4

Vague by Design: Performance Evaluation and Learning from Wages

TTW: 7 September 2023

Franz Ostrizek, Sciences Po

Introduction

- Performance evaluation is a key aspect of labor contracts and organization design
  - Many ways to evaluate: Shop floor control, consumer scores, product testing, sales,...
  - Digitization provides a growing number of possibilities
- Performance evaluations are an important source of information in the workplace
- Inform the firm about the worker's performance
  - Necessary basis of incentivizing effort via performance pay
  - Classic results show more information is better Holmström '79, Grossman&Hart '83
- Inform the worker about his performance
  - Learn about ability/match with the job
  - Confidence in his capability to succeed and sense of agency

This Paper

Dual role of performance evaluation: basis of *incentives* and agent *learning*

- How do these two aspects interact?
- How to optimally design performance evaluation when it shapes worker confidence?

This Paper

Dual role of performance evaluation: basis of *incentives* and agent *learning*

- How do these two aspects interact?
- How to optimally design performance evaluation when it shapes worker confidence?

- Two-period model of moral hazard with uncertain match-specific ability
- Principal designs evaluation of output and contingent wages
  - Fully flexible evaluation: Could observe true contribution to profits
  - Commitment to performance pay
- Learning about the agent's ability based on these evaluations

This Paper: Preview of Results

Dual role of performance evaluation: basis of ***incentives*** and agent *learning*

Dual role of performance evaluation: basis of *incentives* and agent ***learning***

- Agent learning imposes a cost on the principal

This Paper: Preview of Results

Dual role of performance evaluation: basis of *incentives* and agent *learning*

- Agent learning imposes a cost on the principal
- General case: information only about effort always used

This Paper: Preview of Results

Dual role of performance evaluation: basis of *incentives* and agent *learning*

- Agent learning imposes a cost on the principal
- General case: information only about effort always used
- 1d case: learning more costly at the top/bottom $\implies$ upper/lower censorship

This Paper: Preview of Results

Dual role of performance evaluation: basis of *incentives* and agent *learning*

- Agent learning imposes a cost on the principal
- General case: information only about effort always used
- 1d case: learning more costly at the top/bottom $\Longrightarrow$ upper/lower censorship
- Binary case: Under strong complementarities, optimal evaluation is noisy & tough

Road Map

- Literature
- General Case
- 1-dimensional Case
- 2x2 Model
    - Terminal Period: Agent Learning Imposes a Cost on the Principal
    - Initial Period: Optimal Evaluation – Noisy and Though
- Extensions

Related Literature

- Design of information
  Kolotilin et al. '22, Doval&Skreta forthcoming, ...
  and performance pay:
  Georgiadis&Szentes '20, Hoffmann et al. '21, Li&Yang '20
- Implicit incentives and information design:
  Ely&Szydlowski '20, Hörner&Lambert '21, Smolin '20
- More information can increase the cost of incentives:
  Fang&Moscarini '05, Jehiel '14, Meyer&Vickers '97, Nafziger '09

General Model

The Model

- Two time periods $t \in \{1,2\}$, common discount factor δ.
- Agent
  - risk averse with utility index $u = \sqrt{2w}$ (this talk) and reservation utility U
  - observable but nonverifiable effort $e_t \in \{0,1\}$ at cost $c \cdot e$
  - time-invariant ability $\theta \in \Theta \subset \mathbb{R}^n$, with prior μ
  - realizes output $y \in \mathbb{R}$ according to $f(\cdot|e,\theta)$, finite support (this talk)
- Principal
  - risk neutral
  - implements high effort

- At the beginning of each period, the principal commits to a contract (S, p, w) consisting of
  - a signal structure $S, p(s|y_t)$, and
  - wages w as a function the signal.

Information, Contracts and Commitment

- At the beginning of each period, the principal commits to a contract (S, p, w) consisting of
    - a signal structure $S, p(s|y_t)$, and
    - wages w as a function the signal.
- Agent observes the contract and makes participation and effort decision

- At the beginning of each period, the principal commits to a contract (S, p, w) consisting of
  - a signal structure $S, p(s|y_t)$, and
  - wages w as a function the signal.
- Agent observes the contract and makes participation and effort decision
- Output is not observed
- Principal and agent observe the signal realization, wages, and effort
- Update beliefs to $\hat{\mu}(s)$

The Contracting Problem

First Period

$$\Pi_1 = \max_{S,p,w} \mathbb{E}[y|e=1,\mu] + \int_S p(s|e=1,\mu)\Big(\Pi_2(\hat{\mu}(s)) - w(s)\Big)\,\mathrm{d}s \tag{1}$$

$$\text{s.t.} \int_S p(s|e=1,\mu)u(w(s))\,\mathrm{d}s - c \geq U \tag{P$_1$}$$

$$\int_S p(s|e=1,\mu)u(w(s))\,\mathrm{d}s - c \geq \int_S p(s|e=0,\mu)u(w(s))\,\mathrm{d}s \tag{IC$_1$}$$

Second Period

$$\Pi_2(\hat{\mu}) = \max_{S,p,w} \mathbb{E}[y|e=1,\hat{\mu}] - \int_S p(s|e=1,\hat{\mu})w(s)\,\mathrm{d}s \tag{2}$$

$$\text{s.t.} \int_S p(s|e=1,\hat{\mu})u(w(s))\,\mathrm{d}s - c \geq U \tag{P$_2$}$$

$$\int_S p(s|e=1,\hat{\mu})u(w(s))\,\mathrm{d}s - c \geq \int_S p(s|e=0,\hat{\mu})u(w(s))\,\mathrm{d}s \tag{IC$_2$}$$

