Vague by Design: Performance Evaluation and Learning from Wages

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Introduction

- Performance evaluation is a key aspect of labor contracts and organization design
  - Many ways to evaluate: Shop floor control, consumer scores, product testing, sales,...
  - Digitization provides a growing number of possibilities
- Performance evaluations are an important source of information in the workplace
- Inform the firm about the worker's performance
  - Necessary basis of incentivizing effort via performance pay
  - Classic results show more information is better Holmström '79, Grossman&Hart '83
- Inform the worker about his performance
  - Learn about ability/match with the job
  - Confidence in his capability to succeed and sense of agency

This Paper

Dual role of performance evaluation: basis of *incentives* and agent *learning*

- How do these two aspects interact?
- How to optimally design performance evaluation when it shapes worker confidence?

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- How do these two aspects interact?
- How to optimally design performance evaluation when it shapes worker confidence?

- Two-period model of moral hazard with uncertain match-specific ability
- Principal designs evaluation of output and contingent wages
  - Fully flexible evaluation: Could observe true contribution to profits
  - Commitment to performance pay
- Learning about the agent's ability based on these evaluations

This Paper: Preview of Results

Dual role of performance evaluation: basis of ***incentives*** and agent *learning*

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- Agent learning imposes a cost on the principal

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- Binary case: Optimal evaluation is noisy & tough

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Dual role of performance evaluation: basis of *incentives* and agent *learning*

- Agent learning imposes a cost on the principal
- Binary case: Optimal evaluation is noisy & tough
- General case:
  - lower censorship
  - use information about effort, shroud information about ability

Road Map

- Literature
- 2x2 Model
  - Transformation to Belief Space
  - Terminal Period: Agent Learning Imposes a Cost on the Principal
  - Initial Period: Optimal Evaluation – Noisy and Though
- General Case
- Extensions

Related Literature

- Design of information
  Kolotilin et al. '22, Doval&Skreta forthcoming
  and performance pay:
  Georgiadis&Szentes '20, Hoffmann et al. '21, Li&Yang '20
- Implicit incentives and information design:
  Ely&Szydlowski '20, Hörner&Lambert '21, Smolin '20
- More information can increase the cost of incentives:
  Fang&Moscarini '05, Jehiel '14, Nafziger '09, Meyer&Vickers '97

2x2 Model

The Model

- Two time periods $t \in \{1, 2\}$, common discount factor δ.
- Agent
  - risk averse with utility index $u = \sqrt{w}$ (this talk) and reservation utility U
  - observable but nonverifiable effort $e_t \in \{0, 1\}$ at cost $c \cdot e$
  - time-invariant ability $\theta \in \{\theta_H, \theta_L\}$, high with prior probability μ.
- Principal
  - risk neutral
  - implements high effort

- Output is
  - high or low, $y_t \in \{y_L, y_H\}$, high with probability

type \ effort	$e_t = 0$	$e_t = 1$
$\theta = \theta_L$	a	$a + b$
$\theta = \theta_H$	$a + \Delta a$	$a + b + \Delta a + \Delta b$

- Effort is productive: $b \geq 0$
- Ability is productive: $\Delta a \geq 0$
- Complementarities: Δb
  Log-Supermodular: $\frac{\Delta b}{b} > \frac{\Delta a}{a}$

Technology

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  - a signal structure $S, p(s|y_t)$, and
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Information, Contracts and Commitment

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  - a signal structure $S, p(s|y_t)$, and
  - wages w as a function the signal.
- Agent observes the contract and makes participation and effort decision
- Output is not observed
- Principal and agent observe the signal realization, wages, and effort
- Update beliefs to $\hat{\mu}(s) \in [\underline{\mu}, \bar{\mu}]$

The Contracting Problem

First Period

$$\Pi_1 = \max_{S,p,w} \mathbb{E}[y|e=1,\mu] + \int_S p(s|e=1,\mu)\Big(\Pi_2(\hat{\mu}(s)) - w(s)\Big)\,\mathrm{d}s \tag{1}$$

$$\text{s.t.} \int_S p(s|e=1,\mu)u(w(s))\,\mathrm{d}s - c \geq U \tag{P$_1$}$$

$$\int_S p(s|e=1,\mu)u(w(s))\,\mathrm{d}s - c \geq \int_S p(s|e=0,\mu)u(w(s))\,\mathrm{d}s \tag{IC$_1$}$$

