Screening while Controlling an Externality*

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Abstract

We propose a tractable framework to introduce externalities in a screening model. Agents differ in both payoff-type and influence-type (ranking how beneficial their actions are for others). Applications range from pricing network goods to regulating industries that create externalities. Inefficiencies arise only if the payoff-type is unobservable. When both dimensions are unobserved, the optimal allocation satisfies lexicographic monotonicity: increasing along the payoff-type to satisfy incentive compatibility, but tilted towards influential agents to move the externality in the socially desirable direction. In particular, the allocation depends on a private characteristic that is payoff-irrelevant for the agent. We characterize the solution through a two-step ironing procedure that addresses the nonmonotonicity in virtual values arising from the countervailing impact of payoff- and influence-type. Rents from influence can emerge but only indirectly, i.e. when the observed level of influence is used as a signal of the unobserved payoff-type.

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1 Introduction

In many settings ranging from production with a polluting factor to the consumption of a network good, individual activities affect the payoff of others and their willingness to act. A planner or monopolist designing policy in such an environment is often affected by adverse selection as well: A network good monopolist may price discriminate to exploit unobserved heterogeneity in consumers' valuations while providing large quantities to influential consumers in order to increase the willingness to pay in the population. Likewise, a regulator that controls pollution through production quotas or taxes may need to discriminate among heterogeneously productive firms. Given the attention received by the study of externalities and screening separately and the abundance of applications that are affected by both forces, it is perhaps surprising that their interaction has only received limited attention in the literature.[1] A possible explanation is that the most natural framework for conducting this analysis is a screening model with at least two dimensions of heterogeneity: agents differ both in their taste for the activity and in their impact on others. Multidimensional screening problems, however, present notorious difficulties and often require a case-by-case analysis of the particulars of the setting.[2]

In this paper, we propose a tractable framework for analyzing the interplay of screening and externalities. The principal faces a population of agents whose payoffs are interdependent through a global externality. Agents can be characterized by a two-dimensional type, one parametrizing the returns from the activity as in a standard screening model (payoff-type), the other parametrizing the impact of his activity on the externality (influence-type). There is no aggregate uncertainty, the principal knows the distribution of types and costs or benefits from the externality in aggregate, but is ignorant of each individual agent's type. The crucial assumption is that even though both dimensions of the agents' types enter the principal's objective, only the payoff-type affects their utility. Apart from single-crossing, we are permissive on the functional form of the utility and externality function as well as on the correlation between payoff- and influence-type.[3] We provide bounds ensuring the existence of a solution and derive its properties in the general case, and illustrate our results in specific settings.

We study how the observability of each of the dimensions affects the allocation of

[1] Such situations are of course analyzed in terms of mechanism design, although this literature typically focuses on situations with aggregate uncertainty about the value of the externality and finding the (constrained) efficient allocation.

[2] Rochet and Stole (2003) survey the literature on multidimensional screening, highlighting tractable cases and the source of the general difficulty of such problems, namely the lack of fixed order on types along which sorting constraints bind. This also justifies the moniker of multidimensional screening for our discrete setting.

[3] This flexibility in the correlation structure effectively weakens the assumption that the influence-type does not enter the utility, as it allows us to reinterpret as influence all heterogeneity that enters the principal's objective without affecting the agent's utility. The substantive assumption is the single-dimensionality (single-crossing) of the appropriately redefined payoff-type and residual. We conduct such a transformation in the context of pollution in production in Section 2.2.1.

the activity and rents as well as the aggregate externality. Clearly, if both characteristics are observed by the principal, she can implement the efficient contract and extract all surplus. This result holds as long as the payoff-type is observed: As the influence-type does not affect an agent's utility, the sorting constraints only require that utility is flat along this dimension; the first best (through full surplus extraction) satisfies this feasibility condition, and hence is optimal.

The main contribution of this paper is the characterization of the optimal contract when both dimensions are unobserved (*full screening* contract). A crucial intermediate step is to show that – despite the apparent multidimensionality – there is a fixed total order of types determining the binding sorting constraints and that the optimal allocation must be increasing along the lexicographic order where the payoff-type is the dominant dimension (*lexicographic monotonicity*, Theorem 1). As in a standard screening problem, incentive compatibility drives monotonicity in the payoff-type. Within a payoff-type slice, the principal provides a higher allocation to more influential consumers in order to move the externality in the socially desirable direction: With positive consumption externalities, influencers are made to consume more, while with pollution externalities green firms are made to produce more, for example. Such tilting to increase total surplus is always profitable for the principal even though it also increases information rents. The principal's control of the externality, however, is constrained by asymmetric information and lexicographic monotonicity emerges as a way to resolve the tension between screening and externality provision.

From an applied perspective, Theorem 1 shows both the potential and the limitations of screening for two characteristics using just a single instrument. Agents with the same payoff-type who would be assigned to the same action in a standard setting instead choose from a range of options. By giving them such flexibility, the principal allows agents to sort along a dimension that does not enter into their utility directly, but impacts the social value of individual actions through the externality. Nevertheless, lexicographic monotonicity imposes tight restrictions: It is impossible to force high consumption by influential agents that do not care about the activity (have low payoff-types) unless virtually every consumer has equally high or even higher consumption; likewise, productive firms that pollute a lot must produce (weakly) more that green but unproductive firms. The effect of screening on the aggregate externality is ambiguous: On the one hand, it induces the familiar downward distortion in allocations; on the other hand, lexicographic monotonicity makes it more costly to reduce the allocation of low influence agents when they are bunched with the preceding high-influence types. For the case of pollution, we show that these effects can lead to perplexing comparative statics: It is exactly when the principal cares a lot about pollution that the screening contract features a higher level of pollution than the first-best. For an intermediate degree of concern, the two coincide; when the principal cares little about pollution, its second-best level falls short of the first best.

From a theoretical perspective, Theorem 1 helps to clarify the role of private information that is non-verifiable and not payoff-relevant for the agent in adverse selection problems. Such information arises naturally in a setting with externalities (see also Shi and Xing, 2020), and when agents have private information relevant only for the payoffs of other agents (Jehiel and Moldovanu, 2001) or about the welfare weight the principal desires to ascribe to them (Akbarpour ⓡ al., 2020). It is commonly argued that such private information cannot be elicited and used by an incentive compatible mechanism. We show that this impossibility result relies crucially on a continuum type space assumption (Lemma 0).

Despite making the problem one-dimensional, lexicographic monotonicity does not render its solution straightforward. Even under the usual regularity conditions, the virtual value typically is nonmonotonic in the lexicographic order for two reasons, both arising around the switching types (types with the highest level of influence, who are adjacent to an agent with higher payoff-type): First, only the allocation of these types directly causes information rents and hence only their virtual value is downward distorted. Second, the subsequent type in the lexicographic order has the lowest influence; this downward jump is a source of nonmonotonicity because the virtual value is increasing in influence. We generalize standard techniques (Myerson, 1981; Toikka, 2011) to take into account that the network effect creates interdependence among individuals' virtual values and provide a two-step ironing procedure (Theorem 2) to characterize the allocations and aggregate activity that solve the full screening problem. In contrast to efficiency at the boundary results (e.g. Rochet and Choné, 1998), we show that bunching can occur even for agents with the highest payoff-type, and that every bunching region contains agents of the highest-influence type.

One feature shared by the efficient and full screening contract is that individuals' influence is not rewarded: In the former case, full surplus extraction leaves everyone without any rent, while in the latter case incentive compatibility prevents any rent from emerging along a dimension (influence) that does not directly affect individuals' utility. This implication seems at odds with evidence of large rents enjoyed by influencers inside a network and with many results in the literature (Candogan et al., 2012; Bloch and Quérou, 2013; Fainmesser and Galeotti, 2020). To investigate when such rents can emerge in our setting, we study the problem with observable influence (but unobserved payoff-type) in our linear-quadratic application to the consumption of network goods. In this case, a condition on primitives ensures that the optimal contract exhibits rents for influential consumers:[4] Even when influencers are atomistic and have no market power, they can gain from their position but *only if it is observable and only indirectly* as the observed level of influence is informative about the unobserved payoff-type.

The paper proceeds as follows. We conclude this introductory section by discussing the related literature. Section 2 presents the general model and three applications, which will also be used as running examples to illustrate our results. As a benchmark,

[4]In particular, the condition puts an upper bound on the affiliation between the payoff- and influence-type.

we characterize the decentralized equilibrium of the game if the technology is available to every agent and the efficient allocation in Section 3, and show that the efficient allocation is implementable as long as the payoff-type is observed. We then analyze the full screening problem in Section 4. Section 5 analyses the problem when influence is observed but payoff-types are private information, Section 6 concludes. We gather all proofs in Appendix A and derivations for the examples in Appendix B.

1.1 Literature

There is a growing literature on monopolist screening with network effects. Weber (2006) characterizes the implementable allocations in a general multidimensional model with externalities and provides a set of necessary conditions for the optimal control problem characterizing the optimal screening contract. We focus on a setting where despite its underlying multidimensionality the problem can be reduced to a single-dimensional screening problem and we use this tractability to provide a tight characterization of the solution and its properties. Sundararajan (2004) and Csorba (2008) study screening with externalities in consumption when consumers have private information only about their valuation of the good. In a contemporaneous paper, Shi and Xing (2020) study screening for payoff-type and influence-type with a linear-quadratic demand specification (see also Section 2.2.2). Because they study a continuum setting, their optimal allocation is constant along the influence-dimension which enables them to obtain a tight characterization that delivers implications on networks, while our assumption of discrete types allows us to investigate how the allocation and rents depend on influence. Zhang and Chen (2020) consider an explicit stochastic network formation model with two specifications, screening for a consumers' in- or out-degree. Depending on this choice, their model can generate both quantity discounts and premia. Gramstad (2016) consider screening in a undirected network when network effects only depend on the number of neighbors that adopted the good, rather than their intensity of consumption. We analyze both dimensions of private information – payoff-type and influence – at the same time and study their interaction in screening.

We contribute to the literature on multidimensional screening (for a survey, see Rochet and Stole, 2003) by studying a setting in which there is a total order on the agents utility functions, but the principals payoff depends on the full multidimensional type.

Our model also relates to the classic literature on contracting and mechanism design with externalities (Segal, 1999, 2003; Jehiel et al., 1999; Winter, 2004). A recent contribution is Jadbabaie and Kakhbod (2019), who compare bilateral and multilateral contracting in a known finite network. This literature focuses on the externality of contracting with finitely many agents and on the interplay between contracting and the strategic interaction among agents. We focus on public contracting with a population of atomistic agents and select the principal-preferred equilibrium as is typical in the adverse selection literature.

Finally, there is a large literature studying the targeting of interventions in networks (see e.g. Ballester et al., 2006; Banerjee et al., 2019; Galeotti et al., 2020). We contribute to this literature by demonstrating the limitations of targeting when there is only coarse information about the network structure and this information has to be elicited.

We discuss the literature related specifically to our applications in the respective subsections.

2 Model and Applications

We construct a parsimonious model of screening with externalities and two dimensions of (potentially unobserved) heterogeneity: an influence-type and a payoff-type. Agents choose actions which have aggregate effects. The influence type determines the impact of individual action in the creation of this externality, while the payoff-type parametrizes the surplus from the individual action and the aggregate effect.

2.1 Setup and Primitives

There is a unit mass of agents characterized by a type $\theta \in \Theta$ distributed according to a full support distribution F. Each agent takes an action $x \in \mathbb{R}_+$ whose payoff is subject to network (or aggregate) effects: the attractiveness of the action is dependent on an aggregate variable, $\overline{x}$. For a given $\overline{x}$, an agent of type θ derives utility

$$u(x, \theta, \overline{x}) - t \tag{1}$$

from a action x and transfer $t \in \mathbb{R}$. The aggregate effect $\overline{x}$ is a weighted average of individual actions

$$\overline{x} = \int v(x(\theta), \theta) \mathrm{d}F(\theta) \tag{2}$$

We assume that the individual payoff characteristics and externality production can each be summarized by a one-dimensional type. In other words, we can write

$$u(x, \theta, \overline{x}) = u(x, k(\theta), \overline{x}) \tag{3}$$

$$v(x, \theta) = v(x, l(\theta)) \tag{4}$$

for a pair of functions $(l, k) : \Theta \to [k_0, K] \times [l_0, L] \subset \mathbb{R}^2$. We assume concavity in the agent's action ($u_{xx} < 0$) and $u_x(0, k, 0) > 0$ to ensure the existence of an interior optimum and the following properties

$$u_k \geq 0, \quad u_{kx} \geq 0, \quad u_{xxk} \geq 0 \tag{5}$$

$$u_{\overline{x}} \geq 0, \quad u_{\overline{x}\overline{x}} \leq 0, \quad u_{x\overline{x}} \geq 0, u_{\overline{x}\overline{x}k} \geq 0 \tag{6}$$

The payoff-type k behaves as in a standard screening model with single-crossing and the usual condition ensuring convexity of the information rent.[5] The global externality $\overline{x}$ describes a (weakly) positive externality with diminishing returns that also increases the marginal payoff from the activity x. For the influence function v, we maintain

$$v_{xx} < 0, \quad v_l > 0, \quad v_{xl} > 0 \tag{7}$$

This setup allows us to encompass both negative and positive externalities from the agents' actions in a common framework. If $v_x > 0$, the activity produces a positive shift in individual payoffs (per $u_{\overline{x}} \geq 0$) and we assume that this technology is concave. If $v_x < 0$, we have a negative externality of the activity with convex costs ($v_{xx} < 0$). This also allows for a game with strategic substitutes between agents. In either case, agents with high influence-type l are the "good types", because they produce either a larger positive or smaller negative externality as $v_{xl} > 0$.

Although our type space is two-dimensional, only one of the coordinates enters the agents' utility. As we will show, the principal can – and will indeed find it optimal to – elicit this payoff-irrelevant dimension and implement different contracts along it, a feature of this environment that is subdued in a double-continuum type space but of key interest to us.

Lemma 0. *Suppose that $k(\Theta)$ is uncountable, that the induced distribution $\mathrm{F} \circ k^{-1}$ is atomless, and that $x(k,l)$ is strongly increasing in k. Then, for almost all k, $x(k,l)$ is constant in l.*

Proof. Consider $x(k) = \min_l x(k,l)$. This has to be an increasing function and it has an upwards jump wherever $x(k,l)$ is not constant in l. As an increasing function can have at most countably many points of discontinuity, we have the result. $\square$

Note the Lemma applies to the allocation in the full screening problem, as it has to be increasing in k by incentive compatibility. Consequently, l can affect the allocation only for countably many k, which is irrelevant for the principal in the double continuum case.[6]

As our interest lies in exploring the impact of heterogeneous influence on screening, we instead consider finite types. Let $\mathcal{K} \times \mathcal{L}$ denote the induced types space. By abuse of notation, we denote the induced distribution by F. We assume that $\mathcal{K} \times \mathcal{L}$ is finite and – as a normalization – let $\mathcal{K} \times \mathcal{L} = \{k_0, k_0 + 1, \dots, \mathrm{K}\} \times \{l_0, l_0 + 1, \dots, \mathrm{L}\}$.[7] Hence, we write $f_{k,l}$ for the probability mass and $x_{k,l}$ for the allocation of type k, l, etc. Apart from full

[5] Since the function $k(\theta)$ is a derived object, this assumption simply implies that there exists such a linear order on Θ, which implicitly defines k.

[6] For similar results establishing that payoff-irrelevant information cannot be used in a model with a continuum type-space, see e.g. Jehiel and Moldovanu (2001), Shi and Xing (2020) and Akbarpour ⓡ al. (2020).

[7] Note that this is for simplicity. We could allow for continuum $\mathcal{L}$ and – assuming a distribution with atoms – even continuum $\mathcal{K}$ at the cost of more complex notation.

support, we make no restriction on $f_{k,l}$. Importantly, we do not impose any correlation structure.

Principal

The agents' action is produced at zero marginal cost to the planner. The planner offers a menu of contracts $\{(x_{k,l}, t_{k,l})\}_{k,l \in \mathcal{K}\times\mathcal{L}}$ to maximize expected transfers plus possibly a direct payoff from the aggregate action, $\kappa(\overline{x})$, subject to sorting and participation constraints.[8] We assume that $\kappa' \geq 0$ and $\kappa'' \leq 0$. This aggregate term captures the impact of the externality that is not mediated through the payoffs of the agents; for example the impact of pollution on society at large. Finally, we assume as a non-triviality condition that the externality has an impact on the principal's problem, either directly or indirectly through some type's utility,

$$\forall \overline{x}, \quad \max\left\{\max_{k\in\mathcal{K}}\{u_{\overline{x}}(0,k,\overline{x})\}, \kappa'(\overline{x})\right\} > 0. \tag{NT}$$

We will consider several planner problems, each corresponding to a different assumption on which consumer characteristics are observable. Throughout these problems, the objective, the aggregate network effect and the participation constraints will remain the same. Depending on the misreports that are feasible (i.e. which characteristics are verifiable), the problem will have different sorting constraints. We identify the sorting constraint with the associated (pair of) types. Accordingly, the set of feasible deviations is denoted by $A \subset (\mathcal{K}\times\mathcal{L})^2$, where $(k,l),(k',l') \in A$ means that type k,l can imitate type k',l' and therefore a feasible allocation must satisfy the sorting constraint

$$u(x_{k,l}, k, \overline{x}) - t_{k,l} \geq u(x_{k',l'}, k, \overline{x}) - t_{k',l'}. \tag{IC$_{k,l\to k',l'}$}$$

The problem corresponding to a set of feasible deviations A is

$$\pi(A) := \max_{\overline{x},\{(x_{k,l},t_{k,l})\}_{k,l\in\mathcal{K}\times\mathcal{L}}} \sum f_{k,l} t_{k,l} + \kappa(\overline{x}) \tag{8}$$

$$\text{s.t.} \qquad \overline{x} = \sum f_{k,l} v(x_{k,l}, l) \tag{9}$$

$$\forall k,l: \quad u(x_{k,l}, k, \overline{x}) - t_{k,l} \geq 0 \tag{P$_{k,l}$}$$

$$\forall ((k,l),(k',l')) \in A: \quad u(x_{k,l}, k, \overline{x}) - t_{k,l} \geq u(x_{k',l'}, k, \overline{x}) - t_{k',l'} \geq \tag{IC$_{k,l\to k',l'}$}$$

To save on notation, we suppress the non-negativity constraints $x_{k,l} \geq 0$. Table 1 specifies the set of feasible deviations associated with each observability assumption. Throughout the paper, we will let ζ denote the Lagrange multiplier associated with the constraint (9), i.e. the marginal increase in the principal's objective associated with an

[8] Note that every set of contracts induces a game among the consumers at the consumption stage, as aggregate consumption is endogenous. Without loss of generality, we restrict attention to menus inducing a pure strategy equilibrium. This is implied by the concavity of the planer problem, which we impose throughout.

	payoff-type observable	payoff-type not observable
influence observable	$\varnothing$	$\bigcup_{l\in\mathcal{L}} (\mathcal{K}\times l)^2$
influence not observable	$\bigcup_{k\in\mathcal{K}} (k\times\mathcal{L})^2$	$(\mathcal{K}\times\mathcal{L})^2$

Table 1: Four different observability assumptions as sets of feasible deviations.

exogenous increase in the externality.

2.2 Applications

We now present three economic applications that fit our general framework. We will return to simple 2-by-2 examples of these models to illustrate our results and their implications throughout the paper.

