APLICACIONES

Ley de Gauss

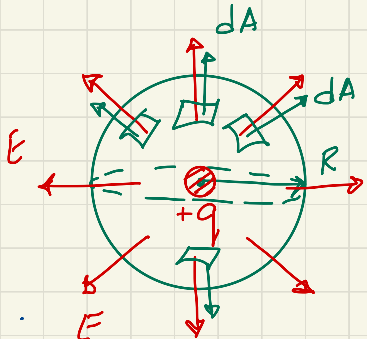
--- simetría ---

ESFÉRICA Y CILÍNDRICA

EJERCICIOS
• PARA RESOLVER •
EN CLASE

ing. Claudia
Contreras

Ley de Gauss



$$\vec{E} = \frac{kq}{R^2}\hat{r}$$

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E\,dA\cos 0°$$

$$= \frac{kQ}{R^2}\oint dA = \frac{kQ}{R^2}A = \frac{kQ}{\cancel{R^2}} * 4\pi \cancel{R^2}$$

$$= kQ\,4\pi \qquad \text{recuerde} \quad k = \frac{1}{4\pi\varepsilon_0}$$

$$= \frac{1}{\cancel{4\pi}\varepsilon_0} * Q\,\cancel{4\pi} = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$$

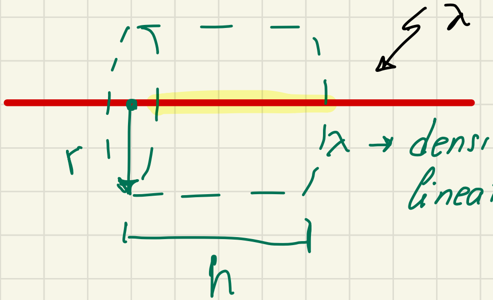
$$\boxed{\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}}$$

Ley de Gauss

	esf.	cilindro
Área	$4\pi r^2$	$2\pi r h$
Volumen	$\frac{4}{3}\pi r^3$	$\pi r^2 h$

Tipos de distribuciones de carga

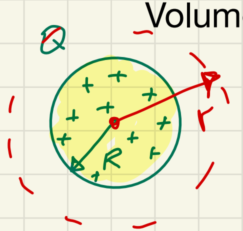
Lineales



$\lambda \rightarrow$ densidad lineal

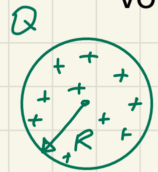
$$q_{enc} = \lambda \, \text{longitud} = \lambda h$$

Volumétricas



$$q_{enc} = \rho \frac{4}{3}\pi R^3$$

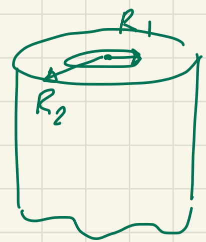
Volumétricas



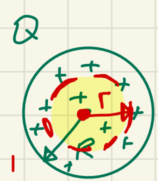
$\rho \rightarrow$ densidad volumétrica de carga

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} \quad \frac{C}{m^3}$$

$$\rho = \frac{Q}{\pi R^2 h} \quad \frac{C}{m^3}$$



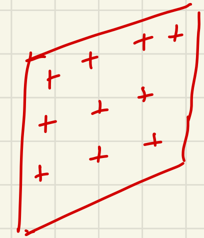
$$\rho = \frac{Q}{\pi R_2^2 h - \pi R_1^2 h}$$



