Indian Institute of Technology Patna
Date: 28^{th} September 2024
Midsem CS5101/CS514/MC5101: Algorithm Design
Duration: 2 hours Full Marks = 50

Instructions:

- **Sections I and II should be answered in the main copy provided (not in the question paper).**
- **There is no negative marking**
- **No doubts will be entertained.**

Section-I Full Marks: 25

1. What is the main idea behind the backtracking approach? [1 mark]
   (a) Explore all possible solutions and keep the first one.
   (b) Divide the problem into smaller subproblems and solve each recursively.
   (c) Build solutions incrementally and abandon partial solutions that don't satisfy the constraints.
   (d) Use dynamic programming to find the optimal solution.

2. Topological sorting is only possible for which type of graph? [1 mark]
   (a) Undirected graph
   (b) Directed Acyclic Graph (DAG)
   (c) Directed Cyclic Graph
   (d) Weighted Graph

3. A simple acyclic path between source and sink which pass through only positive weighted edges is called? [1 mark]
   (a) augmenting path
   (b) critical path
   (c) residual path
   (d) maximum path

4. The data structure used to implement recursive function calls ___ [1 mark]
   (a) Array
   (b) Linked List
   (c) Queue
   (d) Stack

5. Which of the following is true about Breadth First Search (BFS) in an unweighted graph? [1 mark]
   (a) BFS always finds the longest path from the source to a node
   (b) BFS can get stuck in a loop without visiting all nodes
   (c) BFS always finds the shortest path from the source to a node
   (d) BFS does not visit every vertex in a connected graph

6. Which of the following is NOT a characteristic of the backtracking algorithm? [1 mark]
   (a) Recursive approach
   (b) Breadth-first exploration
   (c) Depth-first exploration
   (d) Trial and error

7. The recurrence relation capturing the time required for move in Tower of Hanoi problem with n discs and 3 tower is. [1 mark]
   (a) $T(n) = 2T(n-2) + 2$
   (b) $T(n) = 2T(n-1) + n$
   (c) $T(n) = 2T(n/2) + 1$
   (d) $T(n) = 2T(n-1) + 1$

8. In Big-M/Two-phase method, artificial variables are introduced so that [1 mark]
   (a) they can serve as initial basic variables
   (b) <= constraints can be converted to equality constraints
   (c) slack variables remain always positive
   (d) excess variables remain always positive

9. Which of the following statements regarding basic feasible solutions (bfs) of simplex method is incorrect [1 mark]
   (a) always associated with some corner point corresponding to polygon covering the feasible region
   (b) We can get optimal solution of an LP by enumerating all bfs's.
   (c) for all bfs, all basic variables are always >= 0.
   (d) Optimal solution to LP might not be associated with any bfs.

10. In DFS, what is the time complexity for traversing a graph with V vertices and E edges? [1 mark]
    (a) $O(V^2)$
    (b) O(ElogV)
    (c) O(E+V)
    (d) $O(E^2)$

11. In a directed graph, a Strongly Connected Component (SCC) is: [1 mark]
    (a) A subgraph where every vertex has the same in-degree and out-degree
    (b) A subgraph where every vertex is reachable from every other vertex within the subgraph
    (c) A maximal subgraph where every vertex is reachable from every other vertex within the subgraph
    (d) A subgraph where no cycles exist

12. In a max-flow network, assume L and R represent partitions associated with min-cut (see Fig. 1) where e is an edge from L to R and e' is an edge from R to L. Capacity of e and e' are Cap_e and $Cap_{e'}$ respectively. When max-flow is achieved, then flow in e and e' are $Flow_e$ and $Flow_{e'}$ respectively. [2 marks]

    (a) $Flow_e$=Cap_e and $Flow_{e'}$=$Cap_{e'}$

    (b) $Flow_e$=0 and $Flow_{e'}$=$Cap_{e'}$

    (c) $Flow_e$=0 and $Flow_{e'}$=0

    (d) $Flow_e$=Cap_e and $Flow_{e'}$=0

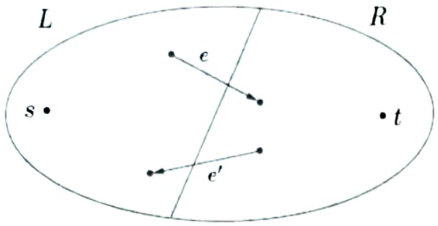

Figure 1: Min-Cut

13. As a corporation mayor, you want to ensure smooth traffic, and hence, you want to employ traffic police at least at one end of every road segment. However, you want to employ a minimum number of traffic police. You meet 4 engineers. They all said this can be viewed as a graph problem where junctions are nodes and road segments joining two junctions are edges. However, they differ in solution. Following are the solutions from 4 engineers. Which one is correct? [2 marks]

    (a) Number of police required is the minimum vertex cover of this graph.

    (b) The number of police required is the size of the maximum independent set of this graph.

    (c) The number of police required is the size of the max-clique of this graph.

    (d) We should check for a Hamiltonian path, and in every alternate junction of that path, we should employ police.

14. Assume that you are given an efficient algorithm to find out if a graph G has any independent set of size k. You are asked to find out an efficient algorithm for vertex cover of size k. You come up with a polynomial time reduction algorithm that will convert the vertex cover problem instance to an independent set problem instance, and for every yes input of the vertex cover problem, the independent set should output Yes, and for every no input of the vertex cover problem, the independent set should output No. Which of the following statements is correct? [2 marks]

    (a) Make a complement of graph G, say G'. If G' has an independent set of size k, then G has a vertex cover of size k.

