## **Abstracting Abstract Machines**

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## **Abstract**

We describe a derivational approach to abstract interpretation that yields novel and transparently sound static analyses when applied to well-established abstract machines. To demonstrate the technique and support our claim, we transform the CEK machine of Felleisen and Friedman, a lazy variant of Krivine's machine, and the stack-inspecting CM machine of Clements and Felleisen into abstract interpretations of themselves. The resulting analyses bound temporal ordering of program events; predict return-flow and stack-inspection behavior; and approximate the flow and evaluation of by-need parameters. For all of these machines, we find that a series of well-known concrete machine refactorings, plus a technique we call store-allocated continuations, leads to machines that abstract into static analyses simply by bounding their stores. We demonstrate that the technique scales up uniformly to allow static analysis of realistic language features, including tail calls, conditionals, side effects, exceptions, first-class continuations, and even garbage collection.

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## 1. Introduction

Abstract machines such as the CEK machine and Krivine's machine are first-order state transition systems that represent the core of a real language implementation. Semantics-based program analysis, on the other hand, is concerned with safely approximating intensional properties of such a machine as it runs a program. It seems natural then to want to systematically derive analyses from machines to approximate the core of realistic run-time systems.

Our goal is to develop a technique that enables direct abstract interpretations of abstract machines by methods for transforming a given machine description into another that computes its finite approximation.

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resulting in the CESK, CESK\*, and time-stamped CESK\* machines, respectively. The time-stamps encode the history (context) of the machine's execution and facilitate context-sensitive abstractions. We then demonstrate that the time-stamped machine abstracts directly into a parameterized, sound and computable static analysis.

We demonstrate that the technique of refactoring a machine with **store-allocated continuations** allows a direct structural abstraction<sup>1</sup> by bounding the machine's store. Thus, we are able to convert semantic techniques used to model language features into static analysis techniques for reasoning about the behavior of those very same features. By abstracting well-known machines, our technique delivers static analyzers that can reason about by-need evaluation, higher-order functions, tail calls, side effects, stack structure, exceptions and first-class continuations.

The basic idea behind store-allocated continuations is not new. SML/NJ has allocated continuations in the heap for well over a decade [28]. At first glance, modeling the program stack in an abstract machine with store-allocated continuations would not seem to provide any real benefit. Indeed, for the purpose of defining the meaning of a program, there is no benefit, because the meaning of the program does not depend on the stack-implementation strategy. Yet, a closer inspection finds that store-allocating continuations eliminate recursion from the definition of the state-space of the machine. With no recursive structure in the state-space, an abstract machine becomes eligible for conversion into an abstract interpreter through a simple structural abstraction.

To demonstrate the applicability of the approach, we derive abstract interpreters of:

- a call-by-value λ-calculus with state and control based on the CESK machine of Felleisen and Friedman [13].
- a call-by-need λ-calculus based on a tail-recursive, lazy variant of Krivine's machine derived by Ager, Danvy and Midtgaard [1], and
- a call-by-value λ-calculus with stack inspection based on the CM machine of Clements and Felleisen [3];

and use abstract garbage collection to improve precision [25].

## Overview

In Section 2, we begin with the CEK machine and attempt a structural abstract interpretation, but find ourselves blocked by two recursive structures in the machine: environments and continuations. We make three refactorings to:

- 1. store-allocate bindings,
- 2. store-allocate continuations, and
- 3. time-stamp machine states;

<sup>1</sup> A structural abstraction distributes component-, point-, and member-wise.

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