Constitutive Models and Materials

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Questions

- New assignment on simulation released
- Anything from last lecture ?
- Reminder: Start thinking about projects

Today

- Material Models

First Piola-Kirchhoff Stress

- Simple formula to convert from P to σ

$$\mathbf{P} = J\sigma\mathbf{F}^{-T}$$

- Using hyperelastic models in FEM
  - Compute P
  - Convert to σ
  - Proceed as normal

Simple Hyperelastic Models

St. Venant-Kirchhoff

$$\mathbf{P} = \mathbf{F}\left[2\boxed{\mu}\mathbf{E} + \boxed{\lambda}\mathrm{tr}(\mathbf{E})\mathbf{I}\right]$$

Neo Hookean

$$\mathbf{P} = \boxed{\mu}(\mathbf{F} - \mathbf{F}^{-T}) + \boxed{\lambda}\log(J)\mathbf{F}^{-T}$$

Each model as 2 parameters:

Simple Hyperelastic Models

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Each model as 2 parameters:

μ are λ Lame parameters

Simple Hyperelastic Models

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Neo Hookean

$$\mathbf{P} = \boxed{\mu}(\mathbf{F} - \mathbf{F}^{-\mathsf{T}}) + \boxed{\lambda}\log(J)\mathbf{F}^{-\mathsf{T}}$$

Each model as 2 parameters:
μ are λ Lame parameters

They are related to the fundamental physical parameters:

The Poisson's Ratio
The Young's Modulus (Stiffness)

Online conversion tool: http://www.efunda.com/formulae/solid_mechanics/mat_mechanics/calc_elastic_constants.cfm

Types of Materials

- There are many types of materials
  - Elastic ← Done
  - Plastic ← Now
  - Composites
  - Cellular Materials
  - Lattice Structures
- Each one has different mechanical properties
- When we fabricate things we exploit these properties to achieve optimal results

Plastic Materials

- Defining Properties:
  - Object reference shape changes
  - Object does not always return to its original shape

Example: Crushing a Coke Can

Old Reference State

New Reference State

Example: Crushing a Van

A Simple Model For Plasticity

- Recall our model for strain: $\frac{1}{2}\left(\mathbf{F}^T\mathbf{F} - \mathbf{I}\right)$
- Let's consider how to encode a change of reference shape into this metric
- We want to exchange $\mathbf{F}$ with ${}^{w}_{p}\mathbf{F}$ a deformation gradient that takes into account the new shape of our object

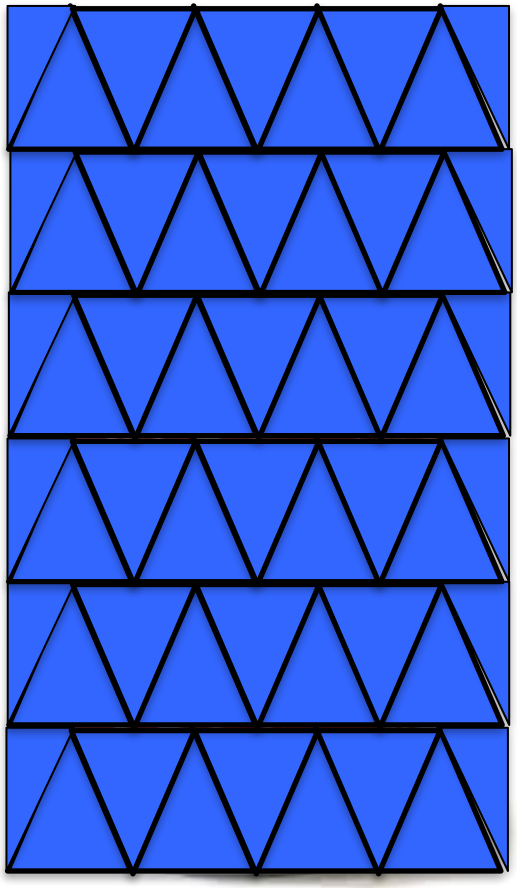
New Reference State

Example: Crushing a Coke Can

Old Reference State

New Reference State

Example: Crushing a Coke Can

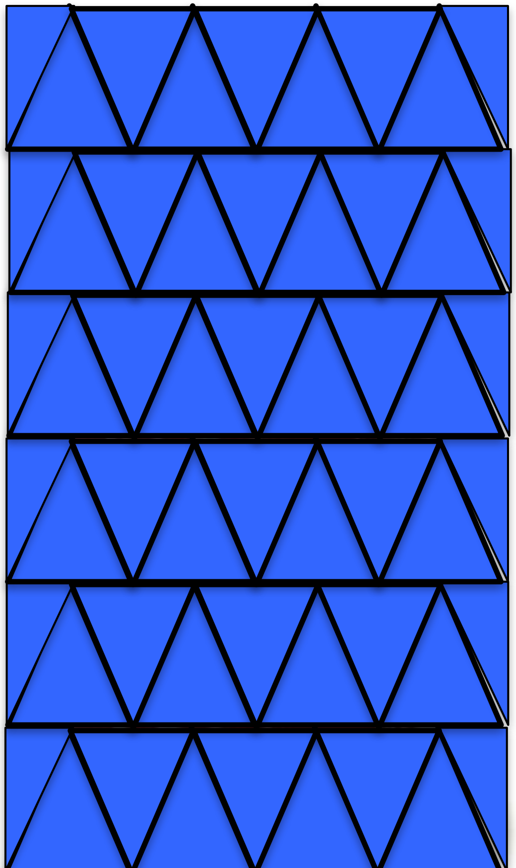


Mesh Lives Here!!!!