$$q_{enc} = \rho \, \text{Volumen}$$

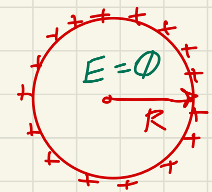
$$q_{enc} = \rho \frac{4}{3}\pi r^3$$

Superficiales

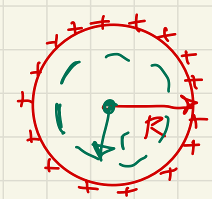


σ = densidad superficial

$$\sigma = \frac{Q}{\text{Area}} \quad \frac{C}{m^2}$$



$$\sigma = \frac{Q}{4\pi R^2}$$

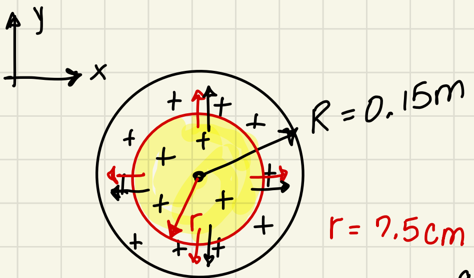


$$q_{enc} = \sigma \, \text{Area}$$

$$\rightarrow q_{enc} = 0$$

Problema 1.

Una esfera aislante con un radio de $0.15m$, tiene una densidad uniforme de carga de $\rho = 7.50nC/m^3$. $= \dfrac{Q}{\frac{4}{3}\pi R^3}$
Calcule el campo eléctrico a las siguientes distancias de su centro: a) inmediatamente afuera de su superficie. b) a $0.30m$ c) en el interior a $0.075m$



c) $E\,(r = 0.075m)$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

izq de la ecuación SIEMPRE AREA GAUSIANA

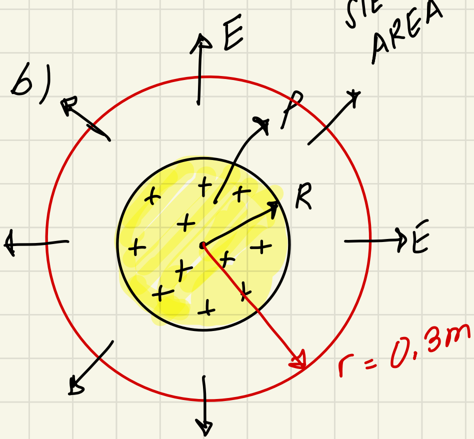
$$E(\cancel{4\pi} r^{\cancel{2}}) = \frac{\rho\, \frac{\cancel{4}}{3}\cancel{\pi}\, r^{\cancel{3}}}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho r}{3\varepsilon_0}\,\hat{r}$$

$$\vec{E} = \frac{(7.5\times10^{-9})(0.075)}{3(8.85\times10^{-12})}$$

$$\vec{E} = 21.19\,\frac{N}{C}\,\hat{r}$$

b)



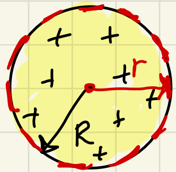
$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(\cancel{4\pi} r^2) = \frac{\rho\left(\frac{\cancel{4}}{3}\cancel{\pi} R^3\right)}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho R^3}{3\varepsilon_0 r^2}\,\hat{r}$$

$$\vec{E} = \frac{7.5\times10^{-9}(0.15)^3}{3(8.85\times10^{-12})(0.3)^2}$$

$$\vec{E} = 10.59\,\frac{N}{C}\,\hat{r}$$

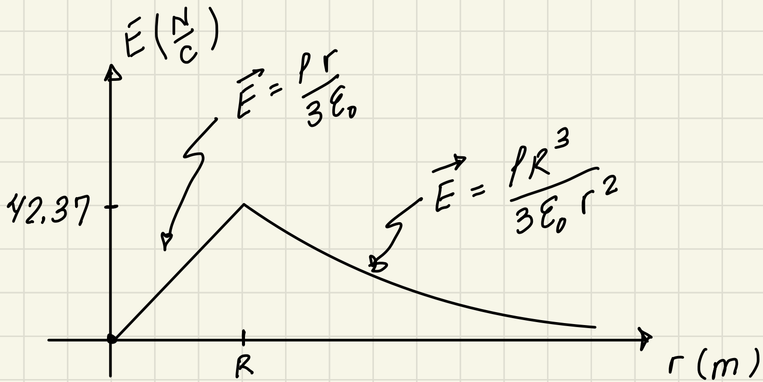


$\longrightarrow$ $r = R$

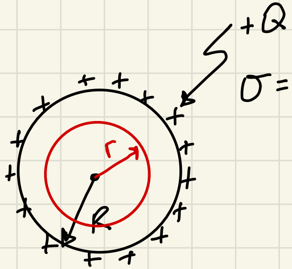
a) $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

