Accelerated Dynamics in HMC Simulations of Lattice Field Theory

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Introduction

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- What results have we got?
  - Successfully reproduced harmonic and enharmonic oscillator properties.

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- How are we doing it?
  - Using MCMC (Markov chain Monte Carlo) methods.
- What results have we got?
  - Successfully reproduced harmonic and enharmonic oscillator properties.
- Why are we doing it?
  - Can be used for calculations in lattice field theory.

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Table 1: Expectation Values for quantum harmonic oscillator with $\mu^2 = 1$, lattice spacing $= 1$, lattice size $= 1000$

Results - Harmonic Oscillator Potential

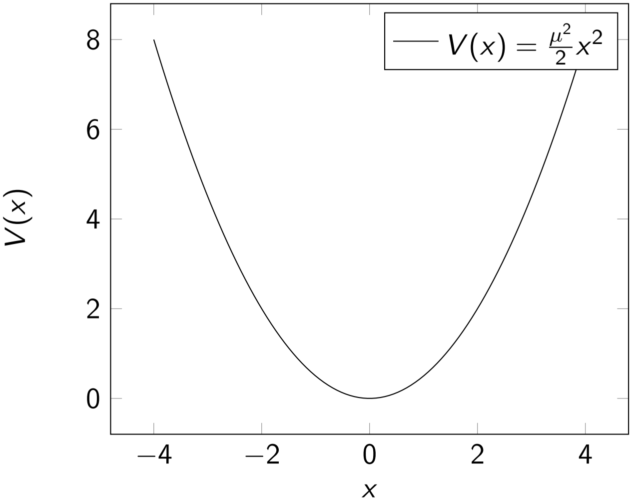

Figure 1: Harmonic Oscillator Potential with $\mu^2 = 1$.

Results - Harmonic Oscillator Wave Function

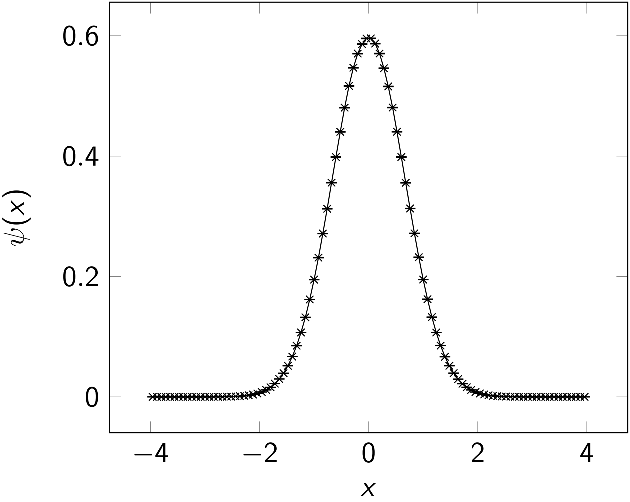

Figure 2: Continuum, discrete and measured wave functions for the harmonic oscillator with $\mu^2 = 1, m = 1, a = 1, L = 1000, d = 0.1, N = 10$, configurations $= 100000$, burn period $= 1000$.

Results - Harmonic Oscillator Typical Trajectory

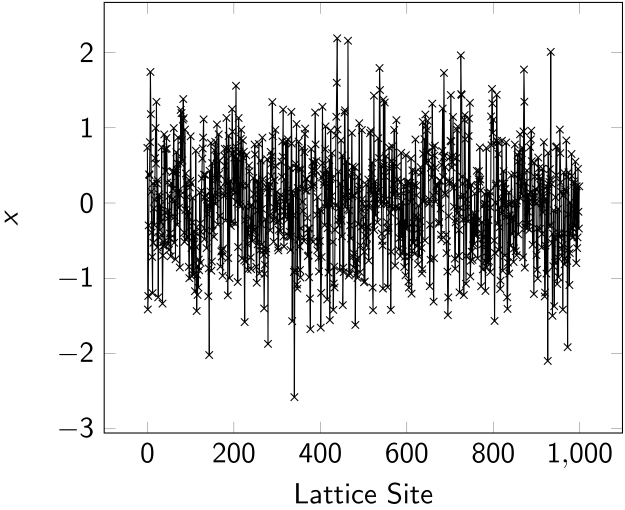

Figure 3: Typical configuration for the harmonic oscillator with $\mu^2 = 1, m = 1, a = 1, L = 1000, d = 0.1, N = 10$, configurations $= 100000$, burn period $= 1000$.

