Optimal detection of ultra-weak light fluxes with an EMCCD : statistical model and experiments

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Abstract

The present work deals with the detection of ultra-weak light emitted by extended 2D or 3D objects, using an EMCCD camera in the binary photon counting mode, and assuming a uniform flux of photo-electrons on the detector. To discriminate this flux signal from all noise sources, we provide an extensive characterization of the detector combined with a statistical model of the signal-to-noise ratio as a function of exposure time. We find optimal detection conditions, in which steady-state fluxes as low as typically $1 photon.sec^{-1}cm^{-2}$ can be detected with a single image. In these optimal conditions, experiments also show that variations of the temperature of the black-body radiation can be detected in the range of 1*C at ambiant temperature. Finally, for the case non-stationary fluxes, we describe how flux fluctuations are statistically filtered at both short and long times, and our model predicts the optimal time-scale to best estimate these fluctuations.

1 Introduction

In the recent years, increasingly sensitives detectors are used to detect localised sources, like small objects or single molecules, but those low noise point detectors are unsuitable for extended sources or diluted emitters. Infra-noise light fluxes produced by an extended surface needs larges light detectors ; however increasing the surface of a detector increases its noise, and its noise's variance.

Among all detectors, there are many advantages of EMCCDs, such as sub-electron read-out noise and low dark current, for low light applications. Nowadays their low areal noise and high detection capacities permit to reach single photon detection ; however, detecting a few photon per hour stays a challenge, more so when emitted by a large surface. To achieve a sufficient signal over noise ratio (SNR) for significative detection, one must carefully consider and explore the operating mode of the camera, to reach a detection's optimum for a given light flux.

In our work, we address the detection a light flux intensity (1 photon/sec/cm^2) given by a large area, as compared to the sensor's surface, which is lower than our detector equivalent areal noise.

Concerning light flux detection, 3 situations have to be considered :

- constant uniform flux,
- variable uniform flux : statistic filtering,
- non uniform flux : detection of an object's presence.

Here, we propose a statistical method, combined with an EMCCD detector, for an optimised detection of constant, uniform, and low light fluxes emitted by dim extended objects. We present the camera we choose and its use's mode, describe the statistical model of the detector, and show the experimental pixel by pixel characterisation and behaviour of the detector. Then we build a model to simulate the detector's response to a low, constant, uniform flux, and to extract the SNR as well as two characteristics exposure times, in order to use the detector optimally.

2 Camera Presentation

The most suitable detectors for low light fluxes applications are photomultipliers and EMCCDs. To detect a low, uniform light flux, photomultipliers seem adapted because of their low dark count ; however, they usually present a smaller detection surface than EMCCDs. For extended source emission, the wider the detector the better : the areal noise is the critical factor of detector choice.

We compared the areal noise of several PMT's and EMCCDs which didn't need complex cooling systems, and found one very sensitive EMCCD camera best suited for our application because of its unique noise reducing technology, leading to a background signal 2 to 10 times lower than other EMCCDs cameras and competing PMT's areal noise. Moreover, EMCCDs are suitable to image objects, and permits non-uniform light fluxes imaging as well.

tableau comparaison bruit detectors / surface

EMCCDs are devices capable of sub-electron read-out noise, together with a high quantum efficiency (QE). However, the gain operation is plagued by an excess noise factor (ENF). In order to get rid of the ENF, the photon counting (PC) operation is mandatory, with the drawback of counting only one photon per pixel per frame. In this mode, the clock induced charges (CIC), produced in the process of shifting charges from one pixel to the next, by the shift clock signal, and multiplied by the gain, dominates the noise budget of the EMCCD for small time of exposures. In our camera h-nü512, a new EMCCD controller, the CCD Controller for Counting Photons, permits a new clocking and reduces the CIC. Both are commercialised by Nüvü cameras inc. ref The Darkest EMCCD ever The second noise source, the dark current, corresponds to the thermal production of photo-electrons in each pixel, accumulates linearly with time, and is amplified by the pixel multiplication gain.