Old Reference State

New Reference State

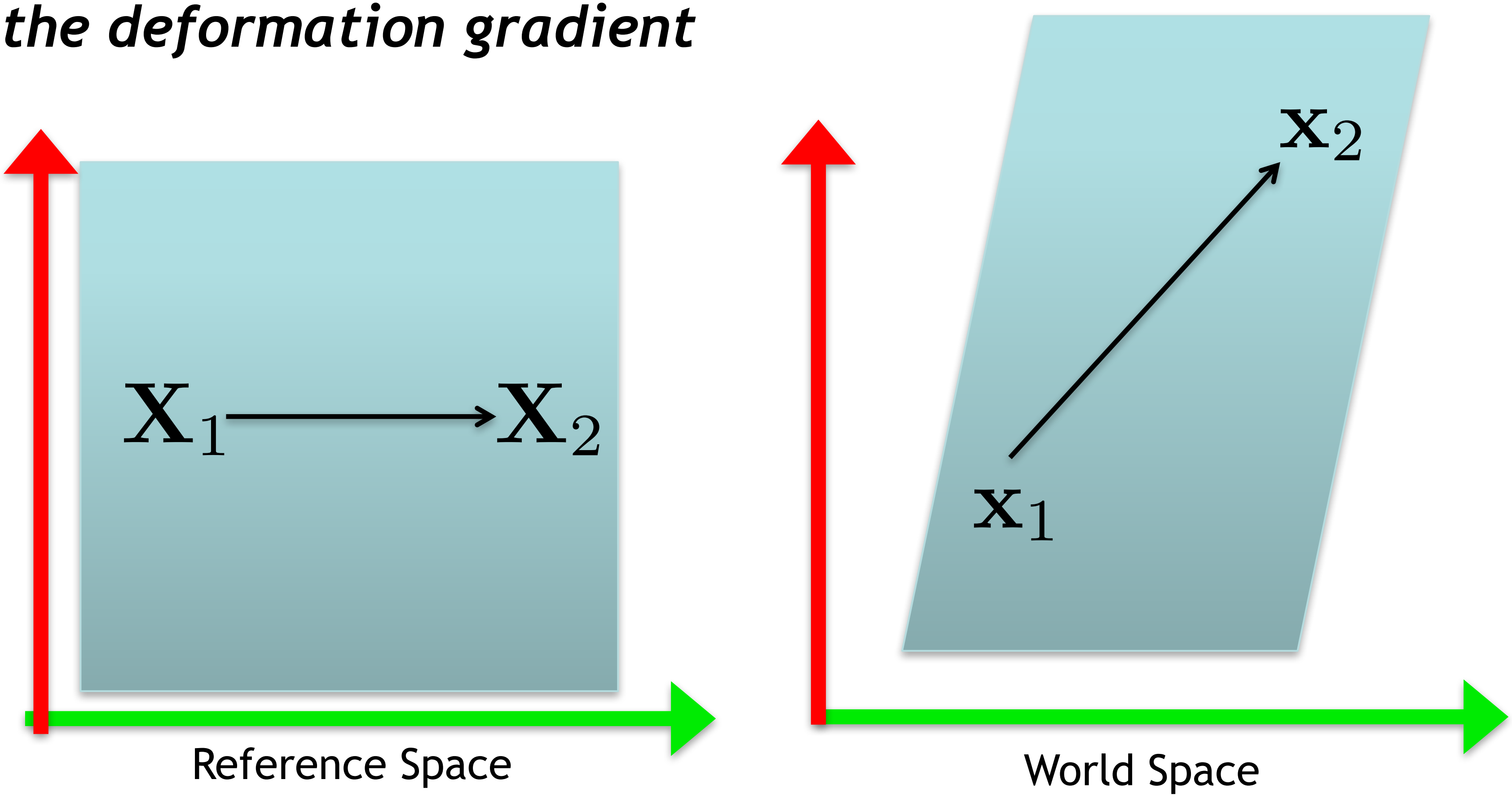
Example: Crushing a Coke Can



How can we encode shape change without changing the mesh?

Continuum Mechanics: Deformation

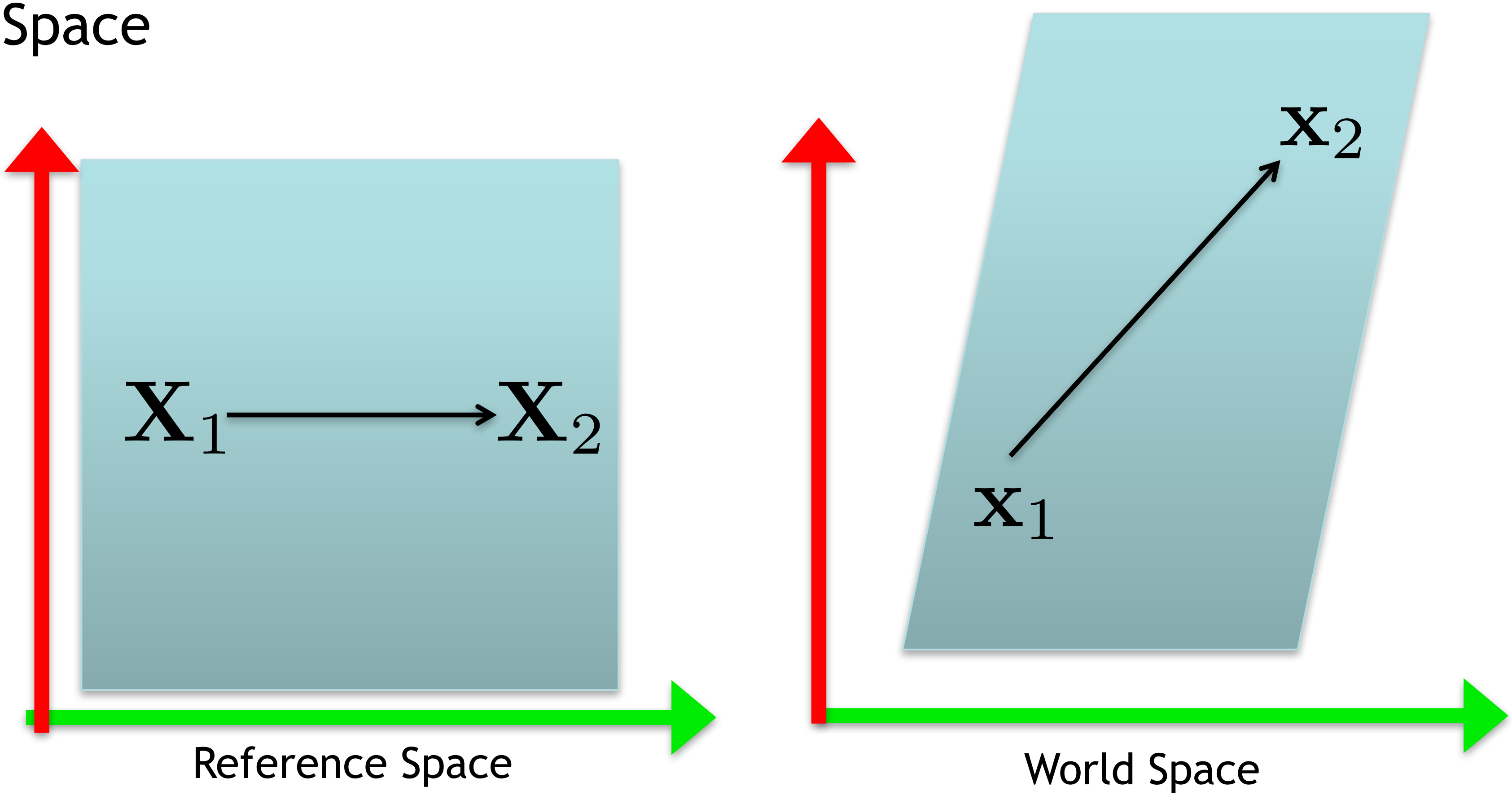
- $\mathbf{F}$ is our deformation measure called ***the deformation gradient***



