A NEW EFFICIENT ELLIPSE DETECTION METHOD

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ABSTRACT

In this paper, we introduce a new method for ellipse detection. This method takes the advantages of major axis of an ellipse to find ellipse parameter fast and efficiently. It only needs an one-dimensional accumulator array to accumulate the length information for minor axis of the ellipse. This method does not require the evaluation of the tangents or curvatures of the edge contours, which are generally very sensitive to noise working conditions. No complicated mathematical computation is involved in the implementation and the required computational storage space is much cheaper compared to the current methods. Experiments with both synthetic and real images indicate the effectiveness of the proposed method.

1. INTRODUCTION

Ellipse detection is one of the key problems in image processing. Its importance is widely recognized and it has been researched using a good variety of methods (see Tsuji and Matsumoto [6], Davies [2], Yip et al. [8], Aguado et al. [1], Ji et al. [4], and Lei and Wong [5]). Since five parameters are required to fully define an ellipse, a generalized Hough Transform (HT) needs a five dimensional parameter space to detect an ellipse. That is very memory and time consuming. Some geometric constraints have to be used to reduce the complexity of the parameter space.

In order to overcome the excessive time and space requirements for ellipse extraction, most of previous techniques decompose the five-dimensional parameter space into several sub spaces of fewer dimensions. The decomposition is achieved by using geometric features that define constraints in the organization of edge data. These constraints include distance and angular relationships that define relative positions between a set of edge points. Hence, the parameters are computed after labeling the points that satisfy the constraints in a computational intensive approach.

In this paper, a new efficient ellipse detection approach is introduced. This method takes the advantages of major axis of an ellipse to find ellipse parameter fast and efficiently. Since we only need a one-dimensional accumulator array to accumulate the length of the minor axis of the ellipse, the required computational storage space is much cheaper than the previous methods.

2. PREVIOUS WORKS

In most current-reported ellipse detection research works, researchers try to collect ellipse information by using less points and construct simple parameter space. Yin and Chen [7] introduced a new approach that is based on the geometrical property. The edge points in an input image were classified into several subimages. Ellipses with different symmetric axes lie in different subimages. In each subimage, symmetry was applied again to obtain those sets of five points that possibly lie on the same ellipse. Ho and Chen [3] constituted two midpoint arrays by considering pairs of edge points in the same horizontal and vertical positions. From these two arrays the straight lines are detected separately by the HT and their intersections provide possible centers for ellipses. Then three other parameters of the angle and their major and minor axes remain to be estimated. Aguado et al. [1] proposed a method that a mapping for ellipse extraction had been developed, which includes edge tangent information. In order to decompose the space required for the HT and retains the original advantages of the HT they combined this mapping with local information computed from pair of edge points. Since edge direction information was included, the proposed method only involved pairs of points without any geometric constraints. Lei and Wong [6] proposed an approach to detect ellipses from symmetric contours in a picture efficiently. The idea is based on the detection of the symmetric axes from contours in a Hough-based approach, so as to transform a high-dimensional problem into two two-dimensional ones. From the idea, they found symmetric axes from contours first and then find ellipse from the contours.

3. FINDING ELLIPSE PARAMETERS

For an arbitrary ellipse, there are five unknown parameters, (x_0, y_0) for the center, α for the orientation, (a, b) for the major and minor axes. Usually we need a set of 5 edge pixels to calculate all parameters. If we use additional information at each edge pixel and/or choose special pixels, we require fewer pixels to determine the position of an ellipse. In the following we introduce a new approach to achieve this.

For each pair of pixels (x_1, y_1) and (x_2, y_2), we assume they are two vertices on the major axis of an ellipse. We also can assume they are two vertices on the minor axis of an ellipse, but it will increase computational time because we need to check pixels in a larger range of the image. Then we can calculate four parameters for the assumed ellipse as following:

$$x_0 = (x_1 + x_2)/2 \quad (1)$$

$$y_0 = (y_1 + y_2)/2 \quad (2)$$

$$a = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}/2 \quad (3)$$

$$\alpha = atan\ [(y_2 - y_1)/(x_2 - x_1)], \quad (4)$$

Where (x_0, y_0) is the center of the assumed ellipse, *a* the half-length of the major axis and α the orientation of the ellipse.

