Determining the Accuracy and Precision of Different Methods of Determining the Speed of Light in a Vacuum, c

Modern day physics require that different values are known to precise accuracies and are accepted all over the globe. Using three methods to obtain a value for c two were found to be the most reliable giving values of $(3.04 \pm 0.09) \times 10^8\ m\ s^{-1}$ and $(3.11 \pm 0.05) \times 10^8 m\ s^{-1}$ for c. The first value coming from a method that plotted phase change against path difference and used the gradient to find c, the second coming from a method of calculating the refractive index of water.

1. Introduction

For many centuries it was thought that the speed of light was infinite, in these times there was no way of "seeing" any evidence for the finite speed of light. One way that experiment that was thought of as evidence for this theory was observing the shadow of the Earth on the Moon [1]. Scientists believed that, if the speed of light was finite, the Earth's shadow on the moon would have a lag relative to the actual position of the Earth. No lag time was observed so the speed of light was thought to be infinite.

The 1st person to calculate a finite speed of light was Olaus Roemer [2]. He observed that there was a noticeable difference in the predicted times of when Jupiter's moons would eclipse and the actual time that they did eclipse and concluded that this was due to the varying length that light has to travel (depending on the relative positions of the Earth and Jupiter) and obtained a value for c of $2.14 \times 10^8\ m\ s^{-1}$ which is 28.6% away from the current accepted value of $2.99792458 \times 10^8\ m\ s^{-1}$.

After this there were many attempts to establish a more accurate value for the speed of light in a vacuum, c, and with the publication of "A Dynamical Theory of the Electromagnetic Field" [4] the speed of light was now defined using two constants: the electric permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}\ m^{-3} kg^{-1} s^4 A^2\ (3s.f)$ and the magnetic permeability of free space, $\mu_0 = 1.26 \times 10^{-6}\ m\ kg\ s^{-2} A - 2$ in the following equation:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{1}$$

This means that by accurately knowing ε_0 and μ_0 it was possible to know c to a greater accuracy.

This is exactly what Rosa and Dorsey did to achieve the value: $c = 2.99788 \times 10^8\ m\ s^{-1}$ which is 0.0015% away from the accepted value [5].

Since then there have been values of improved accuracies with the most accurate being found in 1972: $c = 299792457 \pm 1\ m\ s^{-1}$ [6]. However since 1983 c has taken to be a constant used in the definition of a metre: "The metre is the length of the path travelled by light in a vacuum during a time interval of $1/299792458$ of a second" [3] which fixes the accepted value for c to 299792458 m s^{-1}.

Since the value of c is now accepted to be a fixed value (in a vacuum) the following experiment is not to try and find a more accurate value for c but is to explore different methods of obtaining this value in order to evaluate how accurate the equipment being used is. This is important because some of the equipment being used is basic or slightly old; correctly carrying out this experiment will determine how accurate the equipment being used is.

Two of the methods used for measuring c involve measuring the path difference and the shift of two waves. The following equation relates the speed of light, c, to the frequency, f, and wavelength, λ, of the light [7]:

$$c = f\lambda \tag{2}$$

By considering two identical sources of light incident on a detector, one travelling a distance of x length and the other travelling a distance of x-Δx, the two light beams will not arrive at the detector at the same time which might mean they are not in phase. The two beams will only be in phase if Δx is an integer multiple of the wavelength of the light:

$$\Delta x = n\lambda \tag{3}$$

The phase difference between the two waves is given by:

$$\Delta\phi = \omega\Delta t \tag{4}$$

Where $\Delta\phi$ is the phase change, ω is the angular frequency ($\omega = 2\pi f$) and Δt is the shift in the wave. The phase difference is proportional to the path difference Δx and includes the speed of light in the following equation:

$$c = \frac{\Delta x}{\Delta\phi}\omega_m \tag{5}$$

Where ω_m is the modulation angular frequency.

Another equation involving the speed of light also involves the refractive index, n:

$$n = \frac{c}{v} \tag{6}$$

By introducing different media with a refractive index greater than one the light will take longer to travel the same distance. This introduces the idea of an optical path length, OPL, which will increase (even if the actual path length remains constant) if such media are added to the path:

$$OPL = \sum_i n_i x_i \tag{7}$$

Where x_i is the length of the part of the path having a refractive index n_i. Introducing a transparent medium will change the speed of light and also the phase (compared to the same situation without the medium). This observation is used to calculate the refractive index if the medium using:

$$n = 1 + \frac{\Delta x}{l} \tag{8}$$

Where l is the distance travelled in the medium.