$$E\left(\cancel{4\pi r^2}\right) = \frac{\rho \frac{\cancel{4}}{3}\cancel{\pi} R^{\cancel{3}}}{\varepsilon_0}$$

$$E = \frac{\rho R}{3\varepsilon_0}\hat{r} = \frac{7.5\times10^{-9}(0.15)}{3(8.85\times10^{-12})} = \underline{42.37\,\frac{N}{C}\,\hat{r}}$$



¿Qué sucede si tengo una esfera solamente cargada superficialmente?



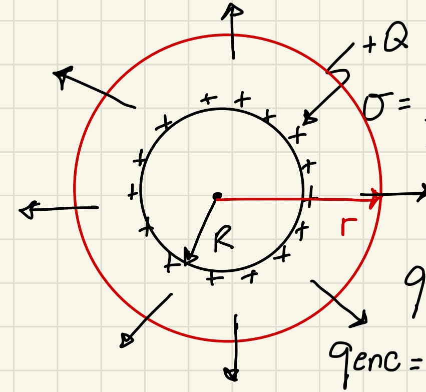
$$\sigma = \frac{Q}{4\pi R^2}$$

a) $\vec{E}(r<R)$

$$q_{enc} = 0$$

$$\vec{E} = \emptyset \ \frac{N}{C}$$

b) $E\ (r>R)$



$$\sigma = \frac{Q}{4\pi R^2}$$

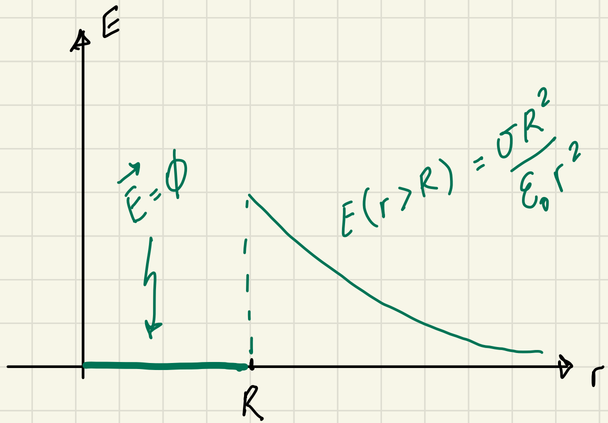
$$q_{enc} = Q$$

$$q_{enc} = \sigma\, 4\pi R^2$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

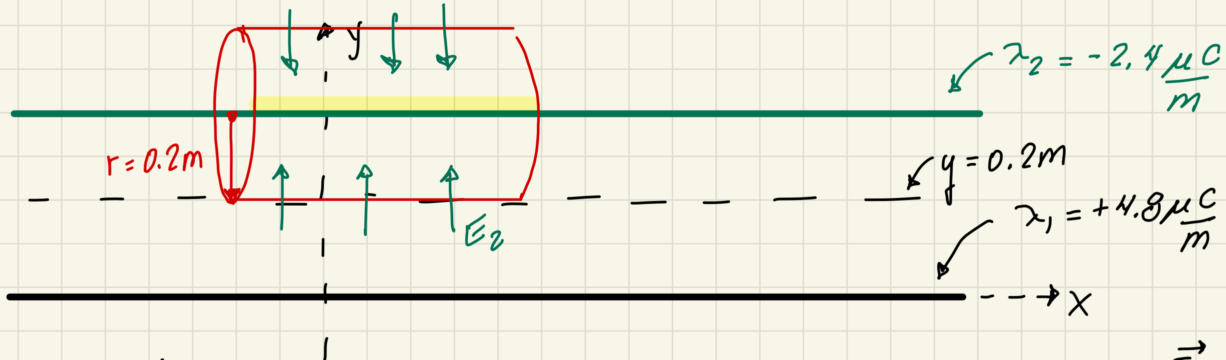
$$E\,(\cancel{4\pi} r^2) = \frac{\sigma\, \cancel{4\pi} R^2}{\varepsilon_0}$$

$$\vec{E} = \frac{\sigma R^2}{\varepsilon_0 r^2}\ \hat{r}$$



<u>Problema 2</u>.

