TP de méthodes variationnelles

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1.1

The denoising by quadratic regularization works by minimizing an energy function that includes a regularization term given by:

$$\|\lambda u\|^2.$$

In the function `resoud_quad_fourier`, this is achieved by rewriting the problem as:

$$\arg\min \|K * u - V\|^2 + \left\|\frac{\partial u}{\partial x}\right\|^2 + \left\|\frac{\partial u}{\partial y}\right\|^2.$$

This function can also be rewritten as:

$$\arg\min \|\delta\delta\delta_{\delta * u - V\|^2 + \|K_1 * u\|^2 + \|K_2 * u\|^2},$$

where K_1 is the kernel that takes the derivative of u in the x-direction, and K_2 is the kernel that takes the derivative in the y-direction.

This function is implemented by defining K as a vector:

$$K = \begin{bmatrix} (1, -1) \\ (1, -1)^T \\ \text{ImpulsoGirak} \end{bmatrix}.$$

While V is defined as:

$$V = \begin{bmatrix} 0 \\ 0 \\ \text{Im} \end{bmatrix}.$$

The minimization of energy is therefore achieved by finding u such that:

$$\arg\min \sum \|K_i * u - V_i\|^2.$$

1.2

If λ is too small, the denoising will not be effective, while if λ is too large, the image will appear blurry.

1.3

We have that:

$$\|\tilde{u} - v\|^2 \approx \|u - v\|^2,$$

therefore:

$$\frac{\|\tilde{u} - v\|^2}{N} \approx \frac{\|u - v\|^2}{N} = \frac{\|b\|^2}{N} = 25,$$

since the variance of the Gaussian noise is 25.

Therefore, we want to find $\tilde{u}$ such that:

$$\frac{\|\tilde{u} - v\|^2}{N} = 25.$$

This can be achieved using a dichotomy-based algorithm, leading to a value of:

$$\lambda = 0.333.$$

1.4

A loop that varies the value of λ and selects the one that minimizes $\|\tilde{u} - u\|$ is sufficient to solve the problem. The optimal value found was:

$$\lambda = 0.115,$$

for noise with a standard deviation of 5.

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2.1

As long as λ is not too large, causing the algorithm to diverge, the resulting minimum energy remains the same.

2.2

The algorithm runs faster and the final result is much better.

3

For the quadratic regularization method, the optimal λ was found to be approximately:

$$\lambda = 1.175.$$

For the Total Variation (TV) method, the optimal value was closer to:

$$\lambda = 41.3.$$

The TV method yielded superior results, as it preserved image details better and introduced significantly less blurring.