], \quad [C_2C_{\parallel,\infty}||m_{001}]. \quad (\text{S3})$$

Here, $C_{\parallel,\infty}$ is the rotation along spin direction. However, we find for all the four mirror operators, there is no spin group that can accommodate mirror-AFE, as there always exists a symmetry preserving each mirror sector that causes the polarization of the mirror sector to be zero.

For magnetic systems with SOC, we search through the 125 magnetic layer-point groups (see Sec. VII), which describe the point-group symmetry of the 2D systems that has a finite thickness along out-of-plane direction, and find only 2 groups that can accommodate mirror-AFE. These candidate groups are listed in Table S2. We observe that as expected, these two groups that can host mirror-AFE correspond to magnetic systems. Particularly, both groups are PT -symmetric antiferromagnetics (AFMs), where X correspond to PT symmetry.

These results (Table S2) further demonstrate that the type-II AFE materials are intrinsically multiferroic, with both AFE and AFM orderings.

B. Lattice model for mirror-AFE

We construct a simple lattice model for mirror-AFE with SOC. We choose a 2D orthorhombic lattice belonging to magnetic layer-point group $m'mm$ (MLG 37.7.236) (see Table S2). This lattice is defined on the x - y plane, as shown

TABLE S2: Magnetic layer-point groups for mirror-AFE, corresponding to the magnetic systems with SOC. The last column presents the corresponding magnetic layer groups (MLGs).

\mathcal{G}	Generators of \mathcal{G}_0	Symmetry X	Magnetic order	\mathcal{Q}	MLGs
$2'/m$	m_{001}	PT	PT -AFM	$(\mathcal{Q}_1, \mathcal{Q}_2, 0)$	6.3.23 37.7.236, 47.7.336
$m'mm$	$m_{001}, 2_{010}T$	PT	PT -AFM	$(0, \mathcal{Q}_2, 0)$	40.7.269, 40.9.271 44.7.313, 44.7.313

in Fig. S8(a). For simplicity, we assume that the lattice constants for this lattice is $a = b = 1$. Each unit of the lattice contains two sites at positions $A = (0, 0, z)$ and $B = (0, 0, -z)$, where the value of z is not relevant here. The two sites each has two spin orbitals, and feature an AFM order, as illustrated in Fig. S8(a).

The lattice Hamiltonian is constrained by the generators of MLG 37.7.236, which can be chosen as m_{001} , PT and $2_{010}T$. With the basis states in Fig. S8(a), the symmetry operators are represented as

$$m_{001} = -i\tau_1\sigma_3, \quad PT = i\tau_1\sigma_2\mathcal{K}, \quad 2_{010}T = -\tau_1\sigma_0\mathcal{K}, \quad (\text{S4})$$

where \mathcal{K} represents the complex conjugation operator, τ 's and σ 's are Pauli matrices acting on site and spin spaces respectively. τ_0 (σ_0) represents the 2×2 identity matrix. Following the standard approach [12, 13], the symmetry-allowed lattice Hamiltonian can be established as [14, 15]

$$\begin{aligned} \mathcal{H}(\mathbf{k}) = & (t_1 \cos k_x + t_2 \cos k_y + t_3 \sin k_x)\tau_0\sigma_0 + t_4 \sin k_y\tau_3\sigma_1 + t_5\tau_1\sigma_0 \\ & + (\Delta_0 + t_6 \cos k_y + t_7 \cos k_x + t_8 \sin k_x)\tau_3\sigma_2, \end{aligned} \quad (\text{S5})$$

with $t_{1,2,\dots,8}$ the hopping parameters and Δ_0 denoting the exchange term associated with AFM order. From the exchange term $\Delta_0\tau_3\sigma_2$, one knows that the Néel vector is along y direction. Under a unitary transformation with transformation matrix

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}, \quad (\text{S6})$$

the Hamiltonian (S5) becomes a block-diagonalized matrix, expressed as

$$\begin{aligned} \mathcal{H}_m(\mathbf{k}) = U\mathcal{H}(\mathbf{k})U^\dagger = & (t_1 \cos k_x + t_2 \cos k_y + t_3 \sin k_x)\tau'_0\sigma'_0 - t_4 \sin k_y\tau'_0\sigma'_1 + t_5\tau'_3\sigma'_3 \\ & + (\Delta_0 + t_6 \cos k_y + t_7 \cos k_x + t_8 \sin k_x)\tau'_0\sigma'_2, \end{aligned} \quad (\text{S7})$$

where τ' 's and σ' 's are still Pauli matrices, but no longer act on site and spin spaces. Notice that here τ' 's act on mirror space. For simplification, we set $t_1 = t_2 = t_3 = t_8 = 0$, and the Hamiltonian (S7) reduces to a simple form

$$\mathcal{H}_m(\mathbf{k}) = (\Delta_0 + t_6 \cos k_y + t_7 \cos k_x)\tau'_0\sigma'_2 - t_4 \sin k_y\tau'_0\sigma'_1 + t_5\tau'_3\sigma'_3. \quad (\text{S8})$$

Since the first term in (S8) is odd under T symmetry, we can rewrite (S8) to highlight the role of AFM order. The modified Hamiltonian reads

$$\mathcal{H}_m(\mathbf{k}) = \Delta_0(1 + \chi_1 \cos k_y + \chi_2 \cos k_x)\tau'_0\sigma'_2 - t_4 \sin k_y\tau'_0\sigma'_1 + t_5\tau'_3\sigma'_3, \quad (\text{S9})$$

where $\chi_{1(2)}$ are dimensionless constants. The band structure of model (S9) is shown in Fig. S8(b), showing the characteristic spin degeneracy for PT -AFM.

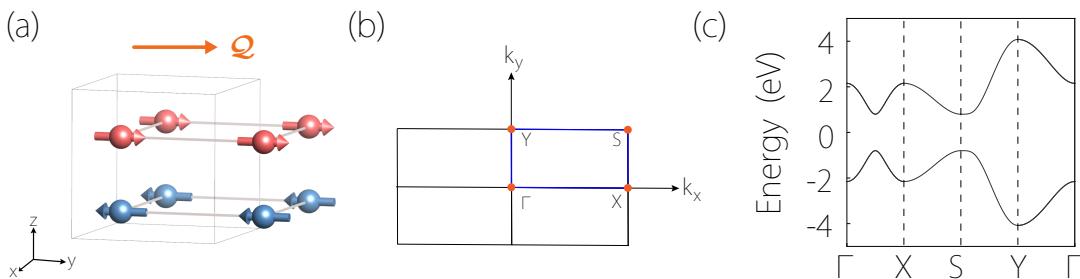


FIG. S8: (a) Illustration of the mirror-AFE lattice model (S9). The red and blue arrows denote the magnetic moments on the sites. (b) BZ and (c) band structure of lattice model (S9). The bands are spin degenerate due to the PT symmetry. Here, we set $\Delta_0 = -1$ eV, $t_4 = 0.8$ eV, $t_5 = 0.8$ eV, $\chi_1 = -1$ and $\chi_2 = 2$.

After computing the polarization of each mirror subspace, we find that the mirror-AFE order \mathcal{Q} is along the y axis, with $\mathcal{Q}_2 = 0.075$ eb (b is the lattice constant) [see Fig. S9(a)]. Similarly, nonzero Δ_0 is also essential for finite

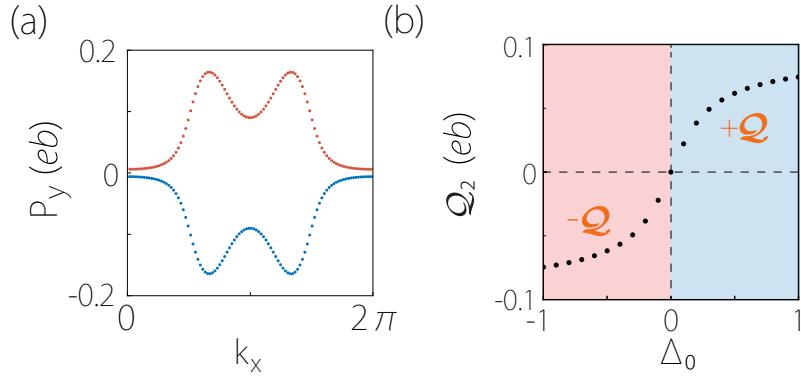


FIG. S9: (a) The k resolved polarization $P_y = \int_0^{2\pi} \mathcal{A}_y(k_x) dk_y$ for different k_x , obtained from model (S9). It takes opposite values for the two mirror subspaces, leading to mirror-AFE. The red (blue) dots denotes the results for $m = i$ ($m = -i$) subspace. (b) The AFE order Q_2 vs the AFM order Δ_0 . Here, we set $\Delta_0 = -1$ eV, $t_4 = 0.8$ eV, $t_5 = 0.8$ eV, $\chi_1 = -1$ and $\chi_2 = 2$ in (a-b) and $\Delta_0 = 1$ eV in (b).

mirror-AFE order [see Fig. S9(b)], demonstrating that mirror-AFE results from the AFM order. By reversing the Néel vector, the mirror-AFE order \mathcal{Q} is also reversed [see Fig. S9(b)], showing that mirror-AFE materials are intrinsically multiferroic with strong magnetoelectric coupling.

Besides, if we break the $2_{010}T$ symmetry but preserve m_{001} and PT , the magnetic layer-point group of the lattice model becomes $2'/m$, and then the model will have finite AFE polarization along x direction. To directly show it, we add a $2_{010}T$ -breaking term in the Hamiltonian (S9), and the new Hamiltonian is

$$\mathcal{H}'_m(\mathbf{k}) = \mathcal{H}_m(\mathbf{k}) + t' \sin k_x \tau'_0 \sigma'_1 - t' \cos k_x \tau'_0 \sigma'_2. \quad (\text{S10})$$

The calculated band structure and k resolved polarization (Wilson loops) for different mirror subsystems are shown in Fig. S10. By integrating the Wilson loops, we obtain the AFE polarization, which is $\mathcal{Q} = (0.078ea, 0.058eb, 0)$.

C. Material candidates of mirror-AFE

Guided by Table S2, we find several material candidates for mirror-AFE, including Ruddlesden-Popper related iron oxyhalides: monolayer $Sr_3X_2Cl_2O_5$ ($X = Fe, Ru, Os$). The 3D bulk material $Sr_3Fe_2Cl_2O_5$ has already been experimentally synthesized and proven to be AFM at 300 K [16–18].

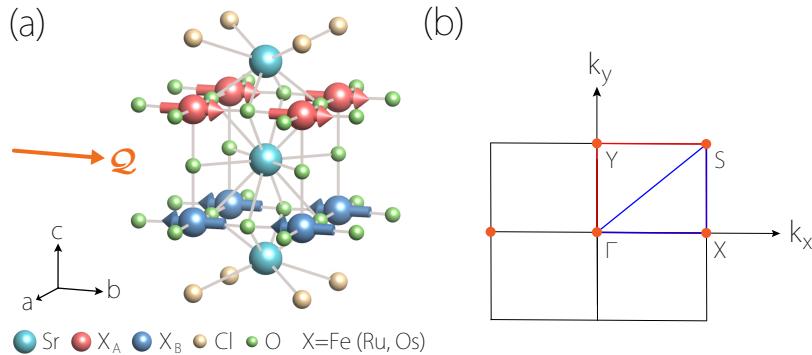


FIG. S11: (a) Crystalline structure and (b) BZ of monolayer $Sr_3X_2Cl_2O_5$ $X = (Fe, Ru, Os)$. The red and blue arrows denote the magnetic moments on the X sites.

Monolayer $Sr_3X_2Cl_2O_5$ ($X = Fe, Ru, Os$) has a square lattice structure, consisting of seven atomic layers in the sequence of Cl-Sr-X(O)-Sr(O)-X(O)-Sr-Cl, as shown in Fig. S11(a). Transition metal ions with partially filled d shells are typical sources for magnetism. In the monolayer $Sr_3X_2Cl_2O_5$ ($X = Fe, Ru, Os$), the magnetism is mainly from X ions. To determine the magnetic order, we compare the energies of different magnetic configurations, which

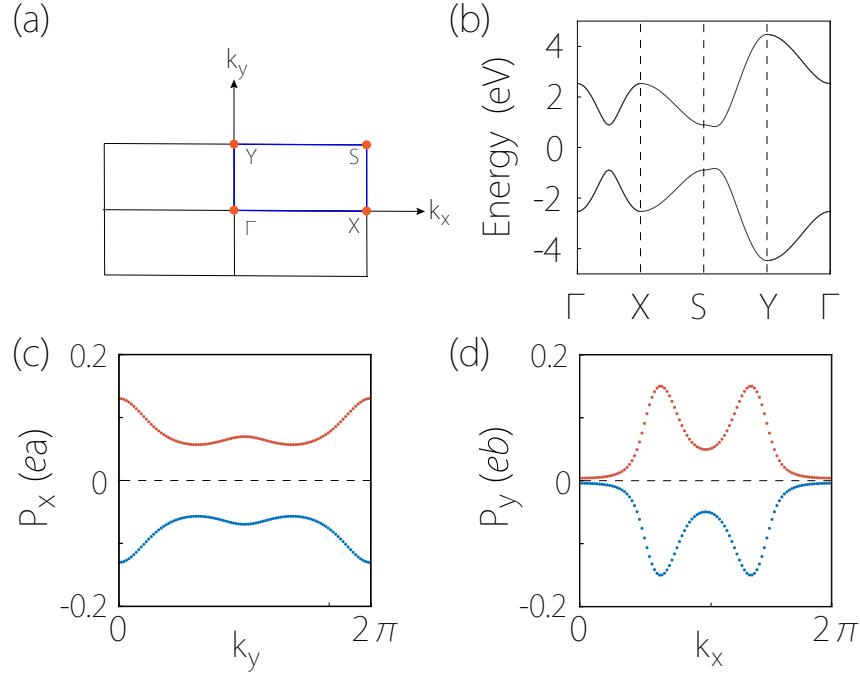


FIG. S10: (a) BZ and (b) band structure of the lattice model (S10), which are spin degenerate. (c) The k resolved polarization $P_x = \int_0^{2\pi} \mathcal{A}_x(k_y) dk_x$ for different k_y . (d) $P_y = \int_0^{2\pi} \mathcal{A}_y(k_x) dk_y$ for different k_x . Both P_x and P_y take opposite values for the two mirror subspaces, leading to mirror-AFE. The red (blue) dots denotes the results for $m = i$ ($m = -i$) subspace. Here, we set $\Delta_0 = -1$ eV, $t_4 = 0.8$ eV, $t_5 = 0.8$ eV, $\chi_1 = -1$, $\chi_2 = 2$ and $t' = 0.4$ eV.

010-AFM, 001-ferromagnetic (FM), 010-FM, 001-AFM (see Fig. S12). Our calculations show that the 010-AFM configuration as the lowest-energy state across all U values, as shown in Tables S3-S5.

In the 010-AFM state, the magnetic layer-point group of monolayer $\text{Sr}_3\text{X}_2\text{Cl}_2\text{O}_5$ is $m'mm$ (magnetic layer group 37.7.236). According to Table S2, the systems are PT -AFM, which allows AFE order \mathbf{Q} in the y direction (b axis).

