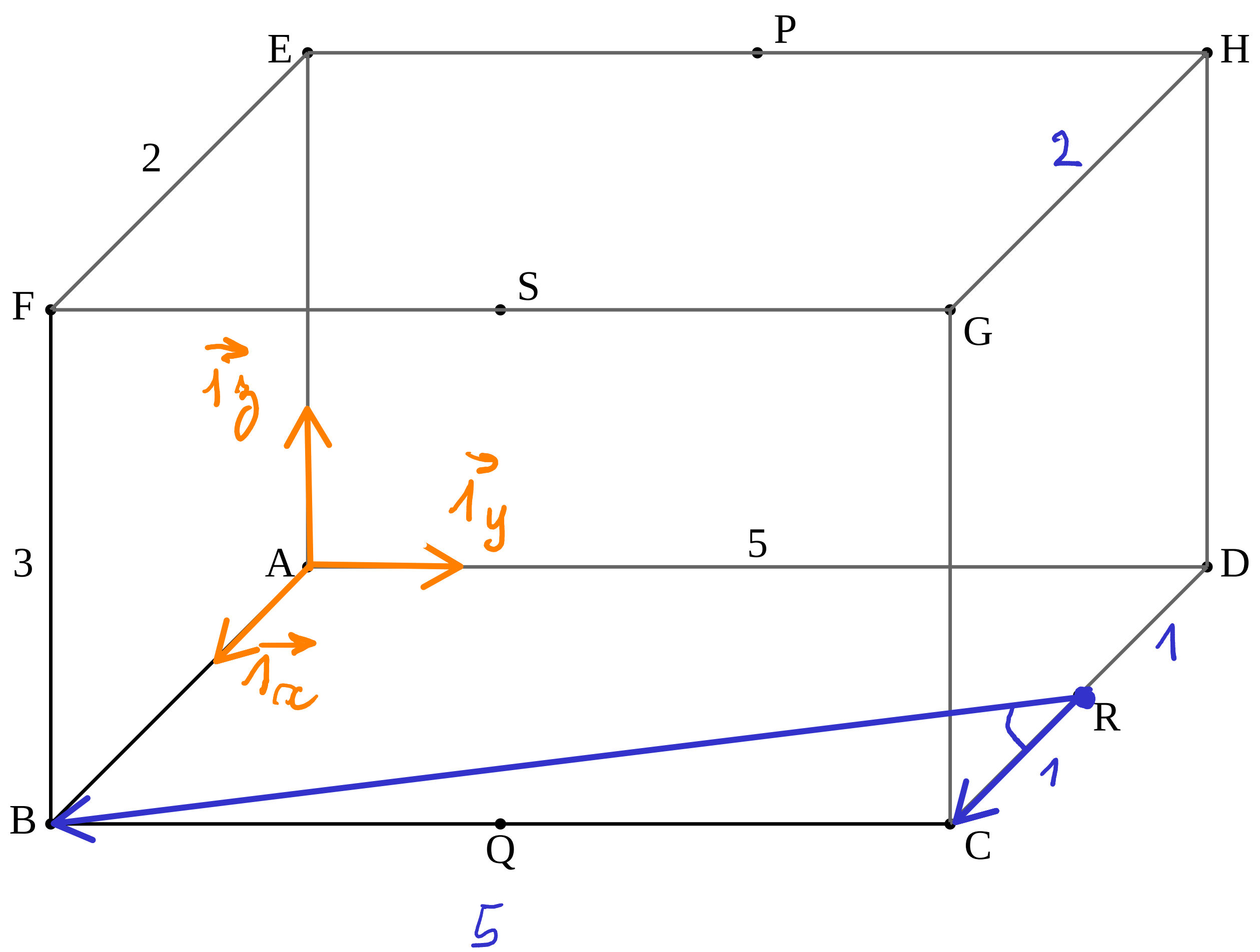
PREPA-ES-GEOMETRIE VECTORIELLE



1. Calculez les angles suivants à l'aide du produit scalaire

$$\widehat{BRC} \quad \widehat{FPQ} \quad \widehat{GSD}$$

$\vec{RB}\begin{pmatrix}1\\-5\\0\end{pmatrix}$

$\vec{RB} \cdot \vec{RC} = \|\vec{RB}\| \cdot \|\vec{RC}\| \cdot \cos \hat{R}$

$\vec{RC}\begin{pmatrix}1\\0\\0\end{pmatrix}$

2. Orthogonality Questions

1. Given the vectors $\vec{u}_1 = (p+5, 1, 5)$ and $\vec{v}_1 = (3, -2p, 5)$, determine the possible values of p so that these vectors are orthogonal.