The Final Period

- Pure incentive problem, no motive to shape learning
- Classic result:

Proposition

The optimal evaluation in the final period is fully informative.

$$\max_{S,p,w} \mathbb{E}[y|e=1] + \mathbb{E}_p\left(\Pi_2(\hat{\mu}(s)) - w(s)\right)$$

$$\text{s.t. } \mathbb{E}_p u(w(s)) - c \geq U \tag{P$_1$}$$

$$\mathbb{E}_p \frac{p(s|e=1) - p(s|e=0)}{p(s|e=1)} u(w(s)) \geq c \tag{IC$_1$}$$

Posterior Space

$$\max_{S,p,w} \mathbb{E}[y|e=1] + \mathbb{E}_p\Big(\Pi_2(\hat{\mu}(s)) - w(s)\Big)$$

$$\text{s.t. } \mathbb{E}_p u(w(s)) - c \geq U \tag{P$_1$}$$

$$\mathbb{E}_p \frac{p(s|e=1) - p(s|e=0)}{p(s|e=1)} u(w(s)) \geq c \tag{IC$_1$}$$

$$\begin{tikzcd} y \arrow[r, "p"] & s \arrow[r] & \Delta(Y) \arrow[dl] \arrow[dr] & \\ & \text{posterior} \in \Delta(\Theta) & & \text{score} \in \mathbb{R} \end{tikzcd}$$

Posterior Space

$$
\begin{aligned}
\max_{S,p,w}\ & \mathbb{E}[y|e=1] + \mathbb{E}_p\Big(\Pi_2(\hat{\mu}(s)) - w(s)\Big) \\
\text{s.t. } & \mathbb{E}_p u(w(s)) - c \geq U && (\mathrm{P}_1) \\
& \mathbb{E}_p \frac{p(s|e=1) - p(s|e=0)}{p(s|e=1)} u(w(s)) \geq c && (\mathrm{IC}_1)
\end{aligned}
$$

$$
\begin{tikzcd}
& \Delta\Delta(Y) \arrow[dl] \arrow[dr] & \\
\Delta\Delta(\Theta) & & \Delta(\mathbb{R})
\end{tikzcd}
$$

Posterior Space

- Rewrite the program as a choice of $\Phi \in \Delta\Delta Y$
- Let $\boldsymbol{\mu} \in [0,1]^{|Y|}$, $\boldsymbol{x} \in \mathbb{R}^{|Y|}$ denote the vector of posteriors and scores given y

$$\max_{w,\Phi} \mathbb{E}\left(\Pi_2(\boldsymbol{\mu} \cdot \phi) - w(\phi)\right)$$

$$\text{s.t. } \mathbb{E}\, u(w(\phi)) - c \geq U \tag{P$_1$}$$

$$\mathbb{E}\,(\boldsymbol{x} \cdot \phi)\, u(w(\phi)) \geq c \tag{IC$_1$}$$

$$\mathbb{E}\,\phi = f(\cdot|1, \mu_0)$$

Optimal Evaluation: Geometry

- Can learn about the optimal structure of evaluation from looking at $\boldsymbol{\mu}$ and $\boldsymbol{x}$

Optimal Evaluation: Geometry

- Can learn about the optimal structure of evaluation from looking at $\boldsymbol{\mu}$ and $\boldsymbol{x}$

Proposition (Learning about effort only is always desirable)

Suppose $\boldsymbol{\mu}(y) = \boldsymbol{\mu}(y')$ and $\boldsymbol{x}(y) \neq \boldsymbol{x}(y')$. Then y and y' are not pooled in any optimal evaluation structure.

- Sketch of proof
  - Suppose not. Split the signal. Set $u(w(s)) \pm \epsilon$.
  - No impact on Π_2, second order loss in w, first order gain in (IC)

Optimal Evaluation: Geometry

- Can learn about the optimal structure of evaluation from looking at $\boldsymbol{\mu}$ and $\boldsymbol{x}$

Proposition (Learning about effort only is always desirable)

Suppose $\boldsymbol{\mu}(y) = \boldsymbol{\mu}(y')$ and $\boldsymbol{x}(y) \neq \boldsymbol{x}(y')$. Then y and y' are not pooled in any optimal evaluation structure.

- Sketch of proof
    - Suppose not. Split the signal. Set $u(w(s)) \pm \epsilon$.
    - No impact on Π_2, second order loss in w, first order gain in (IC)
- This restricts pooling a lot, especially if Θ is small
    - Suppose $|\Theta| = n$, then there is an optimal evaluation in which the support of any signal is at most n
    - 'generically' for all optimal evaluations

$$\Delta\Delta(\Theta) \longleftrightarrow \Delta(\mathbb{R})$$

- Would be nice to ”cut out the middle man”

$$\Delta\Delta(\Theta) \longleftrightarrow \Delta(\mathbb{R})$$

- Would be nice to "cut out the middle man"
- Suppose $\Theta = \{\theta_h, \theta_l\}$

Proposition

The following are equivalent

- $x \in \mathrm{span}(\boldsymbol{\mu}, \mathbf{1})$
- *there is a bijection between posteriors and scores*
- $f = f(\cdot|0,0) + g(e,\theta)\big(f(\cdot|1,1) - f(\cdot|0,0)\big)$