Second Period

$$\Pi_2(\hat{\mu}) = \max_{S,p,w} \mathbb{E}[y|e=1,\hat{\mu}] - \int_S p(s|e=1,\hat{\mu})w(s)\,\mathrm{d}s \tag{2}$$

$$\text{s.t.} \int_S p(s|e=1,\hat{\mu})u(w(s))\,\mathrm{d}s - c \geq U \tag{P$_2$}$$

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Posterior Space

$$\max_{S,p,w} \mathbb{E}[y|e=1] + \mathbb{E}_p\left(\Pi_2(\hat{\mu}(s)) - w(s)\right)$$

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$$\begin{tikzcd}
y \arrow[r, "p"] & s \arrow[r] & \Delta(Y) \arrow[dl] \arrow[dr] & \\
& \text{posterior} \in \Delta(\Theta) & & \text{score} \in \mathbb{R}
\end{tikzcd}$$

Posterior Space

$$
\begin{aligned}
\max_{S,p,w}\ & \mathbb{E}[y|e=1] + \mathbb{E}_p\left(\Pi_2(\hat{\mu}(s)) - w(s)\right) \\
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$$\Delta\Delta(\Theta) \longleftrightarrow \Delta(\mathbb{R})$$

Proposition

The contracting problem can be written as a choice of a distribution over posteriors, m, with mean μ and support on $[\underline{\mu}, \bar{\mu}]$ and a mapping from posteriors to wages.

- Wages deterministic as a function of the posterior: $w(s) \mapsto w(\hat{\mu})$
- Probability over signal realizations on-path: $p(s|e_t = 1, \mu) \mapsto m(\hat{\mu})$
- Score:

$$\begin{aligned} p(s|e_t = 1, \mu) - p(s|e_t = 0, \mu) &\mapsto \big(b + \Delta b\mu\big)\big(p(s|y_H) - p(s|y_L)\big) \\ &\mapsto \big(b + \Delta b\mu\big)\frac{\hat{\mu} - \mu}{(\Delta a + \Delta b)\mu(1 - \mu)} m(\hat{\mu}) \end{aligned}$$

$$\Pi_1 = \max_{m,w} \mathbb{E}[y|e=1,\mu] + \int m(\hat{\mu})\left(\delta \Pi_2(\hat{\mu}) - w(\hat{\mu})\right)\,\mathrm{d}\hat{\mu} \tag{3}$$

$$\text{s.t.} \int u(w(\hat{\mu}))m(\hat{\mu})\,\mathrm{d}\hat{\mu} - c \geq U \tag{P$_1$}$$

$$\int (b + \Delta b\mu)\frac{\hat{\mu}-\mu}{(\Delta a + \Delta b)\mu(1-\mu)} u(w(\hat{\mu}))m(\hat{\mu})\,\mathrm{d}\hat{\mu} \geq c \tag{IC$_1$}$$

$$\int \hat{\mu}\, m(\hat{\mu})\,\mathrm{d}\hat{\mu} = \mu\,; \quad \mathrm{supp}(m) \subset [\underline{\mu}, \bar{\mu}] \tag{BP}$$

The Final Period

- Pure incentive problem, no motive to shape learning
- Classic result of Grossman and Hart (1983):

Proposition

The optimal evaluation in the final period is fully informative.

The Impact of Information

- The last period induces the continuation value $\int \Pi_2(\hat{\mu}) m(\hat{\mu}) \, \mathrm{d}\hat{\mu}$
- What is the impact of more information about the agent's type?

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   scales with Δa: impact of ability

2. Agent has more information when choosing effort
   - More expensive to satisfy incentive compatibility
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   scales with Δb: interaction of effort and ability

- Second-period IC:

$$u(w_H) - u(w_L) = \frac{c}{b + \hat{\mu}\Delta b}$$

The Impact of Agent Learning

- Second-period IC:

$$u(w_H) - u(w_L) = \frac{c}{b + \hat{\mu}\Delta b}$$

- Required bonus inversely proportional to a linear function of beliefs
    - Optimistic agent cheaper to motivate
    - Uncertain agent is cheaper to motivate
    - Given change in belief has a larger effect at low beliefs

Learning is Costly

Proposition

If the technology is log-supermodular, Π_2, is strictly concave in the posterior belief. Furthermore, it is more concave at low posteriors, $\Pi_2''' > 0$.