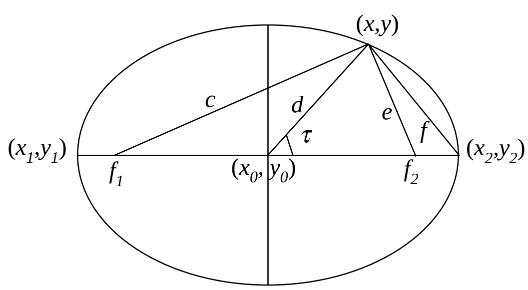

Figure 1. Ellipse geometry

Figure 1 shows ellipse geometry. f_1 and f_2 are foci of the ellipse and (x, y) is the third point used to calculate the fifth parameter. The distance between (x, y) and (x_0, y_0) should be less than the distance between (x_1, y_1) and (x_0, y_0) or between (x_2, y_2) and (x_0, y_0). So the half-length of the minor axis can be estimated by the following equation

$$b^2 = (a^2d^2sin^2\tau)/(a^2-d^2cos^2\tau) \quad (5)$$

where $cos\tau$ is

$$cos\tau = (a^2 + d^2 - f^2)/(2ad) \quad (6)$$

and *d* is the distance between (x, y) and (x_0, y_0).

Consequently, by using equations (1)-(6) it is possible to calculate all five parameters of an ellipse. Since we only need to vote on the half-length of the minor axis, we can use a one-dimensional accumulator array. If the votes reach a threshold, an ellipse is found and we output the parameters for this detected ellipse and remove all pixels on this ellipse from the image. After this pair of pixels is checked, we clear the accumulator array and go to the next pair.

4. ELLIPSE DETECTION STEPS

Based on the approach we discussed above, the following steps are used to detect ellipse on the image. The input image is edge image, on which black pixels represent background and white pixels are image pixels.

(1) Store all edge pixels in a one dimensional array.
(2) Clear the accumulator array .
(3) For each pixel (x_1, y_1), carry out the following steps from (4) to (14).
(4) For each other pixel (x_2, y_2), if the distance between (x_1, y_1) and (x_2, y_2) is greater than the required least distance for a pair of pixels to be considered then carry out the following steps from (5) to (14).
(5) From the pair of pixels (x_1, y_1) and (x_2, y_2), using equations (1) to (4) to calculate the center, orientation and length of major axis for the assumed ellipse.
(6) For each third pixel (x, y), if the distance between (x, y) and (x_0, y_0) is greater than the required least distance for a pair of pixels to be considered then carry out the following steps from (7) to (9).
(7) Using equations (5) and (6) to calculate the length of minor axis.
(8) Increment the accumulator for this length of minor axis by 1.
(9) Loop until all pixels are computed for this pair of pixels.
(10) Find the maxium element in accumulator array. The related length is the possible length of minor axis for assumed ellipse. If the vote is greater than the reqired least number for assumed ellipse, one ellipse is detected.
(11) Output ellipse parameters.
(12) Remove the pixels on the detected ellipse from edge pixel array.
(13) Clear accumulator array.
(14) Loop until all pairs of pixels are computed.
(15) Superimpose detected ellipses on the original image.
(16) End.

5. EXPERIMENT RESULTS

The performance of the above algorithm is demonstrated by synthetic and real images. Six synthetic images were used to test the proposed algorithm (see Figure 2.a-f). The size of synthetic images is 256x256. Some of them include noise pixels. No false ellipses were detected for those tests. Table 1 shows the test results for synthetic images. From the results we can see this method is very accurate, even for the noisy image and multiple ellipses image.

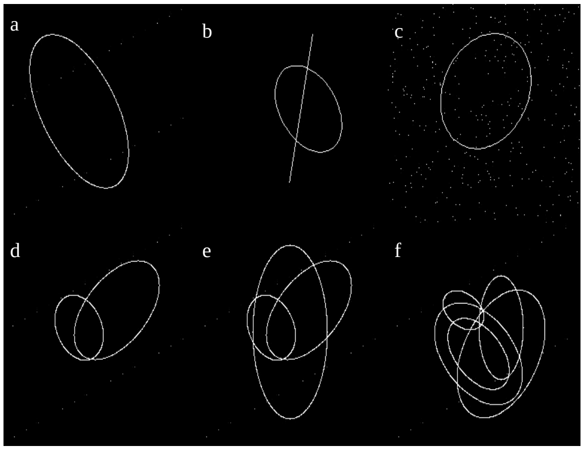

Figure 2. Synthetic test images

Table 1. Results of Detected Ellipses Positions v.s. the Actual Ellipse Positions for Synthetic Images in Figure 2.