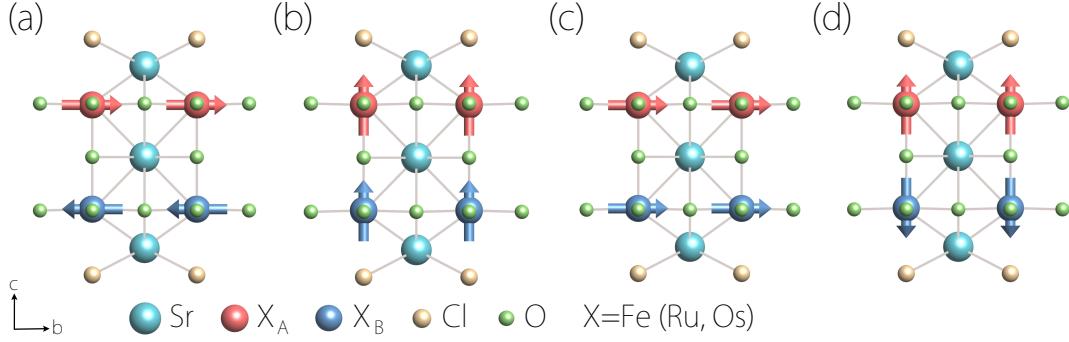


FIG. S12: Illustrations of typical magnetic configurations. The ground state has 010-AFM ordering. (a) 010-AFM, (b) 001-FM, (c) 010-FM, (d) 001-AFM.

TABLE S3: The relative energies (in unit of eV/Fe atom) of monolayer $\text{Sr}_3\text{Fe}_2\text{Cl}_2\text{O}_5$ for 010-AFM, 001-FM, 010-FM and 001-AFM. The ground-state energy for 010-AFM is chosen as a reference.

$U(\text{eV})$	010-AFM	001-FM	010-FM	001-AFM
4.3	0	0.109031	0.108855	0.000335
4.8	0	0.100806	0.100656	0.000306
5.3	0	0.092925	0.092797	0.000273

TABLE S4: The relative energies (in unit of eV/Ru atom) of monolayer $\text{Sr}_3\text{Ru}_2\text{Cl}_2\text{O}_5$ for 010-AFM, 001-FM, 010-FM and 001-AFM. The ground-state energy for 010-AFM is chosen as a reference.

$U(eV)$	010-AFM	001-FM	010-FM	001-AFM
5.3	0	0.112986	0.196976	0.374659
5.5	0	0.144614	0.478063	0.383122
6.0	0	0.155520	0.446599	0.350848

TABLE S5: The relative energies (in unit of eV/Os atom) of monolayer $\text{Sr}_3\text{Os}_2\text{Cl}_2\text{O}_5$ for 010-AFM, 001-FM, 010-FM and 001-AFM. The ground-state energy for 010-AFM is chosen as a reference.

$U(eV)$	010-AFM	001-FM	010-FM	001-AFM
3.2	0	0.127072	0.126574	0.002055
3.5	0	0.182906	0.223384	0.201736
3.8	0	0.240260	0.256092	0.282765

Next, we use monolayer $\text{Sr}_3\text{Os}_2\text{Cl}_2\text{O}_5$ as an example to check the thermal stability of monolayer $\text{Sr}_3\text{X}_2\text{Cl}_2\text{O}_5$. The thermal stability of monolayer $\text{Sr}_3\text{Os}_2\text{Cl}_2\text{O}_5$ is systematically examined through *ab initio* molecular dynamics (AIMD) simulations conducted at 300 K using a $3 \times 3 \times 1$ supercell. Structural snapshots captured during the 5 ps simulation (see Fig. S13) reveal thermally induced lattice distortions with variable amplitudes, yet exhibit no observable bond rupture or structural fragmentation. This quantitative analysis conclusively demonstrates that the monolayer $\text{Sr}_3\text{Os}_2\text{Cl}_2\text{O}_5$ has a robust structure and thermodynamic resilience under ambient thermal agitation.

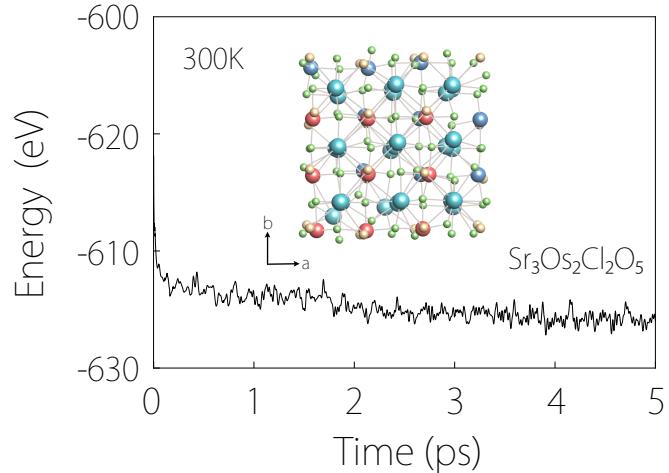


FIG. S13: The total energies with AIMD simulation by using $3 \times 3 \times 1$ supercell at 300 K, and inset shows the snapshots of the crystal structures after 5 ps.

1. Monolayer $\text{Sr}_3\text{Os}_2\text{Cl}_2\text{O}_5$

Monolayer $\text{Sr}_3\text{Os}_2\text{Cl}_2\text{O}_5$ has a square lattice structure with optimized lattice constant $a = b = 4.11 \text{ \AA}$. The local moments are mainly on the Os sites, with a magnitude of $\pm 0.59 \mu_B$ [see Fig. S11(a)].

In Fig. S14(a), we plot the band structure of monolayer $\text{Sr}_3\text{Os}_2\text{Cl}_2\text{O}_5$ in the presence of SOC with $U=3.2 \text{ eV}$. One observes it is a magnetic semiconductor with an indirect band gap $\sim 0.15 \text{ eV}$. The spin degeneracy in the band structure can be clearly seen, manifesting its *PT*-AFM character. Our calculations confirm that \mathbf{Q} is along y direction, with a value $\sim 2.39 \text{ pC/m}$ [see Fig. S14(b)]. As a comparison, the ferroelectric WTe₂ with polarization of 0.16 pC/m is detected in experiments [19]. In addition, we find that the type-II AFE order of monolayer $\text{Sr}_3\text{Os}_2\text{Cl}_2\text{O}_5$ vanishes above

magnetic transition, and is flipped under the reversal of Néel vector. These results reveal monolayer $\text{Sr}_3\text{Os}_2\text{Cl}_2\text{O}_5$ as an PT -AFM mirror-AFE with sizable AFE and strong coupling between AFM and AFE orders.

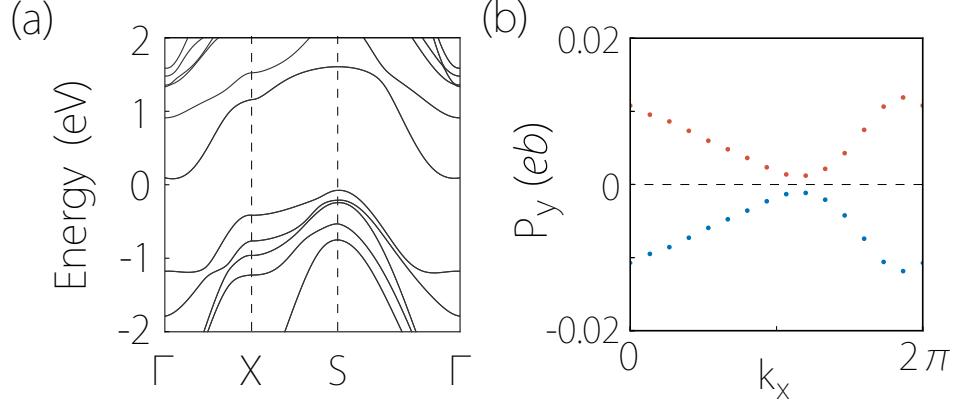


FIG. S14: (a) Band structure of monolayer $\text{Sr}_3\text{Os}_2\text{Cl}_2\text{O}_5$ with SOC for $U=3.2$ eV, which are spin degenerate. (c) The k resolved polarization $P_y = \int_0^{2\pi} \mathcal{A}_y(k_x) dk_y$ for different k_x . It takes opposite values for the two mirror subspaces, leading to AFE. The red (blue) dots denotes the results for $m = i$ ($m = -i$) subspace.

2. Monolayer $\text{Sr}_3\text{Fe}_2\text{Cl}_2\text{O}_5$

Monolayer $\text{Sr}_3\text{Fe}_2\text{Cl}_2\text{O}_5$ has a square lattice structure with optimized lattice constant $a = b = 3.97$ Å. For monolayer $\text{Sr}_3\text{Fe}_2\text{Cl}_2\text{O}_5$, the magnetic moments are mainly on the Fe atoms with a magnitude of $\pm 4.325 \mu_B$ for $U = 5.3$ eV. The band structure of monolayer $\text{Sr}_3\text{Fe}_2\text{Cl}_2\text{O}_5$ with SOC for $U = 5.3$ eV is shown in Fig. S15(a), and the mirror-AFE order parameter Q_2 is calculated as 0.046 pC/m.

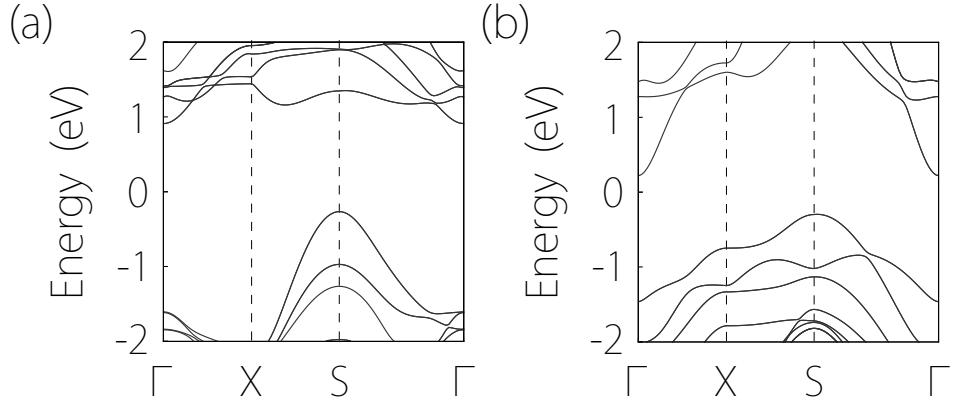


FIG. S15: Band structure of (a) monolayer $\text{Sr}_3\text{Fe}_2\text{Cl}_2\text{O}_5$ and (b) monolayer $\text{Sr}_3\text{Ru}_2\text{Cl}_2\text{O}_5$ with SOC, which are spin degenerate. Here, we set $U=5.3$ eV for both materials.

3. Monolayer $\text{Sr}_3\text{Ru}_2\text{Cl}_2\text{O}_5$

The lattice constant of monolayer $\text{Sr}_3\text{Ru}_2\text{Cl}_2\text{O}_5$ is $a = b = 4.09$ Å. The magnetic moment of this material are mainly on the Ru atoms with a magnitude of $\pm 2.416 \mu_B$ for $U = 5.3$ eV. The band structure of monolayer $\text{Sr}_3\text{Ru}_2\text{Cl}_2\text{O}_5$ with SOC for $U = 5.3$ eV is shown in Fig. S15(b), and the mirror-AFE order parameter Q_2 is calculated as 0.687 pC/m.

VI. OPERATORS OF 2D AND 3D COLLINEAR SPIN-POINT GROUPS

A. Operators of 94 2D collinear spin layer-point groups

TABLE S6: 63 collinear spin layer-point group \mathcal{G} : $\mathcal{G} = (\mathcal{G}_0 + [C_2T||E]\mathcal{G}_0) \ltimes SO(2)$ for AFM. Only nontrivial operators in \mathcal{G}_0 are listed. The generators of \mathcal{G}_0 are marked in blue. The groups allowing for 2D spin-AFE are highlighted by red. The subscript i in the second column ($[...i]$) indicates the i -th collinear spin layer-point group obtained from a same 3D collinear spin point group.

No.	Notation	Operators in \mathcal{G}_0 [$\mathcal{G} = (\mathcal{G}_0 + [C_2T E]\mathcal{G}_0) \ltimes SO(2)$]
1	$\bar{1}\bar{1}^{\infty m}1$	$[C_2 P], [E E]$
2	$[\bar{1}2^{\infty m}1]_1$	$[C_2 2_{001}], [E E]$
3	$[\bar{1}2^{\infty m}1]_2$	$[C_2 2_{100}], [E E]$
4	$[\bar{1}m^{\infty m}1]_1$	$[C_2 m_{001}], [E E]$
5	$[\bar{1}m^{\infty m}1]_2$	$[C_2 m_{100}], [E E]$
6	$[^12/\bar{1}m^{\infty m}1]_1$	$[E 2_{001}], [C_2 P], [E E], [C_2 m_{001}]$
7	$[^12/\bar{1}m^{\infty m}1]_2$	$[E 2_{100}], [C_2 P], [E E], [C_2 m_{100}]$
8	$[\bar{1}2/^1m^{\infty m}1]_1$	$[C_2 P], [E m_{001}], [E E], [C_2 2_{001}]$
9	$[\bar{1}2/^1m^{\infty m}1]_2$	$[C_2 P], [E m_{100}], [E E], [C_2 2_{100}]$
10	$[\bar{1}2/\bar{1}m^{\infty m}1]_1$	$[C_2 2_{001}], [E P], [E E], [C_2 m_{001}]$
11	$[\bar{1}2/\bar{1}m^{\infty m}1]_2$	$[C_2 2_{100}], [E P], [E E], [C_2 m_{100}]$
12	$[\bar{1}m^{\bar{1}}m^12^{\infty m}1]_1$	$[C_2 m_{100}], [E 2_{001}], [E E], [C_2 m_{010}]$
13	$[\bar{1}m^{\bar{1}}m^12^{\infty m}1]_2$	$[C_2 m_{100}], [E 2_{010}], [E E], [C_2 m_{001}]$
14	$[^1m^{\bar{1}}m^{\bar{1}}2^{\infty m}1]_1$	$[E m_{010}], [C_2 2_{001}], [E E], [C_2 m_{100}]$
15	$[^1m^{\bar{1}}m^{\bar{1}}2^{\infty m}1]_2$	$[E m_{001}], [C_2 2_{010}], [E E], [C_2 m_{100}]$
16	$[^1m^{\bar{1}}m^{\bar{1}}2^{\infty m}1]_3$	$[E m_{100}], [C_2 2_{010}], [E E], [C_2 m_{001}]$
17	$[^12\bar{1}2\bar{1}2^{\infty m}1]_1$	$[E 2_{001}], [C_2 2_{010}], [E E], [C_2 2_{100}]$
18	$[^12\bar{1}2\bar{1}2^{\infty m}1]_2$	$[E 2_{010}], [C_2 2_{001}], [E E], [C_2 2_{100}]$
19	$[^1m^{\bar{1}}m^{\bar{1}}m^{\infty m}1]_1$	$[E P], [C_2 m_{010}], [C_2 m_{100}], [E E], [E 2_{001}], [C_2 2_{100}], [C_2 2_{010}], [E m_{001}]$
20	$[^1m^{\bar{1}}m^{\bar{1}}m^{\infty m}1]_2$	$[E P], [C_2 m_{010}], [C_2 m_{001}], [E E], [E 2_{100}], [C_2 2_{001}], [C_2 2_{010}], [E m_{100}]$
21	$[^1m^1m^{\bar{1}}m^{\infty m}1]_1$	$[E m_{001}], [E m_{010}], [C_2 P], [E E], [E 2_{100}], [C_2 2_{010}], [C_2 2_{001}], [C_2 m_{100}]$
22	$[^1m^1m^{\bar{1}}m^{\infty m}1]_2$	$[E m_{100}], [E m_{010}], [C_2 P], [E E], [E 2_{001}], [C_2 2_{010}], [C_2 2_{100}], [C_2 m_{001}]$
23	$\bar{1}m^{\bar{1}}m^{\bar{1}}m^{\infty m}1$	$[C_2 m_{001}], [C_2 m_{010}], [C_2 P], [E E], [C_2 m_{100}], [E 2_{100}], [E 2_{010}], [E 2_{001}]$
24	$\bar{1}4^{\infty m}1$	$[C_2 4_{001}^+], [E E], [E 2_{001}], [C_2 4_{001}^-]$
25	$\bar{1}\bar{4}^{\infty m}1$	$[C_2 \bar{4}_{001}^+], [E E], [E 2_{001}], [C_2 \bar{4}_{001}^-]$
26	$\bar{1}4/^1m^{\infty m}1$	$[C_2 4_{001}^+], [E P], [E E], [E 2_{001}], [E m_{001}], [C_2 4_{001}^-], [C_2 \bar{4}_{001}^+], [C_2 \bar{4}_{001}^-]$
27	$\bar{1}4/\bar{1}m^{\infty m}1$	$[C_2 4_{001}^+], [C_2 P], [E E], [C_2 m_{001}], [E 2_{001}], [E \bar{4}_{001}^+], [E \bar{4}_{001}^-], [C_2 4_{001}^-]$
28	$^14/\bar{1}m^{\infty m}1$	$[E 4_{001}^+], [C_2 P], [E E], [E 2_{001}], [E 4_{001}^-], [C_2 m_{001}], [C_2 \bar{4}_{001}^+], [C_2 \bar{4}_{001}^-]$
29	$^14\bar{1}2\bar{1}2^{\infty m}1$	$[E 4_{001}^+], [C_2 2_{100}], [C_2 2_{\bar{1}10}], [E E], [C_2 2_{010}], [C_2 2_{110}], [E 2_{001}], [E 4_{001}^-]$
30	$\bar{1}4^12\bar{1}2^{\infty m}1$	$[C_2 4_{001}^+], [E 2_{100}], [C_2 2_{\bar{1}10}], [E E], [E 2_{010}], [C_2 2_{110}], [E 2_{001}], [C_2 4_{001}^-]$
31	$^14\bar{1}m^{\bar{1}}m^{\infty m}1$	$[E 4_{001}^+], [C_2 m_{100}], [C_2 m_{\bar{1}10}], [E E], [E 2_{001}], [E 4_{001}^-], [C_2 m_{010}], [C_2 m_{110}]$

