Modelling diffraction of a coherent source of light through single and multiple slits

In recent decades the physics of single and multiple slit diffraction has revolutionised physics as a whole with it being the building blocks of many different research areas. Physicist in these areas therefore require accurate models of these different types of diffraction with which to move even further forward in their respective findings. My aim in this experiment is to accurately model single and multiple slit diffraction computationally. The outcome of this experiment is that the alignment of the instruments used to obtain an accurate model is integral to the experiment and thus proved to be the dominant error withholding me from more accurate models.

1. Introduction

Single and multiple slit diffraction is used in many areas of physics such as crystallography and holography. One example of the more recent uses of crystallography is structural analysis of DNA [1] which uses X-ray crystallography. Holography can be used to record and reproduce an image to form a new three-dimensional image which involves interference patterns and diffraction [2].

The more accurately diffraction can be modelled the more effective it can be used alongside advances in physics which involves diffraction. This is why my report focuses on modelling diffraction using a different number of slits and changing the slit width in an attempt further understand the basic principles defining more recent complex studies.

The amplitude of the electric field due to single slit diffraction, E_P, is given by [3]:

$$E_P = \frac{E_0 \sin\left(\frac{\beta}{2}\right)}{(\frac{\beta}{2})} \tag{1}$$

Where E_0 is the arc length of the curved trail of path difference between the top and bottom strips of an interference pattern and β is the total phase difference between the top and bottom strips. Since the intensity, I, at each point is proportional to the square of the amplitude and if I_0 is defined as the intensity when θ is zero and β is also zero then the intensity at any point of a single slit diffraction pattern is given by:

$$I = I_0 \left(\frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}} \right)^2 \tag{2}$$

Given that functions in the form $\frac{sinx}{x} = sincx$ this gives the following equation:

$$I = I_0 sinc^2 (\frac{\beta}{2}) \tag{3}$$

This shows that for a single slit the pattern on the screen will show a bright fringe at $\beta = 0$, and also therefore $\theta = 0$ (since $\beta = \frac{2\pi a}{\lambda}\sin(\theta)$ can be shown geometrically where θ is the angle between the normal to the screen and the vector pointing towards the slit, λ is the wavelength and a is the width of the slit), with multiple less intense fringes being shown the more you increase the angle θ.

For double slit diffraction the intensity, I, can be shown to be equal to [3]:

$$I = I_0 \cos^2\left(\frac{\varphi}{2}\right) sinc^2\left(\frac{\beta}{2}\right) \tag{4}$$

Where $\varphi = \frac{2\pi d}{\lambda}\sin(\theta)$, and d is the slit separation.

This equation shows that, in multiple slit diffraction, the intensity can be represented as a product of the intensity from single slit diffraction and a squared cosine function.

These relationships are only available to us because of Fraunhofer diffraction [3]. This is a case where the source of light, the slit and the image are far enough away that the light from the source to the slit can be considered parallel; also the light from the slit to the image can be considered parallel (therefore the angle that these rays of light create with the light that travels straight through the slit without diffracting can be considered equal). This simplifies the relationship considerably as Fresnel diffraction (where the source, slit and image are relatively close) involves double integrals to obtain the electric field diffraction pattern [5].

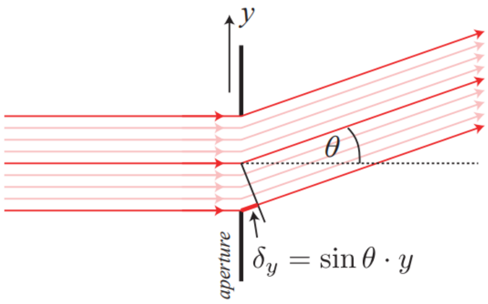

Figure. 1. A diagram explaining Fraunhofer diffraction where the beams of light are approximated to be parallel [4].

The aim of this is experiment is to accurately model the diffraction patterns of a class II semiconductor diode laser when passed through different single and multiple slits in order to understand what effects different characteristics of the diffraction pattern.