Una línea con carga uniforme y muy larga tiene una carga por unidad de longitud de $+4.8\mu C/m$ yace a lo largo del eje "x". Una segunda línea con carga tiene una carga por unidad de longitud de $-2.4\mu C/m$ y es paralela al eje "x" en $y = 0.4m$. ¿Cuál es el campo eléctrico producido por ambas líneas en $y = 0.2m$?



$$\vec{E}_R = \vec{E}_1 + \vec{E}_2$$

para línea 2

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

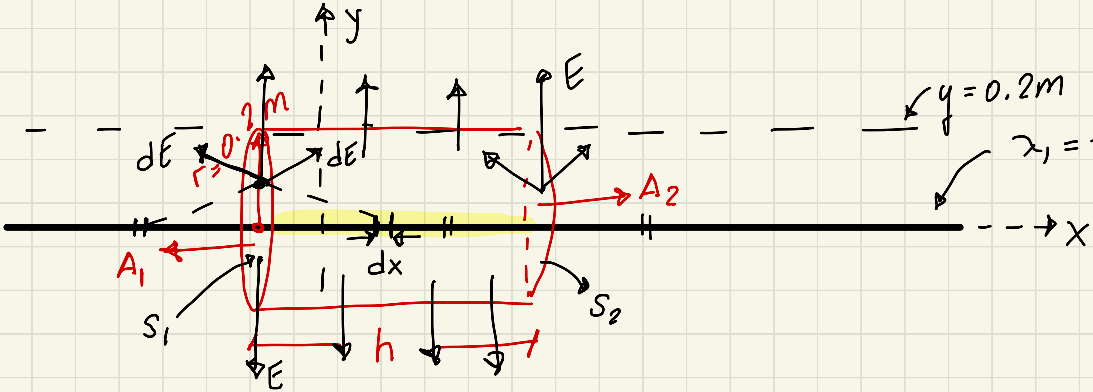
$$E(2\pi r \cancel{h}) = \frac{\lambda \cancel{h}}{\varepsilon_0}$$

$$\vec{E}_2 = \frac{\lambda_2}{2\pi \varepsilon_0 r}\hat{r} = \frac{-2.4\times10^{-6}}{2\pi * 8.85\times10^{-12} * 0.2} = -215.8\,\frac{kN}{C}\,\hat{r}$$

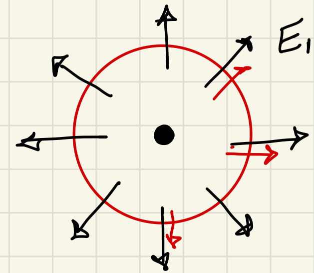
$$\vec{E}_2 = 215.8\,\frac{kN}{C}\,(+\hat{j})$$

$$\vec{E}_R = \left[431.6\,\frac{kN}{C} + 215.8\,\frac{kN}{C}\right]\hat{j}$$

$$\boxed{\vec{E}_R = 647.4\,\frac{kN}{C}\,\hat{j}}$$



$q_{enc} = \lambda_1 h$



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\cancel{\int_{\substack{CIRC. \\ IZQ}} \vec{E} \cdot d\vec{A}_1} + \cancel{\int_{\substack{CIRC \\ DER}} \vec{E} \cdot d\vec{A}_2} + \int_{CURVA} \vec{E} \cdot d\vec{A}_3 = \frac{q_{enc}}{\varepsilon_0}$$

$$\downarrow$$

$$E_1 (2\pi r \cancel{h}) = \frac{\lambda_1 \cancel{h}}{\varepsilon_0}$$

$$\vec{E}_1 = 431.6 \frac{kN}{C} (+\hat{j})$$

$$\vec{E}_1 = \frac{\lambda_1}{2\pi \varepsilon_0 r} \hat{r} = \frac{4.8 \times 10^{-6}}{2\pi (8.85 \times 10^{-12})(0.2)} \hat{r} = 431.6 \frac{kN}{C} \hat{r}$$

