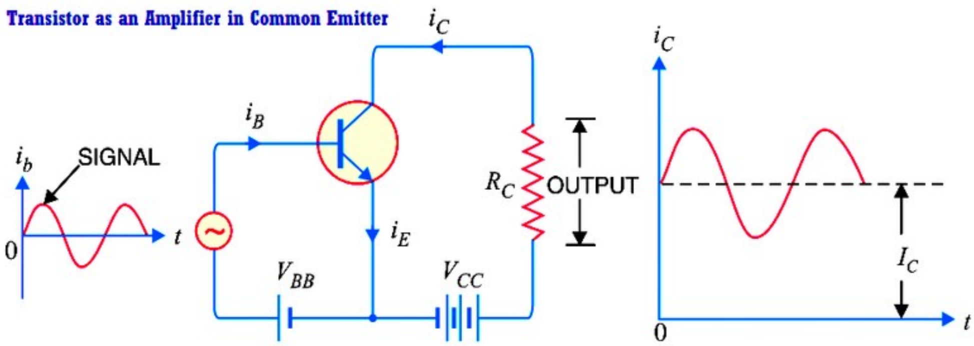
Bipolar transistors: Dynamic model

1. Transistor as an Amplifier in Common Emitter

The below Fig. shows the common emitter NPN amplifier circuit. Note that a battery V_{BB} is connected in the input circuit in addition to the signal voltage. This DC voltage is known as *bias voltage* and its magnitude is such that it always keeps the emitter-base junction forward biased regardless of the polarity of the signal source.



1.1. Operation

During the positive half-cycle of the signal, the forward bias across the emitter-base junction is increased. Therefore, more electrons flow from the emitter to the collector *via* the base. This causes an increase in collector current. The increased collector current produces a greater voltage drop across the collector load resistance R_C. However, during the negative half-cycle of the signal, the forward bias across emitter-base junction is decreased. Therefore, collector current decreases. This results in the decreased output voltage (in the opposite direction). Hence, an amplified output is obtained across the load.

1.2. Analysis of collector currents

When no signal is applied, the input circuit is forward biased by the battery V_{BB}. Therefore, a DC collector current I_C flows in the collector circuit. This is called *zero signal collector current*. When the signal voltage is applied, the forward bias on the emitter-base junction increases or decreases depending upon whether the signal is positive or negative. During the positive half-cycle of the signal, the forward bias on emitter-base junction is increased, causing total collector current i_C to increase. Reverse will happen for the negative half-cycle of the signal.

Above Fig. (right) shows the graph of total collector current i_{Ctot} versus time. From the graph, it is clear that total collector current consists of two components, namely;

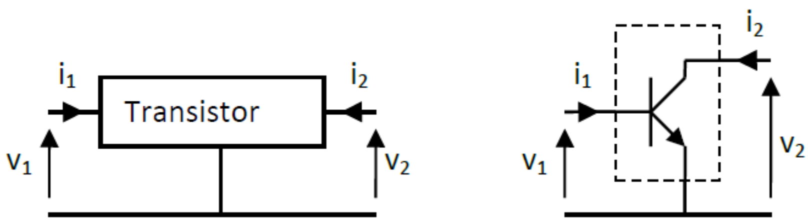
- The DC collector current I_C (zero signal collector current) due to bias battery V_{BB}. This is the current that flows in the collector in the absence of signal.
- The AC collector current i_C due to signal.

Total collector current, $i_{Ctot}=i_C+I_C$

The useful output is the voltage drop across collector load R_C due to the AC component i_C. The purpose of zero signal collector current is to ensure that the emitter-base junction is forward biased at all times.

2. Dynamic model (small signals)

In linear regime, for small signals, the transistor can be considered as a linear quadrupole (see diagram below). However, the transistor has only 3 poles. One of its terminals will therefore be common at the input and output of the quadrupole. Hence we can find, Common-Emitter, Common-Base and Common-Collector circuits. The Common-Emitter is most known and used one.