    (b) Make a complement of graph G, say G'. If G' has an independent set of size n (total number of vertex) - k, then G has a vertex cover of size k.

    (c) If G has an independent set of size n - k, th a vertex cover of size k.

    (d) If G has an independent set of size k, then G vertex cover of size k.

15. If f is a flow in graph G and G_f is an induced graph by flo f on G and f' is a flow in G_f, then which of the following statements is incorrect? [2 marks]

    (a) There will be a flow in G where the amount of flow will be f+f'

    (b) Max flow in G is greater than or equal to f+f'

    (c) Max flow in G is at most f+f'

    (d) Max flow in G is always greater than f

16. Suppose a polynomial time algorithm is discovered that correctly computes the largest clique in a given graph. In this scenario, which one of in Figure 2 represents the correct Venn diagram of the complexity classes P, NP, and NP-complete (NPC)? [2 marks]

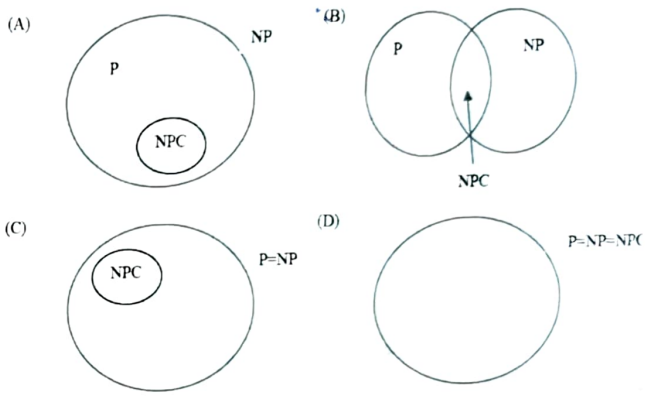

Figure 2:

    (a) A

    (b) B

    (c) C

    (d) D

17. Consider two decision problems, Q1 and Q2, such that Q1 reduces in polynomial time to 3-SAT and 3-SAT reduces in polynomial time to Q2. Yes input of Q1 can be verified in polynomial time. Then which one of the following is consistent with the above statement? [2 marks]

    (a) Q1 is in NP, Q2 is NP-hard

    (b) Q2 is in NP, Q1 is NP-hard

    (c) Both Q1 and Q2 are in NP

    (d) Both Q1 and Q2 are in NP-hard

18. Consider the following function,

```
int unknown(int n) {
    int i, j, k = 0;
    for (i  = n/2; i <= n; i++)
        for (j = 2; j <= n; j = j * 2)
            k = k + n/2;
    return k;
}
```

What is the **time complexity** of the function? [2 marks]

(a) n^2

(b) n(log n)

(c) n^3

(d) n^3(log n)

Section-II

Full Marks:25

1. Find the Maximum flow using Push-Relabel and mention the height and overflow for each node at each step. No need for any explanation. [10 marks]

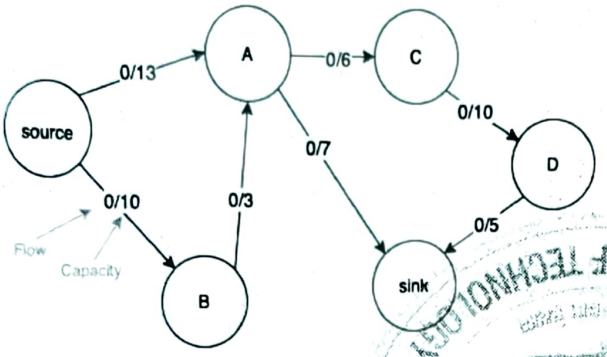

Figure 3:

One such table is given below. Follow the same format for each step.

Node	Height	Overflow
source	6	-
A	0	-
B	0	-
C	0	-
D	0	-
sink	0	-

Table 1:

2. Three products – A, B, and C - are produced in three machine centers X, Y, and Z. All three products require a part of their manufacturing operation at each of the machine centers. The time required for each operation on various products is indicated in Table 2. Only 100, 77, and 80 hours are available at machine centers X, Y, and Z, respectively. The profit per unit from A, B, and C is Rs.12, Rs.3, and Re.1, respectively.

   (a) Formulate the problem. [1 mark)

   (b) Convert it to standard form, identify all the types of variables used, and define them. [2 marks]

   (c) Determine a suitable product mix so as to maximize the profit using Simplex. *(Note: There is part marking for each step.)* [7 marks]

Product	Machine Centres			Price per unit
	X	Y	Z	
A	10	7	2	12
B	2	3	4	3
C	1	2	1	1
Available hours	100	77	80	

Table 2: Caption

3. (a) For the graph given in Figure 4, perform DFS, and whenever there's a choice of vertices, pick the one that is alphabetically first. Classify each edge as a tree edge, forward edge, back edge, or cross edge. [3 marks]

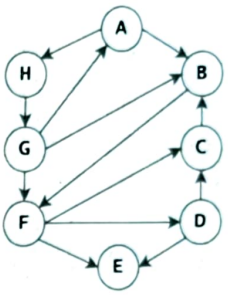

Figure 4:

(b) Explain the BFS algorithm with its pseudo code. Also, justify the time complexity of the algorithm. (7 marks)