No.	Notation	Operators in \mathcal{G}_0 [$\mathcal{G} = (\mathcal{G}_0 + [C_2 T E] \mathcal{G}_0) \ltimes SO(2)$]
32	${}^{\bar{1}}\bar{4}^1 m^{\bar{1}} m^{\infty m} 1$	$[C_2 4_{001}^+], [C_2 m_{100}], [E m_{\bar{1}10}], [E E], [E 2001], [C_2 4_{001}^-], [C_2 m_{010}], [E m_{110}]$
33	${}^1\bar{4}^{\bar{1}} 2^{\bar{1}} m^{\infty m} 1$	$[E \bar{4}_{001}^+], [C_2 2_{100}], [C_2 m_{\bar{1}10}], [E E], [C_2 2_{010}], [E 2001], [C_2 m_{110}], [E \bar{4}_{001}^-]$
34	${}^{\bar{1}}\bar{4}^{\bar{1}} 2^1 m^{\infty m} 1$	$[C_2 \bar{4}_{001}^+], [C_2 2_{100}], [E m_{110}], [E E], [C_2 2_{010}], [E 2001], [E m_{\bar{1}10}], [C_2 \bar{4}_{001}^-]$
35	${}^{\bar{1}}\bar{4}^1 2^{\bar{1}} m^{\infty m} 1$	$[C_2 \bar{4}_{001}^+], [E 2_{100}], [C_2 m_{110}], [E E], [E 2_{010}], [E 2001], [C_2 m_{\bar{1}10}], [C_2 \bar{4}_{001}^-]$
36	${}^{\bar{1}}\bar{4}/{}^{\bar{1}} m^{\bar{1}} m^1 m^{\infty m} 1$	$[C_2 4_{001}^+], [C_2 P], [C_2 m_{100}], [E m_{\bar{1}10}], [E E], [C_2 m_{001}], [E 2_{100}], [E 2_{010}], [E 2001], [C_2 2_{\bar{1}10}], [E 4_{001}^-], [E 2_{110}], [C_2 m_{010}], [E m_{110}], [E \bar{4}_{001}^+], [E \bar{4}_{001}^-]$
37	${}^1\bar{4}/{}^{\bar{1}} m^1 m^1 m^{\infty m} 1$	$[E 4_{001}^+], [C_2 P], [E m_{100}], [E m_{\bar{1}10}], [E E], [C_2 m_{001}], [C_2 2_{100}], [C_2 2_{010}], [E 2001], [C_2 2_{\bar{1}10}], [E 4_{001}^-], [C_2 2_{110}], [E m_{010}], [E m_{110}], [C_2 \bar{4}_{001}^+], [C_2 \bar{4}_{001}^-]$
38	${}^1\bar{4}/{}^1 m^1 m^{\bar{1}} m^{\infty m} 1$	$[C_2 4_{001}^+], [E P], [C_2 m_{100}], [E m_{\bar{1}10}], [E E], [E m_{001}], [C_2 2_{100}], [C_2 2_{010}], [E 2001], [E 2_{\bar{1}10}], [C_2 4_{001}^-], [E 2_{110}], [C_2 m_{010}], [E m_{110}], [C_2 \bar{4}_{001}^+], [C_2 \bar{4}_{001}^-]$
39	${}^1\bar{4}/{}^1 m^{\bar{1}} m^{\bar{1}} m^{\infty m} 1$	$[E 4_{001}^+], [E P], [C_2 m_{100}], [C_2 m_{\bar{1}10}], [E E], [E m_{001}], [C_2 2_{100}], [C_2 2_{010}], [E 2001], [C_2 2_{\bar{1}10}], [E 4_{001}^-], [C_2 2_{110}], [C_2 m_{010}], [C_2 m_{110}], [E \bar{4}_{001}^+], [E \bar{4}_{001}^-]$
40	${}^1\bar{4}/{}^{\bar{1}} m^{\bar{1}} m^{\bar{1}} m^{\infty m} 1$	$[E 4_{001}^+], [C_2 P], [C_2 m_{100}], [C_2 m_{\bar{1}10}], [E E], [C_2 m_{001}], [E 2_{100}], [E 2_{010}], [E 2001], [E 2_{\bar{1}10}], [E 4_{001}^-], [E 2_{110}], [C_2 m_{010}], [C_2 m_{110}], [C_2 \bar{4}_{001}^+], [C_2 \bar{4}_{001}^-]$
41	${}^{\bar{1}}\bar{3}^{\infty m} 1$	$[C_2 \bar{3}_{001}^+], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 \bar{3}_{001}^-], [C_2 P]$
42	${}^1\bar{3}^{\bar{1}} 2^{\infty m} 1$	$[E 3_{001}^+], [C_2 2_{100}], [E E], [E 3_{001}^-], [C_2 2_{110}], [C_2 2_{010}]$
43	${}^1\bar{3}^{\bar{1}} m^{\infty m} 1$	$[E 3_{001}^+], [C_2 m_{100}], [E E], [E 3_{001}^-], [C_2 m_{110}], [C_2 m_{010}]$
44	${}^1\bar{3}^{\bar{1}} m^{\infty m} 1$	$[E \bar{3}_{001}^+], [C_2 m_{100}], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 2_{100}], [C_2 2_{110}], [C_2 2_{010}], [E P], [E \bar{3}_{001}^-], [C_2 m_{110}], [C_2 m_{010}]$
45	${}^{\bar{1}}\bar{3}^1 m^{\infty m} 1$	$[C_2 \bar{3}_{001}^+], [E m_{100}], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 2_{100}], [C_2 2_{110}], [C_2 2_{010}], [C_2 P], [C_2 \bar{3}_{001}^-], [E m_{110}], [E m_{010}]$
46	${}^{\bar{1}}\bar{3}^{\bar{1}} m^{\infty m} 1$	$[C_2 \bar{3}_{001}^+], [C_2 m_{100}], [E E], [E 3_{001}^+], [E 3_{001}^-], [E 2_{100}], [E 2_{110}], [E 2_{010}], [C_2 P], [C_2 \bar{3}_{001}^-], [C_2 m_{110}], [C_2 m_{010}]$
47	${}^{\bar{1}}\bar{6}^{\infty m} 1$	$[C_2 \bar{6}_{001}^+], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 m_{001}], [C_2 \bar{6}_{001}^-]$
48	${}^{\bar{1}}6^{\infty m} 1$	$[C_2 6_{001}^+], [E E], [E 3_{001}^+], [C_2 2_{001}], [E 3_{001}^-], [C_2 6_{001}^-]$
49	${}^1\bar{6}^{\bar{1}} 2^{\bar{1}} 2^{\infty m} 1$	$[E 6_{001}^+], [C_2 2_{100}], [C_2 2_{\bar{1}10}], [E E], [E 3_{001}^+], [E 3_{001}^-], [E 2_{001}], [E 6_{001}^-], [C_2 2_{110}], [C_2 2_{120}], [C_2 2_{010}], [C_2 2_{210}]$
50	${}^{\bar{1}}6^1 2^{\bar{1}} 2^{\infty m} 1$	$[C_2 6_{001}^+], [E 2_{100}], [C_2 2_{\bar{1}10}], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 2_{001}], [C_2 6_{001}^-], [E 2_{110}], [C_2 2_{120}], [E 2_{010}], [C_2 2_{210}]$
51	${}^{\bar{1}}6/\bar{1} m^{\infty m} 1$	$[C_2 6_{001}^+], [E P], [E E], [C_2 m_{001}], [E 3_{001}^+], [C_2 2_{001}], [E 3_{001}^-], [C_2 6_{001}^-], [C_2 \bar{6}_{001}^+], [E \bar{3}_{001}^+], [E \bar{3}_{001}^-], [C_2 \bar{6}_{001}^-]$
52	${}^{\bar{1}}6/{}^1 m^{\infty m} 1$	$[C_2 6_{001}^+], [C_2 P], [E E], [E m_{001}], [E 3_{001}^+], [C_2 2_{001}], [E 3_{001}^-], [C_2 6_{001}^-], [E \bar{6}_{001}^+], [C_2 \bar{3}_{001}^+], [C_2 \bar{3}_{001}^-], [E \bar{6}_{001}^-]$
53	${}^1\bar{6}/{}^{\bar{1}} m^{\infty m} 1$	$[E 6_{001}^+], [C_2 P], [E E], [C_2 m_{001}], [E 3_{001}^+], [E 2_{001}], [E 3_{001}^-], [E 6_{001}^-], [C_2 \bar{6}_{001}^+], [C_2 \bar{3}_{001}^+], [C_2 \bar{3}_{001}^-], [E \bar{6}_{001}^-]$
54	${}^1\bar{6}^{\bar{1}} m^{\bar{1}} m^{\infty m} 1$	$[E 6_{001}^+], [C_2 m_{100}], [C_2 m_{\bar{1}10}], [E E], [E 3_{001}^+], [E 2_{001}^+], [E 3_{001}^-], [E 6_{001}^-], [C_2 m_{110}], [C_2 m_{120}], [C_2 m_{010}], [C_2 m_{210}]$
55	${}^{\bar{1}}6^1 m^{\bar{1}} m^{\infty m} 1$	$[C_2 6_{001}^+], [E m_{100}], [C_2 m_{\bar{1}10}], [E E], [E 3_{001}^+], [C_2 2_{001}^+], [E 3_{001}^-], [C_2 6_{001}^-], [E m_{110}], [C_2 m_{120}], [E m_{010}], [C_2 m_{210}]$

No.	Notation	Operators in \mathcal{G}_0 [$\mathcal{G} = (\mathcal{G}_0 + [C_2 T E] \mathcal{G}_0) \ltimes SO(2)$]
56	${}^1\bar{6}^{\bar{1}}m^{\bar{1}}2^{\infty m}1$	$[E \bar{6}_{001}^+], [C_2 m_{100}], [C_2 2_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 2_{120}], [C_2 2_{210}], [E m_{001}], [E \bar{6}_{001}^-], [C_2 m_{110}], [C_2 m_{010}]$
57	${}^1\bar{6}^{\bar{1}}m^{\bar{1}}2^{\infty m}1$	$[C_2 \bar{6}_{001}^+], [E m_{100}], [C_2 2_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 2_{120}], [C_2 2_{210}], [C_2 m_{001}], [C_2 \bar{6}_{001}^-], [E m_{110}], [E m_{010}]$
58	${}^1\bar{6}^{\bar{1}}m^12^{\infty m}1$	$[C_2 \bar{6}_{001}^+], [C_2 m_{100}], [E 2_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 3_{001}^-], [E 2_{120}], [E 2_{210}], [C_2 m_{001}], [C_2 \bar{6}_{001}^-], [C_2 m_{110}], [C_2 m_{010}]$
59	${}^1\bar{6}/{}^1\bar{6}^{\bar{1}}m^1m^{\infty m}1$	$[C_2 6_{001}^+], [E P], [E m_{100}], [C_2 m_{1\bar{1}0}], [E E], [E 3_{001}^+], [C_2 2_{001}], [E 3_{001}^-], [C_2 6_{001}^-], [C_2 2_{1\bar{1}0}], [E 2_{110}], [C_2 2_{120}], [E 2_{010}], [C_2 2_{210}], [C_2 m_{001}], [C_2 \bar{6}_{001}^+], [E \bar{3}_{001}^+], [E \bar{3}_{001}^-], [C_2 \bar{6}_{001}^-], [E m_{110}], [C_2 m_{120}], [E m_{010}], [C_2 m_{210}], [E 2_{100}]$
60	${}^1\bar{6}/{}^1\bar{6}^{\bar{1}}m^1m^{\infty m}1$	$[C_2 6_{001}^+], [C_2 P], [C_2 m_{100}], [E m_{1\bar{1}0}], [E E], [E 3_{001}^+], [C_2 2_{001}], [E 3_{001}^-], [C_2 6_{001}^-], [C_2 2_{1\bar{1}0}], [E 2_{110}], [C_2 2_{120}], [E 2_{010}], [C_2 2_{210}], [E m_{001}], [E \bar{6}_{001}^+], [C_2 \bar{3}_{001}^-], [C_2 \bar{3}_{001}^+], [E \bar{6}_{001}^-], [C_2 m_{110}], [E m_{120}], [C_2 m_{010}], [E m_{210}], [E 2_{100}]$
61	${}^1\bar{6}/{}^1\bar{6}^{\bar{1}}m^1m^{\infty m}1$	$[E 6_{001}^+], [E P], [C_2 m_{100}], [C_2 m_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 2_{001}], [E 3_{001}^-], [E 6_{001}^-], [C_2 2_{1\bar{1}0}], [C_2 2_{110}], [C_2 2_{120}], [C_2 2_{010}], [C_2 2_{210}], [E m_{001}], [E \bar{6}_{001}^+], [E \bar{3}_{001}^+], [E \bar{3}_{001}^-], [E \bar{6}_{001}^-], [C_2 m_{110}], [C_2 m_{120}], [C_2 m_{010}], [C_2 m_{210}], [C_2 2_{100}]$
62	${}^1\bar{6}/{}^1\bar{6}^{\bar{1}}m^1m^{\infty m}1$	$[E 6_{001}^+], [C_2 P], [E m_{100}], [E m_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 2_{001}], [E 3_{001}^-], [E 6_{001}^-], [C_2 2_{1\bar{1}0}], [C_2 2_{110}], [C_2 2_{120}], [C_2 2_{010}], [C_2 2_{210}], [C_2 m_{001}], [C_2 \bar{6}_{001}^+], [C_2 \bar{3}_{001}^+], [C_2 \bar{3}_{001}^-], [C_2 \bar{6}_{001}^-], [E m_{110}], [E m_{120}], [E m_{010}], [E m_{210}], [C_2 2_{100}]$
63	${}^1\bar{6}/{}^1\bar{6}^{\bar{1}}m^1m^{\infty m}1$	$[E 6_{001}^+], [C_2 P], [C_2 m_{100}], [C_2 m_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 2_{001}], [E 3_{001}^-], [E 6_{001}^-], [E 2_{1\bar{1}0}], [E 2_{110}], [E 2_{120}], [E 2_{010}], [E 2_{210}], [C_2 m_{001}], [C_2 \bar{6}_{001}^+], [C_2 \bar{3}_{001}^+], [C_2 \bar{3}_{001}^-], [C_2 \bar{6}_{001}^-], [C_2 \bar{6}_{001}^-], [C_2 m_{110}], [C_2 m_{120}], [C_2 m_{010}], [C_2 m_{210}], [E 2_{100}]$

TABLE S7: 31 collinear spin layer-point group \mathcal{G} : $\mathcal{G} = (\mathcal{G}_0 + [C_2T||E]\mathcal{G}_0) \ltimes SO(2)$ for FM. Only nontrivial operators in \mathcal{G}_0 are listed. The generators of \mathcal{G}_0 are marked in blue. The subscript i in the second column ($[...]$) indicates the i -th collinear spin layer-point group obtained from a same 3D collinear spin point group.

