Condition of Two Parallel Lines

Two lines in a plane are said to be **parallel** if their direction vectors are scalar multiples of each other.

Let's define the **direction vector** of a line. A direction vector $\vec{d}$ of a line is any vector that is parallel to the line. If two lines have direction vectors $\vec{d_1}$ and $\vec{d_2}$, then the lines are parallel if and only if:

$$\vec{d_1} = k \cdot \vec{d_2}$$

where k is a scalar (a real number). This means that the components of $\vec{d_1}$ and $\vec{d_2}$ must be proportional.

Example 1:

Consider two lines with direction vectors $\vec{d_1} = 2\vec{i} + 3\vec{j}$ and $\vec{d_2} = 4\vec{i} + 6\vec{j}$. Are these lines parallel?

Solution: Notice that $\vec{d_2} = 2 \cdot \vec{d_1}$. Since $\vec{d_2}$ is a scalar multiple of $\vec{d_1}$, the lines are parallel.

Example 2:

Determine if the lines with direction vectors $\vec{d_1} = 3\vec{i} - 2\vec{j}$ and $\vec{d_2} = -6\vec{i} + 4\vec{j}$ are parallel.

Solution: Observe that $\vec{d_2} = -2 \cdot \vec{d_1}$. Since $\vec{d_2}$ is a scalar multiple of $\vec{d_1}$, the lines are parallel.