2.2.1 Nonlinear Taxation of Externality Producing Goods

Consider a setting in which firms produce goods using a polluting process. They differ in both their productivity and their pollution intensity. Pollution creates an atmospheric externality (Meade, 1952). A regulator desiring to control the aggregate level of externality while raising tax revenue designs nonlinear production taxes.

A firm of type θ produces quantity x employing a perfect complement decreasing returns technology $x = \left(\min\left\{\frac{1}{\theta_1}z_1, \frac{1}{\theta_2}z_2\right\}\right)^{\frac{1}{2}}$, where z_1 is a clean factor (e.g. labor) and z_2 is a pollutant factor (e.g. gasoline). Let w_1, w_2 denote the factor prices, and we normalize the price of output to one. Hence, profits are given by $x-(w_1\theta_1+w_2\theta_2)x^2$, and the externality is $-\theta_2 x^2$. The planner faces a disutility of pollution $\kappa(\bar{x}) = \kappa\bar{x}$ for $\kappa > 0$. This fits our framework with $k(\theta) = \frac{2}{w_1\theta_1+w_2\theta_2} - 1$, $u(x,k,\bar{x}) = x - \frac{1}{2(k+1)}x^2$, $l(\theta) = \max\{\theta_2\} - \theta_2$, and $v(x,l) = -(\max\{\theta_2\} - l)x^2$.[9] A natural benchmark in this setting is a Pigouvian tax on pollution or, equivalently, a tax on the pollutant factor. However, such a tax or alternative policies such as mandates to use cleaner inputs or processes are not feasible when pollution, input use, or the technology are not verifiable or such measures are not politically viable. As an example, consider a firm sourcing raw materials from mines located at various distances from the production facilities. Tightly controlling the sourcing and transportation expenses is difficult, especially in an emerging market context. The principal instead sets a menu of output levels and transfers, screening both productivity (to generate revenue) and "greenness" (to curb pollution).

The fundamental parameters of the production function determine the payoff and influence type. In particular, the pollutant factor requirement θ_2 enters both k and

[9] Note that this joint definition of k, l rules out a rectangular type space with full support unless $w_2 = 0$. We can restore rectangularity with a more cumbersome parametrization.

Perfect complements and decreasing returns by contrast are required to fit the general framework: decreasing returns for concavity and no substitutability to keep the payoff-type one-dimensional. Under a more permissive functional form, we would obtain $\pi(\theta, w, x) = \sum w_i z_i(\theta, w, x)$, which cannot in general be written as $\pi(k(\theta), x, w)$ for a single-dimensional type $k(\theta)$. Indeed, this fails if $w_1 \cdot w_2 \neq 0$ whenever any substitutability across factors is permitted.

l. The correlation between the two is determined by factor prices and is typically non-zero even if the fundamental parameters θ are independently distributed.

2.2.2 Sale of a Network Good

Consider a good with externalities in consumption, caused either by direct interaction on the platform as in the case of Dropbox and Facebook or indirectly through the availability of accessories as in the case of operating systems or interchangeable-lens cameras (Farrell and Saloner, 1985; Katz and Shapiro, 1985). We follow the network formation model formulated and applied in Galeotti and Goyal (2009) and Fainmesser and Galeotti (2016).[10] There is a continuum of consumers connected by a directed network. A consumer's marginal utility of consumption increases as others who influence her increase their consumption. Formally, the influence parameter l coincides with the agent's in-degree (the number of consumers he influences), while the payoff parameter k is his out-degree (the number of consumers he is influenced by, which can also be interpreted as his susceptibility to the network effect). When making consumption choices, consumers do not know the network structure, but only their in- and out-degree. They take expectations over their realized utility conditional on this information alone. Accordingly, the utility is expressed as

$$u(x,\overline{x},k) = x + \gamma k\overline{x}x - \frac{1}{2}x^2 - t \tag{10}$$

where $\overline{x} = \sum_{k,l} f_{k,l} \frac{l}{\mathbb{E}[l]} x_{k,l}$. When forming expectations, individuals take account of the fact that they are more likely to link to influential individuals, which consequently need to be over-counted relative to their frequency in determining the expected consumption of a neighbor. Clearly, this specification fits into our general framework with

$$u\left(x,k,\overline{x}\right) = \left(1+\gamma\overline{x}k\right)x - \frac{1}{2}x^2, \qquad v\left(x,l\right) = \frac{l}{\mathbb{E}\left(\boldsymbol{l}\right)}x, \qquad \kappa(\overline{x}) = 0. \tag{11}$$

This reduced from can also be interpreted as an aggregate network effect: Agents directly care about the weighted population average of x, e.g. due to a desire to conform; they differ in both their desire to conform k and their visibility or social status l.

2.2.3 Human Capital

Human capital accumulated through learning-by-doing in industry is often proposed as a propellant of economic development (Lucas, 1988). Consider the following highly stylized steady-state model of the labor market. Firms and workers are matched randomly for one period. Work at time t produces human capital that is carried over

[10] A recent literature focuses on the use of network information by a monopolist, both in the case of an explicit finite network (e.g. Bloch and Quérou, 2013; Candogan et al., 2012) and when consumers only know their level of susceptibility and influence (Fainmesser and Galeotti, 2016). We adopt the demand and interaction specification developed in the latter in our example, but focus on the screening problem.

into the next period; firms are heterogeneous in both how much they rely on human capital and how much human capital they bestow to workers that are employed at their establishments.[11]

Let $\overline{h}$ denote the average human capital in the economy. The effective labor units of a firm with productivity k employing a measure h of workers are $k\overline{h}h$. The firm operates a Cobb-Douglas technology with decreasing returns ($\alpha < 1$), generating profits

$$u(h, k, \bar{h}) = \left(k\overline{h}h\right)^{\alpha} - wh \tag{12}$$

for a wage rate w. We assume that w is unaffected by the level of aggregate human capital (there is a reserve army of the unemployed working in the traditional sector). We parameterize the human capital formed by employment in a firm of type l by $v(h, l) = hl$. The ministry of economic development chooses a nonlinear employment subsidy to maximize human capital subject to a cost of funds λ, i.e. we write $\kappa(\bar{h}) = \frac{\bar{h}}{\lambda}$.

A similar model can be employed to study task allocation in a big corporation where employees are allocated across heterogeneous divisions by the headquarter: Compared to the headquarter, individual divisions care more about their revenue and less about the development of human capital as workers will be reassigned.

3 Benchmark Allocations

We first characterize the decentralized solution. We then turn to the efficient allocation. Clearly, the latter is implemented by a principal who can observe the payoff and influence types, i.e. it solves $\pi(\emptyset)$. Finally, we show that this solution is implemented even if the principal can observe only the payoff-type. The unobservability of influence does not create any rents and distortions in this case.

3.1 The Decentralized Solution

As a benchmark, consider the case where every agent has access to the production technology and chooses $x_{k,l}^{\mathrm{D}}$ to maximize $u(x, k, \overline{x})$.[12] Observe that $x_{k,l}^{\mathrm{D}} = x_k^{\mathrm{D}}$, as influence does not enter the utility function. A decentralized allocation solves

$$u_x(x_k^{\mathrm{D}}, k, \overline{x}) = 0, \quad \overline{x}^{\mathrm{D}} = \sum f_{k,l} v(x_k^{\mathrm{D}}, l) \tag{13}$$

For any given $\overline{x}$, the privately optimal allocation is unique by concavity. We provide an existence condition, which bounds the strength of positive externalities, as Lemma 4 in the appendix. An equilibrium $\left(\boldsymbol{x}^{\mathrm{D}}, \overline{x}^{\mathrm{D}}\right)$ may not be unique and the equilibria are Pareto-ranked in $\overline{x}$, as higher aggregate activity increases private utility.

[11] Indeed, Arellano-Bover and Saltiel (2021) show that returns to experience are highly heterogeneous across firms and are not well explained by firm observables, supporting the assumption that such characteristics are not easily observed by the planner.

[12] Equivalently, consider $n > 1$ firms competing in price schedules.

Example (**Human Capital**)**.** In the setting of Section 2.2.3, the decentralized conditional labor demand is given by

$$h_k^{\mathrm{D}} = \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \left(k\overline{h}\right)^{\frac{\alpha}{1-\alpha}} \tag{14}$$

notice that the non-triviality condition (NT) does not rule out a degenerate decentralized equilibrium. A non-trivial equilibrium exists if $\alpha < \frac{1}{2}$ and is given by

$$\overline{h} = \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \mathbb{E}\left[lk^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1-\alpha}{1-2\alpha}} \tag{15}$$

Example (**Network Good**)**.** In the setting of Section 2.2.2, there is a unique decentralized equilibrium given by

$$x_k^{\mathrm{D}} = 1 + \gamma k \overline{x}^{\mathrm{D}}, \qquad \overline{x}^{\mathrm{D}} = \frac{1}{1 - \gamma \frac{\mathbb{E}[kl]}{\mathbb{E}[l]}} \tag{16}$$

The externality in the decentralized case is merely a byproduct of privately-chosen consumption. Fixing the marginal distribution over k and l, it is increasing in the covariance of payoff- and influence-type. The decentralized outcome coincides with the first best (which we characterize in the subsequent section) if $\gamma = 0$. Aggregate consumption and total surplus in the decentralized outcome fall short of their first-best values when $\gamma > 0$.

3.2 The First Best

The efficient allocation solves the principal's problem without incentive compatibility constraints, $\pi(\varnothing)$. In order to guarantee the existence of a solution, it is not sufficient to assume concavity of the agents' utility function. Instead, concavity of the planner's problems arises jointly from the agents' utility and the aggregate externality. The following Lemma leverages the fact that we can focus the attention only on two dimensions of the allocation, the subspace along which externalities are produced and consumed.

Lemma 1. *The first-best planner problem is globally concave if and only if the value of the maximization problem*[13]

$$\max y^{\mathrm{T}} \left(\mathrm{dg}\left([u_{xx}] + (\mathbb{E}u_{\overline{x}} + \kappa')\,[v_{xx}]\right) + (\mathbb{E}u_{\overline{x}\overline{x}} + \kappa'')\left[\sqrt{f}\,v_x\right]\left[\sqrt{f}\,v_x\right]^{\mathrm{T}} + 2\,\mathrm{Sym}\left(\left[\sqrt{f}\,u_{\overline{x}x}\right]\left[\sqrt{f}\,v_x\right]^{\mathrm{T}}\right)\right) y \tag{17}$$

$$\text{s.t. } y \in \mathrm{span}\left(\left[\sqrt{f}\,u_{\overline{x}x}\right], \left[\sqrt{f}\,v_x\right]\right), \quad \|y\| = 1$$

is strictly negative for all $\boldsymbol{x} \geq 0$ and bounded away from zero for $\|\boldsymbol{x}\|$ large. Then, the first-best

[13] $\mathrm{dg}(a)$ denotes the diagonal matrix with entries provided by the vector a, $\mathrm{Sym}(A) = \frac{A + A^{\mathrm{T}}}{2}$, $\left[\sqrt{f}\,v_x\right]$ denotes a column vector with typical element $\sqrt{f_{k,l}}\,v_x(x_{k,l}, l)$ and so on.

contract induces a pure strategy equilibrium among agents.

Note that the all the expressions in (17) depend implicitly on the full allocation x. The condition provides a tight bound for the general non-parametrized case and simplifies to a familiar upper bound on the degree of complementarities in the application to network goods,

$$\gamma < \frac{\mathbb{E}[l]}{\sqrt{\mathbb{E}[k^2]\mathbb{E}[l^2]} + \mathbb{E}[kl]}. \tag{18}$$

In the pollution case, we have $u_{\overline{x}} = 0$ and hence the condition is implied by the concavity of u and v. From now on, we assume that the condition of Lemma 1 is met.

The efficient allocation maximizes individual utility with the adjustment term $v_x\left(x^\star_{k,l}, l\right)\zeta^\star$ taking the externality into account. This adjustment is proportional to the shadow value of $\overline{x}$, which corresponds to the surplus generated by the externality.

Proposition 1. *The efficient allocation* $(x^\star_{k,l})_{k,l\in\mathcal{K}\times\mathcal{L}}$ *solves*

$$0 = u_x\left(x^\star_{k,l}, k, \overline{x}^\star\right) + v_x\left(x^\star_{k,l}, l\right)\zeta^\star \tag{19}$$

$$\zeta^\star = \sum f_{k,l} u_{\overline{x}}\left(x^\star_{k,l}, k, \overline{x}^\star\right) + \kappa'(\overline{x}^\star) \tag{20}$$

$$\overline{x}^\star = \sum f_{k,l} v(x^\star_{k,l}, l) \tag{21}$$

A monopolist observing both the payoff- and influence-type implements the efficient allocation and extracts all surplus. Agents receive the same level of utility (zero) in the optimal contract, in particular influence is neither rewarded nor punished.

Example (**Network Good cont'd**)**.** For the sale of a network good, we can solve (19) in closed form,

$$x^\star_{k,l} = 1 + \gamma\overline{x}^\star k + \zeta^\star \frac{l}{\mathbb{E}[l]}. \tag{22}$$

In order to produce the efficient level of $\overline{x}$, the planner induces all types to overconsume relative to their privately optimal level $1 + \gamma\overline{x}k$. This is especially pronounced at high levels of influence. Such "influencers" are not compensated with higher utility, but they are held indifferent through lower unit prices,

$$\frac{t^\star_{k,l}}{x^\star_{k,l}} = \frac{1}{2}\left(1 + \gamma\overline{x}^\star k - \frac{l}{\mathbb{E}[l]}\zeta^\star\right). \tag{23}$$

3.3 Observable Payoff-Type

We now turn to the case where the principal observes the payoff-type of the agents, while their influence is private information. The planner problem needs to satisfy the sorting constraints $\bigcup_{k\in\mathcal{K}}(k\times\mathcal{L})^2$. As l does not directly enter the utility function, sorting is equivalent to the requirement that the utility of type (k,l) in their respective contract is independent of their level of influence l. Otherwise, every agent of payoff-type k would mimic the type k, l' whose contract delivers the highest level of utility. Formally,

Lemma 2. *A menu of contracts satisfies the $\bigcup_{k \in \mathcal{K}} (k \times \mathcal{L})^2$ sorting constraints if and only if for each k, l, l'*

$$u(x_{k,l}, k, \overline{x}) - t_{k,l} = u(x_{k,l'}, k, \overline{x}) - t_{k,l'} \tag{H}$$

Henceforth, let $U_k := u(x_{k,l}, k, \overline{x}) - t_{k,l}$ denote the utility of agents with payoff-type k as a function of the contract.

The first-best contract with full rent extraction satisfies condition (H) since all types receive zero utility. Therefore, it is feasible and hence optimal for the principal. Even though the problem has a full dimension of private information, it collapses for a given k as influence does not interact with the contract terms. Eliciting influence by itself does not introduce distortions and information rents.

Proposition 2. *The efficient allocation with full extraction solves the problem with known payoff-type.*

No Underreporting of Payoff-Type

Suppose that agents cannot underreport their payoff-type. This may occur due to a technological constraint or because the seller has correct information about a lower bound of their payoff-type. Such a constraint is reasonable in social media if the payoff-type is tightly linked to the time spent on the social network, which is identifiable by the provider and cannot be easily hidden or split across multiple accounts.

Agents can still exaggerate their payoff-type or misreport their influence, which must therefore be ruled out by sorting constraints. Formally, the monopolist faces the problem $\pi(\bigcup_{k \in \mathcal{K}} (k \times \mathcal{L}) \times (k^+ \times \mathcal{L}))$ where $k^+ := \{k' \in \mathcal{K} : k' \geq k\}$. As in a standard model without an externality, only the downward sorting constraints will bind in the second-best problem of Section 4. Prohibiting this deviation makes the first best implementable and allows the principal to extract all surplus just as with observable k, even if the influence type l remains unobserved.

Remark 1. The first-best allocation also solves $\pi(\bigcup_{k \in \mathcal{K}} (k \times \mathcal{L}) \times (k^+ \times \mathcal{L}))$.

4 Full Screening

We now turn to the case in which both payoff- and influence-type are not observed. The principal then solves the two-dimensional screening problem with consumption externality $\pi(\mathcal{K} \times \mathcal{L})^2$.

We first characterize the implementable allocations. Following the usual argument combining upward and downward incentive compatibility between two types, any implementable allocation has to satisfy monotonicity along the payoff-type. Importantly, this is required for any combination of influence types. Conversely, for any such allocation, we can find transfers that satisfy the incentive compatibility and participation constraints.

Proposition 3. *There exists a vector of transfers implementing* $\boldsymbol{x}$ *if and only if it satisfies k-monotonicity, namely for every* k, k', l, l',

$$(x_{k,l} - x_{k',l'})(k - k') \geq 0. \tag{24}$$

In contrast to a well-behaved screening problem without externalities, the first-best may fail to be implementable in our setting. In the first best, the allocation of agents with high influence but low payoff-type is inflated in order to create the externality. Consequently, the allocation may violate k-monotonicity, as we illustrate in the following example.

Example (**Pollution** 2×2)**.** Consider the setting of Section 2.2.1 with $\mathcal{K} = \{0, 1\}$ and $\mathcal{L} = \{0, 1\}$: Apart from differences in productivity, there is one polluting sector $l = 0$ and one green sector $l = 1$, i.e. the green sector does not pollute at all. As $\overline{x}$ does not enter firm profits and the marginal social cost of pollution is constant, we have $\zeta^\star \equiv \kappa$ and the first best is given by

$$(x_{0,0}, x_{0,1}, x_{1,0}, x_{1,1})^\star = \left(\frac{1}{1+\kappa}, 1, \frac{1}{\frac{1}{2}+\kappa}, 2 \right) \tag{25}$$

The first best is implementable if and only if $\kappa \leq \frac{1}{2}$. Intuitively, when damage from pollution is limited, dirty high-productivity firms are allowed to produce more than clean low-productivity firms, which is required by incentive compatibility in the full screening problem.

Remark 2. The decentralized solution $\boldsymbol{x}^{\mathrm{D}}$, by contrast, is always implementable as it is flat in l and increasing in k.

Extremal Sorting

Towards characterizing the full screening allocation, we start by simplifying the set of constraints. Since the problem contains all sorting constraints along the influence dimension, Lemma 2 implies that utility has to be constant along the l dimension whereby condition (H) remains necessary in the full screening problem. Then, a slice $k \times \mathcal{L}$ of the type space can be treated as a single type for the purpose of outward deviations. In addition, we can rank the attractiveness of contracts in each $k \times \mathcal{L}$ slice by quantity alone: Higher payoff-types will prefer the contract with the highest allocation, while lower payoff-types will prefer the contract with the lowest allocation. Furthermore, as a consequence of single-crossing, we can restrict attention to local misrepresentation of the payoff-type. Consequently, for types in $k \times \mathcal{L}$, the relevant downward deviation is towards the contract giving the highest consumption in the $k-1 \times \mathcal{L}$ slice, whereas the relevant upward deviation is towards the contract giving the lowest consumption in the $k+1 \times \mathcal{L}$ slice. If it is not profitable to deviate to the contract with the largest (smallest) level of consumption in the slice, it is not profitable to deviate into the slice at all.