The transistor as a quadrupole: case of Common Emitter assembly

***Note**: The currents i_1 and i_2, and the voltages v_1 and v_2 are small signals.*

Every quadrupole is characterized by two linear equations linking the four input and output signals. The transistor, in small signals will be characterized by hybrid parameters:

$$v_1=h_{11}i_1+h_{12}v_2$$

$$i_2=h_{21}i_1+h_{22}v_2$$

h_{11}, h_{12}, h_{21} and h_{22} are the dynamic parameters of the quadrupole, so in the case considered, those of the transistor in Common Emitter assembly.

For Common Emitter assembly, we have: $v_1=v_{BE}, i_1=i_B, v_2=v_{CE}$ and $i_2=i_C$

The system then becomes:

$$v_{BE}=h_{11e}i_B+h_{12e}v_{CE}$$

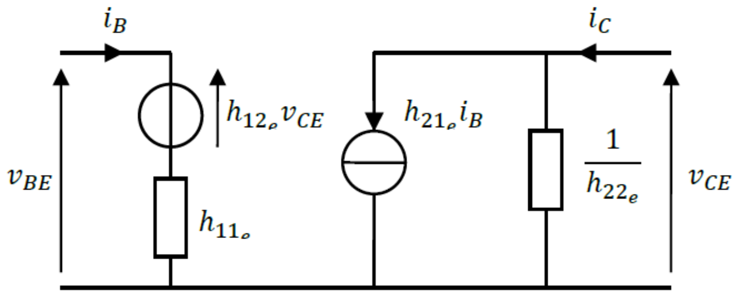
$$i_C=h_{21e}i_B+h_{22e}v_{CE}$$

The index "e" means that these are the dynamic parameters of the transistor in Common Emitter assembly.

For $v_{CE}=0$, we get: $h_{11e}=\frac{v_{BE}}{i_B}$ *and* $h_{21e}=\frac{i_C}{i_B}$

For $i_B=0$, we get: $h_{12e}=\frac{v_{BE}}{v_{CE}}$ *and* $h_{22e}=\frac{i_C}{v_{CE}}$

We thus obtain the following small signal diagram:



2.1. Calculation of dynamic parameters

2.1.1. Calculation of h_{12e}

h_{12_e} is the "reverse transfer coefficient". It can be neglected because its value is very low.

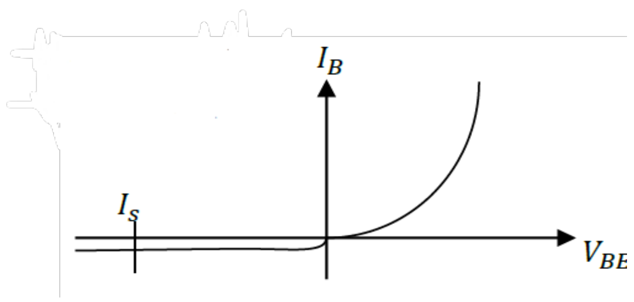
$(h_{12_e} = 10^{-3}$ *to* $10^{-4} \approx 0)$.

***Note**: We have already seen that V_{CE} has very little influence on V_{BE} (Chapter I)*

2.1.2. Calculation of h_{11e}

h_{11_e} is the input resistor of the transistor.

As $v_{BE} = dV_{BE}$ and $i_B = dI_B$ then $h_{11_e} = \left(\frac{dV_{BE}}{dI_B}\right)_{(v_{CE}=0)}$



Taking up the characteristic $V_{BE} = f(I_B)$ of Chapter I, we see that the characteristic is that of a PN junction. It can therefore be approximated by the equation of Ebers and Moll: $I_B = I_S\left(e^{\frac{V_{BE}}{mV_T}} - 1\right)$ (see diodes chapter)

$$\frac{dI_B}{dV_{BE}} = \frac{I_S}{mV_T} \cdot e^{\frac{V_{BE}}{mV_T}} = \frac{1}{mV_T}\left(I_S \cdot e^{\frac{V_{BE}}{mV_T}}\right) \approx \frac{1}{mV_T} I_{B_0} \quad \text{because} \quad I_S \ll I_{B_0} \quad (I_S \approx 10^{-15} A)$$

So $\boxed{h_{11_e} = \frac{mV_T}{I_{B0}} = r}$, with: m: empirical coefficient, m = 2

$V_T = 26\,mV$ thermal potential)

I_{B0} : Base polarization current.