- Rewrite the program as a choice of $m \in \Delta\Delta\Theta$

$$\Pi_1 = \max_{m,w} \mathbb{E}[y|e=1,\mu] + \int m(\hat{\mu})\left(\delta\Pi_2(\hat{\mu}) - w(\hat{\mu})\right)\,\mathrm{d}\hat{\mu} \tag{3}$$

$$\text{s.t.} \int u(w(\hat{\mu}))m(\hat{\mu})\,\mathrm{d}\hat{\mu} - c \geq U \tag{P$_1$}$$

$$\int \frac{g(1,\mu_0) - g(0,\mu_0)}{g(1,1) - g(1,0)} \frac{\hat{\mu} - \mu_0}{\mu_0} u(w(\hat{\mu}))m(\hat{\mu})\,\mathrm{d}\hat{\mu} \geq c \tag{IC$_1$}$$

$$m \precsim_{MPS} \boldsymbol{\mu} \tag{BP}$$

Solution Sketch

- Optimal evaluation depends on shape of value, in particular third derivative
- With $u = \sqrt{2w}$, only continuation value determines this shape, and $\Pi_2(\mu) = -\frac{1}{2}\frac{1}{\mathbb{V}[x]}$

Theorem

There exists an optimal evaluation structure with upper (lower) censorship if $\frac{\partial^3}{\partial \mu^3}\frac{1}{\mathbb{V}[x]} > (<)0$.

- Kolotilin et al. '22, Kleiner et al. '21
- not unique
- censorship in x-space, with MLRP in y-space
- shape: Cramer-Rao bound

The 2x2 Model

Technology

- Output is
  - high or low, $y_t \in \{y_L, y_H\}$, high with probability

effort / type	$e_t = 0$	$e_t = 1$
$\theta = \theta_L$	a	$a + b$
$\theta = \theta_H$	$a + \Delta a$	$a + b + \Delta a + \Delta b$

- Effort is productive: $b \geq 0$
- Ability is productive: $\Delta a \geq 0$
- Complementarities: Δb
  Log-Supermodular: $\frac{\Delta b}{b} > \frac{\Delta a}{a}$

TECHNOLOGY

- Output is
  - high or low, $y_t \in \{y_L, y_H\}$, high with probability

effort / type	$e_t = 0$	$e_t = 1$
$\theta = \theta_L$	a	$a + b$
$\theta = \theta_H$	$a + \Delta a$	$a + b + \Delta a + \Delta b$

- Effort is productive: $b \geq 0$
- Ability is productive: $\Delta a \geq 0$
- Complementarities: Δb
  Log-Supermodular: $\frac{\Delta b}{b} > \frac{\Delta a}{a}$

- Output is
  - high or low, $y_t \in \{y_L, y_H\}$, high with probability

effort / type	$e_t = 0$	$e_t = 1$
$\theta = \theta_L$	a	$a + b$
$\theta = \theta_H$	$a + \Delta a$	$a + b + \Delta a + \Delta b$

- Effort is productive: $b \geq 0$
- Ability is productive: $\Delta a \geq 0$
- Complementarities: Δb
  Log-Supermodular: $\frac{\Delta b}{b} > \frac{\Delta a}{a}$

- Output is
  - high or low, $y_t \in \{y_L, y_H\}$, high with probability

type \ effort	$e_t = 0$	$e_t = 1$
$\theta = \theta_L$	a	$a + b$
$\theta = \theta_H$	$a + \Delta a$	$a + b + \Delta a + \Delta b$

- Effort is productive: $b \geq 0$
- Ability is productive: $\Delta a \geq 0$
- Complementarities: Δb

  Log-Supermodular: $\frac{\Delta b}{b} > \frac{\Delta a}{a}$

Technology

- Output is
  - high or low, $y_t \in \{y_L, y_H\}$, high with probability

effort / type	$e_t = 0$	$e_t = 1$
$\theta = \theta_L$	a	$a + b$
$\theta = \theta_H$	$a + \Delta a$	$a + b + \Delta a + \Delta b$

- Effort is productive: $b \geq 0$
- Ability is productive: $\Delta a \geq 0$
- Complementarities: Δb
  Log-Supermodular: $\frac{\Delta b}{b} > \frac{\Delta a}{a}$

$$\Pi_1 = \max_{m,w} \mathbb{E}[y|e=1,\mu] + \int m(\hat{\mu})\left(\delta\Pi_2(\hat{\mu}) - w(\hat{\mu})\right)\,\mathrm{d}\hat{\mu} \tag{4}$$

$$\text{s.t.} \int u(w(\hat{\mu}))m(\hat{\mu})\,\mathrm{d}\hat{\mu} - c \geq U \tag{P$_1$}$$

$$\int (b + \Delta b\mu)\frac{\hat{\mu} - \mu}{(\Delta a + \Delta b)\mu(1-\mu)} u(w(\hat{\mu}))m(\hat{\mu})\,\mathrm{d}\hat{\mu} \geq c \tag{IC$_1$}$$

$$\int \hat{\mu}\, m(\hat{\mu})\,\mathrm{d}\hat{\mu} = \mu\,; \quad \operatorname{supp}(m) \subset [\underline{\mu}, \bar{\mu}] \tag{BP}$$

The Impact of Information

- The last period induces the continuation value $\int \Pi_2(\hat{\mu}) m(\hat{\mu}) \, \mathrm{d}\hat{\mu}$
- What is the impact of more information about the agent's type?

The Impact of Information

- The last period induces the continuation value $\int \Pi_2(\hat{\mu}) m(\hat{\mu})\, \mathrm{d}\hat{\mu}$
- What is the impact of more information about the agent's type?

