Spacetime Cloaks

Undergraduate Literature Review

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Space Time Cloaks Literature Review

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Spacetime cloaking was recently predicted, along with a theoretical design, as an extension of the idea of spatial cloaking which was proposed over a decade ago [McCall et al., 2011, Pendry, 2006]. It is an exciting new advancement in this area, and a major new type of cloak, rather than a new design. Spacetime cloaking gives rise to a whole range of new effects we could achieve, like the ability to give the appearance of teleportation or edit causality. The cloaks are able to completely obscure a non-emitting event within the window of the cloak. In this review I explain the origins of spacetime cloaks from spatial cloaks and discuss potential applications for such devices as well as the experiments that have so far been conducted. Paying particular interest to electromagnetic cloaks. Much of the literature speculating about spacetime cloaks is by M. McCall and P. Kinsler, although I have tried to include a wider range of sources.

1 Introduction

Humanity has long been fascinated with the notion of invisibility, having the ability to conceal something in the open. We have developed tricks and methods to approximate invisibility but until recently, being able to truly cloak an object was just a fantasy. With the development of a whole new branch of manmade materials (metamaterials) we have finally been able to make invisibility cloaks a reality, if just for very small objects.

That certain media affect light, bending it as it passes into the material is something we have known for millennia, it is apparent with a pool of water. For centuries we have even had the ability to subtly but purposefully, manipulate light with mirrors, prisms and lenses. Now we have unprecedented control over the materials we can manufacture and use. This has allowed us to create "metamaterials" with fascinating properties not found in nature, such as negative refractive indices [1, 2].

Just over a decade ago it was suggested that we could build an electromagnetic cloak with metamaterials if we assumed we could have absolute control of the refractive properties of the material [3, 4] and since then, there has been much interest in the field and it has advanced rapidly with many demonstrations of cloaks working for several different bandwidths of light, including the visible spectrum [5, 6, 7, 8, 9, 10] and even designs for ultra-thin [11] and carpet cloaks [12], each with their own potential applications. Six years ago an entirely new direction for electromagnetic cloaks was proposed: a spacetime cloak (STC) [13]. This kind of cloak would hide a specific, finite event from history, rather than hide an object or region of space indefinitely. Most of the research on cloaking devices is done on electromagnetic cloaks since the technology to produce such cloaks is available and there are already many potential applications; however any waves are potentially suitable for a cloaking device and acoustic cloaks have been devised [14].

In this report I wish to discuss the exciting results we are seeing in from research into the new field of spacetime cloaking and explore the benefits and applications of such cloaks.

2 Concept of Cloaking

Firstly I will introduce the concept of 'ordinary' spatial cloaking and the theory behind opening up an electromagnetic hole in a medium where an object can be concealed. Then I will extend this hole into spacetime and thus produce a theoretical STC, before discussing how we might move this from conjecture and theory into reality.

2.1 Creating an electromagnetic hole

The theory to produce spatial cloaking relies on the tool of transformation optics [3, 4]. Under normal conditions light follows a straight path through Minkowski spacetime. If we need to alter this, we can employ a trick: we can model it as a vector in space, and allow it to propagate normally but use a changed coordinate space so that the light follows a different geometry, as you can see in the transformation in figure 1, where the straight line vectors no longer lie along the grid. Pendry likens this to deforming a rubber medium, where the light is made to travel in specific directions through an optical system [4]. We then want to find the different refractive indices through the material that will reflect this change in geometry, so that the material bends the light as we want. With this tool we can distort the path of light through the real space, from a straight line to one of our choosing. By giving us the values of ϵ and μ we need through the medium to make the EM space the same as the transformed coordinate space, meaning light will propagate as we intended.

