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UFR de sciences Economiques, de Gestion, Mathématique et d'Informatique

(UFR-SEGMI)

Application of ARMA-GARCH model and Support Vector Regression in Financial Time Series Forecasting

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Abstract

Time series modeling and forecasting has fundamental importance to various practical domains. Thus a lot of active research works is going on in this subject during several years. Many important models have been proposed in literature for improving the accuracy and efficiency of time series modeling and forecasting.

The use of classical statistical models and machine learning techniques for forecasting financial time series has been proved extremely successful in recent times. In this project, we apply the finite mixture of ARMA-GARCH model instead of AR or ARMA models to compare with support vector machines (SVM) in forecasting financial time series. We analyze three stock market indices S&P 500 from New York Stock Exchange, FTSE 100 from London Stock Exchange and CAC 40 from Paris Stock Exchange with daily returns data. The data cover the period between 01 July 1996 and 01 July 2013. To improve the forecasting ability and make it possible, we transform this data into daily returns.

To evaluate forecast accuracy, we use the three performance criteria MAE, RMSE and MSE. Our experiment shows that the SVM model outperforms the finite mixture of ARMA-GARCH and concludes that SVMs provide a promising alternative to time series forecasting.

To have authenticity as well as clarity in our discussion about time series modeling and forecasting, we take the help of various published research works from reputed journals and some standard books.

Résumé

En pratique, la modélisation et la prévision des séries temporelles a une importance fondamentale dans de différents domaines. Ainsi, un grand nombre des travaux de recherche en cours traitent ce sujet et cela durant plusieurs années. De multiples modèles ont été proposés dans la littérature pour améliorer la précision et l'efficacité de la modélisation et la prévision des séries temporelles.

Ces derniers temps, l'utilisation des modèles statistiques classiques et des techniques d'apprentissage automatique, pour la prévision des séries temporelles financières, a apporté d'excellents résultats. Dans ce projet, nous appliquons le modèle à erreur conditionnellement hétéroscédastique ARMA-GARCH au lieu des modèles autorégressifs AR ou modèles autorégressifs et moyenne mobile ARMA et le comparons aux machines à vecteurs de support ou séparateurs à vaste marge (SVM) pour la prévision de séries temporelles financières. De ce fait, nous analysons trois indices boursiers S & P 500 de la Bourse de New York, le FTSE 100 de la Bourse de Londres et le CAC 40 de la Bourse de Paris. Les données sont quotidiennes, et couvrent la période allant du 01 Juillet 1996 au 01 Juillet 2013. Pour améliorer la capacité de prévision et la rendre possible, on transforme ces données en rendements quotidiens.

Pour évaluer la précision des prévisions, on utilise les trois critères de performance MAE, RMSE et MSE. Notre expérience montre que les SVM surpassent le modèle ARMA-GARCH et conclut que les SVM constituent une alternative prometteuse pour la prévision des séries temporelles.

Par souci d'authenticité ainsi que de clarté dans notre étude sur la modélisation et la prévision des séries temporelles financières, de divers travaux de recherche, d’ouvrages, de revues réputées et de livres classiques ont été sollicités.

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Introduction

The prediction of the financial market is a major challenge in both academic and business world and is currently receiving considerable attention from the research community. Indeed, in the economic and business environment, it is very important to forecast various kinds of financial variables precisely, to develop appropriate strategies, provides concrete data for investment decisions and avoid the risk of large potential losses. Because of their unstructured and noisy nature, non-stationary and hidden relationships [34], financial time series are among the most difficult signals to predict, which naturally leads to the debate on the predictability of the market among academics and market practitioners. However, a large number of economists and professional investors, challenge assumptions, and are widely convinced that the financial market is predictable to a certain degree. In addition, researchers in the machine learning and data mining community have also tried to predict the financial market, using various learning algorithms.

Historically there has been a heated debate on the predictability of the stock returns, thus, some classical methods, have been developed in predicting financial time series. We quote, among them, One of the most popular and frequently used stochastic time series models is the Autoregressive Integrated Moving Average (ARIMA) [6, 9, 11] model also know as the Box and Jenkins ARMA. This method support a constant variance, while in the real economic and financial series, we find periods of unusual large volatility, followed by periods of relative tranquillity. This behaviour indicates a form of heteroskedasticity. Engle [21] proved the possibility of modelling the mean and variance of such series at the same time a model, named autoregressive conditional heteroskedastic (ARCH); whereas Box and Jenkins [5] modelled only the mean series. After that, Bollerslev [31] extended Engle's original work by developing a technique that allows the conditional variance to be an ARMA process and that extended process is known as the GARCH process. Wong *et al.* [40] adopted the well-known GARCH model in the form of the so-called mixture of AR-GARCH model in exchange rate prediction. Again, Tang *et al.* explored the mixture of ARMA-GARCH model for stock price prediction.

Researchers in the machine learning and data mining community have also tried to forecast the financial market, using various learning algorithms. The support vector machine (SVM) is one of the tools that have shown promising results, while the evidence on the forecasting ability of the GARCH model is not completely sure. A major breakthrough in the area of time series forecasting occurred with the development of Vapnik's support vector machine (SVM) concept [8, 14, 32, 33]. Vapnik and his co-workers designed SVM at the AT & T Bell laboratories in 1995 [14, 29, 37]. Based on the structural risk

minimization (SRM) principle, SVM achieves a balance between the training error and generalization error, leading to better forecasting. Selecting the best model in SVM is equivalent to solving a quadratic programming, which gives SVM another merit of a unique global solution
The initial aim of SVM was to solve pattern classification problems but afterwards they have been widely applied in many other fields such as function estimation, regression (SVR), signal processing and time series prediction problems [14, 33, 43]. The remarkable characteristic of SVM is that it is not only destined for good classification but also intended for a better generalization of the training data. For this reason the SVM methodology has become one of the well-known techniques, especially for time series forecasting problems in recent years.

Many researchers have already published huge number of papers comparing autoregressive (AR) or ARMA or GARCH models with SVM in financial time-series forecasting. There are some recent papers in this combination. For example, Chen *et al.* [41] compared SVMs and BPs taking AR as a benchmark in forecasting the six major Asian stock markets, and Chen *et al.* [26] proposed recurrent SVR based on the GARCH model compared with moving average (MA),recurrent NN and parametric GARCH model in terms of their ability to forecast volatility. But they did not take account of the finite mixture ARMA-GARCH model in this combination. However, Santos *et al.* [1] compared ARMA-GARCH model with ARMA, ANNs and fuzzy systems in forecasting exchange rates, but they did not consider SVM in this combination, also Altaf and Hossain [3] and Mohammed Nasser did a Comparison of the finite mixture of ARMA-GARCH, back propagation neural networks and support-vector machines in forecasting financial returns but did not uses the RegSMOImproved [27] that we used for the Support Vector Regression.

Our project is or is organized as follows. The previous researches are reviewed in the first section. The second section contains the stock market presentation and the indexes that we will model their returns. The third section is dedicated to the presentation of all the aspects of the ARMA- GARCH model. The section 4 is about SVMs, Section 5 gives a set of definitions of the empirical used tests and Section 6 contains the application of ARMA-GARCH and SVR models on the returns of the chosen indexes. The comparison of the results of the three series is in Section 7 which is followed by the conclusion.

1. Previous researches

So many studies used SVM methods and traditional statistical methods to predict the behaviour of prices of financial products and used to compare the results of each method with another in order to find the best forecasting model. For example, Altaf Hossain and Mohammed Nasser [3] did a Comparison of the finite mixture of ARMA-GARCH, back propagation neural networks and support-vector machines in forecasting financial returns, where SVM outperforms the two other models. Ince and Trafalis [7] proposed a two-stage forecasting model which incorporates, in one hand, parametric techniques such as ARIMA, vector autoregressive and co-integration techniques, and in the other hand non-parametric techniques such as Support Vector Regression and Artificial Neural Networks. Their comparison showed that the SVR outperformed the ANN. Viviana Fernandez [38]applied two techniques to forecast the value of US manufacturing shipments, wavelets and support vector machines (SVM), these two methodologies were compared with well-known time series techniques: multiplicative seasonal autoregressive integrated moving average (ARIMA).

Other studies tested the power of machine learning techniques, for example Application of support vector machines in financial time series forecasting, where Francis E.H. Tay and Lijuan Cao [14] examined the feasibility of SVM in financial time-series forecasting by comparing it with a multilayer back-propagation neural network, and the results showed that SVM outperforms them. Also, Kim and Han [13]showed that SVM provides a promising alternative to stock market prediction comparing it with BPNNs. Konstantinos Theofilatos, Spiros Likothanassis and Andreas Karathanasopoulos [12] presented a paper that aims in investigating the performance of machine learning techniques (K-Nearest Neighbors algorithm, Naïve Bayesian Classifier, Artificial Neural Networks, Support Vector Machines and Random Forests) for market predictions. Christian L. Dunis, Rafael Rosillo, David de la Fuente and Raul Pino [4]did a research which aims at examining the application of support vector machines (SVMs) to the task of forecasting the weekly change in the Madrid IBEX-35 stock index and compared it with The most popular architecture of Neural networks, the MLP. Wun-Hua Chen and Jen-Ying Shih [41] applied Support-Vector Machines (SVMs) and Back Propagation (BP) neural networks for six Asian stock markets and the results showed the superiority of both models, compared to the precedent researches. Wei Huang, Yoshiteru Nakamori and Shou-Yang Wang [42] investigate the predictability of financial movement direction with SVM by forecasting the weekly movement direction of NIKKEI 225 index, and compared its results with those of Linear Discriminant Analysis, Quadratic Discriminant Analysis and Elman Back propagation Neural Networks. While HUSEYIN INCE and THEODORE B. TRAFALIS [36] noted that there is no difference between MLP networks and SVR techniques when we compare their mean square error values.

The other papers were about traditional statistical methods, for example forecasting Financial Time Series, written by Peter Princ, Sára Bisová and Adam Borovička [18]. They compare two different approaches for one-step ahead forecasting in stock market indices with parameters from normal and two-piece normal (TPN) distribution. Philippe Lambert and Sébastien Laurent [19] innovate by modeling financial time series using GARCH-type models with a skewed Student distribution. Again, Tang and *al.* extended the mixture of AR–GARCH model to the mixture of ARMA-GARCH model and this last outperformed the mixture of AR–GARCH models in prediction or Wong *and al.who* [40] proposed the finite mixture of AR–GARCH model and it performed better than pure GARCH in financial prediction.

Other authors' did not compare between two methods but treated each method individually in order to improve this one. For example, Li Wang and Ji Zhu [15], who proposed a two-step kernel learning method based on Support Vector Regression (SVR) for financial time series forecasting.
Martin Sewell and John Shawe-Taylor [17] wrote a paper which employs kernel methods to forecast foreign exchange rates, and aims to earn a positive return. Lijuan Cao proposes a study using the support vector machines (SVMs) experts for time series forecasting [16].

In one hand we found that most of the studies above showed that SVM models outperformed both Artificial Neural Networks and traditional statistical models like the GARCH type. And in the other hand we noted that simple neural learning procedures easily outperformed the best practice of traditional statistical models.
So all these studies proves us the craze which carries a considerable number of researchers the fact of returning the models of forecasts of the financial time series, in particular the indices, more and more successful and realistic.

2. The stock market

Financial globalization, also called globalization can be defined as the huge increase in financial flows between countries around the world. Today, financial globalization is provided by a high mobility of capital internationally, itself made possible by the widespread liberalization of capital movements born in the 1980s. The principle of free movement of capital is reflected in the prohibition of all restrictions on capital movements and the prohibition of all restrictions on payments (for the purchase of a good or service).

The stock market likes indices and stock too. So much so that their number is estimated at more than 100,000! They are very large - such as the MSCI World Index - or very close - as the CAC Next 20 - these tools are extremely useful for many practical applications. Thus, fund managers compare their annual income with that of their "benchmark", the famous "Benchmark" means any investor which regularly speak. Moreover, the indices are the basis for a multitude of derivatives developed by companies such as NYSE Euronext market. Finally, the creation of many ETFs put once again the importance and multiplicity of indices that are their underlying. Yet despite their importance, the French-language literature on this subject is very limited...

2.1 Definition of a stock index

An index can be defined as a synthetic indicator of the performance of the stock market from a given date. It assesses the direction and magnitude of price movements quoted from a basket of securities considered significant titles. The method of selection of these securities is different depending on the role that is assigned to the index. To ensure a good representation of the index, the basket must be regularly updated. The index is primarily characterized by its level indicated in points. The level of the index reflects the performance of the basket since the base date. From one day to the next, an index varies and this variation is measured in points or percentage. This raw data can be enhanced by describing the number of shares and increase the number of shares down [ ].

The indices represent a given financial center, an economic area like the euro area, a sector (raw materials, industry, media, etc..) Or a subset of securities classified by size. Manufacturing indices has become an industry, and these clues abound. Many of them serve as underlying listed financial products themselves

2.2 Indices’ utility

The proliferation index well reflects the demand from investors. In the context of information explosion, the indices provide a comprehensive measure of aggregate information carrier. Specifically, the indices are:

- To benchmark (performance standard) for fund managers
- On a tool for analyzing global trends or detailed by sector or size of business
- Supportive of derivatives such as options or futures contracts that can be covered, arbitrate or speculate.

2.3 Indices producers'

Companies specializing in the manufacture of indices: Dow Jones, MSCI (Morgan Stanley Capital International) Standart Poor's, but also companies jointly held by media and such Exchange FTSE (Financial Times Stock Exchange) owned by The Financial Times the London Stock Exchange that calculates some 120,000 daily equity indices, fixed income and hedge funds! Other exchanges such NYSE Euronext, Deutsche Boerse, the Tokyo Stock Exchange have also developed its own index families.

2.4 S&P 500, the main stock index in U.S.A

The S&P (Standard & Poor's) 500 Index is an index that represents the 500 largest companies listed on U.S. exchanges. Standard & Poor's introduced its first stock index in 1923. The S&P 500 index in its present form began on March 4, 1957. Technology has allowed the index to be calculated and disseminated in real time. The S&P 500 is widely used as a measure of the general level of stock prices, as it includes both growth stocks and value stocks.

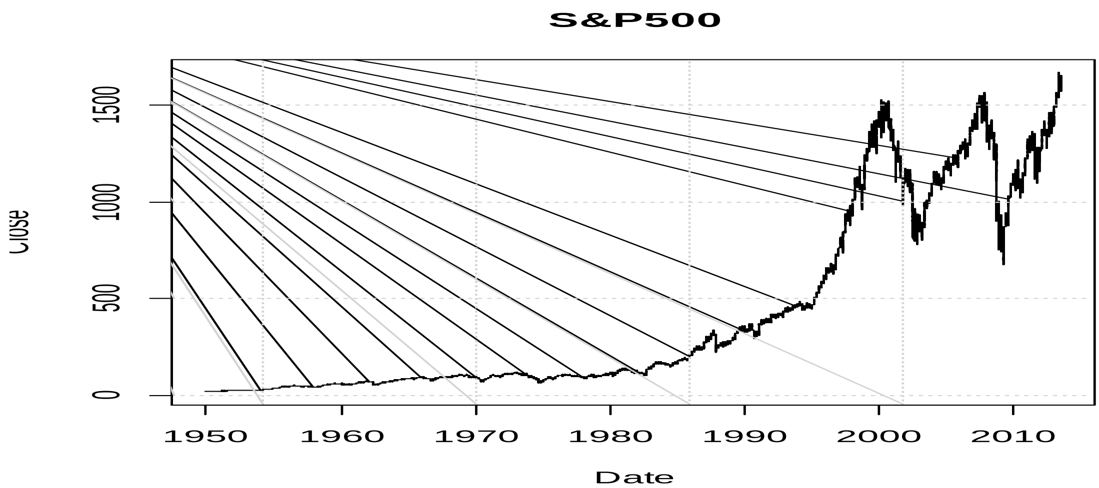


Figure 2.1 - S&P500 evolution from its creation

The S & P is the U.S. equity benchmark, before the Dow Jones which includes 30 largest companies, and the NASDAQ, which is the benchmark of U.S. technology stocks. Even at the global level, the S&P 500 is considered a benchmark because it includes a large number of companies, not necessarily American, but next on the U.S. markets that serve as "barometer" of the global economy. This is the most followed by fund managers and stakeholders of finance in general, representativeness nearly 80% of the total capitalization of the U.S. market index. Note that 84.4% of the total capitalization of the S & P500 from the NYSE, NASDAQ and 15.5%, 0.1%, the AMEX (American third markets).

2.5 CAC 40, the main stock index in France

The CAC 40 (Continuous Assisted Quotation) is the main stock index instead of Paris. It is one of Europe's most watched indicators International, two-thirds of the activity of the CAC 40 companies it is estimated that approximately is located outside France.

Created with 1000 basis points at December 31, 1987 by the Company of stockbrokers, the CAC 40 index is determined from the course of forty shares listed on the continuous market among the first one hundred companies whose trade is the most abundant on Euronext Paris as part of Euronext, the first European exchange. These companies, representing different branches of activities, in principle reflect the overall trend of the economy of large French companies and their list is reviewed regularly in order to maintain this representation.

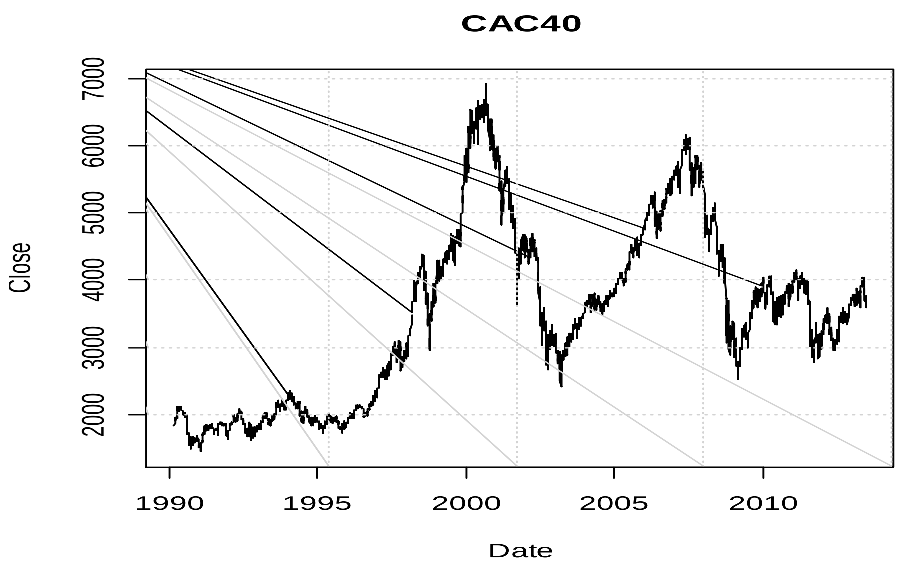


Figure 2.2 - CAC40 evolution from its creation

The CAC 40 index is calculated continuously by the weighted by market capitalization of each mean value. Any member of the CAC 40 can see the trading in its shares suspended for 15 minutes if it varies by more than 10%, 5% and twice in the same direction. We say that the title is "upward reserved " or "downward reserved." Each of the CAC 40 index weighted according to the number of shares available on the market. The weights vary from one company to another depending on its market capitalization and trading occurred on value. So when the price of a security rises, its weight in the CAC 40 increases.

2.6 FTSE 100, the main stock index in U.K

The FTSE 100, better known as the Footsie is a stock market index made up of one hundred highly capitalized UK companies to the London Stock Exchange. These companies represent more than 70% of the London market. FTSE stands for "Financial Times Stock Exchange" since the index was created by the Financial Times, the famous British business daily. It was launched January 3, 1984 on the basis of 1000 points and reached its highest in December 1999 to 6950.60 points. Today, it is the benchmark index of the London Stock Exchange. And even if the FTSE All-Share Index is more comprehensive index, the FTSE 100 is the most widely used as an indicator of the economic health of the UK.

After falling during the financial crisis of 2007-2010 to below 3500 in March 2009, the index recovered to a peak of 6091.33 on 8 February 2011, fell under the 5000 mark on the morning of 23 September 2011, but reached 6,840.27 (its highest since September 2000) on 22 May 2013.