1. Principal can tailor the contract to the agent's ability
   - Filter out the impact of ability: contract less risky
   - Increases continuation profit
2. Agent has more information when choosing effort
   - More expensive to satisfy incentive compatibility
   - Decreases continuation profit

The Impact of Information

- The last period induces the continuation value $\int \Pi_2(\hat{\mu}) m(\hat{\mu}) \, \mathrm{d}\hat{\mu}$
- What is the impact of more information about the agent's type?

1. Principal can tailor the contract to the agent's ability
   - Filter out the impact of ability: contract less risky
   - Increases continuation profit

   scales with Δa: impact of ability

2. Agent has more information when choosing effort
   - More expensive to satisfy incentive compatibility
   - Decreases continuation profit

   scales with Δb: interaction of effort and ability

- Second-period IC:

$$u(w_H) - u(w_L) = \frac{c}{b + \hat{\mu}\Delta b}$$

The Impact of Agent Learning

- Second-period IC:

$$u(w_H) - u(w_L) = \frac{c}{b + \hat{\mu}\Delta b}$$

- Required bonus inversely proportional to a linear function of beliefs
    - Optimistic agent cheaper to motivate
    - Uncertain agent is cheaper to motivate
    - Given change in belief has a larger effect at low beliefs

Learning is Costly

Proposition

If the technology is log-supermodular, Π_2, is strictly concave in the posterior belief. Furthermore, it is more concave at low posteriors, $\Pi_2''' > 0$.

- Strong complementarity of effort and ability: Agent learning dominates
- Principal has an incentive to conceal information
- Information aversion strongest at the bottom: avoid pessimistic agents

Solving the Full Problem

- First period: Incentives and learning
  - Incentives: More informative evaluation *decreases* agency cost *this period*
  - Learning: More informative evaluation *increases* agency cost *next period*

Solving the Full Problem

- First period: Incentives and learning
    - Incentives: More informative evaluation *decreases* agency cost *this period*
    - Learning: More informative evaluation *increases* agency cost *next period*
- Information design problem, with:
    - Endogenous payoffs (wages are designed)
    - Additional constraints (participation and incentive compatibility)
- Binary state does not guarantee binary evaluation (Le Treust&Tomala, 2019)

The Optimal Contract

Theorem

The optimal contract in the first period is (essentially) unique, with a binary ($S = \{G, B\}$) and tough evaluation structure. Let $\sigma \in [0, 1)$ denote the degree of vagueness. The optimal contract consists of

$$\begin{tikzcd} y_H \arrow[r, "1-\sigma"] \arrow[dr, "\sigma"] & G \\ y_L \arrow[r] & B \end{tikzcd}$$

- a good evaluation and associated high wage, only if output was good,
- a bad evaluation and associated low wage: always after output was bad, with prob. σ after output was good.

Proof of Theorem 1: Outline

$$\mathcal{L}(w, m; \underbrace{(\lambda_P, \lambda_{IC})}_{\lambda})$$

Lagrangian of the contracting problem including (P) and (IC)

Concavification of the partially maximized Lagrangian (Georgiadis&Szentes '20)

Proof of Theorem 1: Outline

$$\mathcal{L}(w, m; \lambda)$$

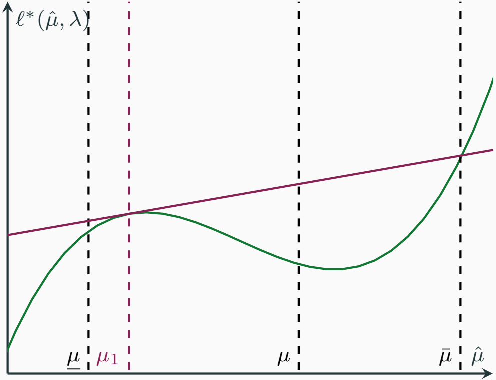
Optimal Wages given m, λ: Standard moral hazard problem $\mapsto w^*(\hat{\mu}; \lambda)$

objective is an expectation given λ: $\mathcal{L}(w^*(\hat{\mu}; \lambda), m; \lambda) = \int \ell^*(\hat{\mu}; \lambda) m(\hat{\mu}) \,\mathrm{d}\hat{\mu}$

Information Design given λ: Concavification of $\ell^* \mapsto m^*(\hat{\mu}; \lambda)$

$m^*(\cdot; \lambda)$ binary and tough for all λ

Duality: $\mapsto$ Solution exists and features of m^* hold in the optimal contract



- Unconstrained information design
- Payoff $\ell^*(\hat{\mu}; \lambda)$
- $\frac{\partial^3}{\partial \hat{\mu}^3} \ell^*(\hat{\mu}; \lambda) > 0$
    - Convex $\implies$ *m* fully informative
    - Concave-convex: For low μ, learning effect dominates $\implies$ *m* tough
- This for given λ, but $\lambda(m)$!

Binary and Noisy

- Optimal evaluation is binary
    - Motive to control learning does not add complexity to the evaluation
    - Data aggregated into a pass-fail signal
- Noisy evaluation can be optimal
    - Preserve agent’s uncertainty
- The optimal noise is asymmetric
    - Tough evaluation: Avoid unwarranted praise, embrace unwarranted reprimand
    - “Drill-sergeant mentality” is part of optimal organization design
- Prevent very low posteriors
    - Costly to motivate, change in posterior has a large effect
- Result of joint design of evaluation and wages

What if effort and ability are not strong complements?