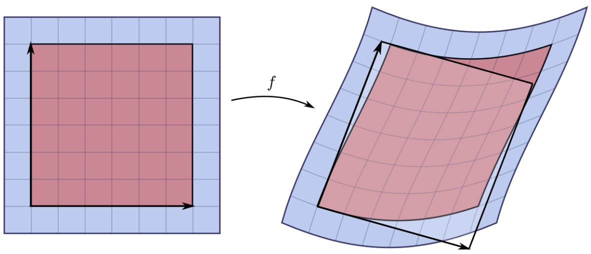

Figure 1: *A graph showing the effect of a Jacobian transformation on a 2D Cartesian plane intended to show how straight lines can curve under a transformation*

Transformation optics is analogous to gravitation in general relativity, causing light to bend around massive objects by warping spacetime [15]. Instead transformation optics describes a warped EM space in the medium with particular permittivities and permeabilities. This analogy is particularly pertinent in the case of gravitational lensing, where light bends around an object allowing one to see objects that would classically be hidden behind it. To produce an electromagnetic cloak we need a medium that will cause light not just to bend around a volume but to seamlessly flow all the way around an object from any point and join together to flow onward at the other side as if the light had simply followed the expected trajectory [3].
We can use tensor calculations, as with relativity, to describe the transformations. By applying a diffeomorphism to the Minkowski metric ($\eta_{\mu\nu}$) which is equal to the identity, except in the neighbourhood of the region we wish to cloak [15]. In a medium the distances light 'sees' as it travels and thus the path a ray takes is dependent on the differences in refractive index. Since refractive index is a feature of the electromagnetic permittivity and permeability of the medium [16], if we can specify these values through the material, we can precisely control the geometry the light experiences: like bending EM space.

$$\mathbf{B} = \mu_0 \mathbf{H}, \qquad \mathbf{E} = \epsilon_0 \mathbf{D} \tag{1}$$

$$\mathbf{B}' = \mu_0 \mu_\alpha \mathbf{H}, \qquad \mathbf{E}' = \epsilon_0 \epsilon_\alpha \mathbf{D} \tag{2}$$

Where ϵ_0, μ_0 are the electromagnetic permittivity and permeability of free space, respectively. The constitutive equations give us the electric and magnetic fields $(\mathbf{E}, \mathbf{B})$, in a time independent medium, given ϵ and μ, which are the identity in the case of a vacuum. Since $\mathbf{D}$ and $\mathbf{H}$ are the same in the medium as in a vacuum.
Since a real cloak will require the electric permittivity and magnetic permeability of our material to be particularly specified, we need to find manipulations of Maxwell's equations that will satisfy the transformation and give us the values for ϵ and μ. If we take a normal volume of empty space we can model it as a 3D Cartesian coordinate space in (x, y, z), which will make the problem easier than if we work out transformations in 1+3D spacetime. Light in this vacuum will travel along a vector through the coordinate system. It is possible to do a coordinate transformation on the space $\overrightarrow{r} \rightarrow \overrightarrow{r}'$ to bend the straight lines to curved ones [3, 17, 13]. When we substitute the new coordinate system we find Maxwell's equations with differing values of ϵ and μ, which gives the changing refractive index we need.
We then change the values of μ and ϵ to those specified by our coordinate transformation, which would require metamaterials for a working cloak [3]. Let us imagine a spherical cloak about the origin of our space; by transforming the coordinates so that the origin point becomes a volume where the space around it is compressed to the edge of the sphere as you can see in the 2D representation in figure 2A. The light will look as if it has traveled straight through when in reality its path is being distorted around a volume, as can be seen in 3D in figure 2B.

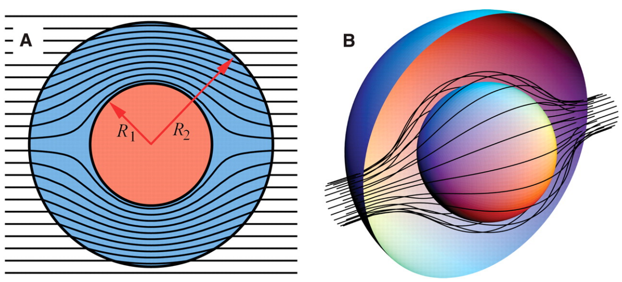

Figure 2: *A: A 2D representation of light paths through the system where they enter the blue annulus and bend around the red centre, leaving the other side along the path of their original trajectory.*
B: A 3D view of the output of a ray tracing program tracing the path of the rays in the cloak
(Taken from [3])

Spatial cloaks have received a lot of attention and many practical examples have been built. Cloaks capable of working at optical wavelengths have been demonstrated [9], and alternative designs like carpet cloaks have also been proposed.
There are however some issues with spatial cloaking, the light must follow spacelike trajectories in the model achieve the cloak [18], since they will have to travel around a volume while behaving as it it had just passed through. Something that is obviously impossible as light can only travel at speeds $\leq c$, although such superluminal trajectories are compatible with special relativity [13]. Looking at spatial cloaks as 4D systems may help with this, as is considered in [18].