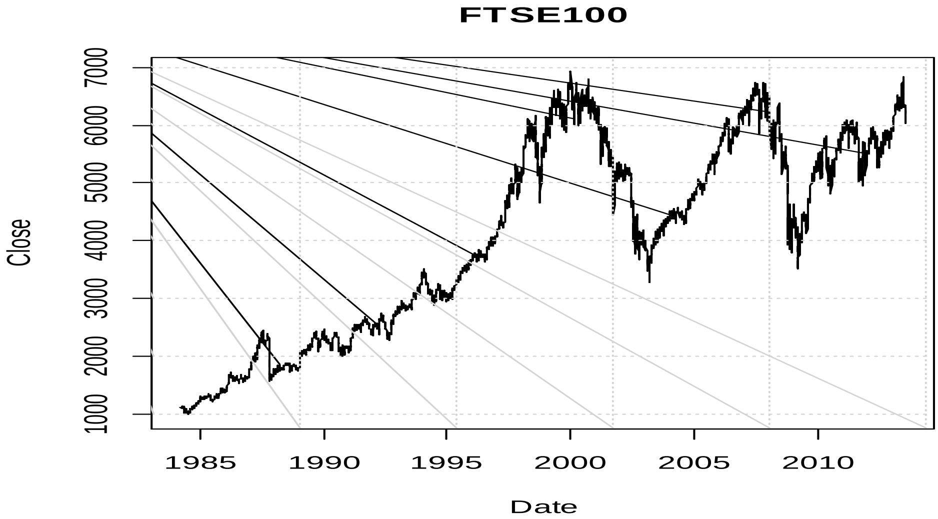


Figure 2.3 - FTSE100 evolution from its creation

The Footsie consists of titles hundred largest companies listed on the London Stock Exchange companies. Among the titles that make up the FTSE 100, there may be mentioned: Royal Dutch Shell, BP, Vodafone Group, HSBC, GlaxoSmithKline, AstraZeneca, British American Tobacco, BG Group, Tesco, BHP Billiton, Aviva, Barclays, Lloyds Banking Group, Marks & Spencer, Rolls-Royce Group, and Unilever.

3. Basic concepts of time series modeling

It speaks of a time series to describe a set of numeric values observed on a phenomenon that evolves over time. For several years the time series have been studied and analyzed in order to interpret their behavior over time, whether to include past trends or to predict behavior in the future. They can be represented mathematically and give an idea of the dynamics of variables represented, most often using probabilistic and statistical concepts [28, 35, 22].

3.1 Definition of a time series

A time series is a sequence of real numbers, indexed by the integers as time. For each instant of time, the value of the quantity studied $\mathbf{X_t}$ is called a random variable. All values $\mathbf{X_t}$ varies when **t** is called random process: **[$\mathbf{X_t}$, t ∈ Z]**. So a time series is the realization of a random process.

A time series is any sequence of observations corresponding to the same variable: there may be macroeconomic data (GDP of a country, inflation, exports ...), micro (sales of a given company , the number of employees, the income of an individual, the number of children a woman ...), financial (the CAC40, the price of an option to buy or sell, the price of a share), weather (rainfall, the number of sunny days per year ...), political (the number of voters, votes received by a candidate ...), demographic (the average size of population, age ...).

In practice, all that is quantifiable and varies with time. The time dimension is important here because it is the analysis of a historical chronicle of changes in one variable over time in order to understand the dynamics. The periodicity of the series does not matter, however: it may be, daily measurements monthly, quarterly, annual ... even without periodicity.

3.2 Definition of stationarity

Given a time series Xt, t = 1,...,T (T is the number of observations in the series). Before performing specific tests on this series and look at the model, several preliminary steps are necessary. We should study its stochastic characteristics, such as its expectation and variance. Especially before applying conventional methods of time series (ex. ARMA model), it is necessary to ensure that for the series studied, the mean and variance remain stable over time.

White noise definition: The time series ε_t is said to be a white noise with mean zero and variance σ_t^2 written as:

$$\epsilon \sim WN(0, \sigma_\epsilon^2)$$

If and only if ε_t has zero mean and covariance function as:

$$\gamma_\epsilon(h) = \left\{ \begin{array}{lll} \sigma_\epsilon^2 & \text{if} & h = 0 \\ 0 & \text{if} & h \neq 0 \end{array} \right\}$$

It is clear that a white noise process is stationary. Note that white noise assumption is weaker than identically independent distributed assumption.

3.3 Time series characteristics

Let us enumerates the most important characteristics of a time series

3.3.1 *Mean and variance*

Given a stationary time series X_t, t=1,...,T. The mean and variance expressions' are:

- Mean: $E(X_t)=\frac{1}{T}\sum_{t=1}^{T} X_t$ (3.1)
- Variance: $V(X_t)=\frac{1}{T}\sum_{t=1}^{T}[X_t - E(X_t)]^2$ (3.2)

3.3.2 *Auto-covariance function*

The auto-covariance function $\boldsymbol{\gamma_h}$ is:

- $\boldsymbol{\gamma_h} = \boldsymbol{Cov}(X_t, X_{t+h}) = E[[X_t - E(X_t)][X_{t+h} - E(X_{t+h})]]$ (3.3)

3.3.3 *Auto-correlation function "ACF"*

The autocorrelation function allows measuring the temporal connections between the different components of the series X_t, in fact:

- $\boldsymbol{\rho_h}=\frac{\boldsymbol{Cov}(X_t, X_{t+h})}{\sigma_{X_t}\ \sigma_{X_{t+h}}}=\frac{\boldsymbol{\gamma_h}}{\sqrt{\boldsymbol{\gamma_0}}\sqrt{\boldsymbol{\gamma_0}}}=\frac{\boldsymbol{\gamma_h}}{\boldsymbol{\gamma_0}}$ (3.4)

3.3.4 *Partial auto-correlation function*

The partial autocorrelation of a time series at lag k is denoted α_k and is found as follows:

- Fit a linear regression of y_t to the first k lags (i.e. fit an AR(k) model to the time series):

  $yt = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_k y_{t-k} + \varrho_t$ (3.5)

- Then $\alpha_k = \phi_k\hat{}$, the fitted value of ϕ_k from the regression (Least Squares).

The set of partial autocorrelations at different lags is called the partial autocorrelation function (PACF) and is plotted like the ACF.

3.4 Main statistical models for the study of time series

Here we present the main families of models used to process time series.

3.4.1 *Auto-Regressive model AR (p)*

They were introduced by Yule in 1927. It takes into account a linear dependence of the process has its own password:

$$AR(p): \quad \boldsymbol{X_t} = \alpha_1 X_{t-1} + \ \dots \ + \alpha_p X_{t-p} + \varepsilon_t \qquad (3.\ 6)$$

Where $p \in N^*$ is the order process, $\alpha_1 \dots \alpha_p$ are the real constants and $(\varepsilon_t)_{t\in Z}$ is a white noise.

3.4.2 *Moving-Average model MA (q)*

They, also, were introduced in 1927 by Slutsky. A moving average process is the sum of white noise and its delays:

$$MA(q): \quad \boldsymbol{X}_t \quad = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} \qquad (3.7)$$

Where q ∈ N* is fixed and $\theta_1 \ldots \theta_q$ are real constants.

3.4.3 *Auto-Regressive Moving Average model ARMA (p, q)*

Developed by Box and Jenkins in 1970, the ARMA models are a combination of autoregressive and moving average models:

$$ARMA(p,q): \quad \boldsymbol{X}_t - \alpha_1 X_{t-1} - \ldots - \alpha_p X_{t-p} = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} \qquad (3.8)$$

The ARIMA (Auto-Regressive Integrated Moving Average) and SARIMA (seasonal Auto-Regressive Integrated Moving Average), which is an ARMA process integrated with a seasonal component, then was developed in order to modelize a large number of real phenomena which present trends and / or seasonality.

3.4.4 *Auto-Regressive Conditional heteroskedasticity model ARCH*

Conventional forecasting models based on ARMA time series models assume constant variance (homoscedasticity assumption). This model therefore neglects possibly the information contained in the residual factor of chronic. ARCH models allow chronic model (the most of the time financial ones) which have a volatility (or variance or variability) instant which depends on the past. It is possible to develop a dynamic forecasting chronic in terms of mean and variance. Initially introduced by Engle (1982), these models have been very important developments and applications in the future.

ARCH models were introduced by Engle in 1982 and generalized in GARCH models by Bollerslev in 1986. The analogy between the discrete-time ARCH models and models of continuous time diffusion was established in 1990 by Nelsen. This analogy has, in particular, the development of stochastic volatility models. One of the contributions of ARCH models was better fit to the data (particularly financial data) than did the ARMA models.

3.4.4.1 Presentation of the ARCH models

Given $(\boldsymbol{X}_t)$ a process AR (1), as:

$$X_t = \alpha X_{t-1} + \varepsilon_t$$

Where$\boldsymbol{\varepsilon_t} \sim \boldsymbol{N}(\mathbf{0}, \boldsymbol{\sigma}^2)$, so:

$$(X_t) = \frac{1}{1-\alpha^2}\sigma^2 \text{ et } V(X_t|X_{t-1}) = \sigma^2,$$

I.e. the variance and the conditional variance do not depend on time. Given $(\boldsymbol{X}_t)$ ARCH process (1), as:

$$X_t = \varepsilon_t \sqrt{\alpha_0 + \alpha_1 X_{t-1}^2}$$

Where$\boldsymbol{\varepsilon_t} \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{\sigma}^2)$, so:

$$V(X_t|X_{t-1}) = \left[\alpha_0 + \alpha_1 X_{t-1}^2\right]\sigma^2.$$

Definition: We say that the process $(\boldsymbol{X}_t)$ follows an ARCH (p) process if it is defined by an equation of the form:

$$X_t = \varepsilon_t \sqrt{h_t} \text{ où } h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2, \qquad (3.9)$$

Where, $\boldsymbol{\varepsilon_t}$ is a Gaussian centered white noise with variance $\boldsymbol{\sigma}^2$, as $\boldsymbol{\varepsilon_t} \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{\sigma}^2)$. This could be written:

$$X_t^2 = \varepsilon_t \left[\alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2\right]. \qquad (3.10)$$

3.4.4.2 The ARCH (p) process

The ARCH (p) are extensions of the ARCH (1), this last is:

$$X_t = \varepsilon_t h_t \text{ où } h_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2 \text{ et } \varepsilon_t \sim \mathcal{N}(0, \sigma^2),$$

and ARCH (p) involve several delays,

$$X_t = \varepsilon_t h_t \text{ où } h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2 \text{ et } \varepsilon_t \sim \mathcal{N}(0, \sigma^2). \qquad (3.11)$$

Volatility at time "**t**" is then a function of the squared deviations from the mean observed in the recent past. If the coefficients $\boldsymbol{\alpha_i}$ are all positive (and large enough), there is a persistence of volatility levels: periods of high volatility followed by periods of low volatility is observed. Then we can write:

$$\mathbb{E}\left(X_t|\underline{X_{t-1}}\right) = 0, \text{ et } \mathbb{E}\left(X_t^2|\underline{X_{t-1}}\right) = h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2. \qquad (3.12)$$

The marginal variance (or unconditional) for such a process is:

$$\mathrm{var}(\epsilon_t) = \frac{\omega}{1 - \sum_{i=1}^{p} \alpha_i}. \qquad (3.13)$$

3.4.4.3 The GARCH (p,q) process

These models were introduced by Bollerslev in 1986, inspired by the approach of Box and Jenkins, with an autoregressive dynamic.

$$X_t = \varepsilon_t h_t \text{ où } h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}^2 \text{ et } \varepsilon_t \sim \mathcal{N}\left(0, \sigma^2\right). \quad (3.14)$$

It may be noted first that, in this form, the coefficients **p** and **q** are not similar to those of ARMA models: in particular **q**, which corresponds to the autoregressive nature of the process h_t^2 . Then we have:

$$\mathbb{E}\left(X_t | \underline{X_{t-1}}\right) = 0 \text{ et } \mathbb{E}\left(X_t^2 | \underline{X_{t-1}}\right) = h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i}^2 + \sum_{i=1}^{q} \beta_j h_{t-j}^2.$$

The marginal variance (or unconditional) for such a process is:

$$\mathrm{var}(\epsilon_t) = \frac{\omega}{1 - (\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j)}. \quad (3.15)$$

3.4.4.4 Models conditionally heteroscedastic error

We presented the ARCH and GARCH models as white noise conditionally heteroskedastic. Such white noise can be itself the error x_t of series obeying such ARMA:

$$x_t = c + \sum_{i=1}^{m} a_i x_{t-i} + \sum_{j=1}^{n} b_j \epsilon_{t-j} + \epsilon_t \quad (3.16)$$

Where, ε_t follows a GARCH (p, q) described by (3.14). We called **ARMA / GARCH** this type models.

3.4.5 Long memory models

The models considered mostly models are called `a short memory: two minutes' distant process have only very little interaction between them. There is additional modeling for a “long memory process”, such as FARIMA processes (fractional ARIMA) introduced in 1980 by Granger.

3.4.6 Multivariate models

We are sometimes forced to model a multiple phenomenon, or more series having strong relationships between them. The vector autoregressive models are an example of a multivariate model:

$$VAR: \quad \boldsymbol{X_t} = AX_{t-1} + E_t \qquad \boldsymbol{X}_t = (X_t^1, X_t^2) \quad (3.17)$$

Where A is a constant square matrix and $(\boldsymbol{E_t})_{t \in Z}$ is a multidimensional white noise.

3.4.7 *Non-parametric models*

All models are parametric previously considered: the estimation of one or more parameters is sufficient to determine the temporal relationship of a process. However, we can consider that the link function is not set, and then we try to determine a function **"f"** in classes adapted reflecting the temporal relationship, for example:

$$X_t = f\left(X_{t-1}, \ldots, X_{t-p}\right) + \varepsilon_t. \qquad (3.18)$$

3.4.8 *Semi-parametric models*

The non-parametric models suffer from the "scourge of dimension." A non-parametric modeling of one or more combination of variables is then used, for example:

$$X_t = f\left(\theta_1 X_{t-1} + \ldots + \theta_p X_{t-p}\right) + \varepsilon_t, \qquad (3.19)$$

Where p >= 1 is fixed and $\theta_1 \ldots \theta_p$ are the constants.

3.5 Specific problems of time series

Since the first studies on time series, various problems have emerged at this level, and are the subject of several studies to date. Among the major difficulties in processing time series include the prediction of future values of a series, the temporal prediction is currently being used in various fields, it proves a satisfactory means of horizons accuracy, and however it is constrained by increasing the error rate with the expansion of the prediction horizon.

"There are a range of issues specific to time series that are not foreign practitioners and descriptive statistics that will require the development of a number of techniques to econometric treatment (that is to say, probabilistic foundations). This is the primary reason for the development of time series econometrics. These problems are: prediction, identification and removal of the trend, seasonally adjusted, break detection, separation of the short-term and long-term study of agents' expectations ...»

Source: Sébastien LECHEVALIER, "An introduction to time series econometrics", University of Paris I

4. Support Vector Machines

Data mining is the search for relevant information (the "nuggets" of information) for decision support and forecasting. It sets statistical and machine learning techniques, taking into account the specificity of large to very large data (Big data) [30,27].

The development of information technology and computing allows storage (databases), processing and very large sets of data analysis. More recently, the development of software and interfaces available to users, gives the possibility of implementing these simple methods. This development and popularization of new algorithmic techniques (neural networks, support vector machines ...) and graphical tools, led to the development and marketing of software (Enterprise miner, Clementine, Insightfull undermine ...) incorporating a subset of algorithmic and statistical methods used in the terminology of Data Mining.

In those techniques, we distinguish two types of problems: the presence or absence of a response variable **Y** and a form to acknowledge that has been together with **X**, observed on the same objects. In the first case there is indeed a problem of modeling and supervised learning: find a function **f** may at best as a criterion to define, reproduce **Y** having observed **X**.

$$Y = f(X) + \varepsilon \qquad (4.1)$$

Where, ε is the measurement's noise or error.

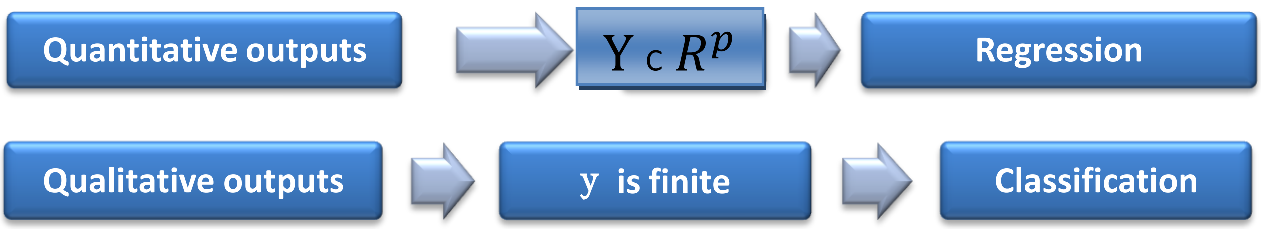
If the error is multiplicative a logarithmic transformation brings us to the previous problem. Otherwise, in the absence of a variable to explain, then it is said to unsupervised learning.

In our study, SVM, we will mainly interest us in supervised learning, where learning consists of observation type input-output data are available from a set:

$$d_1^n = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_n, y_n)\} \qquad (4.2)$$

With $x_i \in X$ (often $=R^p$) and $y_i \in Y$ for $i = 1 \ldots n$.

The aim is to build from this training sample, a model that will allow us to predict the output **y** associated with new entry (or predictor) **X.** The output **y** can be quantitative (price of a stock, electric consumption's curve, pollution's card ...) or qualitative (occurrence of cancer, recognition of numbers ...).



4.1 Definition of SVM

Support Vector Machines classifier (Machines à vecteurs supports) are part of supervised learning techniques. Introduced by Vapnik (1995) to solve classification problems, they have been known as a great success used extensively in various fields: pattern recognition, OCR, bioinformatics ... The use has also spread to problem solving regression as well managed as classifications.

This technique uses a data set called "learning" whose bodies contain a target value also called "class label" and several attributes representing all observed variables, for produce a model to predict the targets of another set called "test" by not providing that model the attributes of test data. In other words, if we consider a data set, divided into two groups, one group for the known examples and another for unknown examples, the goal of SVM is to learn a function that reflects the behavior of known examples to predict the targets of unknown samples.

The main idea of SVM when applied to classification problems is to find a canonical hyper plane which maximally separates the two given classes of training samples. Let us consider two sets of linearly separable training data points in $\boldsymbol{R^P}$ which are to be classified into one of the two classes C_1 and C_2 using linear hyper planes, (i.e. straight lines). From an infinite number of separating hyper planes the one with maximum margin is to be selected for best classification and generalization. Below we present a diagrammatic view of this concept:

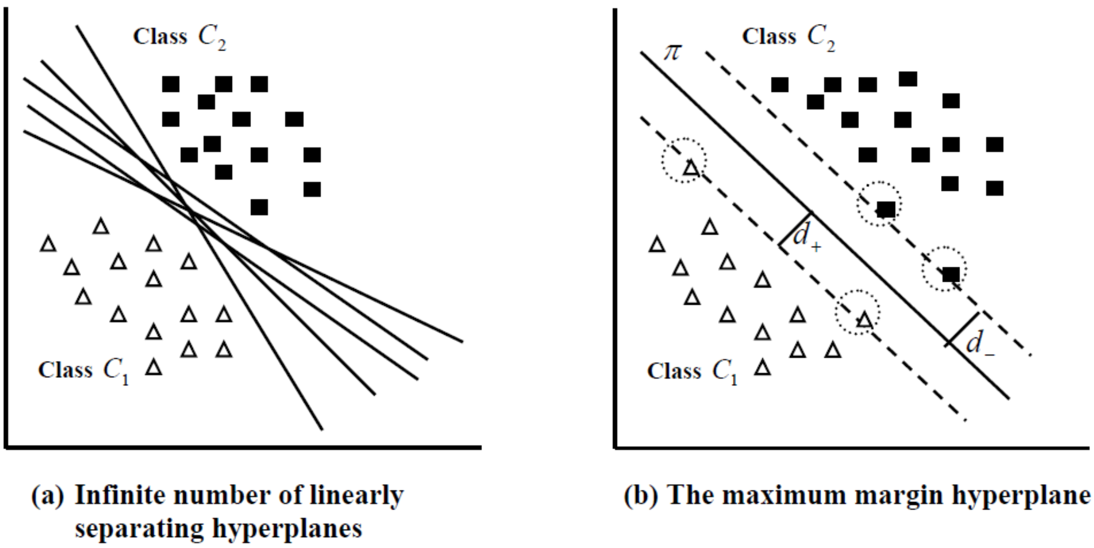


Figure 4.1- Support vectors for linearly separable data points

In **Fig.4.1 (a)** it can be seen that there are an infinite number of hyper planes, separating the training data points. As shown in **Fig.4.1 (b),** d_+ and d_- denote the perpendicular distances from the separating hyper plane to the closest data points of C_1 and C_2 respectively. Then either of the distances d_+ or d- is termed as the margin, and the total margin is M = d_+ + d-. For accurate classification as well as best generalization, the hyper plane which maximizes the total margin is considered as the optimal one and is known as the Maximum Margin Hyper plane. Off course for this optimal hyper plane d_+ = d-. The data points from either of the two classes which are closest to the maximum margin hyper plane are known as Support Vectors. In **Fig.4.1 (b)** π denotes the optimal hyper plane and circulated data points of both the classes represent the support vectors.