Definition 1. A menu of contracts $\{(x_{kl}, t_{k,l})\}_{kl \in \mathcal{K}\times\mathcal{L}}$ satisfies **extremal sorting (ES)** if, for each k,

$$U_k \geq u(\min_l x_{k+1,l}, k, \overline{x}) - \max_l t_{k+1,l} \tag{ES-A$_k$}$$

$$U_k \geq u(\max_l x_{k-1,l}, k, \overline{x}) - \min_l t_{k-1,l} \tag{ES-B$_k$}$$

Finally, by the sorting constraints, it is sufficient to consider the participation constraint of the lowest payoff-type as all other participation constraints will be implied.

$$U_{k_0} \geq 0 \tag{P}$$

Formalizing this discussion, we have

Proposition 4. *An allocation satisfies k-monotonicity, (H), (P), and extremal sorting if and only if it satisfies all participation and incentive constraints* $(\mathcal{K}\times\mathcal{L})^2$.

Lexicographic Monotonicity

The next step is to identify the extremal types (i.e. those that have highest and lowest allocation in an optimal contract) within a slice $k \times \mathcal{L}$. Let $\succ$ be the lexicographic order on $\mathcal{K}\times\mathcal{L}$ where $\mathcal{K}$ is the dominant dimension, i.e.

$$(k,l) \succ (k',l') \iff k > k' \text{ or } k = k', l > l' \tag{26}$$

and denote by $M := \{\boldsymbol{x} \geq 0 : \boldsymbol{x} \text{ is weakly increasing in } \succ\}$ the set of lexicographic-monotonic allocations.

Theorem 1 (**Lexicographic Monotonicity**). *If* $\boldsymbol{x}$ *solves* $\pi(\mathcal{K}\times\mathcal{L})^2$, *then* $\boldsymbol{x} \in M$.

The proof of the theorem proceeds in three steps. First, we show that the upward sorting constraints are implied in the optimal allocation. We then write the problem in utility space (anticipating Proposition 5 below) and establish that the principal always benefits from a marginal (windfall) increase in $\overline{x}$, all

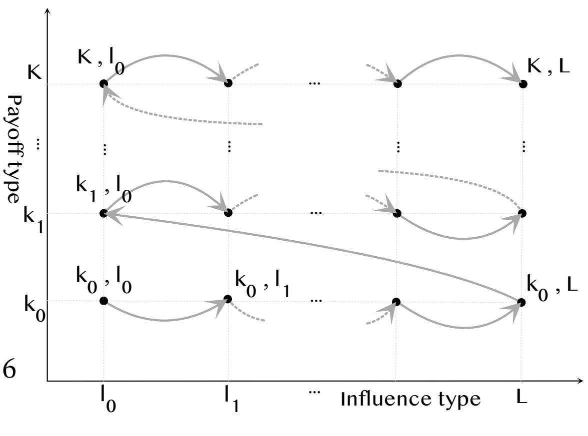

Figure 1: The Lexicographic Order $\succ$.

other things being equal. In other words, the associated increase in surplus dominates the increase in information rents and we have $\zeta > 0$.[14] Consequently, we establish with a variational argument that within a $k \times \mathcal{L}$ slice the principal always desires to allocate higher levels of consumption to more influential types – a change that increases $\overline{x}$ while even increasing the surplus generated within the slice. Since Proposition 4 implies that only the largest $x_{k,l}$ in a slice $k \times \mathcal{L}$ is relevant for deviation, adding the sorting constraints does not alter this property. Combining this fact with k-monotonicity, we conclude that the optimal allocation has to be lexicographic monotonic.

The Relaxed Problem

Using the results derived in the previous section, we can rewrite the principal's problem as a monotonicity constrained optimization in terms of virtual values. This problem is one-dimensional along the lexicographic order.

Proposition 5. *The problem* $\pi(\mathcal{K} \times \mathcal{L})^2$ *is equivalent to*

$$\max_{\boldsymbol{x} \in \mathrm{M}} \sum f_{k,l} \left[u\left(x_{k,l}, k, \overline{x}\right) - \chi_{l=\mathrm{L}} \left\{ \frac{1 - \mathrm{F}(k)}{f_{k,l}} \int_k^{k+1} u_k\left(x_{k,l}, j, \overline{x}\right) \mathrm{d}j \right\} \right] + \kappa(\overline{x}) \tag{UP}$$

$$s.t.\ \overline{x} = \sum f_{k,l} v\left(x_{k,l}, l\right) \tag{ζ}$$

where $\mathrm{F}(k) = \sum_{l, j \leq k} f_{j,l}$ *denotes the c.d.f. on the payoff-type dimension.*

In the appendix (Lemma 7), we provide general conditions that ensure that the principal's problem is strictly concave. As for the first best (Lemma 1), they involve maximizing a two-dimensional quadratic form. Under this condition, the full screening contract induces a pure strategy equilibrium between agents. In the sale of a network good from Section 2.2.2, we obtain a bound on the degree of complementarities

$$\gamma < \frac{\mathbb{E}[l]}{\sqrt{\left[\sum_k \frac{(1-\mathrm{F}_k)^2}{f_{k,\mathrm{L}}} + \mathbb{E}[k]\right] \mathbb{E}[l^2]} + \left[\mathbb{E}[kl] - \mathrm{L}\left(\mathbb{E}[k] - k_0\right)\right]} \tag{27}$$

In the pollution case the condition is implied since $u_{\overline{x}} = 0$ and $u_{kxx} = 0$. Throughout this section, we assume that the concavity condition is satisfied. We therefore have a unique solution to our program, which we denote by $\boldsymbol{x}^{\mathrm{FS}}$.

[14] A positive marginal value of the aggregate action for the monopolist is a natural yet not immediate result. In contrast to the symmetric information benchmark, under asymmetric information $\overline{x}$ impacts revenues in two opposing ways: On the one hand, increasing $\overline{x}$ increases total surplus; on the other hand, it increases the information rents. We show that the first force dominates.

4.1 The Ironing Procedure

To achieve a closer characterization of the optimal allocation we need to deal with the monotonicity constraints. As opposed to the textbook screening model, a simple condition on primitives is not sufficient to rule out violations of monotonicity. Instead, by the nature of our problem, there are two sources of monotonicity violations in this candidate allocation. First, sorting constraints only affect types with $l = \mathrm{L}$ directly: screening distortions on the whole $k \times \mathcal{L}$ slice accumulate on the k, L type. The resulting downward distortion will typically be propagated along the l-dimension by lexicographic monotonicity. This would happen even if single-crossing and a monotone hazard rate are satisfied. The second source of violations of monotonic virtual values is the jump between type k, L and $k+1, l_0$. One the one hand, the latter has a higher payoff-type; on the other hand, he is less influential, which depresses his virtual value. The strength of those distortions depends on the endogenous objects $\zeta, \overline{x}$ and no general conditions consistent with our analysis can ensure the lexicographic monotonicity of the candidate allocation.

Example (**Pollution** 2×2)**.** Let us illustrate these issues in the pollution setting where we can solve for the allocation in closed form. The pointwise maximizer of the objective (UP) is

$$(\check{x}_{0,0}, \check{x}_{0,1}, \check{x}_{1,0}, \check{x}_{1,1}) = \left(\frac{1}{1+\kappa}, 1 - \frac{1 - f_{0,0} - f_{0,1}}{1 - f_{0,0} + f_{0,1}}, \frac{1}{\frac{1}{2} + \kappa}, 2 \right) \tag{28}$$

The pointwise maximum violates lexicographic monotonicity between $(0,0)$ and $(0,1)$ if

$$f_{0,1} < \frac{1 - f_{0,0}}{2\kappa} \tag{29}$$

i.e. whenever the downward distortion due to the sorting constraint is larger than the reduction in output due to pollution for type $(0,0)$. Monotonicity is violated if $\kappa \approx 0$ or if the downward distortion is large as $f_{0,1} \approx 0$. In this case, the two low-productivity types will be bunched.

There is a non-monotonicity induced by the downward jump in influence between $(0,1)$ and $(1,0)$ if

$$f_{0,1} > \frac{1 - f_{0,0}}{1 + 2\kappa} \tag{30}$$

i.e. whenever the downward distortion due to sorting is smaller than the reduction in output due to pollution for type $(1,0)$. Combining both inequalities, we see that the pointwise maximizer $\check{\boldsymbol{x}}$ solves the full screening problem if and only if

$$\kappa \in \left[\frac{1 - f_{0,0}}{2 f_{0,1}} - \frac{1}{2}, \frac{1 - f_{0,0}}{2 f_{0,1}} \right]. \tag{31}$$

The resulting ironing procedure has important consequences for the impact of screening on pollution, as illustrated in the second panel of Figure 2. In the unconstrained case, the downward distortion only affects the "green" type $(0,1)$. Therefore,

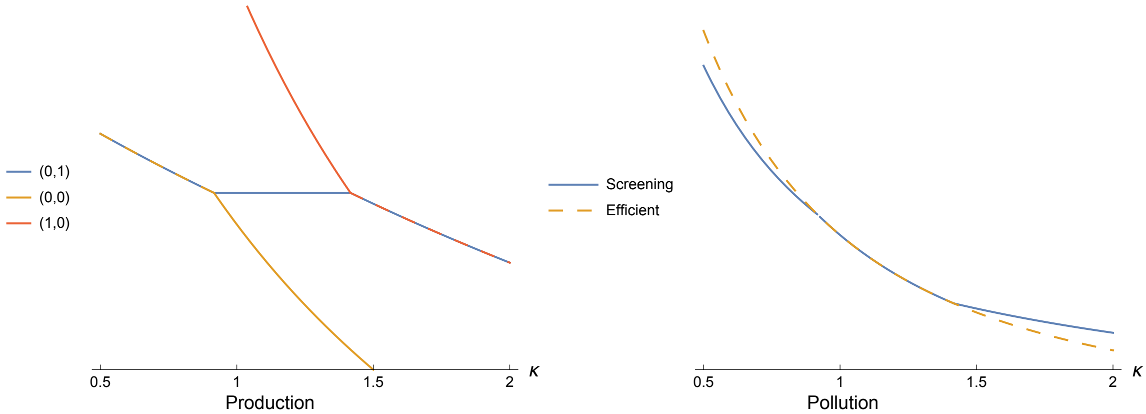

Figure 2: Production and aggregate pollution as a function of social cost κ.

the pollution mitigation and rent extraction motive are independent and $\overline{x}^{\star}$ and $\overline{x}^{\mathrm{FS}}$ coincide. This is the case when the costs of pollution for the principal are intermediate. When the monotonicity constraint between $(0,0)$ and $(0,1)$ is binding, the "dirty" type is distorted downwards and the rent extraction motive leads to pollution lower than in the first best. This is the case when the cost of pollution for the principal are low. When instead the monotonicity constraint between $(0,1)$ and $(1,0)$ is binding, the need for screening high- from low- productivity types depresses the output of the inefficient green type in favor of the efficient dirty type, which makes a reduction in pollution more costly in terms of total surplus than in the unconstrained case. This leads to pollution in excess of the first-best level exactly when the costs of pollution are high for the principal.

In the pollution example, ζ is pinned down exogenously since we have a constant marginal social benefit of $\overline{x}$ and the aggregate outcome does not directly enter the agents' utility. In general, the ironing conditions depend not only on primitives, but also on the endogenous objects $\overline{x}$ and ζ. For example, in the sale of a network good the solution requires ironing unless for all k

$$\zeta\frac{\mathrm{L}-l_0}{\mathbb{E}[l]} < \gamma\overline{x}\left(1+\frac{1-\mathrm{F}(k)}{f_{k,\mathrm{L}}}\right) < \gamma\overline{x}+\zeta\frac{1}{\mathbb{E}[l]}. \tag{32}$$

The ironing procedure therefore proceeds in two steps. We first solve for the candidate allocation for fixed aggregate variables $(\overline{x},\zeta)$ and then solve for the aggregate quantities. We write the objective function as the weighted sum of virtual values $\mathrm{J}(x_{k,l},k,l,\overline{x},\zeta)$.

$$\sum_{k,l} f_{k,l}\underbrace{\left[u\left(x_{k,l},k,\overline{x}\right)-\chi_{l=\mathrm{L}}\left\{\frac{1-\mathrm{F}(k)}{f_{k,l}}\left[u\left(x_{k,l},k+1,\overline{x}\right)-u\left(x_{k,\mathrm{L}},k,\overline{x}\right)\right]\right\}+\zeta v\left(x_{k,l},l\right)\right]}_{:=\mathrm{J}(x_{k,l},k,l,\overline{x},\zeta)} \tag{33}$$

which defines a set of proposed allocations $\check{x}_{k,l}$ maximizing the (rescaled) virtual value

J. By concavity of the virtual value, $\check{x}_{k,l}(\overline{x},\zeta)$ solves $\mathrm{J}_x(\check{x}_{k,l},k,l,\overline{x},\zeta)=0$. Clearly, if $\check{x}\in \mathrm{M}$, then it is the solution, $\check{x}(\overline{x},\zeta)=x^{\mathrm{FS}}(\overline{x},\zeta)$. As discussed above, this generally will not be the case.

Allocation Conditional on Aggregate Variables

We adapt standard techniques from Toikka (2011) to the problem rendered one-dimensional in the lexicographic order by virtue of Theorem 1.

Let $k(q),l(q):[0,1]\mapsto \mathcal{K}\times\mathcal{L}$ trace out the distribution f on $\mathcal{K}\times\mathcal{L}$ along the lexicographic order. In other words, if $q\in\left[\sum_{i,j<k,l} f_{i,j},\sum_{i,j\lesssim k,l} f_{i,j}\right)$, we have $k(q)=k$ and $l(q)=l$. Denote an inverse by $q(k,l)=\sum_{i,j<k,l} f_{i,j}$. The cumulative virtual value is given by

$$\mathrm{H}(x,q)=\int_0^q \mathrm{J}_x(x,k(r),l(r),\overline{x},\zeta)\mathrm{d}r \tag{34}$$

It follows from Toikka (2011) that ironing the original problem is equivalent to convexifying H (see Figure 3). For every x, let $\mathrm{G}(x,\cdot):=\mathrm{Conv}\,\mathrm{H}(x,\cdot):=\max\{g(x,\cdot)\le \mathrm{H}(x,\cdot)|g \text{ is convex}\}$ which is continuously differentiable almost everywhere on $[0,1]$.

$$\bar{\mathrm{J}}(x,k,l,\overline{x},\zeta)=\mathrm{J}(0,k,l,\overline{x},\zeta)+\int_0^x \mathrm{G}_q(y,q(k,l))\,\mathrm{d}y \tag{35}$$

The conditionally optimal allocation then solves

$$x^{\mathrm{FS}}(\overline{x},\zeta):=\arg\max_{x\ge 0}\sum \bar{\mathrm{J}}(x,k,l,\overline{x},\zeta) \tag{36}$$

Remark 3. As our type space is finite, the convexification is easy to compute. A simple algorithm proceeds downwards in the lexicographic order and "greedily" irons out violations of convexity as it encounters them. It finishes in at most $|\mathcal{K}\times\mathcal{L}|+1$ steps.

If $\mathrm{H}(x_{k,l},q)<\mathrm{G}(x_{k,l},q)$, the lexicographic monotonicity constraints are active at the corresponding type and there is bunching. Indeed, since H is piece-wise linear, with kinks where $k(q),l(q)$ jumps between types, we obtain an intuitive characterization of $x^{\mathrm{FS}}(\overline{x},\zeta)$.

Lemma 3. *The ironing procedure induces a partition of types $\mathcal{B}$, with typical element* B*, ordered by $\succ$. The optimal allocation for a given $(\overline{x},\zeta)$ is constant within cells and strictly increasing across cells. For interior $x_{k,l}$, $(k,l)\in\mathrm{B}$ implies that $x_{k,l}=x_{\mathrm{B}}$, solving*

$$u_x(x_{\mathrm{B}},k_{\mathrm{B}},\overline{x})+\zeta\mathbb{E}[v_x(x_{\mathrm{B}},l)|\mathrm{B}]-\frac{\sum_{k,l>\mathrm{B}} f_{k,l}}{\sum_{k,l\in\mathrm{B}} f_{k,l}}[u_x(x_{\mathrm{B}},k_{\mathrm{B}}+1,\overline{x})-u_x(x_{\mathrm{B}},k_{\mathrm{B}},\overline{x})]=0 \tag{37}$$

where $k_{\mathrm{B}}=\min_{(k,l)\in\mathrm{B}}k$ and $k^{\mathrm{B}}=\max_{(k,l)\in\mathrm{B}}k$.

In the network good application, types coincide with marginal utility and the marginal externality and hence condition (37) simplifies to

$$x_{\mathrm{B}} = \max\left\{1 + \gamma\overline{x}\left(\mathbb{E}[k|\mathrm{B}] - \frac{\sum_{k,l>\mathrm{B}} f_{k,l}}{\sum_{(k,l)\in\mathrm{B}} f_{k,l}}\right) + \zeta\frac{\mathbb{E}[l|\mathrm{B}]}{\mathbb{E}[l]}, 0\right\} \tag{38}$$

Agents are allocated consumption according to the expected payoff-type and expected externality in their partition cell.

The bunching regions have the following properties

Proposition 6.

1. *There is no bunching at the top of the lexicographic order:* $\{(\mathrm{K},\mathrm{L})\} \in \mathcal{B}$.
2. *Every nontrivial cell of* $\mathcal{B}$ *contains a switching type in* $\succ$*:* $|\mathrm{B}| > 1 \implies \exists\,(k,\mathrm{L}) \in \mathrm{B}$.
3. *Suppose that the externality is positive* ($v_x \geq 0$)*. There is active influence tilting within a payoff slice only if the higher influence-type consumes more than his decentralized allocation:* $x^{\mathrm{FS}}_{k,l} > x^{\mathrm{FS}}_{k,l-1} \implies u_x\left(x^{\mathrm{FS}}_{k,l}, k, \overline{x}\right) < 0$.

The first property provides a weak analogue to the "no distortion at the top" results common across screening models: For a given externality $\overline{x}$ and value of the externality ζ, the highest type is not affected by the ironing procedure. His consumption, however, is distorted relative to the decentralized and first-best consumption *even given* $\overline{x}$, as the value of the externality for the monopolist generally differs from $\zeta^\star$.[15] Agents with the highest payoff-type but lower influence, by contrast, can be affected by ironing as shown in the above example. The second property says that every bunching region includes an agent with the highest influence. It is around these types that the nonmonotonic virtual values can arise: Either their action is heavily downward distorted to reduce information rents of higher types and they are bunched with less influential agents of the same (or lower) payoff-type, or their action is distorted upward to promote the externality and they are bunched with less influential agents of higher payoff-type. Bunching regions "strictly within" a payoff slice are never optimal as virtual values are locally increasing. As for the third property, notice that when the externality is positive, $u_x\left(x^{\mathrm{FS}}_{k,l}, k, \overline{x}\right) < 0$ implies that the decentralized solution associated to the full screening aggregate activity, $x^{\mathrm{D}}_k(\overline{x})$, is greater than $x^{\mathrm{FS}}_{k,l}$. Whenever the optimal menu offers flexibility for self-selection based on influence, the more influential agents consume more than their privately optimal level. In other words, flexibility is only optimal if the provision of the externality overpowers the usual downward-distortion motive. When the externality is negative, by contrast, all types consume less than their privately optimal level as both the screening as well as the externality-provision distortions push in the same direction.

[15] In the pollution example, we have $u_{\overline{x}} \equiv 0$ and linear κ, and therefore always $\zeta = \kappa$. Hence, if $\overline{x}^\star = \overline{x}^{\mathrm{FS}}$ – which is the case for an open set of parameters, see (31) – the highest type produces at the (unconditionally) efficient level.