2.2 Extending the hole into spacetime

I will not go into a full treatment of 1+3D spacetime since there have been no experimental demonstrations nor suggested methods for producing a full STC. The calculations involved are also more arduous and utilise rank 4 tensors [13]. Instead I will approach the 1+1D problem which is more easily solvable and has been experimentally produced multiple times.

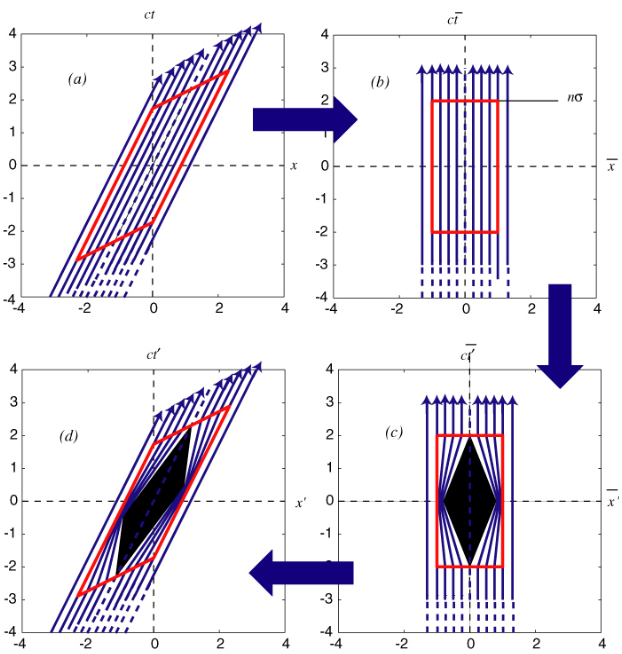

Figure 3: *Construction of the hole as a curtain cloak, in a path of light rays through spacetime*

STCs are in some senses conceptually much simpler than a purely spatial cloak since, in a fundamental sense, they rely on speeding up and slowing down light to create a leading and trailing part [19]. There is a gap in the middle and by knowing the time it will pass over a region of the path, one can time an event to occur in the interval thus leaving the light undisturbed and the event undetected. A non-emitting event would be completely hidden, but an emitting event would leak from the cloak; in a perfect cloak an emitting event would be compressed into a single momentary flash of light [13]. In an imperfect cloak the light would be compressed but could be visible as a sort of sped up history [19]. While a STC might in some way seem to be an abstruse variant of a spatial cloak it is in fact a distinctly new type of cloak requiring spacetime transformations and only cloaking the region of some finite period [13, 20, 19]. It's actualisation will require a time-dependent electromagnetic medium for the light to pass through [13]. Since the light will need to be transformed in spacetime not solely in space.
For the spacetime cloak we will consider the transformation $(x, t) \rightarrow (x', t')$ where the light propagates in the positive x direction, the light rays must follow the straight line $x = ct + k$. Then we consider spacetime transformations of Maxwell's equations, that are invariant in $t \rightarrow t'$. Since changing electric and magnetic fields will interfere, a linking term is required in this time-dependent variant that models how $\mathbf{E}$ and $\mathbf{B}$ depend on both ϵ and μ.

$$\begin{pmatrix} B_y \\ B_z \end{pmatrix} = \beta(x,t) \begin{pmatrix} -E_z \\ E_y \end{pmatrix} + \mu(x,t) \begin{pmatrix} H_y \\ H_z \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} E_y \\ E_z \end{pmatrix} = \epsilon(x,t) \begin{pmatrix} D_y \\ D_z \end{pmatrix} + \beta(x,t) \begin{pmatrix} H_z \\ -H_y \end{pmatrix} \tag{4}$$

As with spatial cloaks the values of ϵ and μ are specified in a spacetime varient of transformation optics (see appendix of [13]). We use this spacetime varying electromagnetic medium to alter the beam as it passes along our cloaking device. We could generalise this idea to a higher dimensional space.
Since the light in this model only propagates in 1D space it clearly doesn't curve in space, but it does in spacetime; the spacetimne curvature corresponds to a slowing down and speeding up of the wave. This opens a curtain as can be seen in figure 3 with a null on centred on the origin in which an event can occur. After the event, the curtain closes again so that there is no trace of the cloak. This kind of cloak is later referred to by Kinsler and McCall as a speed cloak [19].
A free-space spacetime cloak is possible to design /citeMcCall2011,Kinsler2014 but the calculations are arduous as noted above, I am more concerned in this review with discussing the potential and the demonstrations. For any interested reader: Kinsler and McCall do a good job of explaining the multidimensional spacetime cloak in [19], I could not do any better nor make it more concise.