1.2 Materials and Methods

This experiment utilises a convex lens to focus the diffraction patterns on to a CCD (Charge-coupled device) camera in order to observe the diffraction pattern on the programme Leybold videocom. This was integral to the experiment as observing the pattern on a screen would have increased the errors of the measurements dramatically. Figure 2 represents the order of the optical components used during the experiment:

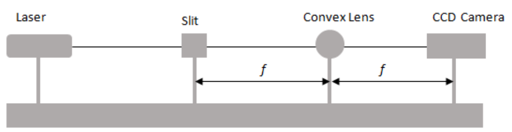

Figure. 2. A diagram of the components used in the

experiment where f is the focal length of the lens. Both the slit and the camera are a length of f away from the lens.

When observing the single slit diffraction a lens of focal length 200mm was used and slits of width 0.4mm, 0.15mm and 0.075mm were used with a light wavelength of 65nm. The experiment was done in a dark room to minimise background light. Before any slit was used the laser had to be aligned with the CCD camera, without the slit or the lens in the way; which proved to be difficult since the laser was easy to knock out of place even when reasonable alignment had been reached; this proved to be one of the largest sources of errors as it relied on a high level of precision when using imprecise instruments. After this the slit and the lens can be aligned with the laser making sure the laser goes through the centre of both objects. Now when using the videocom intensity application the diffraction pattern should be too large for the screen. Now intensity blockers have to be used in order to observe the pattern properly: By over-blocking the laser, and making the pattern too small, it is much easier to centre on the computer screen using the height and side to side displacement controls for the CCD camera. Once this is done unblock the laser until the maximum intensity is the maximum it can whilst remaining fully on the screen. Now a theoretical curve can be plotted by inputting values for the wavelength, focal length and slit width into the application and a theoretical curve will be plotted on the same graph as the experimental curve.

However, even when the laser is refocused and almost perfectly aligned with the other objects the experimental pattern is always slightly off from the theoretical pattern, this is due to not only the alignment process but also the fact that there was some background light even in the dark room the experiment took place. To get a more appropriate pattern the data was exported into a Microsoft excel spreadsheet and the solver add in was used to vary parameters of the theoretical pattern in order to make it as close as possible to the experimental one. Once this was achieved a line graph of the raw data was created with both patterns for each slit on the graph.

For the multiple slit diffraction each slit grating has slit widths of 0.15mm and slit separations of 0.25mm with 2,3,4 and 5 slits being used (the number of slits shall now be given the symbol *N*). A different convex lens was also used which had a focal length of 50mm. The same procedure was used to align the laser and the objects which leads to the dominant error also being this process.

A theoretical pattern was found using the same method as the single slit diffraction and proved to be more useful in this case as these experimental patterns were even further away than the single slit ones. This suggests that the increased focal length and possibly the new diffraction gratings added to the error. It is more likely that the increased focal length is the main source of added errors as the lens is there to focus the diffraction pattern which becomes even harder to align when you more than double the distance between three of the four instruments being used. The successive peaks in the patterns where calculated using origin's graph reader function and the error in these values were calculated by obtaining the lowest possible range of where the peak can be (visually) and halving this value.

1.3 Results

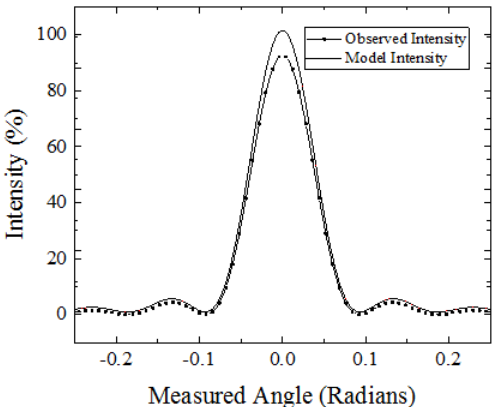

Figure. 3. Single slit diffraction with a slit width of 0.4mm.

The model peak intensity, $I_{M0} = 101.6 \pm 0.2\%$ with the experimental peak intensity, $I_{E0} = 92.4 \pm 0.2\%$. The 1st order maximum intensity for the model, $I_{M1} = 5.6 \pm 0.2\%$ with the experimental pattern having $I_{E1} = 4.3 \pm 0.2\%$ both having an angle of $\theta = \pm 0.132 \pm 0.004\ Radians$.