2 Materials and Methods

The apparatus used was an optical workbench (with a length scale across the bench), an LED source, a focusing lens a receiver, an oscilloscope and a tube of water. The receiver created the phase shift by splitting the 60 MHz signal from the LED into two beams (one with an unchanged frequency and one with a frequency of $59.9Mhz$) and combining them. This creates a resultant beam of about $100KHz$ which allows me to obtain the phase shift of the waves when projected onto an oscilloscope with the original source.

For my first method uses equation (5) and does not involve the tube of water. The LED is at projected on to the lens (which are both on the optical bench), which focuses the light on the receiver, which gives the oscilloscope a signal. The phase change was obtained by measuring Δt for the two signals and converting this into a phase shift using equation (4), this was done for multiple positions; from which Δx was obtained, and a graph of phase change against path difference (Δx) was plotted, from which c was obtained using equation (5) and by calculating ω_m using $f_m = 60MHz$.

The second method is the same as the first, except that the phase change was measured using the Lissajous figure on the oscilloscope.

The third method uses equation (8). The tube of water is now placed between the LED and the focusing lens on the optical bench, however the tube of water does not have any water in it at this point. The LED or receiver phase control knob are adjusted until the two signals are in phase (using the Lissajous method). Now the tube is filled up with water and placed back into its original position and the LED is moved along the optical bench until the two signals are back in phase; the difference in these two positions (Δx) is found from which the refractive index of water can be found using equation (8). This value can be inserted into equation (6) with the speed of light in water to obtain the c.

3 Results

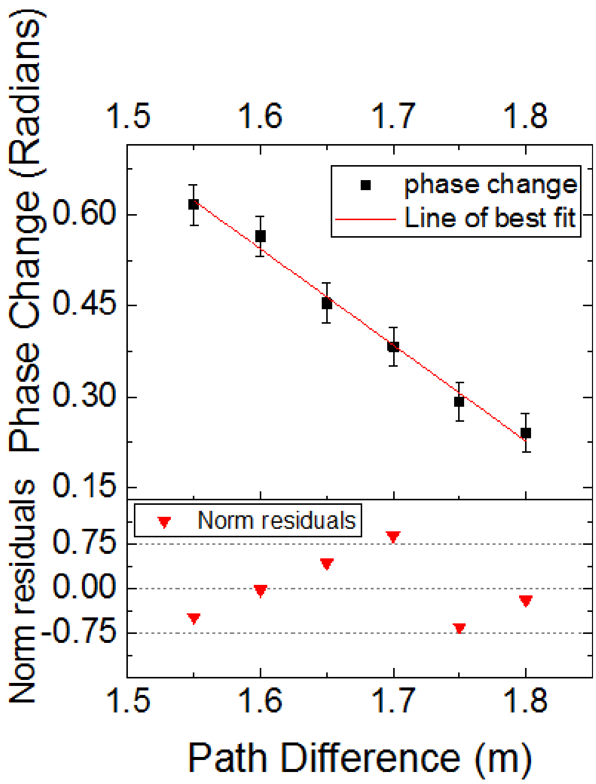

Figure. 1. A graph of Phase change against Path Difference with normalised residuals from method one, the gradient is equal to $-1.58 \pm 0.07\ Rad\ m^{-1}$.

From equation (5) it is clear that the gradient of this graph, m, is equal to: $\frac{\omega m}{c}$. From method one: $c = (2.4 \pm 0.1) \times 10^8\ m\ s^{-1}$.
Since the accepted value is not within this error there could be a source of systematic errors from the equipment being used, or maybe the chosen errors are too small (the error on c was calculated using the functional approach).

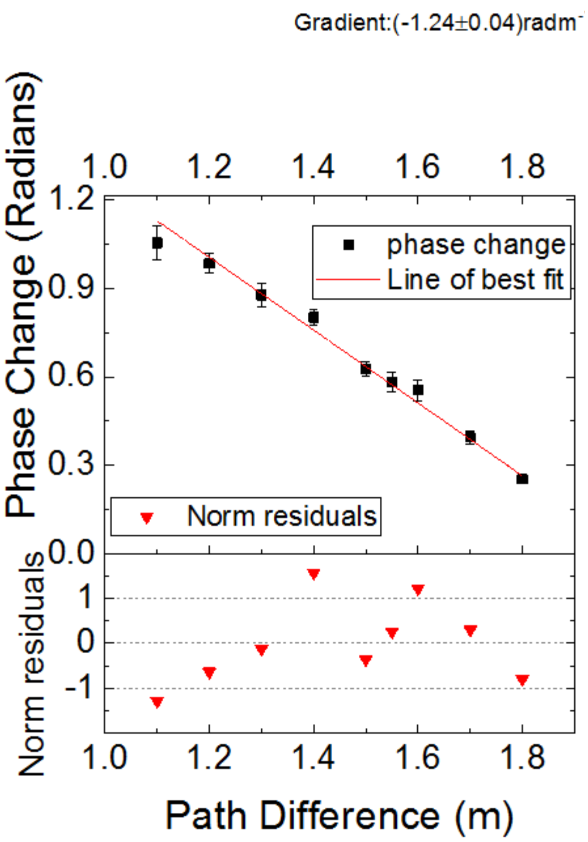

Figure. 2. A graph of Phase change against Path Difference with normalised residuals from method two, the gradient is equal to $-1.24 \pm 0.04\ Rad\ m^{-1}$.