In PC mode, cooled at -85c, and at gain 3000 (use according Nüvü cameras inc. recommendations), the h-nü512 presents specifications such as :

- dark current (Id) : 0.0002 e^{-}/pixel/s
- clock induced charges (CIC) : 0.001 e^{-}/pixel/frame
- quantum efficiency QE $> 90\%$
- 512$\times$512 pixels, 16$\times$16

To image a few photons per hour, the common strategy admitted, while using such an high frame rate EMCCD, is to enhance the SNR by acquiring and stacking multiple short exposures instead of one long exposure. Indeed, long exposure times may not result in the highest possible SNR for a given total integration time : because of the PC mode, pixels saturate at very long time of exposure because of the noise and the growing impact of cosmic rays.

The experimental SNR study done by Daigle and al ref Astronomical Imaging With EMCCDs Using Long Exposures shows the advantages of their PC mode that shortens the exposure times (minutes compared to hours) compared to a CCD displaying a Read Out Noise (RON), but also that shortening the exposure time to several seconds doesn't impair the SNR while providing better dynamic range.

During our first acquisitions, we empirically set the time of exposure to 1s, then to 40 seconds after an experimental estimation of the SNR, without being able to detect our signal. Therefore, we built a statistical model, based on the detector's individual pixel's noise, to predict the best way to use our detector. From this model, we simulated the SNR for over and under-noise uniforms, constants light fluxes, extracted the optimal time of exposure depending on the signal's intensity, and the expected dynamic range of the measure.

During the acquisitions, the experimental environment must be kept in the dark ; therefore, we designed a light-tight black box, with a darkness judged by the camera background level, and located in a permanently dark room.

3 Statistical model

A SNR model was built to evaluate the sensitivity to detect a uniform light flux impinging on the detector. To this end, the measured observable is simply the total number of counts on the detector N1, i.e. the number of pixels that light up. It amounts to using the whole detector surface as a single big pixel, and provides us with an outstanding sensitivity.

As the detector is operated in the photon-counting (PC) mode, the output of each pixel, 0 and 1, is a simple Bernoulli random variable, X_{pixel}, if considering independent repetitions over time. Each pixel having its own CIC and I_{dark}, the probability to light up can be broken into the probability to deliver a CIC count $p(X = 1)$ or a dark count $p(X_{dark} = 1)$. A simple reasoning leads to :

$$p(X_{pixel} = 1) = 1 - p(X_{pixel} = 0) = 1 - p(X_{CIC} = 0) * p(X_{dark} = 0)$$

X_{CIC} is considered as a simple stationery Bernoulli variable, and X_{dark} as a Bernoulli variable depending on the exposure time. More precisely, the dark count during a given exposure time is assumed to be poissonian, from which X_{dark}, which is binary, is truncated. As a consequence :

$$p(X_{pixel} = 0) = (1 - p_{CIC}) * e^{-\lambda\tau}$$

where λ is the frequency of counts as a result of the additive effect of the dark current and the light flux $\phi = \phi_{I_{dark}} + \phi_{signal}$. Since this is a Bernoulli variable, the variance is :

$$\sigma^2_{Id+CIC} = P_{Id+CIC} * (1 - P_{Id+CIC})$$

and the SNR reads :

$$SNR = \frac{\Delta_{(signal+noise)-noise}}{\sqrt{\sigma^2_{signal+noise} + \sigma^2_{noise}}}$$

The SNR for the sum of all pixels N1 is represented as a of function of the time of exposure for different values of $\epsilon = \phi_{signal}/\phi_{I_d}$, ratio of the signal flux over the equivalent dark flux.

4 Single pixel response's characterisation

When considering the image obtained from a very dim object, the problem is to distinguish if a pixel or an area of pixels is significantly higher (or not) than the neighbours. In other words, is a pattern of pixels (zeros and ones) consistent with the random effect of a perfectly uniform optical field, or is it produced by a contrasted object ? To address an heterogeneous light flux detection, it is essential to know the background and the fluctuations of each pixel one by one.

We studied the camera pixels with the following steps. First, studying the global response of the detector (ones' frequency per frame for a given exposure time), gives a preliminary approach of the real CIC and I_d of our camera (**??**). Second, we determined for each pixel the CIC and the I_d by a linear regression for exposure times that stay away from saturation (**??**). For the linear regression, we weighted the estimated variance of each data point, and were satisfied with the goodness of fit (χ^2, data not shown.) Then we observed the distribution of the CIC and the I_d parameters across all pixels (**??**), leading to a histogram (**??**, **??**) and cumulative distribution (**??**, **??**) and an image (data not shown) for each parameter. Abnormal pixels were localised on the camera chip, and subsequently discarded when needed.

