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Option Pricing on Stocks in Mergers and Acquisitions

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ABSTRACT

We develop an arbitrage-free and complete framework to price options on the stocks of firms involved in a merger or acquisition deal allowing for the possibility that the deal might be called off at an intermediate time, creating discontinuous impacts on the stock prices. Our model can be a normative tool for market makers to quote prices for options on stocks involved in such deals and also for traders to control risks associated with such deals using traded options. The results of tests indicate that the model performs significantly better than the Black–Scholes model in explaining observed option prices.

THERE WERE 279 MERGER AND ACQUISITION deals announced during the calendar year 2001, of which 177 deals involved firms where either the target or the acquirer had traded options and 67 deals involved firms where both the target and the acquirer had traded options. With the growing number of stocks that have listed options and the accompanying growth in the volume of trading in options, the problem of correctly valuing options on the stocks of firms involved in such deals is increasingly relevant and important. Valuing options on the stocks of merging firms is not straightforward because the stock price processes after deal announcement are affected by the pending deal, and there is also the possibility that the deal might be called off, which may have discontinuous impacts on the stock prices. As a result, pricing options on the stocks of merging firms in the Black–Scholes framework may be inappropriate because it requires the underlying stock price processes to be continuous diffusions.[1]

In this paper, we develop an arbitrage-free and complete model in continuous time to price options on the stocks of firms involved in merger and acquisition

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[1] Fischer Black (1992) first emphasized the limitations of the Black–Scholes framework in pricing options on stocks involved in merger deals and the need for a more consistent and accurate pricing model.

deals. The model is intended to be a tool both for market makers to quote prices for options on such stocks and for options traders and risk arbitrageurs to gauge the fair value of such options and use them effectively in controlling risks involved in merger deals. The framework we propose can be especially useful in situations where options on one or both stocks are illiquid thus necessitating the formulation of a consistent model to quote prices. The results of tests of the model on actual option data demonstrate that it performs well in explaining observed option prices.

There are some studies in the literature that investigate the behavior of implied volatilities around merger or acquisition announcements. Jayaraman, Mandelker, and Shastri (1991) examine the possibility of leakage of information regarding a merger prior to the announcement of the first bid for the target firm. Their tests for the existence of market anticipation that are based on the behavior of variances implied in the premia of call options on the target conclude that the evidence is consistent with the hypothesis that the market anticipates an acquisition prior to the first announcement. Levy and Yoder (1993, 1997) find that the implied standard deviations of target firms increase significantly three days prior to the announcement of a merger or acquisition while the bidding firm's implied standard deviations are not affected. Barone-Adesi, Brown, and Harlow (1994) argue that in an efficient market, option prices of the bidder and especially the target should reflect the market's expectation of the deal's consummation. They predict that implied volatilities should decline for options with successively longer maturities for deals that are expected to be completed and find empirical support for this prediction. The objectives of these studies, however, differ significantly from ours in that they do not seek to build a consistent pricing model that explains observed option prices. Moreover, the notion of an "implied volatility" presupposes a model where the underlying stock price processes are continuous diffusions.

There has been little theoretical work done so far in valuing options on the stocks of firms involved in a merger deal *after* the merger announcement has been made and *before* the deal either goes through or is called off. We consider the situation where a merger deal between two firms has been announced and the stock prices of the two firms have reacted to the announcement. The deal is expected to go through on the closing date unless it is called off for reasons including shareholder disapproval and regulatory considerations. In the scenario where the deal is still pending, we model the stock price processes as continuous diffusions. However, we allow for the possibility that the prices may jump discontinuously if the deal is called off. This is consistent with the findings of several empirical papers that have reported statistically significant abnormal returns in the prices of the stocks, especially the target stock, immediately after the cancellation of a pending merger deal.[2] Hence, in effect, we model the stock price processes as jump diffusions.

There are a number of studies in the literature that investigate option pricing and hedging problems when there are jumps in the underlying stock price

[2] See, for example, Dodd (1980), Asquith (1983), and Bradley, Desai, and Kim (1983).

processes.[3] The present paper is the first (to our knowledge) to apply the general methodology of the theory of option pricing on jump diffusion processes to the problem of developing a suitable model to price options on stocks involved in merger deals.

The model proposed in the paper may be applied to various types of merger deals. However, for concreteness and expositional convenience, we restrict our discussion in this paper to *stock for stock* deals. Moreover, a majority of the merger or acquisition deals recently announced have been stock for stock deals. Of the 279 deals announced in 2001, 140 were stock for stock deals. We assume that if the deal is called off at any intermediate time, the two stock prices jump respectively to the prices of some *marketed* securities or baskets of securities. For expositional convenience, we refer to these processes as the *base price processes* of the stocks. Under the assumption that the jump proportions are bounded, we show that the base price processes must be perfectly correlated with the respective stock price processes in the situation where the deal is still pending. We then show that the jump proportions must in fact be deterministic and have a specific functional form to preclude the existence of arbitrage in the financial market. We also show that the proportions by which the two stock prices would jump if the deal were called off must have a specific analytical relationship to each other to prevent arbitrage opportunities. We then obtain analytical expressions in closed form for European option prices on the stocks. We demonstrate the completeness of our model provided the investor can trade not only in the underlying stocks and a risk-free bond, but also in securities (or baskets) representing the base prices of the stocks. In other words, we show that any contingent claim on either stock may be replicated with a dynamic trading strategy in the underlying stocks, the "base securities" and the bond.

We discuss the numerical implementation of the model to derive option prices. We then carry out extensive tests of our model on all the stock for stock merger deals announced in the calendar year 2001 where both the target and the acquirer had traded options. Specifically, we test our model on all near the money, short maturity options on both stocks where the Black–Scholes model is an appropriate benchmark. We find that the prices predicted by our model are always significantly closer to observed prices.

Finally, if the jump risk premium is known, we show how the model may be used to infer the probability of success of a pending deal from the information contained in stock and option prices. Under the assumption that the jump risk premium is zero, the results of empirical tests of this facet of the model show that it assigns significantly greater probabilities of success to pending deals that eventually succeed than to those that eventually fail even in the early periods of deals. These results indicate that the market's perception of the outcome of a pending deal is reflected in stock and option prices even in the early period of the deal.

[3] See Carr, Geman, and Madan (2001), Gukhal (2001), Madan and Milne (1991), Naik and Lee (1990), Schweizer (1991), Scott (1997).

In summary, this paper extends the existing literature by proposing a consistent and parsimonious theoretical framework based on fundamental principles of *no arbitrage* and *market completeness* to price options on the stocks of firms involved in merger or acquisition deals after such an announcement has been made. Our primary objective is to provide the simplest possible extension of the Black–Scholes model that accommodates the effects of pending merger deals. This could provide the foundation for more complex and realistic models.

The plan for the paper is as follows. In Section I, we present the general model under consideration. In Section II, we present our main results for the case where the proposed deal is a stock for stock deal. Section III is devoted to demonstrating the completeness of our market model when there are baskets of securities representing the base prices of the stocks available for trading. In Section IV, we discuss the numerical implementation of the model and present the results of extensive tests of our model. Section V concludes the paper. All detailed proofs appear in the Appendix.

I. The Model

We consider a probability space (Ω, F, P) equipped with a complete, right continuous filtration $\{F_t\}$, and a time horizon T^* with $F_{T^*} \equiv F$. We assume that the probability space is large enough to accommodate four F_t-adapted Brownian motions W_1, W_2, W_3, W_4, and an F_t-adapted *single jump* process N with parameter λ and jump size equal to one, *independent* of the Brownian motions. The Brownian motions W_1, W_2 are independent of each other and the Brownian motions W_3, W_4. However, W_3, W_4 may have a nonzero correlation. We assume that Ω has the form $\Omega = \Omega' \times \Omega''$ and the Brownian motions are defined on Ω' and the jump process on Ω'' and the filtration F_t is the usual completed and augmented product filtration. We abuse notation by writing $W_i(t, (\omega', \omega'')) = W_i(t, \omega')$ and $N(t, (\omega', \omega'')) = N(t, \omega'')$ where $(\omega', \omega'') \in \Omega' \times \Omega'' = \Omega$. We assume that P is the risk-neutral or pricing probability in our financial market.

We assume that a merger or acquisition announcement is made at time $t = 0$ and the deal is expected to go through at the time horizon T. However, there is a nonzero probability that the deal could be called off at some time in the interval $[0, T)$ and this breakdown process is the jump process N, that is, the risk-neutral probability that the deal is called off during a time interval $[t, t + dt] \subset [0, T)$ provided it hasn't yet been called off is $\lambda\, dt$. Therefore, the risk-neutral probability that the deal will be called off at some time before the deal date is $1 - \exp(-\lambda T)$. We assume that the process $N(\cdot)$ jumps at time T if it has not already jumped at a prior instant. Therefore, the risk-neutral probability that the process $N(\cdot)$ jumps at time T (or alternatively the risk-neutral probability that the deal is successful) is $\exp(-\lambda T)$.[4]

[4] We suppress the explicit dependence on the probability space parameter ω in the following for notational convenience. All the stochastic processes defined in the following are *corlol*, that is, they are right continuous with finite left limits.

A. Model for the Stock Price Processes

The two stock price processes when the merger has not yet been called off are assumed to satisfy

$$dS_1(t)1_{N(t)=0} = 1_{N(t)=0}[((\mu_1(t-) - d_1)S_1(t-)\,dt + \sigma_1' S_1(t-)\,dW_1(t)], \tag{1}$$

$$\begin{aligned} dS_2(t)1_{N(t)=0} = 1_{N(t)=0}[(\mu_2(t-) - d_2)S_2(t-)\,dt + \sigma_2' S_2(t-)\,dW_1(t) \\ + \sigma_2'' S_2(t-)\,dW_2(t)] \end{aligned} \tag{2}$$

under the risk-neutral pricing measure P. Thus, when the deal is still on (that is, $(N(t) = 0)$), the stock price processes are continuous diffusions just as in the Black–Scholes model. The processes $\mu_1(\cdot)$ and $\mu_2(\cdot)$ are assumed to be random but F_t-progressively measurable processes. The dividend yields of the respective stocks d_1, d_2 are assumed to be constant for convenience of exposition. We also assume throughout the paper that the dividend yields are not affected if the deal is called off.[5]

If the deal is called off at time $t < T$ (that is, $\Delta N(t) = N(t) - N(t-) = 1$), the two price processes experience jumps given by

$$S_1(t)1_{\Delta N(t)=1} = 1_{\Delta N(t)=1}\beta_1(t-)S_1(t-) \tag{3}$$

$$S_2(t)1_{\Delta N(t)=1} = 1_{\Delta N(t)=1}\beta_2(t-)S_2(t-) \tag{4}$$

where $\Delta N(t) = N(t) - N(t-)$ and $\beta_1(\cdot)$ and $\beta_2(\cdot)$ are F_t-adapted processes such that

$$\beta_i(t)1_{N(t)=1} = 1, \quad \beta_i(T) = 1. \tag{5}$$

The above conditions imply that there can be a jump in the respective stock price processes. However, the jump factors $\beta_i(\cdot)$ equal one at the deal date T, that is, even if the process $N(\cdot)$ jumps at time T, there is no resulting jump in the stock prices since the deal has gone through. After the jumps, the price processes evolve according to the equations

$$dS_1(t)1_{N(t)=1} = 1_{N(t)=1}[(\mu_1(t-) - d_1)S_1(t-)\,dt + \sigma_1 S_1(t-)\,dW_3(t)], \tag{6}$$

$$dS_2(t)1_{N(t)=1} = 1_{N(t)=1}[(\mu_2(t-) - d_2)S_2(t-)\,dt + \sigma_2 S_2(t-)\,dW_4(t)] \tag{7}$$

[5] We assume that dividends are paid continuously for analytical tractability even though they are actually paid at discrete instants of time. This assumption as well as our assumption that dividend yields are constant regardless of the outcome of the deal can be relaxed without significantly altering our conclusions, but have been imposed for notational and expositional simplicity.

under the pricing probability P. Since W_3, W_4 may have a nonzero correlation, the stock prices may be correlated with each other after the deal has been called off. Since the stock price processes are continuous after the process $N(\cdot)$ has jumped (that is, either at time $t < T$ if the deal is called off or at time T if the deal goes through) and the evolutions (6) and (7) are under the risk-neutral measure, we must have $\mu_1(t)1_{N(t)=1} = \mu_2(t)1_{N(t)=1} = r1_{N(t)=1}$ where r is the risk-free interest rate. Therefore, $\mu_1(\cdot)$ and $\mu_2(\cdot)$ in (6) and (7) above are equal to r. The volatilities of the stocks after the deal has been called off (σ_1, σ_2) may be different from the volatilities (σ_1', σ_2') when the deal is still pending. If the deal is still pending, the drifts of the processes $\mu_1(\cdot) - d_1, \mu_2(\cdot) - d_2$ in (1) and (2) may be different from $(r - d_1)$ and $(r - d_2)$, respectively, due to the possibility of jumps in the two stock prices subsequent to the deal being called off.