***Note:** r is inversely proportional to the polarization current of the base.*

2.1.3. Calculation of h_{21e}

h_{21e} is the transfer coefficient of the dynamic base current. As $i_C = dI_C$ and $i_B = dI_B$ then

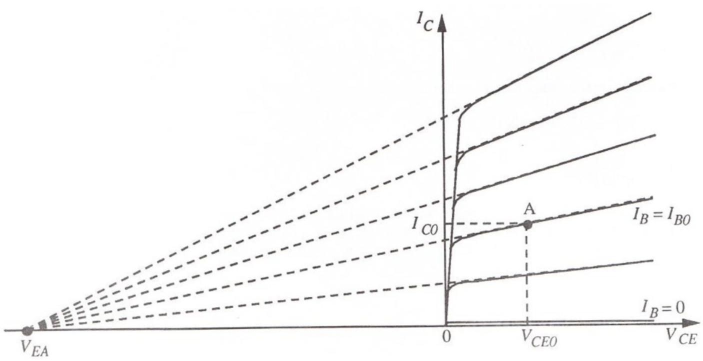
$$h_{21e} = \left(\frac{dI_C}{dI_B}\right)_{(v_{CE}=0)}$$

and $I_C = \beta . I_B$, therefore $dI_C = \beta . dI_B + I_B d\beta$. We often take β=$constant$ (which is not entirely accurate but an approximation permitting to say $d\beta = 0$). Therefore $dI_C = \beta dI_B \Rightarrow \frac{dI_C}{dI_B} = \beta$

therefore $\boxed{h_{21e} = \beta}$

2.1.4. Calculation of h_{22e}

$\frac{1}{h_{22e}}$ is the output resistor of the transistor. This resistor is due to the Early effect.



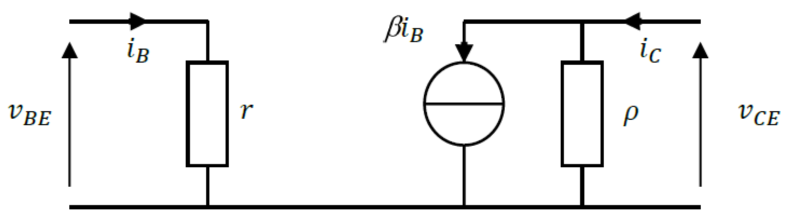
V_{EA} is called the Early's voltage

As $i_C = dI_C$ and $v_{CE} = dV_{CE}$, $h_{22e} = \left(\frac{dI_C}{dV_{CE}}\right)_{(I_B=0)}$. So h_{22e} represents the slope of the curve at the point of polarization. The transistor output resistor is called $\rho = \frac{1}{h_{22e}}$

Knowing the point of polarization (I_{C0}, V_{CE0}) and the Early's voltage, we can calculate ρ since it corresponds to the slope of the curve.

We obtain $\rho = \frac{|(V_{EA})| + V_{CE0}}{I_{C0}}$. ρ is inversely proportional to the polarization current. In addition, its value is very large (→ ∞). $h_{22e} = \frac{1}{\rho}$ will therefore often be neglected.

