The Noise is in the Mind: Existence of Trading Equilibria with Transparent Prices

Franz Ostrizek and Elia Sartori

Sciences Po and Napoli-Federico II

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Introduction

- Classic problem of equilibrium existence in models of informed trading

Price Paradox: If prices perfectly reflect the information contained in individual actions, then individuals will ignore their private information (and cease to acquire any), thereby eliminating the very source of information contained in prices.

- The usual approach to restore existence: Noise trading makes the price noisy

Behavioral foundation of informed trade

What misperceptions and biases give existence in a workhorse model of informed trade?

Existence (if and) only if Imperfection

- **Fisher Black**, AFA Presidential Address 1985, *Noise*

> *“Noise makes financial markets possible, but also makes them imperfect”*

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- **Behavioral Finance**, literature on overconfidence, underinference from prices...

Behavioral biases make financial markets imperfect ...

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> *Behavioral biases make financial markets imperfect ...*

- This paper: does BF flip FB?

> *... do they also make them possible?*

Model (finite N)

Objective Environment: Kyle '89 w/o Noise

- **Fundamental** Single risky asset with liquidation value $\nu \sim \mathcal{N}\left(0, \tau_0^{-1}\right)$
- **Agents** N (ex-ante) identical CARA-ρ traders
- **Market** Traders submit demand schedule $x_i(\cdot)$, p determined by $0 = \sum x_i(p)$
- **Information** Traders observe private signals $s_i = \nu + \epsilon_i$, $\epsilon_i \sim \mathcal{N}\left(0, \tau_s^{-1}\right)$, $\epsilon_i \perp \epsilon_j$

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- **Timing**
  1. Private information arrives
  2. Traders submit schedules x_i
  3. Trade occurs at clearing price p
  4. Liquidation value (payoffs) realize: $x_i\left(\nu - p\right)$

Perceived Economy $\vartheta = (\Sigma, \Lambda)$

- Trader i thinks that the signals are distributed according to

$$\begin{pmatrix} s_i \\ \boldsymbol{s}_{-i} \end{pmatrix} \sim \mathcal{N}\left(\nu, \tau_s^{-1} \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right)$$

where Σ is a valid covariance matrix (psd).

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where Σ is a valid covariance matrix (psd).

- The trader uses her perceived market clearing rule

$$\begin{pmatrix} \Lambda_1, & \boldsymbol{\Lambda_2}, & \Lambda_z \tau_s^{-1/2} \end{pmatrix} \cdot \begin{pmatrix} x_i \\ \boldsymbol{x}_{-i} \\ z \end{pmatrix} = 0$$

(with $z \sim \mathcal{N}(0,1)$ a perceived shifter) to interpret price and price impact.

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- Rational economy: $\vartheta^R = (I_{N \times N}, (1, \mathbf{1}, 0))$. Kyle economy: $\vartheta^K = (I_{N \times N}, (1, \mathbf{1}, 1))$

Misperceived Information: Σ Examples

- **Precision relative to others**: $\Sigma_{11} = \xi^{-1}$, $\Sigma_{22} = \delta \begin{pmatrix} 1 & \phi & \cdots & \phi \\ \phi & 1 & & \phi \\ \vdots & & \ddots & \vdots \\ \phi & \phi & \cdots & 1 \end{pmatrix}$, where
  - $\xi \in (0, \infty)$ parametrizes **under/overconfidence** in the precision of own signal
  - $\delta \in (1, \infty)$ parametrizes **dismissal** and
  - $\phi \in [-\frac{1}{N-2}, 1]$ parametrizes **correlation delusion**

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- **Information projection**: i believes j's signal is

$$s_j = \alpha s_i + (1-\alpha)\nu + \sqrt{1-\alpha^2}\epsilon_j$$

$\alpha = 0 \Rightarrow$ rational; $\alpha = 1 \Rightarrow s_j = s_i$, others have no (residual) information.