We assume that if the deal is called off, the two stock prices jump to the prices of *marketed* securities or baskets of securities. More precisely, we assume the existence of two marketed processes $S_1^*(\cdot), S_2^*(\cdot)$ such that if the deal is called off at time $t < T$ (that is, $\Delta N(t) = 1$), the price of stock i will jump from $S_i(t-)$ to $S_i^*(t)$. Therefore, in our notation,

$$S_i(t-)\beta_i(t-)1_{N(t-)=0} = S_i^*(t)1_{N(t-)=0} \quad \text{for} \quad i = 1, 2 \text{ and } 0 \le t < T \tag{8}$$

For subsequent expositional convenience, we refer to these processes as the *base price processes*. Since the base prices are the prices of the stocks in the *absence* of the deal, they are continuous in the traditional Black–Scholes framework. Moreover, since they are the prices of traded (or tradable) baskets of securities in the market, the corresponding discounted gains processes (discounted at the risk-free rate) must be martingales under the risk-neutral measure in order to preclude the existence of arbitrage in the market (see Duffie (1998)). Hence, in our notation,

$$S_i^*(t)/\exp[(r - d_i)t] \tag{9}$$

must be martingales under the risk-neutral measure P for $0 \le t < T$ and $i = 1, 2$. The dividend yields for $S_1^*(\cdot), S_2^*(\cdot)$ are the same as for $S_1(\cdot), S_2(\cdot)$ since we have assumed that the dividend yields of the respective stocks are not affected if the deal is called off.[6] The base prices could be the prices of baskets of closely related stocks not involved in the merger deal. These would be good proxies for the prices of the two stocks if the deal were called off. In this scenario, one may view the deal as representing a complex option on the base prices with a nonzero value as long as the deal is still pending.[7] In summary, S_1 and S_2 satisfy the following stochastic differential equations under the risk-neutral probability for $0 \le t < T$

[6] This assumption can easily be relaxed without significantly altering our results, but has been imposed throughout the paper for notational simplicity.

[7] We would like to emphasize here that it is not necessary to actually identify the baskets that represent the base prices. It is enough to assume that they exist.

$$
\begin{aligned}
dS_1(t) &= 1_{\{N(t)=0\}}[(\mu_1(t-) - d_1)S_1(t-)\,dt + \sigma_1' S_1(t-)\,dW_1(t)] \\
&\quad + 1_{\{N(t)=1\}}[(r - d_1)S_1(t-)\,dt + \sigma_1 S_1(t-)\,dW_3(t)] \\
&\quad + (\beta_1(t-) - 1)S_1(t-)\,dN(t) \\
dS_2(t) &= 1_{\{N(t)=0\}}[(\mu_2(t-) - d_2)S_2(t-)\,dt + \sigma_2' S_2(t-)\,dW_1(t) \\
&\quad + \sigma_2'' S_2(t-)\,dW_2(t)] + 1_{\{N(t)=1\}}[(r - d_2)S_2(t-)\,dt \\
&\quad + \sigma_2 S_2(t-)\,dW_4(t)] + (\beta_2(t-) - 1)S_2(t-)\,dN(t)
\end{aligned}
\tag{10}
$$

In addition, we assume throughout the paper that the jump factors $\beta_i(\cdot)$ are uniformly bounded, that is, if the deal is called off at an intermediate date, the proportions by which the two stock prices would jump instantaneously are uniformly bounded. This assumption is quite reasonable from an economic standpoint since it is unlikely that stock prices would jump *instantaneously* by unbounded proportions if the deal were to be called off.

B. Model for the Base Price Processes

We model the two base price processes $S_1^*(\cdot), S_2^*(\cdot)$ under the risk-neutral measure as follows.

$$
\begin{aligned}
dS_1^*(t) &= S_1^*(t)\big[(r - d_1)\,dt + \sigma_1^*\,dW_1^*(t)\big] \\
dS_2^*(t) &= S_2^*(t)\big[(r - d_2)\,dt + \sigma_2'^*\,dW_1^*(t) + \sigma_2''^*\,dW_2^*(t)\big],
\end{aligned}
\tag{11}
$$

We note that the drifts of the base price processes are equal to $(r - d_1)$ and $(r - d_2)$, respectively, reflecting the fact (see (9)) that the discounted gains processes must be martingales under the risk-neutral pricing measure. The base price processes are only relevant in determining how the stock prices would jump if the deal were called off before the deal date. After the stock prices have jumped, their subsequent evolutions are geometric Brownian motions with volatilities that may be different from those in (11) as described in equation (10).

C. Evolution after the Deal Date

We need to distinguish between two cases, the situation where the deal goes through, that is, $\Delta N(T) = 1$ and the situation where the deal is called off before the deal date, that is, $\Delta N(T) = 0$. For times $t \geq T$, we have

$$
\begin{aligned}
dS_1(t) &= 1_{\{\Delta N(T)=1\}}[(r - d_1)S_1(t)\,dt + \sigma_1' S_1(t)\,dW_1(t)] \\
&\quad + 1_{\{\Delta N(T)=0\}}[(r - d_1)S_1(t)\,dt + \sigma_1 S_1(t)\,dW_3(t)] \\
dS_2(t) &= 1_{\{\Delta N(T)=1\}}[(r - d_1)S_2(t)\,dt + \sigma_1' S_2(t)\,dW_1(t)] \\
&\quad + 1_{\{\Delta N(T)=0\}}[(r - d_2)S_2(t)\,dt + \sigma_2 S_2(t)\,dW_4(t)].
\end{aligned}
\tag{12}
$$

The above expressions represent the assumption that if the deal is not called off before time T, the deal goes through and there are no subsequent jumps in the stock prices. If the deal goes through, the volatilities of the two stocks and their dividend yields must, of course, be the same as described in the equations above. This completes the description of the model.

II. Stock for Stock Deals

In this section, we shall consider the scenario wherein the two firms make an announcement to merge and the terms of the deal are such that each shareholder of the second firm will receive α shares of the stock of the combined firm. In this case, we see that in order to prevent the existence of trivial arbitrage opportunities in the market, the prices of the two stocks at the horizon (deal date) T in the situation where the deal goes through must satisfy[8]

$$S_2(T)1_{\Delta N(T)=1} = \alpha S_1(T)1_{\Delta N(T)=1}. \tag{13}$$

We can now state without proof the following well-known result that characterizes the drifts $\mu_1(\cdot)$, $\mu_2(\cdot)$ of the two stock price processes for a pending deal under the risk-neutral measure.

PROPOSITION 1: *In the absence of arbitrage, we must have for* $0 \leq t < T$,

$$\begin{aligned} \mu_1(t-)1_{N(t-)=0} &= [r + \lambda(1 - \beta_1(t-))]1_{N(t-)=0} \\ \mu_2(t-)1_{N(t-)=0} &= [r + \lambda(1 - \beta_2(t-))]1_{N(t-)=0} \end{aligned} \tag{14}$$

in the notation of equations (3) and (4).

The difference of the drifts from the risk-free rate is exactly the compensator of the jump process or the jump risk premium under the risk-neutral probability. We can now state the following proposition that basically says that the volatilities of the stocks of both firms must converge after a merger announcement and the two stock prices must be perfectly correlated.

PROPOSITION 2: *The ratio of the prices of the two stocks in the situation where the deal has not yet been called off before time T must be a deterministic function of time, that is, in equations (1) and (2), we must have* $\sigma_2' = \sigma_1'$ *and* $\sigma_2'' = 0$.

Proof: The proof of Proposition 2 is given in the Appendix.

The intuition for the result of the above proposition is the following. Within our framework, the ratio of the two stock prices when the deal is still pending is a process that has a constant value equal to the stock exchange ratio α at the deal maturity date. This is due to the fact that if the deal is not called off before the deal date, it goes through. If this ratio process is stochastic, then it must be a lognormal process by (10). However, this directly implies that it cannot be a constant at the deal maturity date. Therefore, the ratio process must be

[8] If the deal goes through the process $N(\cdot)$ jumps at time T so that $\Delta N(T) = N(T) - N(T-) = 1$.

deterministic and this directly implies that the two stocks must be perfectly correlated and their volatilities must coincide.

As a consequence of the above proposition, it follows that in the situation where the deal has not been called off, we have for $0 \le t < T$

$$d\gamma(t)1_{N(t)=0} = \gamma(t)1_{N(t)=0}(\mu_2(t-) - d_2 - \mu_1(t-) + d_1)\,dt; \quad \gamma(t) = S_2(t)/S_1(t). \tag{15}$$

We carry out an empirical analysis of all the 32 stock for stock merger deals announced in the calendar year 2001 where both the target and the acquirer had traded options to test the principal results of the above proposition, that is, the volatilities of the two stocks converge after a merger announcement and the two stock price processes are perfectly correlated. These results are displayed in Table I. We display the ratios of the volatilities of the relevant stocks before and after the corresponding merger announcements and the correlation of the stock price processes before and after the announcements. The mean, median, and standard deviation of the correlations before the announcement dates are 0.408, 0.388, and 0.31, respectively, and after the announcement dates are 0.9, 0.93, and 0.086, respectively. The mean, median, and standard deviation of the volatility ratios for all the deals before the announcement dates are 1.526, 1.245, and 0.955, respectively, and after the announcement dates are 1.025, 1.007, and 0.169, respectively.

After deal announcement, 40.6 percent of the correlations are greater than 0.95, 62.5 percent are greater than 0.9, and 81.3 percent are greater than 0.8. Similarly, after deal announcement, 40.6 percent of the volatility ratios lie in the interval (0.95, 1.05), 68.8 percent lie in the interval (0.9, 1.1) and 90.6 percent lie in the interval (0.8, 1.2). These results provide empirical support for the theoretical results of Proposition 2. In fact, it is possible to use the nonparametric chi-square goodness of fit test to show that the sample distributions of correlations and volatility ratios are statistically consistent with population distributions that imply higher proportions of deals for which the correlations and volatility ratios after deal announcement are "close" to one. This provides further empirical support for the results of Proposition 2. We do not display these results for the sake of brevity.[9]

We can now state the following proposition that relates the base price processes to the corresponding stock price processes. In particular, we show that the base prices and stock prices must be perfectly correlated for the jumps in the stock prices to be bounded subsequent to the deal being called off.

PROPOSITION 3: *If the stock price and base price processes for $0 \le t < T$ are modeled by (10) and (11) respectively, then*

$$\begin{aligned} W_1^*(\cdot) &\equiv W_1(\cdot), \quad W_2^*(\cdot) \equiv W_2(\cdot) \\ \sigma_1^* &= \sigma_1', \quad \sigma_2'^* = \sigma_2', \quad \sigma_2''^* = \sigma_2''. \end{aligned} \tag{16}$$

[9] The results are available upon request.

Table I

Summarized Correlation/Volatility Ratio Results for Stock for Stock Mergers Announced in 2001

The table shows the correlations and volatility ratios of the stocks involved in each deal before and after the announcement date (**Ann Date**).