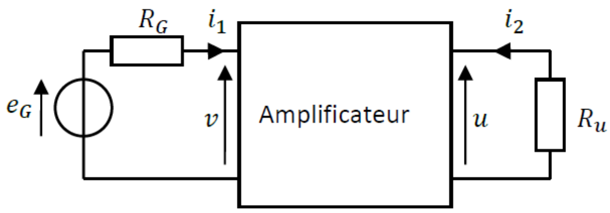
The following dynamic diagram is obtained:



3. Transistor amplifier assemblies

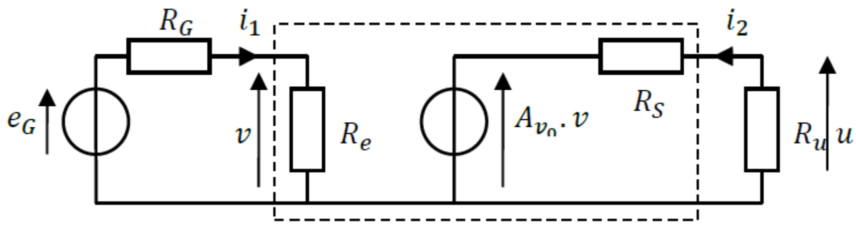
3.1. General information about amplifiers

The amplifier is a circuit intended to amplify the power of a signal. The signal is applied to the input of the amplifier by a source represented by a voltage generator e_G having an internal resistance R_G. The load can be represented by a resistor R_u. The amplification can be achieved by amplifying the input voltage v or the input current i_G (i_1 in the diagram below) or both.



The amplifier must be a linear function. By increasing the amplitude of the signal, its shape must be preserved. If the shape of the amplifier's output signal is different from the shape of the input signal, there is a distortion of the information carried by the signal. To avoid distortions, the transistor must be used in the linear sections of the characteristics curves.

3.1.1. Symbolization of amplifiers

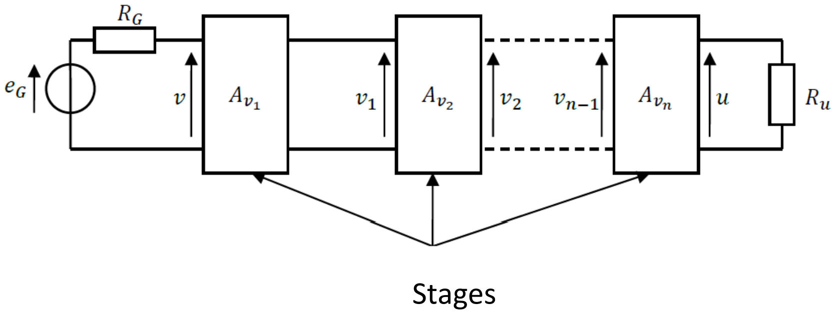


Amplifier

- Input resistor: $R_e = \frac{v}{i_1}$
- Voltage amplification: $A_v = \frac{u}{v}$ (Note: Empty gain: $A_{v0} = \frac{u}{v}$ when $R_u = +\infty$)
- Output resistor: $R_S = \frac{u}{i_2}$ when $e_G = 0$
- Current amplification: $A_i = \frac{i_2}{i_1} = \frac{i_2}{i_1} \cdot \frac{u}{u} \cdot \frac{v}{v} = \frac{i_2}{u} \cdot \frac{u}{v} \cdot \frac{v}{i_1} = \left(-\frac{1}{R_u}\right) . A_v . R_e = -A_v \cdot \frac{R_e}{R_u}$
- Power amplification: $A_p = \frac{P_u}{P_e} = \frac{u.(-i_2)}{v.i_1} = -A_v . A_i = A_v{}^{2} \cdot \frac{R_e}{R_u}$, where P_u is the power absorbed by R_u and P_e is the power absorbed at the input of the amplifier.

