The variables used in the Active Inference Thermostat agent:

1. *temperature*: the current temperature of the system being controlled by the Active Inference Thermostat agent.
2. T: The target temperature, which is the temperature that the thermostat is trying to maintain.
3. P: The prior belief about the temperature, which is used to update the posterior belief based on new sensory information.
4. E: The energy or prediction error, which is the difference between the predicted temperature and the actual temperature.
5. H: The entropy or uncertainty about the prediction error, which represents the degree of confidence in the prediction error.
6. F: The free energy, which is the difference between the entropy and the energy. It is a measure of the surprise or unexpectedness of the sensory input.
7. k: The control precision, which is the inverse of the expected variance of the control signal. It determines the strength of the control action.
8. γ: The precision parameter of the noise model, which represents the precision of the sensory input.
9. ϕ: The free energy contribution from the noise model, which represents the cost of acquiring sensory information.
10. μ: The mean of the posterior distribution, which represents the best estimate of the temperature.
11. σ: The standard deviation of the posterior distribution, which represents the uncertainty or precision of the estimate.
12. α: The learning rate, which determines the speed at which the prior parameters are updated based on new sensory information.

Here are the key equations that describe the operation of the Active Inference Thermostat agent.

1. Generative model: The generative model takes in the current temperature (*temperature*) and the parameters of the prior distribution (T, P, μ, and σ) and outputs the precision-weighted prediction error (pe_{pw}). Mathematically, we can express this as:

$$pe_{pw} = \frac{1}{\sigma^2}(T\mu + P - temperature) \tag{1}$$

2. Free energy: The free energy is calculated based on the precision-weighted prediction error (pe_{pw}), the parameters of the noise model (ϕ and γ), and the control parameter (k). Mathematically, we can express this as:

$$E = \frac{1}{2} pe_{pw}^2 \tag{2}$$

$$H = \frac{\gamma}{2} \ln(2\pi) + \frac{\gamma}{2} \ln(k^2) + \frac{\phi}{2\gamma} \tag{3}$$

$$F = E + H \tag{4}$$

3. Active inference update: The active inference update takes in the generative model, the current free energy (F), the prior parameters (T, P, μ, and σ), the control parameter (k), the current temperature (*temperature*), and the learning rate (α), and outputs updated values for the prior parameters and the free energy. Mathematically, we can express this as:

$$\frac{\partial F}{\partial T} = -\frac{pe_{pw}}{k^2} \mu \tag{5}$$

$$\frac{\partial F}{\partial P} = -\frac{pe_{pw}}{k^2} \tag{6}$$

$$\frac{\partial F}{\partial \mu} = \frac{pe_{pw}}{k^2} T \tag{7}$$

$$\frac{\partial F}{\partial \sigma} = -\frac{pe_{pw}}{k^2} \sigma + \frac{\gamma}{\sigma} \tag{8}$$

$$T_{new} = T - \alpha \frac{\partial F}{\partial T} \tag{9}$$

$$P_{new} = P - \alpha \frac{\partial F}{\partial P} \tag{10}$$

$$\mu_{new} = \mu - \alpha \frac{\partial F}{\partial \mu} \tag{11}$$

$$\sigma_{new} = \sigma - \alpha \frac{\partial F}{\partial \sigma} \tag{12}$$

$$pe_{pw,new} = \frac{1}{\sigma_{new}^2} (T_{new} \mu_{new} + P_{new} - temperature) \tag{13}$$

$$F_{new} = E_{new} + H \tag{14}$$

4. Control action: The control action is based on the current free energy and the set point temperature (*setpoint*). If the free energy is greater than a threshold value ($F_{threshold}$), the control action decreases the temperature by a fixed amount (Δ). Otherwise, the control action increases the temperature by a fixed amount (Δ). Mathematically, we can express this as:

$$\Delta T = \begin{cases} -\Delta, & F > F_{threshold} \\ \Delta, & F \leq F_{threshold} \end{cases}$$

$$T_{new} = T + \Delta T$$

Once the control action has been calculated, the updated value of the temperature (T_{new}) is passed back to the generative model, which in turn generates a new prediction error ($pe_{pw,new}$). This prediction error is then used in the active inference update step to compute updated values for the prior parameters and the free energy, and the process repeats.