$$\mathbf{dx} \approx \mathbf{FdX}$$

Continuum Mechanics: Deformation

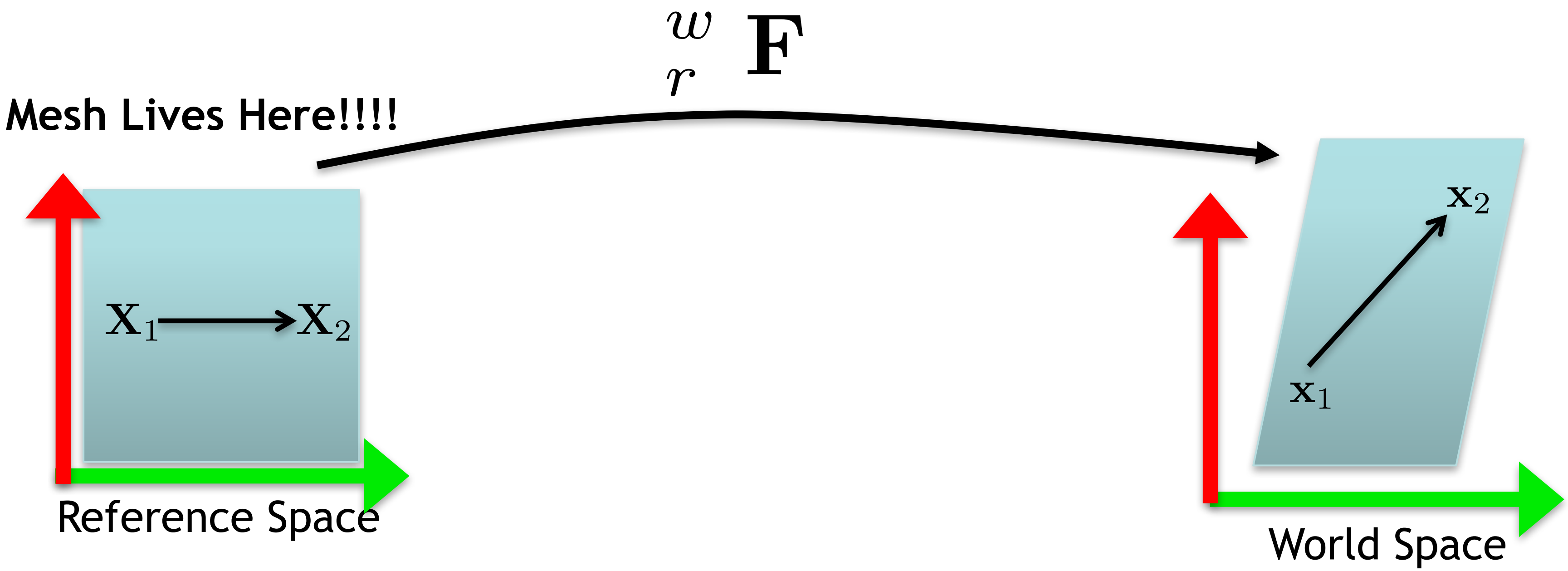
- $\mathbf{F}$ transforms a vector from Reference Space to World Space