No.	Notation	Operators in \mathcal{G}_0 [$\mathcal{G} = (\mathcal{G}_0 + [C_2T E]\mathcal{G}_0) \ltimes SO(2)$]
1	$\infty^m 1$	$[E E]$
2	${}^1\bar{1}^{\infty^m} 1$	$[E P]$, $[E E]$
3	$[{}^12^{\infty^m} 1]_1$	$[E 2_{001}]$, $[E E]$
4	$[{}^12^{\infty^m} 1]_2$	$[E 2_{100}]$, $[E E]$
5	$[{}^1m^{\infty^m} 1]_1$	$[E m_{001}]$, $[E E]$
6	$[{}^1m^{\infty^m} 1]_2$	$[E m_{100}]$, $[E E]$
7	$[{}^12/{}^1m^{\infty^m} 1]_1$	$[E 2_{001}]$, $[E P]$, $[E E]$, $[E m_{001}]$
8	$[{}^12/{}^1m^{\infty^m} 1]_2$	$[E 2_{100}]$, $[E P]$, $[E E]$, $[E m_{100}]$
9	$[{}^1m^1m^12^{\infty^m} 1]_1$	$[E m_{100}]$, $[E 2_{001}]$, $[E E]$, $[E m_{010}]$
10	$[{}^1m^1m^12^{\infty^m} 1]_2$	$[E m_{100}]$, $[E 2_{010}]$, $[E E]$, $[E m_{001}]$
11	${}^12^12^12^{\infty^m} 1$	$[E 2_{010}]$, $[E 2_{100}]$, $[E E]$, $[E 2_{001}]$
12	${}^1m^1m^1m^{\infty^m} 1$	$[E P]$, $[E m_{010}]$, $[E m_{100}]$, $[E E]$, $[E 2_{100}]$, $[E 2_{010}]$, $[E 2_{001}]$, $[E m_{001}]$
13	${}^14^{\infty^m} 1$	$[E 4_{001}]$, $[E E]$, $[E 2_{001}]$, $[E 4_{001}^-]$
14	${}^1\bar{4}^{\infty^m} 1$	$[E \bar{4}_{001}^+]$, $[E E]$, $[E 2_{001}]$, $[E \bar{4}_{001}^-]$
15	${}^14/{}^1m^{\infty^m} 1$	$[E 4_{001}^+]$, $[E P]$, $[E E]$, $[E 2_{001}]$, $[E 4_{001}^-]$, $[E m_{001}]$, $[E \bar{4}_{001}^+]$, $[E \bar{4}_{001}^-]$
16	${}^14^12^12^{\infty^m} 1$	$[E 4_{001}^+]$, $[E 2_{100}]$, $[E 2_{110}]$, $[E E]$, $[E 2_{001}]$, $[E 2_{1\bar{1}0}]$, $[E 2_{010}]$, $[E 4_{001}^-]$
17	${}^14^1m^1m^{\infty^m} 1$	$[E 4_{001}^+]$, $[E m_{100}]$, $[E m_{110}]$, $[E E]$, $[E 2_{001}]$, $[E 4_{001}^-]$, $[E m_{010}]$, $[E m_{1\bar{1}0}]$
18	${}^1\bar{4}^12^1m^{\infty^m} 1$	$[E \bar{4}_{001}^+]$, $[E 2_{100}]$, $[E m_{110}]$, $[E E]$, $[E 2_{010}]$, $[E 2_{001}]$, $[E m_{1\bar{1}0}]$, $[E \bar{4}_{001}^-]$
19	${}^14/{}^1m^1m^1m^{\infty^m} 1$	$[E 4_{001}^+]$, $[E P]$, $[E m_{100}]$, $[E m_{110}]$, $[E E]$, $[E 2_{100}]$, $[E 2_{010}]$, $[E 2_{001}]$, $[E 2_{1\bar{1}0}]$, $[E m_{001}]$, $[E m_{1\bar{1}0}]$, $[E \bar{4}_{001}^+]$, $[E \bar{4}_{001}^-]$, $[E 4_{001}^-]$
20	${}^13^m 1$	$[E 3_{001}^+]$, $[E E]$, $[E 3_{001}^-]$
21	${}^1\bar{3}^{\infty^m} 1$	$[E \bar{3}_{001}^+]$, $[E E]$, $[E 3_{001}^+]$, $[E P]$, $[E 3_{001}^-]$, $[E \bar{3}_{001}^-]$
22	${}^13^12^{\infty^m} 1$	$[E 3_{001}^+]$, $[E 2_{100}]$, $[E E]$, $[E 3_{001}^-]$, $[E 2_{110}]$, $[E 2_{010}]$
23	${}^13^1m^{\infty^m} 1$	$[E 3_{001}^+]$, $[E m_{100}]$, $[E E]$, $[E 3_{001}^-]$, $[E m_{110}]$, $[E m_{010}]$
24	${}^1\bar{3}^1m^{\infty^m} 1$	$[E \bar{3}_{001}^+]$, $[E m_{100}]$, $[E E]$, $[E 3_{001}^+]$, $[E 3_{001}^-]$, $[E 2_{100}]$, $[E 2_{110}]$, $[E 2_{010}]$, $[E P]$, $[E m_{110}]$, $[E m_{010}]$, $[E \bar{3}_{001}^-]$
25	${}^1\bar{6}^{\infty^m} 1$	$[E \bar{6}_{001}^+]$, $[E E]$, $[E 3_{001}^+]$, $[E 3_{001}^-]$, $[E m_{001}]$, $[E \bar{6}_{001}^-]$
26	${}^16^{\infty^m} 1$	$[E 6_{001}^+]$, $[E E]$, $[E 3_{001}^+]$, $[E 2_{001}]$, $[E 3_{001}^-]$, $[E 6_{001}^-]$
27	${}^16^12^12^{\infty^m} 1$	$[E 6_{001}^+]$, $[E 2_{100}]$, $[E 2_{1\bar{1}0}]$, $[E E]$, $[E 3_{001}^+]$, $[E 2_{001}]$, $[E 3_{001}^-]$, $[E 6_{001}^-]$, $[E 2_{110}]$, $[E 2_{010}]$, $[E 2_{120}]$
28	${}^16/{}^1m^{\infty^m} 1$	$[E 6_{001}^+]$, $[E P]$, $[E E]$, $[E 3_{001}^+]$, $[E 2_{001}]$, $[E 3_{001}^-]$, $[E 6_{001}^-]$, $[E m_{001}]$, $[E \bar{6}_{001}^+]$, $[E \bar{6}_{001}^-]$, $[E \bar{3}_{001}^+]$

No.	Notation	Operators in \mathcal{G}_0 [$\mathcal{G} = (\mathcal{G}_0 + [C_2 T E] \mathcal{G}_0) \ltimes SO(2)$]
29	${}^1\bar{6}^1 m^1 m^{\infty m} 1$	$[E 6_{001}^+], [E m_{100}], [E m_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 2_{001}], [E 3_{001}^-], [E 6_{001}^-], [E m_{110}], [E m_{010}], [E m_{210}], [E m_{120}]$
30	${}^1\bar{6}^1 m^1 2^{\infty m} 1$	$[E \bar{6}_{001}^+], [E m_{100}], [E 2_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 2_{120}], [E 3_{001}^-], [E \bar{6}_{001}^-], [E 2_{210}], [E m_{001}], [E m_{110}], [E m_{010}]$
31	${}^1\bar{6}/^1 m^1 m^1 m^{\infty m} 1$	$[E 6_{001}^+], [E P], [E m_{100}], [E m_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 2_{001}], [E 3_{001}^-], [E 6_{001}^-], [E 2_{120}], [E 2_{110}], [E 2_{210}], [E 2_{010}], [E 2_{1\bar{1}0}], [E m_{001}], [E \bar{6}_{001}^+], [E \bar{3}_{001}^+], [E \bar{3}_{001}^-], [E \bar{6}_{001}^-], [E m_{110}], [E m_{120}], [E m_{010}], [E m_{210}], [E 2_{100}]$

B. Operators of 90 3D collinear spin point groups

TABLE S8: 58 collinear spin point group \mathcal{G} : $\mathcal{G} = (\mathcal{G}_0 + [C_2T||E]\mathcal{G}_0) \ltimes SO(2)$ for AFM. Only nontrivial operators in \mathcal{G}_0 are listed. The generators of \mathcal{G}_0 are marked in blue. The groups allowing for 3D spin-AFE are highlighted by red.

No.	Notation	Operators in \mathcal{G}_0 [$\mathcal{G} = (\mathcal{G}_0 + [C_2T E]\mathcal{G}_0) \ltimes SO(2)$]
1	$\bar{1}\bar{1}^{\infty m}1$	$[C_2 P], [E E]$
2	$\bar{1}2^{\infty m}1$	$[C_2 2_{010}], [E E]$
3	$\bar{1}m^{\infty m}1$	$[C_2 m_{010}], [E E]$
4	${}^12/\bar{1}m^{\infty m}1$	$[E 2_{010}], [C_2 P], [E E], [C_2 m_{010}]$
5	$\bar{1}2/{}^1m^{\infty m}1$	$[C_2 P], [E m_{010}], [E E], [C_2 2_{010}]$
6	$\bar{1}2/\bar{1}m^{\infty m}1$	$[C_2 2_{010}], [E P], [E E], [C_2 m_{010}]$
7	$\bar{1}m\bar{1}m^12^{\infty m}1$	$[C_2 m_{100}], [E 2_{001}], [E E], [C_2 m_{010}]$
8	${}^1m\bar{1}m^12^{\infty m}1$	$[E m_{010}], [C_2 2_{001}], [E E], [C_2 m_{100}]$
9	${}^12\bar{1}2\bar{1}2^{\infty m}1$	$[E 2_{001}], [C_2 2_{010}], [E E], [C_2 2_{100}]$
10	${}^1m\bar{1}m^1\bar{1}m^{\infty m}1$	$[E P], [C_2 m_{010}], [C_2 m_{100}], [E E], [E 2_{001}], [C_2 2_{100}], [C_2 2_{010}], [E m_{001}]$
11	${}^1m^1\bar{1}m^{\infty m}1$	$[E m_{001}], [E m_{010}], [C_2 P], [E E], [E 2_{100}], [C_2 2_{010}], [C_2 2_{001}], [C_2 m_{100}]$
12	$\bar{1}m\bar{1}m^1\bar{1}m^{\infty m}1$	$[C_2 m_{001}], [C_2 m_{010}], [C_2 P], [E E], [C_2 m_{100}], [E 2_{100}], [E 2_{010}], [E 2_{001}]$
13	$\bar{1}4^{\infty m}1$	$[C_2 4_{001}^+], [E E], [E 2_{001}], [C_2 4_{001}^-]$
14	$\bar{1}\bar{4}^{\infty m}1$	$[C_2 \bar{4}_{001}^+], [E E], [E 2_{001}], [C_2 \bar{4}_{001}^-]$
15	$\bar{1}4/{}^1m^{\infty m}1$	$[C_2 4_{001}^+], [E P], [E E], [E 2_{001}], [E m_{001}], [C_2 4_{001}^-], [C_2 \bar{4}_{001}^+], [C_2 \bar{4}_{001}^-]$
16	$\bar{1}4/\bar{1}m^{\infty m}1$	$[C_2 4_{001}^+], [C_2 P], [E E], [C_2 m_{001}], [E 2_{001}], [E \bar{4}_{001}^+], [E \bar{4}_{001}^-], [C_2 4_{001}^-]$
17	${}^14/\bar{1}m^{\infty m}1$	$[E 4_{001}^+], [C_2 P], [E E], [E 2_{001}], [E 4_{001}^-], [C_2 m_{001}], [C_2 \bar{4}_{001}^+], [C_2 \bar{4}_{001}^-]$
18	${}^14\bar{1}2\bar{1}2^{\infty m}1$	$[E 4_{001}^+], [C_2 2_{100}], [C_2 2_{\bar{1}10}], [E E], [C_2 2_{010}], [C_2 2_{110}], [E 2_{001}], [E 4_{001}^-]$
19	$\bar{1}4^12\bar{1}2^{\infty m}1$	$[C_2 4_{001}^+], [E 2_{100}], [C_2 2_{\bar{1}10}], [E E], [E 2_{010}], [C_2 2_{110}], [E 2_{001}], [C_2 4_{001}^-]$
20	${}^14\bar{1}m\bar{1}m^{\infty m}1$	$[E 4_{001}^+], [C_2 m_{100}], [C_2 m_{\bar{1}10}], [E E], [E 2_{001}], [E 4_{001}^-], [C_2 m_{010}], [C_2 m_{110}]$
21	$\bar{1}4^1m\bar{1}m^{\infty m}1$	$[C_2 4_{001}^+], [C_2 m_{100}], [E m_{\bar{1}10}], [E E], [E 2_{001}], [C_2 4_{001}^-], [C_2 m_{010}], [E m_{110}]$
22	${}^1\bar{4}\bar{1}2\bar{1}m^{\infty m}1$	$[E 4_{001}^+], [C_2 2_{100}], [C_2 m_{\bar{1}10}], [E E], [C_2 2_{010}], [E 2_{001}], [C_2 m_{110}], [E \bar{4}_{001}^-]$
23	$\bar{1}4\bar{1}2^1m^{\infty m}1$	$[C_2 \bar{4}_{001}^+], [C_2 2_{100}], [E m_{110}], [E E], [C_2 2_{010}], [E 2_{001}], [E m_{\bar{1}10}], [C_2 \bar{4}_{001}^-]$
24	$\bar{1}4^12\bar{1}m^{\infty m}1$	$[C_2 \bar{4}_{001}^+], [E 2_{100}], [C_2 m_{110}], [E E], [E 2_{010}], [C_2 2_{\bar{1}10}], [C_2 \bar{4}_{001}^-]$
25	$\bar{1}4/\bar{1}m\bar{1}m^1m^{\infty m}1$	$[C_2 4_{001}^+], [C_2 P], [C_2 m_{100}], [E m_{\bar{1}10}], [E E], [C_2 m_{001}], [E 2_{100}], [E 2_{010}], [E 2_{001}], [C_2 2_{\bar{1}10}], [E 4_{001}^-], [E 2_{110}], [C_2 m_{010}], [E m_{110}], [E \bar{4}_{001}^+], [E \bar{4}_{001}^-]$
26	${}^14/\bar{1}m^1m^1m^{\infty m}1$	$[E 4_{001}^+], [C_2 P], [E m_{100}], [E m_{\bar{1}10}], [E E], [C_2 m_{001}], [C_2 2_{100}], [C_2 2_{010}], [E 2_{001}], [C_2 2_{\bar{1}10}], [E 4_{001}^-], [E 2_{110}], [C_2 m_{010}], [E m_{110}], [E \bar{4}_{001}^+], [C_2 \bar{4}_{001}^-]$
27	$\bar{1}4/{}^1m^1m\bar{1}m^{\infty m}1$	$[C_2 4_{001}^+], [E P], [C_2 m_{100}], [E m_{\bar{1}10}], [E E], [E m_{001}], [C_2 2_{100}], [C_2 2_{010}], [E 2_{001}], [E 2_{\bar{1}10}], [C_2 4_{001}^-], [C_2 m_{010}], [E m_{110}], [E \bar{4}_{001}^+], [C_2 \bar{4}_{001}^-]$
28	${}^14/{}^1m^1m\bar{1}m^{\infty m}1$	$[E 4_{001}^+], [E P], [C_2 m_{100}], [C_2 m_{\bar{1}10}], [E E], [E m_{001}], [C_2 2_{100}], [C_2 2_{010}], [E 2_{001}], [C_2 2_{\bar{1}10}], [E 4_{001}^-], [C_2 2_{110}], [C_2 m_{010}], [C_2 m_{110}], [E \bar{4}_{001}^+], [E \bar{4}_{001}^-]$
29	${}^14/\bar{1}m\bar{1}m^1\bar{1}m^{\infty m}1$	$[E 4_{001}^+], [C_2 P], [C_2 m_{100}], [C_2 m_{\bar{1}10}], [E E], [C_2 m_{001}], [E 2_{100}], [E 2_{010}], [E 2_{001}], [E 2_{\bar{1}10}], [E 4_{001}^-], [E 2_{110}], [C_2 m_{010}], [C_2 m_{110}], [C_2 \bar{4}_{001}^+], [C_2 \bar{4}_{001}^-]$