2. Given the vectors $\vec{u}_2 = (1, p, 3)$ and $\vec{v}_2 = (p, -1, 0)$, find the values of p such that they are orthogonal.

3. Given the vectors $\vec{u}_3 = (2, 3, p)$ and $\vec{v}_3 = (-p, 1, 2)$, determine if there exists a value of p that makes the vectors orthogonal.

$$\vec{RB} \cdot \vec{RC} = 1 \cdot 1 + (-5) \cdot 0 + 0 \cdot 0 = 1$$

$$\|\vec{RB}\| = \sqrt{1^2 + (-5)^2 + 0^2} = \sqrt{26}$$

$$\|\vec{RC}\| = \sqrt{1^2 + 0^2 + 0^2} = 1$$

$$\Rightarrow \frac{1}{\sqrt{26}} = \frac{\sqrt{26} \cdot 1}{\sqrt{26}} \cdot \cos \hat{R} \Rightarrow$$

$$\Rightarrow \cos R = \frac{1}{\sqrt{26}}$$

$$\Rightarrow \hat{R} = \arccos\left(\frac{1}{\sqrt{26}}\right) = 78{,}7^{\circ}$$

2) $\vec{u}_1 = (p+5, 1, 5)$ and $\vec{v}_1 = (3, -2p, 5)$.

$\vec{u}_1 \perp \vec{v}_1$ și produit scalaire $\vec{u}_1 \cdot \vec{v}_1 = 0$

$$\vec{u}_1 \cdot \vec{v}_1 = (p+5) \cdot 3 + 1 \cdot (-2p) + 5 \cdot 5$$

$$= 3p + 15 - 2p + 25$$

$$= p + 40 = 0 \Rightarrow \boxed{p = -40}$$

$\vec{u}_2 = (1, p, 3)$ and $\vec{v}_2 = (p, -1, 0)$.

$$\vec{u}_2 \cdot \vec{v}_2 = 1 \cdot p - p + 0$$

$$= 0$$

<u>Réponse</u> : $p \in \mathbb{R}$

1. Determine the possible values of p such that the vectors $\vec{u}_1 = (p^2, 2, 4)$ and $\vec{v}_1 = (1, 4, 8)$ are collinear.

$$\vec{v}_1 = k \cdot \vec{u}_1 \qquad (k \neq 0)$$

$$\frac{8}{4} = 2 \qquad \frac{4}{2} = 2 \qquad \cancel{p^2} \cdot \frac{1}{\cancel{p^2}} = 2 \cdot p^2$$

$$x^2 = 4$$

$x = -2$ ou $x = 2$

$$1 = 2p^2$$

$$\Rightarrow \frac{1}{2} = p^2$$

$$p = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

ou

$$p = -\frac{\sqrt{2}}{2}$$

3. Collinearity Questions

1. Determine the possible values of p such that the vectors $\vec{u}_1 = (p^2, 2, 4)$ and $\vec{v}_1 = (1, 4, 8)$ are collinear.

2. Find the values of p such that the vectors $\vec{u}_2 = (3p, 6, 9)$ and $\vec{v}_2 = (1, 2, 3)$ are collinear.

3. Prove that the vectors $\vec{u}_3 = (p, p, p)$ and $\vec{v}_3 = (1, 1, 1)$ are collinear for any value of $p \neq 0$.

4. Explique un procédé pour déterminer l'orthogonalité de deux droites dont on connait les équations
5. Explique un procédé pour déterminer le parallélisme de deux droites dont on connait les équations
6. Démontre que dans un triangle quelconque triangle KLM on a :

$$KL^2 = KM^2 + LM^2 - 2.KM.LM.cos\widehat{M}$$

à l'aide du produit scalaire

7. Si dans ce triangle on a $KM = 7$, $KL = 6$ et $LM = 5$, calcule $\widehat{M}$