$$\frac{}{n : Nat}$$

$$\frac{e_1 : Nat \qquad e_2 : Nat}{e_1 + e_2 : Nat} \qquad \frac{e_1 : Nat \qquad e_2 : Nat}{e_1 \doteq e_2 : Nat} \quad \text{usw.}$$

$$\frac{}{n \Downarrow n} \qquad \frac{e_1 \Downarrow n_1 \qquad e_2 \Downarrow n_2}{e_1 + e_2 \Downarrow n_1 + n_2} \qquad \frac{e_1 \Downarrow n_1 \qquad e_2 \Downarrow n_2}{e_1 * e_2 \Downarrow n_1 \cdot n_2}$$

$$\frac{e_1 \Downarrow n+k \qquad e_2 \Downarrow n}{e_1 \doteq e_2 \Downarrow k} \qquad \frac{e_1 \Downarrow n \qquad e_2 \Downarrow n+k}{e_1 \doteq e_2 \Downarrow 0}$$

$$\frac{e_1 \Downarrow n_1 \qquad e_2 \Downarrow n_2}{e_1 \div e_2 \Downarrow q} \quad n_1 = q \cdot n_2 + r \text{ und } r < n_2$$

$$\frac{e_1 \Downarrow n_1 \qquad e_2 \Downarrow n_2}{e_1 \,\%\, e_2 \Downarrow r} \quad n_1 = q \cdot n_2 + r \text{ und } r < n_2$$

$$\frac{}{false : Bool} \qquad \frac{}{true : Bool}$$

$$\frac{e_1 : Bool \qquad e_2 : t \qquad e_3 : t}{\mathit{if}\ e_1\ \mathit{then}\ e_2\ \mathit{else}\ e_3 : t}$$

$$\frac{e_1 : Nat \qquad e_2 : Nat}{e_1 < e_2 : Bool} \qquad \frac{e_1 : Nat \qquad e_2 : Nat}{e_1 \leqslant e_2 : Bool} \quad \text{usw.}$$

$$\frac{}{false \Downarrow false} \qquad \frac{}{true \Downarrow true}$$

$$\frac{e_1 \Downarrow true \qquad e_2 \Downarrow v}{\mathit{if}\ e_1\ \mathit{then}\ e_2\ \mathit{else}\ e_3 \Downarrow v} \qquad \frac{e_1 \Downarrow false \qquad e_3 \Downarrow v}{\mathit{if}\ e_1\ \mathit{then}\ e_2\ \mathit{else}\ e_3 \Downarrow v}$$

$$\frac{e_1 \Downarrow n+k \qquad e_2 \Downarrow n}{e_1 < e_2 \Downarrow false} \qquad \frac{e_1 \Downarrow n \qquad e_2 \Downarrow n+k+1}{e_1 < e_2 \Downarrow true} \quad \text{usw.}$$

$$\frac{\Sigma \vdash e_1 : t_1 \qquad \Sigma \vdash e_2 : t_2}{\Sigma \vdash (e_1, e_2) : t_1 * t_2} \qquad \frac{\Sigma \vdash e : t_1 * t_2}{\Sigma \vdash \mathit{fst}\ e : t_1} \qquad \frac{\Sigma \vdash e : t_1 * t_2}{\Sigma \vdash \mathit{snd}\ e : t_2}$$

$$\frac{\delta \vdash e_1 \Downarrow v_1 \qquad \delta \vdash e_2 \Downarrow v_2}{\delta \vdash (e_1, e_2) \Downarrow (v_1, v_2)} \qquad \frac{\delta \vdash e \Downarrow (v_1, v_2)}{\delta \vdash \mathit{fst}\ e \Downarrow v_1} \qquad \frac{\delta \vdash e \Downarrow (v_1, v_2)}{\delta \vdash \mathit{snd}\ e \Downarrow v_2}$$

$$\frac{\Sigma \vdash e : t \qquad p \sim t : \Sigma'}{\Sigma \vdash (\mathit{let}\ p = e) : \Sigma'}$$