No.	Notation	Operators in \mathcal{G}_0 [$\mathcal{G} = (\mathcal{G}_0 + [C_2T E]\mathcal{G}_0) \ltimes SO(2)$]
30	$\bar{\mathbf{3}}^{\infty m} \mathbf{1}$	$[C_2 \bar{\mathbf{3}}_{001}^+], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 \bar{\mathbf{3}}_{001}^-], [C_2 P]$
31	$\mathbf{1}\bar{\mathbf{3}}^{\bar{\mathbf{1}}} \mathbf{2}^{\infty m} \mathbf{1}$	$[E 3_{001}^+], [C_2 2_{100}], [E E], [E 3_{001}^-], [C_2 2_{110}], [C_2 2_{010}]$
32	$\mathbf{1}\bar{\mathbf{3}}^{\bar{\mathbf{1}}} m^{\infty m} \mathbf{1}$	$[E 3_{001}^+], [C_2 m_{100}], [E E], [E 3_{001}^-], [C_2 m_{110}], [C_2 m_{010}]$
33	$\bar{\mathbf{3}}^{\bar{\mathbf{1}}} m^{\infty m} \mathbf{1}$	$[E \bar{\mathbf{3}}_{001}^+], [C_2 m_{100}], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 2_{100}], [C_2 2_{110}], [C_2 2_{010}], [E P], [E \bar{\mathbf{3}}_{001}^-], [C_2 m_{110}], [C_2 m_{010}]$
34	$\bar{\mathbf{3}}^{\bar{\mathbf{1}}} m^{\infty m} \mathbf{1}$	$[C_2 \bar{\mathbf{3}}_{001}^+], [E m_{100}], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 2_{100}], [C_2 2_{110}], [C_2 2_{010}], [C_2 P], [C_2 \bar{\mathbf{3}}_{001}^-], [E m_{110}], [E m_{010}]$
35	$\bar{\mathbf{3}}^{\bar{\mathbf{1}}} m^{\infty m} \mathbf{1}$	$[C_2 \bar{\mathbf{3}}_{001}^+], [C_2 m_{100}], [E E], [E 3_{001}^+], [E 3_{001}^-], [E 2_{100}], [E 2_{110}], [E 2_{010}], [C_2 P], [C_2 \bar{\mathbf{3}}_{001}^-], [C_2 m_{110}], [C_2 m_{010}]$
36	$\bar{\mathbf{6}}^{\infty m} \mathbf{1}$	$[C_2 \bar{\mathbf{6}}_{001}^+], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 m_{001}], [C_2 \bar{\mathbf{6}}_{001}^-]$
37	$\bar{\mathbf{6}}^{\infty m} \mathbf{1}$	$[C_2 \bar{\mathbf{6}}_{001}^+], [E E], [E 3_{001}^+], [C_2 2_{001}], [E 3_{001}^-], [C_2 6_{001}^-]$
38	$\mathbf{1}\bar{\mathbf{6}}^{\bar{\mathbf{1}}} \mathbf{2}^{\bar{\mathbf{1}}} \mathbf{2}^{\infty m} \mathbf{1}$	$[E 6_{001}^+], [C_2 2_{100}], [C_2 2_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 3_{001}^-], [E 2_{001}], [E 2_{100}], [C_2 2_{120}], [C_2 2_{010}], [C_2 2_{210}]$
39	$\bar{\mathbf{6}}^1 \mathbf{2}^{\bar{\mathbf{1}}} \mathbf{2}^{\infty m} \mathbf{1}$	$[C_2 6_{001}^+], [E 2_{100}], [C_2 2_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 2_{001}], [C_2 6_{001}^-], [E 2_{110}], [C_2 2_{120}], [E 2_{010}], [C_2 2_{210}]$
40	$\bar{\mathbf{6}}/\bar{\mathbf{1}} m^{\infty m} \mathbf{1}$	$[C_2 6_{001}^+], [E P], [E E], [C_2 m_{001}], [E 3_{001}^+], [C_2 2_{001}], [E 3_{001}^-], [C_2 6_{001}^-], [C_2 \bar{\mathbf{6}}_{001}^+], [E \bar{\mathbf{3}}_{001}^+], [E \bar{\mathbf{3}}_{001}^-], [C_2 \bar{\mathbf{6}}_{001}^-]$
41	$\bar{\mathbf{6}}/\mathbf{1} m^{\infty m} \mathbf{1}$	$[C_2 6_{001}^+], [C_2 P], [E E], [E m_{001}], [E 3_{001}^+], [C_2 2_{001}], [E 3_{001}^-], [C_2 6_{001}^-], [E \bar{\mathbf{6}}_{001}^+], [C_2 \bar{\mathbf{3}}_{001}^+], [C_2 \bar{\mathbf{3}}_{001}^-], [E \bar{\mathbf{6}}_{001}^-]$
42	$\mathbf{1}\bar{\mathbf{6}}/\bar{\mathbf{1}} m^{\infty m} \mathbf{1}$	$[E 6_{001}^+], [C_2 P], [E E], [C_2 m_{001}], [E 3_{001}^+], [E 2_{001}], [E 3_{001}^-], [E 6_{001}^-], [C_2 \bar{\mathbf{6}}_{001}^+], [C_2 \bar{\mathbf{3}}_{001}^+], [C_2 \bar{\mathbf{3}}_{001}^-], [E \bar{\mathbf{6}}_{001}^-]$
43	$\mathbf{1}\bar{\mathbf{6}}^1 \mathbf{m}^{\bar{\mathbf{1}}} m^{\infty m} \mathbf{1}$	$[E 6_{001}^+], [C_2 m_{100}], [C_2 m_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 2_{001}^+], [E 3_{001}^-], [E 6_{001}^-], [C_2 m_{110}], [C_2 m_{120}], [C_2 m_{010}], [C_2 m_{210}]$
44	$\bar{\mathbf{6}}^1 \mathbf{m}^{\bar{\mathbf{1}}} m^{\infty m} \mathbf{1}$	$[C_2 6_{001}^+], [E m_{100}], [C_2 m_{1\bar{1}0}], [E E], [E 3_{001}^+], [C_2 2_{001}^+], [E 3_{001}^-], [C_2 6_{001}^-], [E m_{110}], [C_2 m_{120}], [E m_{010}], [C_2 m_{210}]$
45	$\bar{\mathbf{6}}^{\bar{\mathbf{1}}} \mathbf{m}^{\bar{\mathbf{1}}} 2^{\infty m} \mathbf{1}$	$[E \bar{\mathbf{6}}_{001}^+], [C_2 m_{100}], [C_2 2_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 2_{120}], [C_2 2_{210}], [E m_{001}], [E \bar{\mathbf{6}}_{001}^-], [C_2 m_{110}], [C_2 m_{010}]$
46	$\bar{\mathbf{6}}^{\bar{\mathbf{1}}} \mathbf{m}^{\bar{\mathbf{1}}} 2^{\infty m} \mathbf{1}$	$[C_2 \bar{\mathbf{6}}_{001}^+], [E m_{100}], [C_2 2_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 3_{001}^-], [C_2 2_{120}], [C_2 2_{210}], [C_2 m_{001}], [C_2 \bar{\mathbf{6}}_{001}^-], [E m_{110}], [E m_{010}]$
47	$\bar{\mathbf{6}}^{\bar{\mathbf{1}}} m^1 2^{\infty m} \mathbf{1}$	$[C_2 \bar{\mathbf{6}}_{001}^+], [C_2 m_{100}], [E 2_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 3_{001}^-], [E 2_{120}], [E 2_{210}], [C_2 m_{001}], [C_2 \bar{\mathbf{6}}_{001}^-], [C_2 m_{110}], [C_2 m_{010}]$
48	$\bar{\mathbf{6}}/\bar{\mathbf{1}} m^1 \mathbf{m}^{\bar{\mathbf{1}}} m^{\infty m} \mathbf{1}$	$[C_2 6_{001}^+], [E P], [E m_{100}], [C_2 m_{1\bar{1}0}], [E E], [E 3_{001}^+], [C_2 2_{001}], [E 3_{001}^-], [C_2 6_{001}^-], [C_2 2_{1\bar{1}0}], [E 2_{110}], [C_2 2_{120}], [E 2_{010}], [C_2 2_{210}], [C_2 m_{001}], [C_2 \bar{\mathbf{6}}_{001}^+], [E \bar{\mathbf{3}}_{001}^+], [E \bar{\mathbf{3}}_{001}^-], [C_2 \bar{\mathbf{6}}_{001}^-], [E m_{110}], [C_2 m_{120}], [E m_{010}], [C_2 m_{210}], [E 2_{100}]$
49	$\bar{\mathbf{6}}/\mathbf{1} m^{\bar{\mathbf{1}}} \mathbf{m}^1 m^{\infty m} \mathbf{1}$	$[C_2 6_{001}^+], [C_2 P], [C_2 m_{100}], [E m_{1\bar{1}0}], [E E], [E 3_{001}^+], [C_2 2_{001}], [E 3_{001}^-], [C_2 6_{001}^-], [C_2 2_{1\bar{1}0}], [E 2_{110}], [C_2 2_{120}], [E 2_{010}], [C_2 2_{210}], [E m_{001}], [E \bar{\mathbf{6}}_{001}^+], [C_2 \bar{\mathbf{3}}_{001}^+], [C_2 \bar{\mathbf{3}}_{001}^-], [E \bar{\mathbf{6}}_{001}^-], [C_2 m_{110}], [C_2 m_{120}], [E m_{010}], [C_2 m_{210}], [E 2_{100}]$