***Note**: From R_e, R_s and A_v we determine all the other parameters.*

3.1.2. Amplification cascading



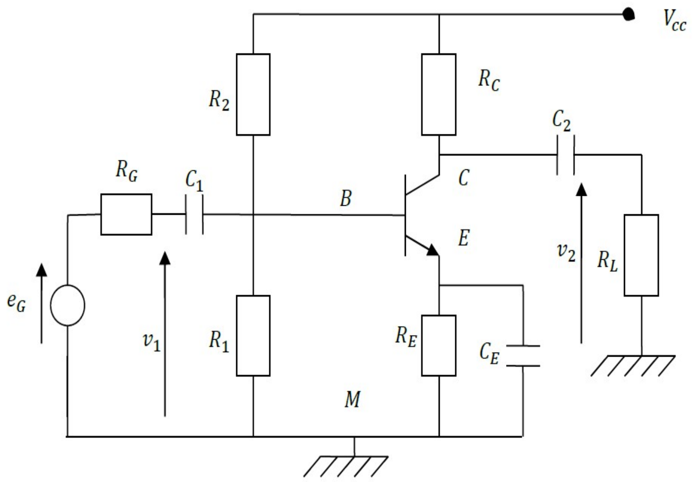
$$A_{v_1} = \frac{v_1}{v} \quad A_{v_2} = \frac{v_2}{v_1} \quad A_{v_n} = \frac{v_n}{v_{n-1}}$$

Total voltage amplification : $A_v = A_{v_1}.A_{v_2}.\dots.A_{v_n} = \prod_{i=1}^{n} A_i$

> ***Note****: The output resistor R_S of each intermediate stage acts as the resistance of the generator R_G for the next stage. The input resistor R_e of each intermediate stage plays the role of Load resistor R_u for the previous stage.*

3.1.3. Common Emitter assembly

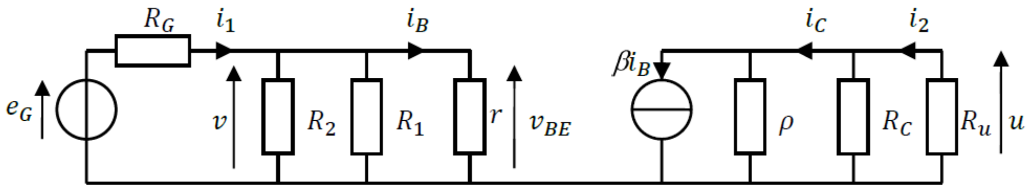
3.1.3.1. Polarization



The capacitors C_1 and C_2 are used to separate the circuit from the DC input source. These are link capacitors.

The capacitor C_E is used to "short-circuit" the resistor R_E in respect to small signals. Indeed, the impedance of the capacitor in sinusoidal alternating regime is $Z_{CE} = \frac{1}{j C_E \omega}$, therefore the greater the frequency, the lower the impedance. The Emitter is therefore connected (from the point of view of small signals) to ground, so we have a Common Emitter assembly. C_E is a decoupling capacitor.

3.1.3.2. Small signals equivalent diagram:



- Input resistor: $R_e = \frac{v}{i_1} = R_1 \,//\, R_2 \,//\, r$
- Output resistor: $R_S = \frac{u}{i_2}$, when $E_G = 0$, so $R_S = \rho \,//\, Rc$
- Voltage amplification: $A_v = \frac{u}{v}$ and $i_B = \frac{v}{r}$, Moreover, $u = -\beta . I_B\ (\rho // R_C // R_u)$, so
  $u = -\beta . \frac{v}{r}\ (\rho \,//\, R_c \,//\, R_u)$ $\qquad A_v = -\frac{\beta}{r}\ (\rho \,//\, R_c \,//\, R_u)$
- Current Amplification: $A_i = \frac{i_2}{i_1} = -A_v . \frac{R_e}{R_u} = \frac{\beta(\rho \,//\, R_c \,//\, R_u)(R_1 \,//\, R_2 \,//\, r)}{r.R_u}$
- Power Amplification: $A_p = \frac{P_u}{P_e} = -A_v . A_i = \frac{\beta^2(\rho \,//\, R_c \,//\, R_u)^2(R_1 \,//\, R_2 \,//\, r)}{r^2.R_u}$