MATH3024 Project

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Introduction

This is the project for Fraser Paterson

Questions

Question 1

The system for question 1 is as follows:

$$\dot{x} = \alpha(y - x - f(x))$$
$$\dot{y} = x - y + z$$
$$\dot{x} = -\beta y$$

where

$$f(x) = bx + \frac{1}{2}(a-b)[|x+1| - |x-1|]$$

and $\alpha = 10.0, \beta = 14.87, a = -1.27, b = -0.68$

1.a This is as described in my interem report.

1.b For an x to x coupling we take a copy of the entire system and add the difference in the x coordinates, multiplied by the coupling strength: σ, to both x coordinates in each respective system. Let each coordinate dynamics be given by a primed value, hence we have:

$$\dot{x} = \alpha(y - x - f(x)) + \sigma(x' - x)$$
$$\dot{y} = x - y + z$$
$$\dot{x} = -\beta y$$
$$\dot{x'} = \alpha(y' - x' - f(x')) + \sigma(x - x')$$
$$\dot{y'} = x' - y' + z'$$

$$\dot{x'} = -\beta y'$$

where

$$f(x') = bx + \frac{1}{2}(a-b)[|x'+1| - |x'-1|]$$

and the parameters remain as above.

To analyse the stability of the coupling for any given $\sigma \in [0, 10.0]$ we define the following errors in each respective component of the coupled dynamics:

$$e_x = x - x'$$
$$e_y = y - y'$$
$$e_z = z - z'$$

Substituting in each respective expression for x and x' into these error equations, we have:

$$e_x = \alpha(e_y - e_x - (f(x) - f(x'))) - 2\sigma e_x$$

Note, that by the intermidiate value theorem $f(x) - f(x') = f'(\eta)(x - x')$ we may express the above as:

$$e_x = \alpha(e_y - e_x - f'(\eta)e_x) - 2\sigma e_x$$

Now, $f'(\eta) =$ a or b. Hence let $f'(\eta) = \xi$ and so we have: $e_x = (-\alpha - \alpha\xi - 2\sigma)e_x + \alpha e_y$. Performing a similar set of substitutions for the y and z errors yields the full error dynamics:

$$e_x = (-\alpha - \alpha\xi - 2\sigma)e_x + \alpha e_y$$
$$e_y = e_x - e_y + e_z$$
$$e_z = -\beta e_y$$

Expressing the error dynamics as a matrix equation:

$$\begin{pmatrix} \dot{e_x} \\ \dot{e_y} \\ \dot{e_z} \end{pmatrix} = \begin{pmatrix} -\alpha - \alpha\xi - 2\sigma & \alpha & 0 \\ 1 & -1 & 0 \\ 0 & -\beta & 0 \end{pmatrix} \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}$$

NO NEED TO CONDUCT EXACT VALUE ON σ MERELY NEED TO REFERENCE THE GRAPHS If we are to have stable synchronisation, we require σ such that all the eigenvalues of the above matrix are real and negative, thus we find the roots of the corresponding characteristic polynomial:

$$\lambda^3 + (\alpha + \alpha\xi + 2\sigma + 1)\lambda^2 + (\alpha\xi + 2\sigma + \beta)\lambda + (\alpha\beta + \alpha\xi\beta + 2\sigma\beta) = 0$$

Now for any polynomial of the form:

$$\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$$

The Routh-Hurwitz criterion specifies the following 2 conditions on the coefficents, sufficent to gurantee that all roots are real and negative. These are:

$$a_0 > 0, a_1 > 0 \;\; and \;\; a_2 a_1 - a_0 > 0$$

Applying these conditions to the characteristic polynomial in question, substituting in the values for α, β, a and b and $f'(\eta) = \xi = a$ we yield the following condition on sigma to ensure stable synchronisation of the x to x coupling:

$$\sigma > 5.56$$

1.c In the case of a y to y and z to z coupling, we perform the same series of operations as in the above x to x coupling, making sure to add the coupling term to the correct dynamical component

For the y to y coupling we have:

$$\dot{x} = \alpha(y - x - f(x))$$
$$\dot{y} = x - y + z + \sigma(y - y')$$
$$\dot{x} = -\beta y$$
$$\dot{x'} = \alpha(y' - x' - f(x'))$$
$$\dot{y'} = x' - y' + z' + \sigma(y' - y)$$
$$\dot{x'} = -\beta y'$$

and $f(x')$ is as before. Matrix equation Characteristic polynomial Ruth Hurwitz Condition on σ

Question 2

The maths for question 2 The following is a graph:

Question 3

Maths for question 3

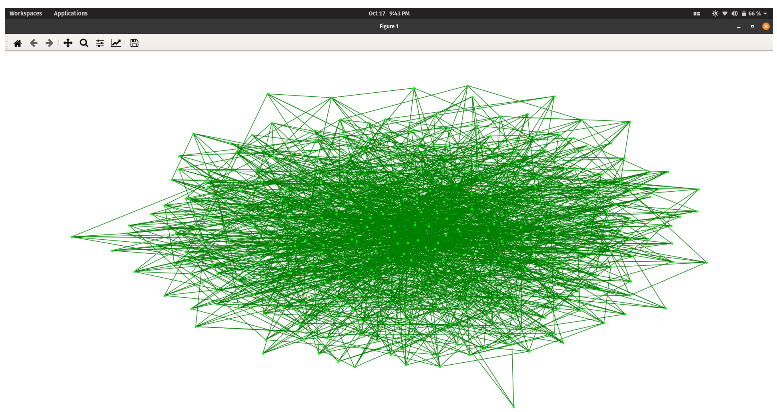

Figure 1: A random graph with 300 vertices