Filtros digitales

2) LP Butter de orden 2: con $F = 1\,kHz \rightarrow \omega = 1$; $H(s) = \dfrac{1}{s^2 + \sqrt{2}\, s + 1}$

a) $F_s = 100\,kHz$ Asumo sin prewarping: $k = 2F_s \rightarrow s = k\,\dfrac{z-1}{z+1}$

$$H(z) = \frac{1}{k^2\left(\frac{z-1}{z+1}\right)^2 + \sqrt{2}\,k\,\frac{z-1}{z+1} + 1} = \frac{1}{k^2\,\frac{(z-1)^2}{(z+1)^2} + \sqrt{2}\,k\,\frac{z-1}{z+1} + 1}$$

$$H(z) = \frac{1}{\frac{k^2(z-1)^2 + \sqrt{2}\,k\,(z-1)(z+1) + (z+1)^2}{(z+1)^2}}$$

$$H(z) = \frac{z^2 + 2z + 1}{k^2 z^2 - k^2 2z + k^2 + \sqrt{2}\,k\,z^2 - \sqrt{2}\,k + z^2 + 2z + 1}$$

$$H(z) = \frac{z^2 + 2z + 1}{z^2\left(k^2 + \sqrt{2}\,k + 1\right) + z\left(2 - 2k^2\right) + \left(k^2 - \sqrt{2}\,k + 1\right)}$$

$$\Omega = \frac{\omega}{f_s} = \frac{2\pi f_0}{f_s} = 2\pi\,\frac{1\,kHz}{100\,kHz} = \frac{2\pi}{100} = \Omega_c$$

$w_c = 1$ Normalizado $\rightarrow w_c = k \tan\left(\dfrac{\Omega_c}{2}\right) \rightarrow k = \dfrac{1}{\tan\left(\frac{\Omega_c}{2}\right)} = \dfrac{1}{\tan\frac{\pi}{100}} = 31{,}82$

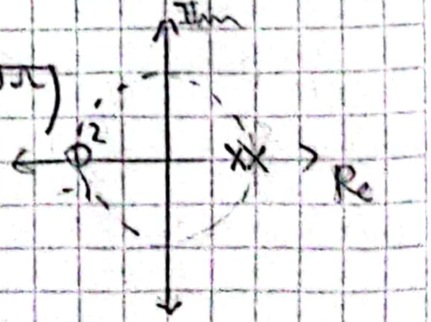
$$H(z) = \frac{z^2 + 2z + 1}{z^2\,1060 - z\,2023{,}02 + 968{,}57} = \frac{1}{1060}\,\frac{z^2 + 2z + 1}{z^2 - 1{,}9z + 0{,}9}$$

$$H(e^{j\Omega}) = \frac{1}{1060}\,\frac{e^{2j\Omega} + 2e^{j\Omega} + 1}{e^{j\Omega\,2} - 1{,}9\,e^{j\Omega} + 0{,}9} = \frac{\cancel{e^{j\Omega}}\left(e^{j\Omega} + 2 + e^{-j\Omega}\right)}{e^{j\Omega}\left(e^{j\Omega} - 1{,}9 + e^{-j\Omega} - 0{,}1\,e^{-j\Omega}\right)}$$

!!

$$H(e^{j\Omega}) = \frac{1}{1060} \cdot \left(\frac{e^{j\Omega} + e^{-j\Omega}}{2} + \frac{2}{2}\right)\left(2 \cdot \frac{1}{\frac{e^{j\Omega} + e^{-j\Omega}}{2} - \frac{1{,}9}{2} - \frac{0{,}1\,e^{-j\Omega}}{2}}\right)$$

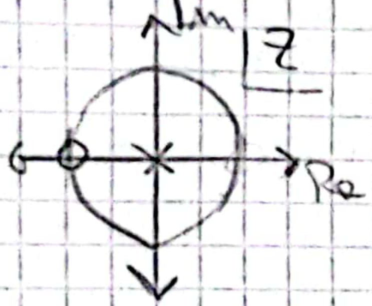
$$H(e^{j\Omega}) = \frac{1}{1060}\,\frac{\cos\Omega + 1}{\cos\Omega - \frac{1{,}9}{2} - 0{,}05\,e^{-j\Omega}}$$



