Additional Practice Exercises

April 2018

1 Perfect Bayesian Equilibrium

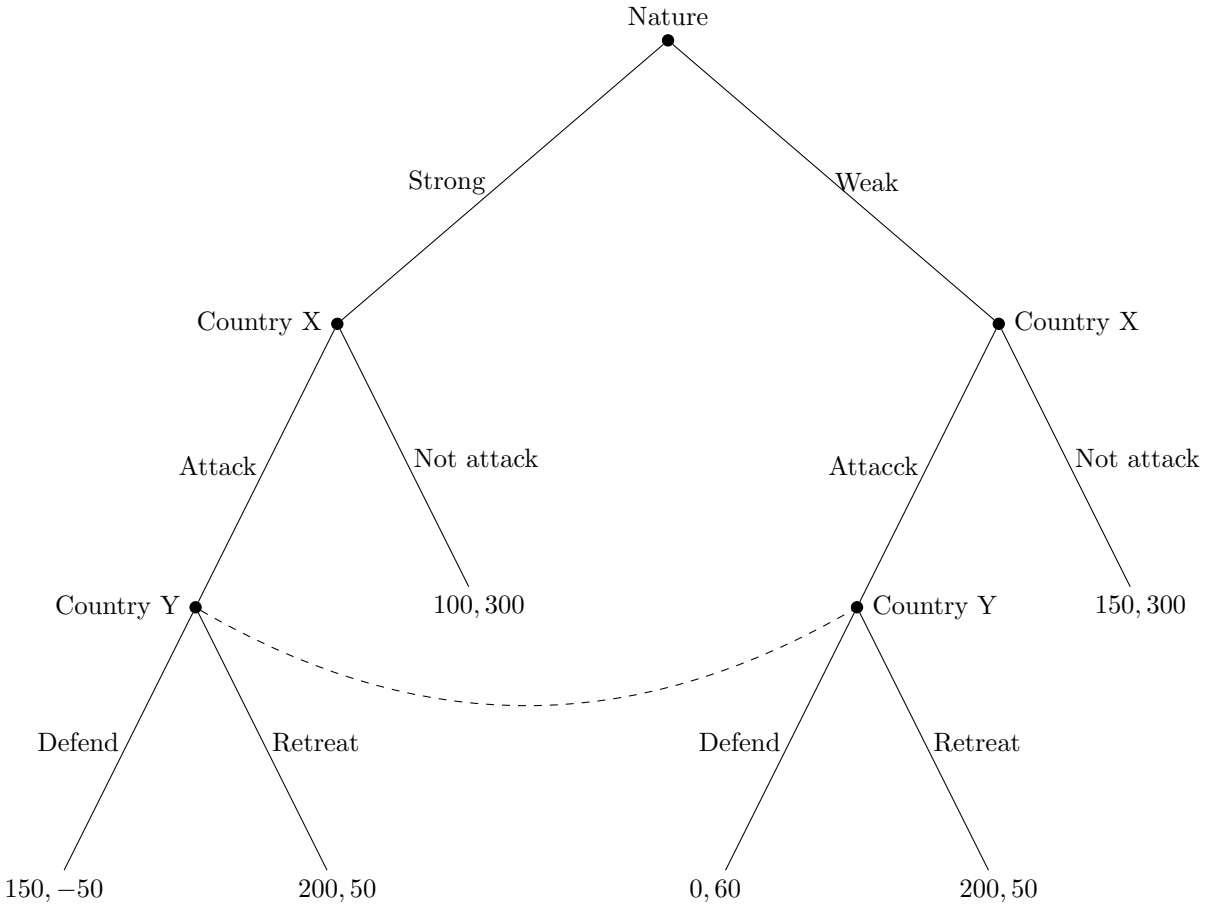
Country X is considering invading Country Y. Country X knows whether its army is strong or weak. Country Y does not know that. If Country X attacks, Country Y can either defend its territory, or retreat. Country Y would prefer to retreat if the attacker is strong, but prefer to defend if the attacker is weak. The payoffs are given in the game tree below.