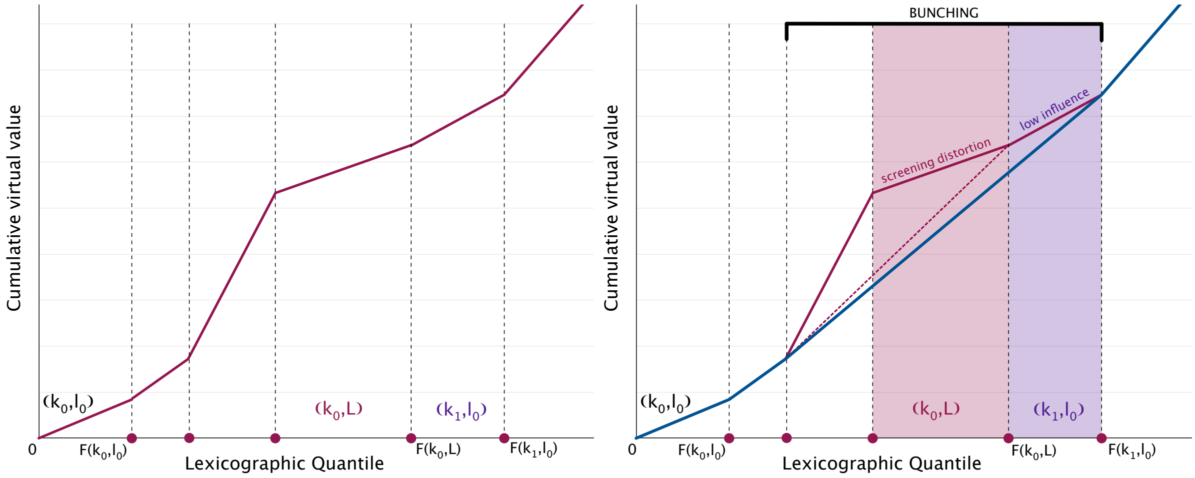

Figure 3: The convexification (blue) of the cumulative virtual value (crimson) results in a bunching region around a switching type.

Endogenizing Aggregate Variables

The final step in the derivation of the optimal contract is to solve for $\overline{x}$. For this purpose, we set up an auxiliary fixed point problem using the conditionally-ironed allocation $x^{\mathrm{FS}}(\overline{x}, \zeta)$ and show that a solution to this two-step procedure corresponds to the solution of the relaxed problem. Although the self-map may not be a contraction, existence and uniqueness are guaranteed by the properties of the relaxed problem.

Theorem 2. *The allocation* $\boldsymbol{x}^{\mathrm{FS}}$ *solving* $\pi(\mathcal{K} \times \mathcal{L})^2$ *satisfies*

$$\boldsymbol{x}^{\mathrm{FS}} = x^{\mathrm{FS}}(\overline{x}^{\mathrm{FS}}, \zeta^{\mathrm{FS}}) \tag{39}$$

where $(\overline{x}^{\mathrm{FS}}, \zeta^{\mathrm{FS}})$ *is the unique fixed point of the self-map* $\Gamma : \mathbb{R}^2 \mapsto \mathbb{R}^2$ *given by*

$$\Gamma\begin{pmatrix} \overline{x} \\ \zeta \end{pmatrix} = \begin{pmatrix} \sum f_{k,l}\, v\left(x^{\mathrm{FS}}_{k,l}(\overline{x}, \zeta), l\right) \\ \sum f_{k,l}\left(u_{\overline{x}}(x^{\mathrm{FS}}_{k,l}(\overline{x}, \zeta), k, \overline{x}) - \chi_{l=\mathrm{L}} \frac{1-\mathrm{F}_k}{f_{kl}} \int_k^{k+1} u_{k\overline{x}}\left(x^{\mathrm{FS}}_{k,l}(\overline{x}, \zeta), j, \overline{x}\right) \mathrm{d}j\right) + \kappa'(\overline{x}) \end{pmatrix} \tag{40}$$

Establishing that the two-step procedure yields the unique solution of the general problem $\pi(\mathcal{K} \times \mathcal{L})^2$ proves that all properties of the optimal allocation conditional on aggregate variables $\overline{x}, \zeta$ also characterize the solution to $\pi(\mathcal{K} \times \mathcal{L})^2$. In particular, Proposition 6 characterizes unconditionally optimal bunching regions. We illustrate those properties in a 2×2 network good application.

Example (**Network Good** 2×2)**.** Consider the setting of Section 2.2.2 with $\mathcal{K} = \{0, 1\}$ and $\mathcal{L} = \{0, 1\}$: agents with payoff-type 0 are not susceptible at all to the network effect, $u_{\overline{x}}(0, \cdot) \equiv 0$; agents with influence type 0 do not create any consumption externality, $v(0, \cdot) \equiv 0$. Given this parametrization three cases emerge as a full screening solution, depending on complementarities γ and the distribution of types f.

1. Low susceptibility agents are bunched and excluded, $x_{1,1} > x_{1,0} > x_{0,1} = x_{0,0} = 0$.

2. Low susceptibility agents are bunched at a positive level of consumption, $x_{1,1} > x_{1,0} > x_{0,1} = x_{0,0} > 0$

3. The allocation satisfies strict monotonicity along the lexicographic order, $x_{1,1} > x_{1,0} > x_{0,1} > x_{0,0} = x^{\star}_{0,0} = 1$.

Figure 4 displays these possible regions. For every distribution, at $\gamma = 0$ every type consumes 1 (the first-best allocation) as there are no externalities and differences between payoff-types. For low γ, there is always bunching of the non-susceptible agents; local to $\gamma = 0$, this level is decreasing in the degree of complementarity as the differences between payoff-types also increase. As γ approaches its upper bound γ^{FS}, given in (27), two things can happen (depending on the distribution of types): either the bunching level drops to 0 and the non-susceptible agents are excluded, or it bends back to 1 and the allocation is strictly monotonic. Notice that all properties in Proposition 6 hold: Type $(1,1)$ is never bunched. The second property holds vacuously in a 2×2 example. To check the third property, recall from Example 3.1 that $x^{\mathrm{D}}_k(\overline{x}) = 1 + k\gamma\overline{x}$. Agents that are not susceptible are separated in the full screening solution if and only if $x_{1,0} > 1 = x^{\mathrm{D}}_0$, namely for large γ in the right panel of Figure 4. Agents that are susceptible are always separated since

$$x^{\mathrm{FS}}_{1,1} > 1 + \gamma\overline{x} + \frac{\zeta}{\mathbb{E}[l]} > 1 + \gamma\overline{x} = x^{\mathrm{FS}}_{1,0} = x^{\mathrm{D}}_1(\overline{x}) \tag{41}$$

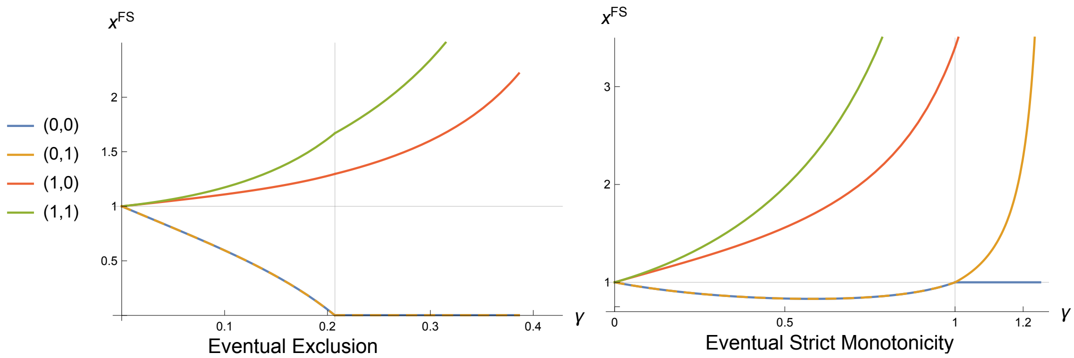

Figure 4: Consumption in the 2×2 example as a function of complementarities. Type distributions $f = (.1, .1, .2, .6)$ (left, switching from region 2 to 1) and $f = (.3, .3, .3, .1)$ (right, from region 2 to 3).

As for the aggregate variables and welfare, recall that aggregate consumption is inefficiently low in the decentralized outcome. The ranking of total surplus between the decentralized and second-best allocation is ambiguous. On the one hand, the screening motive of the principal induces a downward distortion, on the other hand, the principal internalizes the aggregate externality. We show by means of example (Figure 5) that the decentralized solution dominates the screening solution in terms of total surplus and consumer surplus for low γ, but screening performs better for sufficiently high γ, even in terms of consumer surplus.[16]

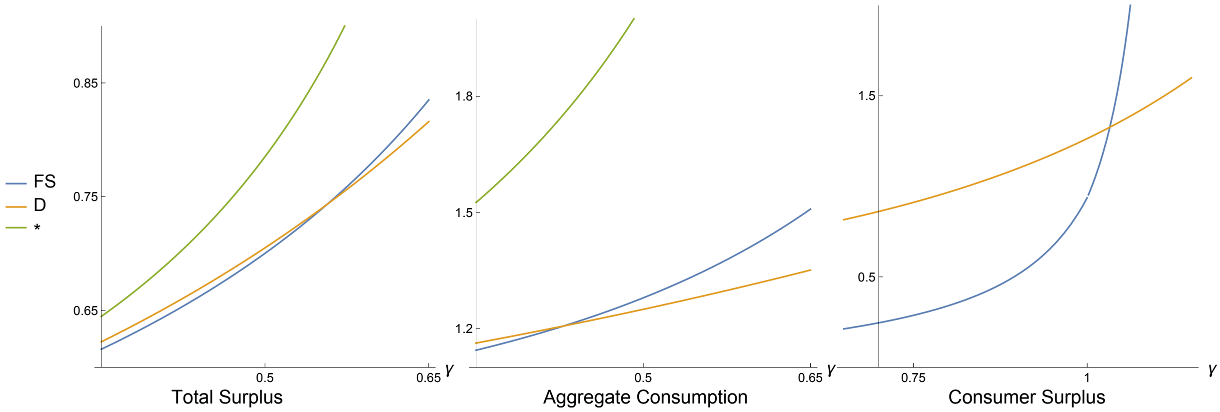

Figure 5: Aggregate measure in the 2x2 consumption example ($f = (.45,.3,.05,.2)$) as a function of complementarities.

As we have seen in the previous examples, $\overline{x}^{\mathrm{FS}}$ can exceed (2x2 pollution example) or fall short of (2x2 pollution and 2x2 network good examples) its first-best level. This results from the interplay of the downward distortion commonly associated with screening contracts, the impact of $\overline{x}$ on information rents and the ironing procedure. This interplay also makes it difficult to obtain general conditions, whereby a case-by-case analysis is required. In the network good example, we show in the appendix (B.2) that when the full screening solution does not require ironing, we always have $\overline{x}^{\mathrm{FS}} < \overline{x}^{\star}$.

5 Observable Influence

Neither the efficient nor the full screening contract produce influence rents: either surplus is fully extracted or the horizontal sorting condition (H) implies that utility has to be constant in a payoff-type slice. Influence rents, however, seem to characterize some markets in which influencers often receive lucrative deals to promote products. In a model where such influential agents have market power those rents can emerge as a result of bargaining. Can a model like ours where such market power is excluded (as there is a continuum of agents in each type) still generate rents from influence? To answer this question, we investigate the final observability assumption, i.e. the monopolist observes the influence type l but the payoff-type k is private information. Motivated by the application and to simplify the exposition, we will focus on the pricing of a network good with linear quadratic utility (Section 2.2.2).

Now, the monopolist can condition consumption on the observable l but has to ensure types k sort into their contract. Therefore – for a given $\overline{x}$ – the planner is solving a sequence of $|\mathcal{L}|$ one-dimensional screening problems. As per standard

[16]Clearly, the latter result depends crucially on the distribution of types. Regardless of the externality, if the type of the agent is almost known, there will be almost full extraction. In the example, type $(0,1)$ is relatively abundant, linking information rents to the creation of the externality.

arguments, we can rewrite each as the maximization of virtual value subject to a monotonicity constraint. The problems for different l are however coupled through aggregate consumption $\overline{x}$.

Proposition 7. *The maximization problem* $\pi\left(\bigcup_{l\in\mathcal{L}}(\mathcal{K}\times l)^2\right)$ *is equivalent to*

$$\max_{\boldsymbol{x},\overline{x}} \sum_{k,l} f_{k,l}\left\{\left(1+\gamma\overline{x}\left(k-\frac{\mathrm{F}(\mathrm{K}|l)-\mathrm{F}(k|l)}{f(k|l)}\right)\right)x_{kl}-\frac{1}{2}x_{kl}^2\right\} \tag{42}$$

subject to the aggregate effect, non-negativity and monotonicity conditional on l.

In this case, violations of monotonicity are solely the mechanical consequence of a nonmonotonic inverse hazard rate of the conditional type distribution. We hence restrict attention to the regular case in which the monotonicity constraints are slack.

Assumption 1. *For every l, the virtual value $k-\frac{\mathrm{F}(\mathrm{K}|l)-\mathrm{F}(k|l)}{f(k|l)}$ is nonnegative and increasing in k.*

Analogous to the full information case, the first-order conditions of this problem have two components. The first part is the familiar screening formula, the second adjusts consumption upward for influential individuals in order to provide a stronger network effect.

$$x_{k,l}^{\mathrm{OI}} = \underbrace{1+\gamma\overline{x}^{\mathrm{OI}}\left(k-\frac{\mathrm{F}(\mathrm{K}|l)-\mathrm{F}(k|l)}{f(k|l)}\right)}_{\text{optimal screening for fixed } \overline{x}} + \underbrace{\frac{l}{\mathbb{E}[l]}\zeta^{\mathrm{OI}}}_{\text{provide public good } \overline{x}} \tag{43}$$

Agents receive information rents for their payoff-type k. The magnitude of these rents depends on the level of consumption of agents with the same l but lower k. Therefore, the rent of type k,l is dependent on his (observable) level of influence. We say that there are *rents from influence* if, for every fixed k, the information rent is increasing in l. There are *expected rents from influence* if the expected rent is increasing in l.

The information rent of type k,l can be written as $\gamma\overline{x}\sum_{j<k}x_{j,l}^{\mathrm{OI}}$. Influence affects optimal consumption and hence information rents through two channels. First, more influential individuals consume more and high levels of consumption cause high rents. Second, influence has an effect on the downward distortion of consumption. If payoff- and influence-type are affiliated, the downward distortion is larger for influential consumers since there are more high susceptibility types among them. The outcome depends on the balance of these two forces, whose relative strength is determined by a moment Ξ of the type distribution measuring both the scale of externalities and the (unsigned) association between k,l, i.e. how informative the observable influence is

about the payoff-type of the agent. Formally, the rents of type k, l are proportional to

$$\xi(k,l) := \underbrace{k\frac{l}{\mathbb{E}[l]}\Xi}_{\text{provision of } \overline{x}} - \underbrace{\sum_{j=0}^{k-1}\frac{1-\mathrm{F}(j|l)}{f(j|l)}}_{\text{screening distortion}}, \tag{44}$$

$$\text{where } \Xi := \gamma\left(\mathbb{E}[k] + \mathbb{E}[\left(\frac{1-\mathrm{F}(k|l)}{f(k|l)}\right)^2]\right) > 0. \tag{45}$$

Proposition 8. *Suppose that the nonnegativity constraints are slack.*

1. *There are rents from influence if and only if, for all k, $\xi(k,l)$ is increasing in l.*
2. *There are expected rents from influence if and only if $\mathbb{E}[\xi(k,l)|l]$ is increasing in l.*

If k and l are independent, the screening distortion is independent of l and we have both rents from influence and expected rents from influence. In general, one can happen without the other, as we illustrate in our running example.

Example (**Network Good** 2×2 **cont'd**). We further parameterize the 2×2 network good setting by the covariance of k and l, letting $f_{0,0} = f_{1,1} = .25 + \rho$, $f_{0,1} = f_{1,0} = .25 - \rho$.[17] We highlight some features of the solution (detailed in B.3) in Figure 6.

In the first two panels we compare the full screening contract with the observable influence contract. Aggregate consumption is smaller with observable influence when there is moderate positive correlation. This results from the large downward distortion of $x_{0,1}$ chosen in order to depress the information rents of the relatively common type $(1,1)$, a motive that is attenuated when l is not observed. Clearly, the profit of the seller is weakly higher when influence is observable, with equality only when $x^{\mathrm{OI}}_{0,0} = x^{\mathrm{OI}}_{0,1}$, i.e. when the observable-l contract is incentive compatible in the full screening problem (the tangency point in the top right panel).

In the bottom panels we plot rents and expected rents from influence. For $\rho < 0$, there are always (pointwise) rents from influence. Both the relative abundance of low payoff-types and the motive to provide the consumption externality push towards a relatively high $x^{\mathrm{OI}}_{0,1}$, which results in these rents. There is a cutoff $\bar{\rho} > 0$ above which the high-payoff, low-influence type obtains a higher rent. The question of expected influence rents is more subtle, as there is the additional composition effect: as ρ increases, the influential agents also become more abundant relative to the non-influential ones (in the slice of high-payoff agents that receive some rents). As long as γ is not too large, this composition effect dominates for moderately negative correlation. Although type $(1,1)$ obtains a higher rent, the relative abundance of $(0,1)$ types means that on average high-influence consumers have a lower rent. This highlights the

[17] Note that Assumption 1 is satisfied trivially in a two payoff-type example and that

$$Cov(k,l) = f_{11} - \left(f_{0,1} + f_{1,1}\right)\left(f_{1,0} + f_{1,1}\right) = .25 + \rho - (.5)^2 = \rho$$

interaction of the conditional rent above (which is positive if correlation is negative) and the shift in relative mass from low- to high-payoff types (which favors rents for high-influence types if there is positive correlation, at least initially). The relative strength of these effects is mediated by γ as it scales up the magnitude of rents: For large γ, expected rents are in line with pointwise rents.

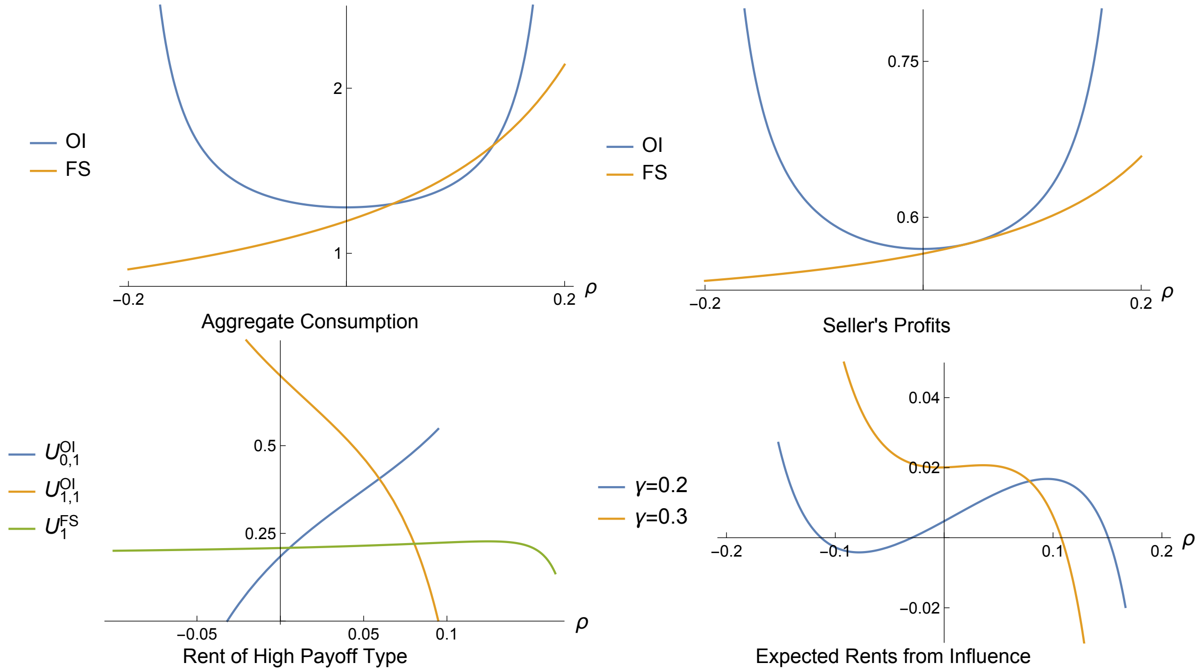

Figure 6: Observable influence in the 2x2 consumption example.