NOTA

a) $h_1(k) \to h(k) = (\underset{b_0}{1}, \underset{b_1}{1}) \Rightarrow y(k) = b_0 x(1) + b_1 x(0) = x(k-1) + x(k)$

$H(z) = 1 + z^{-1} = \frac{1+z}{z}$

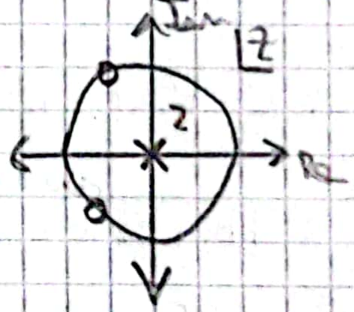


$H(e^{j\Omega}) = \frac{1+e^{j\Omega}}{e^{j\Omega}} = \frac{e^{j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}} + e^{-j\frac{\Omega}{2}}\right)}{e^{j\Omega}}$

$H(e^{j\Omega}) = \underbrace{\cos\frac{\Omega}{2}}_{\text{Modulo}} \; \underbrace{e^{-j\frac{\Omega}{2}}}_{\text{Fase}}$

$h_2(k) = (1,1,1) \to y(k) = b_0 x(2) + b_1 x(1) + b_2 x(0) = x(k-2) + x(k-1) + x(k)$

$H(z) = 1 + z^{-1} + z^{-2} = \frac{z^2+z+1}{z^2} \to \frac{z^2+z+1}{z^2}$

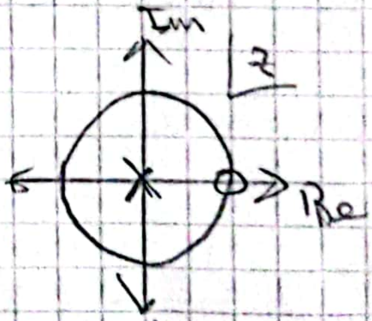


$H(e^{j\Omega}) = \frac{e^{j2\Omega} + e^{j\Omega} + 1}{e^{j2\Omega}} = \frac{e^{j\Omega}\left(e^{j\Omega} + 1 + e^{-j\Omega}\right)}{e^{j2\Omega}} =$

$H(e^{j\Omega}) = (1 + 2\cos(\Omega))\, e^{-j\Omega}$

b) $h_1(k) = (1,-1) \to y(k) = b_0 x(1) + b_1 x(0) \to x(k-1) - x(k)$

$H(z) = z^{-1} - 1 = \frac{1-z}{z}$

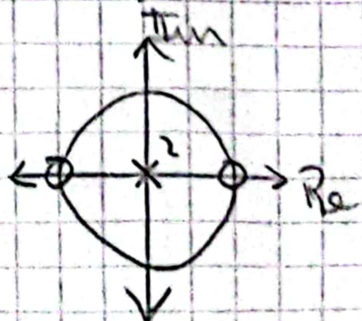


$H(e^{j\Omega}) = \frac{1-e^{j\Omega}}{e^{j\Omega}} = \frac{-e^{j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}\right)}{e^{j\Omega}}$

$H(e^{j\Omega}) = -e^{-j\frac{\Omega}{2}}\, j2 \operatorname{sen}\frac{\Omega}{2}$ Fase $= \frac{\pi}{2} - \frac{\Omega}{2}$

$h_2(k) = (1,0,-1) = x(k-2) - x(k)$

$H(z) = z^{-2} - 1 = \frac{1-z^2}{z^2}$



$H(e^{j\Omega}) = \frac{1-e^{2j\Omega}}{e^{2j\Omega}} = \frac{-e^{j\Omega}\left(e^{j\Omega} - e^{-j\Omega}\right)}{e^{j2\Omega}}$

$H(e^{j\Omega}) = -e^{-j\Omega} j \operatorname{sen}(\Omega).2 = -e^{j\frac{\pi}{2} - \Omega}\, 2 \operatorname{sen}(\Omega)$

NOTA