Derivatives Course

Join our course to understand derivatives!

Function	Derivative
$f(x) = c$	$f'(x) = 0$ Constant Rule
$f(x) = x$	$f'(x) = 1$
$f(x) = x^n$	$f'(x) = nx^{n-1}$) Power rule
$f(x) = e^x$	$f'(x) = e^x$
$f(x) = \ln(x)$	$f'(x) = \frac{1}{x}$
$f(x) = \sin(x)$	$f'(x) = \cos(x)$
$f(x) = \cos(x)$	$f'(x) = -\sin(x)$
$f(x) = \tan(x)$	$f'(x) = \sec^2(x)$

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Function	Derivative	Name
$f(x) = c \cdot g(x)$	$f'(x) = c \cdot g'(x)$	Constant Multiple Rule
$f(x) = g(x) + h(x)$	$f'(x) = g'(x) + h'(x)$	Sum Rule
$f(x) = g(x) \cdot h(x)$	$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$	Product Rule
$f(x) = \frac{g(x)}{h(x)}$	$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h(x)^2}$	Quotient Rule
$f(x) = g(h(x))$	$f'(x) = g'(h(x)) \cdot h'(x)$	Chain Rule

Derivative Exercises

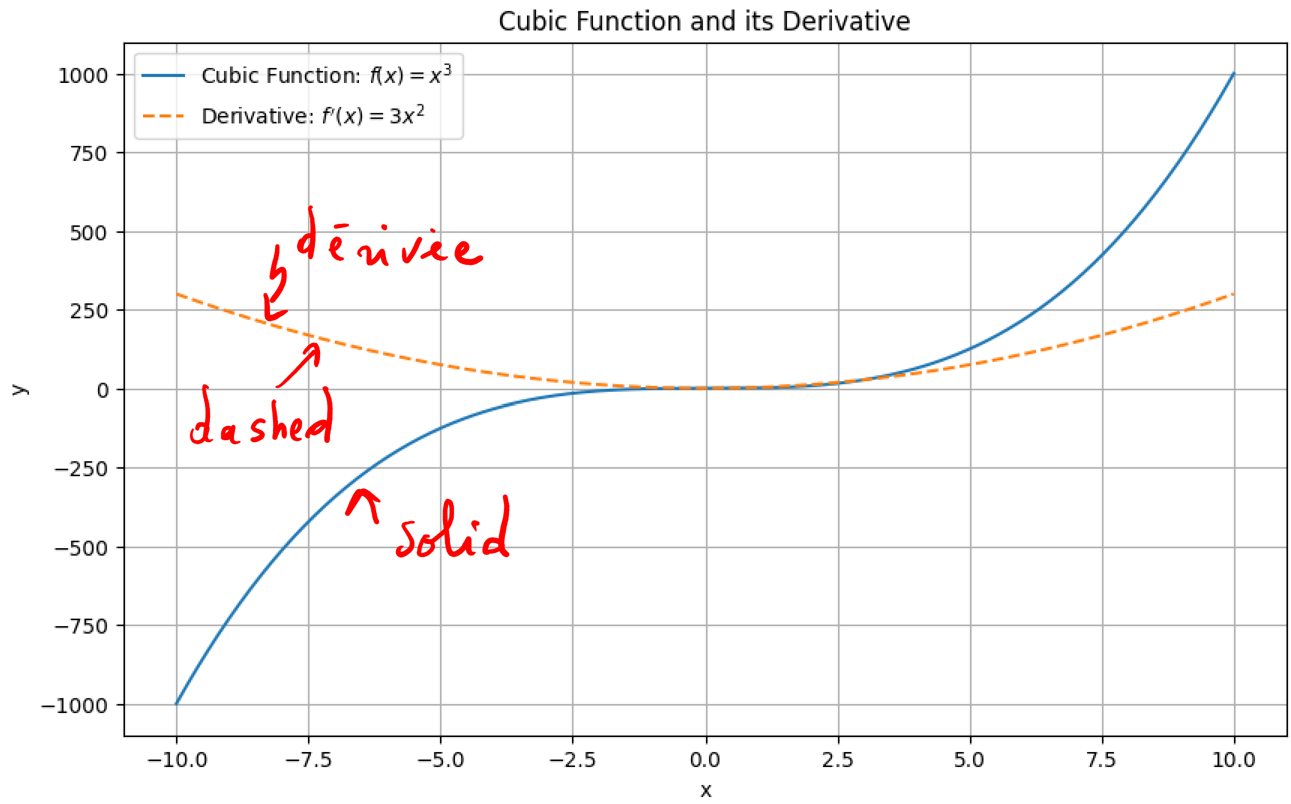
1. **Basic Derivatives:**
   - Calculate the derivative of the following functions:
     - $f(x) = 2x^3 - 5x^2 + 3x - 7$
     - $g(x) = \sin(x) + \cos(x)$
     - $h(x) = e^x - \ln(x)$
2. **Product and Quotient Rules:**
   - Use the product rule to find the derivative of $f(x) = (3x^2 + 2x)(x^3 - x + 1)$.
   - Use the quotient rule to find the derivative of $g(x) = \frac{x^2+1}{x^3-2x}$.
3. **Chain Rule:**
   - Find the derivative of $f(x) = \sin(x^2 + 3x)$ using the chain rule.

