Prescriptive analytics - hands on session 2

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Goals this afternoon

After this lecture you can:

1. Recognize prescriptive analytics problems.
2. Formulate a portfolio selection model.
3. Implement a portfolio selection model in python.

Outline

11.30 Recap facility location model.

12.00 Formulating portfolio selection model.

12.30 *lunch break*

13.30 Hands-on: implementing portfolio selection model.

14.45 Wrap up and final remarks.

15.00 End of session.

Recap: Facility Location

Figure: 37 major European cities as potential distribution centers.

Lessons from game

Some elements that made the game difficult:

- Hard to go from one candidate solution to another (a lot of work to switch).
- Unfathomable number of possibilities. How to systematically try them?
- When do you know you cannot do better than your current solution?

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- Hard to go from one candidate solution to another (a lot of work to switch).
- Unfathomable number of possibilities. How to systematically try them?
- When do you know you cannot do better than your current solution?

Model formulation

N: number of cities
i, j: index of cities

Variables

x_i	Equals 1 if distribution is open at city i, 0 otherwise.
y_{ij}	Pallets shipped from city i to city j.

Parameters

F_i	Facility at city i (if opened) in euros.
S_i	Shipping capacity out of city i in pallets.
d_i	Demand at city i.
c_{ij}	Shipping costs from city i to city j per pallet in euros.

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$$
\begin{aligned}
\min_{x,y} \quad & \sum_{i=1}^{N} F_i x_i + \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} y_{ij} \\
\text{subject to} \quad & \sum_{i=1}^{N} y_{ij} = d_i \qquad j = 1, \ldots, N \\
& \sum_{j=1}^{N} y_{ij} \leq S_i x_i \qquad i = 1, \ldots, N \\
& y_{ij} \geq 0 \qquad i = 1, \ldots, N, \; j = 1, \ldots, N \\
& x_i \in \{0, 1\} \qquad i = 1, \ldots, N.
\end{aligned}
$$

Switching to jupyter notebook

Switch to present jupyter notebook solution...

Portfolio optimization

We are given data on the daily returns of 100 assets from the last 10 years.