4.2 Theory of SVMS in regression approximation

SVMs can also be applied to regression problems by the introduction of an alternative loss function, (Smola, 1996). The loss function must be modified to include a distance measure. Figure 5.1 illustrates four possible loss functions.

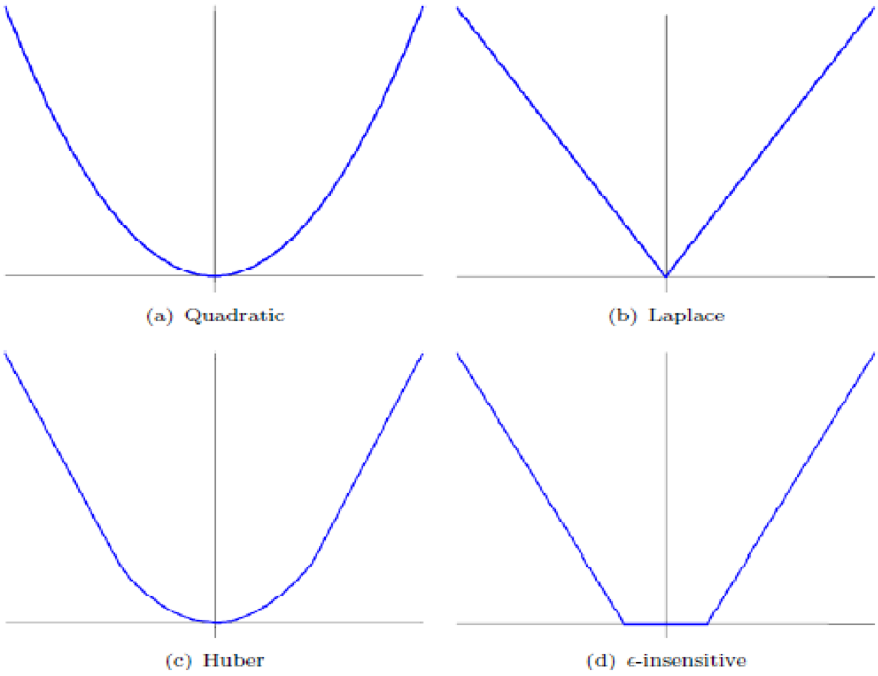


Figure 4.2- Loss Functions

The loss function in **Figure 4.2(a)** corresponds to the conventional least squares error criterion. The loss function in **Figure 4.2(b)** is a Laplacian loss function that is less sensitive to outliers than the quadratic loss function. Huber proposed the loss function in **Figure 4.2(c)** as a robust loss function that has optimal properties when the underlying distribution of the data is unknown. These three loss functions will produce no sparseness in the support vectors. To address this issue Vapnik proposed the loss function in **Figure 4.2(d)** as an approximation to Huber's loss function that enables a sparse set of support vectors to be obtained.

In the case of linear regression, the variable to predict $\boldsymbol{y_i}$ is a real number. Given a set of data points $G = \{(x_i, d_i)\}_i^n$ (**x**$_i$ is the input vector, **d**$_i$ is the desired value and **n** is the total number of data patterns), SVMs approximate the function using the following:

$$y = f(x) = w\phi(x) + b, \tag{4.3}$$

Where $\emptyset(\boldsymbol{x})$ is the high dimensional feature space which is non-linearly mapped from the input space **x**. To handle the case where the data are not perfectly linear, a margin of error of more or less ε is allowed for prediction. **Figure 4.3** illustrates the principle [27].

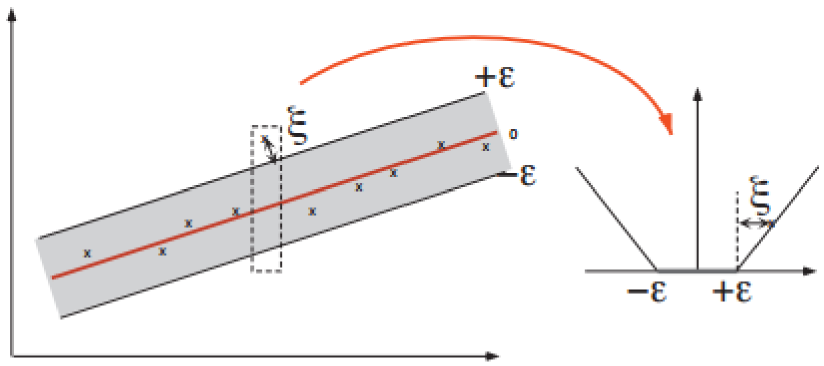


Figure 4.3- SVR regression $\varepsilon - insensitive$

The coefficients **w** and **b** are estimated by minimizing:

$$R_{\mathrm{SVMs}}(C) = C\frac{1}{n}\sum_{i=1}^{n} L_{\varepsilon}(d_i, y_i) + \frac{1}{2}||w||^2, \tag{4.4}$$

$$L_{\varepsilon}(d, y) = \begin{cases} |d - y| - \varepsilon & |d - y| \geqslant \varepsilon \\ 0 & \text{otherwise} \end{cases}. \tag{4.5}$$

In the regularized risk function given by (4.4), the first term:

$$C\frac{1}{n}\sum_{i=1}^{n} L_{\varepsilon}(d_i, y_i) \tag{4.6}$$

Is the empirical error (risk). They are measured by the $\varepsilon - insensitive$ loss function given by (4.5). This loss function provides the advantage of enabling one to use sparse data points to represent the decision function given by (4.3) The second term :

$$\frac{1}{2}||w||^2 \tag{4.7}$$

Is the regularization term "C" is referred to as the regularized constant and it determines the trade-off between the empirical risk and the regularization term. Increasing the value of C will result in the relative importance of the empirical risk with respect to the regularization term to grow. ε Is called the tube size and it is equivalent to the approximation accuracy placed on the training data points. Both C and ε are user-prescribed parameters.

4.3 Kernel function

SVM can be applied only on linearly separable data or could be linearly separated with a reasonable amount of error. However, problems answering to these constraints are rare. Indeed, the majority of problems are composed of linked data from a class, or more generally has a target variable, complex relationships cannot be modeled linearly. There is an approach which addresses these problems. It consists to find a space in which data can be linearly separable. We call such a space, feature space or attributes space. Once this space defined via a shift function that combines $\emptyset$ has given its image in the feature space, it becomes possible to use all linear algorithms in the new space. **Figures 4.4**(a) and (b) provide an illustration of this principle. The difficulty in this approach lies in the definition of the shift function which may require a good knowledge of the data.

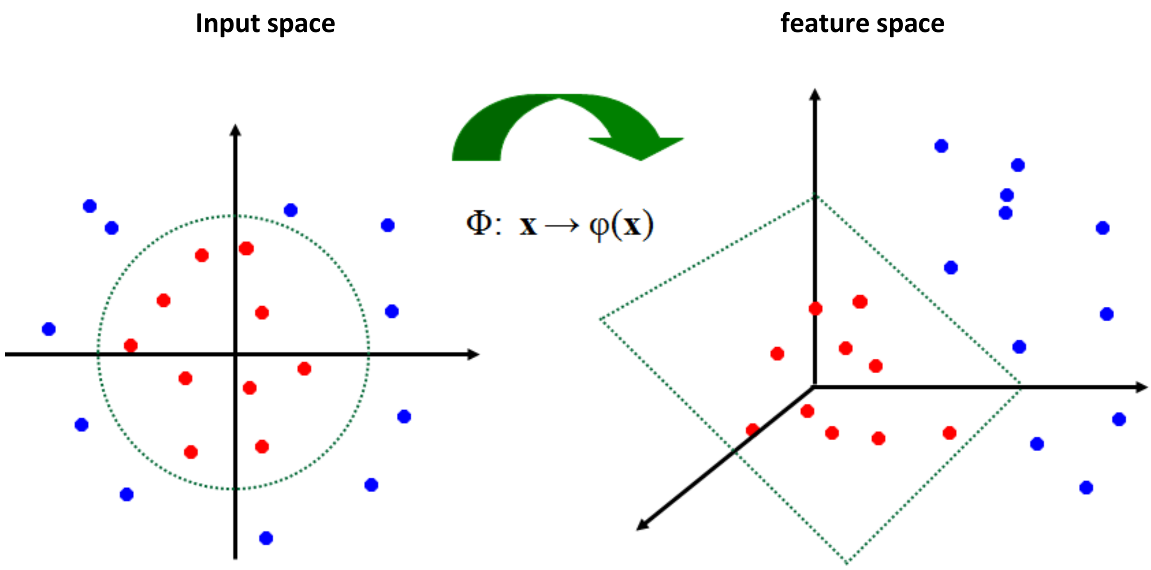


Figure 4.4- Passing data from the input space to a feature space where the data are linearly separable.

General idea: the original input space can always be mapped to some higher-dimensional feature space where the training set is separable

$K(x_i, x_j)$ is defined as the kernel function. The value of the kernel is equal to the inner product of two vectors $\mathbf{X}_i$ and $\mathbf{X}_j$ in the feature space $\emptyset(x_i)$ and $\emptyset(x_j)$, that is:

$$K(x_i, x_j) = \emptyset(x_i) * \emptyset(x_j) \tag{4.8}$$

The typical examples of kernel function are the polynomial kernel:

$$K(x, y) = (x * y + 1)^d \tag{4.9}$$

And the Gaussian kernel (RBF):

$$K(x, y) = \exp(-1/\delta^2(x - y)^2) \tag{4.10}$$

Where d is the degree of polynomial kernel and δ^2 is the bandwidth of the Gaussian kernel. The kernel parameter should be carefully chosen as it implicitly defines the structure of the high dimensional feature space $\emptyset(x)$and thus controls the complexity of the final solution.

4.4 Sequential minimal optimization

SMO was proposed (Platt 1999) that puts chunking to the extreme by iteratively selecting subsets only of size 2 and optimizing the target function with respect to them. It has been reported to have good convergence properties and it is easily implemented. The key point is that for a working set of 2 the optimization sub problem can be solved analytically without explicitly invoking a quadratic optimizer.

Note that the reasoning only applies to SV regression with the ε insensitive loss function—for most other convex cost functions an explicit solution of the restricted quadratic programming problem is impossible. Yet, one could derive an analogous non quadratic convex optimization problem for general cost functions but at the expense of having to solve it numerically.

Recently, Smola and Schölkopf [2] proposed an iterative algorithm, called sequential minimal optimization (SMO), for solving the regression problem using SVM. This algorithm is an extension of the SMO algorithm proposed by Platt for SVM classifier design. Computational speed and ease of implementation are some of the noteworthy features of the SMO algorithm. Recently, some improvements to Platt's SMO algorithm for SVM classifier design were suggested. For our study, we will use the extend ideas to Smola and Schölkopf's SMO algorithm for regression. The **Reg SMOImproved** [27] enhances the value of the original SMO for regression even further.

5. Empirical tests

Here we will give the tests that we have required during our study [28, 35, 22].

5.1 Criterion of pattern comparison

It often happens that after several tests, several models show resistant. To choose the best among them, we can use criteria for comparing models. These are numerous and strong play, sometimes, an important role in econometrics.

5.1.1 Standard criteria

A fundamental challenge while predicting is how to determine prediction error and appropriate technique for a series under study. The term accuracy refers to how best the model fits a given series, Makridakis and wheelwright (1989). All the forecast accuracy measures used are obtained as follows;

- Mean error

$$ME = \frac{1}{n}\sum_{i=1}^{n} e_t \tag{5.1}$$

- Mean Square Error

$$\text{MSE} = \frac{1}{n}\sum_{t=1}^{n}(\hat{Y}_t - Y_t)^2 \tag{5.2}$$

- Root Mean Square Error

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{Y}_t - Y_t)^2} \tag{5.3}$$

- Mean percentage Error

$$\text{MPE} = \frac{1}{n}\sum_{t=1}^{n}\frac{(\hat{Y}_t - Y_t)}{\hat{Y}_t} x100 \tag{5.4}$$

- Mean Absolute Error

$$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n}|(\hat{Y}_t - Y_t)| \tag{5.5}$$

- Mean Absolute Percentage Error is also calculated as

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\frac{|(\hat{Y}_t - Y_t)|}{\hat{Y}_t} x100 \tag{5.6}$$

5.1.2 Information criteria

This approach was introduced by Akaike in 1969. This measure, of the difference between the proposed model and the real rule, can be obtained using the amount of Kullback information.

Definition:

f_0 is the unknown density of observations, and $\{ f_{(.)}, f \in F \}$ family densities of which have been estimated. The difference between the real law and the model is given by:

$$I(f_0, \mathcal{F}) = \min_{f \in \mathcal{F}} \int \log \frac{f_0(x)}{f(x)} . f_0(x)\, dx \tag{5.7}$$

This quantity is always positive and vanishes only if f_O belongs to F. This is unknown since f_O is unknown, try to minimize an estimate of I, $\ddot{I}$. Several estimator amount of information has been proposed in the case of models ARMA (p, q) from T observations,

Aikaïke (1969) :

$$AIC\,(p,q) = \log \widehat{\sigma}^2 + 2\frac{p+q}{T} \tag{5.8}$$

Schwarz (1977) :

$$BIC\,(p,q) = \log \widehat{\sigma}^2 + [p+q]\,\frac{\log T}{T} \tag{5.9}$$

Hanna-Quinn (1979) :

$$\phi\,(p,q) = \log \widehat{\sigma}^2 + [p+q]\,c\frac{\log(\log T)}{T} \qquad c > 2 \tag{5.10}$$

5.2 Portmanteau Test (whiteness test)

The two well known one are:

5.2.1 Box-Pierce test

Financial applications often require testing jointly that several autocorrelations of r_t are zero. Box and Pierce (1970) propose the Portmanteau statistic:

$$Q^*(m) = T\sum_{\ell=1}^{m} \hat{\rho}_\ell^2 \tag{5.11}$$

As a test statistic for the null hypothesis H_0: $\rho 1 = \cdots = \rho m = 0$ against the alternative hypothesis

H_a: ρi _= 0 for some $i \in \{1, \ldots, m\}$.

Under the assumption that $\{r_t\}$ is an iid sequence with certain moment conditions, *Q*(m)* is asymptotically a chi-squared random variable with *m* degrees of freedom.

5.2.2 Ljung and Box test

Ljung and Box (1978) modify the *Q*(m)* statistic as below to increase the power of the test in finite samples:

$$Q(m) = T(T+2)\sum_{\ell=1}^{m} \frac{\hat{\rho}_\ell^2}{T-\ell}. \tag{5.12}$$

The decision rule is to reject *Ho* if $Q(m) > X_{\alpha}^{2}$, where X_{α}^{2} denotes the 100(1 − α)th percentile of a chi-squared distribution with *m* degrees of freedom. Most software packets will provide the *p*-value of *Q(m)*. The decision rule is then to reject *Ho* if the *p*-value is less than or equal to *α*, the significance level.

5.3 Conditional heteroskedasticity test

Since the ARCH model has the form of an autoregressive model, Engle (1982) proposed **the Lagrange Multiplier (LM) test**, in order to test for the existence of ARCH behavior based on the regression. The test statistic is given by TR^2, where R is the sample multiple correlation coefficient computed from the regression of ε_t^2 on a constant and $\varepsilon_{t-1}^2, \ldots, \varepsilon_{t-q}^2$, and T is the sample size. Under the null hypothesis that there is no ARCH effect, the test statistic is asymptotically distributed as chi-square distribution with q degrees of freedom.

5.4 Normality tests

Here the two tests that we have used in the study:

5.4.1 Jarque-Bera test

The Jarque-Bera test is a goodness-of-fit measure of departure from normality based on the sample kurtosis and skew. In other words, JB determines whether the data have the skew and kurtosis matching a normal distribution. The test is named after Carlos M. Jarque and Anil K. Bera. The test statistic for JB is defined as:

$$JB = \frac{n}{6}\left(S^2 + \frac{K^2}{4}\right) \sim X_{v=2}^{2}$$

Where

$\mathrm{S}=$the sample skew

$\mathrm{K}=$the sample excess kurtosis

$\mathrm{n}=$the number of non-missing values in the sample

$\mathrm{JB}=$the test statistic; JB has an asymptotic chi-square distribution

5.4.2 Shapiro-Wilk test

Shapiro-Wilk test W and its approximation W.

$$W = \frac{\left(\sum_{i=1}^{n}\left(a_i x_{(i)}\right)\right)^2}{\sum_{i=1}^{n}(x_{(i)} - \overline{X})^2}$$

Where

- $X_{(i)} =$ the i^{th} order (smallest number in the sample)
- $a_i =$ a constant given by

  $$(a_1, a_2, \dots, a_n) = \frac{m^T V^{-1}}{\sqrt{(m^T V^{-1} V^{-1} m)}}$$

- $m=$ the expected values of the order statistics of independent and identical distributed random variables sampled from Gaussian distribution
- $V=$ the covariance matrix of m order statistics

5.5 Skweness and Kurtosis

Skweness: indicator used in distribution analysis as a sign of asymmetry and deviation from a normal distribution. Its interpretation:

- Skewness > 0 - Right skewed distribution - most values are concentrated on left of the mean, with extreme values to the right.
- Skewness < 0 - Left skewed distribution - most values are concentrated on the right of the mean, with extreme values to the left.
- Skewness = 0 - mean = median, the distribution is symmetrical around the mean.

Kurtosis: indicator used in distribution analysis as a sign of flattening or "peakedness" of a distribution. Interpretation:

- Kurtosis > 3 - Leptokurtic distribution, sharper than a normal distribution, with values concentrated around the mean and thicker tails. This means high probability for extreme values.
- Kurtosis < 3 - Platykurtic distribution, flatter than a normal distribution with a wider peak. The probability for extreme values is less than for a normal distribution, and the values are wider spread around the mean.
- Kurtosis = 3 - Mesokurtic distribution - normal distribution for example.

5.6 Other statistics

Type	Optimal	Acceptable
R^2 and Adjusted R^2	$\rightarrow 1$	$> 0,8$
J-statistic	$\rightarrow 0$	$< 0,1$
Mean dependant variable	$\rightarrow +\infty$	> 100
S.E. of Regression	$\rightarrow 0$	Choose the lower value (comparison)
Residual sum of squares	$\rightarrow 0$	Choose the lower value (comparison)
Prob(F-statistic)	$\rightarrow 0$	$< 0,05$
Durbin-Watson statistic	$\rightarrow 2$	$1.8 < DW < 2.2$ (Under conditions)
Determinant residual covariance	$\rightarrow 0$	Choose the lower value (comparison)
Log-Likelihood	$\rightarrow +\infty$	$> 10^3$
Average Log-Likelihood	$\rightarrow +\infty$	> 10
AIC	$\rightarrow -\infty$	Choose the lower value (comparison)
SIC	$\rightarrow -\infty$	Choose the lower value (comparison)
HQIC	$\rightarrow -\infty$	Choose the lower value (comparison)

Table 5.1- Summary of some common statistics and their level of significance

6. Study of the tree series “CAC40”, “S&P500” and “FTSE100”

Modeling of financial time series is a complex problem. This complexity is not only due to the wide variety of series used (price action, interest rates, exchange rates, etc...), the importance of the observation frequency (second, minute, hour, day, etc.) or the availability of very large sample size. It is mainly due to the existence of statistical regularities (' stylized facts ') common to a large number of financial and difficult to reproduce series artificially from stochastic models.