			Correlations		Vol Ratios	
Ann Date	Target	Acquirer	Before	After	Before	After
1/23/2001	NETOPIA INC	PROXIM INC	0.387	0.976	1.626	0.917
1/29/2001	DALLAS SEMICONDUCTOR CORP	MAXIM INTEGRATED PRODUCTS	0.797	0.958	0.721	0.989
2/4/2001	TOSCO CORP	PHILLIPS PETROLEUM CO	0.299	0.921	1.12	0.908
2/20/2001	NEW ERA OF NETWORKS INC	SYBASE INC	0.32	0.972	4.086	1.054
3/12/2001	CAMBRIDGE TECHNOLOGY PRTNRS	NOVELL INC	0.39	0.785	1.455	1.131
3/12/2001	UNITED DOMINION INDUSTRIES	SPX CORP	0.365	0.959	0.964	0.998
3/13/2001	C I T GROUP INC	TYCO INTERNATIONAL LTD NEW	0.242	0.967	0.968	1.029
3/16/2001	BERGEN BRUNSWIG CORP	AMERISOURCE HEALTH CORP	0.442	0.761	1.113	0.945
3/19/2001	TRUE NORTH COMMUNICATIONS	INTERPUBLIC GROUP COS INC	−0.007	0.96	0.931	0.914
3/22/2001	KENT ELECTRONICS CORP	AVNET INC	0.548	0.915	1.47	1.041
3/26/2001	C CUBE MICROSYSTEMS INC NEW	L S I LOGIC CORP	0.807	0.983	0.942	1.017
3/27/2001	ALZA CORP	JOHNSON & JOHNSON	−0.106	0.927	3.657	1.166
4/11/2001	AUTOWEB COM INC	AUTOBYTEL COM INC	0.23	0.763	2.644	1.659
4/16/2001	WACHOVIA CORP NEW	FIRST UNION CORP	0.813	0.703	0.775	1.262
5/7/2001	NOVA CORP GA	U S BANCORP DEL	0.175	0.862	0.721	0.581
5/15/2001	SAWTEK INC	TRIQUINT SEMICONDUCTOR INC	0.772	0.933	0.634	0.945
5/23/2001	MARINE DRILLING COS INC	PRIDE INTERNATIONAL INC DEL	0.729	0.849	0.876	0.838
6/1/2001	MESSAGEMEDIA INC	DOUBLECLICK INC	−0.037	0.865	2.366	1.155

6/25/2001	HOMESTAKE MINING CO	BARRICK GOLD CORP	0.809	0.94	1.658	1.017
6/29/2001	DURAMED PHARMACEUTICALS	BARR LABORATORIES INC	−0.017	0.966	1.895	1.022
7/16/2001	S C I SYSTEMS INC	SANMINA CORP	0.387	0.976	1.626	0.917
7/31/2001	GENERAL SEMICONDUCTOR INC	VISHAY INTERTECHNOLOGY INC	0.334	0.905	1.39	1.016
8/2/2001	GENRAD INC	TERADYNE INC	0.127	0.867	1.381	1.131
8/3/2001	SENSORMATIC ELECTRONICS CORP	TYCO INTERNATIONAL LTD NEW	0.669	0.97	1.294	0.992
8/29/2001	WESTVACO CORP	MEAD CORP	0.741	0.939	0.981	1.094
9/4/2001	COMPAQ COMPUTER CORP	HEWLETT PACKARD CO	0.389	0.848	1.195	0.962
9/4/2001	GLOBAL MARINE INC	SANTA FE INTERNATIONAL CORP	0.83	0.869	0.899	0.978
10/24/2001	P R I AUTOMATION INC	BROOKS AUTOMATION INC	0.752	0.981	1.195	0.988
11/18/2001	CONOCO INC	PHILLIPS PETROLEUM CO	−0.171	0.711	1.842	1.099
12/3/2001	AVIRON	MEDIMMUNE INC	0.628	0.995	1.307	0.884
12/3/2001	AVANT CORP	SYNOPSYS INC	−0.02	0.779	4.431	1.193
12/6/2001	COR THERAPEUTICS INC	MILLENNIUM PHARMACEUTICALS	0.446	0.993	0.669	0.959
		Mean	0.408	0.9	1.526	1.025
		Median	0.388	0.93	1.245	1.007
		Standard Deviation	0.31	0.086	0.955	0.169

Therefore, the stock price processes and the base price processes are perfectly correlated in the situation where the deal has not been called off. Moreover, the jump proportions $\beta_1(\cdot)$, $\beta_2(\cdot)$ must in fact be deterministic functions of time in the situation where the deal is still pending.

Proof: The proof of Proposition 3 is given in the Appendix.

We can intuitively understand the result of the above proposition by the following argument. The diffusion terms driving the stock price processes represent the fluctuations caused by market conditions *external* to the merger deal. Since the base processes represent the prices of the respective stocks in the absence of the merger deal, we would expect external market conditions to affect the stock price processes and the base price processes in the same way. The stock price and base price processes are therefore perfectly correlated when the deal is still on and the jump proportions must be deterministic functions of time. The results of the previous propositions imply the following result that provides a complete analytical characterization of the jump proportions $\beta_i(\cdot)$.

PROPOSITION 4: *$\beta_1(t)$ and $\beta_2(t)$* for $0 \leq t < T$ must be given by

$$\begin{aligned}\beta_i(t-)1_{N(t-)=0} &= \frac{A_i \exp(-(\lambda)t)}{1 + A_i \exp(-(\lambda)t)} 1_{N(t-)=0} \\ \beta_i(t)1_{N(t)=1} &= 1_{N(t)=1}\end{aligned} \tag{17}$$

where A_i is a deterministic constant.

Proof: The proof of Proposition 4 is given in the Appendix.

The above propositions demonstrate that the jump proportions for the two stocks must not only be deterministic but must also have a specific analytical form to preclude the existence of arbitrage opportunities in our model. This allows us to obtain closed form analytical expressions for the option prices.

D. Pricing of European Options

We begin by showing that the parameters defining the jumps of the stock prices if the deal is called off must have a specific analytical relationship. From equation (15), we see that in the situation where the merger has not been called off

$$\begin{aligned}\gamma(t)1_{N(t)=0} &= \gamma(0)1_{N(t)=0} \exp\left[(d_1 - d_2)t + \int_0^t (\mu_2(s-) - \mu_1(s-))\, ds\right] \\ &= \gamma(0)1_{N(t)=0} \exp[(d_1 - d_2)t] \exp\left[\lambda\left(\int_0^t (\beta_1(s-) - \beta_2(s-)\right) ds\right].\end{aligned} \tag{18}$$

Since $\gamma(T)1_{\Delta N(T)=1} = \gamma(T)1_{N(T-)=0} = \alpha$, we have

$$1_{\Delta N(T)=1}\int_0^T (\beta_1(s-) - \beta_2(s-))\,ds = \left[\frac{\log(\alpha/\gamma(0)) - (d_1 - d_2)T}{\lambda}\right]1_{\Delta N(T)=1} \tag{19}$$

where $\gamma(0) = S_2(0)/S_1(0)$. We can evaluate the integral above explicitly to conclude that

$$\frac{\exp(\lambda T) + A_1}{\exp(\lambda T) + A_2}\cdot\frac{1 + A_2}{1 + A_1} = \frac{\alpha}{\gamma(0)}\exp[-(d_1 - d_2)T\,]. \tag{20}$$

Thus, given the terms of the deal (α, T), the above expression demonstrates that the parameters characterizing the jumps of the two stocks are not independent of each other. We now derive explicit closed form expressions for option prices on the two stocks. For notational convenience, the terms $\mu_1(\cdot)$, $\mu_2(\cdot)$, $\beta_1(\cdot)$, $\beta_2(\cdot)$ appearing in the following expressions refer to their values in the situation when the deal is still on. We first consider a call option on the first stock with strike K maturing at a time $T_0 \le T$. Then we can see that the price of the option at time 0 is given by

$$\begin{aligned}
&P_1(0, T_0, K)\\
&\quad = \exp(-rT_0)\Bigg\{\exp(-\lambda T_0)E\Bigg[\Bigg(S_1(0)\exp\Bigg[\int_0^{T_0}\big(\mu_1(s-) - d_1 - (1/2)\sigma_1'^2\big)\,ds\\
&\qquad + \sigma_1'\int_0^{T_0} dW_1(s)\Bigg] - K\Bigg)^+\Bigg] + \int_0^{T_0} ds\,\lambda\exp(-\lambda s)\beta_1(s-)\\
&\qquad \times E\Bigg[\Bigg(S_1(0)\exp\Bigg[\int_0^s\big(\mu_1(t-) - d_1 - (1/2)\sigma_1'^2\big)\,dt + \int_0^s \sigma_1' dW_1(t)\\
&\qquad + \big(r - d_1 - (1/2)\sigma_1^2\big)(T_0 - s) + \sigma_1 W_3(T_0 - s)\Bigg] - (K/\beta_1(s-))\Bigg)^+\Bigg]\Bigg\}.
\end{aligned} \tag{21}$$

The first term above represents the expected discounted payoff (under the risk-neutral measure) if the deal is not called off until the time T_0, and the second term represents the expected discounted payoff if the deal is called off at some time in the interval $(0, T_0)$. The expression above is clearly unchanged if we replace $W_3(T_0 - s)$ by $W_1(T_0 - s)$. Therefore,

$$\begin{aligned}
&P_1(0, T_0, K)\\
&\quad = \exp(-rT_0)\Bigg\{\exp(-\lambda T_0)E\Bigg[\Bigg(S_1(0)\exp\Bigg[\int_0^{T_0}\big(\mu_1(s-) - d_1 - (1/2)\sigma_1'^2\big)\,ds\\
&\quad + \sigma_1' W_1(T_0)\Bigg] - K\Bigg)^+\Bigg] + \int_0^{T_0} ds\,\lambda e^{-\lambda s}\beta_1(s-)
\end{aligned}$$

$$\times E\left[\left(S_1(0)\exp\left[\int_0^s \left(\mu_1(t-) - d_1 - (1/2)\sigma_1'^2\right)dt + \sigma_1' W_1(s)\right.\right.\right.$$
$$\left.\left.\left.+ \left(r - d_1 - (1/2)\sigma_1^2\right)(T_0 - s) + \sigma_1 W_1(T_0 - s)\right] - (K/\beta_1(s-))\right)^+\right]\Bigg\}. \qquad (22)$$

We can use the explicit analytical expressions obtained for $\beta_1(\cdot)$ (and therefore $\mu_1(\cdot)$) to obtain closed form analytical expressions for option prices. We first evaluate the integral in the first term above and the inner integral in the second term above to obtain

$$P_1(0, T_0, K) = e^{-\lambda T_0}\left\{S_1(0)e^{-d_1 T_0}\frac{e^{\lambda T_0} + A_1}{1 + A_1}N(\alpha_1') - Ke^{-rT_0}N(\alpha_2')\right\}$$
$$+ \frac{\lambda S_1(0)e^{-d_1 T_0}A_1}{1 + A_1}\int_0^{T_0} dse^{-\lambda s}N(\alpha_1(s))$$
$$- \lambda Ke^{-rT_0}\int_0^{T_0} dse^{-\lambda s}N(\alpha_2(s)). \qquad (23)$$

In the above,

$$\alpha_1' = \frac{\log\left[\frac{S_1(0)(e^{\lambda T_0} + A_1)}{K(1 + A_1)}\right] + \left(r - d_1 + \frac{\sigma'^2}{2}\right)T_0}{\sigma'\sqrt{T_0}};$$
$$\alpha_2' = \frac{\log\left[\frac{S_1(0)(e^{\lambda T_0} + A_1)}{K(1 + A_1)}\right] + \left(r - d_1 - \frac{\sigma'^2}{2}\right)T_0}{\sigma'\sqrt{T_0}}$$
$$\alpha_1(s) = \frac{\log\left[\frac{S_1(0)(A_1)}{K(1 + A_1)}\right] + (r - d_1)T_0 + \frac{\sigma'^2 s + \sigma^2(T_0 - s)}{2}}{\sqrt{\sigma'^2 s + \sigma^2(T_0 - s)}}; \qquad (24)$$
$$\alpha_2(s) = \frac{\log\left[\frac{S_1(0)(A_1)}{K(1 + A_1)}\right] + (r - d_1)T_0 - \frac{\sigma'^2 s + \sigma^2(T_0 - s)}{2}}{\sqrt{\sigma'^2 s + \sigma^2(T_0 - s)}}.$$