Comparing the results in this section to the previous one, we see that influence affects an agent's utility in our setting *only if it is observable and only indirectly: through its impact on information rents.* Other models on the pricing of network goods with observable influence do find rents as a direct result of influence, by contrast. With a known finite network, this is the result of a stochastic outside option (Bloch and Quérou, 2013) or the restriction to linear contracts (Candogan et al., 2012). With a similar demand specification as ours, Fainmesser and Galeotti (2020) find rents for influence when two firms compete for influential consumers using price discounts.

6 Conclusion

We analyze a screening problem with externalities. Agents have private information about their payoff-type and their influence on the externality. The principal provides a menu of actions and transfers, trading off revenues with the direct impact of the externality on her payoff. Several problems fall into this framework; for example, a monopolist using nonlinear pricing when there are consumption externalities or a government designing a tax when there are externalities between firms, positive through external economies of scale or negative through pollution. Although the problem is two-dimensional at the surface and contracts are linked "globally" through the externality, we show that it is nevertheless tractable. Eliciting influence is for free: As long as the payoff-type is observable, the principal can implement the first best. If both characteristics are unobservable, we show that the problem can be rendered one-dimensional along the lexicographic order, with the payoff-type as the dominant dimension. In the full screening contract, the principal screens along the payoff-type while tilting the allocation along the influence-type to control the externality. We characterize the solution through a two-step ironing procedure that addresses the nonmonotonicity in virtual values that arises naturally along the lexicographic order. As all violations of monotonicity originate at the highest influence types, every bunching region must include at least one of them. By offering a given payoff-type the flexibility to sort along a payoff-irrelevant dimension, the principal is able to increase total surplus (sometimes even the welfare of all agents) and reap some of those gains. There are rents for high payoff-types, but no rents for influence. If influence is observable, we obtain a family of one-dimensional screening problems coupled through the externality. Influence affects utility only if it is observed and even then only indirectly, through its effect on information rents. Highly influential consumers obtain higher rents if payoff- and influence-type are not too strongly affiliated.

A Proof Appendix

Proof of Lemma 1: Consider the Hessian of total surplus.[18]

$$\mathrm{H}^\star = \mathrm{dg}[f \odot (u_{xx} + \mathbb{E}[u_{\overline{x}} + \kappa']v_{xx})] + \mathbb{E}[u_{\overline{xx}} + \kappa''][f \odot v_x][f \odot v_x]^{\mathrm{T}} + 2\,\mathrm{Sym}\Big([f \odot u_{\overline{x}x}][f \odot v_x]^{\mathrm{T}}\Big) \tag{46}$$

Let $\mathrm{S} = \mathrm{dg}[\sqrt{f}]$. Then $\mathrm{H}^\star = \mathrm{S}\widehat{\mathrm{H}}^\star\mathrm{S}$, where

$$\widehat{\mathrm{H}}^\star = \mathrm{dg}[(u_{xx} + \mathbb{E}[u_{\overline{x}} + \kappa']v_{xx})] + \mathbb{E}[u_{\overline{xx}} + \kappa'']\Big[\sqrt{f} \odot v_x\Big]\Big[\sqrt{f} \odot v_x\Big]^{\mathrm{T}} + 2\,\mathrm{Sym}\Big(\Big[\sqrt{f} \odot u_{\overline{x}x}\Big]\Big[\sqrt{f} \odot v_x\Big]^{\mathrm{T}}\Big) \tag{47}$$

Since S is positive definite, the Hessian is negative definite whenever $\widehat{\mathrm{H}}^\star$ is n.d.. The first two summands are n.d. since $u_{xx} < 0$, $u_{\overline{x}} \geq 0$, $\kappa' \geq 0$ $v_{xx} \leq 0$, $u_{\overline{xx}} \leq 0$, and $\kappa'' \leq 0$. Hence, the only threat to concavity comes from the two final terms. Note that the two matrices annihilate the component of any vector outside of $\mathrm{span}\Big(\Big[\sqrt{f} \odot u_{\overline{x}x}\Big], \Big[\sqrt{f} \odot v_x\Big]\Big)$. To establish concavity it is hence sufficient to show that the quadratic form defined by $\mathrm{H}^\star$ is negative for unit vectors of the form $x = \alpha\Big[\sqrt{f} \odot u_{\overline{x}x}\Big] + \beta\Big[\sqrt{f} \odot v_x\Big]$.

First, note that

$$\|x\| = \alpha^2\mathbb{E}[u_{\overline{x}x}^2] + \beta^2\mathbb{E}[v_x^2] + 2\alpha\beta\mathbb{E}[u_{\overline{x}x}v_x] \tag{48}$$

The quadratic form evaluates to

$$\mathrm{Q}(\alpha, \beta) := y(\alpha, \beta)^{\mathrm{T}}\widehat{\mathrm{H}}^\star y(\alpha, \beta) \tag{49}$$

[18]Recall that $\mathrm{dg}(a)$ denotes the diagonal matrix with entries provided by the vector a, $\mathrm{Sym}(\mathrm{A}) = \frac{\mathrm{A}+\mathrm{A}^{\mathrm{T}}}{2}$, $\odot$ denotes element-wise multiplication of vectors and $[v_x]$ denotes the column vector with typical element $v_x(x_{k,l}, l)$.

$$
\begin{aligned}
=&\alpha^2\mathbb{E}[u_{\overline{x}x}^2(u_{xx}+\mathbb{E}[u_{\overline{x}}+\kappa']v_{xx})]+2\alpha\beta\mathbb{E}[u_{\overline{x}x}v_x(u_{xx}+\mathbb{E}[u_{\overline{x}}+\kappa']v_{xx})] \\
&+\beta^2\mathbb{E}[v_x^2(u_{xx}+\mathbb{E}[u_{\overline{x}}+\kappa']v_{xx})]+\mathbb{E}[u_{\overline{x}\overline{x}}+\kappa'']\Big\{\alpha^2\mathbb{E}[u_{x\overline{x}}v_x]^2+2\alpha\beta\mathbb{E}[u_{x\overline{x}}v_x]\mathbb{E}\Big[v_x^2\Big] \\
&+\beta^2\mathbb{E}\Big[v_x^2\Big]^2\Big\}+2\Big\{\alpha^2\mathbb{E}[u_{\overline{x}x}^2]\mathbb{E}[u_{\overline{x}x}v_x]+\alpha\beta\Big[\mathbb{E}[u_{\overline{x}x}^2]\mathbb{E}[v_x^2]+(\mathbb{E}[u_{\overline{x}x}v_x])^2\Big]+\beta^2\mathbb{E}[u_{\overline{x}x}v_x]\mathbb{E}[v_x^2]\Big\}
\end{aligned} \tag{50}
$$

and we have concavity if the value of

$$
\max_{\alpha,\beta} Q(\alpha,\beta) \tag{51}
$$

$$
\text{s.t.}\,\alpha^2\mathbb{E}[u_{\overline{x}x}^2]+\beta^2\mathbb{E}[v_x^2]+2\alpha\beta\mathbb{E}[u_{\overline{x}x}v_x]=1 \tag{52}
$$

is negative. The solution to this problem is conceptually simple as we are maximizing a quadratic form over an elliptic constraint, but analytically cumbersome. We hence restrict attention to our examples where we can derive meaningful bounds on the parameters.

In the network good example (Section 2.2.2), we have $(u_{xx}+\mathbb{E}[u_{\overline{x}}+\kappa']v_{xx})=-1$ and $\mathbb{E}[u_{\overline{x}\overline{x}}+\kappa'']=0$. Hence we get

$$
\begin{aligned}
Q(\alpha,\beta)=&-\Big(\alpha^2\mathbb{E}[u_{\overline{x}x}^2]+\beta^2\mathbb{E}[v_x^2]+2\alpha\beta\mathbb{E}[u_{\overline{x}x}v_x]\Big) \\
&+2\Big\{\alpha^2\mathbb{E}[u_{\overline{x}x}^2]\mathbb{E}[u_{\overline{x}x}v_x]+\alpha\beta\Big[\mathbb{E}[u_{\overline{x}x}^2]\mathbb{E}[v_x^2]+(\mathbb{E}[u_{\overline{x}x}v_x])^2\Big]+\beta^2\mathbb{E}[u_{\overline{x}x}v_x]\mathbb{E}[v_x^2]\Big\} \qquad (53)\\
=&-1+2\Big\{\mathbb{E}[u_{\overline{x}x}v_x]+\alpha\beta\Big[\mathbb{E}[u_{\overline{x}x}^2]\mathbb{E}[v_x^2]-(\mathbb{E}[u_{\overline{x}x}v_x])^2\Big]\Big\} \qquad (54)
\end{aligned}
$$

by plugging in the constraint. Note that the coefficient of $\alpha\beta$ is nonnegative by Cauchy-Schwartz. Hence, it is sufficient to find $\max\alpha\beta$ subject to the constraint. It follows from straightforward computation that

$$
\max_{\text{s.t.}\,equation\ \textbf{(??)}}\alpha\beta=\frac{1}{2\left(\sqrt{\mathbb{E}[u_{\overline{x}x}^2]\mathbb{E}[v_x^2]}+\mathbb{E}[u_{\overline{x}x}v_x]\right)} \tag{55}
$$

Plugging back and using that $\mathbb{E}[u_{\overline{x}x}^2]=\gamma^2\mathbb{E}[k^2]$, $\mathbb{E}[v_x^2]=\frac{\mathbb{E}[l^2]}{\mathbb{E}[l]^2}$, $\mathbb{E}[u_{\overline{x}x}v_x]=\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}$, after straightforward manipulation we get

$$
\gamma<\frac{\mathbb{E}[l]}{\left(\sqrt{\mathbb{E}[k^2]\mathbb{E}[l^2]}+\mathbb{E}[kl]\right)}. \tag{56}
$$

In the pollution case, we have $u_{\overline{x}}=0$ and hence

$$
Q(\alpha,\beta)=\beta^2\left(\mathbb{E}[v_x^2(u_{xx}+\kappa' v_{xx})]+\kappa''\mathbb{E}\Big[v_x^2\Big]^2\right)=\frac{1}{\mathbb{E}[v_x^2]}\mathbb{E}[v_x^2(u_{xx}+\kappa v_{xx})]<0 \tag{57}
$$

which is always satisfied. □

Lemma 4. *Let $\boldsymbol{x}^{\mathrm{D}}(\overline{x})$ solve $u_x(x,k,\overline{x})=0$. A decentralized equilibrium exists if*

$$
-\sum f_{k,l}v_x\frac{u_{x\overline{x}}}{u_{xx}}\leq c<1 \tag{58}
$$

along $\boldsymbol{x}^{\mathrm{D}}(\overline{x})$ for some $c\in\mathbb{R}$ and $|\overline{x}|>\mathrm{N}\in\mathbb{R}_+$.

Proof of Lemma 4: Consider $\mathrm{D}(\overline{x}):=\sum f_{k,l}v(x_k^{\mathrm{D}}(\overline{x}),l)$. The function is continuous and increasing. Furthermore, $\frac{\mathrm{dD}}{\mathrm{d}\overline{x}}=-\sum f_{k,l}v_x\frac{u_{x\overline{x}}}{u_{xx}}$. Under the condition of the Lemma, D has a fixed point. By construction, for any fixed point $\overline{x}^{\mathrm{D}}$ of D, $\left(\boldsymbol{x}^{\mathrm{D}}(\overline{x}^{\mathrm{D}}),\overline{x}^{\mathrm{D}}\right)$ is a decentralized equilibrium.

Note that in the consumption example, we have

$$\frac{\mathrm{dD}}{\mathrm{d}\overline{x}} = \gamma\mathbb{E}[kl] \tag{59}$$

and the associated condition $\gamma < 1 - \mathbb{E}[kl]$ is tight (see equation (16)). □

Proof of Proposition 1: As the problem is concave, differentiation of the objective – treating $\overline{x}$ as a constraint – yields the desired conditions. □

For the derivation of the unit price in the network good application, note that solving $u(x^{\star}_{k,l}, k, \overline{x}^{\star}) = 0$ yields

$$t^{\star}_{k,l} = \frac{1}{2}\left(1 + \gamma\overline{x}^{\star}k\right)^2 - \frac{1}{2}\left(\frac{l}{\mathbb{E}[l]}\zeta^{\star}\right)^2 = \frac{1}{2}x^{\star}_{k,l}\left(1 + \gamma\overline{x}^{\star}k - \frac{l}{\mathbb{E}[l]}\zeta^{\star}\right). \tag{60}$$

Proof of Lemma 2: Fix an arbitrary k and suppose the set of contracts $\{x_{k,l}, t_{k,l}\}_{l\in\mathcal{L}}$ delivers the same utility $u(x_{k,l}, k, \overline{x}) - t_{k,l}$ for all $l \in \mathcal{L}$. Clearly, there is no incentive to misrepresent the influence-type.

Necessity is immediate from $\mathrm{IC}_{k,l\to k,l'}$ and $\mathrm{IC}_{k,l'\to k,l}$:

$$u(x_{k,l}, k, \overline{x}) - t_{k,l} \geq u(x_{k,l'}, k, \overline{x}) - t_{k,l'} \geq u(x_{k,l}, k, \overline{x}) - t_{k,l} \tag{61}$$

□

Proof of Lemma 2: Recall that the relevant set of constraints for this problem are given by $\bigcup_{k\in\mathcal{K}}(k \times \mathcal{L})$. Consider the first-best allocation. The participation constraints are satisfied and the equilibrium utility is independent of l. Hence, by Lemma 2, the sorting constraints of this problem are satisfied. Clearly, this is the maximal profit the principal can achieve and hence the first-best allocation is the optimal menu of contracts. □

Proof of Remark 1: Suppose an agent with type k, l deviates to k', l' with $k' > k$. Since we have full extraction ($\mathrm{U}_k = 0$), the utility under this deviation is

$$\mathrm{U}_{k'} + u(x^{\star}_{k',l'}, k, \overline{x}) - u(x^{\star}_{k',l'}, k', \overline{x}) = -\int_k^{k'} u_k\left(x^{\star}_{k',l'}, j, \overline{x}\right)\mathrm{d}j \leq 0 = \mathrm{U}_k \tag{62}$$

Hence, it is not profitable. Similarly, there is also no incentive to misrepresent only influence. □

Proofs for Section 4 (Full Screening Problem)

Proof of Proposition 3: Consider the constraints $\mathrm{IC}_{k,l\to k',l'}$ and $\mathrm{IC}_{k',l'\to k,l}$:

$$u(x_{k,l}, k, \overline{x}) - t_{k,l} \geq u(x_{k',l'}, k, \overline{x}) - t_{k',l'} \tag{63}$$

$$u(x_{k',l'}, k', \overline{x}) - t_{k',l'} \geq u(x_{k,l}, k, \overline{x}) - t_{k,l} \tag{64}$$

Taking differences we arrive at

$$u(x_{k,l}, k, \overline{x}) - u(x_{k',l'}, k, \overline{x}) \geq u(x_{k,l}, k', \overline{x}) - u(x_{k',l'}, k', \overline{x}) \tag{65}$$

which implies $k' < k \iff x_{k',l'} < x_{k,l}$ since u has increasing differences in x, k. □

Notation. Fix a menu of contracts $\{(x_{kl}, t_{k,l})\}_{kl \in \mathcal{K}\times\mathcal{L}}$ and, for each k, pick

$$l_k \in \arg\min_{\tilde{l}} x_{k,\tilde{l}}, \quad l^k \in \arg\max_{\tilde{l}} x_{k,\tilde{l}}. \tag{66}$$

Proof of Proposition 4: Consider the sorting constraint from type k,l to type k',l', where $k > k'$. It is implied since

$$\begin{aligned} u(x_{k,l}, k, \overline{x}) - t_{k,l} = \mathrm{U}_k &\geq u(x_{k-1,l^{k-1}}, k, \overline{x}) - t_{k-1,l^{k-1}} && (67)\\ &= \mathrm{U}_{k-1} + u(x_{k-1,l^{k-1}}, k, \overline{x}) - u(x_{k-1,l^{k-1}}, k-1, \overline{x}) && (68)\\ &\geq \cdots \geq \mathrm{U}_{k'} + \sum_{j=k'+1}^{k} \Big(u(x_{j-1,l^{j-1}}, j, \overline{x}) - u(x_{j-1,l^{j-1}}, j-1, \overline{x}) \Big) && (69)\\ &\geq \mathrm{U}_{k'} + \sum_{j=k'+1}^{k} \big(u(x_{k',l'}, j, \overline{x}) - u(x_{k',l}, j-1, \overline{x}) \big) && (70)\\ &\geq u(x_{k',l'}, k, \overline{x}) - t_{k',l'} && (71) \end{aligned}$$

where the first inequality is the extremal downward sorting constraint (ES-B$_k$) and the equalities follow from condition (H). We apply this argument iteratively and estimate the sum of differences using that $\boldsymbol{x}$ is k-monotonic and u has increasing differences. An analogous argument leveraging (ES-A$_k$) establishes the upward IC. Hence, all IC constraints are implied. The sufficiency of (P) for all participation constraints follows from the argument above, noting that the LHS of the penultimate line for $k' = k_0$ is nonnegative by P and $\mathrm{U}_k \geq 0$. □

Lemma 5. *Consider an allocation satisfying the conditions of Proposition 4. If the downward ES-constraints (ES-B$_k$) are binding, the upward ES-constraints (ES-A$_k$) are inactive. Furthermore, in any second best contract, the downward ES-constraints (ES-B$_k$) and participation for k_0, (P), are binding.*

Proof of Lemma: Consider k,l and k',l' with $k < k'$. Then

$$\begin{aligned} u(x_{k',l'}, k, \overline{x}) - t_{k',l'} &= \mathrm{U}_{k'} + u(x_{k',l'}, k, \overline{x}) - u(x_{k',l'}, k', \overline{x}) \qquad (72)\\ &= \mathrm{U}_k + \sum_{j=k+1}^{k'} \Big(u(x_{j-1,l^{j-1}}, j, \overline{x}) - u(x_{j-1,l^{j-1}}, j-1, \overline{x}) \Big) - \big(u(x_{k',l'}, k', \overline{x}) - u(x_{k',l'}, k, \overline{x}) \big)\\ &\geq \mathrm{U}_k + \sum_{j=k+1}^{k'} \big(u(x_{k',l'}, j, \overline{x}) - u(x_{k',l'}, j-1, \overline{x}) \big) - \big(u(x_{k',l'}, k', \overline{x}) - u(x_{k',l'}, k, \overline{x}) \big) = \mathrm{U}_k \end{aligned}$$

where the second line follows by expressing $\mathrm{U}_{k'}$ via the binding downward IC and the inequality follows by (i) increasing differences and (ii) k-monotonicity as $x_{k',l'} \geq x_{j-1,l^{j-1}}$ for all $j \leq k'$. Hence, we have the downward IC.

