Q1

(a) $E(X) = 1\times\frac{1}{3} + 2\times\frac{2}{3}\times\frac{1}{3} + 3\times(\frac{2}{3})^2\times\frac{1}{3} + \cdots + n\times(\frac{2}{3})^{n-1}\times\frac{1}{3}$

$= 1\times\frac{1}{3} + 2\times\frac{2}{3^2} + 3\times\frac{4}{3^3} + \cdots + n\times\frac{2^{n-1}}{3^n}$

so $\frac{2}{3}E(X) = 1\times\frac{2}{3^2} + 2\times\frac{4}{3^3} + \cdots + (n-1)\times\frac{2^{n-1}}{3^n} + n\times\frac{2^n}{3^{n+1}}$

so $\frac{1}{3}E(X) = E(X) - \frac{2}{3}E(X) = 1\times\frac{1}{3} + \frac{2}{3^2} + \frac{2^2}{3^3} + \cdots + \frac{2^{n-1}}{3^n} - n\times\frac{2^n}{3^{n+1}}$

$E(X) = 3\times\frac{1}{3}E(X) = 1 + \frac{2}{3} + (\frac{2}{3})^2 + \cdots + (\frac{2}{3})^{n-1} - \frac{n\cdot 2^n}{3^n}$

~~$E(X) = 3 - \frac{2^n}{3^{n-1}} - \frac{n\cdot 2^n}{3^n}$~~

so $E(X) = \lim_{n\to\infty}(3 - \frac{2^n}{3^{n-1}} - \frac{n\cdot 2^n}{3^n})$

$= 3 - 0 - 0$

$= 3$

the $E(X)$ should be 3

(b) $E(X) = \lim_{n\to\infty}(1\times\frac{1}{N} + 2\times\frac{N-1}{N}\times\frac{1}{N} + \cdots + n(\frac{N-1}{N})^{n-1}\cdot\frac{1}{N})$

so $\frac{N-1}{N}E(X) = 1\times\frac{N-1}{N}\times\frac{1}{N} + 2\times(\frac{N-1}{N})^2\cdot\frac{1}{N} + \cdots + (n-1)(\frac{N-1}{N})^{n-1}\cdot\frac{1}{N} + n(\frac{N-1}{N})^n\cdot\frac{1}{N}$

$\frac{1}{N}E(X) = E(X) - \frac{N-1}{N}E(X) = \frac{1}{N} + \frac{N-1}{N}\cdot\frac{1}{N} + (\frac{N-1}{N})^2\cdot\frac{1}{N} + \cdots + (\frac{N-1}{N})^{n-1}\cdot\frac{1}{N} - n(\frac{N-1}{N})^n\cdot\frac{1}{N}$

$E(X) = N\cdot\frac{1}{N}E(X) = 1 + \frac{N-1}{N} + (\frac{N-1}{N})^2 + \cdots + (\frac{N-1}{N})^{n-1} - n(\frac{N-1}{N})^n$

$E(X) = \lim_{n\to\infty}(N - \frac{(N-1)^n}{N^{n-1}} - \frac{n(N-1)^n}{N^n})$

$= N - 0 - 0$

$= N$

the $E(X)$ should be N