If $\sigma = \sigma'$, that is, the volatility of the stock does not change after the deal is called off, then we can easily evaluate the integrals in (23) to obtain

$$P_1(0, T_0, K) = e^{-\lambda T_0}\left\{S_1(0)e^{-d_1 T_0}\frac{e^{\lambda T_0} + A_1}{1 + A_1}N(\alpha_1') - Ke^{-rT_0}N(\alpha_2')\right\}$$
$$+ (1 - e^{-\lambda T_0})\left\{\frac{S_1(0)e^{-d_1 T_0}A_1}{1 + A_1}N(\alpha_1(0)) - Ke^{-rT_0}N(\alpha_2(0))\right\}. \qquad (25)$$

If $\sigma \neq \sigma'$, evaluating the integrals in (23) is more involved. Expression (23) for the option price can be rewritten as follows:

$$\begin{aligned} P_1(0,T_0,K) &= e^{-\lambda T_0}\left\{S_1(0)e^{-d_1T_0}\frac{e^{\lambda T_0}+A_1}{1+A_1}N(\alpha_1') - Ke^{-rT_0}N(\alpha_2')\right\} \\ &\quad + \frac{\lambda S_1(0)e^{-d_1T_0}e^{\frac{\lambda\sigma^2T_0}{\sigma'^2-\sigma^2}}A_1}{(1+A_1)(\sigma'^2-\sigma^2)}\int_{\sigma^2T_0}^{\sigma'^2T_0} dt\, e^{-\frac{\lambda t}{\sigma'^2-\sigma^2}}N\left(\frac{x+0.5t}{\sqrt{t}}\right) \\ &\quad - \lambda Ke^{-rT_0}\frac{e^{\frac{\lambda\sigma^2T_0}{\sigma'^2-\sigma^2}}}{\sigma'^2-\sigma^2}\int_{\sigma^2T_0}^{\sigma'^2T_0} dt\, e^{-\frac{\lambda t}{\sigma'^2-\sigma^2}}N\left(\frac{x-0.5t}{\sqrt{t}}\right) \end{aligned} \tag{26}$$

where $x = \log[\frac{S_1(0)(A_1)}{K(1+A_1)}] + (r-d_1)T_0$. The integrals above can be expressed in closed form by using the result of the following lemma.[10]

LEMMA 1:

$$\begin{aligned} \int_0^\tau dt\, e^{-\omega t}N\left(\frac{x-\rho t}{\sqrt{t}}\right) &= \frac{1}{\omega}\left\{\frac{1+\operatorname{sgn}(x)}{2} - e^{-\omega\tau}N\left(\frac{x-\rho\tau}{\sqrt{\tau}}\right)\right\} \\ &\quad + \frac{e^{(\rho-\xi)x}}{2\omega}N\left(\frac{x-\xi\tau}{\sqrt{\tau}}\right) + \frac{e^{(\rho+\xi)x}}{2\omega}N\left(\frac{x+\xi\tau}{\sqrt{\tau}}\right) \\ &\quad - \frac{e^{(\rho-\xi)x}+e^{(\rho+\xi)x}}{4\omega}(1+\operatorname{sgn}(x)) \\ &\quad + \frac{\rho e^{(\rho-\xi)x}}{2\xi\omega}N\left(\frac{x-\xi\tau}{\sqrt{\tau}}\right) - \frac{\rho e^{(\rho+\xi)x}}{2\xi\omega}N\left(\frac{x+\xi\tau}{\sqrt{\tau}}\right) \\ &\quad + \frac{\rho(e^{(\rho+\xi)x}-e^{(\rho-\xi)x})}{4\xi\omega}(1+\operatorname{sgn}(x)) \end{aligned}$$

where $\xi = \sqrt{2\omega+\rho^2}$ *and* $\operatorname{sgn}(x) = 1_{x>0} - 1_{x<0}$.

Proof: The proof (due to the anonymous referee) appears in the Appendix.

If $T_0 > T$, we may obtain analytical expressions for option prices using similar arguments that we omit here for the sake of brevity. We obtain similar expressions for options on the second stock using the equations for the evolution of the second stock after the deal date.

III. Completeness of the Market

In the previous sections, we have developed our model for pricing options on mergers and acquisitions by modeling the stock and base price processes under the risk-neutral or equivalent martingale pricing measure. However, we

[10] We thank the anonymous referee for providing us with this lemma and its proof.

have not discussed whether the choice of the equivalent martingale measure is unique, that is, whether there exists another probability measure under which the discounted gains processes associated with traded assets are martingales. This would, in general, imply different prices for options from the ones we have obtained in Section II. In this section, we demonstrate that the equivalent martingale measure is, in fact, unique for our model thereby implying completeness of the market. Therefore, any (suitably regular) contingent claim may be replicated by a self-financing strategy. Such a strategy involves investing in a portfolio in the underlying stocks, cash, and the baskets (or indices) representing the base prices of the respective stocks.

For notational simplicity, we shall assume that the dividend yields of both stocks are zero over the time horizon under consideration. By the results of Section II, the two stock price processes under the risk-neutral measure P can be described by the following equations for $0 \leq t < T$ and $t > T$, respectively,

$$\begin{aligned} dS_1(t) &= S_1(t-)[r\,dt + (\sigma_1' 1_{N(t)=0} + \sigma_1 1_{N(t)=1})\,dW_1(t) + (\beta_1(t-) - 1)\,dM(t)] \\ dS_2(t) &= S_2(t-)[r\,dt + \sigma_1' 1_{N(t)=0} dW_1(t) + \sigma_2 1_{N(t)=1} dW_2(t) \\ &\quad + (\beta_2(t-) - 1)\,dM(t)] \\ dS_1(t) &= S_1(t-)(1_{\Delta N(T)=1}[r\,dt + \sigma_1' dW_1(t)] + 1_{\Delta N(T)=0}[r\,dt + \sigma_1 dW_1(t)]) \\ dS_2(t) &= S_2(t-)(1_{\Delta N(T)=1}[r\,dt + \sigma_1' dW_1(t)] + 1_{\Delta N(T)=0}[r\,dt + \sigma_2 dW_2(t)]) \end{aligned} \tag{27}$$

where $M(t) = N(t) - \int_0^t \lambda 1_{N(s)=0} ds - 1_{t \geq T}(1_{N(T-)=0})$ are martingales. The process $M(\cdot)$ is the compensated jump process associated with the jump process $N(\cdot)$. The base price processes for both stocks are given by

$$\begin{aligned} dS_1^*(t) &= S_1^*(t)[r\,dt + \sigma_1' dW_1(t)] \\ dS_2^*(t) &= S_2^*(t)[r\,dt + \sigma_1' dW_1(t)] \end{aligned} \tag{28}$$

Again, for convenience, let us consider the problem of pricing and hedging contingent claims on the first stock.[11] From (27) and (28), we may see that in order to price contingent claims on the first stock, the relevant information is the information contained in the processes $W_1(\cdot)$ and $M(\cdot)$ (or alternatively $N(\cdot)$), that is, it suffices for us to consider the filtration of the probability space (Ω, F, P) generated by the Brownian motion $W_1(\cdot)$ and the jump process $N(\cdot)$. We shall abuse notation by referring to this filtration (complete and augmented) by F_t. Let the time horizon of our pricing/hedging problem be $[0, T_0]$, that is, we consider the problem of pricing and hedging contingent claims on the first stock with maturities less than or equal to T_0 where $T_0 \geq T$. We can then state the following proposition that provides a characterization for all square integrable F_t-martingales. The proposition also provides a complete characterization of all equivalent martingale measures in our market.

[11] The arguments can be easily generalized to consider contingent claims on the second stock or more complex claims involving both stocks.

PROPOSITION 5: *If Q is a square integrable F_{T_0} measurable random variable, it can be uniquely represented as*

$$Q = E[Q] + \int_0^t x(s)\,dW_1(s) + \int_0^t y(s)\,dM(s) \tag{29}$$

where $x(\cdot), y(\cdot)$ are F_t-predictable processes satisfying $E[\int_0^{T_0} x(s)^2 ds + (\int_0^{T_0} y(s)^2 d[M,M](s)] < \infty$ where $[M,M](\cdot)$ is the quadratic variation process of $M(\cdot)$ (for the definitions of quadratic variation and predictability see Protter (1995)).

Proof: The proof of Proposition 5 is given in the Appendix.

By the result of the above proposition, it follows that $\gamma(\cdot)$ is a square integrable F_t-martingale if and only if it has the unique representation

$$\gamma(t) = \gamma(0) + \int_0^t \eta(s)\,dW_1(s) + \int_0^t \varsigma(s)\,dM(s) \tag{30}$$

where $\eta(\cdot)$, $\varsigma(\cdot)$ are F_t-predictable processes satisfying

$$E\left[\int_0^{T_0} \eta(s)^2 ds + \int_0^{T_0} \varsigma(s)^2 d[M,M](s)\right] < \infty \tag{31}$$

Hence, any F_t-martingale can be expressed as the sum of stochastic integrals with respect to the Brownian motion $W_1(\cdot)$ and the compensated jump process $M(\cdot)$.

A. Uniqueness of the Equivalent Martingale Measure

Let P^* be the actual or real-world probability measure. By definition, P^* is equivalent to P, the risk-neutral measure under consideration. Any equivalent martingale measure P' for our market is clearly equivalent to the measure P since it is equivalent to P^* by definition. Due to the equivalence of the measures, the measure P' can be characterized in terms of the measure P by the strictly positive Radon–Nikodym density process $\gamma(t) = E[dP'/dP \mid F_t]$ (see Chapter 3 of Protter (1995)), that is, if A belongs to the σ-algebra F_t

$$P'(A) = \int_A \gamma(t)\,dP. \tag{32}$$

The process $\gamma(\cdot)$ is a martingale. To avoid unnecessary technicalities, we restrict ourselves to the situation where $\gamma(\cdot)$ is square integrable, that is, we consider the set of "square integrable" equivalent martingale measures.[12] Therefore, by the results obtained, we have the representation

[12] Every square integrable F_t-martingale $\gamma(\cdot)$ can be expressed as $\gamma(t) = E[\Gamma \mid F_t]$ where Γ is a square integrable random variable.

$$\gamma(t) = 1 + \int_0^t \gamma(s-)\eta(s)\,dW_1(s) + \int_0^t \gamma(s-)\varsigma(s)\,dM(s) \tag{33}$$

where $\eta(\cdot)$, $\varsigma(\cdot)$ are square integrable F_t-predictable processes and we have used the fact that $\gamma(\cdot)$ is a strictly positive process.[13] We can now state the following proposition that shows the uniqueness of the equivalent martingale measure in the market.

PROPOSITION 6: *If P' is an equivalent martingale measure with the Radon–Nikodym density process $\gamma(\cdot)$ characterized by* (32) *and* (33), *then $P' \equiv P$, that is, $\eta(\cdot) \equiv \varsigma(\cdot) \equiv 0$ a.e. Therefore, the equivalent martingale measure for our market is unique.*

Proof: The proof of Proposition 6 is given in the Appendix.

The intuition for the above result is that there are two sources of risk in our market: the *diffusion risk* generated by the Brownian motion $W_1(\cdot)$ and the *jump risk* generated by the jump process $M(\cdot)$. However, the investor may choose a portfolio in two independent securities $S_1(\cdot)$ and $S_1^*(\cdot)$ that span the risk in the market.[14] We may explicitly derive a trading strategy in the stock S_1 and the base basket S_1^* that perfectly hedges the payoff of any contingent claim. Since the arguments are quite well known, for the sake of brevity, we shall exclude this discussion here.