Furthermore, suppose that a downward ES-constraint is strictly slack. We can increase the transfer from all affected types without implicating any other constraints, which increases the principal's objective. The same holds if $\mathrm{U}_{k_0} > 0$. □

Lemma 6. *If $\boldsymbol{x}, \boldsymbol{t}, \overline{x}, \zeta$ solves the Lagrangian associated to $\pi(\mathcal{K}\times\mathcal{L})^2$, then $\zeta > 0$.*

Proof of Lemma. By Lemma 5, the downward ES constraints and P are binding and hence we have that

$$\begin{aligned}
U_k &= u(\max_l x_{k-1,l}, k, \overline{x}) - t_{k-1,\arg\max x_{k-1,l}} && (73)\\
&= U_{k-1} + u(\max_l x_{k-1,l}, k, \overline{x}) - u(\max_l x_{k-1,l}, k-1, \overline{x}) && (74)\\
&= \sum_{j=k_0}^{k-1} u(\max_l x_{j,l}, j+1, \overline{x}) - u(\max_l x_{j,l}, j, \overline{x}) && (75)
\end{aligned}$$

Then, we can rewrite the principal's objective as

$$\sum f_{k,l}\left(u(x_{k,l},k,\overline{x}) - U_k\right) = \sum f_{k,l}\left(u(x_{k,l},k,\overline{x}) - \sum_{j=k_0}^{k-1} u(\max_l x_{j,l}, j+1, \overline{x}) - u(\max_l x_{j,l}, j, \overline{x})\right) \quad (76)$$

Consider the Lagrangian with this objective, k-monotonicity, and the ζ constraint. Then, in a candidate optimum, we have

$$0 = \frac{\partial \mathcal{L}}{\partial \overline{x}} = \sum f_{k,l}\left(u_{\overline{x}}(x_{k,l},k,\overline{x}) - \sum_{j=k_0}^{k-1} u_{\overline{x}}(\max_l x_{j,l}, j+1, \overline{x}) - u_{\overline{x}}(\max_l x_{j,l}, j, \overline{x})\right) - \zeta + \kappa'(\overline{x}) \quad (77)$$

$$\begin{aligned}
\zeta &= \sum f_{k,l}\left(u_{\overline{x}}(x_{k,l},k,\overline{x}) - \sum_{j=k_0}^{k-1} u_{\overline{x}}(\max_l x_{j,l}, j+1, \overline{x}) - u_{\overline{x}}(\max_l x_{j,l}, j, \overline{x})\right) + \kappa'(\overline{x}) \quad (78)\\
&= \sum f_{k,l}\left(\underbrace{u_{\overline{x}}(x_{k,l},k,\overline{x}) - u_{\overline{x}}(\max_l x_{k-1,l}, k, \overline{x})}_{\geq 0 \text{ k-mono and } u_{x\overline{x}}\geq 0} + \sum_{j=k_0}^{k-1} \underbrace{u_{\overline{x}}(\max_l x_{j,l}, j, \overline{x}) - u_{\overline{x}}(\max_l x_{j-1,l}, j, \overline{x})}_{\geq 0 \text{ k-mono and } u_{x\overline{x}}\geq 0}\right) + \kappa'(\overline{x})\\
&> 0
\end{aligned}$$

where strictness follows from nontriviality of the allocation and the condition (NT) that either $u_{\overline{x}} > 0$, $u_{x\overline{x}} > 0$ for a positive measure of types, or $\kappa' > 0$. □

Proof of Theorem 1: Suppose $\boldsymbol{x} \notin M$. Then, there exists a $k, l' > l$ such that $x_{k,l'} < x_{k,l}$. Consider

$$x^{\epsilon}_{k,l} = x_{k,l} - \epsilon, \quad t^{\epsilon}_{k,l} = t_{k,l} - \epsilon u_x(x_{k,l}, k, \overline{x}) \quad (79)$$

$$x^{\epsilon}_{k,l'} = x_{k,l'} + \epsilon\frac{f_{k,l}}{f_{k,l'}}, \quad t^{\epsilon}_{k,l'} = t_{k,l} + \epsilon\frac{f_{k,l}}{f_{k,l'}} u_x(x_{k,l'}, k, \overline{x}) \quad (80)$$

To the first order, this change keeps the utility of agents k, l and k, l' unchanged (for fixed $\overline{x}$). Furthermore, this does not tighten any constraints since the range of $x_{k,\cdot}$ contracts while utilities are held constant for type k. Furthermore, consider the expected transfers. we have

$$f_{k,l}t^{\epsilon}_{k,l} + f_{k,l'}t^{\epsilon}_{k,l'} = f_{k,l}t_{k,l} + f_{k,l'}t_{k,l'} + \epsilon f_{k,l}\left(u_x(x_{k,l'},k,\overline{x}) - u_x(x_{k,l},k,\overline{x})\right) > f_{k,l}t_{k,l} + f_{k,l'}t_{k,l'} \quad (81)$$

by concavity of u. For $\overline{x}$, we get

$$\begin{aligned}
\overline{x}^{\epsilon} &= \overline{x} + \epsilon\left(\frac{f_{k,l}}{f_{k,l'}} f_{k,l'} v_x(x_{k,l'}, l') - f_{k,l} v_x(x_{k,l}, l)\right) && (82)\\
&= \overline{x} + \epsilon f_{k,l}\left(v_x(x_{k,l'}, l') - v_x(x_{k,l}, l)\right) > \overline{x} && (83)
\end{aligned}$$

and hence the principal, by judicious adjustment of transfers, can obtain an additional payoff $\zeta\epsilon f_{k,l}\left(v_x(x_{k,l'},l') - v_x(x_{k,l},l)\right)$, establishing that the original allocation was not optimal. □

Proof of Proposition 5: By Lemma 5 and Theorem 1we can write

$$\mathrm{U}_k = \sum_{j=k_0+1}^{k} \left(u(x_{j-1,\mathrm{L}}, j, \overline{x}) - u(x_{j-1,\mathrm{L}}, j-1, \overline{x})\right) \tag{84}$$

and hence the objective of the principal reads

$$\begin{aligned} &\sum f_{k,l}\left(u(x_{k,l},k,\overline{x}) - \sum_{j=k_0+1}^{k}\left(u(x_{j-1,\mathrm{L}}, j, \overline{x}) - u(x_{j-1,\mathrm{L}}, j-1, \overline{x})\right)\right) + \kappa(\overline{x}) = \\ &\sum f_{k,l}\left[u(x_{k,l},k,\overline{x}) - \chi_{l=\mathrm{L}}\frac{1-\mathrm{F}_k}{f_{k,l}}\left(u(x_{k,\mathrm{L}},k+1,\overline{x}) - u(x_{k,\mathrm{L}},k,\overline{x})\right)\right] + \kappa(\overline{x}) \end{aligned} \tag{85}$$

where $\mathrm{F}_k = \sum_{j>k}\sum_l f_{k,l}$. Note that this objective subsumes all participation and sorting constraints, subject to monotonicity and $\overline{x}$, which establishes the proposition. □

Lemma 7. *The relaxed problem (UP) has a unique solution if the value of the quadratic form defined by the matrix*

$$\begin{aligned} \hat{\mathrm{H}}^{\mathrm{FS}} =& \mathrm{diag}[(u_{xx} - \Delta_{xx} + (\mathbb{E}[u_{\overline{x}}] - \mathbb{E}[\Delta_{\overline{x}}])\, v_{xx})] \\ &+ (\mathbb{E}[u_{\overline{xx}}] - \mathbb{E}[\Delta_{\overline{xx}}])\left[\sqrt{f}\, v_x\right]\left[\sqrt{f}\, v_x\right]^{\mathrm{T}} + 2\mathrm{Sym}\left(\left[\sqrt{f}\,(u_{\overline{x}x} - \Delta_{\overline{x}x})\right]\left[\sqrt{f}\, v_x\right]^{\mathrm{T}}\right) \end{aligned} \tag{86}$$

along $\mathrm{span}\left(\left[\sqrt{f}\odot(u_{\overline{x}x} - \Delta_{\overline{x}x})\right], \left[\sqrt{f}\odot v_x\right]\right)$ *is strictly bounded above by* 0*, where* $\Delta_{xx} := \chi_{l=\mathrm{L}}\frac{(1-\mathrm{F}_k)}{f_{l,k}}\int_k^{k+1} u_{kxx}\mathrm{d}s \in \mathbb{R}^{|\mathcal{K}\times\mathcal{L}|}$, $\Delta_{\overline{x}} := \chi_{l=\mathrm{L}}\left(\frac{1-\mathrm{F}_k}{f_{k,l}}\right)\int_k^{k+1} u_{k\overline{x}}\mathrm{d}s$, $\Delta_{\overline{xx}} := \chi_{l=\mathrm{L}}\left(\frac{1-\mathrm{F}_k}{f_{k,l}}\right)\int_k^{k+1} u_{k\overline{xx}}\mathrm{d}s$ *and* $\Delta_{\overline{x}x} := \chi_{l=\mathrm{L}}\frac{(1-\mathrm{F}_k)}{f_{l,k}}\int_k^{k+1} u_{k\overline{x}x}\mathrm{d}s \in \mathbb{R}^{|\mathcal{K}\times\mathcal{L}|}$.

Proof of Lemma: Using the same approach as for the first best above, the Hessian of the principals objective in where we have substituted for $\overline{x}$ is given by$\mathrm{H}^{\mathrm{FS}} = \mathrm{S}\hat{\mathrm{H}}^{\mathrm{FS}}\mathrm{S}$. Note that we have $u_{kxx} \geq 0$ and $u_{k\overline{xx}} \geq 0$. Therefore $\mathbb{E}[\Delta_{\overline{xx}}] \geq 0$. Furthermore $\mathbb{E}[u_{\overline{x}}] - \mathbb{E}[\Delta_{\overline{x}}] > 0$ is implied by our conditions and lexicographic monotonicity: We have by (85) and the proof of Lemma 6

$$\begin{aligned} \mathbb{E}[u_{\overline{x}}] - \mathbb{E}[\Delta_{\overline{x}}] &= \sum f_{k,l}\left[u_{\overline{x}}(x_{k,l},k,\overline{x}) - \chi_{l=\mathrm{L}}\frac{1-\mathrm{F}_k}{f_{k,l}}\left(u_{\overline{x}}(x_{k,\mathrm{L}},k+1,\overline{x}) - u_{\overline{x}}(x_{k,\mathrm{L}},k,\overline{x})\right)\right] \\ &= \sum f_{k,l}\left(u_{\overline{x}}(x_{k,l},k,\overline{x}) - \sum_{j=k_0+1}^{k}\left(u_{\overline{x}}(x_{j-1,\mathrm{L}},j,\overline{x}) - u_{\overline{x}}(x_{j-1,\mathrm{L}},j-1,\overline{x})\right)\right) = \zeta - \kappa'(\overline{x}) \geq 0 \end{aligned} \tag{87}$$

Hence, the first two matrices are negative semi-definite. As in the first-best, we can restrict attention to the subspace $\mathrm{span}\left(\left[\sqrt{f}\odot(u_{\overline{x}x} - \Delta_{\overline{x}x})\right], \left[\sqrt{f}\odot v_x\right]\right)$ as the terminal matrix annihilates all others. Let $y(\alpha,\beta) = \alpha\left[\sqrt{f}\odot(u_{\overline{x}x} - \Delta_{\overline{x}x})\right] + \beta\left[\sqrt{f}\odot v_x\right]$. We have concavity if the value of

$$\begin{aligned} &\max_{\alpha,\beta} y(\alpha,\beta)^{\mathrm{T}}\hat{\mathrm{H}}^{\mathrm{FS}} y(\alpha,\beta) \\ &\text{s.t. } \alpha^2\mathbb{E}[(u_{\overline{x}x} - \Delta_{\overline{x}x})^2] + \beta^2\mathbb{E}[v_x^2] + 2\alpha\beta\mathbb{E}[(u_{\overline{x}x} - \Delta_{\overline{x}x})v_x] = 1 \end{aligned} \tag{88}$$

is negative (for all x).

In the linear case, we can proceed similar to the first-best

$$\hat{\mathrm{H}}^{\mathrm{FS}} = -\mathrm{I} + 2\mathrm{Sym}\left(\left[\sqrt{f} \odot \left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)\right]\left[\sqrt{f} \odot \frac{l}{\mathbb{E}[l]}\right]^{\mathrm{T}}\right) \tag{89}$$

so that the quadratic form evaluates to

$$-1 + 2\mathbb{E}\left[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)\frac{l}{\mathbb{E}[l]}\right]\left[\alpha^2\mathbb{E}\left[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)^2\right] + \beta^2\mathbb{E}\left[\left(\frac{l}{\mathbb{E}[l]}\right)^2\right]\right] \tag{90}$$

$$+ 2\alpha\beta\left\{\mathbb{E}\left[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)^2\right]\mathbb{E}\left[\left(\frac{l}{\mathbb{E}[l]}\right)^2\right] + \mathbb{E}\left[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)\frac{l}{\mathbb{E}[l]}\right]^2\right\} \tag{91}$$

Using the constraint and simplifying, we arrive at

$$y(\alpha,\beta)^{\mathrm{T}}\widehat{\mathrm{H}}^{\mathrm{FS}}y(\alpha,\beta) = -1 + 2\mathbb{E}\left[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)\frac{l}{\mathbb{E}[l]}\right] + \tag{92}$$

$$+ 2\alpha\beta\left\{\underbrace{\mathbb{E}\left[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)^2\right]\mathbb{E}\left[\left(\frac{l}{\mathbb{E}[l]}\right)^2\right] - \mathbb{E}\left[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)\frac{l}{\mathbb{E}[l]}\right]^2}_{\geq 0 \text{ by Cauchy-Schwarz}}\right\}$$

Maximizing, we get

$$\max_{\alpha,\beta:\|y\|=1} \alpha\beta = \frac{1}{2\sqrt{\mathbb{E}[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)^2]\mathbb{E}\left[\left(\frac{l}{\mathbb{E}[l]}\right)^2\right]} + 2\mathbb{E}\left[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)\frac{l}{\mathbb{E}[l]}\right]} \tag{93}$$

which implies the bound

$$\sqrt{\mathbb{E}\left[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)^2\right]\mathbb{E}\left[\left(\frac{l}{\mathbb{E}[l]}\right)^2\right]} + \mathbb{E}\left[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)\frac{l}{\mathbb{E}[l]}\right] < 1 \tag{94}$$

Simplifying by using

$$\mathbb{E}\left(k \cdot \chi_{l=\mathrm{L}}\frac{(1-\mathrm{F}_k)}{f_{\mathrm{L},k}}\right) = \sum_k k \cdot (1-\mathrm{F}_k) = \frac{1}{2}\left[\mathbb{E}\left[k^2\right] - \mathbb{E}[k]\right] \tag{95}$$

and

$$\mathbb{E}\left[\left(\gamma k - \chi_{l=\mathrm{L}}\gamma \frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)\frac{l}{\mathbb{E}[l]}\right] = \frac{\gamma}{\mathbb{E}[l]}\left[\mathbb{E}[kl] - \mathbb{E}\left[\left(\chi_{l=\mathrm{L}}\frac{(1-\mathrm{F}_k)}{f_{l,k}}\right)l\right]\right] \tag{96}$$

$$= \frac{\gamma}{\mathbb{E}[l]}\left[\mathbb{E}[kl] - \mathrm{L}\sum_k (1-\mathrm{F}_k)\right] = \frac{\gamma}{\mathbb{E}[l]}\left[\mathbb{E}[kl] - \mathrm{L}(\mathbb{E}[k] - k_0)\right] \tag{97}$$

we arrive at equation 27 in the text.

$$\gamma < \frac{\mathbb{E}[l]}{\sqrt{\left[\sum_k \frac{(1-\mathrm{F}_k)^2}{f_{k,\mathrm{L}}} + \mathbb{E}[k]\right]\mathbb{E}\left[(l)^2\right]} + \left[\mathbb{E}[kl] - \mathrm{L}(\mathbb{E}[k] - k_0)\right]} \tag{98}$$

□

Lemma 8. *The virtual value* J *is concave in* x *for all* $\overline{x}, \zeta > 0$.