- Find the derivative of $g(x) = e^{x^2+1}$ using the chain rule.

4. **Higher-Order Derivatives:**
   - Calculate the second derivative of $f(x) = x^4 - 3x^3 + 2x^2 - x + 5$.
   - Calculate the second derivative of $g(x) = \ln(x^2 + 1)$.

Derivative Interpretation

$(x^3)' = 3x^2$



1. **Understanding the Function:**
   - What is the mathematical expression for the cubic function defined in the cell? x^3
   - How is the derivative of the cubic function calculated? $3x^2$
2. **Plot Interpretation:**
   - What do the solid and dashed lines represent in the plot? solid → cubic function, dashed → derivative fct
   - How does f' behave when f increases or decreases? f increases $\Rightarrow f' > 0$, f decreases $\Rightarrow f' < 0$
3. **Mathematical Concepts:**
   - What is the significance of the derivative of a function in calculus?
   - What is the mathematical expression for the second derivative of the cubic function?
   - How does f'' behave when f' increases or decreases?

$(x^3)' = 3x^2$

$(x^3)'' = 6x$

la dérivée est la pente de la tangente à la fonction

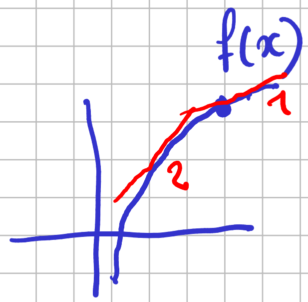
f' increases $\Rightarrow f'' > 0$

f' decreases $\Rightarrow f'' < 0$

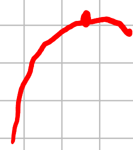
Demonstration of Derivative Functions

Derivative of Simple Functions Using Limits

$f(x) =$



$$\boxed{\begin{array}{l} f'(x) > 0 \\ f''(x) < 0 \end{array}}$$



The derivative of a function $f(x)$ at a point x is defined as the limit of the average rate of change of the function as the interval around x approaches zero. The derivative of a function $f(x)$ at a point x is denoted as $f'(x)$ or $\frac{df}{dx}$.

The derivative of a function $f(x)$ at a point x is defined as:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

To understand the concept of derivatives, let's start with some simple functions and calculate their derivatives using the limit definition.

1. **Derivative of $f(x) = x^2$**

   The derivative of $f(x) = x^2$ using the limit definition is:

   $$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

   Simplifying the expression inside the limit:

   $$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} (2x + h) = 2x$$

2. **Derivative of $f(x) = x$**

   The derivative of $f(x) = x$ using the limit definition is:

   $$f'(x) = \lim_{h \to 0} \frac{(x+h) - x}{h}$$

   Simplifying the expression inside the limit:

   $$f'(x) = \lim_{h \to 0} \frac{x + h - x}{h} = \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1$$

3. **Derivative of $f(x) = c \cdot g(x)$**

   The derivative of $f(x) = c \cdot g(x)$ using the limit definition is:

   $$f'(x) = \lim_{h \to 0} \frac{c \cdot g(x+h) - c \cdot g(x)}{h}$$

   Factoring out the constant c:

   $$f'(x) = c \cdot \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

   By definition, this is:

   $$f'(x) = c \cdot g'(x)$$

- $f(x) = 2x^3 - 5x^2 + 3x - 7$
- $g(x) = \sin(x) + \cos(x)$
- $h(x) = e^x - \ln(x)$

$f'(x) = (2x^3)' - (5x^2)' + (3.x)' - (7)'$ sum rule

$= \underbrace{2\cdot(x^3)' - 5\cdot(x^2)' + 3\cdot(x)'}_{\text{constant multiple rule}} - 0$

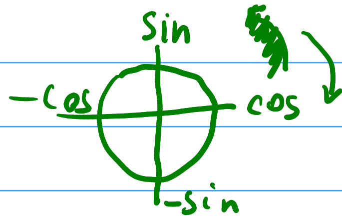
$(x^n)' = n.x^{n-1}$

$(x^3)' = 3x^{3-1}$ $= 2 . 3x^2 - 5 . 2x^1 + 3 . 1$

$f'(x) = 6x^2 - 10x + 3$

$g(x) = \underline{\sin(x)} + \underline{\cos(x)}$

$g'(x) = (\sin(x))' + (\cos(x))'$ sum Rule.



$g'(x) = \cos x + (-\sin x)$

$g'(x) = \cos x - \sin x$

$f(x) = 3x$ notation $f'(x) = \ldots$

$(3x)' = \ldots$

$h(x) = e^x - \ln x$

$h'(x) = (e^x)' - (\ln x)'$ sum Rule.