IV. Numerical Implementation and Empirical Tests

In the previous sections, we have developed a model to price options on the stocks of firms involved in merger or acquisition deals. *A priori*, the model has six basic parameters that are described below.

- σ_1: Volatility of stock 1 after the deal has been called off
- σ_2: Volatility of stock 2 after the deal has been called off
- σ: Volatility of both stocks when the deal is pending.
- λ: Parameter of jump process, that is, risk-neutral probability that the deal is called off over the interval $[t, t + dt] \subset [0, T)$ provided it hasn't been called off is $\lambda\,dt$.
- A_1, A_2: If the deal is called off at time t, the stock $i \in \{1, 2\}$ jumps by a factor $\beta_i(t) = A_i \exp(-\lambda t)/(1 + A_i \exp(-\lambda t))$

The parameters σ_1, σ_2 above may be chosen to be the historical volatilities of the two stocks before the deal was announced or by the at-the-money implied volatilities before the deal was announced. Hence, the model essentially depends on four parameters, $\sigma, \lambda, A_1, A_2$. Given the market prices of the two

[13] We have $\gamma(0) = 1$ due to the equivalence of the measures and the fact that F_0 contains all the null sets since the filtration F_t is assumed to be complete and augmented.

[14] We would like to emphasize here that it is not obvious at the outset that the securities $S_1(\cdot)$ and $S_1^*(\cdot)$ span the risk in the market. This result depends crucially on the *martingale representation* result of Proposition 5.

individual stocks, the parameters λ, A_1, A_2 are, however, related to each other by (20). The model therefore has three independent parameters.

The parameters of the model may be calibrated to the market prices of multiple options on both the stocks of varying strikes and maturities using minimum absolute deviation or least squares fitting. The calibrated parameters may then be used to predict theoretical prices for the options. This would also enable us to evaluate how well the model explains observed market prices of options on both stocks. Since equity options in the U.S. market are American and not European, we implement our model numerically using the standard binomial tree-based methodology to derive theoretical American option prices.[15]

A. Tests of the Model

We test the model on all stock for stock merger deals announced in the calendar year 2001 where both firms involved had listed options and for which historical option price data for both stocks were available.[16] Ours is a pricing model for the period between the deal announcement date and the date the deal either goes through or is called off. For each deal, we therefore test the model on each trading day between the deal announcement date and the date the deal either went through or was called off.

A.1. The Data

The data set used consists of daily stock and option price data for each company involved in a stock for stock merger deal announced during the calendar year 2001 where both the target and the acquirer had traded options. The daily stock price data for 2001 were obtained from the Center for Research in Security Prices (CRSP) and the daily option price data were obtained from T.B.S.P. Inc. (www.tbsp.com), a subscription-based independent market data vendor. For the deals that were pending in 2002, the daily stock and option price data were obtained from Charles Schwab and Co.

There were 26 deals for which we were able to obtain historical option price data. The deal maturity date used is the expected completion date of the deal on the date of announcement. For each deal and for each day, we collect the closing bid and ask quotes for the stocks. We then collect the associated option price quotes (mean of the closing bid and ask prices) for all near the money, short maturity options, (that is, options maturing in less than or equal to three months) that had at least one contract traded over the course of the day. Specifically, for each option maturity date, we pick call and put options with strike prices at or near the stock price, and strike prices immediately above and below this strike price so that we have a maximum of six options (three calls and three puts) for each maturity date. For each deal, our data set therefore consists of a maximum of 18 options on *each* stock for *each* trading day considered between the deal announcement date and the closing date or deal cancellation date.

[15] The details of the numerical implementation are available upon request.

[16] The detailed data on these deals were obtained from press releases.

As has been observed empirically, the prices of deep out of the money or in the money and long dated options display significant skew and kurtosis effects, that is, the Black–Scholes model is not entirely suitable in fitting the market prices of such options (Ball and Torous (1985)). Since our model is an extension of such a model, we use near the money and short maturity options in our tests since the Black–Scholes model performs well in explaining the prices of such options in the absence of any pending deal between the firms. Therefore, the Black–Scholes model is an appropriate benchmark against which to compare our model for the pricing of near the money, short maturity options on stocks involved in merger deals.

We choose the volatilities σ_1, σ_2, that is, the volatilities the two stocks would have if the deal were called off to be the average Black–Scholes implied volatilities of short maturity near the money options on the stocks over the three-month period before the deal was announced. The reasoning behind this choice is apparent from our theoretical framework. Once the deal is called off, the stock prices jump and subsequently evolve as they would normally do in the absence of any deal. Hence, after the deal is called off, short maturity near the money options on both stocks are priced using the traditional Black–Scholes model with the volatility parameter chosen to be the corresponding implied volatility.[17]

A.2. The Methodology

On each day for which option and stock price data is available, the model is calibrated, that is, the parameters of the model are fit to all the option prices. More precisely, we calculate the parameters of the model that minimize the sum of the magnitudes of the differences between the actual option prices and those predicted by the model. We then use the calibrated parameters to obtain the theoretical prices for the corresponding options. These theoretical prices are compared with the market prices for the options by calculating the *Percentage Error of Replication* defined as follows:

$$\frac{\sum_{i=1}^{N} \left| P^i_{theoretical} - P^i_{observed} \right|}{\sum_{i=1}^{N} P^i_{observed}} \tag{34}$$

where N is the total number of options on both the stocks, and $P^i_{theoretical}, P^i_{observed}$ are the theoretical (predicted by the model) and observed prices, respectively,

[17] In reality, however, the stock price volatilities after the deal is called off need not be equal to the volatilities before the deal announcement. In fact, Jayaraman, Mandelker, and Shastri (1991) show that the behavior of implied volatilities before a deal announcement appears to reflect the fact that the market at least partially anticipates the deal announcement. Although this issue does not affect our primary empirical results regarding the ability of the model to broadly explain option prices, it may be addressed by calibrating these additional model parameters to observed option price data.

of option i. We use the measure above since it represents the percentage error of replicating the average option.

We then compare the predictions of our model with the predictions of a traditional Black–Scholes model. For each deal the Black–Scholes model is fitted to the option prices for each stock involved in the deal. More precisely, we calculate the Black–Scholes implied volatilities for each of the stocks thereby obtaining two Black–Scholes parameters for each deal. We then calculate the Black–Scholes percentage error of replication exactly like we do for the model.

A.3. The Results

Table II presents the summarized results of all our tests. For each deal, we display the means and standard deviations of the replication errors of our model and the Black–Scholes model for the period between the deal announcement and deal closing or cancellation dates. We also display the means and standard deviations of the crucial model parameters σ (the volatility of both stocks after the deal is announced) and λ (the intensity of the jump process $N(\cdot)$). We see that for each deal, the model performs significantly better than the Black–Scholes model in explaining observed option price data. The mean (median) of the replication errors of the model for all deals is 8.92 percent (8.98 percent) and for the Black–Scholes model is 27.16 percent (26.12 percent). The results of t-tests indicate that the difference between the Black–Scholes and model errors is positive and significant at the 1 percent level (p-value ~ 0). We would like to emphasize here that our model has only one independent parameter more than the Black–Scholes model. These results demonstrate that the option prices predicted by our model are always significantly closer to observed option prices.

Although our model performs significantly better than the Black–Scholes model in explaining observed option prices on stocks involved in pending merger deals, it is plausible that this may be due to the fact that the model incorporates a possible jump in the underlying stock prices whether or not the jumps are associated with the deal being called off. However, this possibility is at least partly mitigated by the fact that the model assumes that the jumps in the two stock prices occur simultaneously and, moreover, have the specific analytical form derived in Proposition 4. These features are more applicable to the scenario where the jumps are associated with the cancellation of the pending deal.

The forecasted closing date or the deal maturity date is often only an estimated closing date; there may not be a definite date by which any specific deal would go through if it were not called off earlier.[18] The uncertainty associated with the deal date may have a significant effect on option prices and is therefore a potential source of error. Further, in several deals, the options on at least one of the stocks, especially the target stock, are quite illiquid. A partial equilibrium

[18] Moreover, in our tests, we chose the closing date as the expected closing date on the day the deal was announced. As time passes, the expected closing date of a deal is likely to change as more information becomes available.

Table II

Summarized Results of Tests of Model on Deals Announced in 2001

The table shows the **Mean** and **Stdev** of the percentage replication error of **our model** and the **Black–Scholes model** for each deal from the deal announcement date **Ann Date** to the **Effective Date** (the date the deal either went through or was called off). The table also shows the **Mean** and **Stdev** of the model parameters **Sigma** and **Lambda**. The **Deal Terms** are the number of shares of the combined firm or acquirer received for each share of the target. The **Forecasted Closing Date** is the approximate date the deal was expected to go through when it was announced.

Ann Date	Forecasted Closing Date	Effective Date	Target	Acquirer	Mean Sigma	Mean Lambda	Mean Model Error	Mean Black-Scholes Error	Stdev Sigma	Lambda Lambda	Stdev Model Error	Stdev Black-Scholes Error
1/23/2001	4/15/2001	3/23/2001	NTPA	PROX	96.88%	5.245	8.98%	44.24%	36.81%	3.114	2.03%	17.02%
1/29/2001	4/30/2001	4/11/2001	DS	MXIM	75.17%	4.966	6.05%	23.10%	16.58%	1.459	1.18%	7.13%
2/4/2001	9/30/2001	9/17/2001	TOS	P	43.47%	2.314	7.61%	26.12%	11.22%	1.536	1.59%	4.60%
3/12/2001	7/31/2001	7/10/2001	CATP	NOVL	21.34%	0.817	8.55%	21.94%	7.38%	0.705	4.24%	9.96%
3/13/2001	6/30/2001	6/1/2001	CIT	TYC	51.35%	5.124	9.83%	33.33%	10.98%	1.674	2.43%	5.42%
3/16/2001	8/31/2001	8/29/2001	BBC	AAS	52.10%	4.793	9.69%	28.96%	10.65%	3.003	1.81%	5.47%
3/22/2001	6/20/2001	6/8/2001	KNT	AVNT	68.72%	6.187	9.77%	35.07%	16.74%	3.739	1.81%	7.18%
3/27/2001	7/1/2001	6/22/2001	AZA	JNJ	45.82%	0.997	8.83%	27.32%	6.38%	0.582	2.61%	3.81%
4/16/2001	9/15/2001	9/4/2001	WB	FTU	39.20%	2.351	8.70%	19.62%	8.75%	1.880	2.42%	6.65%
5/7/2001	7/31/2001	7/24/2001	NIS	USB	51.23%	1.404	11.16%	16.16%	11.04%	0.985	1.87%	3.29%
5/15/2001	7/31/2001	7/19/2001	SAWS	TQNT	19.15%	7.831	9.59%	35.59%	4.72%	3.864	2.48%	3.07%
5/23/2001	9/30/2001	9/13/2001	MRL	PDE	38.90%	4.130	9.48%	25.01%	11.50%	2.005	1.79%	4.08%
6/25/2001	12/15/2001	12/14/2001	HM	ABX	28.83%	3.054	7.09%	13.69%	3.79%	2.001	1.53%	3.43%
6/29/2001	10/31/2001	10/24/2001	DRMD	BRL	33.76%	5.852	9.06%	35.78%	8.89%	2.632	2.24%	4.64%
7/16/2001	12/31/2001	12/6/2001	SCI	SNM	18.46%	4.688	10.41%	30.01%	4.93%	1.491	1.89%	3.98%
7/31/2001	11/30/2001	11/2/2001	SEM	VSH	42.61%	4.463	8.12%	25.75%	10.33%	1.667	1.54%	3.02%
8/2/2001	10/31/2001	10/29/2001	GEN	TER	18.95%	6.263	7.89%	29.70%	5.43%	2.967	2.57%	5.13%
8/3/2001	11/30/2001	11/14/2001	SRM	TYC	51.13%	1.027	11.99%	27.32%	6.98%	0.408	1.74%	3.12%
8/29/2001	1/31/2002	1/30/2002	W	MEA	63.93%	2.178	10.03%	20.26%	13.14%	1.436	2.38%	5.00%
9/4/2001	5/15/2002	4/25/2002	CPQ	HWP	45.56%	4.679	7.18%	18.42%	6.92%	1.367	1.60%	3.80%
9/4/2001	11/30/2001	11/20/2001	GLM	SDC	28.37%	9.006	9.50%	25.32%	5.45%	4.150	1.67%	4.60%
10/24/2001	5/31/2002	5/15/2002	PRIA	BRKS	46.48%	5.367	10.23%	39.60%	6.21%	1.710	2.03%	3.78%
11/18/2001	8/30/2002	8/30/2002	COC	P	44.23%	2.633	7.40%	15.60%	9.58%	1.867	1.20%	3.00%
12/3/2001	1/31/2002	1/15/2002	AVIR	MEDI	17.11%	6.699	8.43%	35.67%	2.45%	1.339	1.84%	3.71%
12/3/2001	5/17/2002	6/7/2002	AVNT	SNPS	36.07%	5.922	7.40%	25.40%	9.89%	2.445	1.80%	4.20%
12/6/2001	2/15/2002	2/14/2002	CORR	MLNM	43.91%	5.424	8.70%	31.00%	7.06%	1.455	1.50%	5.50%
Mean							8.92%	27.16%				
Median							8.98%	26.12%				