  - Then $\Sigma_{22} = I\left(1-\alpha^2\right) + \alpha^2 \mathbf{1}^T \mathbf{1}$ and $\Sigma_{12} = \alpha \mathbf{1}$ for $\alpha \in (0,1)$.

Misperceived Market Clearing: Λ

- Subjective market clearing is

$$\Lambda_1 x_i + \boldsymbol{\Lambda}_2 \cdot \boldsymbol{x}_{-i} + \Lambda_z \tau_s^{-1} z = 0$$

- If $\boldsymbol{\Lambda}_2 = \Lambda_2 \cdot \mathbf{1}$, then $\frac{\Lambda_1}{\Lambda_2}$ parametrizes own importance relative to others
- Impacts information extraction and perceived market power:
  - $\frac{\Lambda_1}{\Lambda_2}$ large: own action strongly affects price (market moves strongly against me)
  - $\frac{\Lambda_z}{\Lambda_2}$ large: prices perceived to be very noisy (about others' signals)
- Note: Perception of market power and informational impact are consistent; no "schizophrenia" (Hellwig, 1980)

Subjective Best Response

- Each trader is a monopsonist against residual supply curve

$$p = p_i + \lambda x_i \tag{1}$$

- Utility maximization yields best-reply

$$x_i = \frac{\mathbb{E}[\nu|p_i, s_i] - p_i}{2\lambda + \rho\mathbb{V}[\nu|p_i, s_i]} \tag{2}$$

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- $\mathbb{E}$, $\mathbb{V}$, and λ computed under (mis)perceived ϑ det.

Equilibrium

Definition: An equilibrium is a pair (β, γ) such that

i) $x_i = \beta s_i - \gamma p$ matches the best-response condition (2) given the decision rule, price (1) and perception ϑ, and

ii) the second-order condition is satisfied: $2\lambda + \rho \mathbb{V}_\vartheta[\nu | p_i, s_i] > 0$.

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***Rmk**: Nonexistence in the rational economy; existence in the Kyle economy.*

Discussion

- This is just a (parametric) non-common prior model
  - agree to disagree about the fundamental value of the asset ex-interim
- We impose a lot of symmetry
  - primarily for simplicity
  - also helps to interpret an equilibrium in schedules (equilibrium knowledge about β)

Equilibrium: Mechanics

- Matching coefficients: necessary condition ▸ det.

$$\beta = b_1 - b_2 \tau_p(\beta)$$

$$\tau_p^{-1}(\beta) = t_1 + \left(\frac{\Lambda_z}{\beta}\right)^2 t_2$$

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- defines a system whose roots constitutes candidates $\Rightarrow$ check SOC

$$\text{SOC}: \frac{\text{Sthg.} > 0}{\beta} + \rho\mathbb{V} > 0$$

Candidate $\beta > 0$ passes. $\beta = 0^-$ fails: if negative, need "very negative"

Existence: Intuition

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- If $\Lambda_z > 0$, $\Rightarrow \tau_p$ adjusts with $\beta \Rightarrow$ solution of a cubic
  - $\Lambda_z > 0 \Rightarrow \lim_{\beta \searrow 0} \tau_p(\beta) = 0 \Rightarrow$ positive solution $\Rightarrow$ existence

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- If $\Lambda_z = 0$, $\Rightarrow \tau_p$ independent of $\beta \Rightarrow$ unique linear candidate $b(\vartheta) = b_1 - \frac{b_2}{t_1}$
  - price always reveals the average signal, its precision is no longer endogenous!
  - the only question is how agents (mis)interpret it

Existence: Noise, Positive, Negative with SOC

The economy $\vartheta = (\Sigma, \Lambda)$ admits an equilibrium if and only if

- The market clearing rule is perceived with (cognitive) noise, $\Lambda_z > 0$, or
- In the linear economy, either

  $$b\left(\vartheta\right) > 0,$$

  or

  $$b\left(\vartheta\right) < 0 \text{ , and } \Lambda_2 \left(\Sigma_{22} - \mathbf{1}^T \Sigma_{12}\right) \Lambda_2^T < 0.$$