- Strong complementarity of effort and ability: Agent learning dominates
- Principal has an incentive to conceal information
- Information aversion strongest at the bottom: avoid pessimistic agents

Solving the Full Problem

- First period: Incentives and learning
  - Incentives: More informative evaluation *decreases* agency cost *this period*
  - Learning: More informative evaluation *increases* agency cost *next period*

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- First period: Incentives and learning
  - Incentives: More informative evaluation *decreases* agency cost *this period*
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- Information design problem, with:
  - Endogenous payoffs (wages are designed)
  - Additional constraints (participation and incentive compatibility)
- Binary state does not guarantee binary evaluation (Le Treust&Tomala, 2019)

Theorem 1

The optimal contract in the first period is (essentially) unique, with a binary ($S = \{G, B\}$) and tough evaluation structure. Let $\sigma \in [0, 1)$ denote the degree of vagueness. The optimal contract consists of

$$\begin{tikzcd} y_H \arrow[r, "1-\sigma"] \arrow[dr, "\sigma"] & G \\ y_L \arrow[r] & B \end{tikzcd}$$

- a good evaluation and associated high wage, only if output was good,
- a bad evaluation and associated low wage: always after output was bad, with prob. σ after output was good.

Proof of Theorem 1: Outline

$$\mathcal{L}(w, m; \underbrace{(\lambda_P, \lambda_{IC})}_{\lambda})$$

Lagrangian of the contracting problem including (P) and (IC)

Concavification of the partially maximized Lagrangian (Georgiadis&Szentes '20)

Proof of Theorem 1: Outline

$$\mathcal{L}(w, m; \lambda)$$

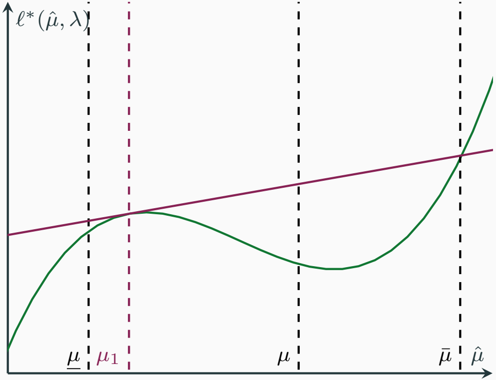
Optimal Wages given m, λ: Standard moral hazard problem $\mapsto w^*(\hat{\mu}; \lambda)$

objective is an expectation given λ: $\mathcal{L}(w^*(\hat{\mu}; \lambda), m; \lambda) = \int \ell^*(\hat{\mu}; \lambda) m(\hat{\mu})\, \mathrm{d}\hat{\mu}$

Information Design given λ: Concavification of $\ell^* \mapsto m^*(\hat{\mu}; \lambda)$

$m^*(\cdot; \lambda)$ binary and tough for all λ

Duality: $\mapsto$ Solution exists and features of m^* hold in the optimal contract



- Unconstrained information design
- Payoff $\ell^*(\hat{\mu};\lambda)$
- $\frac{\partial^3}{\partial \hat{\mu}^3}\ell^*(\hat{\mu};\lambda) > 0$
  - Convex $\implies$ m fully informative
  - Concave-convex: For low μ, learning effect dominates $\implies$ m tough
- This for given λ, but $\lambda(m)$!

Binary and Noisy

- Optimal evaluation is binary
    - Motive to control learning does not add complexity to the evaluation
    - Data aggregated into a pass-fail signal
- Noisy evaluation can be optimal
    - Preserve agent's uncertainty
- The optimal noise is asymmetric
    - Tough evaluation: Avoid unwarranted praise, embrace unwarranted reprimand
    - "Drill-sergeant mentality" is part of optimal organization design
- Prevent very low posteriors
    - Costly to motivate, change in posterior has a large effect
- Result of joint design of evaluation and wages

What if effort and ability are not strong complements?