Proof of Lemma: By direct computation, we have

$$\frac{\partial^2}{\partial x^2} \mathrm{J} = u_{xx} - \chi_{l=\mathrm{L}} \left\{ \frac{1 - \mathrm{F}_k}{f_{kl}} \left[u_{xx}(x, k+1, \overline{x}) - u_{xx}(x, k, \overline{x}) \right] \right\} + \zeta v_{xx} < 0 \tag{99}$$

since $u_{xx} < 0$, $v_{xx} \leq 0$ and $u_{xxk} \geq 0$. □

Proof of Lemma 3: Since we have a finite type space, $\mathrm{H}(x, q)$ is piece-wise linear in q. Therefore, the convexification induces a partition of q which is a coarsening of the partition induced by the map $q(k, l)$. Therefore, we obtain an induced partition of types, which we denote by $\mathcal{B}$ in both spaces by abuse of notation. For $q \in \mathrm{B}$, we have

$$\mathrm{G}_q(x, q) = \int_{r \in \mathrm{B}} \mathrm{H}_q(x, r) \mathrm{d}r = \int_{r \in \mathrm{B}} \mathrm{J}_x(x, k(r), l(r), \overline{x}, \zeta) \mathrm{d}r \tag{100}$$

Since the maximization of $\bar{\mathrm{J}}$ is pointwise, we can impose the nonnegativity constraint pointwise as well. Since $x_{k,l}$ is constant on B, at an interior solution it solves

$$\begin{aligned} 0 = \frac{1}{|\mathrm{B}|} \sum_{k,l \in \mathrm{B}} \bar{\mathrm{J}}(x, k, l, \overline{x}, \zeta) &= \frac{1}{|\mathrm{B}|} \sum_{k,l \in \mathrm{B}} \mathrm{G}_q(x, q(k,l)) = \int_{r \in \mathrm{B}} \mathrm{J}_x(x, k(r), l(r), \overline{x}, \zeta) \mathrm{d}r \\ &= \sum_{k,l \in \mathrm{B}} f_{k,l} \left[u_x(x_{k,l}, k, \overline{x}) - \chi_{l=\mathrm{L}} \left\{ \frac{1 - \mathrm{F}_k}{f_{kl}} \left[u_x(x_{k,l}, k+1, \overline{x}) - u_x(x_{k,\mathrm{L}}, k, \overline{x}) \right] \right\} + \zeta v_x(x_{k,l}, l) \right] \end{aligned} \tag{101}$$

Finally, we divide by $\sum_{k,l \in \mathrm{B}} f_{k,l}$ and rearrange

$$\begin{aligned} &\frac{1}{\sum_{k,l \in \mathrm{B}} f_{k,l}} \sum_{k,l \in \mathrm{B}} f_{k,l} \left\{ u_x(x_{k,l}, k, \overline{x}) - \chi_{l=\mathrm{L}} \left\{ \frac{1 - \mathrm{F}_k}{f_{kl}} \left[u_x(x_{k,l}, k+1, \overline{x}) - u_x(x_{k,\mathrm{L}}, k, \overline{x}) \right] \right\} \right\} = \\ &u_x(x_\mathrm{B}, k_\mathrm{B}, \overline{x}) + \frac{1}{\sum_{k,l \in \mathrm{B}} f_{k,l}} \left\{ \begin{array}{l} \sum_{k,l \in \mathrm{B}} f_{k,l} \left[u_x(x_{k,l}, k, \overline{x}) - u_x(x_\mathrm{B}, k_\mathrm{B}, \overline{x}) \right] \\ -\sum_{k,\mathrm{L} \in \mathrm{B}} (1 - \mathrm{F}_k) \left[u_x(x_{k,l}, k+1, \overline{x}) - u_x(x_{k,\mathrm{L}}, k, \overline{x}) \right] \end{array} \right\} = \\ &u_x(k_\mathrm{B}, x_\mathrm{B}, \overline{x}) + \frac{1}{\sum_{k,l \in \mathrm{B}} f_{k,l}} \left\{ \begin{array}{l} \sum_{k,l \in \mathrm{B}} f_{k,l} \left[u_x(x_{k,l}, k, \overline{x}) - u_x(x_\mathrm{B}, k_\mathrm{B}, \overline{x}) \right] \\ -\left(1 - \mathrm{F}_{k_\mathrm{B}}\right) \left[u_x(x_{k,l}, k_\mathrm{B}+1, \overline{x}) - u_x(x_{k,\mathrm{L}}, k_\mathrm{B}, \overline{x}) \right] \\ -\sum_{k,\mathrm{L} \in \mathrm{B} \setminus \{k_\mathrm{B}, \mathrm{L}\}} (1 - \mathrm{F}_k) \left[u_x(x_{k,l}, k+1, \overline{x}) - u_x(x_{k,\mathrm{L}}, k, \overline{x}) \right] \end{array} \right\} = \\ &u_x(k_\mathrm{B}, x_\mathrm{B}, \overline{x}) + \frac{1}{\sum_{k,l \in \mathrm{B}} f_{k,l}} \left\{ \begin{array}{l} \sum_{k,l \in \mathrm{B}: k,l > k_\mathrm{B}, \mathrm{L}} f_{k,l} \left[u_x(x_{k,l}, k, \overline{x}) - u_x(x_\mathrm{B}, k_\mathrm{B}, \overline{x}) \right] \\ -\sum_{k,l > k_\mathrm{B}, \mathrm{L}} f_{k,l} \left[u_x(x_{k,l}, k_\mathrm{B}+1, \overline{x}) - u_x(x_{k,\mathrm{L}}, k_\mathrm{B}, \overline{x}) \right] \\ -\sum_{k,\mathrm{L} \in \mathrm{B} \setminus \{k_\mathrm{B}, \mathrm{L}\}} (1 - \mathrm{F}_k) \left[u_x(x_{k,l}, k+1, \overline{x}) - u_x(x_{k,\mathrm{L}}, k, \overline{x}) \right] \end{array} \right\} = \\ &u_x(k_\mathrm{B}, x_\mathrm{B}, \overline{x}) + \frac{1}{\sum_{k,l \in \mathrm{B}} f_{k,l}} \left\{ \begin{array}{l} \sum_{k,l \in \mathrm{B}: k,l > k_\mathrm{B}+1, \mathrm{L}} f_{k,l} \left[u_x(x_{k,l}, k, \overline{x}) - u_x(x_\mathrm{B}, k_\mathrm{B}+1, \overline{x}) \right] \\ -\sum_{k,\mathrm{L} \in \mathrm{B} \setminus \{k_\mathrm{B}, \mathrm{L}\}} (1 - \mathrm{F}_k) \left[u_x(x_{k,l}, k+1, \overline{x}) - u_x(x_{k,\mathrm{L}}, k, \overline{x}) \right] \end{array} \right\} \underset{\text{(induction)}}{=} \\ &u_x(k_\mathrm{B}, x_\mathrm{B}, \overline{x}) - \frac{\sum_{k,l > \mathrm{B}} f_{k,l}}{\sum_{k,l \in \mathrm{B}} f_{k,l}} \left[u_x\left(k^\mathrm{B}+1, x_\mathrm{B}, \overline{x}\right) - u_x\left(k^\mathrm{B}, x_\mathrm{B}, \overline{x}\right) \right] \end{aligned} \tag{102}$$

which establishes the first order condition in the text. By Toikka (2011), the solution to the FOC solves the monotonicity constrained problem. □

Proof of Proposition 6: Part 1: By single crossing, $\mathrm{J}_x(x, \mathrm{K}, \mathrm{L}, \overline{x}, \zeta)$ dominates all other types and hence there is no bunching and no distortion (in the weak sense) at the top.

Part 2: Consider a nontrivial cell that does not contain a type k, L. Then, this cell is contained in one $k \times \mathcal{L}$ slice. Within such a slice, except at the switching type, however,

$$\mathrm{J}_x = u_x\left(x_{k,l}, k, \overline{x}\right) + \zeta v_x\left(x_{k,l}, l\right) \tag{103}$$

is increasing in l and decreasing in x, so no ironing is required, a contradiction.

Part 3: Notice that $u_x(x, k, \overline{x}) < 0 \iff x > x_k^{\mathrm{D}}(\overline{x})$. Suppose towards a contradiction that $x_k^{\mathrm{D}}(\overline{x}) \geq x_{k,l} > x_{k,l-1}$. Then, we know that the lexicographic-monotonicity constraint is slack at $x_{k,l-1}$ and hence

$$\frac{\partial \mathcal{L}}{\partial x_{k,l-1}} = f_{k,l-1} \mathrm{J}_x(x_{k,l-1}, k, l-1, \overline{x}, \zeta) > 0 \tag{104}$$

since $\check{x}_{k,l-1} > x_k^{\mathrm{D}}(\overline{x})$. This contradicts the optimality of $x_{k,l-1}$ given $\overline{x}, \zeta$. □

Proof of Theorem 2: Let $\overline{x}^{\mathrm{T}}, \zeta^{\mathrm{T}}$ be a fixed point of Γ and denote $x^{\mathrm{T}} = x^{\mathrm{FS}}(\overline{x}^{\mathrm{T}}, \zeta^{\mathrm{T}})$. Then, since x^{T} solves $\max_{x \in \mathrm{M}} \mathcal{L}(x, \overline{x}^{\mathrm{T}}, \zeta^{\mathrm{T}})$ and this problem is concave-convex (since the objective is concave by Lemma 8 and M is convex), we have $\nabla_x \mathcal{L}(x^{\mathrm{T}}, \overline{x}^{\mathrm{T}}, \zeta^{\mathrm{T}}) \in \mathrm{N_M}(x^{\mathrm{T}})$ where $\mathrm{N_M}(x)$ denotes the outward normal cone to M at x.[19]

Consider now the plugin problem and denote its objective by $\hat{\mathcal{L}}$. By assumption, this is a concave-convex problem and therefore a vector x solves this problem if and only if $\nabla_x \hat{\mathcal{L}}(x) \in \mathrm{N_M}(x)$. We have

$$\begin{aligned}
\nabla_x \hat{\mathcal{L}}(x^{\mathrm{T}}) &= f \odot \Big(\nabla_x \big(u(x^{\mathrm{T}}, \overline{x}^{\mathrm{T}}) + \Delta(x^{\mathrm{T}}, \overline{x}^{\mathrm{T}})\big) + \mathbb{E}[u_{\overline{x}}(x^{\mathrm{T}}, \overline{x}^{\mathrm{T}}) + \Delta_x(x^{\mathrm{T}}, \overline{x}^{\mathrm{T}}) + \kappa'(\overline{x})] \nabla_x v(x^{\mathrm{T}}, \overline{x}^{\mathrm{T}})\Big) && (105)\\
&= f \odot \Big(\nabla_x \big(u(x^{\mathrm{T}}, \overline{x}^{\mathrm{T}}) + \Delta(x^{\mathrm{T}}, \overline{x}^{\mathrm{T}})\big) + \zeta^{\mathrm{T}} \nabla_x v(x^{\mathrm{T}}, \overline{x}^{\mathrm{T}})\Big) && (106)\\
&= \nabla_x \mathcal{L}(x^{\mathrm{T}}, \overline{x}^{\mathrm{T}}, \zeta^{\mathrm{T}}) \in \mathrm{N_M}(x^{\mathrm{T}}) && (107)
\end{aligned}$$

Hence, the solution to the fixed point problem induces a solution to the plugin problem. By uniqueness of this solution, it is unique.

Conversely, let x^{FS} be the solution to $\nabla_x \hat{\mathcal{L}}(x^{\mathrm{FS}}) \in \mathrm{N_M}(x^{\mathrm{FS}})$ and $\overline{x}^{\mathrm{FS}} = \sum f_{k,l} v(x_{k,l}^{\mathrm{FS}}, l)$ and

$$\zeta^{\mathrm{FS}} = \sum f_{k,l} \left(u_{\overline{x}}(x_{k,l}^{\mathrm{FS}}, k, \overline{x}^{\mathrm{FS}}) - \chi_{l=\mathrm{L}} \frac{1 - \mathrm{F}_k}{f_{kl}} \int_k^{k+1} u_{k\overline{x}}\left(x_{k,l}^{\mathrm{FS}}, j, \overline{x}^{\mathrm{FS}}\right) \mathrm{d}j\right) + \kappa'(\overline{x}^{\mathrm{FS}}) \tag{108}$$

Then, following the above chain of equalities backwards it is easy to see that $\nabla_x \mathcal{L}(x^{\mathrm{FS}}, \overline{x}^{\mathrm{FS}}, \zeta^{\mathrm{FS}}) \in \mathrm{N_M}(x^{\mathrm{FS}})$. Hence, $x^{\mathrm{FS}}(\overline{x}^{\mathrm{FS}}, \zeta^{\mathrm{FS}}) = x^{\mathrm{FS}}$ and $\Gamma(\overline{x}^{\mathrm{FS}}, \zeta^{\mathrm{FS}}) = (\overline{x}^{\mathrm{FS}}, \zeta^{\mathrm{FS}})$ by construction. □

Proofs for Section 5 (Observable Influence)

Proof of Proposition 7: We can rewrite the problem in utility space, noting that $u_{k,l} = (1 + \gamma\overline{x}k)x_{k,l} - \frac{1}{2}x_{k,l}^2 - t_{k,l}$ or equivalently $t_{k,l} = (1 + \gamma\overline{x}k)x_{k,l} - \frac{1}{2}x_{k,l}^2 - u_{k,l}$. Then, (P) is equivalent to $u_{k_0,l} = 0$ where equality follows from by the usual argument. IC is equivalent to $u_{k,l} \geq u_{k',l} + \gamma\overline{x}(k - k')x_{k',l}$. Again, by the usual arguments, local downward IC and monotonicity are sufficient and IC are binding, hence $u_{k,l} = \gamma\overline{x}\sum_{j=k_0}^{k-1} x_{j,l}$. Plugging this into the objective and applying summation by parts to the double sum, we arrive at the Proposition. □

[19] Formally, $z \in \mathrm{N_M}(x)$ if $< z, m - x > \leq 0$ for all $m \in \mathrm{M}$.

Proof of Proposition 8: Note that $\zeta = \gamma \sum f_{k,l} \left(k - \frac{\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)}{f_{kl}}\right) x_{k,l}(\zeta, \overline{x})$. Solving further yields

$$\overline{x} = \sum f_{k,l} \frac{l}{\mathbb{E}[l]} \left(1 + \gamma \overline{x} \left(k - \frac{\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)}{f_{kl}}\right) + \frac{l}{\mathbb{E}[l]} \zeta\right) \tag{109}$$

$$= 1 + \gamma \overline{x} \frac{\mathbb{E}[kl]}{\mathbb{E}[l]} - \gamma \overline{x} \frac{\mathbb{E}[kl - k_0 l]}{\mathbb{E}[l]} + \zeta \frac{\mathbb{E}[l^2]}{\mathbb{E}[l]^2} \tag{110}$$

$$= 1 + \gamma \overline{x} k_0 + \zeta \frac{\mathbb{E}[l^2]}{\mathbb{E}[l]^2} \tag{111}$$

where we use that $\sum_{k,l} l\,(\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)) = \mathbb{E}[kl - k_0 l]$.

$$\zeta = \gamma \sum f_{k,l} \left(k - \frac{\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)}{f_{kl}}\right)\left(1 + \gamma \overline{x} \left(k - \frac{\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)}{f_{kl}}\right) + \frac{l}{\mathbb{E}[l]} \zeta\right) \tag{112}$$

$$= \gamma \left(k_0 + \gamma k_0 \zeta + \gamma \overline{x} \left(\mathbb{E}[k] - k_0^2 - 2k_0 + \mathbb{E}[\left(\frac{\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)}{f_{kl}}\right)^2]\right)\right) \tag{113}$$

where we used[20]

$$\sum f_{k,l} \left(k - \frac{\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)}{f_{kl}}\right)^2 = \mathbb{E}[k^2] - 2 \sum k\,(\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)) + \mathbb{E}[\left(\frac{\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)}{f_{kl}}\right)^2] \tag{117}$$

$$= \mathbb{E}[k] - k_0^2 - 2k_0 + \mathbb{E}[\left(\frac{\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)}{f_{kl}}\right)^2]$$

Solving the system defined by (111) and (113) further, we obtain

$$\frac{\zeta}{\gamma \overline{x}} = \Xi = \frac{k_0 + \gamma \left(\mathbb{E}[k] - 2k_0(1 + k_0) + \mathbb{E}\left[\left(\frac{\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)}{f_{kl}}\right)^2\right]\right)}{1 - \left(1 - \frac{\mathbb{E}[l^2]}{\mathbb{E}[l]^2}\right)\gamma k_0} \tag{118}$$

Therefore, $x_{\cdot,l}$ is increasing in l if

$$\frac{l}{\mathbb{E}[l]} \zeta - \gamma \overline{x} \frac{\mathrm{F}_{\mathrm{K}}^l - \mathrm{F}_k^l}{f_{kl}} \propto \frac{l}{\mathbb{E}[l]} \Xi - \frac{\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)}{f_{kl}} = \frac{l}{\mathbb{E}[l]} \Xi - \frac{1 - \mathrm{F}(k|l)}{f(k|l)} \tag{119}$$

is increasing in l. Furthermore, rents are simply $\gamma \overline{x} \sum_{j=0}^{k-1} x_{j,l}$, so the fact that $x_{\cdot,l}$ is increasing in l is sufficient for this to hold. A necessary condition has all cumulative sums increasing.

[20] Solving further,

$$\sum_k (k - k_0) \cdot (1 - \mathrm{F}_k) = \frac{1}{2} \left[\mathbb{E}\left[(k - k_0)^2\right] - \mathbb{E}[k - k_0]\right] \tag{114}$$

$$\sum_k k \cdot (1 - \mathrm{F}_k) = \frac{1}{2} \left[\mathbb{E}\left[k^2\right] - \mathbb{E}[k]\right] + \frac{1}{2} k_0^2 + k_0 \tag{115}$$

and

$$\sum k\,(\mathrm{F}_l(\mathrm{K}) - \mathrm{F}_l(k)) = \frac{1}{2} \left[\mathbb{E}\left[k^2\right] - \mathbb{E}[k]\right] + \frac{1}{2} k_0^2 + k_0. \tag{116}$$

Whenever $l_1 > l_2$ for every k,

$$\sum_{j=k_0}^{k}\left(\frac{l_1}{\mathbb{E}[l]}\Xi - \frac{1-\mathrm{F}(j|l_1)}{f(j|l_1)}\right) > \sum_{j=k_0}^{k}\left(\frac{l_2}{\mathbb{E}[l]}\Xi - \frac{1-\mathrm{F}(j|l_2)}{f(j|l_2)}\right) \tag{120}$$

$$= k\frac{l_1-l_2}{\mathbb{E}[l]}\Xi + \sum_{j=k_0}^{k}\frac{1-\mathrm{F}(j|l_2)}{f(j|l_2)} - \frac{1-\mathrm{F}(j|l_1)}{f(j|l_1)}$$

In particular, if $k_0 = 0$, we have $\Xi = \gamma\left(\mathbb{E}[k] + \mathbb{E}[\left(\frac{\mathrm{F}_l(\mathrm{K})-\mathrm{F}_l(k)}{f_{kl}}\right)^2]\right)$.

For expected rents from influence, we require instead that

$$\sum_{k} f(k|l)\left(\frac{l}{\mathbb{E}[l]}\Xi - \frac{1-\mathrm{F}(k|l)}{f(k|l)}\right) = \frac{l}{\mathbb{E}[l]}\Xi - \mathbb{E}[k-k_0|l] \tag{121}$$

is increasing in l. □

B Examples

B.1 Network Good: Decentralized vs Efficient

For the decentralized case, note that $x_k = 1+\gamma\overline{x}^{\mathrm{D}}k$ and hence

$$\overline{x}^{\mathrm{D}} = \frac{1}{1-\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}}. \tag{122}$$

For the first best, plugging $x^{\star}_{k,l}$ into the definition of $\overline{x}^{\star}$ and $\zeta^{\star}$, we arrive at

$$\zeta^{\star} = \sum_{k,l} f_{kl}\gamma k\left(1+\gamma\overline{x}^{\star}k + \frac{l}{\mathbb{E}[l]}\zeta^{\star}\right) = \gamma\mathbb{E}[k] + \gamma^2\mathbb{E}[k^2]\overline{x}^{\star} + \zeta^{\star}\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]} \tag{123}$$

$$\overline{x}^{\star} = \sum_{k,l} f_{kl}\frac{l}{\mathbb{E}[l]}\left(1+\gamma\overline{x}^{\star}k + \frac{l}{\mathbb{E}[l]}\zeta^{\star}\right) = 1+\gamma\overline{x}^{\star}\frac{\mathbb{E}[kl]}{\mathbb{E}[l]} + \zeta^{\star}\frac{\mathbb{E}[l^2]}{\mathbb{E}[l]^2} \tag{124}$$

Solving this 2x2 linear system gives

$$\overline{x}^{\star} = \frac{1+\frac{\mathbb{E}[l^2]}{\mathbb{E}[l]^2\left[1-\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}\right]}\gamma\mathbb{E}[k]}{1-\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]} - \frac{\gamma^2\mathbb{E}[l^2]\mathbb{E}[k^2]}{\mathbb{E}[l]^2\left[1-\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}\right]}}. \tag{125}$$

where (18) ensures that the denominator is positive.

