Investigating different methods of determining the resonant frequency for a radio

A high proportion of the UK listen to the radio regularly, with many still using analogue radios. In order for a radio to tune into a radio station it has to match its resonant frequency with the frequency of the radio station. The quality of the radio being listened to heavily relies on how close the resonant frequency is to the radio station frequency, this means that how accurately you know the resonant frequency is imperative to an enjoyable listening experience. This experiment compares three different methods of obtaining the resonant frequency, one by obtaining the inductance of the coil of wire being used in the radio, one by finding the resonant time period of the system and one by plotting a graph of power absorbed by the coil of wire against frequency. The method involving resonant time period was the most accurate with the inductance method being the least accurate. In conclusion the time period method is more favourable to use when tuning a radio however further research into more accurate ways to determine inductance would provide more insight into this topic.

1.1 Introduction

In the UK 89.6% of the population listen to the radio at least once every week, with 63% of these people listening to music based radio stations. Of the people that will listen to music based radio 49% of them will use an analogue radio [1]. From this information it is clear that many people listen to the radio Analogue signals are prone to interference which highlights the need to optimise the components of analogue radios. Some analogue radios use a simple LCR circuit (a circuit with an inductor, resistor and a capacitor) where the radio signal will produce a frequency that produces a current of the same frequency in the circuit. The amplitude of the circuit will be highest when the radio frequency is equal to the resonant frequency of the circuit. The resonant frequency can be changed by changing either the capacitance of the circuit or the inductance [2].

The equation that proves this can be derived from Faraday's Law of induction (the induced electromotive force (emf) in a closed loop equals the negative of the time rate of change of magnetic flux through the loop [3]):

$$\varepsilon = -N\frac{d\phi_B}{dt} \tag{1}$$

Where ϕ_B is the magnetic flux, t is the time and ε is the induced emf. We can use this with an equation for self-inductance of a coil of wire [4]:

$$L = \frac{N\phi_B}{I} \tag{2}$$

Where L is the self-inductance of the inductor and I is the current in the inductor. This equation, when you rearrange and derivate with respect to time, gives you:

$$N\frac{d\phi_B}{dt} = L\frac{dI}{dt} \tag{3}$$

This can be substituted into equation (1) to get:

$$\varepsilon = -L\frac{dI}{dt} \tag{4}$$

In an LCR circuit the instantaneous voltage drop across the inductor is given by the above equation and given that:

$$I = \frac{dq}{dt} \tag{5}$$

Where q is the charge across the capacitor in the circuit. This can be used to derive an equation for the voltage drop across a capacitor:

$$V_L = L\frac{d}{dt}(I) = L\frac{d}{dt}\left(\frac{dq}{dt}\right)$$

$$= L\frac{d^2q}{dt^2} \tag{6}$$

And using $V = \frac{q}{C}$ you can get:

$$\frac{d^2q}{dt^2} = -\frac{q}{LC} \tag{7}$$

This equation describes simple harmonic motion (of charge) with a period $T_0 = 2\pi\sqrt{LC}$, where C is the capacitance of the capacitor in the circuit and T_0 is the resonant time period and therefore the resonant frequency, $f_0 = \frac{1}{T_0}$.

Combined with equation (7) this gives:

$$L = \frac{1}{4\pi^2 f_0^2 C} \tag{8}$$

Since the inductance will be needed to find the resonant frequency I will need an equation that will enable me to find the inductance. Considering figure 1 we can assume that the time varying current is sinusoidal (AC circuit) so that:

$$I = I_0\sin(\omega t) \tag{9}$$

Where t is the time and ω is the angular frequency. Using equation (9) and the following equation (derived for equation (6)):

$$V_L = L\frac{dI}{dt} = LI_0\omega\cos(\omega t) \tag{10}$$

Where V_L is the voltage drop across the inductor. Given the fact that the voltage drop across the inductor will be $\frac{\pi}{2}$ out of phase with the voltage drop across the resistor, V_R, and using Ohm's Law we can obtain the following equation:

$$V_R = IR = RI_0\sin(\omega t) \tag{11}$$

And incorporating the phase difference and the fact that $\sin\left(\omega t + \frac{\pi}{2}\right) = \cos(\omega t)$ which gives:

$$I_0 = \frac{V_R}{R cos(\omega t)} \tag{12}$$

Which, when substituted into equation (10) gives:

$$V_L^{MAX} = \left(\frac{\omega L}{R}\right)V_R^{MAX} \tag{13}$$

This shows the maximum voltage drop across each because of the oscillating function of I.