- Intermediate case: Weak complements/substitutes
  - More information increases the continuation value
  - Fully informative evaluation
- Strong substitutes: $\frac{\Delta b}{b} + \frac{\Delta a}{1-a} < 0$
  - More information decreases the continuation value
  - Noisy and *lenient* evaluation
- Lenient Evaluation
  - Let some failures slip (but punish others harshly)
  - Lack of reprimand not very informative: Avoids complacency

plot

Extensions

Extensions →

- Principal can acquire private information
  - Principal-preferred equilibrium: outcome-equivalent to optimal contract

Extensions →

- Principal can acquire private information
  - Principal-preferred equilibrium: outcome-equivalent to optimal contract
- Effort not observable
  - Potentially 3rd, uninformative, evaluation
  - Tough and binary structure preserved conditional on informative evaluation

Extensions →

- Principal can acquire private information
    - Principal-preferred equilibrium: outcome-equivalent to optimal contract
- Effort not observable
    - Potentially 3rd, uninformative, evaluation
    - Tough and binary structure preserved conditional on informative evaluation
- Long-run commitment
    - Robust to commitment to *continuation value*, observed by the agent
    - Full-commitment difficult: belief-manipulation & belief-dependent costs of delay

Extensions →

- Principal can acquire private information
  - Principal-preferred equilibrium: outcome-equivalent to optimal contract
- Effort not observable
  - Potentially 3rd, uninformative, evaluation
  - Tough and binary structure preserved conditional on informative evaluation
- Long-run commitment
  - Robust to commitment to *continuation value*, observed by the agent
  - Full-commitment difficult: belief-manipulation & belief-dependent costs of delay
- Many periods
  - Not analytically tractable: lack of control over shape of continuation value
  - Numerically: Same structure within period; noisier evaluation early in the relationship

Conclusion

- Outcome of performance evaluation is a crucial source of information
  - about effort: Incentives
  - about the agent's ability: Confidence

Conclusion

- Outcome of performance evaluation is a crucial source of information
    - about effort: Incentives
    - about the agent's ability: Confidence
- Tension between those two channels (learning about the importance of effort)
    - As much information as possible about effort
    - As little information as possible about ability

Conclusion →

- Outcome of performance evaluation is a crucial source of information
  - about effort: Incentives
  - about the agent's ability: Confidence
- Tension between those two channels (learning about the importance of effort)
  - As much information as possible about effort
  - As little information as possible about ability
- Optimal Performance Evaluation
  - Noisy, even though wages could condition on true y
  - Strong complementarity: avoid very low posterior beliefs (tough/lower-censorship)

Outlook

- Preference across given information sources: conduct, not results!
  - Salary differences between workers: mostly driven by types, so should be concealed
- Affects task design: Harder/easier to keep agents motivated
- Career Concerns: informationally opposite forces
  - information about effort and ability inseparably intertwined
  - here: source of friction; CC: source of incentives

Thank You!

Utility Function

- Sufficient condition on utility function
- $w = u^{-1}$, "cost of utility"

Assumption 1

1. (No incentives at probability zero) $\frac{w(x)}{x} \to \infty$ as $x \to \infty$.
2. (Decreasing curvature) $w''' \leq 0$.
3. (Bounded changes in curvature) $\frac{w'''(u_H)}{w''(u_H)} \geq -A$.

Condition

- Satisfied for CRRA $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$
    - if $\gamma \leq 1/2$ and U sufficiently large.
    - Always satisfied for $\gamma = \frac{1}{2}$

back

Step 1: Optimal Wages

- Let $\mathcal{L}(m, w; (\lambda_P, \lambda_{IC}))$ denote the Lagrangian associated to the problem.
- Solving for the optimal wage given λ yields

$$w^*(\hat{\mu}, \lambda) = u'^{-1}\left(\left(\lambda_P + \lambda_{IC}\frac{b + \Delta b\mu}{(\Delta a + \Delta b)\mu(1-\mu)}(\hat{\mu} - \mu)\right)^{-1}\right)$$

- Partially maximized Lagrangian, $\sup_w \mathcal{L}(m, w; (\lambda_P, \lambda_{IC}))$, is posterior separable

$$\begin{aligned}\mathcal{L}(m, w^*(\hat{\mu}, \lambda); (\lambda_P, \lambda_{IC})) = \int \Bigg\{ & P^1_\mu Y + \delta\Pi_2(\hat{\mu}) - w^*(\hat{\mu}, \lambda) \\ & + \lambda_P \left(u(w^*(\hat{\mu}, \lambda)) - c - U\right) \\ & + \lambda_{IC}\left(\frac{b + \Delta b\mu}{(\Delta a + \Delta b)\mu(1-\mu)}(\hat{\mu} - \mu)\, u(w(\hat{\mu}, \lambda)) - c\right)\Bigg\} m(\hat{\mu})d\hat{\mu}\end{aligned}$$

back

Step 2: Information Design

- Unconstrained information design problem with payoff $\ell^*(\hat{\mu}; \lambda)$
- The objective is either convex or concave-convex since