$$\frac{}{\_ \sim t : \varnothing} \qquad \frac{}{x \sim t : \{x \mapsto t\}}$$

$$\frac{p_1 \sim t : \Sigma_1 \qquad p_2 \sim t : \Sigma_2}{(p_1 \,\&\, p_2) \sim t : \Sigma_1, \Sigma_2} \qquad \frac{p_1 \sim t_1 : \Sigma_1 \qquad p_2 \sim t_2 : \Sigma_2}{(p_1, p_2) \sim t_1 * t_2 : \Sigma_1, \Sigma_2}$$

$$\frac{\Sigma, \{x_1 \mapsto t_1\} \vdash e_2 : t_2}{\Sigma \vdash (\mathit{let}\ f\ (x_1 : t_1) : t_2 = e_2) : \{f \mapsto t_1 \to t_2\}}$$

$$\frac{\Sigma, \{x_1 \mapsto t_1\} \vdash e : t_2}{\Sigma \vdash (\mathit{fun}\ (x_1 : t_1) \to e) : t_1 \to t_2} \qquad \frac{\Sigma \vdash e : t_1 \to t_2 \qquad \Sigma \vdash e_1 : t_1}{\Sigma \vdash e\ e_1 : t_2}$$

$$\frac{\Sigma \vdash e_1 : t_1}{\Sigma \vdash f\ (e_1) : t_2} \quad f \in dom\,\Sigma \text{ und } \Sigma(f) = t_1 \to t_2$$

$$\frac{}{\delta \vdash (\mathit{fun}\ x \to e) \Downarrow (\delta, x, e)}$$

$$\frac{}{\delta \vdash (\mathit{let}\ f\ (x) = e) \Downarrow \{f \mapsto (\delta, x, e)\}}$$

$$\frac{\delta \vdash e \Downarrow (\delta', x_1, e') \qquad \delta \vdash e_1 \Downarrow v_1 \qquad \delta', \{x_1 \mapsto v_1\} \vdash e' \Downarrow v'}{\delta \vdash e\ e_1 \Downarrow v'}$$

$$\frac{\delta \vdash e_1 \Downarrow v_1 \qquad \delta', \{x_1 \mapsto v_1\} \vdash e \Downarrow v}{\delta \vdash f\ (e_1) \Downarrow v} \quad \delta(f) = (\delta', x_1, e)$$

$$\frac{\delta \vdash e \Downarrow v \qquad p \sim v \Downarrow \delta'}{\delta \vdash \mathit{let}\ p = e \Downarrow \delta'}$$

$$\frac{\Sigma, \{f \mapsto t_1 \to t_2, x_1 \mapsto t_1\} \vdash e_2 : t_2}{\Sigma \vdash (\mathit{let\ rec}\ f\ (x_1 : t_1) : t_2 = e_2) : \{f \mapsto t_1 \to t_2\}}$$

$$\frac{}{\delta \vdash (\mathit{let\ rec}\ f\ x = e) \Downarrow \{f \mapsto (\delta, f, x, e)\}}$$

$$\frac{\delta \vdash e \Downarrow v \qquad \delta \vdash e_1 \Downarrow v_1 \qquad \delta', \{f \mapsto v, x_1 \mapsto v_1\} \vdash e' \Downarrow v'}{\delta \vdash e\ e_1 \Downarrow v'} \quad \text{mit } v = (\delta', f, x_1, e')$$

$$\frac{}{\_ \sim v \Downarrow \varnothing} \qquad \frac{}{x \sim v \Downarrow \{x \mapsto v\}}$$

$$\frac{p_1 \sim v \Downarrow \delta_1 \qquad p_2 \sim v \Downarrow \delta_2}{(p_1 \,\&\, p_2) \sim v \Downarrow \delta_1, \delta_2} \qquad \frac{p_1 \sim v_1 \Downarrow \delta_1 \qquad p_2 \sim v_2 \Downarrow \delta_2}{(p_1, p_2) \sim (v_1, v_2) \Downarrow \delta_1, \delta_2}$$