Prof. Julia Herzen
Prof. Andreas Bausch
Prof. Ulrich Gerland

Introduction to Biophysics
Winter Semester 2025/26, Exercise Sheet 3

Problem P3.1 Coherence

a) What is transverse (spatial) coherence and longitudinal (temporal) coherence?

Solution:

Longitudinal (temporal) coherence length is the distance, after which two waves from originating from the same source point but having slightly different wavelengths will be out of phase by 180 degrees (i.e. being in phase again).

$$\lambda_{coh(longitudinal)} = \frac{\lambda^2}{2\Delta\lambda}$$

Transverse (spatial) coherence length is the distance perpendicular to the propagation direction, after which two waves originating from two different points of a source of finite size will be in phase again.

$$\lambda_{coh(transverse)} = \frac{\lambda \cdot R}{2 \cdot D}$$

For a more detailed explanation of coherence please refer to the 'coherence' chapter of the book by Willmott or by Als-Nielsen.

b) Calculation: In a third generation synchrotron radiation source, the vertical source size is $\approx 100\mu$m, and the experiment may be performed $\approx$ 20 m away.

1. Calculate the transverse coherence length for 1 Å X-rays.

Solution:

$$\lambda_{coh(transverse)} = \frac{\lambda \cdot R}{2 \cdot D}$$

$$\lambda_{coh(transverse)} = \frac{20m \cdot 1 \cdot 10^{-10}m}{2 \cdot 100 \cdot 10^{-6}m} = 10 \cdot 10^{-6}m$$

2. In the experiment hutch, a monochromator is installed (the Bragg crystal is used which has resolution of $\frac{\Delta\lambda}{\lambda} \approx 10^-5$). Calculate the longitudinal coherence length for 1 Å X-rays.

Solution:

$$\lambda_{coh(longitudinal)} = \frac{\lambda^2}{2\Delta\lambda}$$

$$\lambda_{coh(longitudinal)} = \frac{(1 \cdot 10^{-10}m)^2}{2\Delta\lambda}$$
$$\Delta\lambda = \lambda \cdot 10^{-5}$$
$$\lambda_{coh(longitudinal)} = \frac{(1 \cdot 10^{-10}m)^2}{2 \cdot 1 \cdot 10^{-10}m \cdot 1 \cdot 10^{-5}} = 5 \cdot 10^{-6}m$$

Problem P3.2 X-ray phase-contrast imaging

a) Which techniques exist?
Solution: The different techniques can be divided in two categories: 1.) without any optics (the propagation-base method), and 2.) using a kind of optics (e.g. crystals in analyzer-based imaging, gratings in Talbot interferometer, etc.).

b) What is actually measured?
Solution: Independent of the method the measured quantity is the refraction of the X-ray wave at the sample's borders with changing refractive index due to varying electron density of the material. The phase information is lost while detecting the intensity, thus this information needs to be encoded into the intensity and retrieved after the measurement.

c) What are the coherence requirements for the different techniques? Which methods can work with conventional polychromatic X-ray tubes and how?
Solution: All methods benefit from high coherence (both, from transverse and longitudinal coherence), but some of the methods tolerate a lower coherence than others.
1.) The propagation-based method requires a high transverse coherence (i.e. a very small source spot) and tolerates rather broad spectra (e.g. W spectrum of 80 kVp containing photon energies of 15-80 keV still works fine). This technique requires a highly-resolving detector (i.e. effective pixel sizes of 0.3-0.8 μm) to be able to resolve the tiny intensity enhancement at the borders.
2.) The optics-based method using an analyzer crystal requires both, the longitudinal and the transverse coherence. As the crystal uses the Bragg condition for sorting the refracted and the non-refracted part of the beam, the crystal deals as a monochromator of a very limited bandwidth. A high transverse coherence is required to be able to distinguish the refraction angles. A large source spot with a low transverse coherence would smear all these angles.
3.) The grating-based method only requires a transverse coherence, which is large enough to illuminate several grating periods of the phase-shifting grating coherently. This method tolerates a broad spectrum (i.e. low longitudinal coherence) and allows to work with polychromatic sources.
Compared to other methods, the grating-based method is able to work with non-coherent laboratory sources. For that, an additional absorbing grating structure - a so-called source grating - can be introduced, which deals as a series of slits producing many small mutually non-coherent sources. Each of these sources is coherent enough to form the interference pattern by the phase-shifting grating. The intensity of the different slits add up non-coherently.