Linear Case: Egocentrism and Hubris

- If $\Sigma_{12} = 0$, negative candidates never work. Existence iff

$$\underbrace{\frac{\Lambda_2}{\Lambda_1}(N-1)}_{\downarrow \text{ in } \Lambda_1} > \underbrace{\frac{2(N-1)}{\xi\delta(1+(N-2)\phi)}}_{\downarrow \text{ in } \xi\delta} + 1$$

$$\text{Existence} \iff \text{low } \Lambda_1\text{, high } \xi\delta$$

- Underestimate market impact (egocentrism$\downarrow$), overestimate private information (hubris$\uparrow$)

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Existence iff traders do not listen to the price ...

... and/or think that the price does not listen to them

- Allowing $\Sigma_{12} \neq 0$ introduces interaction between info and market clearing misperceptions
  det.

Example: Overconfidence

- $\Sigma_{11} = \xi^{-1}$: each trader thinks that their signal has precision $\xi\tau_s$
  - $\xi > 1$ should help intuitively: instead of inferring from $\sum s_j$, I want to put more weight on s_i
- Candidate

$$b_\xi = \frac{\tau_s}{\rho}\left[\frac{N-2}{N-1}\xi - 2\right]$$

existence iff (Kyle et al., 2018)

$$\xi > 2\frac{N-1}{N-2}$$

  - Impossible in bilateral trade
  - For large N, $\to \xi > 2$
- **Rmk**: Heterogeneous signal precision with common priors doesn't help with existence

Similar intuition for dismissal ▸ det.

Small Noise Economy

- Consider now an economy with small but vanishing noise trading.
  - Existence is guaranteed
- Ratio of informed trade relative to noise floor

$$I(\Lambda_z) = \frac{N\mathbb{E}[|x_i|] - \sqrt{\Lambda_z^2 \tau_s^{-1}}}{\mathbb{E}[|z|]}$$

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Consider an economy $\vartheta_{\Lambda_z} = (\Sigma, (\Lambda_1, \Lambda_2, \Lambda_z))$ with $\Sigma_{12} = 0$.

- If we have existence in ϑ_0, then informed trade is large relative to the noise floor

$$I(\Lambda_z) \to b(\vartheta_0)\Lambda_z^{-1} \text{ as } \Lambda_z \to 0.$$

- If existence fails strictly ($b(\vartheta_0) < 0$), then informed trade is of the same order as the noise floor, $I(\Lambda_z) \to c_1$.
- (If $b(\vartheta_0) = 0$, informed trade grows slowly, $I(\Lambda_z) \to c_2\Lambda_z^{-\frac{1}{3}}$.

- Trade volume: Quantitative manifestation of the existence problem.

Conclusion

- What updating biases (and how strong) are needed to get existence in Kyle economy?
  - **Cognitive** noise: Equivalent for equilibrium play, not equivalent for equilibrium outcomes, e.g. price volatility

$$\mathbb{V}_{\Lambda_z}[p] = \left(\frac{\beta}{\gamma}\right)^2 \frac{\tau_s^{-1}}{N} + \left(\frac{\mathbf{1}}{\boldsymbol{N\gamma}}\right)^{\mathbf{2}} \boldsymbol{\Lambda_z^2 \tau_s^{-1}}$$

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  - Without noise, existence if the (representative) trader...

> has sufficient hubris about her relative information and
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- Source of existence matters for
  - *Tractability*. Adding noise is costly — linear model is simple
  - *Volume*: Break the link between noise and informed volume
    - Noise trader case: might be "essentially no-trade" ($\beta \approx 0$) equilibrium
    - "substantial" informed trade $\iff$ "substantial" part of market behaves in an **erratic but correlated** way that smart money has **no possibility/willingness to know**
- Not today: In the $N \to \infty$ economy, the bias must satisfy limit uncertainty

Thank You!