In an article published in 1963, highlighted a Mandelbrot set such properties. These empirical regularities, verified and supplemented since by many authors appear more or less clearly depending on the frequency of observation of the series and its nature. Some properties are presented below:

- Non stationarity
- Autocorrelations of squared price changes
- volatility clustering
- Thick tails distribution
- Leverage
- Seasonality.

6.1 Analysis tools

Unlike the ARMA-GARCH model, which have several analysis interfaces (R, SAS, IBM SPSS Statistics, XLSTAT, ..) there are few commercial software implementing the SVM, like IBM SPSS Modeler[dm]. Free software is more contrast. On one hand, software dedicated to SVM (SVM^{light}, mySVM, SVMTorch…), and secondly, the most comprehensive software, including SVM function. This is the case of case of Weka that implements the algorithm SMO (sequential minimal optimization) and R whose kernlab package contains the function ksvm which implements several SVM algorithms or the e1071 package for SVR, also in R. all, implement methods for classification and regression, and kernels that we have seen above (linear, polynomial, RBF, simoidal)[10,20,30,44].

Before starting the study, we gone briefly present the set of tools that we have used in our analysis:

- **R software** is statistical software created by Ross Ihaka & Robert Gentleman [LA]. It is both a programming language and a work environment; the commands are executed thanks to instructions code, which are in a relatively simple language. Results are displayed as text and graphics are visualized directly in a window of their own. This software is used to manipulate data, plot graphs and perform statistical analyzes on these data.
- **WEKA** is the product of the University of Waikato(New Zealand) and was first implemented in its modern form in 1997. It uses the GNU GeneralPublic License (GPL). The software is written in the Java™ language and contains a GUI forinteracting with data files and producing visual results. It also has a general API, so we can embed WEKA, like any other library, in our own applications to such things as automated server-side data-mining tasks [we].
- **SPSS Statistics** is a software package used for statistical analysis. Long produced by SPSS Inc., it was acquired by IBM in 2009, and current versions are officially named IBM SPSS Statistics. SPSS is among the most widely used programs for statistical analysis in social science. In addition to statistical analysis, data management and data documentation are features of the base software.

6.2 Data exploration

We first bring our data in R console from yahoo finance: http://fr.finance.yahoo.com/ ; the packages, which were required for this step, are: **its**, **timeSeries and caschrono.**

The code which we used is in **ANNEXE 1,** this last contains the totality of the R code used in the CAC40 ARMA-GARCH study[1]. Other packages were required in R **fBasics, xtable, fGarch** and **FinTS.**

We do our SVR on Weka. We used **SMOreg**, this implements the support vector machine for regression. The parameters can be learned using various algorithms. The algorithm is selected by setting the RegOptimizer. The most popular algorithm "**RegSMOImproved"** is due to Shevade, Keerthi and al **[]** and this is the default RegOptimizer.

[1] To give the code of the three series is not necessary, because the same steps are repeated for each index.

The **Figure 6.1** represents the ACF and PACF of each series, here, numeric statistics are not shown but the graphs, clearly, indicate that our series are not stationary, which suggest that we have to transform our data, in order to make possible the creation of the ARMA-GARCH model.

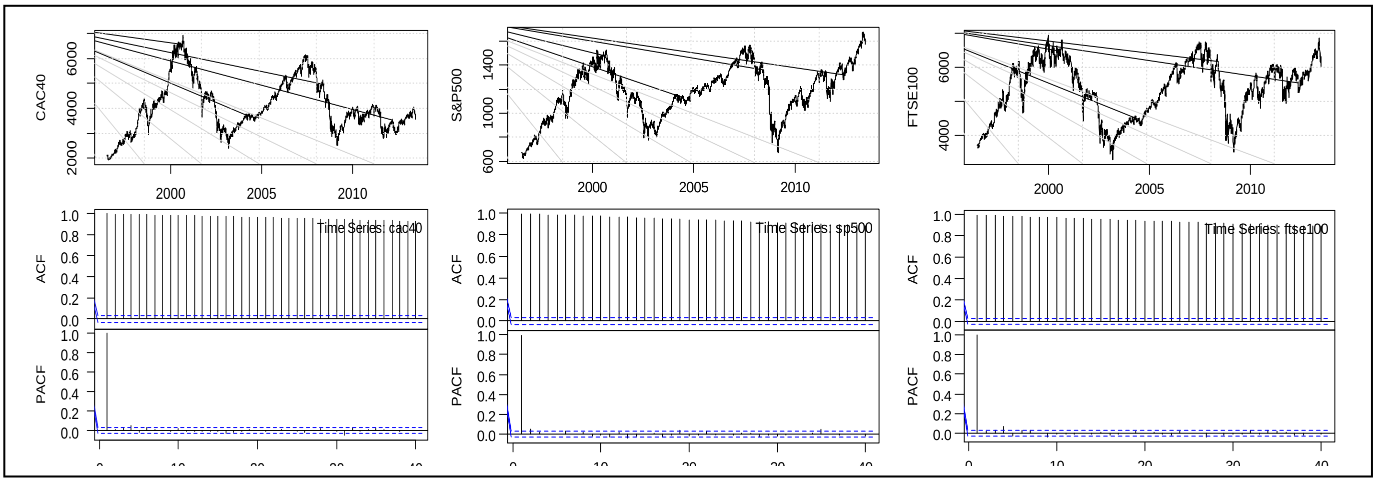


Figure 6.1- ACF and PACF of the three series

6.3 Study of the daily returns

Three stock market indices CAC 40 from Paris Stock Exchange, S&P 500 from New York Stock Exchange and FTSE 100 from London Stock Exchange were chosen for this study. All times series from 01 July 1996 to 01 July 2013 were considered.

In order to eliminate the non-stationarity observed before, we compute the returns of each index.

Returns: given x_t the closing price of of the particular daily index in period t. the simple return is:

$$r_t^* = \frac{x_t - x_{t-1}}{x_{t-1}}, \qquad (*)$$

Thus $x_t = (1+r_t^*)x_{t-1}$ et $\log(x_t) = \log(1+r_t^*) + \log(x_{t-1})$. If $x_t \simeq x_{t-1}$

We obtain $$r_t^* \simeq \Delta \log(x_t). \qquad (**)$$

We call "r_t^*" compound return. Both yields are very similar values. We will work with compound returns.

The daily prices, returns and their squares are plotted in the left side of **Figure 6.2**. In the returns graph, it can readily be seen that the volatility concentrates itself in clusters, that is, periods of high and low volatility can be observed in the data.

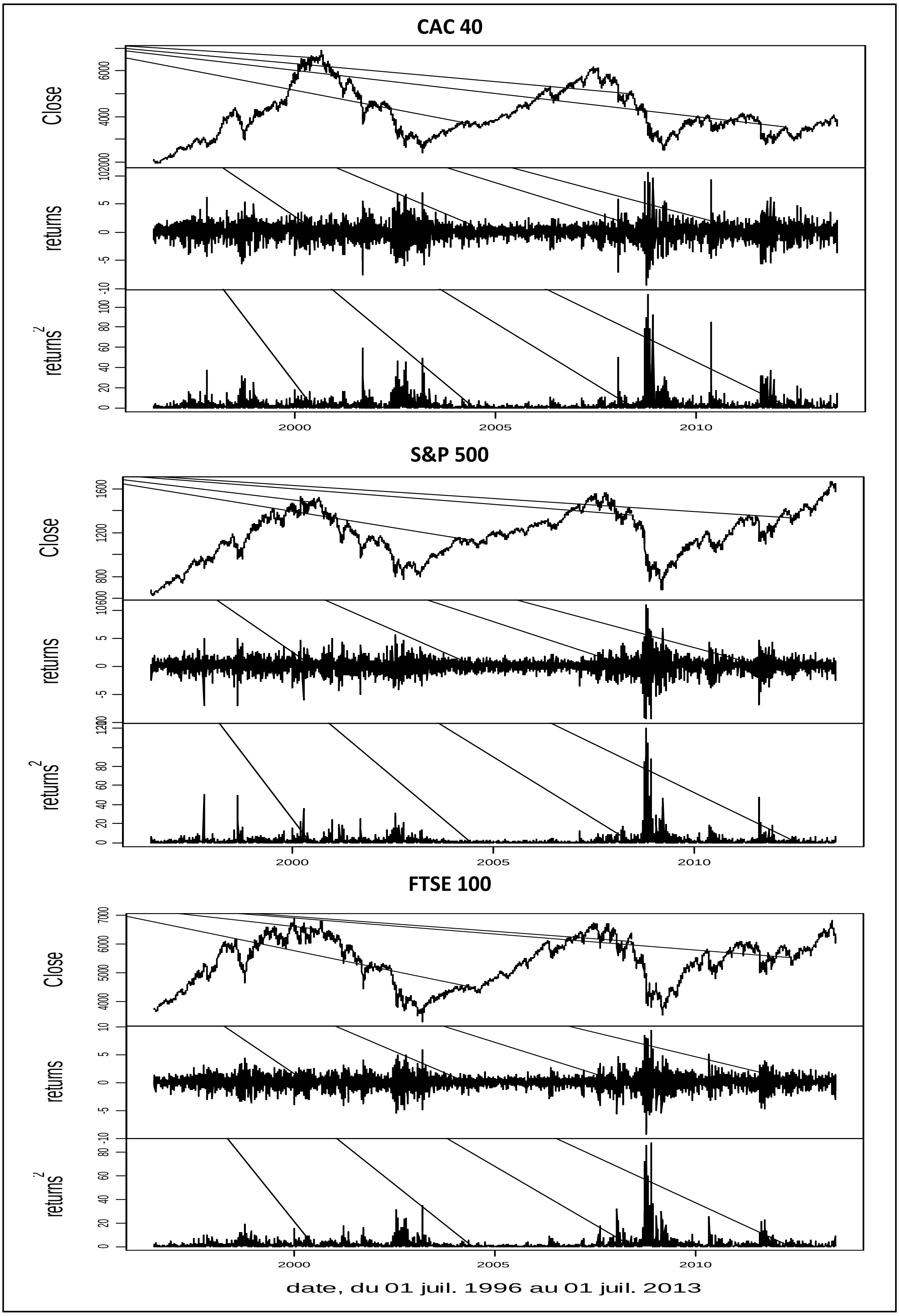


Figure 6.2 – CAC 40, S&P 500 and FTSE 100: daily prices, daily returns and squared returns

We say that a market is efficient if the price of the securities on the market fully reflect all available information. In such a market, it is impossible to predict future returns. In terms of time series, the return on the market is a white noise. White noise series is thus the first requirement for a data series to which we can apply the classical statistical techniques as well as the Artificial Intelligence models for better fitting. This transformation has many advantages. The most important is that the distribution of the transformed data becomes more symmetrical so that it follows a normal distribution more closely.

To illustrate the main empirical properties often observed in high–frequency financial time series, **Table 6.1** contains descriptive statistics, including time period for each market, of the three financial return series observed daily. The time periods cover many important economic events, which we believe are sufficient for the training models.

Table 6 .1 Descriptive statistics on returns distributions

		N	Min.	Max.	Mean	Std. Dev.	Skew		Kurtosis	
Series	**Time period**	**Statistic**	**Statistic**	**Statistic**	**Statistic**	**Statistic**	**Statistic**	**Std. Error**	**Statistic**	**Std. Error**
CAC40	from 01 July 1996 to 01 July 2013	4329	-9,4715	10,5946	,013295	1,5188405	-,013	,037	4,251	,074
S&P500	from 01 July 1996 to 01 July 2013	4278	-9,47	10,96	,0204	1,29578	-,217	,037	7,224	,075
FTSE100	from 01 July 1996 to 01 July 2013	4292	-9,2646	9,3842	,012268	1,2442591	-,135	,037	5,337	,075

In **Table 6.1,** it is possible to observe that all the series show almost zero means[2] and excess kurtosis (always above three) for the normal distribution value.

For the line Skewness, we could know if the distribution is not normal by skeweness and the kurtosis line say if the distribution is not normal by flattening. For the three alternatives, the p-value is low, so we reject the assumption of normality of returns, and this rejection is more due to a flattening than to skewness excess; note that the sign of the skewness indicates that the distribution of returns has left charged tail: there is more very negative returns than positive returns. To improve our understanding of the non-normality, we should examin graphically the distribution of returns. The **Figure 6.3** Superimpose on the same graph of the probability density nonparametrically estimated performance and the density of a normal distribution with the same mean and variance as the yield[3].

[2] The returns are expressed with percentages in order to have clear results, there for to get the real value of the returns we have to divide by one hundred.
[3] The R code of the density plot function is in **ANNEXE1**

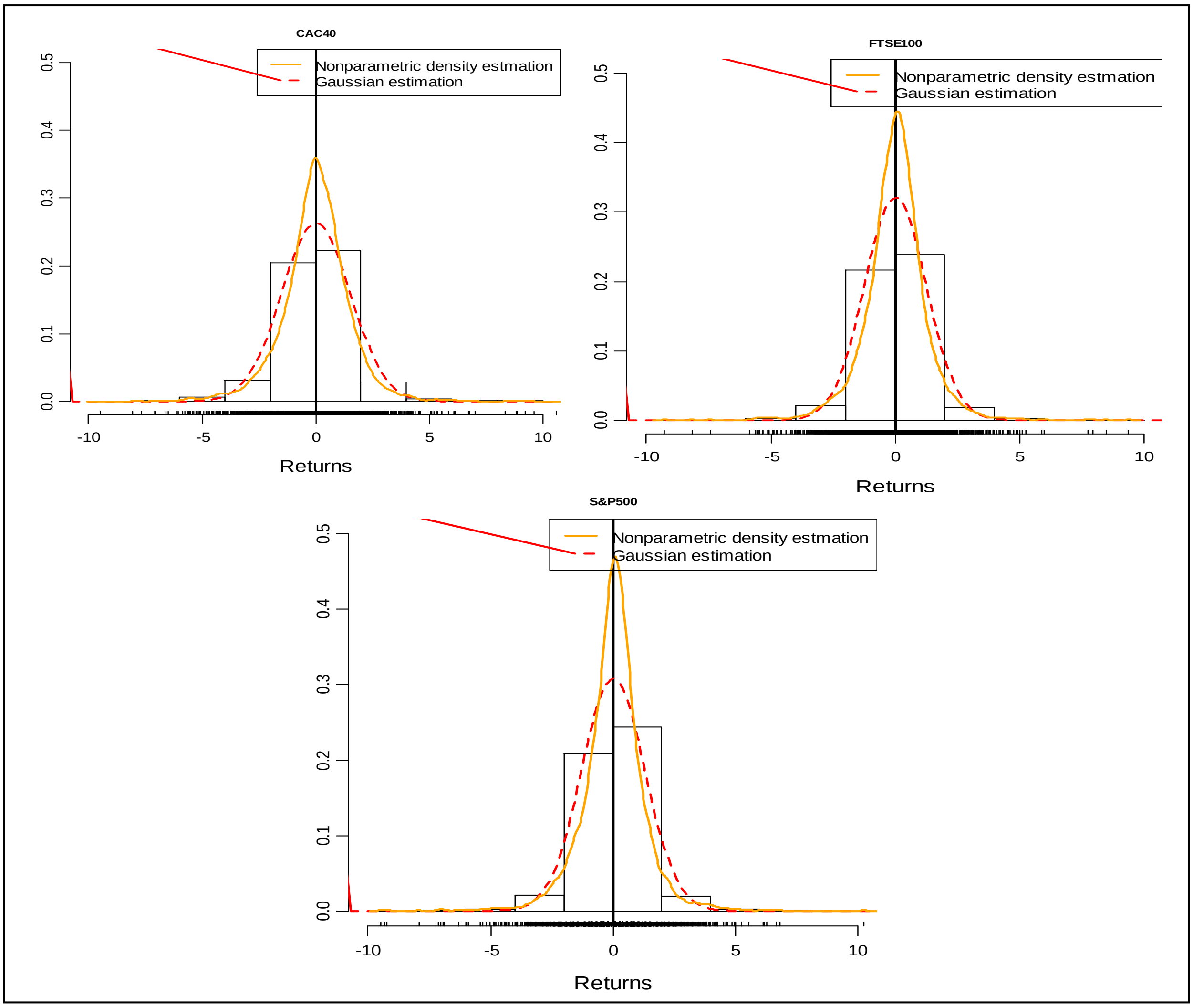


Figure 6.3- Returns of the three indexes

The graph does not clearly identify the skewness of the return distribution, but shows that the distribution is more concentrated around the mean than a normal distribution, which supports our reading of the table of descriptive statistics.

6.3.1 Study of the CAC40 daily returns with ARMA-GARCH

In this step, we will model the daily returns of the CAC40 to make its forecasting. Specifically, we use as a training set "train.r.cac" composed of the returns data least the last 433 values. this number, represents 10% of the total number of obsevations. The observations, excluded from training, will be used for testing "test.r.cac".

6.3.1.1 *Estimation*

The autocorrelation functions (ACFs) and partial autocorrelations functions (PACFs) of the CAC40 daily returns and their squares are depicted in **Figure 6.4**. In non-squared ACF (graphic located in the upper left), almost all the spikes are within the boundary (formed by standard errors), that is, ACF decay very quickly toward zero, whereas almost all the spikes go out of the boundary in the squared ACF(graphic located in the upper right), that is, ACF do not decay toward zero for each market.

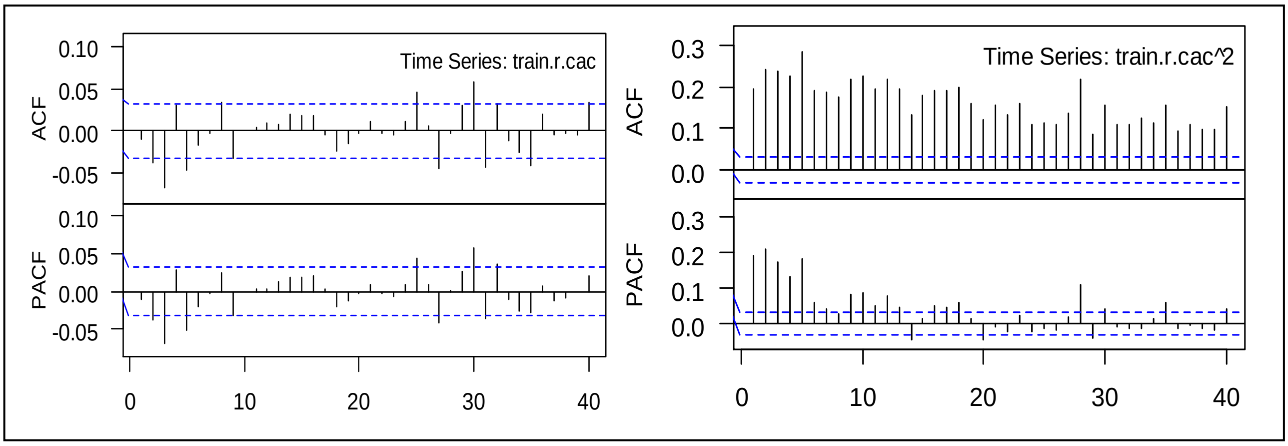


Figure 6.4 - CAC40: ACF and PACF for the daily returns and their squares

Also, **Figure 6.4** indicates that the volatility clustering is reflected in the significant correlations of squared returns. The autocorrelation coefficients of squared returns are larger and last longer (persistent) than those of the return series (non-squared).We must point out that the return series show little or no correlation, but its squares show high correlation, which indicates the ARCH or GARCH effect.