2.3 Achieving the STC

With a STC we can potentially observe some peculiar effects. As the cloak must close at some point any emission from an event within the cloak will escape and would appear speeded up to some degree, in a perfect cloak the light would be compressed to a singular flash but if the closure were less than perfect a glimpse of speeded up history would be visible. Objects moving while the cloak was in operation would appear to have instantaneously teleported, to an observer on the far side of the cloak.

The cloak seems to require at least three elements: one to open the cloak, one to close it and one between to maintain the cloak over the event space. The cloak can utilise nonlinear optical effects to open up an intensity gap in the beam. McCall *et al.* in their 2011 article suggest the use of nonlinear optical fibres which have modulated refractive indices. They suggest that by modulating the index of the fibre producing the cloak from low to high that a spacetime intensity gap will be created that can be closed by another fibre with a refractive index modulated from high to low [13].

This method has issues with it as noted by Fridman *et al.* [21], where they point out that other optical affects caused by pumping the fibres to such high powers could limit the cloaking ability of the setup. They opt for a split-time lens to introduce the non-linear optics into the system, a method that both McCall and Kinsler acknowledge as superior [19].

Since the prediction by McCall *et al.* there have been three practical demonstrations that I will examine in section 5.

3 Advantages of STCs Over Spatial Cloaks

STCs are able to cloak a region of spacetime and do so without detection. Why though should we attempt to produce a STC rather than a spatial cloak, since a spatial cloak will allow us to cloak an area indefinitely? There are advantages peculiar to STCs that can be useful in certain or for all situations. One of the first is that the light does not travel through curved EM space but rather curved spacetime and thus the light experiences only changes to speed and not trajectory. This makes a theoretically perfect cloak impossible to detect while it is possible for spatial cloaks to be detected even if 'perfect' [22, 23].

While there may be ways to detect a STC and currently it is very hard to produce a perfect one, they may be harder to detect; moreover since the cloak isn't operational all of the time but only during brief moments, they would be harder to detect since they require that the detection occurs during the time that the cloak is 'working'.

The other reasons we may wish to use a STC is for cloaking multiple events rather than a continuous region of space: a demonstration of the STC in 2016 found that they could produce a cloak that was sustained over a 5km length of fibre [24]. If we can cloak a series of events it may be more useful than just cloaking a single space. There can be normal communication to the event before it occurs, this may be helpful to us and would be impossible for a spatial cloak.

Spacetime cloaks are intrinsically directional [19], because of this they may be useful where we only want to cloak from one perspective. Generally however, it is a limitation to the cloak, where the spatial cloak has an advantage as being direction-independent, so that the cloaks can work from all angles of view. Spacetime cloaks having a forward and backward direction can only be configured practically to cloak an event from an observer in a particular direction.

4 Applications and potential for STCs

As I have already mentioned there are a number of applications for STCs. They would be useful in digital communications where to hide that some communication has occurred, or for interrupt-without-interrupt operations such as opening a gap in a pulse train to interfere with some signal processing unit (SPU) [19]. This would be advantageous over simply hijacking the SPU since the cloak should be undetectable and allow for stealthy interruption.

They have been shown to work at telecommunications data rates [25, 24], allowing the potential for hiding data transmission from an observer. Being able to communicate without detection could be very useful for operations where communications privacy is extremely important, since the communication would not only be unknown to another observer, they would not even be aware of any communication.

By speeding up and slowing down the light by appropriate amounts we might be able to reverse the beam so that it would appear to show backwards causality [19]. While I am not sure of the practical applications of such a phenomenon, it is nonetheless very interesting. Manipulating the light could be useful to disguise the order of events from an observer.