$$\mathbf{dx} \approx_r^w \mathbf{F}\mathbf{dX}$$

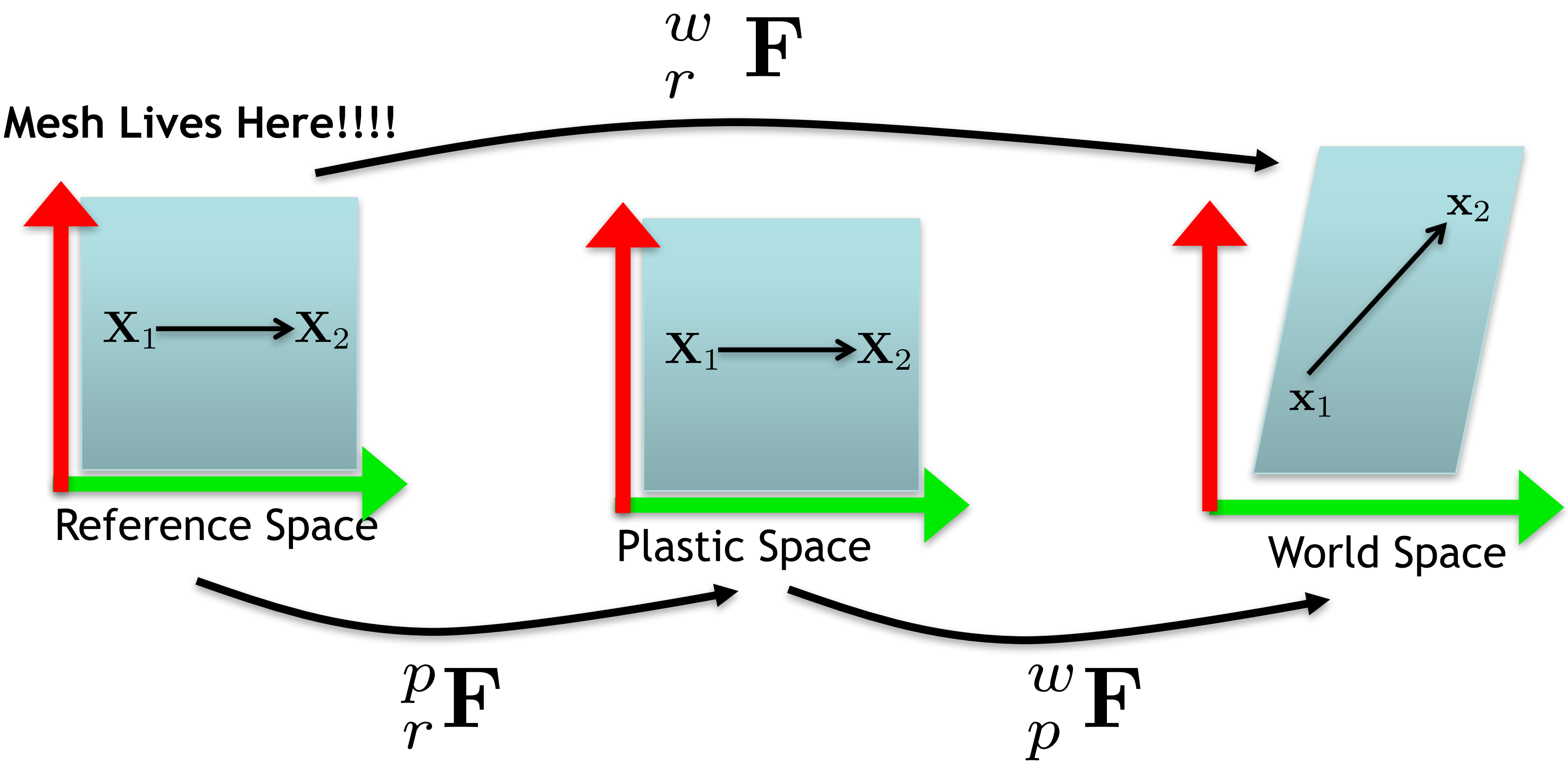
Continuum Mechanics: Deformation

- $\mathbf{F}$ transforms a vector from Reference space to World Space



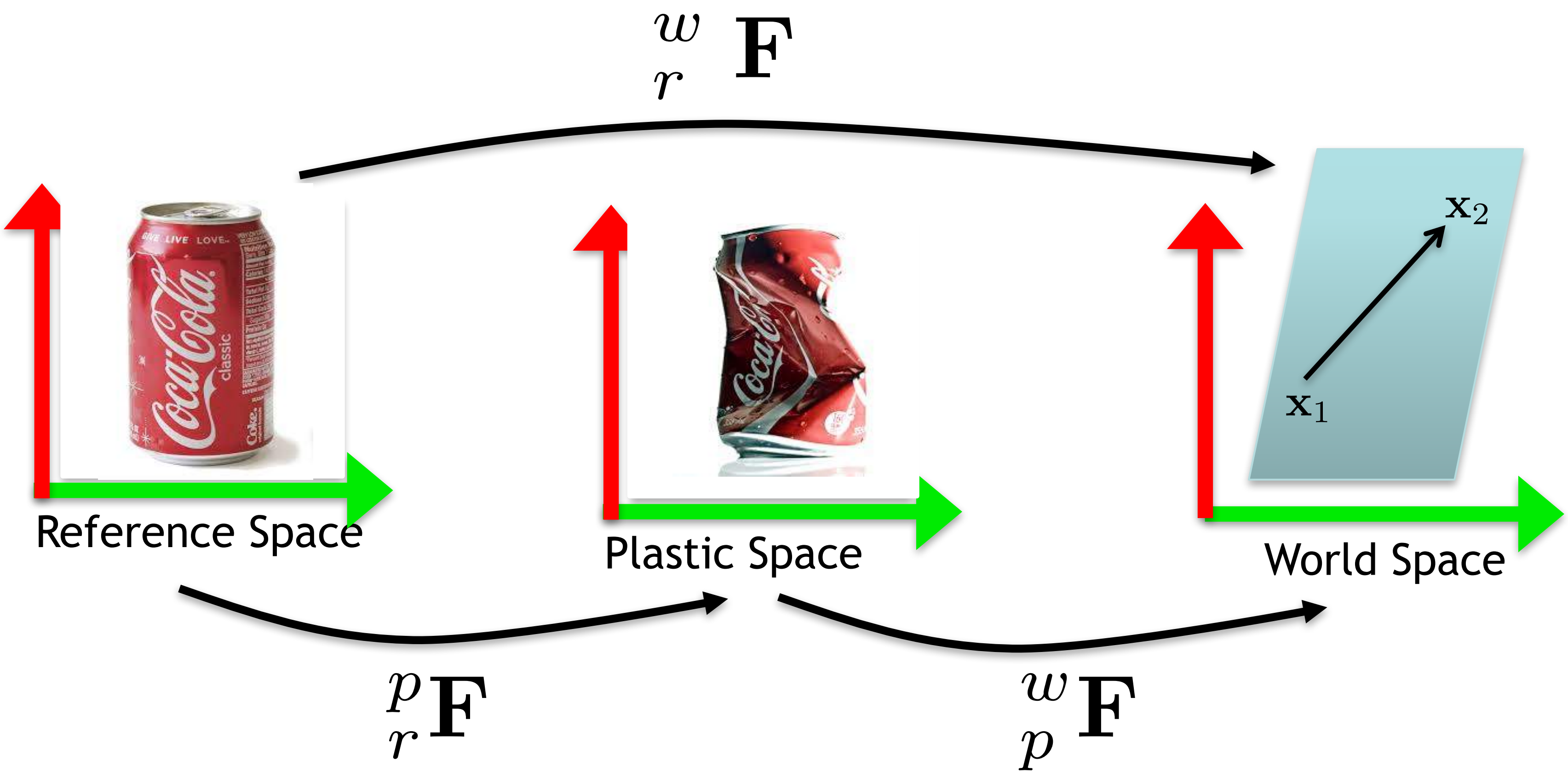
Continuum Mechanics: Deformation

- Introduce a new space