- Intermediate case: Weak complements/substitutes
  - More information increases the continuation value
  - Fully informative evaluation
- Strong substitutes: $\frac{\Delta b}{b} + \frac{\Delta a}{1-a} < 0$
  - More information decreases the continuation value
  - Noisy and *lenient* evaluation
- Lenient Evaluation
  - Let some failures slip (but punish others harshly)
  - Lack of reprimand not very informative: Avoids complacency

plot

The General Case

Posterior Space

Consider now a general $Y \subset \mathbb{R}$ with pdf $f(y|e,\mu)$

$$
\begin{aligned}
&\max_{S,p,w} \mathbb{E}[y|e=1] + \mathbb{E}_p\Big(\Pi_2(\hat{\mu}(s)) - w(s)\Big) \\
&\text{s.t. } \mathbb{E}_p u(w(s)) - c \geq U && (\mathrm{P}_1) \\
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- Rewrite the program as a choice of $\Phi \in \Delta\Delta Y$
- Let $\boldsymbol{\mu} \in [0,1]^{|Y|}$, $\boldsymbol{x} \in \mathbb{R}^{|Y|}$ denote the vector of posteriors and scores given y

$$\max_{w,\Phi} \mathbb{E}\left(\Pi_2(\boldsymbol{\mu}\cdot\phi) - w(\phi)\right)$$

$$\text{s.t. } \mathbb{E}\, u(w(\phi)) - c \geq U \tag{P$_1$}$$

$$\mathbb{E}\,(\boldsymbol{x}\cdot\phi)\, u(w(\phi)) \geq c \tag{IC$_1$}$$

$$\mathbb{E}\,\phi = f(\cdot|1,\mu_0)$$

- Can learn about the optimal structure of evaluation from looking at $\boldsymbol{\mu}$ and $\boldsymbol{x}$

Optimal Evaluation: Geometry

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- **"Learning about effort (only) is always desirable"**
    - Suppose $\boldsymbol{\mu}(y) = \boldsymbol{\mu}(y')$ and $\boldsymbol{x}(y) \neq \boldsymbol{x}(y')$.
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- **The 1-dimensional case**: The following are equivalent
  - $\boldsymbol{x} \in \mathrm{span}(\boldsymbol{\mu}, \mathbf{1})$
  - there is a bijection between feasible distributions over posteriors and feasible distributions over scores
  - $f = f(\cdot|0,0) + g(e,\theta)\big(f(\cdot|1,1) - f(\cdot|0,0)\big)$

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- 1-dim: Sufficiently strong complementarities $\implies \Pi_2''' > 0 \implies$ lower censorship

Extensions

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- Principal can acquire private information
    - Principal-preferred equilibrium: outcome-equivalent to optimal contract

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- Long-run commitment
    - Robust to commitment to *continuation value*, observed by the agent
    - Full-commitment difficult: belief-manipulation & belief-dependent costs of delay
- Many periods
    - Not analytically tractable: lack of control over shape of continuation value
    - Numerically: Same structure within period; noisier evaluation early in the relationship

Conclusion

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    - about effort: Incentives
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  - As much information as possible about effort
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Conclusion →

- Outcome of performance evaluation is a crucial source of information
  - about effort: Incentives
  - about the agent's ability: Confidence
- Tension between those two channels (learning about the importance of effort)
  - As much information as possible about effort
  - As little information as possible about ability
- Optimal Performance Evaluation
  - Noisy, even though wages could condition on true y
  - Strong complementarity: avoid very low posterior beliefs (tough/lower-censorship)

Outlook

- Preference across given information sources: conduct, not results!
    - Salary differences between workers: mostly driven by types, so should be concealed
- Affects task design: Harder/easier to keep agents motivated
- Career Concerns: informationally opposite forces
    - information about effort and ability inseparably intertwined
    - here: source of friction; CC: source of incentives

Thank You!

Utility Function

- Sufficient condition on utility function
- $w = u^{-1}$, “cost of utility”

Assumption 1

1. (No incentives at probability zero) $\frac{w(x)}{x} \to \infty$ as $x \to \infty$.
2. (Decreasing curvature) $w''' \leq 0$.
3. (Bounded changes in curvature) $\frac{w'''(u_H)}{w''(u_H)} \geq -A$.