At the time of writing, metamaterials are not required in cloak design but one may consider the possibility of spacetime metamaterials: for example one based on a time-crystal [26], where the temporal periodicity of the structure could allow for a full spacetime alteration of the material. Kinsler and McCall predict that we could 'blow bubbles in spacetime', particularly inspired by the work of Smolyaninov, where he considers modelling causality with hyperbolic metamaterials [19, 27]. Using Maxwell's fisheye transformation, generalised to spacetime, it is possible to project a hypersphere onto some hypersurface and we could manufacture the hypersurface in question with the mapping built in. Enabling us to create some expanding 'bubble universe' that was time dependant, self-contained. Acting as some cosmological model.

5 Experimental Demonstrations of STCs

In 2011 the spacetime cloak was predicted by McCall *et al.*; in the same year a team at Cornell University demonstrated the first temporal cloak [21]. A couple of years later another team at Cornell managed to create a new type of time lens that was capable of operating at telecommunication data rates and they managed to achieve a 46% cloaking of the entire time axis, concealing pseudorandom digital data at a rate of 12.7Gb/s, by using the time-domain Talbot effect [25], both papers featured in Nature. Since then another demonstration has improved upon the one by Lukens *et al.*, by using an inverse of the Talbot effect. They achieved a cloak of 74% with a continuously cloaking window of 196ps, which could be maintained over 5km of dispersion-compensating fibre [24].

5.1 First Demonstration

Fridman *et al.* took the ideas in the original paper [13] and adjusted them to a different method than that proposed to one using time lenses instead of the original proposal of pumping optical fibres to high power levels [13, 21]. This is because stimulated Raman and Brillouin scattering could potentially disrupt the cloak [21]; the technique was noted by Kinsler and McCall to be an improved one [28].

In this demonstration Fridman *et al.* route a probe beam through a time lens and then a fibre designed to redistribute the chirp in the wavelength to create the gap. There is an analogy to diffraction in the temporal phenomenon of dispersion, this leads to a quadratic phase modulator being equivalent to a 'time lens' [29, 30]. A time lens obeys an equivalent of the lens law in time by producing the quadratic phase shift [21]. The time lens can be achieved by a variety of non-linear optical methods: for example using four-wave mixing, which is a parametric non-linear process, on a chirped pump wave. [21, 30, 31].

Their method to produce the time gap in the wave was to pass the probe beam though a split-time lens (STL), which is composed of two half time lenses connected at the tips; using the principle of four wave mixing on two pump waves, one with a linear chirp and the other with a constant wavelength. This split-time lens imparts two equal and opposite split chirps on the probe beam. The wavelength changes with time such that as the beam passes along a dispersive fibre, it will experience a compression as the chirped wavelengths 'bunch up' (see fig. 4). A gap is opened between the two chirps, pulling their spacetime trajectories apart like a curtain, such that the light will avoid a spatio-temporal region (figure 3), similarly to how the object cloak works in purely spatial dimensions. After the event, the process to create the gap must be performed in reverse so that the probe beam will not show any signs

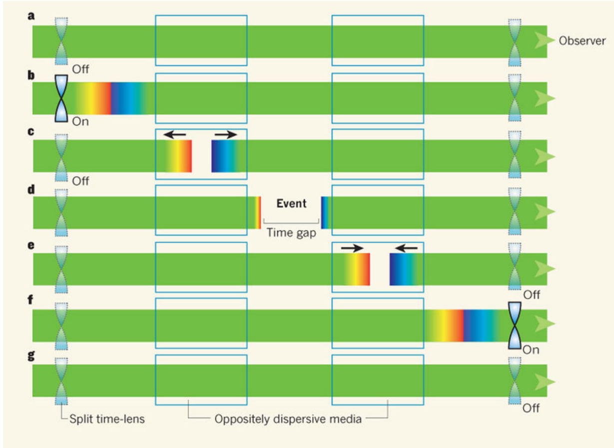

Figure 4: *A schematic for the procedure used by Fridman et al.. a: the beam passes through uninterrupted. b: the first STL is 'turned on' and the beam is chirped. c: the chirped beam passes through the first dispersive medium and a gap forms. d: the beam now has a gap which passes over the space where an event can occur without being 'seen' by the light. e: the beam passes through a second medium which cancels the effects of the first and the gap disappears. f: the beam passes through the second STL and the chirping is 'undone' restoring the light so that no trace of the cloak remains. g: all that is seen by an observer to the right is a continuous stream of light as if nothing had occurred.*
Taken from [32]

of the existence of the cloak, this will require the beam to pass through a medium with the opposite dispersive properties [13] and then a reverse of the first STL.