Equilibrium: Price and Market Power

- Recall

$$x_i = \frac{\mathbb{E}[\nu \mid \{p_i, s_i\}] - p_i}{2\lambda_i + \rho\mathbb{V}[\nu \mid \{p_i, s_i\}]}$$

and

$$x_j = \beta s_j - \gamma p$$

Equilibrium: Price and Market Power

- Recall

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and

$$x_j = \beta s_j - \gamma p$$

- Use conjectured (β, γ) and perceived market clearing to interpret the price

$$\Lambda_1 x_i + \sum_{j \neq i} \Lambda_{2,j} \left(\beta s_j - \gamma p\right) + \Lambda_z {\tau_s}^{-1/2} z = 0$$

yields

$$p = p_i + \lambda_i x_i$$

with

$$\lambda_i = \frac{\Lambda_1}{\gamma \left(\mathbf{1} \Lambda_2^T\right)}, \qquad p_i = \frac{\beta \Lambda_2 \boldsymbol{s}_{-i} + \Lambda_z {\tau_s}^{-1/2} z}{\gamma \left(\mathbf{1} \Lambda_2^T\right)}$$

Matching Coefficients: Details

- Best response yields the matching coefficients condition

$$\beta = \frac{1}{\rho}\left[\hat{\tau}_s\left(1-\frac{\Lambda_1}{\mathbf{1}\Lambda_2^T}\right)-\hat{\tau}_p(\beta)\left(\frac{2\Lambda_1+\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\left(1-\frac{\Lambda_1}{\mathbf{1}\Lambda_2^T}\right)}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)\right]$$

- In the synthetic signal structure

$$\begin{pmatrix} s_i \\ \hat{p}_i \end{pmatrix} \sim \mathcal{N}\left(\boldsymbol{\nu}, \begin{pmatrix} \hat{\tau}_s & 0 \\ 0 & \hat{\tau}_p \end{pmatrix}\right)$$

we have

$$\hat{\tau}_p(\beta) = \tau_s\left[\frac{\Lambda_2\left(\Sigma_{22}-\Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2^T}{\left(\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)^2}+\frac{1}{\left(\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)^2}\left(\frac{\Lambda_z}{\beta}\right)^2\right]^{-1}$$

▸ Back

Price Paradox and its Solution

- Markets with (many) agents $i \in [0,1]$ and payoff-relevant unknown fundamental ν
- Agent observes $s_i = \nu + \epsilon_i$ ($\epsilon_i \perp \epsilon_j \Rightarrow$ diffused knowledge) and p
- Before taking action $p_i(s_i, p)$ that aggregates into $p = \int p_i$

$$[\textbf{Use } s_i] \overset{\textbf{LLN+Invertibility}}{\longrightarrow} [p = \nu] \overset{s_i \perp \nu \ |p}{\longrightarrow} [\textbf{Not Use } s_i] \longrightarrow [p \perp \nu] \longrightarrow [\textbf{Use } s_i]$$

- How to break the implication: p_i uses $s_i \Rightarrow$ aggregation reveals ν?

1. Aggregate noise: Traders observe $p = \int p_j + \eta$ for some $\eta \sim \left(0, \sigma_\eta^2\right)$
   1. Beauty contest $\Rightarrow$ equilibrium iff $\sigma_\eta^2 > 0$. Reducing σ_η^2 crowds out of private information
2. Finite markets: N traders so s_i informative of ν even conditional on $\{s_j\}_{j \neq i} = p$
   1. Break statistical sufficiency at the expense of giving traders market impact Back

Existence Comparative Statics: Egocentrism and Hubris

- Σ_{12} induces interdependence of information and market clearing misperceptions

$$E(\vartheta) = \frac{b_1(\vartheta)}{b_2(\vartheta)}$$

$$H(\vartheta) = \frac{\hat{\tau}_s(\vartheta)}{\hat{\tau}_p(\vartheta)}$$

- (Linear) economy is more *E*gocentric if traders perceives higher market impact.
- (Linear) economy is more *H*ubristic if traders perceive higher precision of private information.

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- (Linear) economy is more Egocentric if traders perceives higher market impact.
- (Linear) economy is more Hubristic if traders perceive higher precision of private information.