No.	Notation	Operators in \mathcal{G}_0 [$\mathcal{G} = (\mathcal{G}_0 + [C_2 T E] \mathcal{G}_0) \ltimes SO(2)$]
50	${}^1\bar{6}/{}^1m^{\bar{1}}m^{\bar{1}}m^{\infty m}1$	$[E 6_{001}^+], [E P], [C_2 m_{100}], [C_2 m_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 2_{001}], [E 3_{001}^-], [E 6_{001}^-], [C_2 2_{1\bar{1}0}], [C_2 2_{110}], [C_2 2_{120}], [C_2 2_{010}], [C_2 2_{210}], [E m_{001}], [E \bar{6}_{001}^+], [E \bar{3}_{001}^+], [E \bar{3}_{001}^-], [E \bar{6}_{001}^-], [C_2 m_{110}], [C_2 m_{120}], [C_2 m_{010}], [C_2 m_{210}], [C_2 2_{100}]$
51	${}^1\bar{6}/{}^{\bar{1}}m^1m^1m^{\infty m}1$	$[E 6_{001}^+], [C_2 P], [E m_{100}], [E m_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 2_{001}], [E 3_{001}^-], [E 6_{001}^-], [C_2 2_{1\bar{1}0}], [C_2 2_{110}], [C_2 2_{120}], [C_2 2_{010}], [C_2 2_{210}], [C_2 m_{001}], [C_2 \bar{6}_{001}^+], [C_2 \bar{3}_{001}^+], [C_2 \bar{3}_{001}^-], [C_2 \bar{6}_{001}^-], [C_2 m_{110}], [E m_{110}], [E m_{120}], [E m_{010}], [E m_{210}], [C_2 2_{100}]$
52	${}^1\bar{6}/{}^{\bar{1}}m^{\bar{1}}m^{\bar{1}}m^{\infty m}1$	$[E 6_{001}^+], [C_2 P], [C_2 m_{100}], [C_2 m_{1\bar{1}0}], [E E], [E 3_{001}^+], [E 2_{001}], [E 3_{001}^-], [E 6_{001}^-], [E 2_{1\bar{1}0}], [E 2_{110}], [E 2_{120}], [E 2_{010}], [E 2_{210}], [C_2 m_{001}], [C_2 \bar{6}_{001}^+], [C_2 \bar{3}_{001}^+], [C_2 \bar{3}_{001}^-], [C_2 \bar{6}_{001}^-], [C_2 m_{110}], [C_2 m_{120}], [C_2 m_{010}], [C_2 m_{210}], [E 2_{100}]$
53	${}^1\bar{2}/{}^{\bar{1}}m^{\bar{1}}\bar{3}^{\infty m}1$	$[E 2_{001}], [C_2 m_{001}], [C_2 \bar{3}_{111}^+], [E E], [E 2_{100}], [E 2_{010}], [E 3_{111}^-], [E 3_{111}^+], [E 3_{1\bar{1}1}^+], [E 3_{1\bar{1}1}^-], [E 3_{11\bar{1}}^-], [E 3_{11\bar{1}}^+], [C_2 P], [C_2 m_{100}], [C_2 m_{010}], [C_2 \bar{3}_{111}^-], [C_2 \bar{3}_{11\bar{1}}^+], [C_2 \bar{3}_{11\bar{1}}^-], [C_2 \bar{3}_{1\bar{1}1}^+], [C_2 \bar{3}_{1\bar{1}1}^-]$
54	${}^{\bar{1}}\bar{4}^1\bar{3}^{\bar{1}}m^{\infty m}1$	$[C_2 \bar{4}_{001}^+], [E 3_{111}^+], [C_2 m_{110}], [E E], [E 2_{100}], [E 2_{010}], [E 3_{111}^-], [E 3_{111}^+], [E 3_{1\bar{1}1}^+], [C_2 \bar{4}_{010}], [E 3_{1\bar{1}1}^-], [E 3_{11\bar{1}}^-], [E 3_{11\bar{1}}^+], [C_2 m_{\bar{1}10}], [C_2 m_{101}], [C_2 \bar{4}_{001}^-], [C_2 m_{011}], [C_2 \bar{4}_{100}], [C_2 \bar{4}_{100}^-], [C_2 m_{\bar{1}01}], [C_2 \bar{4}_{010}], [E 3_{111}^+]$
55	${}^{\bar{1}}4^1\bar{3}^{\bar{1}}2^{\infty m}1$	$[C_2 4_{001}^+], [E 3_{111}^+], [C_2 2_{110}], [E E], [E 2_{100}], [E 2_{010}], [E 3_{111}^-], [E 3_{111}^+], [E 3_{1\bar{1}1}^+], [E 4_{010}^+], [E 3_{1\bar{1}1}^-], [E 3_{11\bar{1}}^-], [E 3_{11\bar{1}}^+], [C_2 2_{\bar{1}10}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{0\bar{1}1}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [E 3_{111}^+], [C_2 2_{\bar{1}01}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{101}], [C_2 4_{010}], [E 3_{111}^+], [C_2 2_{\bar{1}01}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{101}], [C_2 4_{010}], [E 3_{111}^+]$
56	${}^{\bar{1}}4/{}^1m^1\bar{3}^{\bar{1}}2/{}^{\bar{1}}m^{\infty m}1$	$[C_2 4_{001}^+], [E m_{001}], [E \bar{3}_{111}^+], [C_2 2_{110}], [C_2 m_{110}], [E E], [E 2_{100}], [E 2_{010}], [E 3_{111}^-], [E 3_{111}^+], [E 3_{1\bar{1}1}^+], [E 3_{1\bar{1}1}^-], [E 3_{11\bar{1}}^-], [E 3_{11\bar{1}}^+], [C_2 \bar{3}_{111}^+], [E \bar{3}_{111}^-], [E \bar{3}_{11\bar{1}}^+], [E \bar{3}_{11\bar{1}}^-], [C_2 \bar{4}_{010}^+], [E \bar{3}_{11\bar{1}}^+], [E \bar{3}_{11\bar{1}}^-], [E \bar{3}_{1\bar{1}1}^+], [E \bar{3}_{1\bar{1}1}^-], [C_2 2_{\bar{1}10}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{0\bar{1}1}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{101}], [C_2 4_{010}], [E \bar{3}_{111}^+], [C_2 2_{\bar{1}01}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{101}], [C_2 4_{010}], [E \bar{3}_{111}^+]$
57	${}^{\bar{1}}4/{}^{\bar{1}}m^{\bar{1}}\bar{3}^{\bar{1}}2/{}^1m^{\infty m}1$	$[C_2 4_{001}^+], [C_2 m_{001}], [C_2 \bar{3}_{111}^+], [C_2 2_{110}], [E m_{110}], [E E], [E 2_{100}], [E 2_{010}], [E 2_{001}], [E 3_{111}^+], [E 3_{111}^-], [E 3_{1\bar{1}1}^+], [E 3_{1\bar{1}1}^-], [E 3_{11\bar{1}}^-], [E 3_{11\bar{1}}^+], [E 3_{1\bar{1}\bar{1}}^+], [E 3_{1\bar{1}\bar{1}}^-], [E 3_{11\bar{1}}^+], [E 3_{11\bar{1}}^-], [E 3_{1\bar{1}1}^+], [E 3_{1\bar{1}1}^-], [E 3_{111}^+], [E 3_{111}^-], [E 3_{1\bar{1}1}^+], [E 3_{1\bar{1}1}^-], [E 3_{11\bar{1}}^+], [E 3_{11\bar{1}}^-], [E 3_{1\bar{1}\bar{1}}^+], [E 3_{1\bar{1}\bar{1}}^-], [C_2 2_{\bar{1}10}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{0\bar{1}1}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{101}], [C_2 4_{010}], [E \bar{3}_{111}^+], [C_2 2_{\bar{1}01}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{101}], [C_2 4_{010}], [E \bar{3}_{111}^+]$
58	${}^14/{}^{\bar{1}}m^{\bar{1}}\bar{3}^{\bar{1}}2/{}^{\bar{1}}m^{\infty m}1$	$[E 4_{001}^+], [C_2 m_{001}], [C_2 \bar{3}_{111}^+], [E 2_{110}], [C_2 m_{110}], [E E], [E 2_{100}], [E 2_{010}], [E 2_{001}], [E 3_{111}^+], [E 3_{111}^-], [E 3_{1\bar{1}1}^+], [E 3_{1\bar{1}1}^-], [E 3_{11\bar{1}}^-], [E 3_{11\bar{1}}^+], [E 3_{1\bar{1}\bar{1}}^+], [E 3_{1\bar{1}\bar{1}}^-], [E 3_{11\bar{1}}^+], [E 3_{11\bar{1}}^-], [E 3_{1\bar{1}1}^+], [E 3_{1\bar{1}1}^-], [E 3_{111}^+], [E 3_{111}^-], [E 3_{1\bar{1}1}^+], [E 3_{1\bar{1}1}^-], [E 3_{11\bar{1}}^+], [E 3_{11\bar{1}}^-], [E 3_{1\bar{1}\bar{1}}^+], [E 3_{1\bar{1}\bar{1}}^-], [C_2 2_{\bar{1}10}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{0\bar{1}1}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{101}], [C_2 4_{010}], [E \bar{3}_{111}^+], [C_2 2_{\bar{1}01}], [C_2 2_{011}], [C_2 4_{100}], [C_2 4_{100}^-], [C_2 2_{101}], [C_2 4_{010}], [E \bar{3}_{111}^+]$

TABLE S9: 32 collinear spin point group \mathcal{G} : $\mathcal{G} = (\mathcal{G}_0 + [C_2 T]||E]\mathcal{G}_0) \ltimes SO(2)$ for FM. Only nontrivial operators in \mathcal{G}_0 are listed. The generators of \mathcal{G}_0 are marked in blue.

No.	Notation	Operators in \mathcal{G}_0 [$\mathcal{G} = (\mathcal{G}_0 + [C_2 T] E]\mathcal{G}_0) \ltimes SO(2)$]
1	$\infty^m 1$	$[E E]$
2	${}^1\bar{1}^{\infty^m} 1$	$[E P]$, $[E E]$
3	${}^12^{\infty^m} 1$	$[E \textcolor{blue}{2}_{010}]$, $[E E]$
4	${}^1m^{\infty^m} 1$	$[E m_{010}]$, $[E E]$
5	${}^12/{}^1m^{\infty^m} 1$	$[E \textcolor{blue}{2}_{010}]$, $[E P]$, $[E E]$, $[E m_{010}]$
6	${}^1m^1m^12^{\infty^m} 1$	$[E m_{100}]$, $[E \textcolor{blue}{2}_{001}]$, $[E E]$, $[E m_{010}]$
7	${}^12^12^12^{\infty^m} 1$	$[E \textcolor{blue}{2}_{010}]$, $[E \textcolor{blue}{2}_{100}]$, $[E E]$, $[E 2_{001}]$
8	${}^1m^1m^1m^{\infty^m} 1$	$[E P]$, $[E m_{010}]$, $[E \textcolor{blue}{m}_{100}]$, $[E E]$, $[E 2_{100}]$, $[E 2_{010}]$, $[E 2_{001}]$, $[E m_{001}]$
9	${}^14^{\infty^m} 1$	$[E 4_{001}]$, $[E E]$, $[E 2_{001}]$, $[E 4_{001}^-]$
10	${}^1\bar{4}^{\infty^m} 1$	$[E \bar{4}_{001}^+]$, $[E E]$, $[E 2_{001}]$, $[E \bar{4}_{001}^-]$
11	${}^14/{}^1m^{\infty^m} 1$	$[E 4_{001}^+]$, $[E P]$, $[E E]$, $[E 2_{001}]$, $[E 4_{001}^-]$, $[E m_{001}]$, $[E \bar{4}_{001}^+]$, $[E \bar{4}_{001}^-]$
12	${}^14^12^12^{\infty^m} 1$	$[E 4_{001}^+]$, $[E \textcolor{blue}{2}_{100}]$, $[E \textcolor{blue}{2}_{110}]$, $[E E]$, $[E 2_{001}]$, $[E 2_{1\bar{1}0}]$, $[E 2_{010}]$, $[E 4_{001}^-]$
13	${}^14^1m^1m^{\infty^m} 1$	$[E 4_{001}^+]$, $[E m_{100}]$, $[E m_{110}]$, $[E E]$, $[E 2_{001}]$, $[E 4_{001}^-]$, $[E m_{010}]$, $[E m_{1\bar{1}0}]$
14	${}^1\bar{4}^12^1m^{\infty^m} 1$	$[E \bar{4}_{001}^+]$, $[E \textcolor{blue}{2}_{100}]$, $[E \textcolor{blue}{m}_{110}]$, $[E E]$, $[E 2_{010}]$, $[E 2_{001}]$, $[E m_{1\bar{1}0}]$, $[E \bar{4}_{001}^-]$
15	${}^14/{}^1m^1m^1m^{\infty^m} 1$	$[E 4_{001}^+]$, $[E P]$, $[E m_{100}]$, $[E \textcolor{blue}{m}_{110}]$, $[E E]$, $[E 2_{100}]$, $[E 2_{010}]$, $[E 2_{001}]$, $[E 2_{1\bar{1}0}]$, $[E 4_{001}^-]$, $[E m_{001}]$, $[E m_{010}]$, $[E m_{1\bar{1}0}]$, $[E \bar{4}_{001}^+]$, $[E \bar{4}_{001}^-]$
16	${}^13^m 1$	$[E 3_{001}^+]$, $[E E]$, $[E 3_{001}^-]$
17	${}^1\bar{3}^{\infty^m} 1$	$[E \bar{3}_{001}^+]$, $[E E]$, $[E 3_{001}^+]$, $[E P]$, $[E 3_{001}^-]$, $[E \bar{3}_{001}^-]$
18	${}^13^12^{\infty^m} 1$	$[E 3_{001}^+]$, $[E \textcolor{blue}{2}_{100}]$, $[E E]$, $[E 3_{001}^-]$, $[E 2_{110}]$, $[E 2_{010}]$
19	${}^13^1m^{\infty^m} 1$	$[E 3_{001}^+]$, $[E \textcolor{blue}{m}_{100}]$, $[E E]$, $[E 3_{001}^-]$, $[E m_{110}]$, $[E m_{010}]$
20	${}^1\bar{3}^1m^{\infty^m} 1$	$[E \bar{3}_{001}^+]$, $[E \textcolor{blue}{m}_{100}]$, $[E E]$, $[E 3_{001}^+]$, $[E 3_{001}^-]$, $[E 2_{100}]$, $[E 2_{110}]$, $[E 2_{010}]$, $[E P]$, $[E \bar{3}_{001}^-]$, $[E m_{110}]$, $[E m_{010}]$
21	${}^1\bar{6}^{\infty^m} 1$	$[E \bar{6}_{001}^+]$, $[E E]$, $[E 3_{001}^+]$, $[E 3_{001}^-]$, $[E m_{001}]$, $[E \bar{6}_{001}^-]$
22	${}^16^{\infty^m} 1$	$[E 6_{001}^+]$, $[E E]$, $[E 3_{001}^+]$, $[E 2_{001}]$, $[E 3_{001}^-]$, $[E 6_{001}^-]$
23	${}^16^12^12^{\infty^m} 1$	$[E 6_{001}^+]$, $[E \textcolor{blue}{2}_{100}]$, $[E \textcolor{blue}{2}_{1\bar{1}0}]$, $[E E]$, $[E 3_{001}^+]$, $[E 2_{001}]$, $[E 3_{001}^-]$, $[E 6_{001}^-]$, $[E 2_{110}]$, $[E 2_{120}]$, $[E 2_{010}]$, $[E 2_{210}]$
24	${}^16/{}^1m^{\infty^m} 1$	$[E 6_{001}^+]$, $[E P]$, $[E E]$, $[E 3_{001}^+]$, $[E 2_{001}]$, $[E 3_{001}^-]$, $[E 6_{001}^-]$, $[E m_{001}]$, $[E \bar{6}_{001}^+]$, $[E \bar{3}_{001}^+]$, $[E \bar{3}_{001}^-]$, $[E \bar{6}_{001}^-]$
25	${}^16^1m^1m^{\infty^m} 1$	$[E 6_{001}^+]$, $[E \textcolor{blue}{m}_{100}]$, $[E \textcolor{blue}{m}_{1\bar{1}0}]$, $[E E]$, $[E 3_{001}^+]$, $[E 2_{001}]$, $[E 3_{001}^-]$, $[E 6_{001}^-]$, $[E m_{110}]$, $[E m_{010}]$, $[E m_{210}]$, $[E m_{120}]$
26	${}^1\bar{6}^1m^12^{\infty^m} 1$	$[E \bar{6}_{001}^+]$, $[E \textcolor{blue}{m}_{100}]$, $[E \textcolor{blue}{2}_{1\bar{1}0}]$, $[E E]$, $[E 3_{001}^+]$, $[E 2_{120}]$, $[E 3_{001}^-]$, $[E \bar{6}_{001}^-]$, $[E 2_{210}]$, $[E m_{001}]$, $[E m_{110}]$, $[E m_{010}]$
27	${}^16/{}^1m^1m^1m^{\infty^m} 1$	$[E 6_{001}^+]$, $[E P]$, $[E m_{100}]$, $[E \textcolor{blue}{m}_{1\bar{1}0}]$, $[E E]$, $[E 3_{001}^+]$, $[E 2_{001}]$, $[E 3_{001}^-]$, $[E 6_{001}^-]$, $[E 2_{100}]$, $[E 2_{120}]$, $[E 2_{110}]$, $[E 2_{210}]$, $[E 2_{010}]$, $[E 2_{1\bar{1}0}]$, $[E m_{001}]$, $[E \bar{6}_{001}^+]$, $[E \bar{3}_{001}^+]$, $[E \bar{3}_{001}^-]$, $[E \bar{6}_{001}^-]$, $[E \bar{3}_{001}^-]$, $[E \bar{6}_{001}^-]$, $[E m_{110}]$, $[E m_{120}]$, $[E m_{010}]$, $[E m_{210}]$