The heterogeneity of the sensor shows outliers of lower I_d (2‰, 513^e line) and higher I_d, (1‰, randomly spread on the sensor), as well as pixels of higher CIC, patterned in pixels columns, with outliers among them ($\tilde{F}_{CIC} > 5.10^{-3}$: 1‰, 513^e line.) However, 99% of the detector is homogeneous, with a noise variation coefficient c_v such as $c_{vCIC} = 0.32$ and $c_{vId} = 0.34$.

Analytically, a succession of ones and zeros has a frequency p of ones and a variance $\sigma^2 = p(1-p)$ (figure **??** is a graphic representation of σ^2 as a function of p for all camera's pixels.) However, if this result is compatible with a pure Bernoulli, it doesn't inform about the history of pixels's counts.

To see if the noise of the detector is compatible with a poissonian distribution, we have to verify that the events are independents in time - history independent ; thus, we looked at the distribution of time intervalles between two pixel's count, for all pixels, and for different exposure times.(**??**.) As the CIC and I_d parameters are projected on the Bernoulli parameter of the pixel for one given exposure time, we can't extract them analytically from the fit. Also, we can't explain the factor 2 found for the fit parameter during short exposure time. This distribution doesn't show temporal irregularities from minority pixels, fits well with an exponential function, and is therefore compatible with a truncatedPoisson distribution as described in the model.

Those parameters were finally integrated in our SNR model to simulate the response of our specific camera and, when used at its best, its capacity limits to detect low light fluxes.

5 Simulation results and SNR

The simulation of the camera shows the expected total number of counts on the detector N1 and its variance under different uniform and constant light fluxes. The signal is competing with the dark current when the exposure time increases, hence the incident light flux is expressed by a fraction ϵ of the dark current flux equivalent. When the time of exposure increases, the N1 mean reaches a maximum (ie. the total number of functional pixels : 512×512) (**??**) and the N1 variance increases and then collapses to stabilise at a minimum (**??**).

Because of the dynamics linked to the saturation, the SNR presents a maximum, before diminishing with the exposure time. For fluxes that exceed I_d flux equivalent, a "bump" appears at the top of SNR curves : when approaching saturation, i.e. when the Bernoulli probability approaches 1, the distribution is somehow "compressed", and the variance decreases while the mean still increases(**??**.)

6 Constant, Uniform light flux measurement strategy

The SNR per frame presenting a maximum suggests an existing optimal strategy, ie an optimal time of exposure, for a given light flux intensity, under the conditions of acquisition described - gain 3000, PC mode, -85c chilling.

Using a SNR contour representation permits a graphical approach to predict our single frame detection capacity, of a given light flux, for a given exposure time (**??**). For exemple, with 40 photons/cm^2/sec (Id equivalent), the SNR equal 1, for one acquisition of 1 second exposure time.

Going further, on the figure **??**, we observe two main SNR regimes that can be defined as a function of their logarithmic slope. One fellows a logarithmic slope of 1 ("regime 1"), and the other fellows a logarithmic slope of 1/2 ("regime 1/2").

Hence, the fellowing question : should we repeat the same acquisition K times during the total experimentation time t, or should we fix $t = \tau$ if $t < \tau_{SNRmax}$? Studying those SNR regimes will helps us to answer this question.

Indeed, the SNR increases with the number of repetition of acquisitions K in the same experimental conditions, given that the standard error of the mean (SEM) decreases with $\sqrt{K}$, and with the time of exposure τ, until it reaches the time for which the SNR is maximal τ_{SNRmax}. In the "regime 1", the SNR increases proportionally to τ and to $\sqrt{K}$, while in the "regime 1/2", the SNR increases proportionally to $\sqrt{\tau}$ and to $\sqrt{K}$. Therefore, considering one flux intensity ϕ and time of exposure τ, in "regime 1", the SNR increases faster with τ than with K, whereas in "regime 1/2", the SNR increases indifferently with τ and with K.

