Derivatives Course

Join our course to understand derivatives!

Function	Derivative
$f(x) = c$	$f'(x) = 0$
$f(x) = x$	$f'(x) = 1$
$f(x) = x^n$	$f'(x) = nx^{n-1}$
$f(x) = e^x$	$f'(x) = e^x$
$f(x) = \ln(x)$	$f'(x) = \frac{1}{x}$
$f(x) = \sin(x)$	$f'(x) = \cos(x)$
$f(x) = \cos(x)$	$f'(x) = -\sin(x)$
$f(x) = \tan(x)$	$f'(x) = \sec^2(x)$

Function	Derivative	Name
$f(x) = c \cdot g(x)$	$f'(x) = c \cdot g'(x)$	Constant Multiple Rule
$f(x) = g(x) + h(x)$	$f'(x) = g'(x) + h'(x)$	Sum Rule
$f(x) = g(x) \cdot h(x)$	$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$	Product Rule
$f(x) = \dfrac{g(x)}{h(x)}$	$f'(x) = \dfrac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h(x)^2}$	Quotient Rule
$f(x) = g(h(x))$	$f'(x) = g'(h(x)) \cdot h'(x)$	Chain Rule

Derivative Exercises

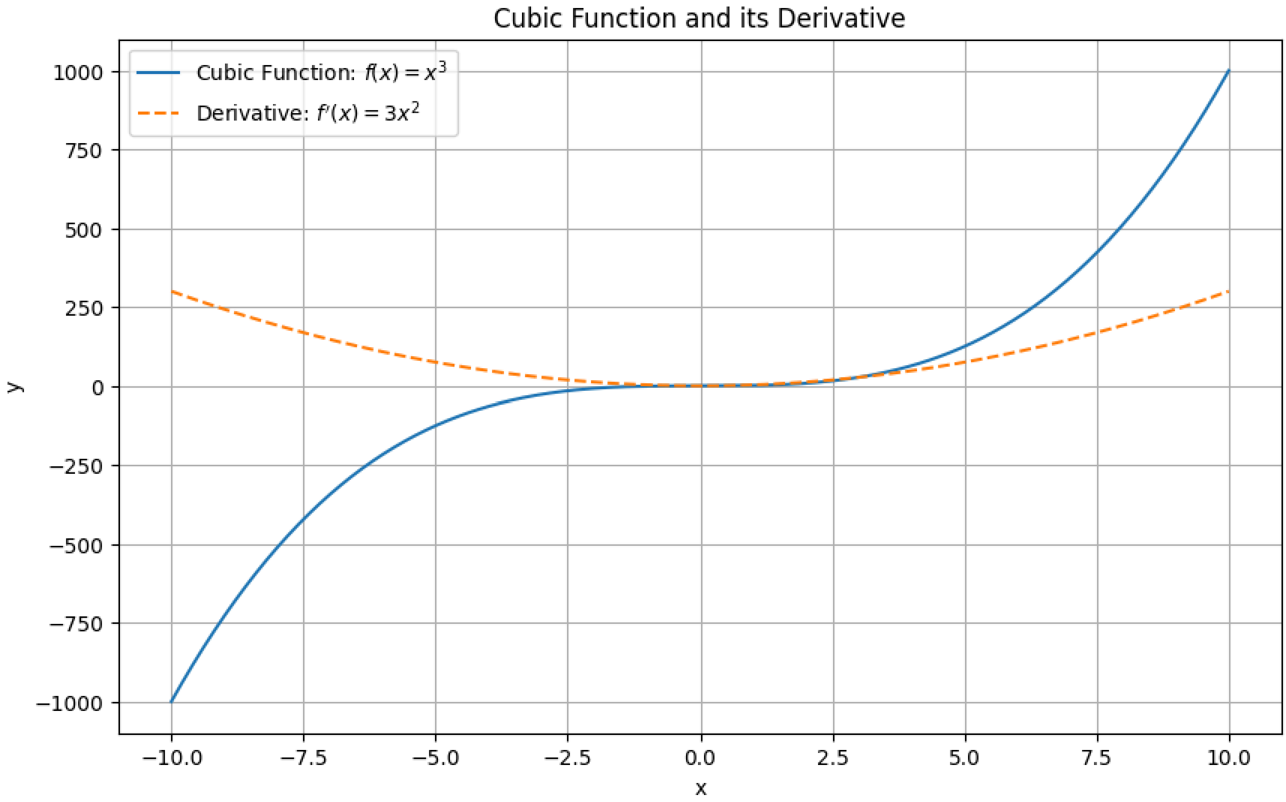
1. **Basic Derivatives:**
   - Calculate the derivative of the following functions:
     - $f(x) = 2x^3 - 5x^2 + 3x - 7$
     - $g(x) = \sin(x) + \cos(x)$
     - $h(x) = e^x - \ln(x)$
2. **Product and Quotient Rules:**
   - Use the product rule to find the derivative of $f(x) = (3x^2 + 2x)(x^3 - x + 1)$.
   - Use the quotient rule to find the derivative of $g(x) = \frac{x^2+1}{x^3-2x}$.
3. **Chain Rule:**
   - Find the derivative of $f(x) = \sin(x^2 + 3x)$ using the chain rule.

- Find the derivative of $g(x) = e^{x^2+1}$ using the chain rule.

4. **Higher-Order Derivatives:**

- Calculate the second derivative of $f(x) = x^4 - 3x^3 + 2x^2 - x + 5$.
- Calculate the second derivative of $g(x) = \ln(x^2 + 1)$.

Derivative Interpretation



1. **Understanding the Function:**

- What is the mathematical expression for the cubic function defined in the cell?
- How is the derivative of the cubic function calculated?

2. **Plot Interpretation:**

- What do the solid and dashed lines represent in the plot?
- How does f' behave when f increases or decreases?

3. **Mathematical Concepts:**

- What is the significance of the derivative of a function in calculus?
- What is the mathematical expression for the second derivative of the cubic function?
- How does f'' behave when f' increases or decreases?

Demonstration of Derivative Functions

Derivative of Simple Functions Using Limits

The derivative of a function $f(x)$ at a point x is defined as the limit of the average rate of change of the function as the interval around x approaches zero. The derivative of a function $f(x)$ at a point x is denoted as $f'(x)$ or $\frac{df}{dx}$.

The derivative of a function $f(x)$ at a point x is defined as:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

To understand the concept of derivatives, let's start with some simple functions and calculate their derivatives using the limit definition.

1. **Derivative of** $f(x) = x^2$

   The derivative of $f(x) = x^2$ using the limit definition is:

   $$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

   Simplifying the expression inside the limit:

   $$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} (2x + h) = 2x$$

2. **Derivative of** $f(x) = x$

   The derivative of $f(x) = x$ using the limit definition is:

   $$f'(x) = \lim_{h \to 0} \frac{(x+h) - x}{h}$$

   Simplifying the expression inside the limit:

   $$f'(x) = \lim_{h \to 0} \frac{x + h - x}{h} = \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1$$

3. **Derivative of** $f(x) = c \cdot g(x)$

   The derivative of $f(x) = c \cdot g(x)$ using the limit definition is:

   $$f'(x) = \lim_{h \to 0} \frac{c \cdot g(x+h) - c \cdot g(x)}{h}$$

   Factoring out the constant c:

   $$f'(x) = c \cdot \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

   By definition, this is:

   $$f'(x) = c \cdot g'(x)$$