Condition

- Satisfied for CRRA $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$
  - if $\gamma \leq 1/2$ and U sufficiently large.
  - Always satisfied for $\gamma = \frac{1}{2}$

back

Step 1: Optimal Wages

- Let $\mathcal{L}(m, w; (\lambda_P, \lambda_{IC}))$ denote the Lagrangian associated to the problem.
- Solving for the optimal wage given λ yields

$$w^*(\hat{\mu}, \lambda) = u'^{-1}\left(\left(\lambda_P + \lambda_{IC}\frac{b + \Delta b\mu}{(\Delta a + \Delta b)\mu(1-\mu)}(\hat{\mu} - \mu)\right)^{-1}\right)$$

- Partially maximized Lagrangian, $\sup_w \mathcal{L}(m, w; (\lambda_P, \lambda_{IC}))$, is posterior separable

$$\begin{aligned}\mathcal{L}(m, w^*(\hat{\mu}, \lambda); (\lambda_P, \lambda_{IC})) = \int \Big\{ & P^1_\mu Y + \delta\Pi_2(\hat{\mu}) - w^*(\hat{\mu}, \lambda) \\ & + \lambda_P\left(u(w^*(\hat{\mu}, \lambda)) - c - U\right) \\ & + \lambda_{IC}\left(\frac{b + \Delta b\mu}{(\Delta a + \Delta b)\mu(1-\mu)}(\hat{\mu} - \mu)\, u(w(\hat{\mu}, \lambda)) - c\right)\Big\} m(\hat{\mu})d\hat{\mu}\end{aligned}$$

back

Step 2: Information Design

- Unconstrained information design problem with payoff $\ell^*(\hat{\mu};\lambda)$
- The objective is either convex or concave-convex since

$$\frac{\partial^3}{\partial\hat{\mu}^3}\ell^*(\hat{\mu};\lambda) = \lambda_{IC}\left(\frac{b+\Delta b\mu}{(\Delta a+\Delta b)\mu(1-\mu)}\right)\frac{\partial^2}{\partial\hat{\mu}^2}u(w(\hat{\mu};\lambda)) + \delta\Pi_2'''(\hat{\mu}) > 0$$

Lemma

For any λ_{IC}, there exists a unique solution to the information design problem. It induces at most two posteriors: the highest feasible posterior $\bar{\mu}$ with probability $m(\bar{\mu}) \in [0, \frac{\mu-\underline{\mu}}{\bar{\mu}-\underline{\mu}}]$ and a low posterior, $\mu^* \in [\underline{\mu}, \mu]$ with $m(\mu^*) \in [\frac{\bar{\mu}-\mu}{\bar{\mu}-\underline{\mu}}, 1]$.

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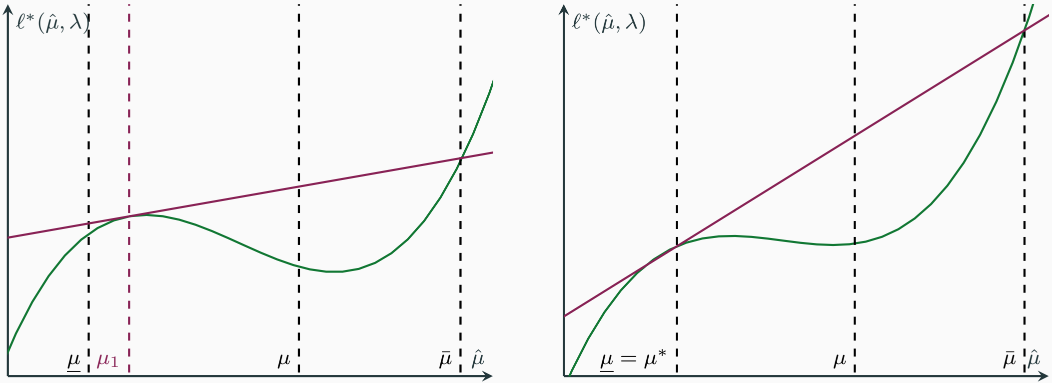

(a) Interior solution.