Those regimes are better viewed with the logarithmic derivative $\delta logSNR/\delta log\tau$ representation (**??**). Here, we noticed that there is only one precise τ for which the logarithmic derivative of the SNR takes the value 1/2, and this τ stands for the "regime 1/2". Below this time of exposure, increasing the τ increases the SNR faster than increasing K ; above it, increasing the K increases the SNR faster than increasing τ. But, repeating an acquisition at the precise time of exposure where $\delta logSNR/\delta log\tau = 1/2$ brings the maximum increasing rate of the SNR once passed the "regime 1". Therefore we called this new characteristic time of exposure the τ of maximal density information ($\tau_{max.d.info}$).

From this analysis, two characteristic exposure times are extracted : τ_{SNRmax} gives the maximum SNR for one frame, and the $\tau_{max.d.info}$ gives the best information density when repeated, both dependant on known or predicted incident light fluxes. In practice, for a long detection of a flux intensity near the dark current level, an repeated exposure time around $500 - 600$ sec is best, while for an inferior flux intensity, the ideal repeated exposure time is slightly higher, around $700 - 800$ sec. This deep study gives us a great tool to use our detector at its best, but also to predict our detection capacity for one uniform, constant light flux emitting source or sample (**??**.)

However other kind of unpredicted fluctuations can impair the model (**??** shows N1 variance excess). Thus, improving experimentally the SNR by a better light collection, and having a *in situ* control with imaging, is also fondamental in ultra weak light flux detection.

(**??**).

7 Fluctuating light flux and heterogeneous emission

The measure of a fluctuating light flux implies a statistic filtration to detect a double stochasticity. In other words, when sampling the acquisition, do the fluctuations exceed the expected stochastic fluctuations of the observable ? If the exposure time is larger than the fluctuation period, we won't determine if there are signal variations, whereas if the exposure time is smaller than the fluctuation period, but the signal is too weak to achieve a sufficient SNR per frame, or by repeating frames, we won't determine if there is a signal at all.

Hence, there is a time resolution limit for a given light flux, depending on the minimal time of exposure to be repeated or to be met to have a sufficient SNR, compared to the variation period and amplitude of the signal.

To image an object emitting an heterogeneous light flux, we have to address the question in part 4 : is a pattern of pixels consistent with the random effect of a perfectly uniform optical field, or is it produced by a contrasted object ? Knowing perfectly the background of each pixels for a given exposure time permits to do tests of spatial randomness to detect an non uniform light field on the detector. A pixel by pixel study is less sensible than a hole detector detection as it will rely uniquely on the number of repetitions of frames to make a significative difference in between pixels.

If we know the image position on the captor, and if the image light flux is uniform, the strategy can be different : to increase the detection sensibility, we can then sum all pixels of the background area and the lit area, as we did for the entire detector, and compare and quantify both observables.

Combining methods as per fit is one's strategic choice to make.

8 Practically

We address here different practical issues or questions about fast SNR simulation, cosmic rays, and and thermal radiation detection

8.1 Fast SNR simulation

Taking into account the 513×512 pixels into a sum of Bernoulli random variable with their own parameter is time consuming, and for the single pixel characterisation, and for the simulation processing. If a subtle knowledge of the camera pixels is not necessary, estimating the SNR by a model is swiftly done by considering the detector homogeneous, with the same CIC and Id parameter for all pixels. The parameters are extracted by doing a simple linear regression ($\tilde{f_{counts}} = \tilde{f_{CIC}} + \tilde{f_{Id}} * \tau$) out of saturation on the global detector response on complete dark conditions (see **??**).

The difference between both homogeneous model (all pixels's noises identical), and the heterogeneous model (all pixels's noises singular), is the variance between pixels on one frame, σ^2_{ij}. Surprisingly enough, this σ^2_{ij} is subtracted to the N1 variance in the heterogeneous model case, which leads to a better SNR (**??**). It is easily demonstrated by :

demonstration de la difference.

However, the difference is small enough (from 1 to 7%) for neglecting it for the sake of simplicity.