The power of the circuit is proportional to the amplitude squared of the output signal for the LCR circuit [5] and given that the definition of the resonant frequency is that it gives the maximum amplitude for a system (the circuit in this case) the resonant frequency should also give the maximum power of the circuit. For small inductances (which will be used in this experiment) we can say that:

$$P = \frac{(V_L)^2}{r} \tag{14}$$

Where P the power is absorbed, V_L^{MAX} is the maximum voltage across the inductor and r is the internal resistance in the inductor.

Therefore by plotting a graph of P against f (frequency) you can determine the peak power as the corresponding power to the resonant frequency.

My task is to compare three ways of experimentally determining f_0 in the hope to improve the accuracy of the tuners of radios that incorporate this method. This will make tuners more accurate as they will have a better estimate for the resonant frequency and will be able to more accurately match their resonant frequency with that of the desired radio signal. The last method will be simply to input values into equation (8).

Of the three different methods, one will involve using equation (8) to find the resonant frequency and the other will involve obtaining the power of the circuit over a range of frequencies and one will involve using a digital oscilloscope to obtain the resonant frequency.

1.2 Materials and Methods:

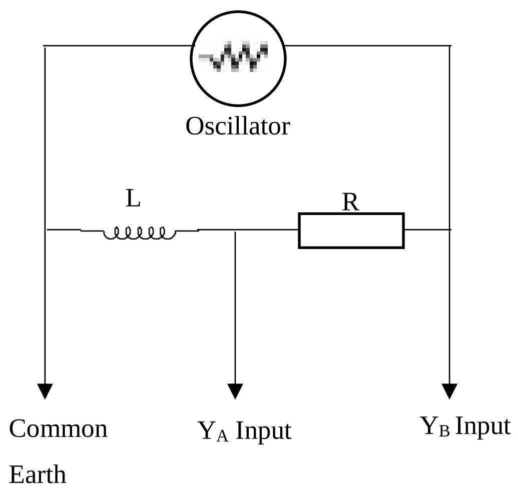

Figure 1. Circuit used to determine L, the inductance of the ferrite coil of wire. R is equal to 100±1kΩ.

First the inductance of the copper coil of wire with a ferrite core was determined. A copper coil with a ferrite core was used because this is what is used in radios. Figure 1 was used to determine this. Note that there is no capacitor in this circuit as it is unnecessary in determining the inductance, L, of the coil of wire. As well as the equipment listed in figure 1, a digital oscilloscope was used (which is where the Y_A and Y_B inputs lead) to display the signal across L (Y_A) and the signal across the resistor (Y_B). Note that the oscilloscope has to be zeroed. The oscillator was set to produce a sinusoidal wave to replicate a radio signal. Values for V_R^{MAX} and V_L^{MAX} can be obtained from the peak to peak voltage readings from the digital oscilloscope with V_R^{MAX} being given by input Y_B. Readings for V_L^{MAX} as a function of frequency were taken and plotted on origin to give the figure 2. The frequency was calculated using the digital reading from the oscilloscope.

Using equation (13) and $\omega = 2\pi f$ the gradient of the voltage-frequency graph can be shown to be $\frac{2\pi V_R^{MAX} L}{R}$ which can lead to a result for the inductance. Errors were calculated for most frequencies using the resolution of the digital scale of the oscilloscope however for lower voltages the 'noise' made the readings from the oscilloscope unreliable and larger than they should have been so another method for obtaining the voltage and errors was used:

By taking an average of the highest and second highest peak of the noise on the positive peak of the larger wave as used as the error in the positive peak measurement, then the same method was applied for the negative peak. These were sum of the squares of these errors was square rooted to get the overall error. Then the individual errors were taken away from the peak to peak voltage to obtain a more accurate measure of V_L. Then the functional approach was used to obtain an error or L.

In the first method used to determine f_0 figure 3 indicates the circuit diagram used, the oscillator was set to produce a square wave. Because of the capacitor there was now a dampened wave input for Y_A shown. From this the resonant time period of a certain number of oscillations (the more the higher the accuracy) which can then be used to get the resonant frequency ($f_0 = \frac{1}{T_0}$). An error for T_0 was found using the resolution of the digital oscilloscope and then the functional approach was used to obtain an error for f_0.