Therefore, our data set is not a white noise, so, we continue with looking for its ARMA model. ACF suggests an MA (5), which the expert modeler of IBM SPSS Statistics confirms. Its application within R gives the following results:

```
. . .
Coefficients:

            ma1        ma2       ma3       ma4       ma5
          -0.0126    -0.0402   -0.0701    0.0244   -0.0411
   s.e.    0.0160     0.0160    0.0164    0.0155    0.0160

sigma^2 estimated as 2.323:  log likelihood=-7168.43
AIC=14348.86   AICc=14348.88   BIC=14386.46

Training set error measures:
```

```
     ME          RMSE         MAE          MAPE         MASE
 0.01178496  1.524209   1.086581    128.104    0.6924167
```

Before getting the p-values of coefficients of the model, which gives us their significance or not, we have to construct a "portmanteau" statistic, because these p- values have meaning, only if the residuals can be considered as white noise.

	Retard	p-value
[1,]	6	0.9732
[2,]	12	0.5946
[3,]	18	0.5589
[4,]	24	0.8003

We note that the Box-Pierce test gives satisfactory results, for the four lags that we have chosen, these results are confirmed in **Figure 6. 5** (the left graphic). Now, we have to check significance of coefficients:

	ma1	ma2	ma3	ma4	ma5
t.stat	-0.7865	-2.5086	-4.2835	1.5800	-2.5613
p.val	0.4316	0.0121	0.0000	0.1141	0.0104

The parameters MA 1 and MA 4 are not significant. Now let's test the absence of conditional heteroskedasticity in the residuals of the model:

```
 ARCH LM-test; Null hypothesis: no ARCH effects

Data:  rr.cac

Chi-squared = 736.3239, df = 20, p-value < 2.2e-16
```

There is conditional heteroskedasticity in the residuals of the model MA (5). Before taking into account the heteroskedasticity, note that, the **Figure6. 5** (right graphic) represents the shape of the distribution of residuals or the density plot, which shows the deformation compared to a normal density.

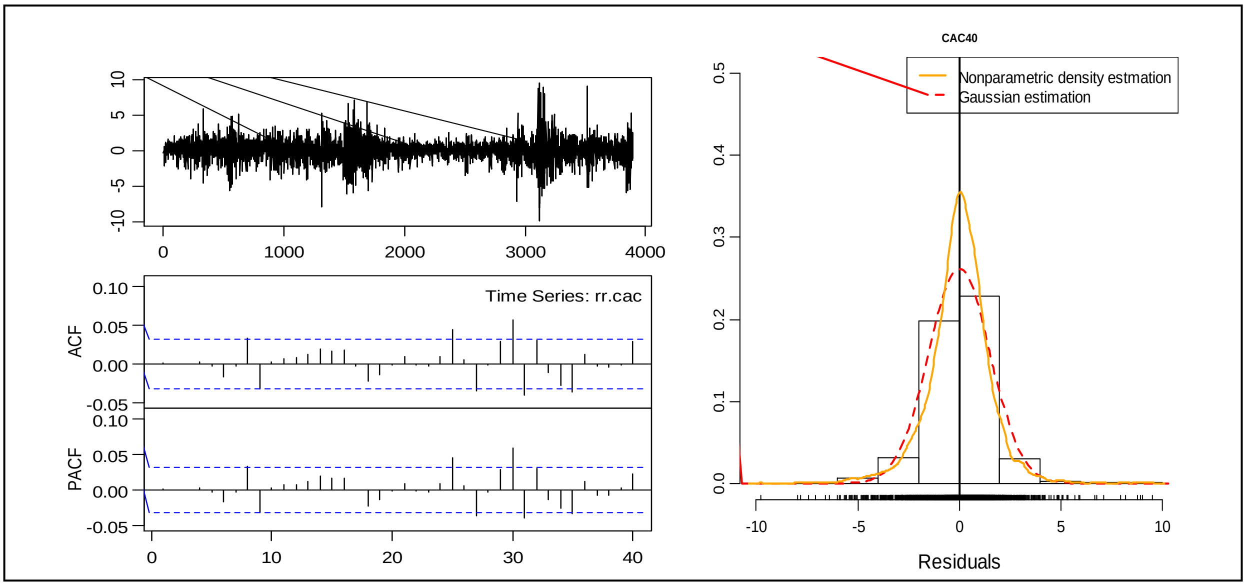


Figure 6.5– CAC40: ACF, PACF and probability density of the ARMA residuals

6.3.1.2 Modeling the heteroskedasticity of the returns

Now, let us model the heteroskedasticity of residuals of ARMA (0, 5) above. After trying a GARCH (1, 1) and GARCH (2, 1), we fit the second one because it gives better AIC.

```
…
Std. Errors:
 based on Hessian

Error Analysis:

            Estimate    Std. Error    t value  Pr(>|t|)
mu          0.074756    0.017677      4.229    2.35e-05  ***
omega       0.024734    0.006021      4.108    3.99e-05  ***
alpha1      0.040378    0.016619      2.430    0.01511   *
alpha2      0.066817    0.020512      3.257    0.00112   **
beta1       0.884868    0.011623      76.128   < 2e-16   ***
---

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 -6534.977    normalized:  -1.677786

Standardised Residuals Tests:

                                    Statistic   p-Value
Jarque-Bera Test    R       Chi^2   134.7958    0
Shapiro-Wilk Test   R       W       0.9933444   1.859943e-12
Ljung-Box Test      R       Q(10)   13.40458    0.2019222
Ljung-Box Test      R       Q(15)   17.23925    0.3047643
Ljung-Box Test      R       Q(20)   18.71385    0.5404897
```

```
Ljung-Box Test      R^2     Q(10)      5.360241    0.865852
Ljung-Box Test      R^2     Q(15)      7.939226    0.9261961
Ljung-Box Test      R^2     Q(20)      12.07946    0.913314
LM Arch Test        R       TR^2       6.575586    0.8843405

Information Criterion Statistics:

     AIC           BIC            SIC          HQIC
  3.358140      3.366185      3.358136      3.360995
```

In Standardized Residuals Tests (a set of tests on residuals) we find:

- Two tests of normality of residuals
- Test whiteness of residuals to the delays 10, 15 and 20
- Conditional homoscedasticity tests, tests of whiteness of the squared residuals and the Lagrange multiplier test.

Aside the non-normality of residuals suggested by the Jarque-Bera, the results are quite satisfactory. Now consider the variance of long-term:

```
Marginal variance:  3.116203
```

The estimate of the marginal variance obtained by substituting in **(3.15)** is positive, which we prefer to obtain. Otherwise, the fitted model is not suitable. Thus, we can combine the two models.

6.3.1.3 Simultaneous returns modeling and their heteroskedasticity

We continue the estimate by combining the two models, the ARMA (0, 5) for the returns evolution and the GARCH (2, 1) for the conditional variance of the error of the MA (5) model evolution.

```
…
Std. Errors:
 based on Hessian

Error Analysis:

             Estimate   Std. Error    t value    Pr(>|t|)
ma1         -0.008416    0.016176     -0.520     0.60289
ma2         -0.030289    0.016899     -1.792     0.07307    .
ma3         -0.046944    0.016735     -2.805     0.00503   **
ma4         -0.007649    0.016519     -0.463     0.64332
ma5         -0.028199    0.016133     -1.748     0.08049    .
omega        0.024016    0.005973      4.021     5.8e-05  ***
alpha1       0.038190    0.016616      2.298     0.02154    *
alpha2       0.066085    0.020501      3.224     0.00127   **
beta1        0.887972    0.011423     77.737     < 2e-16  ***
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 -6539.166    normalized:  -1.678862

Standardised Residuals Tests:

                                  Statistic  p-Value
 Jarque-Bera Test    R    Chi^2   152.1515   0
 Shapiro-Wilk Test   R    W       0.9930167  7.606708e-13
 Ljung-Box Test      R    Q(10)   6.539405   0.7680942
 Ljung-Box Test      R    Q(15)   11.35698   0.7269026
 Ljung-Box Test      R    Q(20)   13.30603   0.8638711
 Ljung-Box Test      R^2  Q(10)   5.497953   0.8555344
 Ljung-Box Test      R^2  Q(15)   7.660052   0.9366876
 Ljung-Box Test      R^2  Q(20)   11.39858   0.935224
 LM Arch Test        R    TR^2    6.326015   0.8987643

Information Criterion Statistics:

   AIC       BIC       SIC      HQIC
3.362344  3.376826  3.362334  3.367485


Marginal variance:  3.097662
```

The normality of the residuals is rejected by the Jarque-Bera test and by the Shapiro-Wilk test. The test results of whiteness and homoscedasticity are satisfactory. The MA1, MA 2, MA 4 and MA 5 parameters are not significant. Finally, the ARMA / GARCH model adjusted for the CAC40 returns is:

$$y_t = \epsilon_t - 0.008416 \ \epsilon_{t-1} - 0.030289 \, \epsilon_{t-2} - 0.046944\epsilon_{t-3} - 0.007649\epsilon_{t-4} - 0.028199 \, \epsilon_{t-5}$$

$$\epsilon_t = \sigma_t z_t$$

$$\sigma_t^2 = 0.024016 + 0.038190\epsilon_{t-1} + 0.066085 \, \epsilon_{t-2} + 0.887972 \ \sigma_{t-1}^2$$

6.3.1.4 Returns forecasting:

We have modeled the returns on a training interval. Consider now the prediction of the series on the horizon 420 (testing set), Based on the combined model of the mean and standard deviation. We calculate the prediction intervals at 95%. In order to compare the predictions, we also calculate the intervals prediction based on Initial modeling MA (5) ignoring heteroskedasticity. Finally, we superimpose the series produced and prediction intervals obtained by the two methods. In order to see the difference between a prediction and confidence interval, we give their definitions:

- **A prediction interval** is an interval associated with a random variable yet to be observed, with a specified probability of the random variable lying within the interval. Here, we might give a 95%

interval for the forecast of the CAC 40 test data. The test data should lie within the interval with probability 0.95.

- **A confidence interval** is an interval associated with a parameter and is a frequentist concept. The parameter is assumed to be non-random but unknown, and the confidence interval is computed from data. Because the data are random, the interval is random. A 95% confidence interval will contain the true parameter with probability 0.95. That is, with a large number of repeated samples, 95% of the intervals would contain the true parameter.

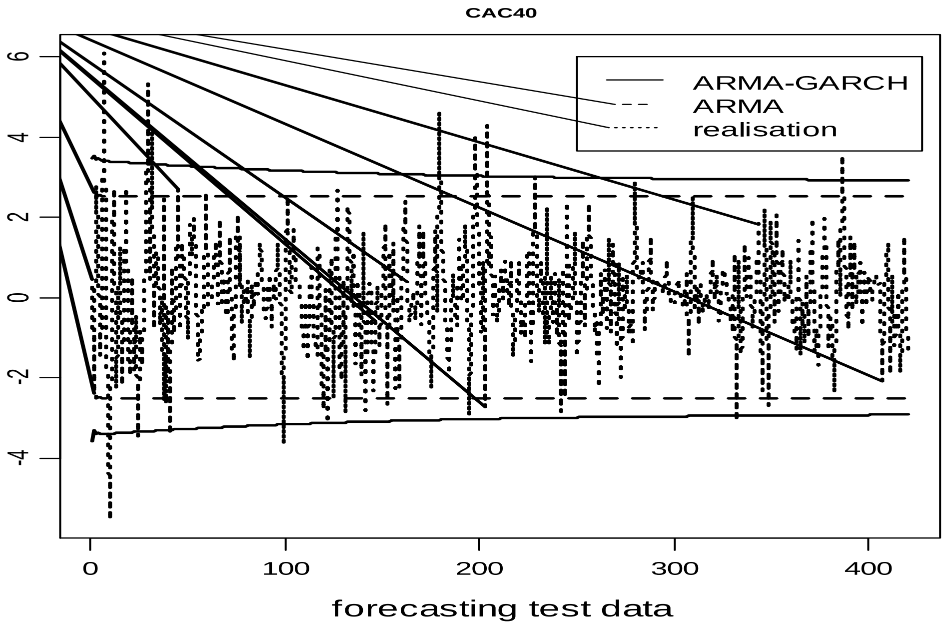


Figure 6.6- Prevision interval 95% and returns realization with ARMA and ARMA-GARCH model

We observe in **Figure 6.6** that the interval that incorporates conditional heteroskedasticity is a sensibly wider than that do not know, but the series remains largely within the two intervals. The interval limits for the ARMA model are straight. The observed proportions are **0.9500476** for ARIMA modeling and **0.9666667** for modeling ARMA GARCH; they are above 0.95.

6.3.2 Study of the S&P500 daily returns with ARMA-GARCH

Now we will model the daily returns of the S&P 500 to make its forecasting. We use as a training set "train.r.sp" composed of the returns data least the last 428 values(10% of the total number of obsevations). The observations, excluded from training, will be used for testing "test.r.sp".

6.3.2.1 Estimation

The autocorrelation functions (ACFs) and partial autocorrelations functions (PACFs) of the S&P 500 daily returns and their squares are depicted in **Figure 6.7**.

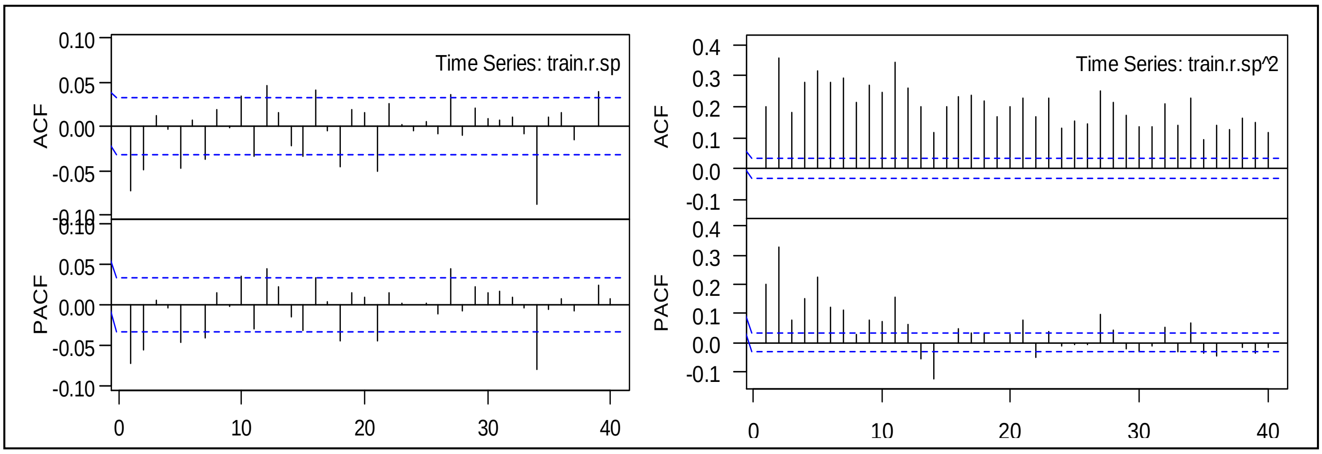


Figure 6.7 – S&P500: ACF and PACF of the daily returns and their squares

The figure above indicates that the autocorrelation coefficients of squared returns are larger and last longer (persistent) than those of the return series (non-squared).We must point out that the return series show little or no correlation, but its squares show high correlation, which indicates the ARCH or GARCH effect.

Therefore, our data set is not a white noise, so, we continue with looking for its ARMA model. The expert modeler of IBM SPSS Statistics proposes an ARMA (1, 5). After some testing around this model, we conclude that, it is the one that has best information criterion. Its application within R gives the following results:

```
…
Coefficients:
              ar1      ma1       ma2       ma3       ma4       ma5
               -
            0.8714   0.7981   -0.1172   -0.0335    0.0013   -0.0430
   s.e.     0.0489   0.0514    0.0208    0.0215    0.0209    0.0166

sigma^2 estimated as 1.744:  log likelihood=-6532.13
AIC=13078.25   AICc=13078.28   BIC=13122.04
Training set error measures:

    ME         RMSE        MAE        MAPE       MASE
0.01744049  1.320688  0.9094536  140.824   0.6723396


            Retard    p-value
 [1,]         6        0.7832
 [2,]        12        0.4768
 [3,]        18        0.3520
```

```
[4,]        24        0.1750

            ar1      ma1      ma2      ma3       ma4       ma5
             -                 -
 t.stat  17.8135  15.5273  5.6255  -1.5632   0.0626   -2.5865
  p.val   0.0000   0.0000  0.0000   0.1180   0.9501    0.0097
```

We note that the Box-Pierce test gives satisfactory results (confirmation on the right graphic of **Figure 6.8**), while MA 4 and MA 5 are not significant. We keep residuals to test the presence of Conditional heteroskedasticity:

```
ARCH LM-test; Null hypothesis: no ARCH effects

data:  rr.sp15
Chi-squared = 1067.404, df = 20, p-value < 2.2e-16
```

We reject the homoscedasticity of the residuals and try to model this conditional heteroskedasticity. Note that, the **Figure 6. 8** (left graphic) represents the shape of the distribution of residuals or the density plot, which shows the deformation compared to a normal density.

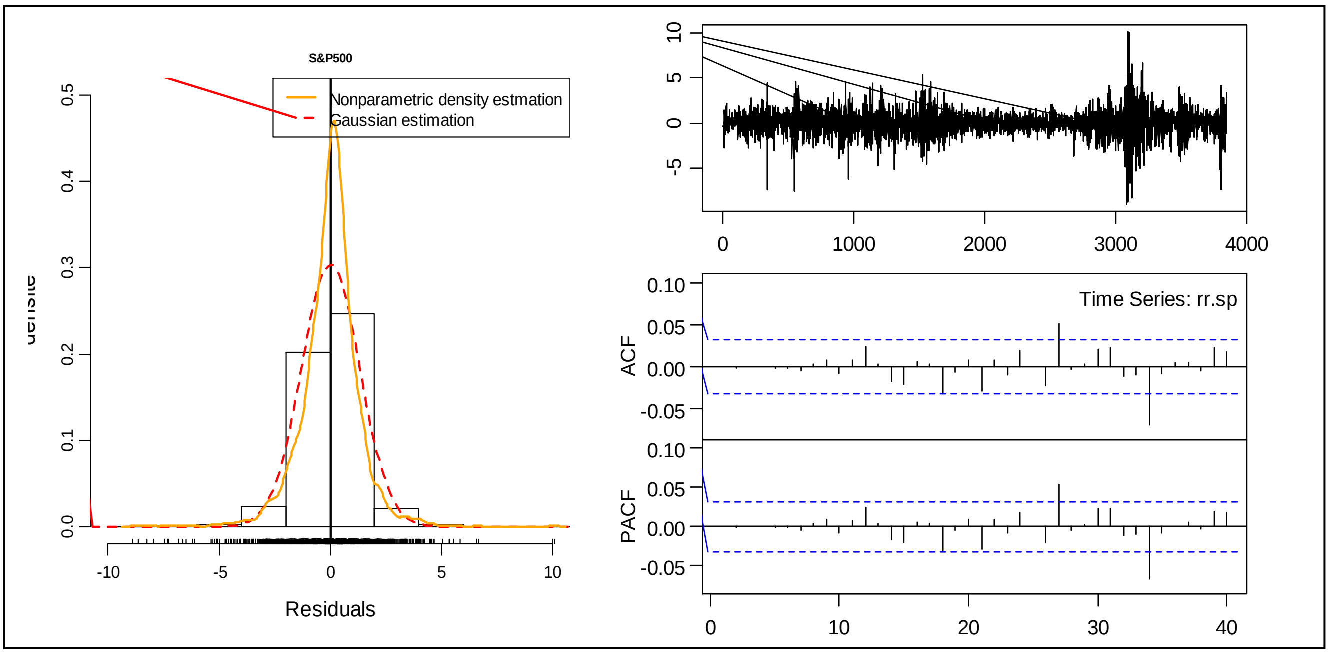


Figure 6.8–S&P500: ACF, PACF and probability density of the model's residuals

6.3.2.2 Modeling the heteroskedasticity of the returns

Now, let us model the heteroskedasticity of residuals of ARMA (1, 5) above. After trying a GARCH (1, 1) and GARCH (2, 1), we fit the second one because it gives better AIC.

```
…
Std. Errors:
 based on Hessian

Error Analysis:

              Estimate   Std. Error   t value    Pr(>|t|)
    mu        0.060039   0.014825     4.050      5.13e-05    ***
    omega     0.022675   0.004767     4.757      1.97e-06    ***
    alpha1    0.020388   0.011442     1.782      0.0748      .
    alpha2    0.091739   0.016308     5.625      1.85e-08    ***
    beta1     0.876256   0.012695     69.025     < 2e-16     ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Log Likelihood:
 -5784.718    normalized:  -1.502915

Standardised Residuals Tests:

                                    Statistic   p-Value
 Jarque-Bera Test    R      Chi^2   534.1536    0
 Shapiro-Wilk Test   R      W       0.9841532   0
 Ljung-Box Test      R      Q(10)   20.96688    0.2132571
 Ljung-Box Test      R      Q(15)   28.7956     0.1708744
 Ljung-Box Test      R      Q(20)   32.88426    0.2474054
 Ljung-Box Test      R^2    Q(10)   8.644943    0.5660982
 Ljung-Box Test      R^2    Q(15)   11.7325     0.6991535
 Ljung-Box Test      R^2    Q(20)   13.25358    0.8662406
 LM Arch Test        R      TR^2    9.238966    0.6823954

Information Criterion Statistics:
   AIC       BIC       SIC       HQIC
3.008427  3.016553  3.008424  3.011313
```

Aside from the non-normality of residuals suggested by the Jarque-Bera test and Shapiro-Wilk test, the results are quite satisfactory, with an “alpha 1” non significant. Now consider the variance of long-term:

```
Marginal variance: 1.951922
```

The estimate is positive. We can combine the two models.