8.2 Cosmic rays

Cosmic rays (CR) are particles of different kinds, carrying a wide range of energies. They arrive from the cosmos and trigger multipixel response patterns on the detector (see **??**). Because these pixel clusters patterns bear some reproducible features (a cluster of connected pixels followed by a "horizontal tail"), we wrote a program to detect a given number of connected pixels, this cut off number of connected pixels being determined by a negligible probability ($< 10^{-6}$) to happen randomly for a given time of exposure. The frequency of events (ie. cluster of connected pixels), as function of the number of pixels in the cluster, was determined by a Monte Carlo simulation, on an arbitrary large enough detector to achieve sufficient statistic, using our model to produce the noise pattern, for different time of exposure. This program create a detector equivalent logical matrix attributing "ones" to the pixels belonging to a cluster's size bigger than the determined cut off. After indexing the concerned pixels, we substitute them by random values drawn from the appropriate Bernoulli distribution. Because the detection of these clusters relies on the recognition of connected pixels, it becomes more difficult to tell them from clusters generated by signal photons when the density of positive pixels is too high, e.g. when the time of exposure exceeds 2000 seconds. However, we could assess the contribution of cosmic rays and we found that, in photon counting mode, it amounts to $4, 1.10^{-6}$ electrons/sec/pixels, that is 1/20 of the dark current (figures **??** and **??**.)

Importantly, this equivalence with an effective photon rate determined in binary mode (photon counting) is physically meaningless, because it does not reflect the actual strength of cosmic rays which saturate analog detectors. Digital treatment makes these high energy perturbations look like regular visible photons, and this can be considered as an additional advantage of using a binary mode to detect low light fluxes.

We consider that the impact of the cosmic ray on our model is negligible, as far as the concerned time of exposure are small enough that their removal doesn't impact significantly the statistic of the wide detector response.

8.3 Black Body Radiation

Given the SNR model a rightful question emerged : what is the sensitivity of our camera to thermal radiation ? Indeed, the signal of interest being so weak, it is fundamental to be able to distinguish it from other kind of radiation and fluctuation linked to temperature. This is even more crucial given that we aim for biological sample that are kept at higher temperature (37 celsius degrees) than room temperature.

Therefore, we modelled the sensitivity of our detector to temperature changes, considering its spectral sensitivity (see figure **??**), and the black body radiation spectra (**??**.).

Our detector is indeed sensitive to temperature, according to the model, and in a much more drastic way that we expected. Given a time of exposure of 600 sec, the SNR is 10 and 100 (according to our SNR model (see figure **??**) for the thermal flux given at a temperature of 23-25c and 37-38c respectively. The saturation of the detector happens for 58c (see figure ??). The Noise Equivalent Temperature Difference (NETD) for a thermal flux given at temperatures 20c, 26c, 32c and 37c is respectively 2, 1, 0.5, 0.2 (figure ??).

It means that for a sample maintained at 37c, the camera would be sensitive to a 0.2c change of temperature. It shows that the measured level of our background for a given τ is actually temperature dependent.

This very recent result raises new challenges for temperature and environment control that are for now unprecedented and was never considered in this field.

9 Material and methods

9.1 background measurements

All background measurements are done in the most complet dark. The internal shutter of the camera is maintained closed, the camera is plugged and a metallic bowl is pressed against the camera facade, sealed by a thorlabs blackout fabric (BK5) in between. The camera is in a light-tight black box, in which all light leaks are plugged, and itself totally recovered by two very black heavy fabrics outside, and inside by the BK5 fabric. The camera and the box temperature is monitored per experiment. The camera temperature varies within 0.05^{o}C and the box stays below 25^{o}C at all times. Variations of the room and bow temperature doesn't impact on the camera temperature and are not correlated to any noise fluctuations to the best of our analysis (data not shown).

9.2 calibrated low fluxes injection

The optical setup shown in figure ?? is used to inject a beam of beam of calibrated low light fluxes. The laser used (Thorlabs HNLS008LEC, 632.8nm, 0.8mW-2mW, polarisation 500:1) passes through different elements to attenuate it before creating an uniform infinite beam to be sent to the camera. Those elements are carefully cleaned before use.

Dielectrical mirrors (Thorlabs, BB1-E02-10) are used to direct the beam through the described elements.