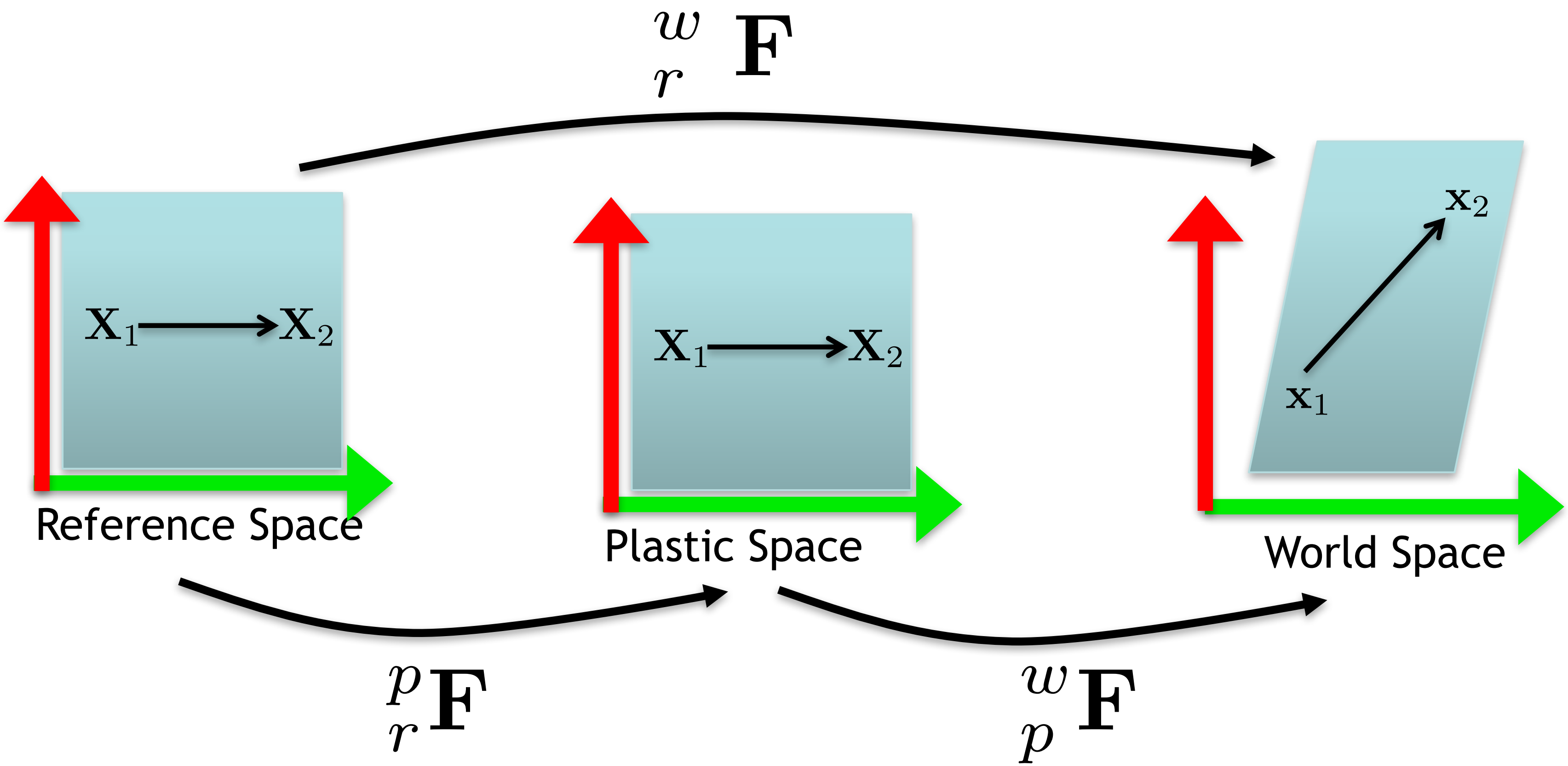
Continuum Mechanics: Deformation

- **Introduce a new space**



Continuum Mechanics: Deformation

- Our goal is to approximate ${}^{w}_{p}\mathbf{F}$ but we only have access to ${}^{w}_{r}\mathbf{F}$



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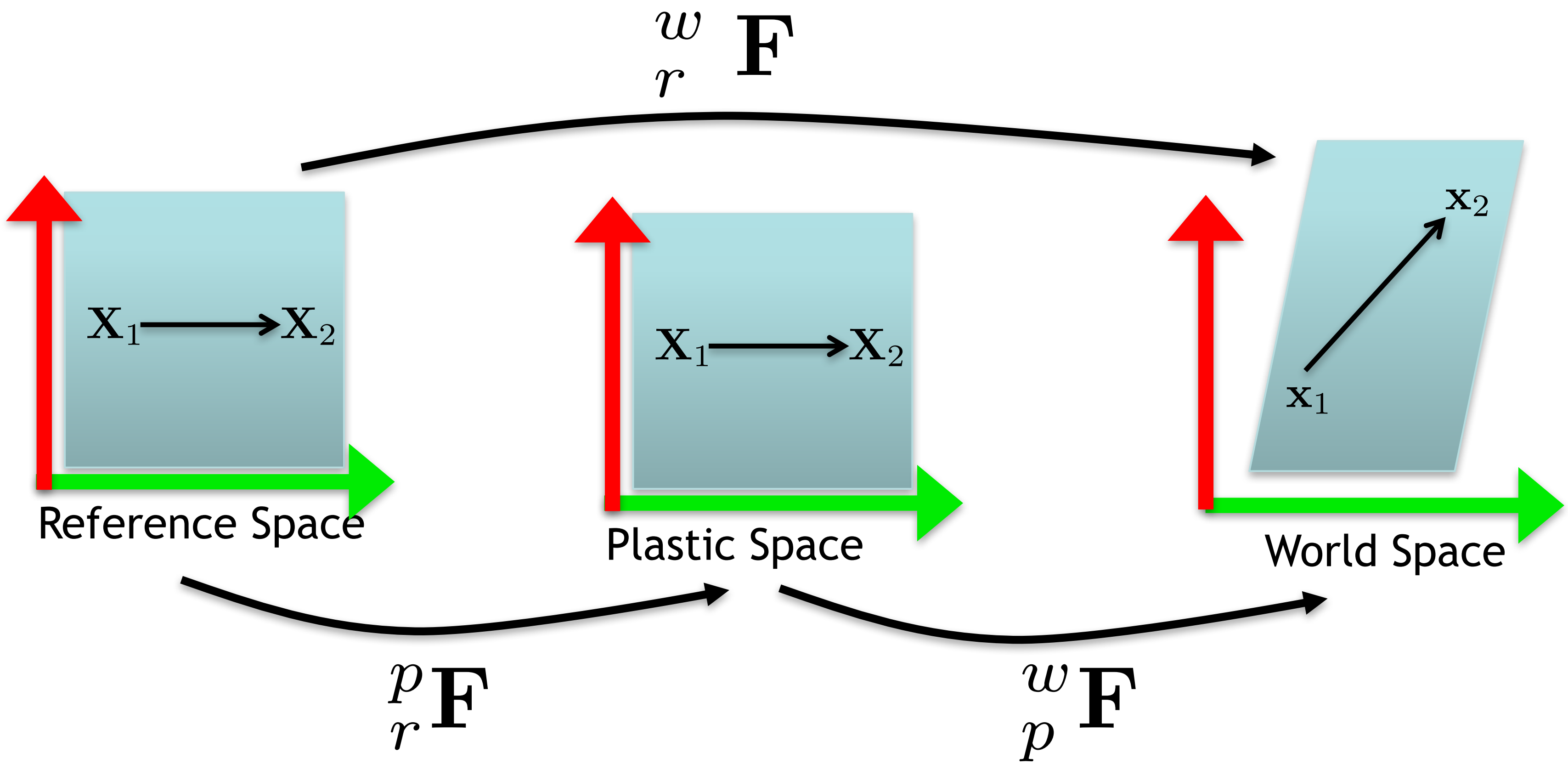