6.3.2.3 Simultaneous returns modeling and their heteroskedasticity

We estimate the ARMA / GARCH model combining returns and their residual’s heteroskedasticity, we get the following results:

…

```
Std. Errors:
 based on Hessian

Error Analysis:
              Estimate   Std. Error  t value    Pr(>|t|)
ar1           0.264573    0.268232    0.986      0.3240
ma1          -0.305230    0.268090   -1.139      0.2549
ma2          -0.009098    0.021171   -0.430      0.6674
ma3          -0.013456    0.018831   -0.715      0.4749
ma4           0.004017    0.018418    0.218      0.8274
ma5          -0.041568    0.016585   -2.506      0.0122     *
omega         0.021476    0.004673    4.595     4.32e-06   ***
alpha1        0.020971    0.010947    1.916      0.0554     .
alpha2        0.088285    0.015960    5.532     3.17e-08   ***
beta1         0.879732    0.012555   70.072     < 2e-16    ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 -5780.51    normalized:  -1.501821

Standardised Residuals Tests:

                              Statistic p-Value
 Jarque-Bera
 Test            R     Chi^2   528.4201  0
 Shapiro-Wilk
 Test            R     W       0.9844986 0
 Ljung-Box Test  R     Q(10)   4.787206  0.9049319
 Ljung-Box Test  R     Q(15)   13.9534   0.5290662
 Ljung-Box Test  R     Q(20)   17.08588  0.6473902
 Ljung-Box Test  R^2   Q(10)   8.202382  0.6090759
 Ljung-Box Test  R^2   Q(15)   11.18247  0.739555
 Ljung-Box Test  R^2   Q(20)   12.82422  0.8847981
 LM Arch Test    R     TR^2    8.691639  0.7290185

Information Criterion Statistics:

    AIC        BIC         SIC         HQIC
 3.008839   3.025091   3.008825    3.014611

Marginal variance: 1.950367
```

Here to, there is non-normality of residuals. The test results of whiteness and homoscedasticity are satisfactory. Sure, alpha 1 is not significant, but the model has not meaningless without such a parameter.

The MA 5 is the only significant parameter of the ARMA (1, 5). Finally, ARMA / GARCH model used for the S&P 500 series is:

$$y_t = 0.264573\, y_{t-1} + \epsilon_t - 0.305230 \quad \epsilon_{t-1} - 0.009098\, \epsilon_{t-2} - \quad 0.013456\epsilon_{t-3} - \quad 0.004017\epsilon_{t-4} - 0.041568\, \epsilon_{t-5}$$

$$\epsilon_t = \sigma_t z_t$$

$$\sigma_t^2 = 0.021476 + 0.020971\, \epsilon_{t-1} + 0.088285\, \epsilon_{t-2} + 0.879732\, \sigma_{t-1}^2$$

6.3.2.4 Returns forecasting:

We have modeled the returns on a training interval. Consider now the prediction of the series on the horizon 420 (testing set), Based on the combined model of the mean and standard deviation. We calculate the prediction intervals at 95%. In order to compare the predictions, we also calculate the intervals prediction based on Initial modeling ARMA (1,5) ignoring heteroskedasticity. Finally, we superimpose the series produced and prediction intervals obtained by the two methods.

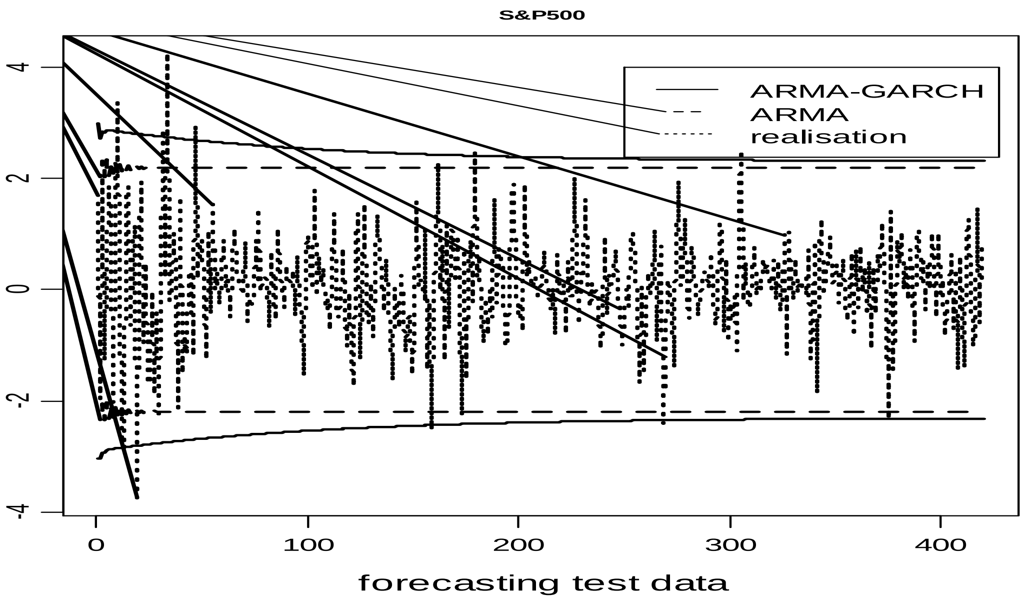


Figure 6.9- Prevision interval 95% and returns realization with ARMA and ARMA-GARCH model

We observe in **Figure 6.9** that the interval that incorporates conditional heteroskedasticity is a bit wider than that do not know, but the series remains largely within the two intervals. The interval limits for the ARMA model are straight. The observed proportions are **0.9642857** for ARIMA modeling and **0.9738095** for modeling ARMA GARCH; they are above 0.95.

6.3.3 Study of the FTSE100 returns with ARMA-GARCH

Now, we model the daily returns of the FTSE 100 to make its forecasting. We use as a training set “train.r.ftse” composed of the returns data least the last 429 values, These will be used to evaluate the predictive quality of model “test.r.ftse”.

6.3.3.1 Estimation

The autocorrelation functions (ACFs) and partial autocorrelations functions (PACFs) of the S&P 500 daily returns and their squares are depicted in **Figure 6.10**.

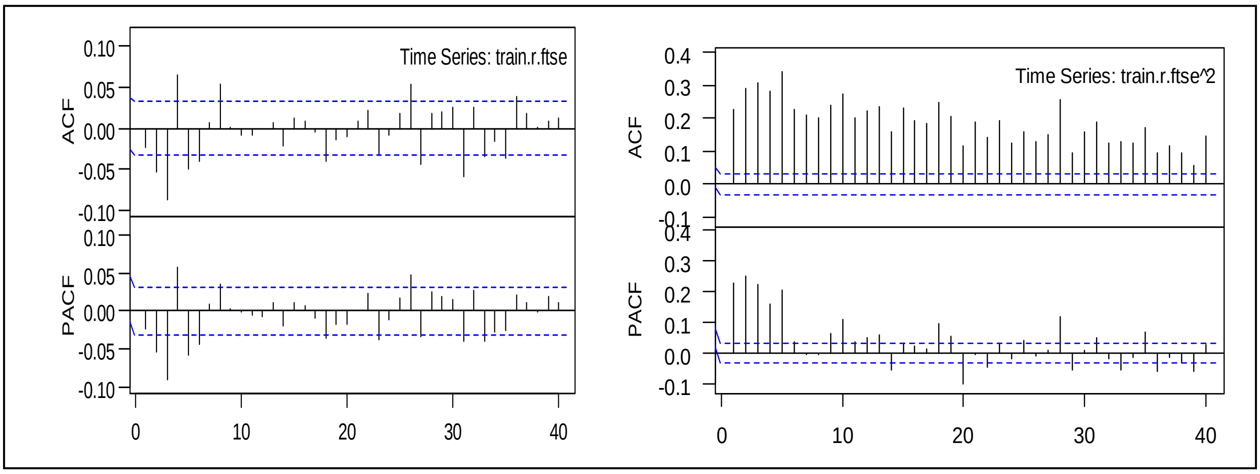


Figure 6.10 – FTSE100: ACF and PACF of the daily returns and their squares

After reading of **Figure 6.10,** we finally point out that the return series show little or no correlation, but its squares show high correlation, which indicates the ARCH or GARCH effect. Therefore, our data set is not a white noise, so, we continue with looking for its ARMA model. The expert modeler of IBM SPSS Statistics proposed an ARMA (2, 5), which we accept after comparing between the AIC of the models suggested by R. The application of the model within R gives the following results:

```
…
Coefficients:
```

	ar1	ar2	ma1	ma2	ma3	ma4	ma5
	0.1709	-0.6830	-0.1930	0.6344	-0.0965	0.0363	-0.1086
s.e.	0.0924	0.1664	0.0932	0.1655	0.0201	0.0260	0.0180

```
sigma^2 estimated as 1.59:  log likelihood=-6375.46
AIC=12766.93   AICc=12766.96   BIC=12817
```

```
Training set error measures:
     ME          RMSE          MAE          MAPE          MASE
0.01173777   1.260956   0.8934456    137.155     0.6946683
```

```
           Retard     p-value
[1,]       6          0.9937
[2,        12         0.9574
[3,]       18         0.1761
[4,]       24         0.0787
```

	ar1	ar2	ma1	ma2	ma3	ma4	ma5
t.stat	1.8493	-4.1053	-2.0704	3.8341	-4.8102	1.3937	-6.0255
p.val	0.0644	0.0000	0.0384	0.0001	0.0000	0.1634	0.0000

We note that the Box-Pierce test gives not very satisfactory results, especially for the lag 24 (confirmation on the left graphic of **Figure 6.11**), while AR 1 and MA 4 are not significant. We keep residuals to test the presence of Conditional heteroskedasticity:

```
ARCH LM-test; Null hypothesis: no ARCH effects

data:  rr.ftse
Chi-squared = 895.6952, df = 20, p-value < 2.2e-16
```

Under the null hypothesis of homoscedasticity, the statistic should be distributed approximately following a $X^2(20)$[4]. Obviously, we reject the homoscedasticity of residuals, and then let us try to model this conditional heteroskedasticity. Note that, the **Figure 6. 11** (right graphic) represents the shape of the distribution of residuals or the density plot, which shows the deformation compared to a normal density.

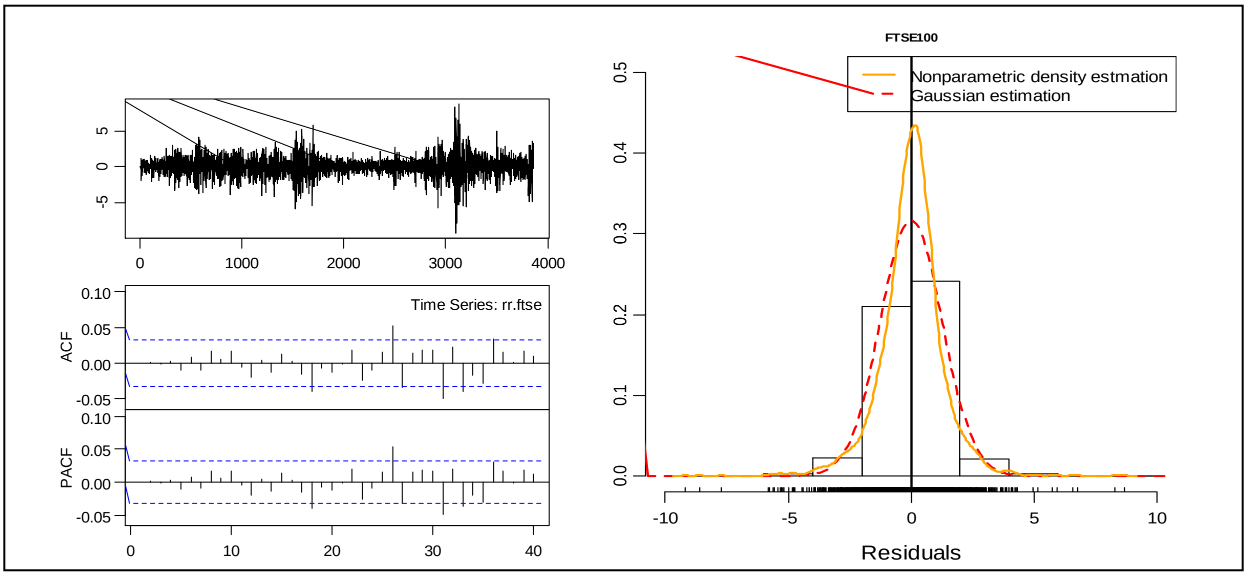


Figure 6.11 – FTSE100: ACF, PACF and probability density of the model's residuals

[4]" 20" is the number of lags that we have chosen to make regression on the squared residuals

6.3.3.2 *Modeling the heteroskedasticity of the returns*

Now, try adjusting a GARCH (1, 1) which has some flexibility without being complicated:

```
...
Error Analysis:

              Estimate   Std. Error  t  value     Pr(>|t|)
mu            0.056304   0.014254      3.950    7.81e-05       ***
omega         0.012208   0.003116      3.918    8.93e-05       ***
alpha1        0.096950   0.009525     10.178    < 2e-16        ***
beta1         0.897907   0.009620     93.337    < 2e-16        ***
---

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 -5672.229    normalized:  -1.468728

Standardised Residuals Tests:
                                          Statistic     p-Value
Jarque-Bera Test     R           Chi^2     100.5627      0
Shapiro-Wilk Test    R           W         0.993793      7.708587e-12
Ljung-Box Test       R           Q(10)     30.55204      0.000695439
Ljung-Box Test       R           Q(15)     35.68007      0.001965595
Ljung-Box Test       R           Q(20)     39.54396      0.005701768
Ljung-Box Test       R^2         Q(10)     17.34329      0.06710475
Ljung-Box Test       R^2         Q(15)     31.71007      0.007049065
Ljung-Box Test       R^2         Q(20)     34.72048      0.02164454
LM Arch Test         R           TR^2      22.19228      0.03542045

Information Criterion Statistics:

    AIC       BIC       SIC      HQIC
 2.939528  2.946011  2.939526  2.941830
```

In addition to the non normality of residuals suggested by Shapiro-Wilk test and the Jarque-Bera test, the results are rather bad, the residuals and their squares show no white noise. Despite of this consider the variance of long-term:

```
Marginal variance 2.373987
```

The estimate is positive. We can combine the two models.

6.3.3.3 *Simultaneous returns modeling and their heteroskedasticity*

We estimate the ARMA / GARCH model combining returns and their residual's heteroskedasticity, we get the following results:

…

```
Error Analysis:

            Estimate    Std. Error    t value    Pr(>|t|)
ar1         0.167811    0.009744      17.223     < 2e-16     ***
ar2         -0.979358   0.010238      -95.661    < 2e-16     ***
ma1         -0.186514   0.019485      -9.572     < 2e-16     ***
ma2         0.953102    0.020085      47.453     < 2e-16     ***
ma3         -0.052930   0.023477      -2.255     0.0242      *
ma4         -0.019310   0.017044      -1.133     0.2572
ma5         -0.039846   0.016788      -2.373     0.0176      *
omega       0.011525    0.002952      3.904      9.46e-05    ***
alpha1      0.095080    0.009103      10.445     < 2e-16     ***
beta1       0.900082    0.009118      98.713     < 2e-16     ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 -5663.747    normalized:  -1.466532

Standardised Residuals Tests:
                                      Statistic    p-Value
 Jarque-Bera Test   R     Chi^2       105.0766     0
 Shapiro-Wilk Test  R     W           0.9937256    6.344812e-12
 Ljung-Box Test     R     Q(10)       4.373887     0.9289082
 Ljung-Box Test     R     Q(15)       9.102003     0.8721239
 Ljung-Box Test     R     Q(20)       11.84668     0.9212521
 Ljung-Box Test     R^2   Q(10)       16.95504     0.07536466
 Ljung-Box Test     R^2   Q(15)       30.86053     0.009170109
 Ljung-Box Test     R^2   Q(20)       33.11803     0.03274788
 LM Arch Test       R     TR^2        22.35267     0.0337521

Information Criterion Statistics:

     AIC        BIC        SIC        HQIC
  2.938243   2.954449   2.938230   2.943998

Marginal variance:  2.381856
```

Here to, there is no normality of residuals. The test results of whiteness and homoscedasticity are half satisfactory. The residuals are white noise but not their squares, and then our residuals are not correlated but not independent. The MA 4 is the only non significant parameter of the ARMA (2, 5). Finally, ARMA - GARCH model used for the FTSE 100 series is:

$$y_t = 0.167811 y_{t-1} - 0.979358 y_{t-2} + \epsilon_t - 0.186514 \ \epsilon_{t-1} + 0.953102\, \epsilon_{t-2} - 0.052930 \epsilon_{t-3} - 0.019310 \epsilon_{t-4} - 0.039846\, \epsilon_{t-5}$$

$$\epsilon_t = \sigma_t z_t$$

$$\sigma_t^2 = 0.011525 + 0.095080\, \epsilon_{t-1} + 0.900082 \ \sigma_{t-1}^2$$

6.3.3.4 Returns forecasting:

We have modeled the returns on a training interval. Consider now the prediction of the series on the horizon 420 (testing set), Based on the combined model of the mean and standard deviation. We calculate the prediction intervals at 95%. In order to compare the predictions, we also calculate the intervals prediction based on Initial modeling ARMA (2, 5) ignoring heteroskedasticity. Finally, we superimpose the series produced and prediction intervals obtained by the two methods.

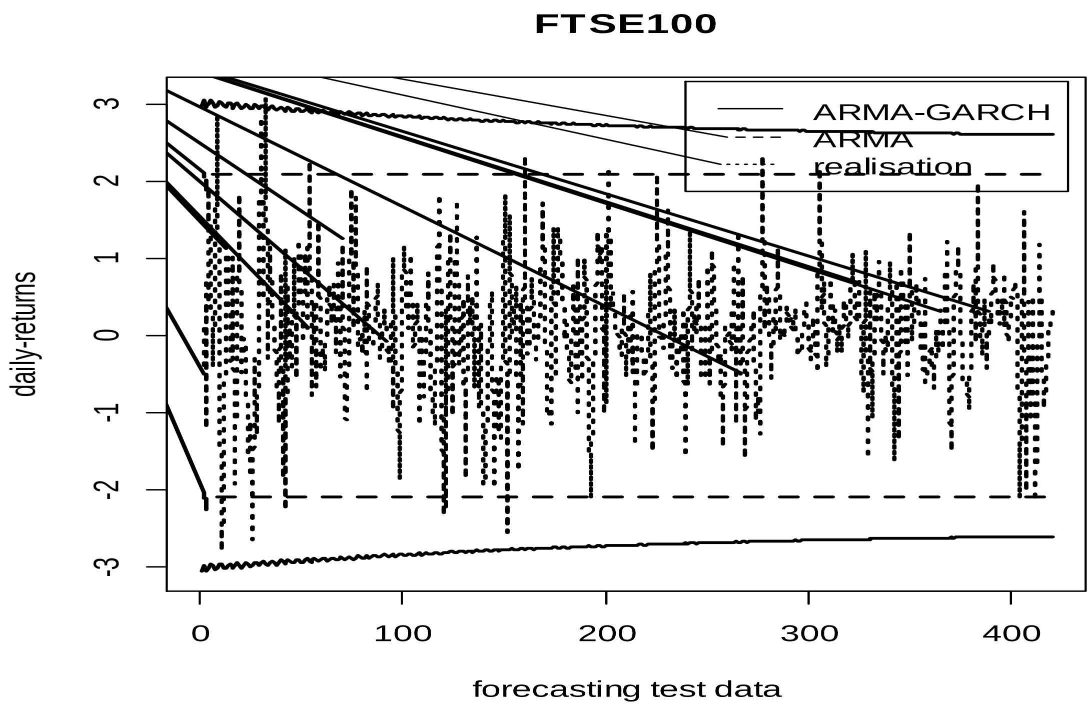


Figure 6.12- Prevision interval 95% and returns realization with ARMA and ARMA-GARCH model

We observe in **Figure 6.12** that the interval that incorporates conditional heteroskedasticity is a sensibly wider than that do not considers it, but the series remains largely within the two intervals. The interval limits for the ARMA model are straight. The observed proportions are **0.9571429** for ARIMA modeling and **0.997619** for modeling ARMA-GARCH, both are above 0.95 and ARMA-GARCH the more.