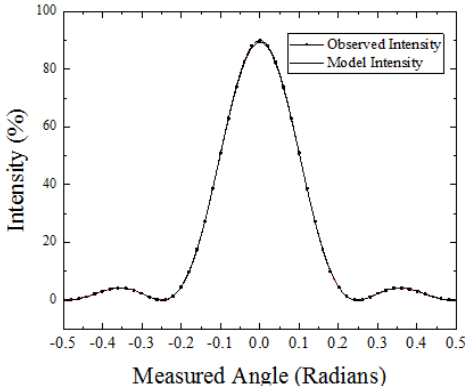

Figure. 4. Single slit diffraction with a slit width of 0.15mm.

$I_{M0} = 89.4 \pm 0.2\%$ $I_{E0} = 89.9 \pm 0.2\%$ $I_1 = 4.3 \pm 0.2\%$ $\theta = \pm 0.355 \pm 0.005\ Radians$.

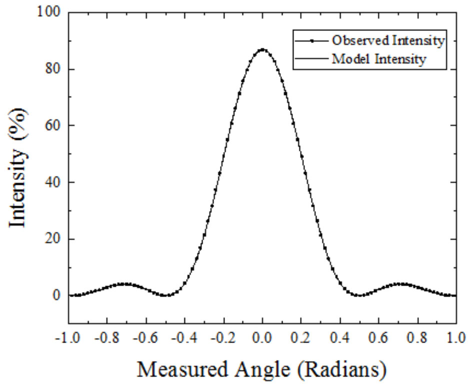

Figure. 5. Single slit diffraction with a slit width of 0.075mm.

$I_0 = 86.9 \pm 0.2\%$ $I_1 = 4.2 \pm 0.2\%$

$\theta = 0.709 \pm 0.014\ Radians$.

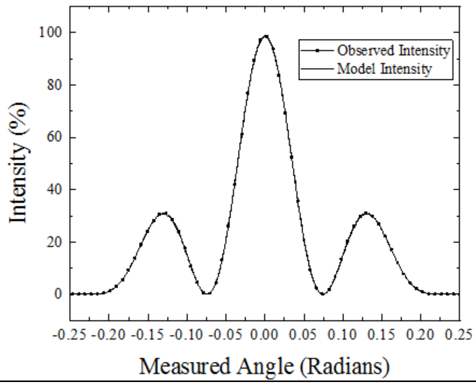

Figure. 6. Multiple slit diffraction with two slits. The slit width is 0.15mm and the slit separation is 0.25mm for

all double slit diffraction patterns.

$I_{M0} = 98.4 \pm 0.2\%$ $I_{E0} = 98.7 \pm 0.2\%$. $I_{M1} = 30.7 \pm 0.2\%$ $I_{E1} = 31.1 \pm 0.2\%$ $\theta_1 = \pm 0.130 \pm 0.004 Radians$.

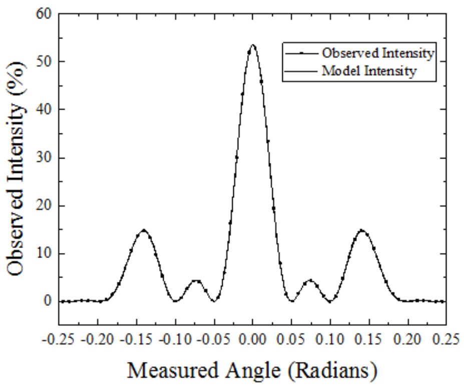

Figure. 7. Multiple slit diffraction with three slits.

$I_0 = 53.5 \pm 0.2\%$ $I_2 = 14.7 \pm 0.2\%$ $\theta_2 = \pm 0.142 \pm 0.005$.

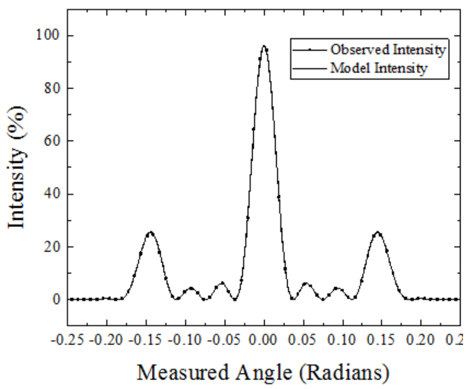

Figure. 8. Multiple slit diffraction with four slits.

$I_{M0} = 95.9 \pm 0.2\%$ $I_{E0} = 96.4 \pm 0.2\%$. $I_{M3} = 25.7 \pm 0.2\%$ $I_{E3} = 25.2 \pm 0.2\%$ $\theta_3 = \pm 0.144 \pm 0.012\ Radians$.