$$ {}^{w}_{p}\mathbf{F} = {}^{w}_{r}\mathbf{F}\,{}^{p}_{r}\mathbf{F}^{-1} $$

Continuum Mechanics: Deformation

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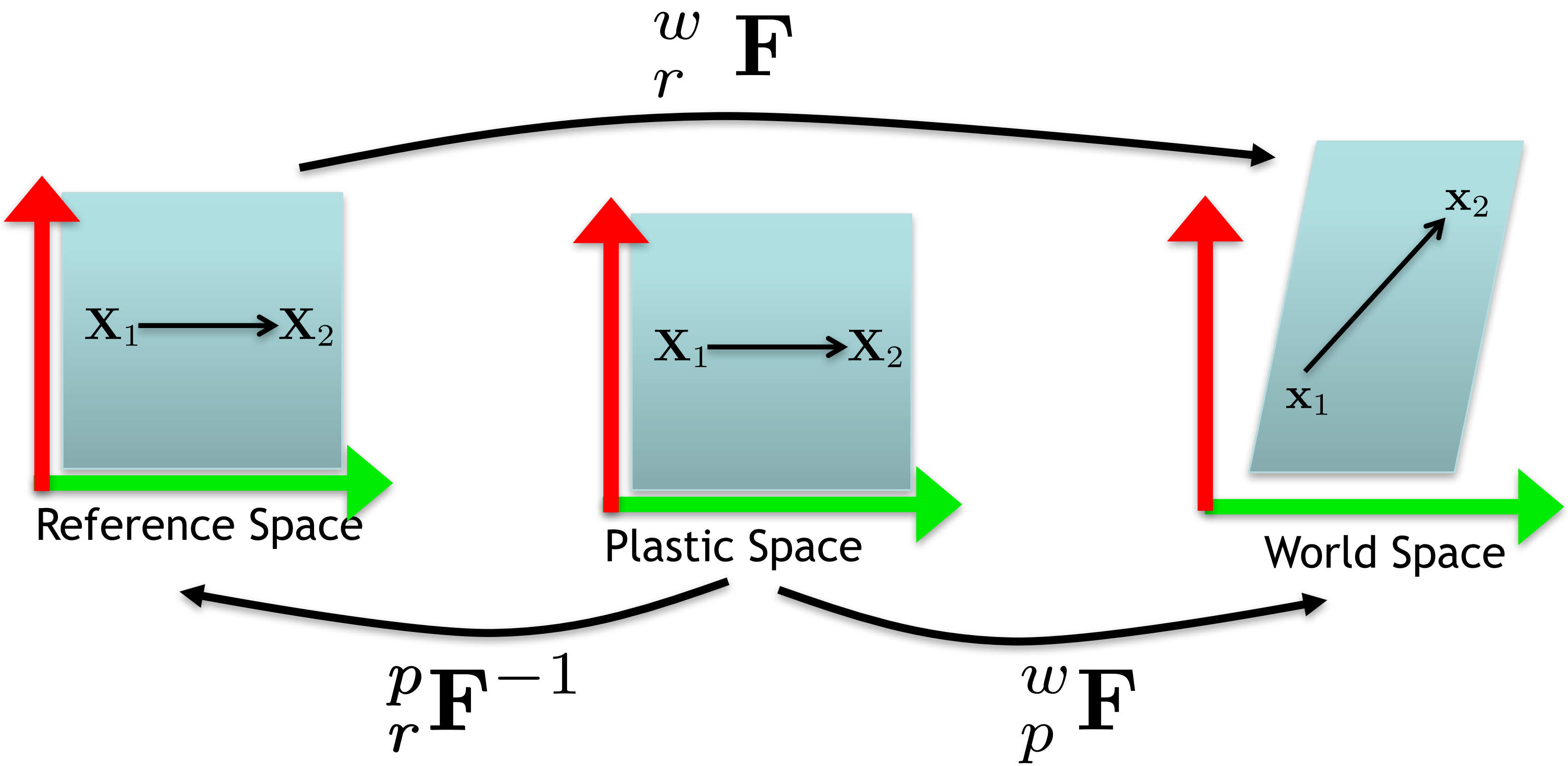
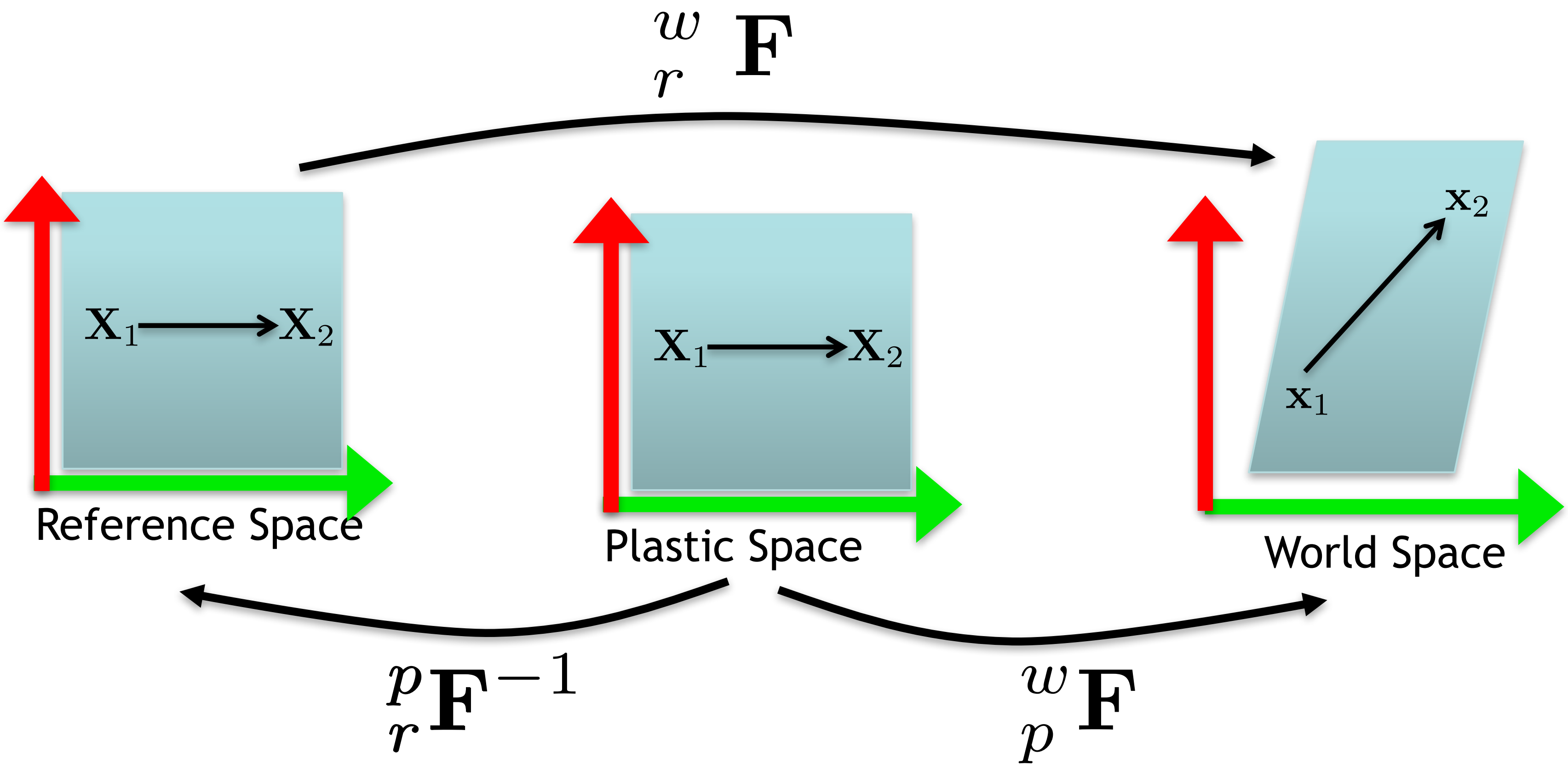