From the spatial perspective, the beam is being modulated by the STL so that the wavelengths along the beam are at different values, in two smooth chirps with a discontinuity between them. When the beam passes through a length of dispersive media, the dispersion causes the light to slow down, the design is such that the parts of the beam will speed and slow respectively to the leading and trailing parts of the chirp, with each front arriving at the event space at two distinct times. A gap will have opened in the middle where the discontinuity between chirps was and this is the gap that will pass over the space, creating the cloak.

5.2 Practical Cloaks

In 2013 Lukens *et al.*, from Cornell had a paper published in Nature for their demonstration of a spacetime cloak that could operate at telecommunication data rates by using the Talbot effect. Thus enabling a range of applications which the previous attempt would be incapable of, as it only covered 10^{-4} of the required cloaking window [25]. In early 2017 there was another publication regarding a demonstration that could cloak an even larger percentage of the data rate and maintain the cloak through a 5km length of optical fibre

[24].

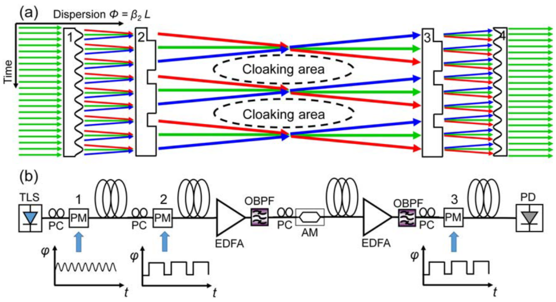

Figure 5: *Schematic of the temporal cloaking. (a) Temporal-ray diagram. (b) Experimental setup. $t - \phi$ diagrams represent the temporal phase modulation patterns and ϕ is the phase. TLS, tunable laser source; PC, polarization controller; PM, phase modulator; DCF, dispersion-compensating fiber; EDFA, Erbium-doped fiber amplifier; OBPF, optical bandpass filter; AM, amplitude modulator; PD, photodetector. (Taken from [24])*

This method was a huge improvement over the first demonstration [21] as that was over the method described in [13]. The most recent cloak was able to open a 196ps window. While this is incredibly short, in comparison to the 20ps and 36ps windows in the prior two demonstrations [21, 25, 24], it is far better. The Talbot effect describes a near field diffraction where the image of the diffraction grating will appear at regular intervals–of the Talbot length–from the grating. There is a temporal analogue of the effect which was utilised by the 2013 demonstration with an effective inverse utilised by the 2016 demonstration [25, 24]. This, as you can see from figure 5, produces cloaks as the radiation propagates and then reproduces the original image after the Talbot length. Data can be passed through the holes in the beam. The method used by Li *et al.* was with a probe from a tuneable laser source at 1557 nm which was phase modulated by a sinusoidal driving signal.
These cloaks are more practical since they are capable of working in the realm of telecommunications data rates, they are also able to cloak multiple events within one 'cloak'. We could potentially hide data transmission through the cloak. But cloaks that last for longer and allow more to be cloaked also allow for other possible applications such as the invisible interruption of an SPU as mentioned earlier.

6 Summary

Spacetime cloaks are very much at the current limits of our abilities. Much research is yet to be done but EM cloaking is an area which has garnered significant interest and STCs are likely to be even more interesting. There has been considerably quick experimental demonstration of STCs which is promising for future developments in the field. Moreover the principles I have talked about in this review can be applied not just to EM cloaks but also to acoustic and electrical cloaks [28, 19, 33]. The field also gives rise to a whole new interest in cloaking and more potential applications, as well as making research into spacetime metamaterials more appealing. Ordinary cloaks also benefit from spacetime cloak research since, as Thompson notes, all cloaks are in spacetime and rely on spacetime transformations of the light.
It is clear that the potential for spacetime cloaks is varied and yet to be fully explored but so far it is proving to be a very interesting subject for research with fairly quick progression and many imaginative concepts such as the playing around with the observed order of events and potential for blowing bubble universes and modelling cosmological models.

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