6.3.4 Study of the three daily returns with SVR

Our regression was performed using the RegSMOImproved of the WEKA Workbench. In this investigation, the Gaussian function RBF is used as the kernel function of the SVMs and that from several typical functions, as it performs well under general smoothness assumptions:

$$K(x, y) = \exp\left(-1/\delta^2 (x - y)^2\right)$$

The set of parameters by default of the (δ^2, C, ε) are (0.001, 1, 0.001). We applied our regression on Weka, with specifying 10% of the data as testing one and the rest is for training and we get the figures below[5]:

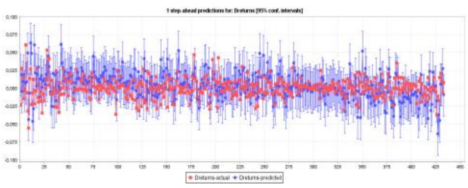


Figure 6.13- CAC40: confidence interval 95% and returns realization with SVR

[5] The numeric results are enormous; in order to have an idea of it we give the CAC 40 numeric results, the prediction value of each observation from the testing data in **ANNEXE 2**.

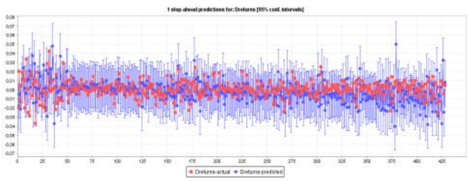


Figure 6.14- S&P 500: confidence interval 95% and returns realization with SVR

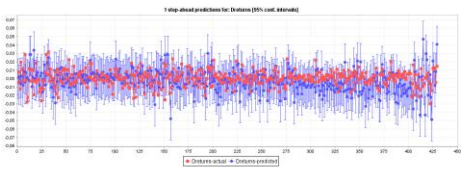


Figure 6.15- FTSE 100: confidence interval 95% and returns realization with SVR

Despite of the easy use of Weka software, unfortunately it do not gives prediction intervals, but plots the value predicted of each observation of the testing set within the confidence interval. For the three indexes we have very close predicted observations to the original ones. While the interval limits for the ARMA model are straight those of SVR are dynamic and have very close to the reality.

So we can say that SVR forecasts consistently over the whole period with time varying property for each series and SVRs are able to forecast stock returns well capturing volatility. That is, SVR holds the long memory property.

7. Results and discussion

In order to compare between the models that we have tested before, we creates **Table 7.1** which regroups the three performance criteria that we have selected. The results of ARMA are not reported in the table to make it simple and clear.

Table 7.1 - Comparison of the results of ARMA-GARCH and SVR

Series	Models	MAE	RMSE	MSE
CAC40	ARMA(0,5)-GARCH(2,1)	1.084792	1.525468	2.327053
	SVR	0.0185	0.0239	0.0006
S&P500	ARMA(1,5)-GARCH(2,1)	0.9082761	1.323738	1.752283
	SVR	0.0115	0.0153	0.0002
FTSE100	ARMA(2,5)-GARCH(1,1)	0.8932009	1.268044	1.607936
	SVR	0.0124	0.016	0.0003

In general, we note that ARMA-GARCH and SVR models outperform the ARMA model. Our experiment also shows that the SVR model outperforms both the finite mixture of ARMA-GARCH models and ARMA of in the three performance criteria, MAE, RMSE and MSE.

It is remarkable that both, of the forecasting of S&P 500 and FTSE 100 returns, which have very close results have performed comparatively better the forecasting of the CAC 40 returns under all used performance criteria. These results suggest that the S&P 500 and FTSE 100 indices are less volatile than the CAC40 index.

Conclusion

In this small project, we have studied the use of ARMA- GARCH and SVR in financial forecasting. This study has concluded that SVMs provide a promising alternative to time series forecasting.

We examined the feasibility of applying an Artificial Intelligence models, SVM/SVR, and one classical statistical model, finite mixture of ARMA-GARCH to financial returns series forecasting.

Our experiments demonstrate that SVR perform better than the ARMA and ARMA-GARCH models for the three performance criteria.

The statistical methods ARMA and ARMA-GARCH require a large number of the sample size for better forecasting, and these models drastically reduce the original sample size when the high-order model is fitted.

To improve the forecasting ability of the ARMA-GARCH model, and make the forecasting possible, we have first transformed our data into daily returns, than look for representative ARMA model for the returns, look for significant GARCH model of the residuals, and then combined the two models to have one model which considerate both returns and residuals which could be so long.

The SVR model is quite simple and has interpretability properties as opposed to the complex GARCH type. The simple AR model is estimated by maximum-likelihood estimation (which is usually affected by potential outliers), while the SVR model is estimated by robust estimation procedure. Also, the RegSMOImproved in Weka facilitates the regression, and gives us very good performance, without having to model the errors, following a multitude of steps, and applying numerous tests (white noise, heteroskedasticity…) even the parameters are the default one. In summary, our result suggests that the simple SVR model could be still used fairly successfully as a long memory model in forecasting financial returns.

Finally we suggest as an extension of this study to try to get the optimal values of parameters of an SVR, many methods exists like Greed Search, Cross Validation, Genetic Algorithms and so on …which certainly will give us better performance criteria . Also, we propose to apply more complex GARCH, or make second differentiation because, as we have seen, our return series were not white noise ones. We also can make more comparison with testing other Artificial Intelligence models like ANN for example.

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ANNEXE 1

R code for the creation of CAC40 ARMA-GARCH model:

```
>  #import the data of the CAC40 from "YAHOO finance" and make the returns
> instruc = c("^FCHI")
> cac40=priceIts(instrument=instruc,start=deb, end=fin, quote="Close")
> colnames(cac40@.Data) = c("CAC40")
> xa=returns(cac40,percentage=TRUE)
> r.cac=xa[complete.cases(xa)==TRUE,]

> #get the ACF and PACF of the CAC40

> xy.acfb(cac40,numer=FALSE)
> #make the figure that represents the daily close price, its returns and their
sqaures
> plot(cac40,xlab="Date",ylab="Close", main="CAC40")
> fa= cbind(cac40,r.cac, r.cac^2 )
> ytr= fa[complete.cases(fa) == TRUE,]
> colnames(ytr)= c("Close", "daily returns","squared returns")
> ab = dimnames(ytr)[[1]]
> abd2 = as.POSIXct(ab)
> abd2.b= format(abd2, "%d %b %Y")
> op = par(oma=rep(0,4),mgp=c(2.5,.7,0),mar=c(4.5,3,2,2),
+ cex.main=.7,cex.axis=.9,cex=.9,cex.lab=1.2)
> plot(zoo(ytr,abd2), xlab= paste('date,
du',abd2.b[1],'au',abd2.b[length(abd2.b)])
,ylab= c("Close", "returns", expression("returns"^2)),main="CAC40")

>  #create the training and testing sets:
> train.r.cac = r.cac[1:(length(r.cac)-434)]
> test.r.cac = r.cac[(length(r.cac)-433):length(r.cac)]
> #get the ACF and PACF of the returns
> xy.acfb(train.r.cac,numer=FALSE)
> #make the ARIMA model
> mod.r.cac<-arima(train.r.cac,order=c(0,0,5),include.mean=FALSE)
> summary(mod.r.cac)

> #extract the residuals of the model
> rr.cac=residuals(mod.r.cac)
> #get the Box-Pierce test of the residuals
> (Box.test.2(rr.cac,ret,type="Box-Pierce",fitdf=3,decim=4))
> #get ACF and PACF of the residuals
> xy.acfb(rr.cac,numer=FALSE)
> #get the statistic that measures the significance of coefficients
> t_stat(mod.r.cac, decim=4)
> #get the Conditional heteroskedasticity test
> ArchTest(rr.cac,lag=20)

> #make the densityplot for the returns and residuals
```

```
> density.plot=function(x,legende=FALSE,...){
+ H<-hist(x, sub = NULL, ylab = "densité", freq=FALSE, ...)
+ abline(v=0,lwd=2)
+ rug(x, ticksize = 0.01)
+ xmin=par()$usr[1];xmax=par()$usr[2]
+ tab<-seq(xmin,xmax,0.1)
+ lines(tab, dnorm(tab, mean(x),sd(x)), col="red", lty=2, lwd=2)
+ lines(density(x),lwd=2, col="orange")
+ if(legende)
+ lg0 =c("Nonparametric density estmation","Gaussian estimation")
+ legend("topright",legend=lg0,lty=c(1,2),lwd=2
+ ,
+ col=c("orange","red"),cex=0.9)
+ }
> density.plot(rr.cac, main="CAC40", xlab = "Residuals", ylim=c(0,0.5),nclass=12,
 xlim=c(-10,10), legende=TRUE )
> density.plot(r.cac, main="CAC40", xlab = "Returns", ylim=c(0,0.5),nclass=12,
 xlim=c(-10,10), legende=TRUE )
```

> #create GARCH model

```
> moda.cac=garchFit(~garch(2,1),data=rr.cac,trace=FALSE,include.mean=TRUE,
na.action=na.pass)
> summary(moda.cac)
```

> #marginal variance

```
> var.marg.est<-function(mod)
+ {param.estim = mod@fit$par
+ std.estim = mod@fit$se.coef
+ k<-which(names(param.estim)=="omega")
+ value = param.estim[k]/(1-sum(param.estim[(k+1):length(param.estim)]))
+ cat("variance marginale : ",value,"\n")
+ }
> var.marg.est(moda.cac)
variance marginale :  3.116203
```

> #create ARMA/GARCH model

```
>mod.cac=garchFit(formula=~arma(0,5)+garch(2,1),data=train.r.cac@.Data,
trace=FALSE,include.mean=FALSE)
> summary(mod.cac)
```

> #get MAE, MASE and RMSE

```
> mae <- function(error)
+ {
+ mean(abs(error))
+ }
> mae(residuals(mod.cac))
[1] 1.084792
> mse <- mean(residuals(mod.cac)^2)
> mse
[1] 2.327053
> r.rmse <- sqrt(mean(residuals(mod.cac)^2))
> r.rmse
[1] 1.525468
```

> #get the forecasting intervals for the ARMA-GARCH model

```
> npred = 420
```

```
> pred.cac=predict(mod.cac,n.ahead=npred,plot=FALSE,nx=0)
> dem.garch = qnorm(0.95)*pred.cac$standardDeviation
> binf.garch = pred.cac$meanForecast-dem.garch
> bsup.garch = pred.cac$meanForecast+dem.garch
> dans2 = (binf.garch < test.r.cac[1:npred])& ( test.r.cac[1:npred] < bsup.garch)
> pcac2 = sum(dans2)/length(dans2)
> pcac2
[1] 0.9666667
```

> #get the forecastiong intervals for ARIMA model

```
> pred.arima=predict(mod.r.cac,n.ahead=npred)
> demi = qnorm(0.95)*pred.arima$se
> binf.arima = pred.arima$pred-demi
> bsup.arima = pred.arima$pred+demi
> dans1 = (pred.arima$pred-demi < test.r.cac[1:npred])&(test.r.cac[1:npred] <
pred.arima$pred+demi)
> pcac1 = sum(dans1)/length(dans1)
> pcac1
[1] 0.9500476
```

> #plot the forecasting intervals for both models

```
> mat.p = cbind(bsup.garch,binf.garch,binf.arima,bsup.arima,test.r.cac[1:npred])
> matplot(1:npred,mat.p,type='l', col='black',
lty=c(1,1,2,2,3),lwd=2,xlab="forecasting test data", ylab="daily-
returns",main="CAC40")
> legend(250,6, leg.txt, lty=c(1,2,3))
```

ANNEXE 2

CAC 40 results on Weka:

inst#	actual	predicted	conf	error
3897	-0.0235	0.0193	-0.011:0.047	0.0428
3898	0.0279	-0.0041	-0.034:0.024	-0.032
3899	0.0154	0.0099	-0.02:0.038	-0.0055
3900	-0.0144	0.0189	-0.011:0.047	0.0334
3901	-0.0015	-0.0027	-0.033:0.025	-0.0013
3902	0.0609	0.0076	-0.022:0.035	-0.0533
3903	-0.006	0.0091	-0.021:0.037	0.015
3904	-0.0321	0.0495	0.019:0.077	0.0816
3905	-0.0553	-0.0435	-0.074:-0.016	0.0119
3906	0.0137	-0.0067	-0.037:0.021	-0.0204
3907	0.0269	0.0634	0.033:0.091	0.0365
3908	-0.0228	0.0489	0.019:0.077	0.0716
3909	-0.0064	-0.046	-0.076:-0.018	-0.0396
3910	0.0127	-0.0284	-0.059:-0.001	-0.0412
3911	-0.0219	0.0608	0.031:0.089	0.0827
3912	-0.0034	0.0216	-0.009:0.049	0.0249
3913	0.0272	0.0032	-0.027:0.031	-0.024
3914	-0.0129	0.0195	-0.011:0.047	0.0324
3915	-0.0194	0.0293	-0.001:0.057	0.0487
3916	0.0052	-0.0126	-0.043:0.015	-0.0178
3917	-0.018	0.0128	-0.017:0.041	0.0308
3918	-0.0044	0.0174	-0.013:0.045	0.0218
3919	-0.0347	0.0011	-0.029:0.029	0.0357
3920	-0.0084	0.0104	-0.02:0.038	0.0188
3921	-0.017	0.013	-0.017:0.041	0.0299
3922	-0.0001	0.0053	-0.025:0.033	0.0054
3923	0.0122	0.0198	-0.01:0.048	0.0075
3924	0.0532	0.0118	-0.018:0.04	-0.0414
3925	0.0046	0.0451	0.015:0.073	0.0405
3926	0.0414	0.0305	0:0.058	-0.0109
3927	-0.0079	-0.0047	-0.035:0.023	0.0031
3928	0.0111	-0.0074	-0.038:0.02	-0.0186
3929	0.0114	-0.025	-0.055:0.003	-0.0364
3930	-0.0068	0.0209	-0.009:0.049	0.0277
3931	-0.0011	-0.0257	-0.056:0.002	-0.0246
3932	-0.0257	0.0183	-0.012:0.046	0.044
3933	0.0245	-0.0174	-0.047:0.01	-0.0419
3934	-0.0264	0.0223	-0.008:0.05	0.0488
3935	-0.0035	-0.0046	-0.035:0.023	-0.0011
3936	-0.0339	0.0026	-0.028:0.03	0.0364
3937	0.0076	-0.0023	-0.032:0.026	-0.0098

3938	-0.0089	0.0612	0.031:0.089	0.07
3939	0.0006	0.0352	0.005:0.063	0.0345
3940	0.0269	0.0284	-0.002:0.056	0.0015
3941	-0.0082	0.0104	-0.02:0.038	0.0185
3942	0.0135	0.0235	-0.007:0.051	0.01
3943	0.0098	0.0132	-0.017:0.041	0.0034
3944	0.0003	-0.0005	-0.031:0.027	-0.0009
3945	-0.0104	0.0215	-0.009:0.049	0.0319
3946	0.0182	0.0149	-0.015:0.043	-0.0033
3947	0.0103	0.0193	-0.011:0.047	0.009
3948	0.0196	0.0285	-0.002:0.056	0.009
3949	0.0071	0.0177	-0.012:0.046	0.0106
3950	-0.0161	0.0079	-0.022:0.036	0.024
3951	-0.0154	-0.0059	-0.036:0.022	0.0095
3952	-0.0024	-0.0003	-0.03:0.028	0.0021
3953	-0.0031	0.0077	-0.022:0.035	0.0108
3954	0.0262	0.013	-0.017:0.041	-0.0132
3955	-0.0019	0.0222	-0.008:0.05	0.0241
3956	-0.0015	0.0142	-0.016:0.042	0.0157
3957	-0.0011	0.0123	-0.018:0.04	0.0134
3958	0.0089	0.0106	-0.019:0.038	0.0018
3959	0.0139	0.0286	-0.001:0.056	0.0148
3960	-0.0015	0.0241	-0.006:0.052	0.0257
3961	0.0194	0.0065	-0.024:0.034	-0.0129
3962	-0.0022	0.017	-0.013:0.045	0.0192
3963	0.0051	0.0115	-0.019:0.039	0.0064
3964	-0.0047	0.0079	-0.022:0.036	0.0126
3965	-0.0031	0.0074	-0.023:0.035	0.0104
3966	0.0152	0.009	-0.021:0.037	-0.0062
3967	-0.0133	0.0162	-0.014:0.044	0.0296
3968	-0.0161	-0.0027	-0.033:0.025	0.0135
3969	0.01	-0.0057	-0.036:0.022	-0.0157
3970	0.0207	0.0151	-0.015:0.043	-0.0056
3971	0.0027	0.045	0.015:0.073	0.0423
3972	0.0151	0.0232	-0.007:0.051	0.0081
3973	-0.0066	0.0168	-0.013:0.045	0.0234
3974	0.0018	0.0153	-0.015:0.043	0.0134
3975	-0.0005	0.012	-0.018:0.04	0.0124
3976	0.0043	0.0098	-0.02:0.038	0.0055
3977	-0.0152	0.013	-0.017:0.041	0.0282
3978	0.0034	0.0084	-0.022:0.036	0.005
3979	-0.0026	0.018	-0.012:0.046	0.0206
3980	0.0043	0.0163	-0.014:0.044	0.0119
3981	0.0009	0.0214	-0.009:0.049	0.0205
3982	0.0136	0.015	-0.015:0.043	0.0014
3983	0.0095	0.03	0:0.058	0.0205