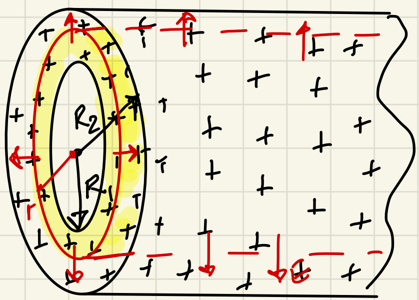
Problema 3. muy largo $q_{enc} = \rho$ Volumen

Un cilindro hueco de radio interior $1mm$ y radio exterior $3mm$ tiene una densidad volumétrica de carga de 80 nC/m^3, distribuida uniformemente en todo su volumen. a) Determine la magnitud del campo eléctrico (en N/C) en un punto localizado a 2 mm del eje del cilindro. b) Determine el campo eléctrico en un punto ubicado a 5 mm del eje del cilindro. c) Determine el campo eléctrico en un punto ubicado a 0.5 mm del eje del cilindro.

a)

$R_1 = 1mm$

$R_2 = 3mm$



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(2\cancel{\pi} r \cancel{h}) = \frac{\rho[\cancel{\pi} r^2 \cancel{h} - \cancel{\pi} R_1^2 \cancel{h}]}{\varepsilon_0}$$

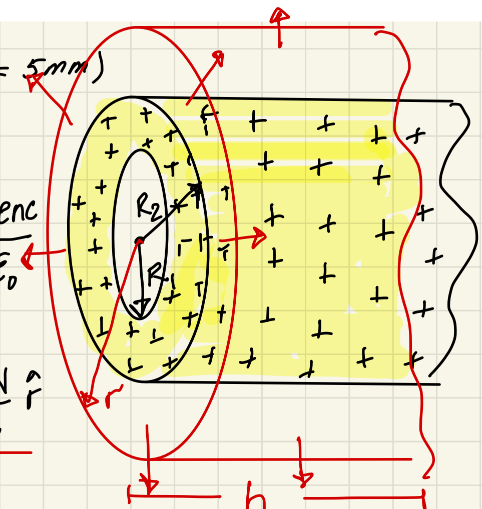
$$\vec{E} = \frac{\rho[r^2 - R_1^2]}{2\varepsilon_0 r}\hat{r}$$

$$= \frac{80\times10^{-9}(0.002^2 - 0.001^2)}{2(8.85\times10^{-12})(0.002)} = 6.78\frac{N}{C}\hat{r}$$

b) $E(r = 5mm)$

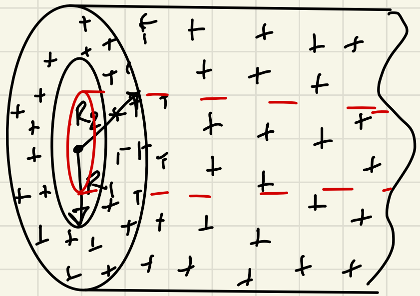
$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\vec{E} = 7.23\frac{N}{C}\hat{r}$$



$$E(2\cancel{\pi} r \cancel{h}) = \frac{\rho(\cancel{\pi} R_2^2 \cancel{h} - \cancel{\pi} R_1^2 \cancel{h})}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0 r} = \frac{80\times10^{-9}(0.003^2 - 0.001^2)}{2(8.85\times10^{-12})(0.005)}$$



c) $E(r = 0.0005m) = \varnothing$

$q_{enc} = \varnothing$

$E(r < R_1) = \varnothing$