model of the type proposed in this paper would not, normally, be expected to explain the prices of such options with as much accuracy as the prices of options that are actively traded. Moreover, the bid-ask spreads in the prices of some options that are not highly liquid are very high (sometimes even 20 percent of the average of the bid-ask prices), and this is a potentially large source of error. In fact, our model might be useful as a normative tool in the *prediction* of the prices of highly illiquid options. Finally, the model proposed is an extension of the Black–Scholes constant volatility model that does not incorporate skew and kurtosis effects. We emphasize again that our primary objective is to propose a consistent parsimonious framework that broadly explains observed option prices. Such a framework could serve as the foundation for more complex and realistic models.

B. The Implied Probability of Success of the Deal

We can, under certain conditions, infer the probability that the market assigns to the success of the deal from implied parameters of our model. If E^* denotes expectation under the real-world probability measure P^* and E denotes expectation under the risk-neutral probability P, then the probability of success of the deal is $E^*[1_{\Delta N(T)=1}]$. If $\Gamma(\cdot)$ is the Radon–Nikodym density process of the measure P^* with respect to the measure P, we have

$$E^*[1_{\Delta N(T)=1}] = E[\Gamma(T)1_{\Delta N(T)=1}] \tag{35}$$

where

$$\Gamma(T) = 1 + \int_0^T \Gamma(s-)\pi(s)\,dW_1(s) + \int_0^T \Gamma(s-)\rho(s)\,dM(s)$$

We may therefore evaluate the expectation in (35) if we know the risk premia parameters $\pi(\cdot)$ and $\rho(\cdot)$. In the situation where the jump risk premium $\rho(\cdot) \equiv 0$, we have

$$E[\Gamma(T)1_{\Delta N(T)=1}] = E\left[\left(1 + \int_0^T \Gamma(s-)\pi(s)\,dW_1(s)\right)1_{\Delta N(T)=1}\right] \tag{36}$$

Since the processes $N(\cdot)$ and $W_1(\cdot)$ are *independent* under the risk-neutral probability P (see the description of the model in Section I), we have

$$E[\Gamma(T)1_{\Delta N(T)=1}] = E[\Gamma(T)]E[1_{\Delta N(T)=1}] = \exp(-\lambda T) \tag{37}$$

where we have used the fact that $E[\Gamma(T)] = 1$ since the measures P and P^* are probability measures and $E[1_{\Delta N(T)=1}] = \exp(-\lambda T)$. Thus, we see that in the situation where the jump risk premium is zero, the actual probability of success of the deal is equal to the risk-neutral probability of success of the deal.

Hence, our model can be used to infer the probability of success of a deal provided the jump risk premium is known. This is because the information contained in the market prices of options only reflects the *risk-neutral* probability of success of the deal through the parameter λ. In general, the only events on which the risk-neutral and actual probabilities agree are those with probabilities 0 or 1. Hence, without making assumptions about the jump risk premium, the only predictions we can make about the outcome of a merger deal are whether it will succeed or fail with certainty.

Under the assumption that the jump risk premium is zero, we now examine the ability of the model to provide information about the outcomes of pending deals. We expand our sample of unsuccessful deals by including those that failed in the previous two years. For each deal and for each day, we use the corresponding calibrated parameter λ to obtain the probability of success of the deal. For each deal, we divide the time period between the deal announcement date and the deal effective date into three approximately equal subperiods and compute the average probability of success over time for each subperiod. The motivation for this is twofold. First, it allows us to capture the effects of the early, middle, and late periods of a deal. Second, the time horizons of two different deals are appropriately scaled so that they become comparable. We also compute the average probabilities of success over the final month of the deal, and over the entire period of the deal. Our results are displayed in Table III.

Table III clearly indicates that for each successful deal, without any exception, the average probability of success increases across the subperiods. This implies that, according to our model, the market assigned greater probabilities to the success of each deal as time passed. The means of the probabilities of success over the first, middle, and final subperiods across all successful deals are 0.473, 0.602, and 0.751, respectively. The mean of the average probabilities of success for all successful deals over the final month, and over the entire period of the deal are 0.763 and 0.604, respectively. On the other hand, the average probabilities of success for the unsuccessful deals do not display a definite monotonic relationship across the three subperiods indicating that the market was not necessarily assigning increasing probabilities of success for the deals as time passed. The means of the averages over each subperiod, the final month, and over the entire deal for the unsuccessful deals are 0.251, 0.296, 0.359, 0.350, and 0.304, respectively. Hence, we see that the mean probabilities of success over each subperiod for all successful deals are greater than the corresponding mean success probabilities for unsuccessful deals. The results of t-tests indicate that these differences are statistically significant at the 1 percent level.

We further examine the model's ability to distinguish between successful and unsuccessful deals by testing the following hypothesis: *Over each subperiod, the average predicted probability of success for a pending deal that eventually succeeds is significantly greater than that for a deal that eventually fails*. For each subperiod, we consider the set of all ordered pairs (p_S, p_U) where p_S, p_U are the average probabilities of success over the subperiod for a successful and unsuccessful deal, respectively, in the sample. The results of sign tests show that, for each subperiod, the proportion of the entire population of possible

Table III

Implied Probabilities of Success of Deals

The table shows the probabilities of success of merger deals predicted by the model where Prob = $\exp(-\lambda^* T)$. We group the deals into successful and unsuccessful deals. For each deal, we display the average probability of success over the first, middle, and final third of the period between the deal announcement and effective date as well as over the final month and the entire period of the deal. We also display the value "q_{-max}" for each sub-period. The proportion of the population of pairs of successful and unsuccessful deals for which the average success probability for the successful deal over the sub-period is greater than that of the unsuccessful deal is greater than q_{-max} at the 5 percent level.

Successful Deals							Unsuccessful Deals						
Target	Acquirer	Average Success Prob (First Third)	Average Success Prob (Middle Third)	Average Success Prob (Final Third)	Average Success Prob (Final Month)	Overall Average	Target	Acquirer	Average Success Prob (First Third)	Average Success Prob (Middle Third)	Average Success Prob (Final Third)	Average Success Prob (Final Month)	Overall Average
DS	MXIM	0.480	0.641	0.786	0.740	0.610	NTPA	PROX	0.327	0.285	0.264	0.290	0.298
TOS	P	0.557	0.793	0.950	0.910	0.810	PRXL	CVD	0.245	0.190	0.301	0.259	0.249
CATP	NOVL	0.785	0.804	0.950	0.930	0.860	AZA	ABT	0.292	0.433	0.410	0.435	0.379
CIT	TYC	0.398	0.632	0.742	0.740	0.580	NR	TBI	0.274	0.296	0.490	0.459	0.352
BBC	AAS	0.380	0.437	0.569	0.600	0.430	CYM	AR	0.298	0.253	0.490	0.491	0.358
KNT	AVNT	0.604	0.646	0.756	0.740	0.680	WLA	AHP	0.010	0.048	0.072	0.077	0.041
AZA	JNJ	0.813	0.851	0.946	0.940	0.870	GLIA	GLFD	0.147	0.468	0.520	0.506	0.374
SEM	VSH	0.485	0.582	0.789	0.770	0.590	REL	LUK	0.493	0.474	0.595	0.527	0.518
WB	FTU	0.431	0.681	0.861	0.910	0.680	FSCO	ZION	0.176	0.215	0.090	0.106	0.169
NIS	USB	0.685	0.932	0.978	0.970	0.870							
SAWS	TQNT	0.443	0.601	0.690	0.640	0.570							
MRL	PDE	0.379	0.613	0.763	0.770	0.550							
HM	ABX	0.403	0.568	0.710	0.820	0.580							
DRMD	BRL	0.249	0.416	0.589	0.580	0.380							

Table III—*Continued*

Successful Deals							Unsuccessful Deals						
Target	Acquirer	Average Success Prob (First Third)	Average Success Prob (Middle Third)	Average Success Prob (Final Third)	Average Success Prob (Final Month)	Overall Average	Target	Acquirer	Average Success Prob (First Third)	Average Success Prob (Middle Third)	Average Success Prob (Final Third)	Average Success Prob (Final Month)	Overall Average
GEN	TER	0.444	0.485	0.633	0.620	0.520							
W	MEA	0.467	0.603	0.790	0.790	0.610							
CPQ	HWP	0.159	0.231	0.584	0.770	0.330							
GLM	SDC	0.327	0.468	0.651	0.640	0.510							
COC	P	0.290	0.346	0.580	0.820	0.405							
AVIR	MEDI	0.682	0.792	0.943	0.850	0.830							
CORR	MLNM	0.502	0.561	0.718	0.670	0.580							
AVNT	SNPS	0.250	0.457	0.620	0.740	0.440							
SRM	TYC	0.852	0.851	0.927	0.910	0.870							
SCI	SNM	0.291	0.451	0.665	0.730	0.450							
Mean		0.473	0.602	0.751	0.763	0.604			0.251	0.296	0.359	0.350	0.304
Median		0.443	0.602	0.749	0.770	0.580			0.274	0.285	0.410	0.435	0.352
q_{-max}		0.833	0.907	0.963	0.981								

pairs of successful and unsuccessful deals for which the average probability of success of the successful deal is greater than that of the unsuccessful deal over the subperiod, is significantly greater than 0.5 at the 1 percent level.

To obtain stronger evidence in support of our hypothesis, we now examine the null hypothesis that the distribution of the signs of the differences between the probabilities of success for a successful and unsuccessful deal is an *asymmetric* binomial distribution with parameter $q > 0.5$. We determine the maximum value of q, say $q_{\max}$, for which the null may be rejected at the 5 percent level. This implies that, for the subperiod, the proportion of the total population of deal pairs for which the average probability of success of the successful deal is greater than that of the unsuccessful deal, is greater than $q_{\max}$ at the 5 percent confidence level. The values for $q_{\max}$, 0.833, 0.907, and 0.963 for the successive subperiods displayed in the table provide further support for the hypothesis that the model is able to distinguish between a deal that is successful, and one that fails, by assigning a significantly greater average probability of success to the successful deal, even in the early periods of the deals. Moreover, these proportions increase over the subperiods.These results provide strong support for the hypothesis that the model is able to distinguish between a pending deal that eventually succeeds and one that eventually fails. Further, as one would expect, the ability to distinguish between successful and unsuccessful deals improves as time passes. Therefore, option prices do appear to reflect the market's perception of the outcome of a pending deal.