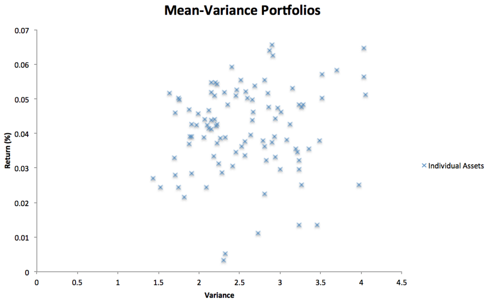
```
This file was created by CMPT_ME_BEME_OP_INV_RETS_DAILY using the 201704 CRSP database.
It contains value- and equal-weighted returns for portfolios formed on ME and INV.

The portfolios are constructed at the end of June.  INV is the change
in total assets from the fiscal year ending in year t-2 to the fiscal year ending in t-1,
divided by t-2 total assets.
Annual returns are from January to December.

Missing data are indicated by -99.99 or -999.

The break points include utilities and include financials.

The  portfolios  include utilities and include financials.


  Average Value Weighted Returns -- Daily
,SMALL LoINV,ME1 INV2,ME1 INV3,ME1 INV4,ME1 INV5,ME1 INV6,ME1 INV7,ME1 INV8,ME1 INV9,SMALL HiINV,ME2 INV1,ME2 INV2,ME2 INV3,ME2 INV4,ME2
INV5,ME2 INV6,ME2 INV7,ME2 INV8,ME2 INV9,ME2 INV10,ME3 INV1,ME3 INV2,ME3 INV3,ME3 INV4,ME3 INV5,ME3 INV6,ME3 INV7,ME3 INV8,ME3 INV9,ME3
INV10,ME4 INV1,ME4 INV2,ME4 INV3,ME4 INV4,ME4 INV5,ME4 INV6,ME4 INV7,ME4 INV8,ME4 INV9,ME4 INV10,ME5 INV1,ME5 INV2,ME5 INV3,ME5 INV4,ME5
INV5,ME5 INV6,ME5 INV7,ME5 INV8,ME5 INV9,ME5 INV10,ME6 INV1,ME6 INV2,ME6 INV3,ME6 INV4,ME6 INV5,ME6 INV6,ME6 INV7,ME6 INV8,ME6 INV9,ME6
INV10,ME7 INV1,ME7 INV2,ME7 INV3,ME7 INV4,ME7 INV5,ME7 INV6,ME7 INV7,ME7 INV8,ME7 INV9,ME7 INV10,ME8 INV1,ME8 INV2,ME8 INV3,ME8 INV4,ME8
INV5,ME8 INV6,ME8 INV7,ME8 INV8,ME8 INV9,ME8 INV10,ME9 INV1,ME9 INV2,ME9 INV3,ME9 INV4,ME9 INV5,ME9 INV6,ME9 INV7,ME9 INV8,ME9 INV9,ME9
INV10,BIG LoINV,ME10 INV2,ME10 INV3,ME10 INV4,ME10 INV5,ME10 INV6,ME10 INV7,ME10 INV8,ME10 INV9,BIG HiINV
19630701,  -0.83,  -0.60,  -0.56,  -0.55,  -0.85,  -0.50,  -1.56,  -1.32,  -0.34,  -0.41,  -0.51,  -0.59,   0.34,  -0.90,   0.17,  -0.98,
-0.42,  -0.99,  -0.79,  -0.72,  -1.00,  -1.66,  -0.69,  -0.41,   0.14,   0.19,  -1.14,  -0.52,  -0.24,  -0.50,  -1.74,  -0.10,  -0.29,
-0.68,  -0.30,  -0.35,  -0.42,  -1.30,  -1.68,  -1.15,  -0.71,  -0.92,  -1.18,  -1.37,  -0.14,  -0.27,  -0.35,  -2.26,  -0.66,  -0.77,
-1.17,  -0.34,  -0.12,  -0.31,  -0.01,  -1.45,   0.15,   0.64,  -1.79,   0.24,  -2.17,   0.26,  -0.35,  -0.07,  -0.54,  -1.17,  -0.11,
-0.71,  -0.70,  -0.08,  -1.32,  -0.95,   0.16,  -0.18,  -1.14,  -0.58,  -0.62,  -1.03,  -0.22,  -1.00,   0.06,  -0.66,  -1.03,  -0.47,
-1.07,  -0.18,  -1.21,  -0.90,  -0.76,  -0.38,  -0.71,  -0.45,  -0.77,  -0.24,  -0.52,  -0.35,  -0.76,  -1.42,  -0.34,  -1.39
19630702,   0.32,   0.35,   0.33,   0.29,   0.93,   1.43,   0.10,   0.34,   0.12,  -0.33,   0.73,   0.16,  -0.51,   0.80,  -0.26,  -0.48,
0.54,   0.92,   0.72,   0.87,   0.79,   0.06,  -0.19,   0.94,   0.04,   0.21,   0.31,   0.44,   0.44,   0.41,   1.33,   0.99,   0.81,
0.75,   0.89,   0.78,   0.75,   1.34,   1.34,   0.24,   0.52,   0.11,   1.00,   1.89,   0.02,   1.46,   0.33,   1.15,   0.32,   0.29,
0.95,   0.24,   0.08,  -0.08,   0.17,   1.22,   0.45,   1.28,   0.70,   0.70,   0.00,   0.17,   1.10,   0.08,   0.20,   0.51,   0.35,
1.03,   0.61,   0.39,   0.68,   1.50,   0.33,   0.64,   0.32,   0.40,   0.85,   0.99,   0.86,   1.85,   1.11,   0.14,   1.60,   0.50,
0.54,   0.26,   1.15,   0.56,   0.54,   0.35,   0.04,   0.74,   0.78,   0.90,   0.86,   0.84,   1.10,   1.36,   0.50,   2.38
19630703,  -0.10,   0.30,   0.38,   0.33,  -0.10,   0.90,   0.20,   0.66,  -0.05,   0.85,   0.40,   1.13,  -0.12,   0.95,  -1.33,  -0.70,
0.44,   0.34,   1.26,   1.23,   0.26,   0.85,   0.86,   0.71,   0.51,   0.29,   1.78,   0.70,   0.75,  -0.56,   0.57,   0.53,  -0.25,
0.42,   0.76,   0.51,   0.37,   0.50,   1.33,   0.01,   0.16,   0.29,  -0.25,   0.40,   0.25,   1.08,  -0.60,   0.53,   0.69,   0.97,
-0.21,   1.19,   0.12,   0.36,   0.31,  -0.40,   0.64,   0.49,   1.06,   0.70,   0.91,   0.93,   0.08,   0.12,   0.78,   1.21,   0.44,
0.01,   0.63,   0.22,   2.33,   0.73,   0.09,   0.16,   0.71,   0.78,   0.58,   0.43,   0.42,   0.73,   3.08,   0.14,   0.10,   0.55,
0.29,   0.80,   0.92,   1.33,   1.24,   0.54,   0.87,  -0.03,   0.30,   0.18,   1.11,   0.79,   0.70,   1.01,   0.29,   1.07
19630705,   0.37,  -0.18,   0.12,  -0.19,   0.46,  -0.13,   0.55,   0.75,   0.88,   0.67,   1.43,   1.26,   0.42,  -0.19,   1.88,   0.89,
0.27,   0.53,   0.77,   0.75,  -0.50,   0.03,  -0.01,   0.85,   0.45,   0.44,   0.49,   0.22,   0.26,   1.42,   0.86,  -0.16,   0.50,
0.82,   0.58,  -0.21,  -0.25,   1.47,   0.87,   0.63,   0.25,   1.09,   0.19,   1.90,   0.22,   0.03,   0.23,   0.90,   0.20,   0.65,
```