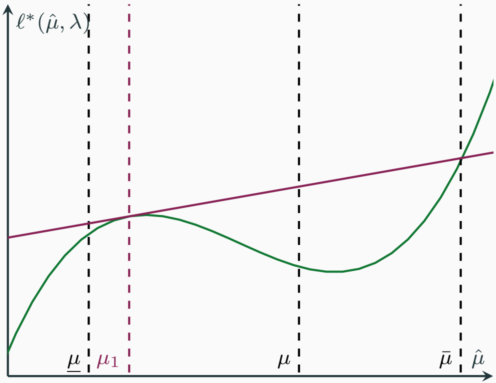
$$\frac{\partial^3}{\partial \hat{\mu}^3} \ell^*(\hat{\mu}; \lambda) = \lambda_{IC} \left(\frac{b + \Delta b \mu}{(\Delta a + \Delta b)\mu(1-\mu)} \right) \frac{\partial^2}{\partial \hat{\mu}^2} u(w(\hat{\mu}; \lambda)) + \delta \Pi_2'''(\hat{\mu}) > 0$$

Lemma

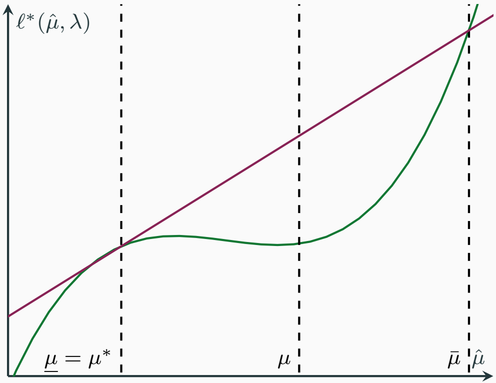
For any λ_{IC}, there exists a unique solution to the information design problem. It induces at most two posteriors: the highest feasible posterior $\bar{\mu}$ with probability $m(\bar{\mu}) \in [0, \frac{\mu - \underline{\mu}}{\bar{\mu} - \underline{\mu}}]$ and a low posterior, $\mu^* \in [\underline{\mu}, \mu]$ with $m(\mu^*) \in [\frac{\bar{\mu} - \mu}{\bar{\mu} - \underline{\mu}}, 1]$.

back

Step 2: Information Design



(a) Interior solution.



(b) Corner solution.

back

Step 3: Strong Duality

- We need to show strong duality in the general problem, i.e.

$$\inf_{\lambda \geq 0} \sup_{w,m \text{ s.t. (BP)}} \mathcal{L}(m, w; \lambda) = \sup_{w,m \text{ s.t. (BP)}} \inf_{\lambda \geq 0} \mathcal{L}(m, w; \lambda)$$

- Two steps: [1] Wages

Lemma

The wage setting problem satisfies strong duality, i.e.

$$\sup_{w} \inf_{\lambda \geq 0} \mathcal{L}(m, w; \lambda) = \inf_{\lambda \geq 0} \sup_{w} \mathcal{L}(m, w; \lambda).$$

back

Step 3: Strong Duality

- We need to show strong duality in the general problem, i.e.

$$\inf_{\lambda \geq 0} \sup_{w,m \text{ s.t. (BP)}} \mathcal{L}(m, w; \lambda) = \sup_{w,m \text{ s.t. (BP)}} \inf_{\lambda \geq 0} \mathcal{L}(m, w; \lambda)$$

- Two steps: [2] Information Design

Lemma

The information design problem satisfies strong duality, i.e.

$$\sup_{m \text{ s.t.(BP)}} \inf_{\lambda \geq 0} \int \ell^*(\hat{\mu}; \lambda) m(\hat{\mu}) \, \mathrm{d}\hat{\mu} = \inf_{\lambda \geq 0} \sup_{m \text{ s.t.(BP)}} \int \ell^*(\hat{\mu}; \lambda) m(\hat{\mu}) \, \mathrm{d}\hat{\mu}.$$

back

A Simplified Problem

- Define a simplified problem, using binary and tough evaluation

$$\max_{m_1,m_2,\mu_1,w_1,w_2} \mathbb{E}[y_1|e=1,\mu] + m_1\big(\Pi_2(\mu_1) - w_1\big) + m_2\big(\Pi_2(\bar{\mu}) - w_2\big) \tag{5}$$

$$\text{s.t.} \quad m_1 u(w_1) + m_2 u(w_2) - c \geq U \tag{P}$$

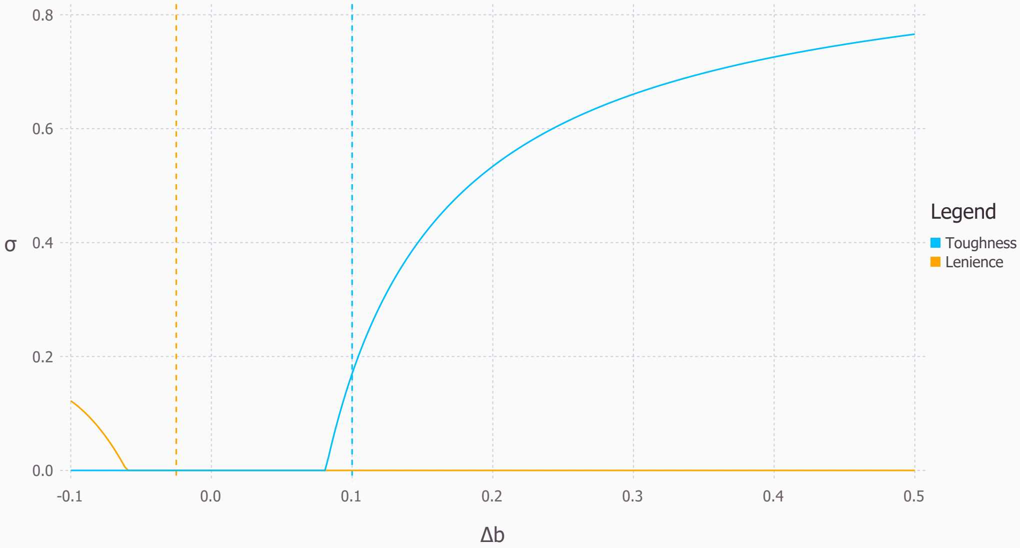
$$\frac{b + \Delta b \mu}{(\Delta a + \Delta b)\mu(1-\mu)} \sum_i m_i(\mu_i - \mu) u(w_i) \geq c \tag{IC}$$

