Filtros digitales

2) LP Butter de orden 2: con $F = 1\,kHz \to \omega = 1$; $H(s) = \dfrac{1}{S^2 + \sqrt{2}\,S + 1}$

a) $F_s = 100\,kHz$ Asumo sin prewarping: $k = 2F_s \to S = k\,\dfrac{z-1}{z+1}$

$$H(z) = \frac{1}{k^2\left(\frac{z-1}{z+1}\right)^2 + \sqrt{2}\,k\,\frac{z-1}{z+1} + 1} = \frac{1}{k^2\,\frac{(z-1)^2}{(z+1)^2} + \sqrt{2}\,k\,\frac{z-1}{z+1} + 1}$$

$$H(z) = \frac{1}{\dfrac{k^2(z-1)^2 + \sqrt{2}\,k\,(z-1)(z+1) + (z+1)^2}{(z+1)^2}}$$

$$H(z) = \frac{z^2 + 2z + 1}{k^2 z^2 - k^2 2z + k^2 + \sqrt{2}\,k\,z^2 - \sqrt{2}\,k + z^2 + 2z + 1}$$

$$H(z) = \frac{z^2 + 2z + 1}{z^2\left(k^2 + \sqrt{2}\,k + 1\right) + z\left(2 - 2k^2\right) + \left(k^2 - \sqrt{2}\,k + 1\right)}$$

$$\Omega = \frac{\omega}{F_s} = \frac{2\pi F_0}{F_s} = 2\pi\,\frac{1\,kHz}{100\,kHz} = \frac{2\pi}{100} = \Omega_c$$

$\omega_c = 1$ Normalizado $\to \omega_c = k \tan\left(\dfrac{\Omega_c}{2}\right) \to k = \dfrac{1}{\tan\left(\frac{\Omega_c}{2}\right)} = \dfrac{1}{\tan\frac{\pi}{100}} = 31{,}82$

$$H(z) = \frac{z^2 + 2z + 1}{z^2\,1060 - z\,2023{,}02 + 968{,}57} = \frac{1}{1060}\;\frac{z^2 + 2z + 1}{z^2 - 1{,}9z + 0{,}9}$$

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