V. Conclusions

In this paper, we present an arbitrage-free and complete framework to price options on the stocks of firms involved in a merger or acquisition deal with the possibility that the deal might be called off thus creating discontinuous impacts on the prices of one or both stocks. We model the stock price processes as jump diffusions and derive explicit relationships between the parameters of the stock price processes. Under the assumption that the jumps are uniformly bounded, we show that they must have specific functional forms to preclude the existence of arbitrage in the market. We use these results to derive analytical formulas for European option prices. We test our model on option data and demonstrate that it performs significantly better than the Black–Scholes framework in explaining observed option prices on stocks involved in merger deals. We also test the model's ability to use observed option prices to infer the outcome of a pending deal. We find that it is able to distinguish between successful and unsuccessful deals even in the early periods of deals. These results suggest that option prices appear to reflect the market's perception of the outcome of a pending deal.

Results similar to those of propositions 2 and 3 may be obtained for more general types of deals, that is, cash deals, collar deals, or combination (stock and cash) deals. The only differences arise in the nature of the boundary conditions on the stock price processes (due to the specific structure of the deal) that affect the determination of the parameters of the jump processes. The model can also be used to help risk arbitrageurs employ options in pursuing their objectives.

Just as recent work has focused on the implications of ruling out "good bets" or "acceptable opportunities" for option prices, it might also be interesting to understand the implications of no risk arbitrage for option pricing.[19]

Finally, it would be interesting to consider the market around the merger announcement, that is, prior to and immediately after the merger announcement, and build a theoretical framework for the determination of *both* stock and option prices. This study of the interaction between stock and option markets would probably necessitate an equilibrium framework as opposed to the partial equilibrium framework adopted in this paper.

Appendix

Proof of Proposition 2: Let $\gamma(t) = S_2(t)/S_1(t)$. From equations (1) and (2), we easily see that in the situation where the deal has not been called off, we must have

$$\begin{aligned}\gamma(t)1_{N(t)=0} = \gamma(0)1_{N(t)=0}\exp\Bigg[&\int_0^t (\mu_2(s-) - d_2 - \mu_1(s-) + d_1)\,ds \\ &+ \big((1/2)(\sigma_2' - \sigma_1')^2 - (1/2)\big(\sigma_2'^2 - \sigma_1'^2\big) - (1/2)\sigma_2''^2\big)t \\ &+ (\sigma_2' - \sigma_1')W_1(t) + \sigma_2'' W_2(t)\Bigg] \qquad \text{(A1)}\end{aligned}$$

By condition (13), we have $\gamma(T)1_{\Delta N(T)=1} = \alpha 1_{\Delta N(T)=1}$ a.s. Therefore

$$\begin{aligned}\log(\alpha/\gamma(0))1_{\Delta N(T)=1} = 1_{\Delta N(T)=1}\int_0^T &\big(\mu_2(s-) - d_2 - \mu_1(s-) + d_1 \\ &+ (1/2)\big(\sigma_2' - \sigma_1'\big)^2 - (1/2)\big(\sigma_2'^2 - \sigma_1'^2\big) - (1/2)\sigma_2''^2\big)\,ds \\ &+ (\sigma_2' - \sigma_1')W_1(T) + \sigma_2'' W_2(T) \qquad \text{(A2)}\end{aligned}$$

Since W_1 and W_2 are independent Brownian motions and $\sigma_1', \sigma_2', \sigma_2'', d_1, d_2$ are constants, we see that the above condition implies that

$$1_{\Delta N(T)=1}\int_0^T (\mu_2(s-) - \mu_1(s-))\,ds = [C + (\sigma_2' - \sigma_1')W_1(T) + \sigma_2'' W_2(T)]1_{\Delta N(T)=1} \qquad \text{(A3)}$$

where C is a bounded constant. Since $\beta_1(\cdot), \beta_2(\cdot)$ are uniformly bounded, $\mu_1(\cdot)$ and $\mu_2(\cdot)$ are bounded by the result of proposition 1. Therefore, if $\sigma_2' \neq \sigma_1'$ or $\sigma_2'' \neq 0$, the above condition implies that a nonzero linear combination of independent normal random variables is bounded, which is false. Therefore, we must have $\sigma_2'' = 0$ and $\sigma_1' = \sigma_2'$. This completes the proof. Q.E.D.

[19] We thank the referee for emphasizing this point.

Proof of Proposition 3: By (10) and (14), we have for $0 \le t < T$,

$$S_1(t)1_{N(t)=0} = S_1(0)1_{N(t)=0} \exp\left[\int_0^t \left(r - d_1 + \lambda(1-\beta_1(s-)) - \frac{1}{2}\sigma_1'^2\right) ds + \sigma_1' W_1(t)\right] \quad \text{(A4)}$$

By (11), we have

$$S_1^*(t) = S_1^*(0)\exp\left[\left(r - d_1 - \tfrac{1}{2}\sigma_1^{*^2}\right)t + \sigma_1^* W_1^*(t)\right] \quad \text{(A5)}$$

By hypothesis,

$$\beta_1(t-)S_1(t-)1_{N(t-)=0} = S_1^*(t)1_{N(t-)=0} \quad \text{(A6)}$$

Therefore,

$$\beta_1(t-)1_{N(t-)=0} = \frac{S_1^*(0)}{S_1(0)}1_{N(t-)=0}\exp\left[\int_0^t \left(\frac{1}{2}\left(\sigma_1'^2 - \sigma_1^{*2}\right) - \lambda(1-\beta_1(s-))\right) ds + \sigma_1^* W_1^*(t) - \sigma_1' W_1(t)\right] \quad \text{(A7)}$$

Since σ_1', σ_1^* are bounded constants and $\beta_i(\cdot)$ is uniformly bounded, we see that equality cannot hold in the expression above if $W_1^*(\cdot) \neq W_1(\cdot)$ and $\sigma_1^* \neq \sigma_1'$ as the left-hand side in the expression above is bounded, and the right-hand side in the expression above would be unbounded due to the fact that a Brownian motion can take unbounded values with positive probability. This contradiction implies that $W_1^*(\cdot) \equiv W_1(\cdot)$ and $\sigma_1^* = \sigma_1'$. Moreover, we now easily conclude that $\beta_1(\cdot)$ must be a deterministic function of time in the situation where the deal has not been called off (see the next proposition). We can use similar arguments to show that $W_2^*(\cdot) \equiv W_2, \sigma_2' = \sigma_2'^*, \sigma_2'' = \sigma_2''^*$ and that $\beta_2(\cdot)$ must also be a deterministic function of time. This completes the proof. Q.E.D.

Proof of Proposition 4: By the definition of the base price processes and the results of the previous propositions, we have

$$\beta_i(t-)1_{N(t-)=0}S_i(0)\exp\left[\int_0^t \mu_i(s-)\,ds - (1/2)\sigma_1'^2 t + \sigma_1' W_1(t)\right] = 1_{N(t-)=0}S_i^*(0)\exp\left[\left(r - (1/2)\sigma_1'^2 t\right) + \sigma_1' W_1(t)\right]. \quad \text{(A8)}$$

Since $\mu_i(s-)1_{N(s-)=0} = (r + \lambda(1-\beta_i(s-))1_{N(s-)=0}$, we easily see that we must have

$$1_{N(t-)=0}\log(\beta_i(t-)) = -1_{N(t-)=0}\left[\int_0^t \lambda(1-\beta_i(s-))\,ds + C_0\right] \tag{A9}$$

where $C_0 = \log(S_i(0)/S_i^*(0))$ is a constant. Therefore, (dropping the factor $1_{N(t-)=0}$ for notational convenience)

$$d\beta_i = -\lambda\beta_i(t)(1-\beta_i(t))\,dt. \tag{A10}$$

Therefore,

$$\int \frac{d\beta_i}{\lambda\beta_i(1-\beta_i)} = -t + C \text{ where } C \text{ is a constant .} \tag{A11}$$

Hence,

$$\frac{\beta_i}{1-\beta_i} = A\exp[-\lambda t] \text{ where } A \text{ is a constant.} \tag{A12}$$

Therefore,

$$\beta_i(t-)1_{N(t-)=0} = \frac{A\exp[-\lambda t]}{1+A\exp[-\lambda t]}1_{N(t-)=0} \tag{A13}$$

This completes the proof. Q.E.D.

Proof of Proposition 5: We start by recalling that the probability space $\Omega = \Omega' \times \Omega''$, with the Brownian motion $W_1(\cdot)$ defined on Ω' and the jump process $N(\cdot)$ defined on Ω'', and the filtration F_t is the completed and augmented product filtration $F_t^{W_1} \otimes F_t^N$. Let Π be the set of square integrable F_{T_0} random variables that have a representation in the form (29).

If $A(\omega')$ is any square integrable $F_{T_0}^{W_1}$-random variable, the martingale representation theorem for Brownian motion (see Protter (1995)) implies the unique representation

$$A = E[A] + \int_0^t \eta(s)\,dW_1(s) \quad \text{with} \quad E\left[\int_0^t \eta(s)^2\,ds\right] < \infty. \tag{A14}$$

Let $B(\omega'')$ be any square integrable $F_{T_0}^N$ random variable. Let τ be the jump time of the process $N(\cdot)$. We can see that τ is an F_t^N-stopping time and that the $\sigma-$ algebra $F_{T_0}^N$ is generated by the events $\{\tau \le t; t < T\}$ and the singleton $\{\tau = T\}$. Any $F_{T_0}^N$-measurable random variable B can therefore be expressed in the form $B = f(\tau) = \int_0^{T_0} f(s)\,dN(s)$ where $f(\cdot)$ is a Borel measurable function on $[0, T_0]$. We now note that

$$\begin{aligned}\Phi(t) &= E\left[B \mid F_t^N\right] = E\left[\int_0^{T_0} f(s)\,dN(s)\middle| F_t^N\right] = \int_0^t f(s)\,dN(s) + 1_{N(t)=0} \\ &\quad \times \left[\int_t^T \lambda f(s)\exp[-\lambda(s-t)]\,ds + f(T)\exp(-\lambda(T-t))\right] \\ &= \int_0^t f(s)\,dN(s) + 1_{N(t)=0} \\ &\quad \times \exp(\lambda t)\left[\int_t^T \lambda f(s)\exp[-\lambda s]\,ds + f(T)\exp(-\lambda T)\right] \end{aligned} \tag{A15}$$

Therefore, we see that

$$\begin{aligned} d\Phi(t) &= f(t)\,dN(t) - dN(t)\exp(\lambda t)\left[\int_t^T \lambda f(s)\exp[-\lambda s]\,ds + f(T)\exp(-\lambda T)\right] \\ &\quad + 1_{N(t-)=0}\lambda\exp(\lambda t)\left[\int_t^T \lambda f(s)\exp[-\lambda s]\,ds\right. \\ &\quad \left. + f(T)\exp(-\lambda T)\right] dt - 1_{N(t-)=0}\lambda f(t)\,dt \end{aligned} \tag{A16}$$

where we have used the fact that $1_{N(t)=0} = 1 - N(t)$. If we set

$$g(t) = f(t) - \exp(\lambda t)\left[\int_t^T \lambda f(s)\exp[-\lambda s]\,ds + f(T)\exp(-\lambda T)\right] \tag{A17}$$

we see that

$$d\Phi(t) = g(t)\,dN(t) - 1_{N(t-)=0}(\lambda g(t))\,dt = g(t)\,dM(t) \tag{A18}$$

by the definition of the compensated jump process $M(\cdot)$. We now note that

$$\begin{aligned} \Phi(0) &= E\left[\int_0^{T_0} f(s)\,dN(s)\right] = E[B] \\ \Phi(T_0) &= B. \end{aligned} \tag{A19}$$

Therefore, from the fact that $d\Phi(t) = g(t)\,dM(t)$, we have

$$B = E[B] + \int_0^{T_0} g(s)\,dM(s). \tag{A20}$$

If the F_{T_0}-random variable Q can be expressed as a product $Q = A(\omega')B(\omega'')$, we see from (A14), (A20) and Ito's lemma that

$$
\begin{aligned}
Q &= E[A]E[B] + E[A]\int_0^{T_0} g(s)\,dM(s) + E[B]\int_0^{T_0} \eta(s)\,dW_1(s) \\
&\quad + \int_0^{T_0} \eta(s)\,dW_1(s)\int_0^{T_0} g(s)\,dM(s) \\
&= E[Q] + E[A]\int_0^{T_0} g(s)\,dM(s) + E[B]\int_0^{T_0} \eta(s)\,dW_1(s) \qquad \text{(A21)} \\
&\quad + \int_0^{T_0}\left(\int_0^{t} \eta(s)\,dW_1(s)\right)_{-} g(t)\,dM(t) \\
&\quad + \int_0^{T_0}\left(\int_0^{t} g(s)\,dM(s)\right)_{-} \eta(t)\,dW_1(t) + \int_0^{T_0} g(s)\eta(s)\,d[W_1, M](s).
\end{aligned}
$$

Since $[W_1, M](\cdot) \equiv 0$, we easily see that Q can be represented in the form (29) with $x(\cdot), y(\cdot)$ predictable. The set of random variables Q that have the product representation above is dense in the space $L^2(\Omega; F_{T_0})$. If $P \in \Pi$, we see that

$$
\|P\|_2^2 = E[P]^2 + E\left[\int_0^{T_0} x(s)^2 ds + \int_0^{T_0} y(s)^2 d[M, M](s).\right. \qquad \text{(A22)}
$$

Thus, the map $P \rightarrow E[P] + \int_0^{T_0} x(s)\,dW_1(s) + \int_0^{T_0} y(s)\,dM(s)$ is an isometry from Π into $L^2(\Omega; F_{T_0})$. This implies that the representation (29) exists for all elements of $L^2(\Omega; F_{T_0})$. The uniqueness of the representation follows easily from the fact that the map above is an L^2 isometry. Q.E.D.