First, a shutter (Thorlabs SH1) controlled by the computer through its controller (Thorlabs Tcube TSC00) and an Arduino, permits to cut off the beam remotely without disturbing the environment, and to synchronize the acquisitions of the camera (internal shutter open or closed) with the presence or not of the beam. Measuring without the beam impacting the camera gives a direct measurement of the ambiant background noise.

ND absorptive filters (Thorlabs NEK 01, diameter 25mm, 400-650nm) are used to limit the maximum possible flux of the laser and to keep it under the saturation of the different detectors used, protecting them from damages.

A telescope 5X (Thorlabs, GBE05-A, 400-650nm) expends the beam before the fiber launch (objective Leica 4X/0.1, Achro ∞/0.17) which conjugates the beam to the entrance of a monomode fiber (Thorlabs 630 A FC1, NA 0.10-0.14, 633-780nm).

The fiber delivers the light through a collimator (Thorlabs F810FC543, focal = 34.74mm, 543nm, NA 0.26) to produces an infinite beam in an optical tube (Thorlabs SM1 tubes).

The beam will be finely attenuated by three polarizers (Thorlabs, LPVISE100A, ε >5000:1 at 535-690nm). The first and last ones are fixed parallely , while the middle one is mounted on a motorized mount (Thorlabs KPRM1E/M and KDC101 K-Cube DC Servo Motor Controller), controlled by a computer. The angle is chosen to obtain a defined output flux.

The attenuated light beam exit the tube by a second collimator that connect to a monomode fiber (Thorlabs 630 A FC1, NA 0.10-0.14, 633-780nm).

The fiber is connected on the other hand to a tube containing fixed optical elements in serie in a thorlab SM1 tube : a convergent lens (Thorlabs, LA1422A, plano convex, focale = 150mm) is placed at a focal distance to put the beam to infinity. With an diaphragm (Thorlabs, SM1D12C), we only select a 7mm diameter central zone of the infinite attenuated beam to have a better average flux uniformity throughout the beam section. The thorlab SM1 tube is fixed on the camera via c-mount and the beam centered on the sensor for acquisition.

We control in real time the fluctuation of the laser by putting a cover slip that reflect a 7% fraction of the initial beam and detected by a powermeter (PM200, detection head : S120C, 400-1100nm, 50nW-50mW). The measured variability of the laser throughout the experiment is 2% (1.38 $\pm$ 0.03mW)

The output flux of the fiber is also controlled before the injection to the NüVü camera with a photomultiplier (Picoquant PMA hybrid, model 40, 3mm diameter detector, Id < 700cps) with a counter (Stanford Research Systems, SR620). For the control we use another ND filter adapted to the performance and saturation of the PMA. For this control, a lens (Thorlabs, LA1951-A, plano convex, focale = 25.4mm) is used to focus all the light emitted out of the fiber to the PMA detector, under the protection of an iris when the PMA is not used.

For each acquisition, we use a signal/camera background subtraction strategy, and the internal shutter of the camera is closed for one image over two. To control the environment background, we also closed the shutter in the optical setup to cut the beam, for each flux condition, while acquiring. We controlled the environment and camera background per acquisition and its evolution (data not shown) and then subtracted the signal to the camera background throughout the experiments, and furthermore the environment background corresponding to each acquisition to the previous corrected signal. Another control is added by comparing through mask analysis (python) the beam illuminated area of the detector compared to the non illuminated area for laser ON and OFF condition for each flux (data not shown). This gives additional information on eventual fluctuations of the detector and environmental background in real time, before subtracting the background to the signal.

The light injection were done as fellowing (one measure corresponds to a sequence of two images shutter open and closed), with 160 seconds of exposure time :

- 4050 ph.cm^{-2}.s^{-1}, 5 measures laser ON, 5 measures laser OFF
- 4.05 ph.cm^{-2}.s^{-1}, 60 measures laser ON, 45 measures laser OFF
- 405 ph.cm^{-2}.s^{-1}, 5 measures laser ON, 5 measures laser OFF
- 40.5 ph.cm^{-2}.s^{-1}, 5 measures laser ON, 5 measures laser OFF
- 0.48 ph.cm^{-2}.s^{-1}, 90 measures laser ON, 90 measures laser OFF

10 Experimental calibration

11 Conclusion