(b) Corner solution.

back

Step 3: Strong Duality

- We need to show strong duality in the general problem, i.e.

$$\inf_{\lambda \geq 0} \sup_{w,m \text{ s.t. (BP)}} \mathcal{L}(m, w; \lambda) = \sup_{w,m \text{ s.t. (BP)}} \inf_{\lambda \geq 0} \mathcal{L}(m, w; \lambda)$$

- Two steps: [1] Wages

Lemma

The wage setting problem satisfies strong duality, i.e.

$$\sup_{w} \inf_{\lambda \geq 0} \mathcal{L}(m, w; \lambda) = \inf_{\lambda \geq 0} \sup_{w} \mathcal{L}(m, w; \lambda).$$

back

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- Two steps: [2] Information Design

Lemma

The information design problem satisfies strong duality, i.e.

$$\sup_{m \text{ s.t.(BP)}} \inf_{\lambda \geq 0} \int \ell^*(\hat{\mu}; \lambda) m(\hat{\mu}) \, \mathrm{d}\hat{\mu} = \inf_{\lambda \geq 0} \sup_{m \text{ s.t.(BP)}} \int \ell^*(\hat{\mu}; \lambda) m(\hat{\mu}) \, \mathrm{d}\hat{\mu}.$$

back

A Simplified Problem

- Define a simplified problem, using binary and tough evaluation

$$\max_{m_1,m_2,\mu_1,w_1,w_2} \mathbb{E}[y_1|e=1,\mu] + m_1\big(\Pi_2(\mu_1) - w_1\big) + m_2\big(\Pi_2(\bar{\mu}) - w_2\big) \tag{4}$$

$$\text{s.t.} \quad m_1 u(w_1) + m_2 u(w_2) - c \geq U \tag{P}$$

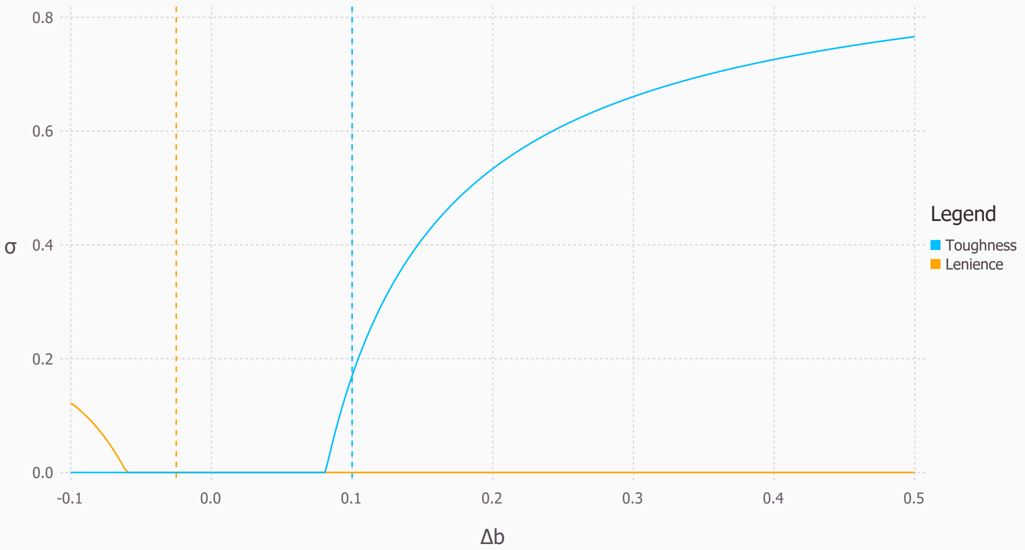
$$\frac{b + \Delta b \mu}{(\Delta a + \Delta b)\mu(1-\mu)} \sum_i m_i(\mu_i - \mu) u(w_i) \geq c \tag{IC}$$

$$m_1\mu_1 + m_2\bar{\mu} = \mu\,; \quad m_1 + m_2 = 1\,; \quad \mu_1 \geq \underline{\mu} \tag{BP}$$

back

COMPLEMENTS AND SUBSTITUTES

BACK



Substitutes: Condition on Utility

BACK

Assumption (1*)

1. *(No incentives at probability zero)* $\frac{w(x)}{x} \to \infty$ *as* $x \to \infty$.
2. *(Increasing curvature)* $w''' \geq 0$.
3. *(Bounded changes in curvature)*

$$\frac{3(b+\mu\Delta b)\Delta b}{c(a\Delta b - b\Delta a)} \geq \frac{w'''(u_L)}{w''(u_L)},$$

where $u_L = U - \frac{a+\mu\Delta a}{b+\mu\Delta b}c$.