Image		x_0	y_0	α	a	b
a	Actual	100	125	60	100	50
	Detected	99.5	124.5	60.2	99.7	51.0
b	Actual	150	125	55	57	38
	Detected	149.5	122.0	54.8	56.3	37.0
c	Actual	130	101	120	70	56
	Detected	130.0	101.5	119.6	70.8	56.0
d	Actual	100	120	60	40	30
		150	100	135	70	40
	Detected	100.0	120.0	59.5	39.4	30.0
		150.0	100.0	135.0	69.3	40.0
e	Actual	100	120	60	40	30
		150	100	135	70	40
		125	125	90	100	50
	Detected	100.0	120.0	59.5	39.4	30.0
		150.0	100.0	135.0	69.3	40.0
		125.0	125.0	90.0	99.0	50.0
f	Real	100	100	30	30	20
		120	150	45	50	30
		150	120	90	60	30
		120	150	45	70	45
		150	150	120	80	50
	Detected	100.0	100.0	31.0	29.2	20.0
		120.0	150.0	45.0	49.5	30.0
		150.0	120.0	90.0	59.0	30.0
		120.0	150.0	45.0	69.3	45.0
		150.0	150.0	120.1	79.8	50.0

Figure 3 shows some results of real world images that used for ellipse detection, with detected ellipses superimposed on the original images. All complete ellipses were detected correctly.

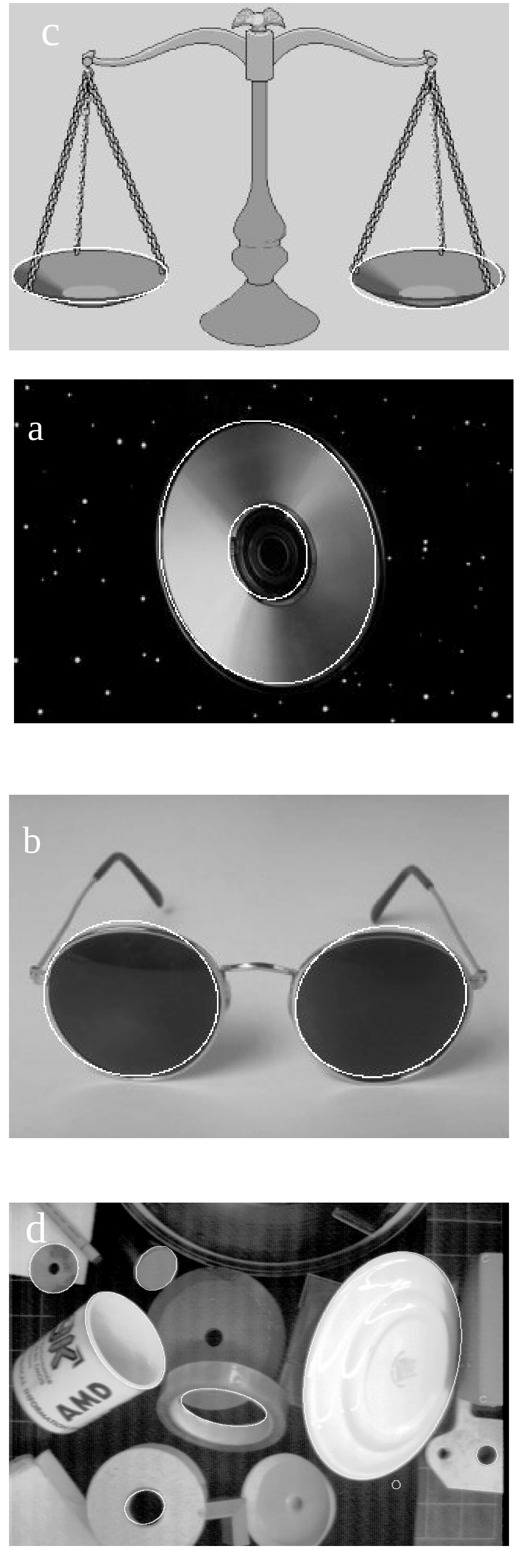

Figure 3. Results on real world images, with detected ellipses superimposed.

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6. CONCLUSIONS

In this paper, we introduce a new efficient ellipse detection method. This method takes the advantages of major axis of an ellipse to find ellipse parameter fast and efficiently.

For each pair of pixels, we assume they are two vertices on the major axis of an ellipse (we can also assume they are two vertices on the minor axis of an ellipse, but it will increase computational time). Then we can calculate four parameters for the assumed ellipse by using those two pixels. Then using another point we will find the last parameter (half-length of the minor axis) of the assumed ellipse. Since we only need to vote on the half-length of the minor axis, a one-dimensional accumulator array is all we need for voting.

This method does not require the evaluation of the tangents or curvatures of the edge contours, which are generally very sensitive to noisy working conditions. However, we can use this information to determine the possible pair of pixels that are vertices on the major axis. No complicated mathematical computation is involved in the implementation and the required computational storage space is much cheaper comparing to the reported methods. One weakness of our approach, as compared with the existing HT techniques for ellipse detection, is rather computationally intense due to the need to enumerate all possible pairs of points for each ellipse long axis. Dynamic HT is employed to reduce the computing time. Experiments with both synthetic and real images indicate the robustness and accuracy of the proposed method.

7. REFERENCES

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