3984	-0.0021	0.025	-0.005:0.053	0.0271
3985	-0.0052	0.0096	-0.021:0.037	0.0147
3986	0	0.0059	-0.024:0.034	0.0059
3987	0.0057	0.0103	-0.02:0.038	0.0046
3988	-0.0074	0.0172	-0.013:0.045	0.0246
3989	0.0036	0.0073	-0.023:0.035	0.0037
3990	-0.0004	0.0142	-0.016:0.042	0.0147
3991	0.0136	0.0164	-0.014:0.044	0.0028
3992	0.0004	0.0272	-0.003:0.055	0.0268
3993	-0.0039	0.0166	-0.014:0.044	0.0205
3994	-0.0365	0.0078	-0.022:0.036	0.0443
3995	0.0088	-0.0157	-0.046:0.012	-0.0245
3996	0.025	0.0233	-0.007:0.051	-0.0017
3997	0.0026	0.0244	-0.006:0.052	0.0218
3998	0.0007	0.0112	-0.019:0.039	0.0105
3999	0.0171	0.0038	-0.026:0.032	-0.0133
4000	0.004	0.0145	-0.016:0.042	0.0104
4001	0.0044	0.0369	0.007:0.065	0.0325
4002	0.0041	0.0067	-0.023:0.035	0.0027
4003	-0.0047	0.0031	-0.027:0.031	0.0078
4004	-0.0132	0.0075	-0.023:0.035	0.0207
4005	-0.001	0.0013	-0.029:0.029	0.0023
4006	-0.0157	0.0098	-0.02:0.038	0.0254
4007	0.0011	0.0024	-0.028:0.03	0.0013
4008	0.0074	0.0097	-0.02:0.037	0.0023
4009	-0.0093	0.0183	-0.012:0.046	0.0276
4010	-0.0114	0.0167	-0.013:0.044	0.0281
4011	-0.0144	0	-0.03:0.028	0.0144
4012	0.0125	0.0079	-0.022:0.036	-0.0046
4013	0.0114	0.0274	-0.003:0.055	0.016
4014	-0.0163	0.0267	-0.003:0.054	0.043
4015	-0.0278	-0.0032	-0.033:0.025	0.0245
4016	0.0019	-0.0132	-0.043:0.015	-0.0152
4017	-0.0313	0.023	-0.007:0.051	0.0542
4018	0.0062	-0.0093	-0.039:0.018	-0.0155
4019	0.0099	-0.0093	-0.039:0.019	-0.0192
4020	-0.025	0.0164	-0.014:0.044	0.0414
4021	0.0051	0.0029	-0.027:0.031	-0.0022
4022	0.0269	-0.0127	-0.043:0.015	-0.0395
4023	-0.016	0.0414	0.011:0.069	0.0574
4024	-0.0207	0.0241	-0.006:0.052	0.0448
4025	0.0046	-0.0158	-0.046:0.012	-0.0203
4026	-0.0287	0.0096	-0.02:0.037	0.0383
4027	0.0226	-0.0021	-0.032:0.026	-0.0248
4028	0.02	-0.0058	-0.036:0.022	-0.0258
4029	-0.0013	0.0074	-0.023:0.035	0.0087

4030	0.0114	0.016	-0.014:0.044	0.0047
4031	-0.0165	0.0035	-0.027:0.031	0.02
4032	0.0042	-0.0101	-0.04:0.018	-0.0143
4033	-0.0009	0.0061	-0.024:0.034	0.007
4034	-0.0192	0.0048	-0.025:0.033	0.024
4035	0.0164	-0.0112	-0.041:0.017	-0.0276
4036	-0.0282	0.0056	-0.024:0.033	0.0339
4037	-0.002	-0.0129	-0.043:0.015	-0.011
4038	0.0037	-0.0081	-0.038:0.02	-0.0118
4039	-0.0001	0.0037	-0.026:0.031	0.0038
4040	-0.0232	0.0401	0.01:0.068	0.0633
4041	-0.0061	0.0019	-0.028:0.03	0.008
4042	0.0031	0.0153	-0.015:0.043	0.0122
4043	-0.0121	0.0239	-0.006:0.052	0.036
4044	-0.0013	0.0073	-0.023:0.035	0.0087
4045	0.0063	0.0007	-0.029:0.028	-0.0057
4046	0.0186	0.0279	-0.002:0.056	0.0093
4047	-0.0266	0.0408	0.011:0.069	0.0674
4048	0.0116	-0.0024	-0.032:0.025	-0.0139
4049	0.0032	0.0189	-0.011:0.047	0.0157
4050	-0.0016	0.0116	-0.018:0.039	0.0132
4051	0.0136	0.0107	-0.019:0.038	-0.003
4052	-0.0227	0.0128	-0.017:0.041	0.0355
4053	0.0005	-0.0197	-0.05:0.008	-0.0202
4054	-0.0223	-0.0022	-0.032:0.026	0.0201
4055	0.0014	-0.015	-0.045:0.013	-0.0164
4056	0.0106	0.0208	-0.009:0.049	0.0102
4057	0.0239	0.0136	-0.016:0.041	-0.0103
4058	0.0042	0.0529	0.023:0.081	0.0488
4059	-0.0064	0.0233	-0.007:0.051	0.0296
4060	-0.0029	-0.0042	-0.034:0.024	-0.0012
4061	0.0014	0.0057	-0.024:0.033	0.0043
4062	-0.0056	0.0065	-0.024:0.034	0.0121
4063	0.0008	0.005	-0.025:0.033	0.0042
4064	0.018	0.0077	-0.022:0.035	-0.0103
4065	-0.007	0.0242	-0.006:0.052	0.0312
4066	0.0167	0.0167	-0.013:0.044	0
4067	0.0028	0.0136	-0.016:0.041	0.0109
4068	-0.0039	0.0042	-0.026:0.032	0.0081
4069	-0.0075	0.0043	-0.026:0.032	0.0119
4070	-0.0227	0.0008	-0.029:0.029	0.0235
4071	-0.003	-0.0166	-0.047:0.011	-0.0136
4072	0.0166	-0.004	-0.034:0.024	-0.0206
4073	-0.0037	0.0195	-0.011:0.047	0.0232
4074	0.0464	0.0168	-0.013:0.045	-0.0296
4075	0.0135	0.028	-0.002:0.056	0.0145

4076	0.0095	0.0099	-0.02:0.038	0.0004
4077	-0.0011	-0.013	-0.043:0.015	-0.0119
4078	-0.0118	-0.016	-0.046:0.012	-0.0041
4079	-0.0189	-0.0282	-0.058:0	-0.0092
4080	-0.0038	-0.0017	-0.032:0.026	0.0021
4081	0.0059	0.0026	-0.027:0.03	-0.0033
4082	-0.0057	0.0123	-0.018:0.04	0.018
4083	-0.007	0.0072	-0.023:0.035	0.0142
4084	0.0144	0.0018	-0.028:0.03	-0.0127
4085	-0.0003	0.0314	0.001:0.059	0.0316
4086	-0.0009	0.0333	0.003:0.061	0.0343
4087	0.0182	0.0043	-0.026:0.032	-0.0139
4088	0.0087	0.0109	-0.019:0.039	0.0023
4089	-0.0216	0.0285	-0.002:0.056	0.0501
4090	-0.0293	-0.0122	-0.042:0.016	0.0171
4091	-0.0087	-0.02	-0.05:0.008	-0.0113
4092	0.0023	0.0004	-0.03:0.028	-0.0019
4093	0.0399	0.0129	-0.017:0.041	-0.0269
4094	0.0225	0.0229	-0.007:0.051	0.0004
4095	0.0123	0.02	-0.01:0.048	0.0078
4096	-0.0088	0.0044	-0.026:0.032	0.0132
4097	0.009	-0.0033	-0.033:0.024	-0.0124
4098	-0.0272	-0.0036	-0.034:0.024	0.0236
4099	0.0429	-0.0173	-0.047:0.01	-0.0602
4100	0.0081	0.0021	-0.028:0.03	-0.006
4101	0.0151	-0.0079	-0.038:0.02	-0.023
4102	-0.0044	0.0039	-0.026:0.032	0.0083
4103	0.0054	-0.0187	-0.049:0.009	-0.0241
4104	-0.0061	-0.0003	-0.03:0.027	0.0058
4105	-0.0027	0.0122	-0.018:0.04	0.0149
4106	0.0069	-0.0132	-0.043:0.015	-0.0201
4107	-0.0003	0.0066	-0.024:0.034	0.0069
4108	0.009	0.0083	-0.022:0.036	-0.0007
4109	0.0023	0.0128	-0.017:0.041	0.0106
4110	-0.0022	0.0083	-0.022:0.036	0.0105
4111	0.0094	0.0085	-0.022:0.036	-0.0009
4112	-0.0148	0.0072	-0.023:0.035	0.0221
4113	-0.0084	-0.0092	-0.039:0.019	-0.0008
4114	0.0002	-0.0102	-0.04:0.018	-0.0104
4115	0.0086	0.0001	-0.03:0.028	-0.0085
4116	-0.0091	0.0292	-0.001:0.057	0.0382
4117	-0.0052	0.0067	-0.023:0.034	0.0118
4118	-0.0102	0.0027	-0.027:0.03	0.0129
4119	0.01	0.0062	-0.024:0.034	-0.0038
4120	0.0118	0.0242	-0.006:0.052	0.0124
4121	-0.016	0.0199	-0.01:0.048	0.0358

4122	0.002	-0.0082	-0.038:0.02	-0.0102
4123	0.0301	-0.0002	-0.03:0.028	-0.0303
4124	0.0026	0.0247	-0.005:0.052	0.0221
4125	-0.0037	0.0288	-0.001:0.057	0.0325
4126	0.0089	-0.0062	-0.036:0.022	-0.015
4127	0.0018	-0.0129	-0.043:0.015	-0.0147
4128	-0.0118	0.023	-0.007:0.051	0.0348
4129	0.0224	-0.0023	-0.032:0.025	-0.0248
4130	-0.0078	0.0116	-0.018:0.039	0.0194
4131	-0.0116	0.0054	-0.025:0.033	0.017
4132	0.0054	-0.0213	-0.051:0.007	-0.0267
4133	-0.0062	-0.0057	-0.036:0.022	0.0005
4134	0.0059	0.0185	-0.012:0.046	0.0126
4135	-0.0095	0.0051	-0.025:0.033	0.0147
4136	0.0047	-0.0067	-0.037:0.021	-0.0115
4137	-0.0286	0.0219	-0.008:0.05	0.0505
4138	0.0071	-0.0151	-0.045:0.013	-0.0222
4139	-0.0249	0.0158	-0.014:0.044	0.0406
4140	0.0236	-0.0123	-0.042:0.016	-0.0359
4141	-0.0061	0.049	0.019:0.077	0.055
4142	-0.0024	0.0155	-0.015:0.043	0.0179
4143	-0.0014	0.0155	-0.015:0.043	0.0169
4144	0.0163	-0.0091	-0.039:0.019	-0.0254
4145	-0.0147	0.046	0.016:0.074	0.0607
4146	-0.007	0.0182	-0.012:0.046	0.0252
4147	-0.005	-0.0091	-0.039:0.019	-0.0041
4148	0.0141	-0.0057	-0.036:0.022	-0.0198
4149	-0.0072	0.0363	0.006:0.064	0.0435
4150	0.0092	0.0124	-0.018:0.04	0.0033
4151	0.0233	0.0165	-0.014:0.044	-0.0069
4152	0.0076	0.0154	-0.015:0.043	0.0078
4153	0.0022	0.0243	-0.006:0.052	0.0221
4154	-0.0087	-0.0026	-0.033:0.025	0.0061
4155	-0.0061	-0.0257	-0.056:0.002	-0.0196
4156	-0.0223	-0.0092	-0.039:0.019	0.0131
4157	0.0059	-0.0208	-0.051:0.007	-0.0267
4158	-0.0044	0.0037	-0.026:0.031	0.0081
4159	0.0069	0.0038	-0.026:0.032	-0.0031
4160	-0.0077	0.0195	-0.011:0.047	0.0271
4161	0.0147	0.0025	-0.028:0.03	-0.0122
4162	-0.0088	0.0384	0.008:0.066	0.0472
4163	0.0134	0.0129	-0.017:0.041	-0.0005
4164	0.0049	0.0154	-0.015:0.043	0.0105
4165	-0.0127	0.0034	-0.027:0.031	0.016
4166	0.0087	-0.0145	-0.045:0.013	-0.0232
4167	-0.0201	-0.0076	-0.038:0.02	0.0125

4168	-0.0006	-0.0258	-0.056:0.002	-0.0253
4169	0.0047	-0.0111	-0.041:0.017	-0.0158
4170	-0.0035	-0.0025	-0.033:0.025	0.001
4171	0.0055	0.0177	-0.012:0.045	0.0122
4172	-0.009	0.001	-0.029:0.029	0.01
4173	-0.0052	-0.0007	-0.031:0.027	0.0045
4174	-0.0122	0.0022	-0.028:0.03	0.0144
4175	0.0289	-0.0124	-0.042:0.015	-0.0413
4176	0.0065	0.0354	0.005:0.063	0.0289
4177	0.0044	0.024	-0.006:0.052	0.0196
4178	0.006	0.0015	-0.029:0.029	-0.0045
4179	0.0087	-0.0098	-0.04:0.018	-0.0185
4180	-0.0079	0.0187	-0.011:0.046	0.0266
4181	0.0003	0.0012	-0.029:0.029	0.0009
4182	0.0037	-0.0109	-0.041:0.017	-0.0147
4183	0.0152	-0.0021	-0.032:0.026	-0.0173
4184	-0.0033	0.0175	-0.013:0.045	0.0208
4185	0.0026	0.0033	-0.027:0.031	0.0007
4186	0.0039	-0.0021	-0.032:0.026	-0.006
4187	0.0028	0.0008	-0.029:0.029	-0.002
4188	0.0031	0.006	-0.024:0.034	0.0029
4189	0.0011	0.0007	-0.029:0.028	-0.0004
4190	0.0018	-0.0035	-0.034:0.024	-0.0053
4191	0.0094	0.0026	-0.027:0.03	-0.0067
4192	0.0001	0.0062	-0.024:0.034	0.006
4193	-0.001	0.002	-0.028:0.03	0.003
4194	0	-0.0046	-0.035:0.023	-0.0047
4195	-0.0014	-0.005	-0.035:0.023	-0.0035
4196	0.0029	0	-0.03:0.028	-0.0029
4197	0.0044	0.0005	-0.03:0.028	-0.0039
4198	0.0006	0.0038	-0.026:0.032	0.0032
4199	-0.0015	0.0046	-0.025:0.032	0.0061
4200	-0.0024	-0.0016	-0.032:0.026	0.0008
4201	0.0059	-0.0027	-0.033:0.025	-0.0086
4202	-0.0148	0.0057	-0.024:0.034	0.0205
4203	0.0057	-0.013	-0.043:0.015	-0.0188
4204	0.0252	-0.0005	-0.031:0.027	-0.0257
4205	-0.0034	0.0149	-0.015:0.043	0.0183
4206	0.0024	0.0117	-0.018:0.039	0.0093
4207	-0.0068	-0.0104	-0.04:0.017	-0.0035
4208	0.0003	-0.022	-0.052:0.006	-0.0223
4209	0.0031	0.0058	-0.024:0.034	0.0027
4210	-0.0039	-0.0001	-0.03:0.028	0.0038
4211	0.0008	-0.0089	-0.039:0.019	-0.0097
4212	0.0006	-0.0022	-0.032:0.026	-0.0028
4213	-0.0029	0.0035	-0.027:0.031	0.0065

4214	0.003	0.0027	-0.027:0.03	-0.0003
4215	0.0096	-0.0004	-0.03:0.027	-0.0099
4216	-0.0007	0.0114	-0.019:0.039	0.0121
4217	0.0057	0.006	-0.024:0.034	0.0003
4218	-0.0059	0	-0.03:0.028	0.0059
4219	-0.004	-0.0084	-0.038:0.019	-0.0044
4220	0.007	-0.0087	-0.039:0.019	-0.0156
4221	0.0069	-0.0033	-0.033:0.025	-0.0102
4222	0.0007	0.0097	-0.02:0.037	0.0089
4223	0.0013	0.0019	-0.028:0.03	0.0006
4224	-0.0054	-0.0044	-0.035:0.023	0.0009
4225	-0.0088	-0.0041	-0.034:0.024	0.0047
4226	0.0109	-0.015	-0.045:0.013	-0.0259
4227	-0.0306	0.0003	-0.03:0.028	0.0308
4228	0.0095	-0.0326	-0.063:-0.005	-0.042
4229	-0.0141	-0.0088	-0.039:0.019	0.0053
4230	-0.0116	-0.0139	-0.044:0.014	-0.0023
4231	0.0134	-0.0078	-0.038:0.02	-0.0212
4232	0.0003	-0.0202	-0.05:0.008	-0.0205
4233	0.0098	0.0344	0.004:0.062	0.0245
4234	0.0032	0.0052	-0.025:0.033	0.002
4235	-0.0079	-0.0058	-0.036:0.022	0.002
4236	-0.0025	0.0033	-0.027:0.031	0.0058
4237	0.0018	-0.0167	-0.047:0.011	-0.0186
4238	0.0186	-0.0073	-0.037:0.02	-0.0259
4239	-0.007	0.0146	-0.015:0.042	0.0216
4240	-0.0232	-0.0041	-0.034:0.024	0.0191
4241	0.0222	-0.0407	-0.071:-0.013	-0.0629
4242	0.0041	-0.0042	-0.034:0.024	-0.0083
4243	-0.0271	0.0224	-0.008:0.05	0.0494
4244	0.019	-0.041	-0.071:-0.013	-0.0601
4245	0.0085	-0.0084	-0.038:0.019	-0.0169
4246	-0.0062	0.0521	0.022:0.08	0.0583
4247	0.0027	-0.0053	-0.035:0.022	-0.008
4248	0.0207	-0.0083	-0.038:0.02	-0.0289
4249	-0.0036	0.0277	-0.002:0.055	0.0312
4250	0.0053	0.0163	-0.014:0.044	0.011
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4260	-0.0144	0.0146	-0.015:0.042	0.029
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4264	-0.0099	0.0266	-0.003:0.054	0.0365
4265	0.0053	-0.0069	-0.037:0.021	-0.0123
4266	0.0196	0.0084	-0.022:0.036	-0.0113
4267	-0.0133	0.0104	-0.02:0.038	0.0237
4268	-0.0077	-0.0051	-0.035:0.023	0.0026
4269	-0.017	-0.0319	-0.062:-0.004	-0.0149
4270	0.0009	-0.0342	-0.064:-0.006	-0.0351
4271	0.0011	0.0026	-0.028:0.03	0.0015
4272	0.0197	-0.0053	-0.035:0.022	-0.025
4273	0.0085	0.0272	-0.003:0.055	0.0187
4274	-0.0124	0.0157	-0.014:0.043	0.0281
4275	-0.0051	-0.0167	-0.047:0.011	-0.0116
4276	-0.0067	-0.0265	-0.057:0.001	-0.0198
4277	-0.0238	-0.0193	-0.049:0.009	0.0045
4278	0	-0.0391	-0.069:-0.011	-0.0391
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4287	0.0005	-0.0062	-0.036:0.022	-0.0067
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4293	0.0064	-0.0113	-0.041:0.016	-0.0177
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4299	0.0054	-0.0087	-0.039:0.019	-0.0141
4300	0.0033	-0.0054	-0.036:0.022	-0.0087
4301	0.0037	-0.0052	-0.035:0.023	-0.0089
4302	-0.0209	-0.0116	-0.042:0.016	0.0094
4303	-0.0026	-0.0388	-0.069:-0.011	-0.0362
4304	0.0097	-0.0287	-0.059:-0.001	-0.0383
4305	0.0138	-0.0064	-0.036:0.021	-0.0202

4306	-0.0191	0.0192	-0.011:0.047	0.0383
4307	0.0056	-0.0193	-0.049:0.008	-0.0249
4308	-0.012	-0.0093	-0.039:0.018	0.0027
4309	-0.0071	-0.0083	-0.038:0.019	-0.0012
4310	0.0013	-0.0238	-0.054:0.004	-0.0251
4311	-0.0189	-0.0201	-0.05:0.008	-0.0012
4312	-0.01	-0.019	-0.049:0.009	-0.009
4313	0.0152	-0.0421	-0.072:-0.014	-0.0573
4314	-0.0021	0.0058	-0.024:0.034	0.008
4315	-0.014	0.0235	-0.007:0.051	0.0375
4316	-0.0044	-0.0311	-0.061:-0.003	-0.0266
4317	0.0011	-0.0223	-0.052:0.005	-0.0235
4318	0.0019	0.0224	-0.008:0.05	0.0205
4319	0.0153	-0.0028	-0.033:0.025	-0.018
4320	-0.0008	0.0033	-0.027:0.031	0.0042
4321	-0.0055	0.0029	-0.027:0.031	0.0084
4322	-0.0373	-0.0172	-0.047:0.011	0.02
4323	-0.0111	-0.0636	-0.094:-0.036	-0.0525
4324	-0.0172	-0.0374	-0.067:-0.01	-0.0201
4325	0.015	-0.0451	-0.075:-0.017	-0.0601
4326	0.0207	0.0058	-0.024:0.034	-0.0149
4327	0.0097	0.0062	-0.024:0.034	-0.0034
4328	-0.0062	0.0276	-0.003:0.055	0.0338
4329	0.0076	-0.0088	-0.039:0.019	-0.0164

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