No.	Notation	Operators in \mathcal{G}_0 [$\mathcal{G} = (\mathcal{G}_0 + [C_2 T E] \mathcal{G}_0) \ltimes SO(2)$]
28	${}^12^13^m1$	$[E 2_{001}], [E 3_{111}^+], [E E], [E 2_{010}], [E 2_{100}], [E 3_{1\bar{1}\bar{1}}^+], [E 3_{1\bar{1}\bar{1}}^+], [E 3_{111}^+], [E 3_{1\bar{1}\bar{1}}^-], [E 3_{1\bar{1}\bar{1}}^-], [E 3_{111}^-]$
29	${}^12/{}^1m^1\bar{3}^m1$	$[E 2_{001}], [E m_{001}], [E 3_{111}^+], [E E], [E 2_{100}], [E 2_{010}], [E 3_{111}^-], [E 3_{111}^+], [E 3_{1\bar{1}\bar{1}}^+], [E 3_{1\bar{1}\bar{1}}^+], [E 3_{111}^-], [E 3_{1\bar{1}\bar{1}}^-], [E P], [E m_{100}], [E m_{010}], [E \bar{3}_{111}^-], [E \bar{3}_{111}^+], [E \bar{3}_{1\bar{1}\bar{1}}^+], [E \bar{3}_{1\bar{1}\bar{1}}^-], [E \bar{3}_{111}^-], [E \bar{3}_{1\bar{1}\bar{1}}^-]$
30	${}^14^13^1m^m1$	$[E \bar{4}_{001}^+], [E 3_{111}^+], [E m_{110}], [E E], [E 2_{100}], [E 2_{010}], [E 2_{001}], [E 3_{111}^-], [E 3_{111}^+], [E 3_{1\bar{1}\bar{1}}^+], [E 4_{010}], [E 3_{1\bar{1}\bar{1}}^-], [E 3_{111}^-], [E 3_{1\bar{1}\bar{1}}^-], [E m_{\bar{1}10}], [E m_{101}], [E \bar{4}_{001}^-], [E m_{0\bar{1}1}], [E m_{011}], [E \bar{4}_{100}^+], [E \bar{4}_{100}^-], [E m_{\bar{1}01}], [E \bar{4}_{010}]$
31	${}^14^13^12^m1$	$[E 4_{001}^+], [E 3_{111}^+], [E 2_{110}], [E E], [E 2_{100}], [E 2_{010}], [E 2_{001}], [E 3_{111}^-], [E 3_{111}^+], [E 3_{1\bar{1}\bar{1}}^+], [E 4_{010}^+], [E 4_{010}^-], [E 3_{1\bar{1}\bar{1}}^-], [E 3_{111}^-], [E 3_{1\bar{1}\bar{1}}^-], [E 2_{\bar{1}10}], [E 4_{001}^+], [E 4_{001}^-], [E 2_{\bar{1}01}], [E 2_{011}], [E 4_{100}^+], [E 4_{100}^-], [E 2_{101}], [E 4_{010}]$
32	${}^14/{}^1m^1\bar{3}^12/{}^1m^m1$	$[E 4_{001}^+], [E m_{001}], [E \bar{3}_{111}^+], [E 2_{110}], [E m_{110}], [E E], [E 2_{100}], [E 2_{010}], [E 2_{001}], [E 3_{111}^-], [E 3_{111}^+], [E 3_{1\bar{1}\bar{1}}^+], [E 3_{1\bar{1}\bar{1}}^-], [E 3_{111}^-], [E 3_{1\bar{1}\bar{1}}^-], [E P], [E m_{100}], [E m_{010}], [E \bar{3}_{111}^-], [E \bar{3}_{111}^+], [E 3_{1\bar{1}\bar{1}}^+], [E 3_{1\bar{1}\bar{1}}^-], [E \bar{3}_{111}^-], [E \bar{3}_{1\bar{1}\bar{1}}^+], [E \bar{3}_{1\bar{1}\bar{1}}^-], [E 4_{010}^+], [E 4_{010}^-], [E 2_{\bar{1}10}], [E 4_{001}^-], [E 2_{011}], [E 4_{100}^+], [E 4_{100}^-], [E 2_{101}], [E 4_{010}^+], [E m_{\bar{1}10}], [E \bar{4}_{001}^+], [E \bar{4}_{001}^-], [E m_{0\bar{1}1}], [E m_{011}], [E \bar{4}_{100}^+], [E \bar{4}_{100}^-], [E m_{\bar{1}01}], [E \bar{4}_{010}^-]$

VII. OPERATORS OF 125 2D MAGNETIC LAYER-POINT GROUPS

TABLE S10: 125 magnetic layer-point groups. Shubnikov (Litvin) notation is presented in second column. The generators for each group are marked in blue. The groups allowing for mirror-AFE are highlighted by red. The superscript ('') in third column indicates the presence of T symmetry. For example, E' denotes the operator of ET .

No.	Notation	Operators
1	1.1 (1.1.1)	E
2	1.1' (1.2.2)	E' , E
3	$\bar{1}$.1 (2.1.3)	P , E
4	$\bar{1}$.1' (2.2.4)	P , E' , E , P'
5	$\bar{1}'$ (2.3.5)	P' , E
6	2.1 (3.1.6)	2_{001} , E
7	2.1 (3.1.6)	2_{100} , E
8	2.1' (3.2.7)	2_{001} , E' , E , $2'_{001}$
9	2.1' (3.2.7)	2_{100} , E' , E , $2'_{100}$
10	2' (3.3.8)	$2'_{001}$, E
11	2' (3.3.8)	$2'_{100}$, E
12	m .1 (4.1.9)	m_{001} , E
13	m .1 (4.1.9)	m_{100} , E
14	m .1' (4.2.10)	m_{001} , E' , E , m'_{001}
15	m .1' (4.2.10)	m_{100} , E' , E , m'_{100}
16	m' (4.3.11)	m'_{001} , E
17	m' (4.3.11)	m'_{100} , E
18	2/ m .1 (5.1.12)	2_{001} , P , E , m_{001}
19	2/ m .1 (5.1.12)	2_{100} , P , E , m_{100}
20	2/ m .1' (5.2.13)	2_{001} , P , E' , E , m_{001} , $2'_{001}$, P' , m'_{001}
21	2/ m .1' (5.2.13)	2_{100} , P , E' , E , m_{100} , $2'_{100}$, P' , m'_{100}
22	$2'/m$ (5.3.14)	m_{001} , $2'_{001}$, E , P'
23	$2'/m$ (5.3.14)	m_{100} , $2'_{100}$, E , P'
24	2/ m' (5.4.15)	2_{001} , P' , E , m'_{001}
25	2/ m' (5.4.15)	2_{100} , P' , E , m'_{100}
26	2' $/m'$ (5.5.16)	P , $2'_{001}$, E , m'_{001}
27	2' $/m'$ (5.5.16)	P , $2'_{100}$, E , m'_{100}
28	222.1 (6.1.17)	2_{100} , 2_{010} , E , 2_{001}
29	222.1' (6.2.18)	2_{100} , 2_{010} , E' , E , 2_{001} , $2'_{100}$, $2'_{010}$, $2'_{001}$
30	2'2'2 (6.3.19)	2_{001} , $2'_{100}$, E , $2'_{010}$
31	2'2'2 (6.3.19)	2_{010} , $2'_{010}$, E , $2'_{001}$
32	mm 2.1 (7.1.20)	2_{001} , m_{100} , E , m_{010}
33	mm 2.1 (7.1.20)	2_{010} , m_{100} , E , m_{001}
34	mm 2.1' (7.2.21)	2_{001} , m_{100} , E' , E , m_{010} , $2'_{001}$, m'_{100} , m'_{010}
35	mm 2.1' (7.2.21)	2_{010} , m_{100} , E' , E , m_{001} , $2'_{010}$, m'_{100} , m'_{001}

No.	Notation	Operators
36	$m'm2' (7.3.22)$	$m_{010}, 2'_{001}, E, m'_{100}$
37	$m'm2' (7.3.22)$	$m_{001}, 2'_{010}, E, m'_{100}$
38	$m'm2' (7.3.22)$	$m_{100}, 2'_{010}, E, m'_{001}$
39	$m'm'2 (7.4.23)$	$2_{001}, m'_{100}, E, m'_{010}$
40	$m'm'2 (7.4.23)$	$2_{010}, m'_{100}, E, m'_{001}$
41	$mmm.1 (8.1.24)$	$2_{100}, 2_{010}, P, E, 2_{001}, m_{100}, m_{010}, m_{001}$
42	$mmm.1' (8.2.25)$	$2_{100}, 2_{010}, P, E', E, 2_{001}, m_{100}, m_{010}, m_{001}, 2'_{100}, 2'_{010}, 2'_{001}, P', m'_{100}, m'_{010}, m'_{001}$
43	$m'mm (8.3.26)$	$2_{100}, m_{010}, P', E, m_{001}, 2'_{010}, 2'_{001}, m'_{100}$
44	$m'mm (8.3.26)$	$2_{001}, m_{010}, P', E, m_{100}, 2'_{010}, 2'_{100}, m'_{001}$
45	$m'm'm (8.4.27)$	$2_{001}, P, 2'_{100}, E, m_{001}, 2'_{010}, m'_{100}, m'_{010}$
46	$m'm'm (8.4.27)$	$2_{100}, P, 2'_{010}, E, m_{100}, 2'_{001}, m'_{010}, m'_{001}$
47	$m'm'm' (8.5.28)$	$2_{100}, 2_{010}, P', E, 2_{001}, m'_{100}, m'_{010}, m'_{001}$
48	$4.1 (9.1.29)$	$4'_{001}, E, 2_{001}, 4'_{001}$
49	$4.1' (9.2.30)$	$4'_{001}, E', E, 2_{001}, 4'_{001}, 2'_{001}, 4'_{001}, 4'_{001}$
50	$4' (9.3.31)$	$4'_{001}, E, 2_{001}, 4'_{001}$
51	$\bar{4}.1 (10.1.32)$	$\bar{4}_{001}, E, 2_{001}, \bar{4}_{001}$
52	$\bar{4}.1' (10.2.33)$	$\bar{4}'_{001}, E, 2_{001}, \bar{4}'_{001}, \bar{4}'_{001}, \bar{4}'_{001}, \bar{4}'_{001}$
53	$\bar{4}' (10.3.34)$	$\bar{4}'_{001}, E, 2_{001}, \bar{4}'_{001}$
54	$4/m.1 (11.1.35)$	$4'_{001}, P, E, 2_{001}, 4'_{001}, m_{001}, \bar{4}'_{001}, \bar{4}'_{001}$
55	$4/m.1' (11.2.36)$	$4'_{001}, P, E', E, 2_{001}, 4'_{001}, m_{001}, \bar{4}'_{001}, \bar{4}'_{001}, 4'_{001}, P', 2'_{001}, 4'_{001}, m'_{001}, \bar{4}'_{001}, \bar{4}'_{001}$
56	$4'/m (11.3.37)$	$4'_{001}, P, E, 2_{001}, m_{001}, 4'_{001}, \bar{4}'_{001}, \bar{4}'_{001}$
57	$4/m' (11.4.38)$	$4'_{001}, P', E, 2_{001}, 4'_{001}, m'_{001}, \bar{4}'_{001}, \bar{4}'_{001}$
58	$4'/m' (11.5.39)$	$4'_{001}, m'_{001}, E, 2_{001}, 4'_{001}, \bar{4}'_{001}, 4'_{001}, P'$
59	$422.1 (12.1.40)$	$4'_{001}, 2_{100}, 2_{110}, E, 2_{001}, 2_{\bar{1}10}, 4'_{001}, 2_{010}$
60	$422.1' (12.2.41)$	$4'_{001}, 2_{100}, 2_{110}, E', E, 2_{001}, 2_{\bar{1}10}, 4'_{001}, 2_{010}, 2'_{100}, 2'_{010}, 2'_{001}, 2'_{\bar{1}10}, 4'_{001}, 4'_{001}, 2'_{110}$
61	$4'22' (12.3.42)$	$4'_{001}, 2_{100}, 2'_{110}, E, 2_{010}, 2_{001}, 2'_{\bar{1}10}, 4'_{001}$
62	$42'2' (12.4.43)$	$4'_{001}, 2'_{100}, 2'_{110}, E, 2'_{010}, 2_{001}, 2'_{\bar{1}10}, 4'_{001}$
63	$4mm.1 (13.1.44)$	$4'_{001}, m_{100}, m_{110}, E, 2_{001}, 4'_{001}, m_{010}, m_{\bar{1}10}$
64	$4mm.1' (13.2.45)$	$4'_{001}, m_{100}, m_{110}, E', E, 2_{001}, 4'_{001}, m_{010}, m_{\bar{1}10}, 4'_{001}, m'_{100}, m'_{110}, 2'_{001}, 4'_{001}, m'_{010}, m'_{\bar{1}10}$
65	$4'm'm (13.3.46)$	$4'_{001}, m'_{100}, m_{110}, E, 2_{001}, 4'_{001}, m'_{010}, m_{\bar{1}10}$
66	$4m'm' (13.4.47)$	$4'_{001}, m'_{100}, m'_{110}, E, 2_{001}, 4'_{001}, m'_{010}, m'_{\bar{1}10}$
67	$\bar{4}2m.1 (14.1.48)$	$\bar{4}'_{001}, 2_{100}, m_{110}, E, 2_{010}, 2_{001}, m_{\bar{1}10}, \bar{4}'_{001}$
68	$\bar{4}2m.1' (14.2.49)$	$\bar{4}'_{001}, 2_{100}, m_{110}, E', E, 2_{010}, 2_{001}, m_{\bar{1}10}, \bar{4}'_{001}, \bar{4}'_{001}, 2'_{100}, m'_{110}, 2'_{010}, 2'_{001}, m'_{\bar{1}10}, \bar{4}'_{001}$
69	$\bar{4}'2'm (14.3.50)$	$\bar{4}'_{001}, 2'_{100}, m_{110}, E, 2'_{010}, 2_{001}, m_{\bar{1}10}, \bar{4}'_{001}$
70	$\bar{4}'2m'.1 (14.4.51)$	$\bar{4}'_{001}, 2_{100}, m_{110}, E, 2_{010}, 2_{001}, m_{\bar{1}10}, \bar{4}'_{001}$
71	$\bar{4}2'm' (14.5.52)$	$\bar{4}'_{001}, 2'_{100}, m'_{110}, E, 2'_{010}, 2_{001}, m'_{\bar{1}10}, \bar{4}'_{001}$
72	$4/mmm.1 (15.1.53)$	$4'_{001}, m_{001}, m_{100}, m_{110}, E, 2_{100}, 2_{010}, 2_{001}, 2_{\bar{1}10}, 4'_{001}, 2_{110}, P, m_{010}, m_{\bar{1}10}, \bar{4}'_{001}, \bar{4}'_{001}$
73	$4/mmm.1' (15.2.54)$	$4'_{001}, m_{001}, m_{100}, m_{110}, E', E, 2_{100}, 2_{010}, 2_{001}, 2_{\bar{1}10}, 4'_{001}, 2_{110}, P, m_{010}, m_{\bar{1}10}, \bar{4}'_{001}, \bar{4}'_{001}, 4'_{001}, m'_{001}, m'_{100}, m'_{110}, 2'_{010}, 2'_{001}, 2'_{\bar{1}10}, 4'_{001}, 2'_{110}, P', m'_{010}, m'_{\bar{1}10}, \bar{4}'_{001}, \bar{4}'_{001}$