$$m_1\mu_1 + m_2\bar{\mu} = \mu\,; \quad m_1 + m_2 = 1\,; \quad \mu_1 \geq \underline{\mu} \tag{BP}$$

back

Complements and Substitutes

BACK



Substitutes: Condition on Utility

BACK

Assumption (1*)

1. *(No incentives at probability zero)* $\frac{w(x)}{x} \to \infty$ *as* $x \to \infty$.
2. *(Increasing curvature)* $w''' \geq 0$.
3. *(Bounded changes in curvature)*

$$\frac{3(b + \mu\Delta b)\Delta b}{c(a\Delta b - b\Delta a)} \geq \frac{w'''(u_L)}{w''(u_L)},$$

where $u_L = U - \frac{a+\mu\Delta a}{b+\mu\Delta b}c$.

Private Information of the Principal

- Principal chooses
  - Evaluation structure: observed by agent, basis of performance pay and learning
  - Private evaluation: not observed by agent, basis of learning only for principal
- Joint distribution over posteriors: $m_P(\mu_P, \hat{\mu})$
  - Agent observes $m(\hat{\mu}) = \int m_P(\mu_P, \hat{\mu})\, \mathrm{d}\mu_P$
- Dynamic game with incomplete information
- Agent updates belief based on
  - First-period evaluation
  - Second-period contract offer

back

Private Information of the Principal

- Unique PBE with passive beliefs: outcome equivalent to optimal contract without private information acquisition
  - Passive beliefs: no updating based on contract offer
  - Principal preferred*
- Private information either revealed or not useful
  - If private information isn't used to adjust second period contract: irrelevant
  - Information used to adjust contract offer: revealed to agent
  - Better to also use it as a basis of performance pay

*Among equilibria that satisfy no-holdup: No rent for the agent in the second period on path.

Private Information of the Principal

- Unique PBE with passive beliefs: outcome equivalent to optimal contract without private information acquisition
  - Passive beliefs: no updating based on contract offer
  - Principal preferred*
- Private information either revealed or not useful
  - If private information isn't used to adjust second period contract: irrelevant
  - Information used to adjust contract offer: revealed to agent
  - Better to also use it as a basis of performance pay
- Remains an equilibrium when principal *has to* acquire private information
- Unique† when private information acquisition strategy observed

back

*Among equilibria that satisfy no-holdup: No rent for the agent in the second period on path.
†Under no-holdup and no-signaling-what-you-don't-know.

Unobservable Effort

- Suppose effort is not observed by the principal
- After a deviation to low effort, signal s
  - Principal has posterior
$$\hat{\mu}(s) = \mu \frac{p(s|y_L) + (a + b + \Delta a + \Delta b)\left[p(s|y_H) - p(s|y_L)\right]}{p(s|y_L) + (a + b + (\Delta a + \Delta b)\mu)\left[p(s|y_H) - p(s|y_L)\right]}$$
  - Agent interprets signal differently:
$$\mu \frac{p(s|y_L) + (a + \Delta a)\left[p(s|y_H) - p(s|y_L)\right]}{p(s|y_L) + (a + \Delta a \mu)\left[p(s|y_H) - p(s|y_L)\right]}$$
- Agent has private information about the posterior

back

Unobservable Effort

- Incentive compatibility in the second period:
  - Slack if agent more optimistic
  - Violated if agent more pessimistic
- "Belief-manipulation motive"
- Double deviations optimal
- First-period IC dynamic: Kink in the principal's objective at prior μ

$$\int \left\{ \frac{(b+\mu\Delta b)}{\mu(1-\mu)\Delta b}\,(\hat{\mu}-\mu)\,u(w(\hat{\mu})) - \left[1 - \frac{(b+\mu\Delta b)}{\mu(1-\mu)\Delta b}\,(\hat{\mu}-\mu)\right] \max\{0, c\Delta b\,\frac{\mu-\hat{\mu}}{b+\hat{\mu}\Delta b}\} \right\} m(\hat{\mu})\,\mathrm{d}\hat{\mu} \geq c$$

back

Unobservable Effort

- Under $u = \sqrt{\cdot}$ and $\Delta a = 0$: At most *three* evaluation outcomes
  - Neutral signal: Not informative about effort and ability[‡]
  - Conditional on informative evaluation: binary and tough
- Intuition: Avoid outcomes that allow generation of private information

back

[‡] In simulations: Never used.

Unobservable Effort

- Under $u = \sqrt{\cdot}$ and $\Delta a = 0$: At most *three* evaluation outcomes
  - Neutral signal: Not informative about effort and ability[‡]
  - Conditional on informative evaluation: binary and tough
- Intuition: Avoid outcomes that allow generation of private information
- More complicated with long-run contracting:
  - Principal can induce a learning motive by providing excessive bonuses in $t = 2$
  - Joint design of information and wages in *both periods*

back

[‡] In simulations: Never used.