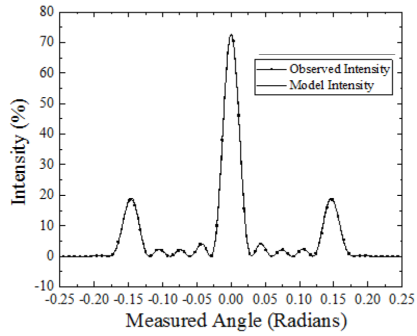

Figure. 9. Multiple slit diffraction with five slits.

$I_{M0} = 72.7 \pm 0.2\%$ $I_{E0} = 72.9 \pm 0.2\%$ $I_{M4} = 19.1 \pm 0.2\%$ $\theta_{M4} = \pm 0.146 \pm 0.006\ Radians$ $I_{E4} = 18.7 \pm 0.2\%$ $\theta_{E4} = \pm 0.149 \pm 0.008\ Raddians$

1.4 Discussion

The model from figure 3 is visually and analytically furthest away from the experimental pattern. It is a minimum of 9.5% (using the different bounds for the intensities) the closest percentage difference between the model and the peek intensities is figure 4 which has a maximum difference of 2.1%. What this suggests is that the 0.4mm slit could have been slightly off with its slit width or that the alignment of the laser was off for this slit. It is more likely that it is the alignment of the slit for a few reasons: When aligning the slit and the laser none of the intensity blockers can be present as you cannot see the laser on the edges of the slit, so when putting the blockers back in the laser might have moved slightly; or even more likely is the likelihood of the laser being miss-aligned with the CCD itself (which was realigned after every slit to make sure the best readings were available).

In terms of the theory behind this experiment it is clear to see that with a decreasing slit width, the more the laser will spread out on the CCD camera. This is an expected result because waves will diffract more around a slit with a width closer to that of the wavelength of the waves [3]. Although this can be seen as trivial it is important to be able to model the simplest of theories in order model the more complex and intricate ones.

A trend shown by looking at the multiple slit diffraction patterns is that as N increases the maxima will get thinner and that there will be one more smaller maximum in between the significant maxima. To be more specific the number of smaller maxima between the significant maxima is equal to N-2.

However the angle of the 2^{nd} significant maxima does seem to stay somewhat constant (varying from $0.149 \pm 0.008\, Radians$ to $0.130 \pm 0.004\, Radians$).

Overall the variation between the model and the experimental patterns don't vary much at all, however this was after using the solver add in for Microsoft excel.

1.5 Conclusion

Whilst the models fit well with the experimental patterns, this was after fiddling with the model so if a more realistic model were to be needed one of the ways of achieving this would be to have an improved method of aligning the instruments and to do the experiment in a darker room with no background light. Another solution to the background light would be to use non visible electromagnetic radiation.

For future study of emission spectroscopy understanding diffraction patterns is essential, one particular use for spectroscopy is to use x-ray emission spectra to analyse the electronic structure of solids and liquids [6] which is one of many more modern uses for spectroscopy that undoubtedly proves how important this area of physics is.

With all this in mind I believe I have accomplished my task with decent success which would be much greater if a few tweaks were made to the experiment as a whole.

References

[1] Laane Jaane et al. "Frontiers and Advances of Molecular Spectroscopy", (Elsevier Science, 2018)

[2] Pascal Picart et al. "New Techniques in Digital Holography", (Wiley, 2015)

[3] Hugh D. Young, Roger A Freedman, "University Physics", (Pearson, 2016)

[4] Waves and Diffraction, [PDF] University of Oxford Department of Physics, page 3. Available at: https://www2.physics.ox.ac.uk/sites/default/files/optics-yr2-5-6.pdf

[5] Rayleigh-Sommerfeld Diffraction Integral of the First Kind Circular Apertures. [PDF] University of Sydney School of Physics, page 2. Available at http://www.physics.usyd.edu.au/teach_res/mp/op/doc/op_rs1_circle.pdf

[6] J. Nordgren, G. Bray, S. Cramm, R. Nyholm, J.-E. Rubensson, and N. Wassdahl. Review of Scientific Instruments 60, 1690 (1989); https://doi.org/10.1063/1.1140929 Published Online: 19 August 1998