Suppose an equilibrium exists in a linear economy. Then it continues to exist when all agents become ***less egocentric*** *and* ***more hubristic****.* Back

Example: Under-Appreciation of the Information of Others

- Misperceive others: $\Sigma_{22} = \delta^{-1} \begin{pmatrix} 1 & \phi & \phi \\ \phi & 1 & \phi \\ \phi & \phi & 1 \end{pmatrix}$

- Setting $\delta \geq 1$, $\phi \geq 0$, we model traders who underestimate the information content of others' signals
  - others have precision $\frac{\tau_s}{\delta}$ and are correlated at ϕ

- Neat to write this in equivalent number of signals, $\hat{\tau}_p = (n-1)\,\tau_s$

- We have existence if traders perceive others to have (a bit less than) half as much information

$$n - 1 < \frac{1}{2} N - 1$$

Back

Examples: Misperceptions of Market Impact

- Misperceiving the relative weight $\lambda = \frac{\Lambda_1}{\Lambda}$

$$b(\vartheta) = \frac{\tau_s}{\rho}\left(1 - \lambda \frac{2N-1}{N-1}\right)$$

- Equilibrium exists if you think you don't have market impact ($\lambda = 0$)
- Generally, sufficient misperception (under correct perception of information)

$$\lambda < \frac{N-1}{2N-1} < \frac{1}{2}$$

Back

Questioning the "Noise Traders" Story

- A "trick" to restore equilibrium existence? Prices contain **Aggregate noise**
  - Market price is no longer statistically sufficient for the fundamental
  - Rational agent uses fundamental information, feeding it into the market price
- A "good story" to invoke the trick? Markets include **Noise traders**
  - Aggregate *i*) non-fundamental liquidity demand or *ii*) sentiments, irrational trading
  - Information-based traders are perfectly rational, also know the noise trading "protocol"
- Either side, and the coexistence, in the **Rational vs. Noise dichotomy** is problematic
  - Noise ⇒ small but many ⇒ aggregate impact needs coordination ⇒ rational should catch it
  - Experimental, survey & market evidence: "informed" agents fall short from Bayesian ideal

▸ Back

Limit Uncertainty and Existence

- In any informative equilibrium ($\beta \neq 0$) of the limit economy, the price reveals the true liquidation value
- Def: An economy has **limit uncertainty** if (according to the subjective belief)

$$\mathbb{E}\left[\nu | \{s_j\}_{j\in\mathbb{N}\setminus\{i\}}\right] \neq \mathbb{E}\left[\nu | \{s_j\}_{j\in\mathbb{N}}\right] \text{ and } \mathbb{V}\left[\nu | \{s_j\}_{j\in\mathbb{N}}\right] > 0$$

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A regular economy admits an **equilibrium in the limit** if and only if it has **limit uncertainty** and a **limit equilibrium**.

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A regular economy admits an **equilibrium in the limit** if and only if it has **limit uncertainty** and a **limit equilibrium**.

- Limit uncertainty is satisfied in the regular case if and only if $\phi > 0$ (correlation illusion)
  - Infinitely many signals of arbitrarily small perceived precision ($\frac{\tau_s}{\delta}$) render a single signal of arbitrarily high perceived precision ($\xi \tau_s$) superfluous
  - Projection also doesn't help: for any fixed level, many others have unbounded precision Back

Mixture Models and Limit Uncertainty

- Cursed updating

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- Although price $=$ fundamental, "part of me" ignores it and s_i remains conditionally informative
- ... but cursedness is not in the class of biases we consider (mixture model)

Not a Literature Slide

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- **Behavioral Finance**
- **Trade vs No-Trade**
- **Underinference from aggregate statistics/prices**: Kagel Levin 02; Weizsäcker 10; Ngangoué Weizsäcker 21
- **Overconfidence**: Dunning Kruger 99; Malmendier Taylor 15; Ortoleva Snowberg 15
- **Misspecified models**