Results - Anharmonic Oscillator Potential

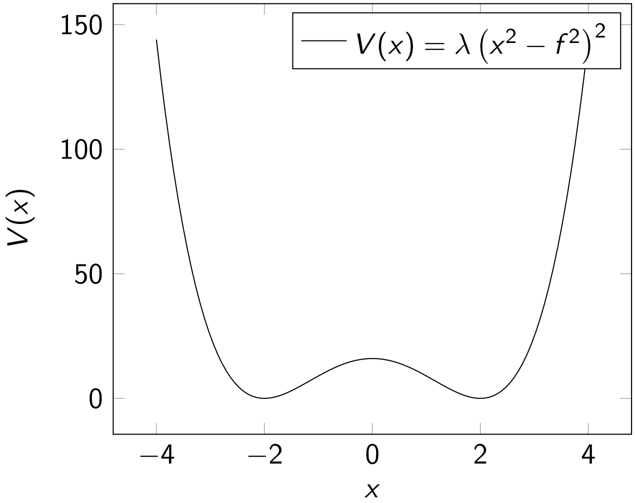

Figure 4: Harmonic Oscillator Potential with $\mu^2 = 1$.

Results - Anharmonic Oscillator Expectation Values

Value	Measured	Reference Values
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Table 2: Expectation Values for quantum anharmonic oscillator with $\mu^2 = 1$, lattice spacing $= 1$, lattice size $= 1000$

Results - Anharmonic Oscillator Wave Function

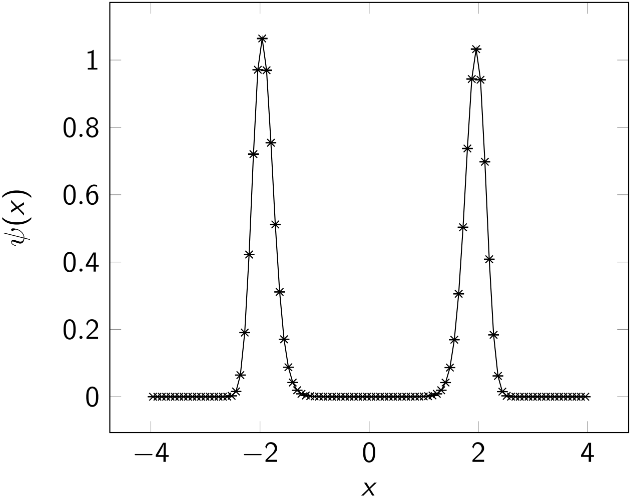

Figure 5: Measured wave function for the harmonic oscillator with $\lambda = 1, f^2 = 4 m = 1, a = 1, L = 1000, d = 0.01, N = 100, \text{configurations} = 100000, \text{burn period} = 1000.$

Results - Anharmonic Oscillator Typical Trajectory

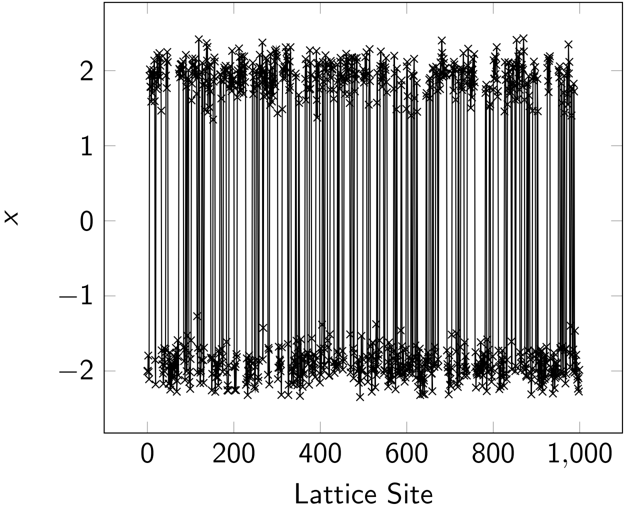

Figure 6: Typical configuration for the anharmonic oscillator with $\lambda = 1, f^2 = 4, m = 1, a = 1, L = 1000, d = 0.01, N = 100,$ configurations $= 100000,$ burn period $= 1000.$