No.	Notation	Operators
74	$4/m'mm$ (15.3.55)	$4_{001}^+, m'_{001}, m_{100}, m_{110}, E, 2'_{100}, 2'_{010}, 2'_{001}, 2'_{\bar{1}10}, 4_{001}^-, 2'_{110}, P', m_{010}, m_{\bar{1}10}, \bar{4}_{001}^{+/-}, \bar{4}_{001}^{-/-}$
75	$4'/mm'm$ (15.4.56)	$4_{001}^{+/-}, m_{001}, m'_{100}, m_{110}, E, 2'_{100}, 2'_{010}, 2'_{001}, 2'_{\bar{1}10}, 4_{001}^{-/-}, 2'_{110}, P, m'_{010}, m_{\bar{1}10}, \bar{4}_{001}^{+/-}, \bar{4}_{001}^{-/-}$
76	$4'/m'm'm$ (15.5.57)	$4_{001}^{+/-}, m'_{001}, m'_{100}, m_{110}, E, 2_{100}, 2_{010}, 2_{001}, 2'_{\bar{1}10}, 4_{001}^-, 2'_{110}, P', m'_{010}, m_{\bar{1}10}, \bar{4}_{001}^+, \bar{4}_{001}^-$
77	$4/mm'm'$ (15.6.58)	$4_{001}^+, m_{001}, m'_{100}, m'_{110}, E, 2'_{100}, 2'_{010}, 2_{001}, 2'_{\bar{1}10}, 4_{001}^-, 2'_{110}, P, m'_{010}, m'_{\bar{1}10}, \bar{4}_{001}^+, \bar{4}_{001}^-$
78	$4/m'm'm'$ (15.7.59)	$4_{001}^+, m'_{001}, m'_{100}, m'_{110}, E, 2_{100}, 2_{010}, 2_{001}, 2'_{\bar{1}10}, 4_{001}^-, 2'_{110}, P', m'_{010}, m'_{\bar{1}10}, \bar{4}_{001}^{+/-}, \bar{4}_{001}^{-/-}$
79	3.1 (16.1.60)	$3_{001}^+, E, 3_{001}^-$
80	$3.1'$ (16.2.61)	$3_{001}^+, E', E, 3_{001}^-, 3_{001}^{+/-}, 3_{001}^{-/-}$
81	$\bar{3}.1$ (17.1.62)	$3_{001}^+, P, E, 3_{001}^-, \bar{3}_{001}, \bar{3}_{001}^{-1}$
82	$\bar{3}.1'$ (17.2.63)	$3_{001}^+, P, E', E, 3_{001}^-, \bar{3}_{001}, \bar{3}_{001}^{-1}, 3_{001}^{+/-}, P', 3_{001}^{-/-}, \bar{3}_{001}^{+/-}, \bar{3}_{001}^{-/-}$
83	$\bar{3}'$ (17.3.64)	$\bar{3}_{001}^{+/-}, E, 3_{001}^+, 3_{001}^-, \bar{3}_{001}^{-/-}, P'$
84	32.1 (18.1.65)	$3_{001}^+, 2_{100}, E, 3_{001}^-, 2_{110}, 2_{010}$
85	$32.1'$ (18.2.66)	$3_{001}^+, 2_{100}, E', E, 3_{001}^-, 2_{110}, 2_{010}, 3_{001}^{+/-}, 2'_{100}, 3_{001}^{-/-}, 2'_{110}, 2'_{010}$
86	$32'$ (18.3.67)	$3_{001}^+, 2'_{100}, E, 3_{001}^-, 2'_{110}, 2'_{010}$
87	$3m.1$ (19.1.68)	$3_{001}^+, m_{100}, E, 3_{001}^-, m_{110}, m_{010}$
88	$3m.1'$ (19.2.69)	$3_{001}^+, m_{100}, E', E, 3_{001}^-, m_{110}, m_{010}, 3_{001}^{+/-}, m'_{100}, 3_{001}^{-/-}, m'_{110}, m'_{010}$
89	$3m'$ (19.3.70)	$3_{001}^+, m'_{100}, E, 3_{001}^-, m'_{110}, m'_{010}$
90	$\bar{3}m.1$ (20.1.71)	$\bar{3}_{001}, m_{100}, E, 3_{001}^+, 3_{001}^-, 2_{100}, 2_{110}, 2_{010}, P, \bar{3}_{001}^-, m_{110}, m_{010}$
91	$\bar{3}m.1'$ (20.2.72)	$\bar{3}_{001}, m_{100}, E', E, 3_{001}^+, 3_{001}^-, 2_{100}, 2_{110}, 2_{010}, P, \bar{3}_{001}^-, m_{110}, m_{010}, \bar{3}_{001}^{+/-}, m'_{100}, 3_{001}^{+/-}, 3_{001}^{-/-}, 2'_{100}, 2'_{110}, P', \bar{3}_{001}^{-/-}, m'_{110}, m'_{010}$
92	$\bar{3}'m$ (20.3.73)	$\bar{3}_{001}^{+/-}, m_{100}, E, 3_{001}^+, 3_{001}^-, 2'_{100}, 2'_{110}, 2'_{010}, P', \bar{3}_{001}^{-/-}, m_{110}, m_{010}$
93	$\bar{3}'m'$ (20.4.74)	$\bar{3}_{001}^{+/-}, m'_{100}, E, 3_{001}^+, 3_{001}^-, 2_{100}, 2_{110}, 2_{010}, P', \bar{3}_{001}^{-/-}, m'_{110}, m'_{010}$
94	$\bar{3}m'$ (20.5.75)	$\bar{3}_{001}, m'_{100}, E, 3_{001}^+, 3_{001}^-, 2'_{100}, 2'_{110}, 2_{010}, P, \bar{3}_{001}^{-1}, m'_{110}, m'_{010}$
95	6.1 (21.1.76)	$6_{001}^+, E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-$
96	$6.1'$ (21.2.77)	$6_{001}^+, E', E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, 6_{001}^{+/-}, 3_{001}^{+/-}, 2'_{001}, 3_{001}^{-/-}, 6_{001}^{-/-}$
97	$6'$ (21.3.78)	$6_{001}^{+/-}, E, 3_{001}^+, 2'_{001}, 3_{001}^-, 6_{001}^-$
98	$\bar{6}.1$ (22.1.79)	$\bar{6}_{001}^+, E, 3_{001}^+, 3_{001}^-, m_{001}, \bar{6}_{001}^-$
99	$\bar{6}.1'$ (22.2.80)	$\bar{6}_{001}^+, E', E, 3_{001}^+, 3_{001}^-, m_{001}, \bar{6}_{001}^-, \bar{6}_{001}^{+/-}, 3_{001}^{+/-}, 3_{001}^{-/-}, m'_{001}, \bar{6}_{001}^{-/-}$
100	$\bar{6}'$ (22.3.81)	$\bar{6}_{001}^{+/-}, E, 3_{001}^+, 3_{001}^-, m'_{001}, \bar{6}_{001}^{-/-}$
101	$6/m.1$ (23.1.82)	$6_{001}^+, m_{001}, E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, P, \bar{6}_{001}^+, \bar{3}_{001}^+, \bar{3}_{001}^-, \bar{6}_{001}^-$
102	$6/m.1'$ (23.2.83)	$6_{001}^+, m_{001}, E', E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, P, \bar{6}_{001}^+, \bar{3}_{001}^-, \bar{6}_{001}^-, 6_{001}^{+/-}, m'_{001}, 3_{001}^{+/-}, 2'_{001}, 3_{001}^{-/-}, 6_{001}^-$
103	$6'/m$ (23.3.84)	$6_{001}^{+/-}, m_{001}, E, 3_{001}^+, 2'_{001}, 3_{001}^-, 6_{001}^-, P', \bar{6}_{001}^+, \bar{3}_{001}^{+/-}, \bar{3}_{001}^{-/-}, \bar{6}_{001}^-$
104	$6/m'$ (23.4.85)	$6_{001}^+, m'_{001}, E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, P', \bar{6}_{001}^+, \bar{3}_{001}^{+/-}, \bar{3}_{001}^{-/-}, \bar{6}_{001}^-$
105	$6'/m'$ (23.5.86)	$6_{001}^{+/-}, m'_{001}, E, 3_{001}^+, 2'_{001}, 3_{001}^-, 6_{001}^-, P, \bar{6}_{001}^+, \bar{3}_{001}^+, \bar{3}_{001}^-, \bar{6}_{001}^-$

No.	Notation	Operators
106	622.1 (24.1.87)	$6_{001}^+, \mathbf{2}_{100}, \mathbf{2}_{210}, E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, 2_{110}, 2_{120}, 2_{010}, 2_{1\bar{1}0}$
107	622.1' (24.2.88)	$6_{001}^+, \mathbf{2}_{100}, \mathbf{2}_{210}, \mathbf{E}', E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, 2_{110}, 2_{120}, 2_{010}, 2_{1\bar{1}0}, 6_{001}^{+'}, 2'_{100}, 2'_{210}, 3_{001}^{+'}, 2'_{001}, 3_{001}^-, 6_{001}^-, 2'_{110}, 2'_{120}, 2'_{010}, 2'_{1\bar{1}0}$
108	6'22' (24.3.89)	$6_{001}^{+'}, \mathbf{2}_{100}, \mathbf{2}_{210}, E, 3_{001}^+, 2'_{001}, 3_{001}^-, 6_{001}^-, 2_{110}, 2'_{120}, 2_{010}, 2'_{1\bar{1}0}$
109	62'2' (24.4.90)	$6_{001}^+, \mathbf{2}'_{100}, \mathbf{2}'_{210}, E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, 2'_{110}, 2'_{120}, 2'_{010}, 2'_{1\bar{1}0}$
110	6mm.1 (25.1.91)	$6_{001}^+, m_{100}, \mathbf{m}_{210}, E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, m_{110}, m_{120}, m_{010}, m_{1\bar{1}0}$
111	6mm.1' (25.2.92)	$6_{001}^+, \mathbf{m}_{100}, \mathbf{m}_{210}, \mathbf{E}', E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, m_{110}, m_{120}, m_{010}, m_{1\bar{1}0}, 6_{001}^{+'}, m'_{100}, m'_{210}, 3_{001}^{+'}, 2'_{001}, 3_{001}^-, 6_{001}^-, m'_{110}, m'_{120}, m'_{010}, m'_{1\bar{1}0}$
112	6'mm' (25.3.93)	$6_{001}^{+'}, m_{100}, \mathbf{m}'_{210}, E, 3_{001}^+, 2'_{001}, 3_{001}^-, 6_{001}^-, m_{110}, m'_{120}, m_{010}, m'_{1\bar{1}0}$
113	6m'm' (25.4.94)	$6_{001}^+, m'_{100}, \mathbf{m}'_{210}, E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, m'_{110}, m'_{120}, m'_{010}, m'_{1\bar{1}0}$
114	$\bar{6}m2.1$ (26.1.95)	$\bar{6}_{001}^+, m_{100}, \mathbf{2}_{210}, E, 3_{001}^+, 3_{001}^-, 2_{120}, 2_{1\bar{1}0}, m_{001}, \bar{6}_{001}^-, m_{110}, m_{010}$
115	$\bar{6}m2.1'$ (26.2.96)	$\bar{6}_{001}^+, \mathbf{m}_{100}, \mathbf{2}_{210}, \mathbf{E}', E, 3_{001}^+, 3_{001}^-, 2_{120}, 2_{1\bar{1}0}, m_{001}, \bar{6}_{001}^-, m'_{100}, 2'_{210}, 3_{001}^{+'}, 3_{001}^-, 2'_{120}, 2'_{1\bar{1}0}, m'_{001}, m'_{110}, m'_{010}, m'_{1\bar{1}0}, \bar{6}_{001}^-, \bar{6}_{001}^{'}, m'_{110}, m'_{010}, m'_{1\bar{1}0}$
116	$\bar{6}'m'2$ (26.3.97)	$\bar{6}_{001}^{+'}, m'_{100}, \mathbf{2}_{210}, E, 3_{001}^+, 3_{001}^-, 2_{120}, 2_{1\bar{1}0}, m'_{001}, \bar{6}_{001}^{'}, m'_{110}, m'_{010}$
117	$\bar{6}'m2'$ (26.4.98)	$\bar{6}_{001}^{+'}, m_{100}, \mathbf{2}'_{210}, E, 3_{001}^+, 3_{001}^-, 2'_{120}, 2'_{1\bar{1}0}, m'_{001}, \bar{6}_{001}^{'}, m_{110}, m_{010}$
118	$\bar{6}m'2'$ (26.5.99)	$\bar{6}_{001}^+, m'_{100}, \mathbf{2}'_{210}, E, 3_{001}^+, 3_{001}^-, 2'_{120}, 2'_{1\bar{1}0}, m_{001}, \bar{6}_{001}^{'}, m'_{110}, m'_{010}$
119	6/mmm.1 (27.1.100)	$6_{001}^+, \mathbf{m}_{001}, \mathbf{m}_{100}, \mathbf{m}_{210}, E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, 2_{100}, 2_{210}, 2_{110}, 2_{120}, 2_{010}, 2_{1\bar{1}0}, P, \bar{6}_{001}^+, \bar{3}_{001}^+, \bar{3}_{001}^-, \bar{6}_{001}^-, m_{110}, m_{120}, m_{010}, m_{1\bar{1}0}, m_{120}, m_{010}, m_{1\bar{1}0}$
120	6/mmm.1' (27.2.101)	$6_{001}^+, \mathbf{m}_{001}, \mathbf{m}_{100}, \mathbf{m}_{210}, \mathbf{E}', E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, 2_{100}, 2_{210}, 2_{110}, 2_{120}, 2_{010}, 2_{1\bar{1}0}, P, \bar{6}_{001}^+, 6_{001}^+, m'_{001}, m'_{100}, m'_{210}, 3_{001}^{+'}, 2'_{001}, 3_{001}^-, 6_{001}^-, 2'_{100}, 2'_{210}, 2'_{110}, 2'_{120}, 2'_{010}, 2'_{1\bar{1}0}, P', \bar{6}_{001}^{'}, \bar{3}_{001}^+, \bar{3}_{001}^{'}, \bar{6}_{001}^{'}, \bar{3}_{001}^+, \bar{3}_{001}^{'}, \bar{6}_{001}^{'}, m_{110}, m_{120}, m_{010}, m_{1\bar{1}0}, m_{120}, m_{010}, m_{1\bar{1}0}$
121	6/ $m'mm$ (27.3.102)	$6_{001}^+, \mathbf{m}'_{001}, \mathbf{m}_{100}, \mathbf{m}_{210}, E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, 2'_{100}, 2'_{210}, 2'_{110}, 2'_{120}, 2'_{010}, 2'_{1\bar{1}0}, P', \bar{6}_{001}^{'}, \bar{3}_{001}^+, \bar{3}_{001}^{'}, \bar{6}_{001}^{'}, m_{110}, m_{120}, m_{010}, m_{1\bar{1}0}$
122	6'/mmm' (27.4.103)	$6_{001}^+, \mathbf{m}_{001}, \mathbf{m}_{100}, \mathbf{m}'_{210}, E, 3_{001}^+, 2'_{001}, 3_{001}^-, 6_{001}^-, 2'_{100}, 2_{210}, 2'_{110}, 2_{120}, 2'_{010}, 2_{1\bar{1}0}, P', \bar{6}_{001}^+, \bar{3}_{001}^+, \bar{3}_{001}^{'}, \bar{6}_{001}^{'}, m_{110}, m'_{120}, m_{010}, m'_{1\bar{1}0}$
123	6'/ $m'mm'$ (27.5.104)	$6_{001}^{+'}, \mathbf{m}'_{001}, \mathbf{m}_{100}, \mathbf{m}'_{210}, E, 3_{001}^+, 2'_{001}, 3_{001}^-, 6_{001}^-, 2_{100}, 2'_{210}, 2_{110}, 2'_{120}, 2_{010}, 2'_{1\bar{1}0}, P, \bar{6}_{001}^{'}, \bar{3}_{001}^+, \bar{3}_{001}^{'}, \bar{6}_{001}^{'}, m_{110}, m'_{120}, m_{010}, m'_{1\bar{1}0}$
124	6/ $mm'm'$ (27.6.105)	$6_{001}^+, \mathbf{m}_{001}, \mathbf{m}'_{100}, \mathbf{m}'_{210}, E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, 2'_{100}, 2'_{210}, 2'_{110}, 2'_{120}, 2'_{010}, 2'_{1\bar{1}0}, P, \bar{6}_{001}^{'}, \bar{3}_{001}^+, \bar{3}_{001}^{'}, \bar{6}_{001}^{'}, m'_{110}, m'_{120}, m'_{010}, m'_{1\bar{1}0}$
125	6/ $m'm'm'$ (27.7.106)	$6_{001}^+, \mathbf{m}'_{001}, \mathbf{m}'_{100}, \mathbf{m}'_{210}, E, 3_{001}^+, 2_{001}, 3_{001}^-, 6_{001}^-, 2_{100}, 2_{210}, 2_{110}, 2_{120}, 2_{010}, 2_{1\bar{1}0}, P', \bar{6}_{001}^{'}, \bar{3}_{001}^+, \bar{3}_{001}^{'}, \bar{6}_{001}^{'}, m'_{110}, m'_{120}, m'_{010}, m'_{1\bar{1}0}$

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