B.2 Network Good: Efficient vs Full Screening

To see that $\overline{x}^{\mathrm{FS}} < \overline{x}^{\star}$ when the solution involves no ironing, consider

$$\overline{x}^{\mathrm{FS}} = \sum f_{k,l}\frac{l}{\mathbb{E}[l]}x_{k,l} \tag{126}$$

$$= 1+\gamma\overline{x}^{\mathrm{FS}}\left(\frac{\mathbb{E}[kl]}{\mathbb{E}[l]} - \sum f_{k,l}\frac{l}{\mathbb{E}[l]}\chi_{l=\mathrm{L}}\frac{1-\mathrm{F}_k}{f_{kl}}\right) + \zeta^{\mathrm{FS}}\frac{\mathbb{E}\left[l^2\right]}{\mathbb{E}[l]}$$

$$\leq 1+\gamma \overline{x}^{\mathrm{FS}}\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}+\zeta^{\mathrm{FS}}\frac{\mathbb{E}[l^2]}{\mathbb{E}[l]^2}$$

and

$$\zeta^{\mathrm{FS}}=\sum f_{k,l}\left(\gamma k x_{k,l}-\chi_{l=\mathrm{L}}\frac{1-\mathrm{F}_k}{f_{kl}}\gamma x_{k,l}\right) \tag{127}$$

$$\leq \sum f_{k,l}\gamma k x_{k,l}=\gamma\mathbb{E}[k]+\gamma^2\mathbb{E}[k^2]\overline{x}^{\mathrm{FS}}+\zeta^{\mathrm{FS}}\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]} \tag{128}$$

Recall that $\overline{x}^\star,\zeta^\star$ solve this system with equality. Hence, we can write $\overline{x}^{\mathrm{FS}}=\overline{x}^\star+\epsilon_x$, $\zeta^{\mathrm{FS}}=\zeta^\star+\epsilon_\zeta$ where

$$\begin{pmatrix}\epsilon_x\\ \epsilon_\zeta\end{pmatrix}\leq\begin{pmatrix}\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]} & \frac{\mathbb{E}[l^2]}{\mathbb{E}[l]^2}\\ \gamma^2\mathbb{E}[k^2] & \gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}\end{pmatrix}\begin{pmatrix}\epsilon_x\\ \epsilon_\zeta\end{pmatrix} \tag{129}$$

Suppose $\epsilon_x>0$, indeed, let WLOG $\epsilon_x=1$. Then, we have

$$\begin{pmatrix}1-\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}-\frac{\mathbb{E}[l^2]}{\mathbb{E}[l]^2}\epsilon_\zeta\\ \left(1-\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}\right)\epsilon_\zeta-\gamma^2\mathbb{E}[k^2]\end{pmatrix}\leq 0 \tag{130}$$

Hence, $\epsilon_\zeta\geq\frac{\left(1-\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}\right)}{\frac{\mathbb{E}[l^2]}{\mathbb{E}[l]^2}}$ and therefore

$$\left(1-\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}\right)\epsilon_\zeta-\gamma^2\mathbb{E}[k^2]\geq\frac{\left(1-\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}\right)^2}{\frac{\mathbb{E}[l^2]}{\mathbb{E}[l]^2}}-\gamma^2\mathbb{E}[k^2]\propto 1-\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}-\frac{\gamma^2\mathbb{E}[k^2]\mathbb{E}[l^2]}{\left(1-\gamma\frac{\mathbb{E}[kl]}{\mathbb{E}[l]}\right)\mathbb{E}[l]^2}>0 \tag{131}$$

a contradiction. Therefore, $\epsilon_x<0$ and we have $\overline{x}^{\mathrm{FS}}<\overline{x}^\star$.

B.3 Network Good: 2x2

Consider the setting of Section (2.2.2) with $\mathcal{K}=\mathcal{L}=\{0,1\}$.

B.3.1 Benchmark Allocations

The decentralized solution has

$$x_0^{\mathrm{D}}=1,\qquad x_1^{\mathrm{D}}=1+\gamma\overline{x} \tag{132}$$

with

$$\overline{x}=\frac{f_{0,1}+f_{1,1}(1+\gamma\overline{x})}{f_{0,1}+f_{1,1}}\implies\overline{x}=\frac{f_{0,1}+f_{1,1}}{(f_{0,1}+f_{1,1}(1-\gamma))} \tag{133}$$

A decentralized equilibrium exists for every $\gamma<1+\frac{f_{0,1}}{f_{1,1}}$.

Computing the efficient allocation is straightforward but yields unwieldy expressions. However, it holds

$$x_{0,1}^\star-x_{1,0}^\star=\frac{\gamma(f_{1,0}-f_{0,1})}{1-f_{0,0}-(1-\gamma)f_{1,0}} \tag{134}$$

Therefore, the first best is implementable if and only if $f_{0,1}\geq f_{1,0}$.

B.3.2 Full Screening

We fully solve for the optimal screening contract in a Mathematica notebook, available upon request. There, we go through all possible bunching scenarios. Only the following scenarios 1, 2, and 5 are possibly optimal.[21]

$$0 < x_{0,0} < x_{0,1} < x_{1,0} < x_{1,1} \tag{SC1}$$

$$0 < x_{0,0} = x_{0,1} < x_{1,0} < x_{1,1} \tag{SC2}$$

$$0 = x_{0,0} = x_{0,1} < x_{1,0} < x_{1,1} \tag{SC5}$$

It can be shown that the first order solution $\check{\boldsymbol{x}}$ is $>_{\mathrm{L}^+}$-monotonic if and only if $\gamma > 1$.[22]

B.3.3 Observable Influence

We have

$$x_{0,0}^{\mathrm{OI}} = 1 - \frac{f_{1,0}}{f_{0,0}}\gamma\overline{x}, \qquad x_{0,1}^{\mathrm{OI}} = 1 - \frac{f_{1,1}}{f_{0,1}}\gamma\overline{x} + \zeta\frac{1}{f_{0,1}+f_{1,1}} \tag{136}$$

$$x_{1,0}^{\mathrm{OI}} = 1 + \gamma\overline{x}, \qquad x_{1,1}^{\mathrm{OI}} = 1 + \gamma\overline{x} + \zeta\frac{1}{f_{0,1}+f_{1,1}} \tag{137}$$

The aggregate variables are

$$\zeta^{\mathrm{OI}} = \gamma\sum f_{k,l}\left(k - \frac{\mathrm{F}_l\left(\mathrm{K}\right) - \mathrm{F}_l\left(k\right)}{f_{kl}}\right)x_{k,l}^{\mathrm{OI}} \tag{138}$$

$$= \gamma\overline{x}^{\mathrm{OI}}\gamma\left[f_{1,0} + f_{1,1} + f_{0,0}\left(\frac{f_{1,0}}{f_{0,0}}\right)^2 + f_{0,1}\left(\frac{f_{1,1}}{f_{0,1}}\right)^2\right] = \gamma\overline{x}^{\mathrm{OI}}\Xi \tag{139}$$

$$\overline{x}^{\mathrm{OI}} = 1 + \zeta^{\mathrm{OI}}\frac{1}{f_{0,1}+f_{1,1}} \tag{140}$$

which we solve for, yielding

$$\overline{x}^{\mathrm{OI}} = \frac{1}{1-\gamma\Xi\frac{1}{f_{0,1}+f_{1,1}}}, \qquad \zeta^{\mathrm{OI}} = \frac{\gamma\Xi}{1-\gamma\Xi\frac{1}{f_{0,1}+f_{1,1}}} > 0. \tag{141}$$

By direct substitution of $f_{i,j} = \frac{1}{4} + (-1)^{\chi_{i\neq j}}\rho$, we see that there are rents from influence if $x_{0,0}^{\mathrm{OI}} < x_{0,1}^{\mathrm{OI}}$ which is the case if $\gamma - 8\rho + 16\gamma\rho^2 < 0$ or, equivalently for $\rho \in [-.25, .25]$, if

[21] The fact that always $x_{0,1} < x_{1,0} < x_{1,1}$ – in other words that only the information rent distortion induces bunching – is a consequence of this particular example in which $\zeta \cdot \mathrm{L}$ is proportional to $\gamma\overline{x}$. If we had a generic network sale we could make the $\mathrm{L} \to l_0$ jump large relative to the expectation and make this relevant.

Because of this bunching structure, the value of x_1 tells us everything: If = 1 we are in strictly monotonic allocation, else the $k = 0$ slice is bunched (sometimes with the 0 bound binding).

[22] Clearly, strict concavity of the first-best problem implies existence of the second best solution. The second-best can exist even if the first-best does not but we have

$$\gamma < \frac{f_{0,1}}{f_{0,1}f_{1,0} - \sqrt{(1-f_{0,0})f_{0,1}(1-f_{0,0}-f_{1,0})(1-f_{0,0}-f_{0,1})}} \tag{135}$$

In particular, this is the case when $x_{0,1}^{\star}$ diverges: The resulting divergence of information rents can reign in the second best value. We use this region to demonstrate that consumer surplus in the screening solution can exceed the decentralized surplus.

$\rho < \bar{\rho} = \frac{1-\sqrt{1-\gamma^2}}{4\gamma}$.

B.4 Pollution 2×2

B.4.1 Benchmark Allocations

Since the aggregate externality does not affect firms' profits, we have $x_0^{\mathrm{D}} = 1, x_1^{\mathrm{D}} = 2$. The equations characterizing the first best are given by

$$0 = 1 - \frac{x_{k,l}}{k+1} - x_{k,l}\,(1-l)\,\zeta^{\star}(\overline{x}) \tag{142}$$

where

$$0 = \sum f_{k,l} u_{\overline{x}}\left(x_{k,l}^{\star}, k, \overline{x}\right) + \kappa'(\overline{x}) - \zeta^{\star}(\overline{x}). \tag{143}$$

implies $\kappa = \zeta^{\star}(\overline{x})$ and by definition $\overline{x}^{\star} = -\frac{1}{2}\left[f_{0,0}x_{0,0}^2 + f_{1,0}x_{1,0}^2\right]$. Plugging those conditions in the FOC delivers $\boldsymbol{x}^{\star} = \left[\frac{1}{1+\kappa}, 1, \frac{1}{\frac{1}{2}+\kappa}, 2\right]$.

B.4.2 Full Screening

The full screening problem reads

$$\max_{\boldsymbol{x}\in\mathrm{M}} f_{0,0}\left[x_{0,0} - \frac{1}{2}\left[1+\kappa\right]x_{0,0}^2\right] + f_{0,1}\left[x_{0,1} - \frac{1}{2}x_{0,1}^2 - \left\{\frac{1-f_{0,0}-f_{0,1}}{f_{0,1}}\left[\frac{1}{4}x_{0,1}^2\right]\right\}\right] + \tag{144}$$

$$+ f_{1,0}\left[x_{1,0} - \frac{1}{2}\left[\frac{1}{2}+\kappa\right]x_{1,0}^2\right] + f_{1,1}\left[x_{1,1} - \frac{1}{4}x_{1,1}^2\right] \tag{145}$$

The first order solution is given by $\check{\boldsymbol{x}} = \left[\frac{1}{1+\kappa}, \frac{2f_{0,1}}{1-f_{0,0}+f_{0,1}}, \frac{1}{\frac{1}{2}+\kappa}, 2\right]$. $\check{\boldsymbol{x}} \in \mathrm{M}$ if and only if

$$\frac{1}{1+\kappa} < \frac{2f_{0,1}}{1-f_{0,0}+f_{0,1}} < \frac{1}{\frac{1}{2}+\kappa} \tag{146}$$

which can be rearranged to deliver the conditions on κ given in the text.

Now suppose $\check{x}_{0,1} = 1 < \frac{1}{1+\kappa} = \check{x}_{0,0}$ so the low-productivity sectors have to be bunched at level $x_{0,0} = x_{0,1} = x_0$. We know that $x_{1,\cdot} = \check{x}_{1,\cdot}$ from Lemma 3. Dropping constant terms, the objective reads

$$f_{0,0}\left(x_0 - \frac{1}{2}\left(1+\kappa\right)x_0^2\right) + f_{0,1}\left(x_0 - \frac{1}{2}x_0^2 - \frac{1-f_{0,0}-f_{0,1}}{4f_{0,1}}x_0^2\right) \tag{147}$$

that delivers

$$x_0 = \frac{f_{0,0}+f_{0,1}}{\kappa f_{0,0} + \frac{1}{2}\left(1+f_{0,0}+f_{0,1}\right)}, \qquad x_{1,\cdot} = \check{x}_{1,\cdot} \tag{148}$$

Finally suppose $\check{x}_{0,1} = 1 > \frac{1}{\frac{1}{2}+\kappa} = \check{x}_{1,0}$ so the low-productivity green sector has to be bunched with the high-productivity dirty sector at level $x_{0,1} = x_{1,0} = x_{\mathrm{B}}$ that maximizes

$$f_{0,1}\left(x_{\mathrm{B}} - \frac{1}{2}x_{\mathrm{B}}^2 - \frac{1-f_{0,0}-f_{0,1}}{4f_{0,1}}x_{\mathrm{B}}^2\right) + f_{1,0}\left(x_{\mathrm{B}} - \frac{1}{2}\left(\frac{1}{2}+\kappa\right)x_{\mathrm{B}}^2\right) \tag{149}$$

that delivers

$$x_{\mathrm{B}} = \frac{f_{0,1} + f_{1,0}}{\frac{1}{2}(1 - f_{0,0} + f_{0,1} + f_{1,0}) + f_{1,0}\kappa} \tag{150}$$

B.4.3 Observable Influence

Notice that in this case we are effectively solving two independent screening problems since the coupling through $\overline{x}$ is missing as it does not enter utility. For $l = 1$ (green sector) we have a standard screening contract with two types that we omit, while for $l = 0$ we have a screening with externality with objective

$$\max_{x_{0,0},x_{0,1}} f_{0,0}\left(x_{0,0} - \frac{1}{2}x_{0,0}^2\right) + f_{1,0}\left(x_{1,0} - \frac{1}{4}x_{1,0}^2 - \frac{1}{4}x_{0,0}^2\right) - \kappa\frac{1}{2}\left(f_{0,0}x_{0,0}^2 + f_{1,0}x_{1,0}^2\right)$$

first order conditions deliver

$$f_{1,0}\left(1 - \left(\frac{1}{2} + \kappa\right)x_{1,0}\right) = 0 \implies x_{1,0} = \frac{1}{\frac{1}{2} + \kappa} = x_{1,0}^{\star}$$

$$\frac{2f_{0,0}}{f_{1,0} + 2f_{0,0}(1+\kappa)} = x_{0,0}$$

and

$$\overline{x} = -\frac{1}{2}\left[f_{0,0}\left(\frac{2f_{0,0}}{f_{1,0} + 2f_{0,0}(1+\kappa)}\right)^2 + f_{1,0}\left(\frac{1}{\frac{1}{2} + \kappa}\right)^2\right] > \overline{x}^{\star}$$

there is suboptimally low pollution since the $(0,1)$ agent produces its efficient level while the $(0,0)$ agent is downward distorted.[23]

References

Akbarpour, Mohammad ⓡ Piotr Dworczak ⓡ Scott Duke Kominers. 2020. "Redistributive Allocation Mechanisms." SSRN 3609182.

Arellano-Bover, Jaime, and Fernando Saltiel. 2021. "Differences in On-the-Job Learning Across Firms." IZA Discussion Paper No. 14473.

Ballester, Coralio, Antoni Calvó-Armengol, and Yves Zenou. 2006. "Who's Who in Networks. Wanted: The Key Player." *Econometrica* 74 (5): 1403–1417.

Banerjee, Abhijit, Arun G Chandrasekhar, Esther Duflo, and Matthew O Jackson. 2019. "Using Gossips to Spread Information: Theory and Evidence from Two Randomized Controlled Trials." *The Review of Economic Studies* 86 (6): 2453–2490.

Bloch, Francis, and Nicolas Quérou. 2013. "Pricing in Social Networks." *Games and Economic Behavior* 80 243–261.

[23] This solution assumes that the planner cares about pollution. Otherwise, she would select $x_{1,0} = 2$ and $x_{0,0} = \frac{2f_{0,0}}{f_{1,0}+2f_{0,0}}$, inducing a level of $\overline{x}$ that can either exceed (for small κ) or fall short of $\overline{x}^{\star}$.

Candogan, Ozan, Kostas Bimpikis, and Asuman Ozdaglar. 2012. “Optimal Pricing in Networks with Externalities.” *Operations Research* 60 (4): 883–905.

Csorba, Gergely. 2008. “Screening Contracts in the Presence of Positive Network Effects.” *International Journal of Industrial Organization* 26 (1): 213–226.

Fainmesser, Itay P., and Andrea Galeotti. 2016. “Pricing Network Effects.” *The Review of Economic Studies* 83 (1): 165–198.

Fainmesser, Itay P., and Andrea Galeotti. 2020. “Pricing Network Effects: Competition.” *American Economic Journal: Microeconomics* 12 (3): 1–32.

Farrell, Joseph, and Garth Saloner. 1985. “Standardization, Compatibility, and Innovation.” *The RAND Journal of Economics* 16 (1): 70–83.

Galeotti, Andrea, Benjamin Golub, and Sanjeev Goyal. 2020. “Targeting Interventions in Networks.” *Econometrica* 88 (6): 2445–2471.

Galeotti, Andrea, and Sanjeev Goyal. 2009. “Influencing the Influencers: A Theory of Strategic Diffusion.” *The RAND Journal of Economics* 40 (3): 509–532.

Gramstad, Arne Rogde. 2016. “Nonlinear Pricing with Local Network Effects.” SSRN 2597638.

Jadbabaie, Ali, and Ali Kakhbod. 2019. “Optimal Contracting in Networks.” *Journal of Economic Theory* 183 1094–1153.

Jehiel, Philippe, and Benny Moldovanu. 2001. “Efficient Design with Interdependent Valuations.” *Econometrica* 69 (5): 1237–1259.

Jehiel, Philippe, Benny Moldovanu, and Ennio Stacchetti. 1999. “Multidimensional Mechanism Design for Auctions with Externalities.” *Journal of Economic Theory* 85 (2): 258–293.

Katz, Michael L., and Carl Shapiro. 1985. “Network Externalities, Competition, and Compatibility.” *The American Economic Review* 75 (3): 424–440.

Lucas, Robert E. 1988. “ON THE MECHANICS OF ECONOMIC DEVELOPMENT.” *Journal of Monetary Economics* 22 3–42.

Meade, J. E. 1952. “External Economies and Diseconomies in a Competitive Situation.” *The Economic Journal* 62 (245): 54.

Myerson, Roger B. 1981. “Optimal Auction Design.” *Mathematics of Operations Research* 6 (1): 58–73.

Rochet, Jean-Charles, and Philippe Choné. 1998. “Ironing, Sweeping, and Multidimensional Screening.” *Econometrica* 66 (4): 783–826.

Rochet, Jean-Charles, and Lars A. Stole. 2003. "The Economics of Multidimensional Screening." In *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress*, edited by Hansen, Lars Peter, Mathias Dewatripont, and Stephen J. Turnovsky Volume 1. of Econometric Society Monographs 150–197, Cambridge: Cambridge University Press.

Segal, Ilya. 1999. "Contracting with Externalities." *The Quarterly Journal of Economics* 114 (2): 337–388.

Segal, Ilya. 2003. "Coordination and Discrimination in Contracting with Externalities: Divide and Conquer?" *Journal of Economic Theory* 113 (2): 147–181.

Shi, Fanqi, and Yiqing Xing. 2020. "Screening with Network Externalities." SSRN 3286427.

Sundararajan, Arun. 2004. "Nonlinear Pricing and Type-Dependent Network Effects." *Economics Letters* 83 (1): 107–113.

Toikka, Juuso. 2011. "Ironing without Control." *Journal of Economic Theory* 146 (6): 2510–2526.

Weber, Thomas A. 2006. "Screening with Externalities." SSRN 931783.

Winter, Eyal. 2004. "Incentives and Discrimination." *American Economic Review* 94 (3): 764–773.

Zhang, Yang, and Ying-Ju Chen. 2020. "Optimal Nonlinear Pricing in Social Networks Under Asymmetric Network Information." *Operations Research* 68 (3): 818–833.