Private Information of the Principal

- Principal chooses
  - Evaluation structure: observed by agent, basis of performance pay and learning
  - Private evaluation: not observed by agent, basis of learning only for principal
- Joint distribution over posteriors: $m_P(\mu_P, \hat{\mu})$
  - Agent observes $m(\hat{\mu}) = \int m_P(\mu_P, \hat{\mu}) \, \mathrm{d}\mu_P$
- Dynamic game with incomplete information
- Agent updates belief based on
  - First-period evaluation
  - Second-period contract offer

back

Private Information of the Principal

- Unique PBE with passive beliefs: outcome equivalent to optimal contract without private information acquisition
  - Passive beliefs: no updating based on contract offer
  - Principal preferred*
- Private information either revealed or not useful
  - If private information isn't used to adjust second period contract: irrelevant
  - Information used to adjust contract offer: revealed to agent
  - Better to also use it as a basis of performance pay

*Among equilibria that satisfy no-holdup: No rent for the agent in the second period on path.

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    - Better to also use it as a basis of performance pay
- Remains an equilibrium when principal *has to* acquire private information
- Unique† when private information acquisition strategy observed

back

*Among equilibria that satisfy no-holdup: No rent for the agent in the second period on path.
†Under no-holdup and no-signaling-what-you-don't-know.

Unobservable Effort

- Suppose effort is not observed by the principal
- After a deviation to low effort, signal s
  - Principal has posterior

$$\hat{\mu}(s) = \mu \frac{p(s|y_L) + (a + b + \Delta a + \Delta b)\left[p(s|y_H) - p(s|y_L)\right]}{p(s|y_L) + (a + b + (\Delta a + \Delta b)\mu)\left[p(s|y_H) - p(s|y_L)\right]}$$

  - Agent interprets signal differently:

$$\mu \frac{p(s|y_L) + (a + \Delta a)\left[p(s|y_H) - p(s|y_L)\right]}{p(s|y_L) + (a + \Delta a \mu)\left[p(s|y_H) - p(s|y_L)\right]}$$

- Agent has private information about the posterior

back

Unobservable Effort

- Incentive compatibility in the second period:
  - Slack if agent more optimistic
  - Violated if agent more pessimistic
- “Belief-manipulation motive”
- Double deviations optimal
- First-period IC dynamic: Kink in the principal’s objective at prior μ

$$\int \left\{ \frac{(b+\mu\Delta b)}{\mu(1-\mu)\Delta b}\,(\hat{\mu}-\mu)\,u(w(\hat{\mu})) - \left[1 - \frac{(b+\mu\Delta b)}{\mu(1-\mu)\Delta b}\,(\hat{\mu}-\mu)\right] \max\{0, c\Delta b\,\frac{\mu-\hat{\mu}}{b+\hat{\mu}\Delta b}\} \right\} m(\hat{\mu})\,\mathrm{d}\hat{\mu} \geq c$$

back

Unobservable Effort

- Under $u = \sqrt{\cdot}$ and $\Delta a = 0$: At most *three* evaluation outcomes
  - Neutral signal: Not informative about effort and ability[‡]
  - Conditional on informative evaluation: binary and tough
- Intuition: Avoid outcomes that allow generation of private information

back

[‡] In simulations: Never used.

Unobservable Effort

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  - Neutral signal: Not informative about effort and ability[‡]
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- Intuition: Avoid outcomes that allow generation of private information
- More complicated with long-run contracting:
  - Principal can induce a learning motive by providing excessive bonuses in $t = 2$
  - Joint design of information and wages in *both periods*

back

[‡] In simulations: Never used.

Long Run Commitment: Continuation Value

- Principal commits to contract: (S, p, w, v)
  - a signal structure $S, p(s|y)$, realization conditional on contemporaneous output
  - wages w, and
  - **continuation value $\mathbf{v}$ as a function the signal.**

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  - **continuation value $\mathbf{v}$ as a function the signal.**
- Assume $u(x) = 2\sqrt{x}$
  - Theorem 1 goes through, delaying *payments* does not affect the mechanism
  - Optimal evaluation: binary and weakly tough

back

Long Run Commitment: Full Commitment

- Principal commits to long-run contract: $(S_1 \times S_2, p, w)$
  - a signal space $S_1 \times S_2$, p progressively measurable wrt y_t,
  - and wages w, progressively measurable wrt s_t.
- Difficult:
  - Agent acquires private info after shirking (effort unobservable to the contract), and
  - the principal can commit to excess bonuses in $t = 2$ (to induce a learning motive).

