Actividad 3

(1) a) $e^{z^3} = e^{r^3} e^{i3\theta}$

$= e^{r^3} (\cos 3\theta + i \operatorname{sen} \theta)$

$= e^{r^3} \cos 3\theta \quad [\cos(r^3 \operatorname{sen} 3\theta) + i \operatorname{sen}(r^3 \operatorname{sen} 3\theta)]$

$= e^{r^3} \cos^3\theta \quad [\cos(r^3 \operatorname{sen} 3\theta) + i \operatorname{sen}(r^3 \operatorname{sen}^3 \theta)]$

b) $e - \frac{\pi}{4} z$: usando $z = x + iy$

$= \frac{e - \pi x - \pi y}{4}$

$= \frac{e - \pi x - \pi y}{4} \left[\cos\left(-\frac{\pi y}{4}\right) + i \operatorname{sen}\left(\frac{-\pi y}{4}\right)\right.$

$= e \frac{- \pi x}{4} \cos\left(\frac{\pi y}{4}\right) - i \operatorname{sen}\left(\frac{\pi y}{4}\right)$

Actividad 3

(1+√i)

2) (a) $\cos(1.7+1.7i)$

$$\frac{e^{i(1.7+1.7i)}+e^{-i(1.7+1.7i)}}{2}$$

$Z=re^{i\theta}$ $r=|z|$ $z=1{,}7+1{,}7i$

$r=\sqrt{(1{,}7)^2+1{,}7}=\sqrt{2(1{,}7)^2}$ $1{,}7\sqrt{2}$

$\theta=\operatorname{arctg}\left(\frac{1{,}7}{1{,}7}\right)=\operatorname{arctg}(1)=\frac{\pi}{4}$

Entonces $1{,}7+1{,}7i=1{,}7\sqrt{2}^{\,i\,\pi/4}$

Euler $e^{i\theta}=\cos(\theta)+i\,\text{sen}(\theta)$

$e^{i(1{,}7+1{,}7i)}=e^{i\cdot1{,}7}\cdot e^{i\cdot1{,}7i}$

$e^{i\cdot1{,}7}=\cos(1{,}7)+i\,\text{sen}(1{,}7)$

$e^{-i\cdot1{,}7}=\cos(-1{,}7)+i\,\text{sen}(-1{,}7)=\cos(1{,}7)-i\,\text{sen}(1{,}7)$

$\cos(1{,}7+1{,}7i)=\cos(1{,}7)\cdot\frac{e^{-1{,}7}+e^{1{,}7}}{2}+i\,\text{sen}(1{,}7)\cdot\frac{e^{-1{,}7}-e^{1{,}7}}{2}$

b $\text{Sen}(2+13i)$

$$\text{Sen}(z)=\frac{e^{iz}-e^{-iz}}{2i}$$

$$\text{Sen}(2+13i)=\frac{e^{i(2+13i)}-e^{-i(2+13i)}}{2i}$$

$$e^{i(2+13i)}=e^{i\cdot2}\cdot e^{-13}$$

$$e^{-i(2+13i)}=e^{-i\cdot2}\cdot e^{13}$$

$e^{i\cdot2}=\cos(2)+i\,\text{sen}(2)$

$e^{-i\cdot2}=\cos(-2)+i\,\text{sen}(-2)=\cos(2)-i\,\text{sen}(2)$

$\text{Sen}(2+3i)=(\cos(2)+i\,\text{sen}(2))\cdot e^{-13}-(\cos(2)-i\,\text{sen}(2))\cdot e^{13}$

Actividad 3

Demostrar que

3). A. $\operatorname{Senh} z = \operatorname{Senh} x \cos y + i \cosh x \operatorname{Sen} y$

$$\operatorname{Senh}(x+iy) = \operatorname{Senh} x \cdot \cosh iy + \operatorname{Senh} iy \cdot \cosh x$$

① $\cosh(iy) = \cos y$

② $\operatorname{Senh}(iy) = \operatorname{sen} y$

$$\operatorname{Senh} z = \operatorname{senh} x \cos y + i \operatorname{Sen} y \cdot \cosh x.$$

B. $\cos z$ es par

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\cos(-z) = \frac{e^{i(-z)} + e^{-i(-z)}}{2}$$

$$\cos(-z) = \frac{e^{-iz} + e^{iz}}{2}$$

$$\cos(z) = \cos(-z)$$

Actividad 3

(4) A) $\cosh^2 z - \operatorname{senh}^2 z = 1$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{senh} x = \frac{e^x - e^{-x}}{2}$$

$$\left(\frac{e^z + e^{-z}}{2}\right)^2 - \left(\frac{e^z - e^{-z}}{2}\right)^2$$

$$\frac{e^{2z} + 2e^z \cdot e^{-z} + e^{-2z}}{4} - \frac{e^{2z} - 2e^z e^{-z} + e^{-2z}}{4}$$

$$\frac{e^{2z} + 2 + e^{-2z} - e^{2z} + 2 - e^{-2z}}{4}$$

$$\frac{4}{4} = 1$$

B) $\operatorname{sen}(z) = \operatorname{sen}(x)\cosh(y) + i\cos(x)\operatorname{senh}(y)$

$\operatorname{sen}(z) = \operatorname{sen}(x + iy)$

$\operatorname{sen}(x+iy) = \operatorname{sen}(x)\cosh(y) + i\cos(x)\operatorname{senh}(y)$

Reorganizar la expresión

$\operatorname{sen}(x)\cosh(y) + i\cos(x)\operatorname{senh}(y) = \operatorname{sen}(x)\cosh(y) + i\cos(x)\operatorname{senh}(y)$

Demostrando la igualdad requerida

$\operatorname{sen}(z) = \operatorname{sen}(x)\cosh(y) + i\cos(x)\operatorname{senh}(y)$