The second method uses equation (14) and figure 3 as the circuit, the oscillator was set back to produce a sinusoidal wave. Frequencies close to that of the previously attained value were used to obtain a range of values of power absorbed by the inductor ($P = \frac{V_L^2}{R}$). Power was plotted against frequency and fitted using the Pearson VII fitting which gave the peak of the graph. Errors were calculated using the resolution of the oscilloscope for the frequency and voltage across L, the functional approach was used to then calculate the error in the power.

The final approach was using equation (8) and plugging in the values for L and C. The errors were obtained using the functional approach.

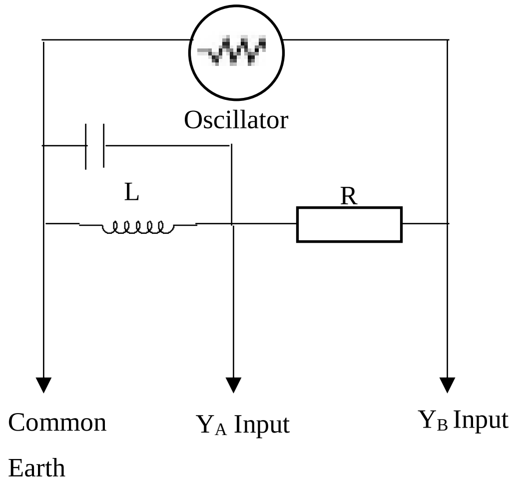

Figure 3. Circuit used to determine f_0 using the first method. R is equal to 100±1kΩ and $C = 0.10 \pm 0.01\mu F$

1.3 Results

<u>Finding the induction:</u>

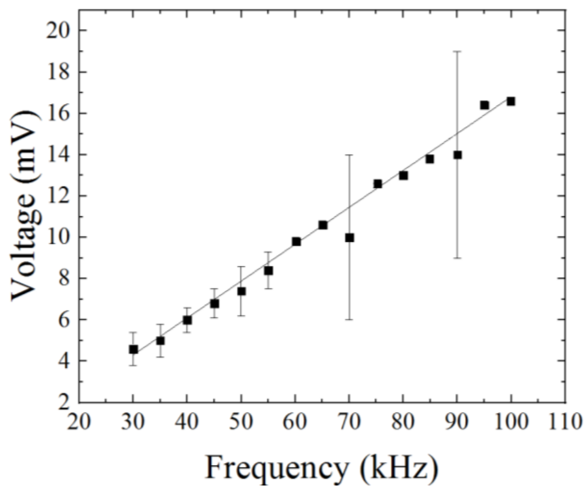

Figure 4. A graph of Voltage against frequency where the gradient is equal to $0.179 \pm 0.005\ mV\ kHz^{-1}$

With the induction being given by:

$$L = \frac{mR}{2\pi V_R^{MAX}}$$

Where m is the gradient of figure 4. With a value the values: $V_R^{MAX} = 4.78 \pm 0.08\, V$ and $R = 100 \pm 1\, k\Omega$ the value for the inductance was: $L = (5.96 \pm 0.17) \times 10^{-4}\, H$.

<u>Finding the resonant frequency:</u>

Method 1:

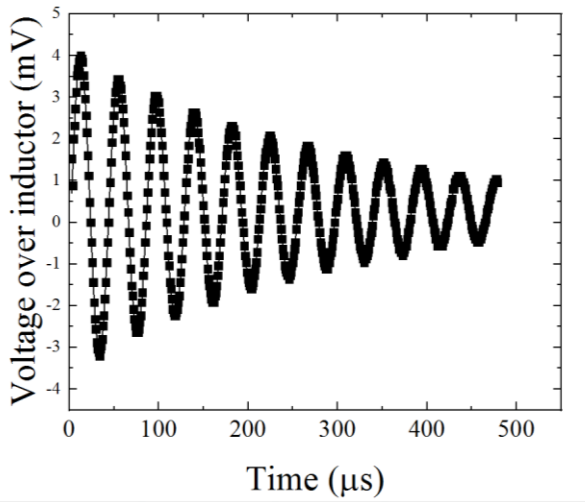

Figure 5. A Voltage-time damping graph which represents the display of the digital oscilloscope. $10T_0$ was measured as $424.0 \pm 0.1\, \mu s$ (errors too small to be seen)