$\implies$ Characterizing the optimum requires joint design in both periods.

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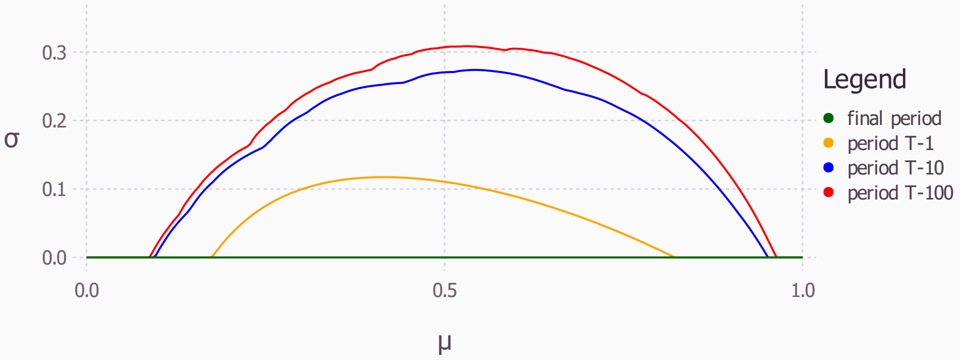
$\implies$ Characterizing the optimum requires joint design in both periods.

- Optimum is not tractable. Effect is still in place:
  - Consider optimal contract without fully informative evaluation
  - Bonus for high output in period 1 optimally split between both periods
  - Principal can *postpone* information, but it is *costly*

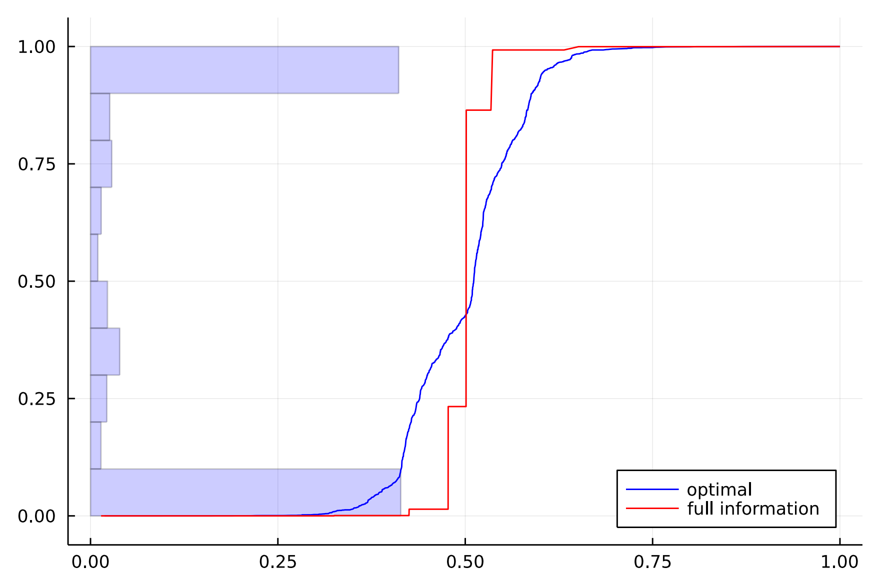
back

Many Periods



Many Periods

BACK



Utility Function

BACK

Assumption (Bounded changes in curvature)

$$\frac{w'''(u_H)}{w''(u_H)} \geq -\frac{3(b+\mu\Delta b)\Delta b}{c((1-a)\Delta b + b\Delta a)},$$

where $u_H = U + \frac{1-a-\mu\Delta a}{b+\mu\Delta b}c.$

Underconfidence: When is Learning costly?

Proposition

The effect of learning on the principal's continuation value is ambiguous.

- *There exists a threshold $\bar{U}$ such that the continuation value is increasing in information if $U \geq \bar{U}$, and*
- *there exists a threshold $\bar{b} > 0$ such that it is decreasing if $b < \bar{b}$.*

back