$$ {}^{w}_{p}\mathbf{F} = {}^{w}_{r}\mathbf{F}\,{}^{p}_{r}\mathbf{F}^{-1} $$

Continuum Mechanics: Deformation

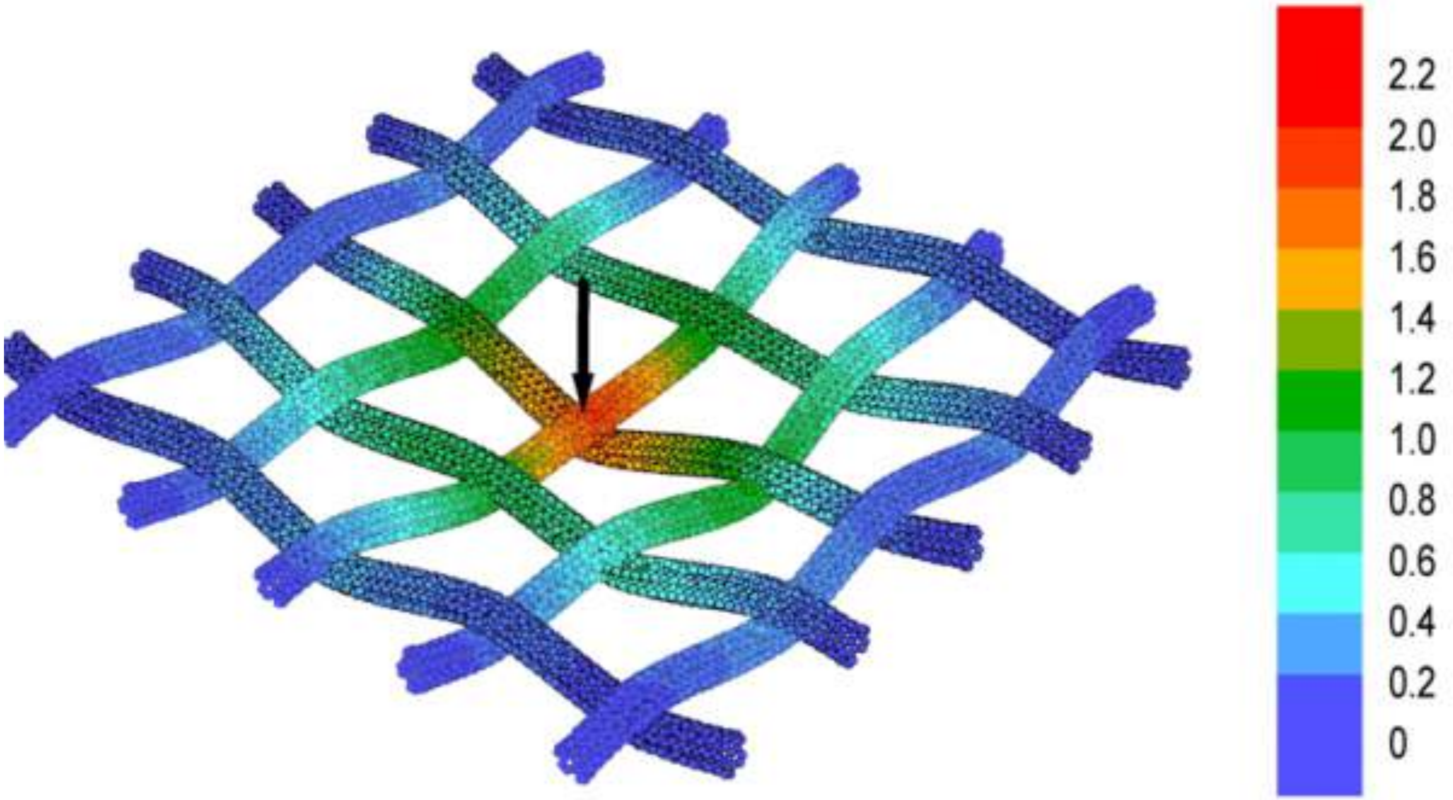
- We can store ${}^{p}_{r}\mathbf{F}^{-1}$ for each triangle in order to keep track of its plastic shape change