Long Run Commitment: Continuation Value

- Principal commits to contract: (S, p, w, v)
  - a signal structure $S, p(s|y)$, realization conditional on contemporaneous output
  - wages w, and
  - **continuation value $\mathbf{v}$ as a function the signal.**

Long Run Commitment: Continuation Value

- Principal commits to contract: (S, p, w, v)
  - a signal structure $S, p(s|y)$, realization conditional on contemporaneous output
  - wages w, and
  - **continuation value $\mathbf{v}$ as a function the signal.**
- Assume $u(x) = 2\sqrt{x}$
  - Theorem 1 goes through, delaying *payments* does not affect the mechanism
  - Optimal evaluation: binary and weakly tough

back

Long Run Commitment: Full Commitment

- Principal commits to long-run contract: $(S_1 \times S_2, p, w)$
  - a signal space $S_1 \times S_2$, p progressively measurable wrt y_t,
  - and wages w, progressively measurable wrt s_t.
- Difficult:
  - Agent acquires private info after shirking (effort unobservable to the contract), and
  - the principal can commit to excess bonuses in $t = 2$ (to induce a learning motive).

$\implies$ Characterizing the optimum requires joint design in both periods.

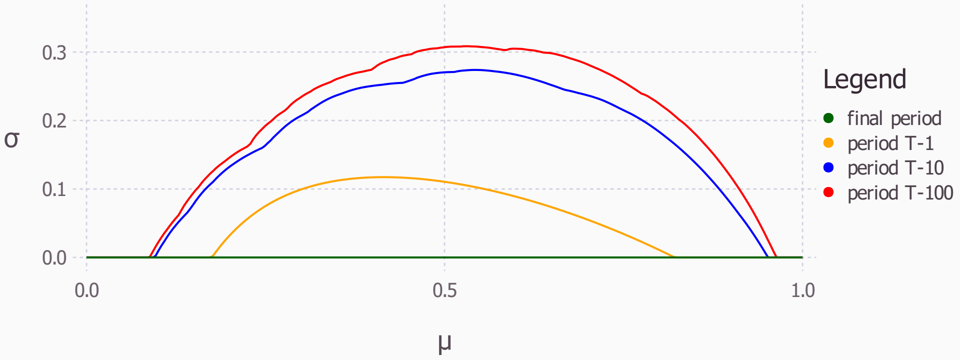
Long Run Commitment: Full Commitment

- Principal commits to long-run contract: $(S_1 \times S_2, p, w)$
  - a signal space $S_1 \times S_2$, p progressively measurable wrt y_t,
  - and wages w, progressively measurable wrt s_t.
- Difficult:
  - Agent acquires private info after shirking (effort unobservable to the contract), and
  - the principal can commit to excess bonuses in $t = 2$ (to induce a learning motive).

$\implies$ Characterizing the optimum requires joint design in both periods.
- Optimum is not tractable. Effect is still in place:
  - Consider optimal contract without fully informative evaluation
  - Bonus for high output in period 1 optimally split between both periods
  - Principal can *postpone* information, but it is *costly*

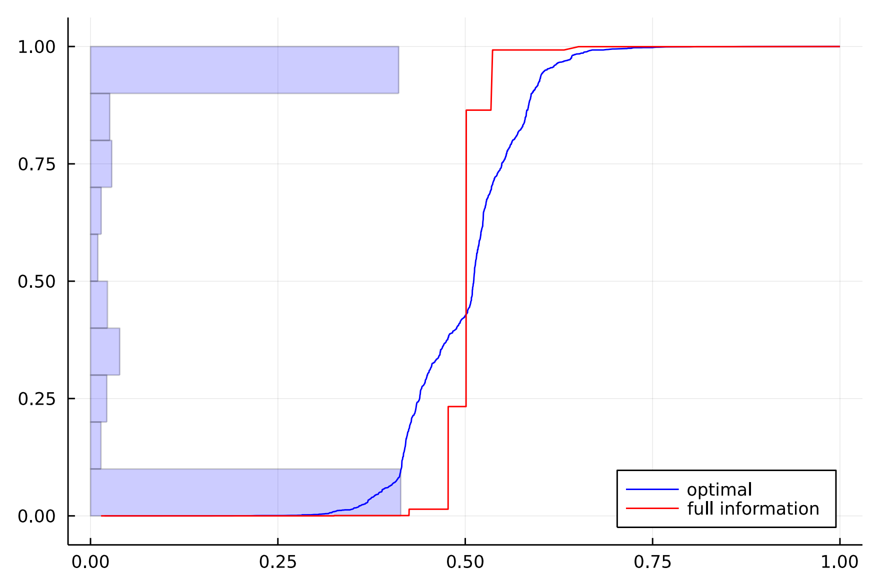
back

Many Periods



Many Periods

BACK



Utility Function

BACK

Assumption (Bounded changes in curvature)

$$\frac{w'''(u_H)}{w''(u_H)} \geq -\frac{3(b+\mu\Delta b)\Delta b}{c((1-a)\Delta b + b\Delta a)},$$

where $u_H = U + \frac{1-a-\mu\Delta a}{b+\mu\Delta b}c.$

Underconfidence: When is Learning costly?

Proposition

The effect of learning on the principal's continuation value is ambiguous.

- *There exists a threshold $\bar{U}$ such that the continuation value is increasing in information if $U \geq \bar{U}$, and*
- *there exists a threshold $\bar{b} > 0$ such that it is decreasing if $b < \bar{b}$.*

back