Results - A Deeper Anharmonic Oscillator Potential

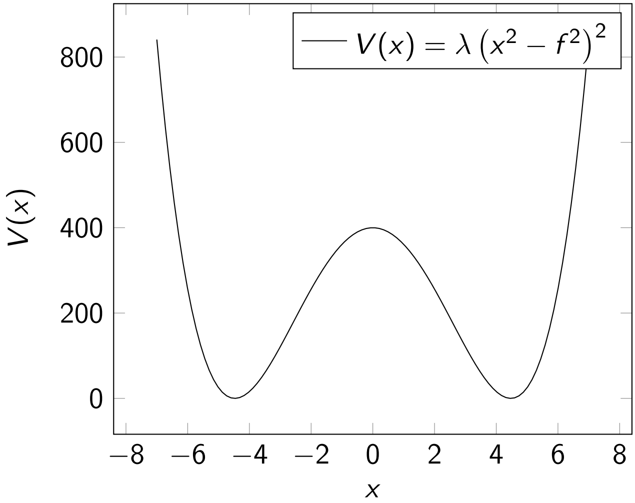

Figure 7: Anharmonic Oscillator Potential with $\lambda = 1$, $f^2 = 20$

Results - A Deeper Anharmonic Oscillator Wave Function

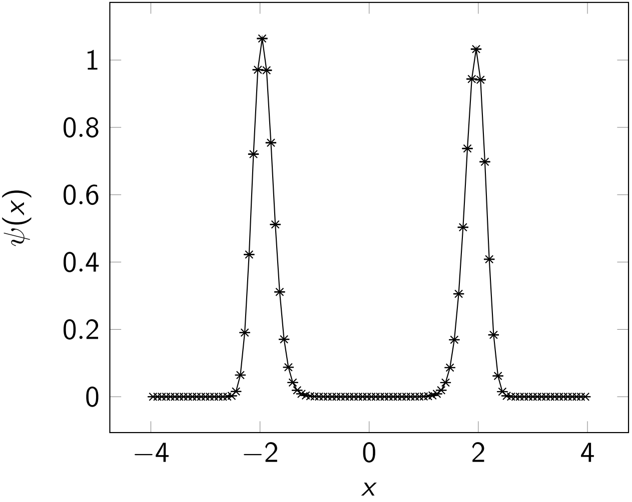

Figure 8: Measured wave function for the harmonic oscillator with $\lambda = 1, f^2 = 4m = 1, a = 1, L = 1000, d = 0.01, N = 100, \text{configurations} = 100000, \text{burn period} = 1000.$

Results - A Deeper Anharmonic Oscillator Typical Trajectory

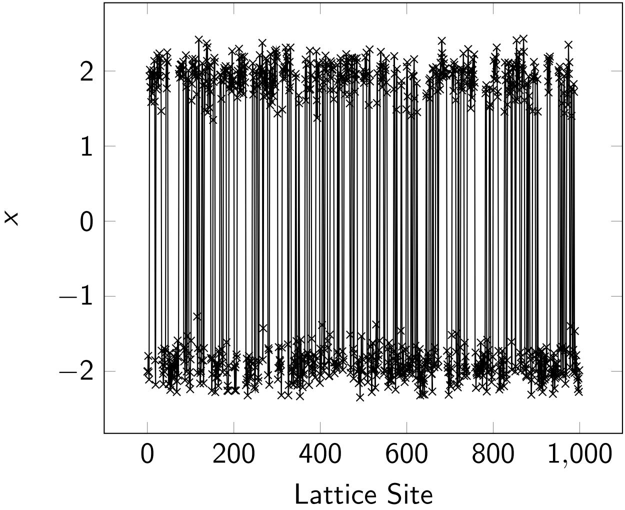

Figure 9: Typical configuration for the anharmonic oscillator with $\lambda = 1, f^2 = 4, m = 1, a = 1, L = 1000, d = 0.01, N =$

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  - Introduce “tempering” into the dynamics to sample from isolated modes.
- Applications of tempering?
  - Potentially applicable to lattice field theory where computation time is far more costly.