Assuming that nature initially chooses troops to be strong with a probability of p(strong) = .6, evaluate the following strategies.

(a) Is (Attack|Strong, Not attack|Weak; Defend|Attack; p(Weak|Attack)=0) a PBE? Why or why not? Show your calculations in detail.

(b) Is (Attack|Strong, Not attack|Weak; Retreat|Attack; p(Strong|Attack)=1) a PBE? Why or why not? Show your calculations in detail.

(c) Is (Attack|Strong, Attack|Weak; Retreat|Attack; p(Strong|Attack)=.6) a PBE? Why or why not? Show your calculations in detail.


Nature
Strong
Weak
Country X
Country X
Attack
Not attack
Attacck
Not attack
Country Y
100, 300
Country Y
150, 300
Defend
Retreat
Defend
Retreat
150, −50
200, 50
0, 60
200, 50

2 Risk Aversion

(a) A company is considering making a campaign donation to Candidate A worth 10 million. If A wins, then the company will receive government contracts worth 100 million. If Candidate A loses, then the company won't get any government deals at all. Candidate A is projected to win with a 60% probability. The company is certain that if they made no donation at all, they would receive the same amount of government contracts as last year, which was 14 million.

What risk attitude does management need to have to choose not to make a donation? Why?

(b) It turns out the government contracts the company would receive if they made a donation and Candidate A won would only total 40 million. Does this change your assessment of the company's risk attitude? Why or why not?

(c) You can either choose to buy lottery tickets worth 10 or not to play the lottery. If you do buy tickets, your chance of winning 1 million is .001%.

What would a risk-neutral person do? Why?

(d) It turns out you miscalculated the odds of winning the lottery. Your chances of winning are actually only .0001%.

What would a risk neutral person do? Why?

3 Mixed Strategies

Look at the following matrix. Players 1 and 2 can either play Heads (H) or Tails (T), and get different payoffs depending on whether their coins match. Is there a mixed strategy equilibrium? Show your calculations in detail.

	H	T
H	3,-3	-2,2
T	-1,1	0,0

Solutions

1. **PBE**
   (a) belief: ✓
   P2's strategy: EP(defend) = -50, EP(retreat) = 50 ✗
   (b) belief: ✓
   P2's strategy: EP(defend) = -50, EP(retreat) = 50 ✓
   P1's strategies: EP(attack|strong) = 200, EP(not attack|strong) = 100 ✓
   EP(not attack|weak) = 150, EP(attack|weak) = 200 ✗
   (c) belief: ✓
   P2's strategy: EP(retreat) = 50, EP(defend) = -6 ✓
   P1's strategies: EP(attack|strong) = 200, EP(not attack|strong) = 100 ✓
   EP(attack|weak) = 200, EP(not attack|weak) = 150 ✓

2. **Risk Aversion**
   (a) EP(certainty) = 14,000,000
   EP(gamble) = 50,000,000
   → risk averse
   (b) EP(certainty) = 14,000,000
   EP(gamble) = 14,000,000
   → still risk averse
   (c) EP(certainty) = 10
   EP(gamble) = 10
   → pick randomly between the two
   (d) EP(certainty) = 10
   EP(gamble) = 1
   → certainty option

3. **Mixed Strategies**
   Player 1's optimal mixed strategy:
   $EP_2(H) = 1 - 4p$, $EP_2(T) = 2p \rightarrow$ p = $\frac{1}{6}$
   $EP_1(H) = 5q - 2$, $EP_1(T) = -q \rightarrow$ q = $\frac{1}{3}$

4. **Auction example from class**
   (a) Strategy: (bid|rich, not bid|poor; leave|bid; p(rich|bid) = 1)
   belief: ✓
   P2's strategy: EP(leave) = 4, EP(counter) = 2 ✓
   P1's strategies: EP(bid|rich) = 7, EP(not bid|rich) = 5 ✓
   EP(not bid|poor) = 5, EP(bid|poor) = 3 ✓