Case

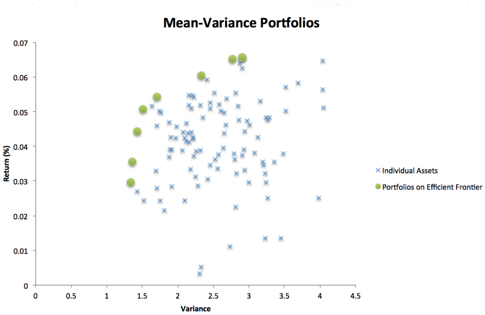
Use optimization to construct the mean-variance portfolio from this data.

Return versus Risk

Average daily returns and (co-)variances calculated from data and plotted.



Efficient Frontier (Markowitz, Noble prize 1990)



All the portfolios on the efficient frontier (in green) dominate all others: there are no portfolios with both *higher return* and *lower variance* (risk).

Model formulation

Parameters	N :	number of assets
	$\bar{r}_i$:	average daily return of asset i
	s_{ij} :	(sample) covariance between daily returns of asset i and j
	α :	Risk-aversion parameter
Variables	x_i :	amount of asset i bought

Model formulation

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Variables

x_i : amount of asset i bought

$$\max_{x_i} \boxed{\sum_{i=1}^{N} \bar{r}_i x_i} - \alpha \boxed{\sum_{i=1}^{N} \sum_{j=1}^{N} s_{ij} x_i x_j}$$

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Variables

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Expected portfolio return

$$\max_{x_i} \boxed{\sum_{i=1}^{N} \bar{r}_i x_i} - \alpha \boxed{\sum_{i=1}^{N} \sum_{j=1}^{N} s_{ij} x_i x_j}$$

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Expected portfolio return Risk-aversion parameter

$$\max_{x_i} \sum_{i=1}^{N} \bar{r}_i x_i - \alpha \sum_{i=1}^{N} \sum_{j=1}^{N} s_{ij} x_i x_j$$

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Variables

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Expected portfolio return   Risk-aversion parameter   Variance of portfolio

$$\max_{x_i} \boxed{\sum_{i=1}^{N} \bar{r}_i x_i} - \alpha \boxed{\sum_{i=1}^{N}\sum_{j=1}^{N} s_{ij} x_i x_j}$$

Model formulation

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Expected portfolio return Risk-aversion parameter Variance of portfolio

$$\max_{x_i} \boxed{\sum_{i=1}^{N} \bar{r}_i x_i} - \alpha \boxed{\sum_{i=1}^{N}\sum_{j=1}^{N} s_{ij} x_i x_j}$$

$$\text{subject to} \sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, \qquad i = 1, \ldots, N.$$

Task

Implement the Mean-variance portfolio optimization model in Python.

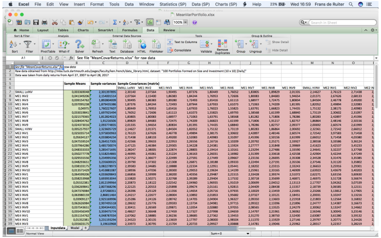

Figure: Excel input data

The jupyter notebook setup and excel files can be found on:

www.fransderuiter.com/JADS.

Conclusions

- Many applications.
- Also some *nonlinear* models can be implemented.
- These models are interpretable.

Interpretable AI

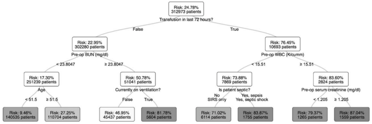

Figure: An example of an optimal decision tree to determine emergency surgery risk.

From Bertsimas, D., Dunn, J., Velmahos, G. C., & Kaafarani, H. M. (2018). Surgical risk is not linear: derivation and validation of a novel, user-friendly, and Machine-learning-based predictive optimal trees in emergency surgery risk (Potter) calculator. **Annals of surgery**, 268(4), 574-583.