Given that $t_0 = 42.40 \pm 0.01 \mu s$ the value for the resonant frequency as obtained as:

$$f_0 = 23.585 \pm 0.005\, kHz$$

Method 2:

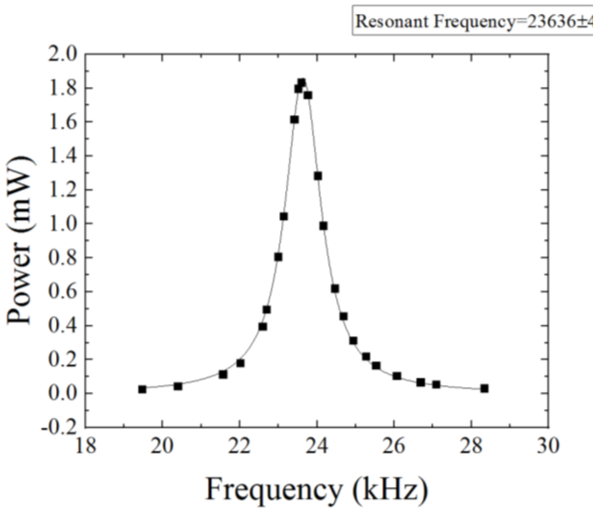

Figure 6. A graph of power against frequency where the peak of the graph is $23636 \pm 4\, Hz$ (errors too small to be seen)

As quoted by the figure caption the peak of the graph, and therefore the resonant frequency, is:

$$f_0 = 23.636 \pm 0.004\, kHz$$

Method 3:

By using $L = (5.96 \pm 0.17) \times 10^{-4}\, H$ and $C = 0.10 \pm 0.01\, \mu F$ the value of f_0 obtained using equation (8) is:

$$f_0 = 21 \pm 1\, kHz$$

1.4 Discussion

My result from method one is very precise ($\pm 0.02\%$), this is due to the minimal error coming from the measurement of T_0 which is due to the oscilloscope having a very low resolution. Also when averaging the wave on the oscilloscope the precision would have increased as it was clearer where the adjacent points of the wave were.

My result for method two is also very precise ($\pm 0.016\%$) and even though the values are not in agreement (from methods one and two) they are still very close (there is a 0.2% difference between the two values) which leads me to believe that both values are accurate. The reason method 2 is precise is also because of the low resolution of the digital oscilloscope however because the scale on the signal generator was $5\ kHz$ per division it was very hard to get values with frequencies close to each other. This means that the fitting on the graph may not have had enough values close to the peak to work out the resonant frequency to the accuracy of the first method.

My result from the third method is not only imprecise ($\pm 5\%$ which is large compared to the other to values) but also appears to be inaccurate (there is a 12% percentage difference in this value and the value from the first method). One of the largest sources of errors from this method (which is obvious from figure 4) is that determining the peak to peak voltages was extremely inaccurate because of the noise of the waves when determining L. Also at 70 and $90\ kHz$ there was an abnormal amount of noise which made the error for these values disproportionally large, which will be due a fault in the oscillator most likely.

A more precise way of obtaining L would be to use the resonant frequency acquired in method one and plug this value into equation (8) however this would have left me with only two methods to determine f_0.

1.5 Conclusion

In conclusion, the first method seems to be the most reliable with the equipment used (even with the smaller error from the second method) which is because of the difficulty in obtaining more than a few values of the power in close proximity to the that of the resonant frequency whilst using the oscillator.

This means that, from my results, it is clear that for more accurate tuning in radios the best method is to decipher the time period of the damping wave created by the capacitor using a digital oscilloscope (or another device that is just as accurate).

Further work could be done in this area on more accurate and precise methods of determining the inductance of the coil wire used within the radio which can then be used in equation (8) to obtain the resonant frequency.

References

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[1] The Communications Market: UK. (2017). [PDF] Ofcom, pages 97, 99. Available at: https://www.ofcom.org.uk/__data/assets/pdf_file/0014/105440/uk-radio-audio.pdf

[2] Hugh D. Young, Roger A Freedman, “University Physics”, page 1060, (Pearson, 2016)

[3] Hugh D. Young, Roger A Freedman, “University Physics”, page 984, (Pearson, 2016)

[4] Hugh D. Young, Roger A Freedman, “University Physics”, page 1018, (Pearson, 2016)

[5] Hugh D. Young, Roger A Freedman, “University Physics”, page 507, (Pearson, 2016)