Proof of Proposition 6: Since P' is an equivalent martingale measure by hypothesis, by definition, $\exp(-r\cdot)\gamma(\cdot)S_1(\cdot)$ and $\exp(-r\cdot)\gamma(\cdot)S_1^*(\cdot)$ must be F_t -martingales. We can apply the general form of Ito's lemma (see p. 71 of Protter (1995)) to (27) and (28) to conclude that

$$
\begin{aligned}
d(\gamma(t)S_1(t)) &= \gamma(t-)\,dS_1(t) + S_1(t-)\,d\gamma(t) + d[\gamma, S_1](t) \\
&= \gamma(t-)S_1(t-)[r\,dt + (\sigma_1 1_{N(t)=1} + \sigma_1' 1_{N(t)=0})\,dW_1(t) \\
&\quad + (\beta_1(t-) - 1)\,dM(t)] + S_1(t-)\gamma(t-)[\eta(t)\,dW_1(t) + \varsigma(t)\,dM(t)] \\
&\quad + \gamma(t-)S_1(t-)[(\sigma_1 1_{N(t)=1} + \sigma_1' 1_{N(t)=0})\eta(t)\,dt \\
&\quad + \varsigma(t)(\beta_1(t-) - 1)\,dN(t)]; \quad 0 \le t < T \\
d(\gamma(t)S_1^*(t)) &= \gamma(t-)S_1^*(t-)[r\,dt + \sigma_1'\,dW_1(t)] + S_1^*(t-)\gamma(t-)[\eta(t)\,dW_1(t) \\
&\quad + \varsigma(t)\,dM(t)] + \gamma(t-)S_1^*(t-)\sigma_1'\eta(t)\,dt. \qquad \text{(A23)}
\end{aligned}
$$

In the above, we have used the fact that $d[W_1, M] = 0$ and $d[M, M] = dN$ for $0 \leq t < T$.

Since $dN(t) = dM(t) + \lambda 1_{N(t-)=0}\, dt$ and $M(\cdot)$ is a martingale, we see from the expressions above that $\exp(-r\cdot)\gamma(\cdot)S_1(\cdot)$ and $\exp(-r\cdot)\gamma(\cdot)S_1^*(\cdot)$ are both F_t local martingales if and only if

$$\left.\begin{aligned} \eta(t)(\sigma_1 1_{N(t)=1} + \sigma_1' 1_{N(t)=0}) + \lambda 1_{N(t-)=0}\varsigma(t)(\beta_1(t-) - 1) &= 0 \\ \eta(t)\sigma_1' &= 0 \end{aligned}\right\} \quad \text{for} \quad 0 \leq t < T. \tag{A24}$$

and

$$\left.\begin{aligned} \eta(t)(\sigma_1 1_{\Delta N(T)=0} + \sigma_1' 1_{\Delta N(T)=1}) &= 0 \\ \eta(t)\sigma_1' &= 0 \end{aligned}\right\} \quad \text{for} \quad t \geq T \tag{A25}$$

which implies that $\eta(t) = \varsigma(t)1_{N(t-)=0} = 0$ for all t. From (33), we now easily see that $P' \equiv P$ and the equivalent martingale measure is unique. This completes the proof. Q.E.D.

Proof of Lemma 1:

Integration by parts gives

$$\begin{aligned} &\int_0^\tau dt\, e^{-\omega t} N\left(\frac{x - \rho t}{\sqrt{t}}\right) \\ &= -\frac{e^{-\omega t}}{\omega} N\left(\frac{x - \rho t}{\sqrt{t}}\right)\Bigg|_0^\tau + \frac{1}{\omega}\int_0^\tau dt\, e^{-\omega t} N'\left(\frac{x - \rho t}{\sqrt{t}}\right)\left(\frac{-x}{2t^{3/2}} - \frac{\rho}{2\sqrt{t}}\right) \\ &= \frac{1}{\omega}\left\{\frac{1 + \operatorname{sgn}(x)}{2} - e^{-\omega\tau} N\left(\frac{x - \rho\tau}{\sqrt{\tau}}\right)\right\} - \frac{x}{2\omega}\int_0^\tau dt\, e^{-\omega t} N'\left(\frac{x - \rho t}{\sqrt{t}}\right)\frac{1}{t^{3/2}} \\ &\quad - \frac{\rho}{2\omega}\int_0^\tau dt\, e^{-\omega t} N'\left(\frac{x - \rho t}{\sqrt{t}}\right)\frac{1}{\sqrt{t}}. \end{aligned} \tag{A26}$$

We use the identities

$$\begin{aligned} e^{-\omega t} N'\left(\frac{x - \rho t}{\sqrt{t}}\right) &= e^{(\rho - \xi)x} N'\left(\frac{x - \xi t}{\sqrt{t}}\right); \quad \xi = \sqrt{2\omega + \rho^2} \\ N'\left(\frac{x - \xi t}{\sqrt{t}}\right) &= e^{2\xi x} N'\left(\frac{x + \xi t}{\sqrt{t}}\right) \end{aligned} \tag{A27}$$

to evaluate both the integrals above. Using (A27) we have

$$
\begin{aligned}
-\frac{x}{2\omega}\int_0^\tau dt\, e^{-\omega t} N'\left(\frac{x-\rho t}{\sqrt{t}}\right)\frac{1}{t^{3/2}}
&= \frac{e^{(\rho-\xi)x}}{\omega}\int_0^\tau dt N'\left(\frac{x-\xi t}{\sqrt{t}}\right)\frac{-x}{2t^{3/2}} \\
&= \frac{e^{(\rho-\xi)x}}{2\omega}\int_0^\tau dt N'\left(\frac{x-\xi t}{\sqrt{t}}\right)\left(\frac{-x}{2t^{3/2}}-\frac{\xi}{2\sqrt{t}}\right) \\
&\quad + \frac{e^{(\rho+\xi)x}}{2\omega}\int_0^\tau dt N'\left(\frac{x+\xi t}{\sqrt{t}}\right)\left(\frac{-x}{2t^{3/2}}+\frac{\xi}{2\sqrt{t}}\right) \\
&= \frac{e^{(\rho-\xi)x}}{2\omega} N\left(\frac{x-\xi t}{\sqrt{t}}\right)\Bigg|_0^\tau + \frac{e^{(\rho+\xi)x}}{2\omega} N\left(\frac{x+\xi t}{\sqrt{t}}\right)\Bigg|_0^\tau \\
&= \frac{e^{(\rho-\xi)x}}{2\omega} N\left(\frac{x-\xi\tau}{\sqrt{\tau}}\right) + \frac{e^{(\rho+\xi)x}}{2\omega} N\left(\frac{x+\xi\tau}{\sqrt{\tau}}\right) \\
&\quad - \frac{e^{(\rho-\xi)x}+e^{(\rho+\xi)x}}{4\omega}(1+\operatorname{sgn}(x)).
\end{aligned}
\tag{A28}
$$

Again, using (A27), we have

$$
\begin{aligned}
-\frac{\rho}{2\omega}\int_0^\tau dt\, e^{-\omega t} N'\left(\frac{x-\rho t}{\sqrt{t}}\right)\frac{1}{\sqrt{t}}
&= \frac{\rho e^{(\rho-\xi)x}}{\xi\omega}\int_0^\tau dt N'\left(\frac{x-\xi t}{\sqrt{t}}\right)\frac{-\xi}{2\sqrt{t}} \\
&= \frac{\rho e^{(\rho-\xi)x}}{2\xi\omega}\int_0^\tau dt N'\left(\frac{x-\xi t}{\sqrt{t}}\right)\left(\frac{-x}{2t^{3/2}}-\frac{\xi}{2\sqrt{t}}\right) \\
&\quad + \frac{\rho e^{(\rho+\xi)x}}{2\xi\omega}\int_0^\tau dt N'\left(\frac{x+\xi t}{\sqrt{t}}\right)\left(\frac{x}{2t^{3/2}}-\frac{\xi}{2\sqrt{t}}\right) \\
&= \frac{\rho e^{(\rho-\xi)x}}{2\xi\omega} N\left(\frac{x-\xi t}{\sqrt{t}}\right)\Bigg|_0^\tau - \frac{\rho e^{(\rho+\xi)x}}{2\xi\omega} N\left(\frac{x+\xi t}{\sqrt{t}}\right)\Bigg|_0^\tau \\
&= \frac{\rho e^{(\rho-\xi)x}}{2\xi\omega} N\left(\frac{x-\xi\tau}{\sqrt{\tau}}\right) - \frac{\rho e^{(\rho+\xi)x}}{2\xi\omega} N\left(\frac{x+\xi\tau}{\sqrt{\tau}}\right) \\
&\quad + \frac{\rho\left(e^{(\rho+\xi)x}-e^{(\rho-\xi)x}\right)}{4\xi\omega}(1+\operatorname{sgn}(x)).
\end{aligned}
\tag{A29}
$$

From (A28) and (A29), we finally obtain

$$\begin{aligned}\int_0^\tau dt\, e^{-\omega t} N\left(\frac{x-\rho t}{\sqrt{t}}\right) = \frac{1}{\omega}\left\{\frac{1+\operatorname{sgn}(x)}{2} - e^{-\omega\tau} N\left(\frac{x-\rho\tau}{\sqrt{\tau}}\right)\right\}\\ + \frac{e^{(\rho-\xi)x}}{2\omega} N\left(\frac{x-\xi\tau}{\sqrt{\tau}}\right) + \frac{e^{(\rho+\xi)x}}{2\omega} N\left(\frac{x+\xi\tau}{\sqrt{\tau}}\right)\\ - \frac{e^{(\rho-\xi)x}+e^{(\rho+\xi)x}}{4\omega}(1+\operatorname{sgn}(x))\\ + \frac{\rho e^{(\rho-\xi)x}}{2\xi\omega} N\left(\frac{x-\xi\tau}{\sqrt{\tau}}\right) - \frac{\rho e^{(\rho+\xi)x}}{2\xi\omega} N\left(\frac{x+\xi\tau}{\sqrt{\tau}}\right)\\ + \frac{\rho\left(e^{(\rho+\xi)x}-e^{(\rho-\xi)x}\right)}{4\xi\omega}(1+\operatorname{sgn}(x)).\end{aligned}$$

This completes the proof of the lemma. Q.E.D.

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