$= e^x - \frac{1}{x}$

$$f(x) = \overbrace{(3x^2+2x)}^{f_1}\cdot\overbrace{(x^3-x+1)}^{f_2}.$$

$$g(x) = \frac{x^2+1}{x^3-2x}. \qquad f_1 = x^2+1, \quad f_2 = x^3-2x$$

$$f'(x) = f'_1(x)\cdot f_2(x) + f_1(x)\cdot f'_2(x)$$ Product Rule

$$= (6x+2)\cdot(x^3-x+1) + (3x^2+2x)\cdot(3x^2-1)$$

$$g'(x) = \frac{f'_1\cdot f_2 - f_1\cdot f'_2}{f_2^2}$$

$$g(x) = \frac{x^2+1}{x^3-2x}. \qquad f_1 = x^2+1, \quad f_2 = x^3-2x$$

Quotient Rule?

$$\frac{f_1'\cdot f_2 - f_1\cdot f_2'}{f_2^2}$$

$$g'(x) = \frac{2x\cdot(x^3-2x) - (x^2+1)\cdot(3x^2-2)}{(x^3-2x)^2}$$

$$(2x)' = 2\cdot 1$$

$$(c\cdot f)' = c\cdot f'$$

$$= \frac{2x^4-4x^2-3x^4+2x^2-3x^2+2}{(x^3-2x)^2}$$

$$= \frac{-x^4-5x^2+2}{(x^3-2x)^2}$$

$f(x) = g(h(x))$

$f(x) = \sin(x^2 + 3x)$ using the chain rule.

$$\begin{cases} f'(x) = g'(h(x)) \cdot h'(x) \\ f(x) = g(h(x)) \end{cases}$$

composée

$g(x) = \sin x$

$h(x) = x^2 + 3x$

étape 1 : identifier g et h

$h'(x) = 2x + 3$

étape 2 : calculer $h'(x)$

$g'(x)$

$g'(h(x))$

$g'(x) = \cos x$

$g'(h(x)) = \cos(x^2+3x)$

étape 3 : j'assemble ma réponse

exemple $f(x) = 2x$

$f(3) = 2 \cdot 3$

$f'(x) = \cos(x^2+3x) \cdot (2x+3)$

$f(x^2) = 2x^2$

$f(x) = e^{x^2+1}$ $\qquad f'(x) = ?$

j'identifie : $g(x) = e^x$ et $h(x) = x^2 + 1$.

je dérive $g'(x) = e^x$ et $h'(x) = 2x$

ensuite : $g'(h(x)) = e^{x^2+1}$

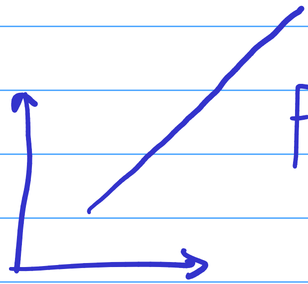
j'assemble : $f'(x) = \underbrace{e^{x^2+1}}_{g'(h(x))} \cdot \underbrace{2x}_{h'(x)}$.

- Calculate the second derivative of $f(x) = x^4 - 3x^3 + 2x^2 - x + 5$.
- Calculate the second derivative of $\underset{f}{\cancel{g}}(x) = \ln(x^2+1)$.

$$f'(x) = 4x^3 - 9x^2 + 4x - 1$$

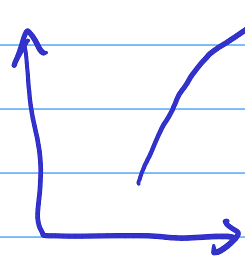
$$f''(x) = 12x^2 - 18x + 4$$

$$f(x) = \ln(x^2+1)$$ Chain Rule!



$$f(x) = g(h(x))$$

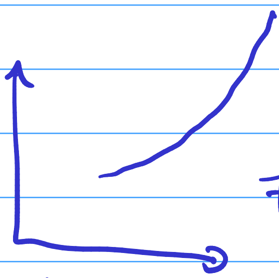
$$g(x) = \ln x$$



$$g'(x) = \frac{1}{x}$$

$$h(x) = x^2+1$$

$$h'(x) = 2x$$



$$g'(h(x)) = \frac{1}{x^2+1}$$

$$f'(x) = g'(h(x)) \cdot h'(x)$$ formule

$$f'(x) = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}.$$

$$f''(x) = \text{Quotient Rule!}$$

$$= \frac{2 \cdot (x^2+1) - 2x \cdot 2x}{(x^2+1)^2}$$

$$= \frac{2x^2+2 - 4x^2}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2}$$

A faire :

1 - dérivée de $f(x) = 3\ln(2x^3+1)$ *

2 - $f(x) = \ln(\cos x)$ *

3 - $f(x) = \dfrac{\sin x}{x}$

4 - $f(x) = 2\sin x \cos x$

5 - $f(x) = (\ln x)^2$ *