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Course: Optimization and Numerical Probability

Newtons Method with four variables

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```
clear;clc
syms X_1 X_2 X_3 X_4;
f = (X_1 + 10*X_2)^2 + 5*(X_3-X_4)^4 + (X_2-2*X_3)^4 + 10*(X_1-X_4)^4
```

f = $10\ (X_1 - X_4)^4 + (X_2 - 2\,X_3)^4 + 5\ (X_3 - X_4)^4 + (X_1 + 10\,X_2)^2$

```
e = 0.01;
```

Initial guess

```
xo = [3,-1,0,1]
```

```
xo = 1×4
     3    -1     0     1
```

```
i = 1;
```

Computing the Gradient

```
dx_1 = diff(f, X_1);
dx_2 = diff(f, X_2);
dx_3 = diff(f, X_3);
dx_4 = diff(f, X_4);
g_func = [dx_1;dx_2;dx_3;dx_4];
g = [subs(dx_1,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)])...
subs(dx_2,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)])...
subs(dx_3,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)])...
subs(dx_4,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)])]
```

g = $\begin{pmatrix} 306 & -144 & -12 & -300 \end{pmatrix}$

Computing the Hessian

```
dxx_1 =  diff(dx_1,X_1);
dxx_12 = diff(dx_1,X_2);
dxx_13 = diff(dx_1,X_3);
dxx_14 = diff(dx_1,X_4);
dxx_2 =  diff(dx_2,X_2);
dxx_23 = diff(dx_2,X_3);
dxx_24 = diff(dx_2,X_4);
dxx_3 =  diff(dx_3,X_3);
dxx_34 = diff(dx_3,X_4);
dxx_4 =  diff(dx_4,X_4);
```

```
dxx_111 = subs(dxx_1,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)]);
dxx_121 = subs(dxx_12,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)]);
dxx_131 = subs(dxx_13,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)]);
dxx_141 = subs(dxx_14,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)]);
dxx_222 = subs(dxx_2,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)]);
dxx_232 = subs(dxx_23,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)]);
dxx_242 = subs(dxx_24,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)]);
dxx_333 = subs(dxx_3,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)]);
dxx_343 = subs(dxx_34,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)]);
dxx_444 = subs(dxx_4,[X_1,X_2,X_3,X_4],[xo(1),xo(2),xo(3),xo(4)]);
h = [dxx_111,dxx_121,dxx_131,dxx_141;dxx_121,dxx_222,dxx_232,dxx_242;...
[dxx_131,dxx_232,dxx_333,dxx_343];[dxx_141,dxx_242,dxx_343,dxx_444]]
```

h =

$$\begin{pmatrix} 482 & 20 & 0 & -480 \\ 20 & 212 & -24 & 0 \\ 0 & -24 & 108 & -60 \\ -480 & 0 & -60 & 540 \end{pmatrix}$$

```
h_inv = inv(h)
```

h_inv =

$$\begin{pmatrix} \frac{1}{27} & -\frac{1}{756} & \frac{29}{1512} & \frac{53}{1512} \\ -\frac{1}{756} & \frac{37}{7560} & \frac{1}{2160} & -\frac{17}{15120} \\ \frac{29}{1512} & \frac{1}{2160} & \frac{607}{30240} & \frac{583}{30240} \\ \frac{53}{1512} & -\frac{17}{15120} & \frac{583}{30240} & \frac{1063}{30240} \end{pmatrix}$$

Objective function evaluated at the initial guess

```
f_x(i) = subs(f,[X_1,X_2,X_3,X_4], [xo(i),xo(i),xo(i),xo(i)]);
```

Implementing the newtion's method

```
while norm(g) > e
  w(1) = xo(1);x(1) = xo(2);y(1) = xo(3);z(1) = xo(4);
  I = [w(i),x(i),y(i),z(i)]'; %%%%Initial values
  w(i+1) = I(1) - h_inv(1,:)*g';
  x(i+1) = I(2) - h_inv(2,:)*g';
  y(i+1) = I(3) - h_inv(3,:)*g';
  z(i+1) = I(4) - h_inv(4,:)*g';
  i = i + 1;
```

updating the gradient

```
  g = [subs(dx_1,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)])...
  subs(dx_2,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)])...
  subs(dx_3,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)])...
```

```
  subs(dx_4,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)])];
```

updating the hessian

```
  dxx_111 = subs(dxx_1,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)]);
  dxx_121 = subs(dxx_12,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)]);
  dxx_131 = subs(dxx_13,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)]);
  dxx_141 = subs(dxx_14,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)]);
  dxx_222 = subs(dxx_2,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)]);
  dxx_232 = subs(dxx_23,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)]);
  dxx_242 = subs(dxx_24,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)]);
  dxx_333 = subs(dxx_3,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)]);
  dxx_343 = subs(dxx_34,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)]);
  dxx_444 = subs(dxx_4,[X_1,X_2,X_3,X_4],[w(i),x(i),y(i),z(i)]);
  h = [dxx_111,dxx_121,dxx_131,dxx_141;dxx_121,dxx_222,dxx_232,dxx_242;...
  [dxx_131,dxx_232,dxx_333,dxx_343];[dxx_141,dxx_242,dxx_343,dxx_444]];
  h_inv = inv(h);
```

Evaluatinf the function for each point

```
  f_x(i) = subs(f,[X_1,X_2,X_3,X_4], [w(i),x(i),y(i),z(i)]);
end
```

Representing the final outcome as a Table

```
itr = 1:i;
f_xk = f_x';
f_xk  = round(double(f_xk),10);
x_1 = w';x_2 = x';x_3 = y';x_4 = z';
iterations = itr';
T = table(x_1,x_2,x_3,x_4,f_xk,iterations)
```

T = 10×6 table

	x_1	x_2	x_3	x_4	f_xk	iterations
1	3.0000	-1.0000	0	1.0000	1170	1
2	2.2222	-0.2222	0.2222	0.8889	32.7901	2
3	1.4815	-0.1481	0.1481	0.5926	6.4771	3
4	0.9877	-0.0988	0.0988	0.3951	1.2794	4
5	0.6584	-0.0658	0.0658	0.2634	0.2527	5
6	0.4390	-0.0439	0.0439	0.1756	0.0499	6
7	0.2926	-0.0293	0.0293	0.1171	0.0099	7
8	0.1951	-0.0195	0.0195	0.0780	0.0019	8
9	0.1301	-0.0130	0.0130	0.0520	3.8476e-04	9
10	0.0867	-0.0087	0.0087	0.0347	7.6002e-05	10