Problem P3.2 Grating-Based Phase-Contrast Imaging

a) Figure 1 shows a grating-based phase-contrast setup at an incoherent X-ray source. Please describe in a few words the function of the three gratings G_0, G_1 and G_2.

b) Explain the procedure to extract the three different signals - attenuation, phase and scattering signal - in grating-based imaging.

Solution:
The analyzer grating G_2 is shifted over one period in several steps (at least three steps to correctly sample a periodic function). Every large detector pixel is detecting intensity at each

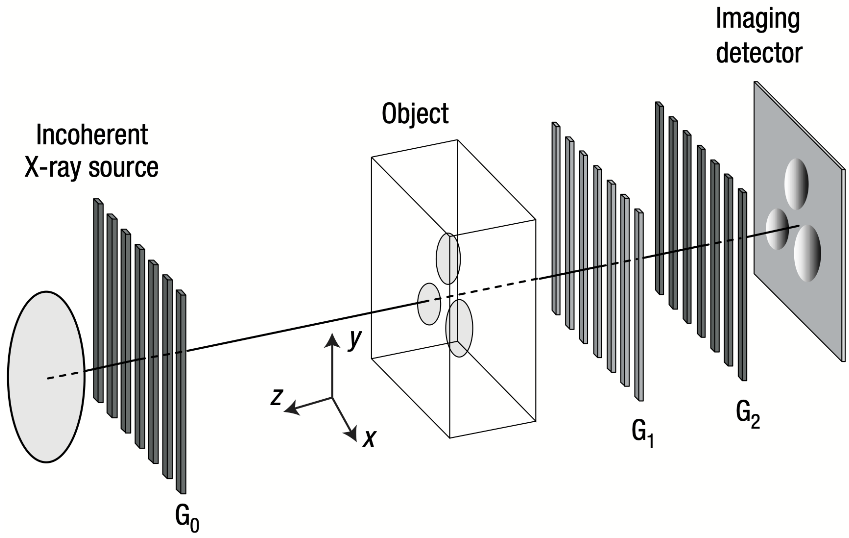

FIGURE 1: Grating-based phase-contrast

Grating	Function
G_0	source grating; deals as an array of slits increasing the transversal coherence of the large source; the different slits are mutually incoherent, but each of the sources has a coherence length long enough to allow interference from the phase grating G_1
G_1	phase grating; causes an interference pattern downstream the x-ray beam in the Talbot distance or a multiple fraction of it the interference pattern has the same periodicity as the phase grating (few micrometers)
G_2	analyzer grating; deals as an array of slits helping to resolve the interference pattern with a much larger pixel (several hundreds of micrometers)

TABLE 1: Function of G_0, G_1 and G_2

step of the analyzer grating. Plotting this intensity of every pixel over the stepping distance will reproduce the interference pattern averaged over several periods in the large pixel - this is called stepping curve. To retrieve the three signals - attenuation, phase and scattering signal, one can fit a sinus function into the measured stepping curve will allow to determine the change of the mean value between the empty interferometer and the one with sample. This difference in the mean value is the amount of attenuation. The same procedure is performed for the phase shift (the angle of the sinus curve) and the scattering, which smears out the interference pattern (amplitude normalized to the mean value). Doing this for every pixel gives a 2D image with three signals.

c) What changes, when a random mask (e.g. sand paper) is used instead of a phase grating? What are the advantages or disadvantages of using random masks?

Solution:
When using a grating interferometer at a coherent source in combination with a highly-resolving detector the phase grating can be replaced by a random mask (e.g. sandpaper). Such a mask imprints a random structure on the wavefront, which then can be analysed

with and without the sample. To retrieve the phase, one method is to track the shifts of the individual structures directly by a correlation algorithm. The advantages are: the mask is very simple (can be purchased in any store); there is no risk of a wrapping phase, as it is not measured modulus 2Pi; The disadvantages: requires coherence and high-resolution detector - rather high dose; less sensitive to small changes in refractive index, since random visibility over the field of view;