For the second method the value of c was found to be $(3.04 \pm 0.09) \times 10^8\ m\ s^{-1}$ (errors calculated using the functional method). The accepted value of c is within this error which leads me to believe that method two does not have a large systematic error as with the first method.

For the third method a value of $\Delta x = 38.8 \pm 0.5\ cm$ with a value of $l = 103.2 \pm 0.5\ cm$. Using this information the value of the refractive index of water, n, was calculated to be: $n = 1.38 \pm 0.02$. The speed of light in water was found by using the actual value for the refractive index of water (1.33)[8] and using equation (6) to obtain a value for v of: $2.25407863158\ \times 10^8\ m\ s^{-1}$.
This leads to a value of c found to be $(3.11 \pm 0.05) \times 10^8 m\ s^{-1}$ which does not include the actual value for c in its error.

4 Discussion

By simply looking at the errors method two stands out as the most effective method for determining a value for c and therefore used the most accurate and precise equipment. The value in the first method deviates by 20% from the accepted value for c and is the least accurate and least precise method. The reason for this can be easily isolated when comparing methods one and two; the methods are identical except for the method for obtaining the phase change: using the oscilloscope to determine Δt and converting this into a phase change was somewhat problematic since the signal on the oscilloscope has a certain amount of noise, also by using the cursor on the oscilloscope the error was only increased as the cursor is relatively thick; this creates some uncertainty in exactly where the cursor is when reading the value off of the oscilloscope. This means that the error given to this measurement may have been too small; this is backed up by the normalised residuals from Figure 1 as all of the values are within one error of the line of best fit (Figure 2 shows exactly two thirds of the normalised residuals within a value of ± 1 which is what should be expected from an accurate and precise experiment). There also seems to be a pattern to the normalised residuals from Figure 1; this suggests that a large systematic error is present. This error is likely to be from the standard axis format of the oscilloscope (Voltage against Time) which significantly decreases when switching to the Lissajous format on the oscilloscope.

This not only presents the possibility of a systematic error from the oscilloscope but it also suggests that the 'ruler' across the optical bench is accurate and precise since the methods of highest accuracy (method two) and precision (method three) obtain realistic values for c and rely on the ruler. This can also be said for using the Lissajous figure; this was apparent during the experiment as obtaining a value for the phase change was made significantly easier when switching to the Lissajous method.

Although method three gives a value of c within two errors of the accepted value the error may have been underestimated. This is because the LED and the focusing lens had wide bases to attach onto the optical bench with the horizontal position of the actual instrument being roughly in the middle of the base. This meant that Δx and l may have had larger errors than originally predicted. This had more of an effect on method three as this only used one value and not a range of values as in method two.

Improvements can be made to all three methods. Method one, in nothing else, was used as a control to ensure that using the Lissajous figure to determine the phase change was more reliable (which was true); so an obvious improvement to this method would be to use the Lissajous figure. However, the aim was to determine the accuracy and precision of the instruments being used are. Therefore this method is completely valid for determining the accuracy of the equipment, but when trying to obtain an accurate value for the speed of light (or any value) this method seems unreliable.

Both methods two and three suggest that accurately measuring Δx was difficult because of the wide bases of the optical instruments so a more accurate way to measure the positions of the instruments is an improvement for all of the methods.

4 Conclusion

I have successfully deconstructed three methods for determining the speed of light in a vacuum and proposed the one which uses the most accurate and precise equipment in the best way. Because of this I believe the experiment has been successful.

The experiment is a very basic insight into what can be done to ensure the instruments being used in modern day physics are as accurate and precise as reasonably possible, but this idea can be taken to extremes to create and test new equipment that will increase our understanding of physics. This can be done by experimentally determining values to previously unprecedented accuracies.

However, this is not limited to testing new equipment, as I have done in this experiment this kind of test can be used to test the reliability of older equipment. This is extremely useful since few people have the resources to create or own the newest of equipment, many people have to use old and potentially faulty equipment.

References

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