$$ {}^{w}_{p}\mathbf{F} = {}^{w}_{r}\mathbf{F}\,{}^{p}_{r}\mathbf{F}^{-1} $$

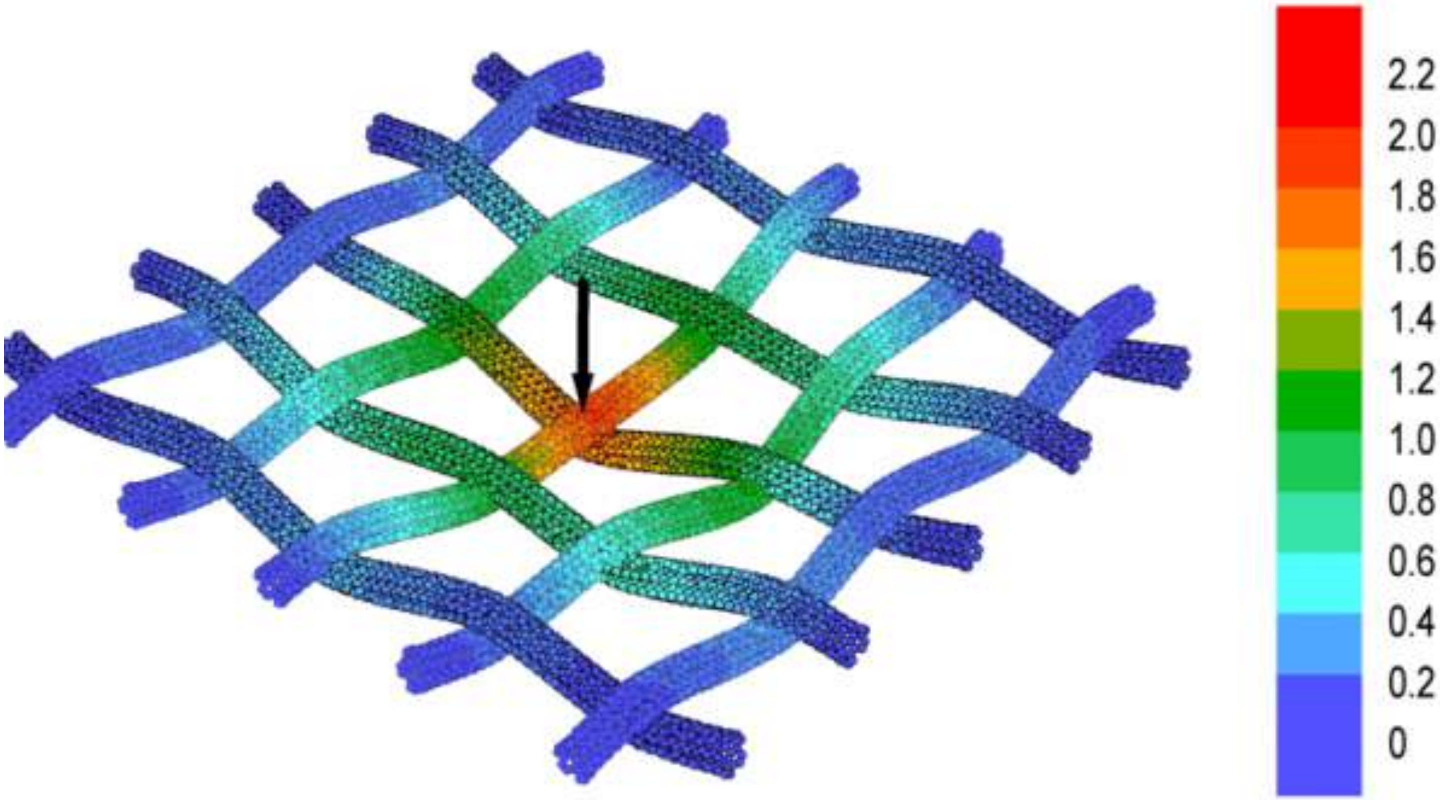
How to Compute the Plastic Deformation Gradient

- We compute the stress on each element during simulation



How to Compute the Plastic Deformation Gradient

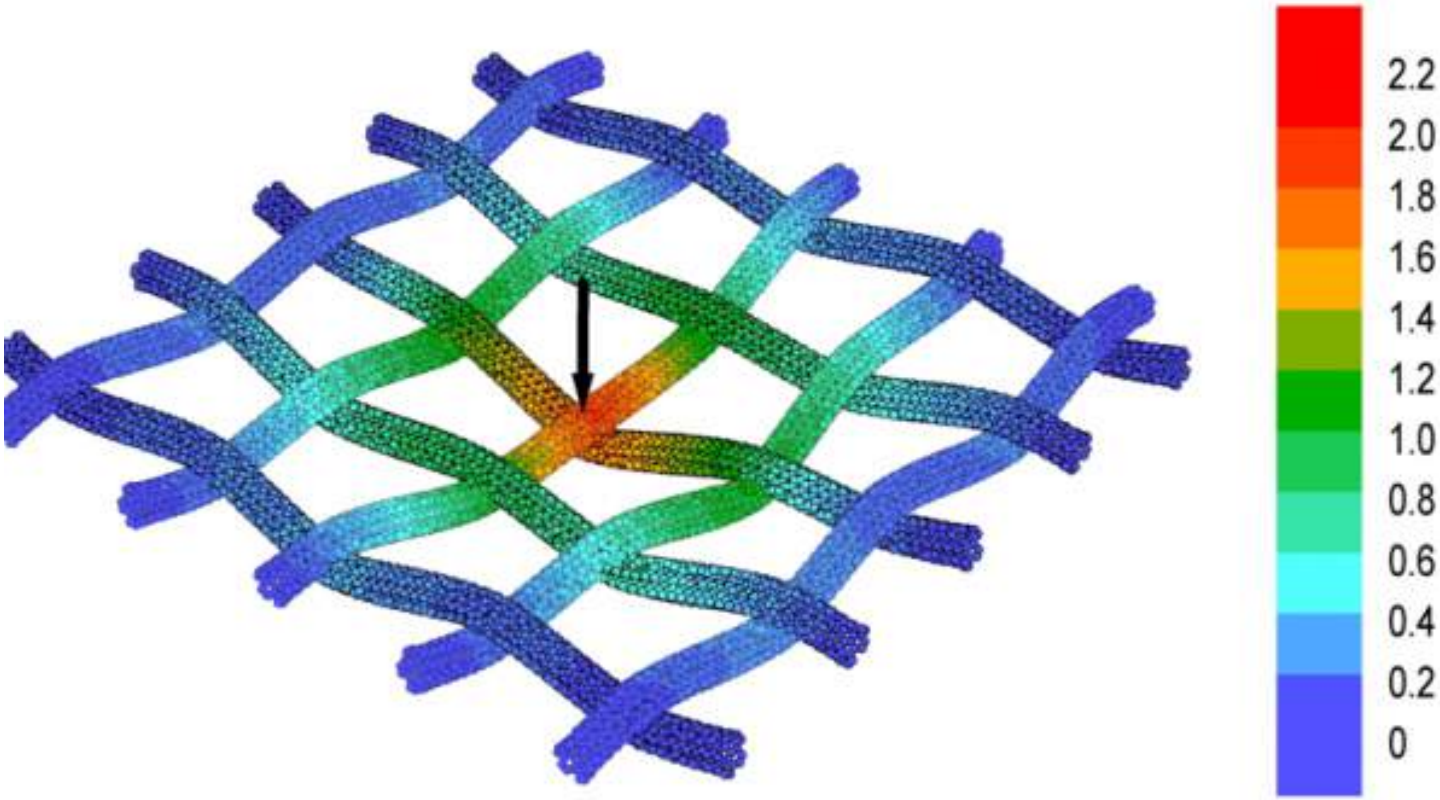
- When the stress in a triangles gets above a certain threshold we store $\mathbf{F}$ as ${}^{p}_{r}\mathbf{F}$



How to Compute the Plastic Deformation Gradient

- Each subsequent simulation step uses $\frac{1}{2}\left({}^{w}_{p}\mathbf{F}^{T}\,{}^{w}_{p}\mathbf{F} - \mathbf{I}\right)$

$${}^{w}_{p}\mathbf{F} = {}^{w}_{r}\mathbf{F}\,{}^{p}_{r}\mathbf{F}^{-1}$$



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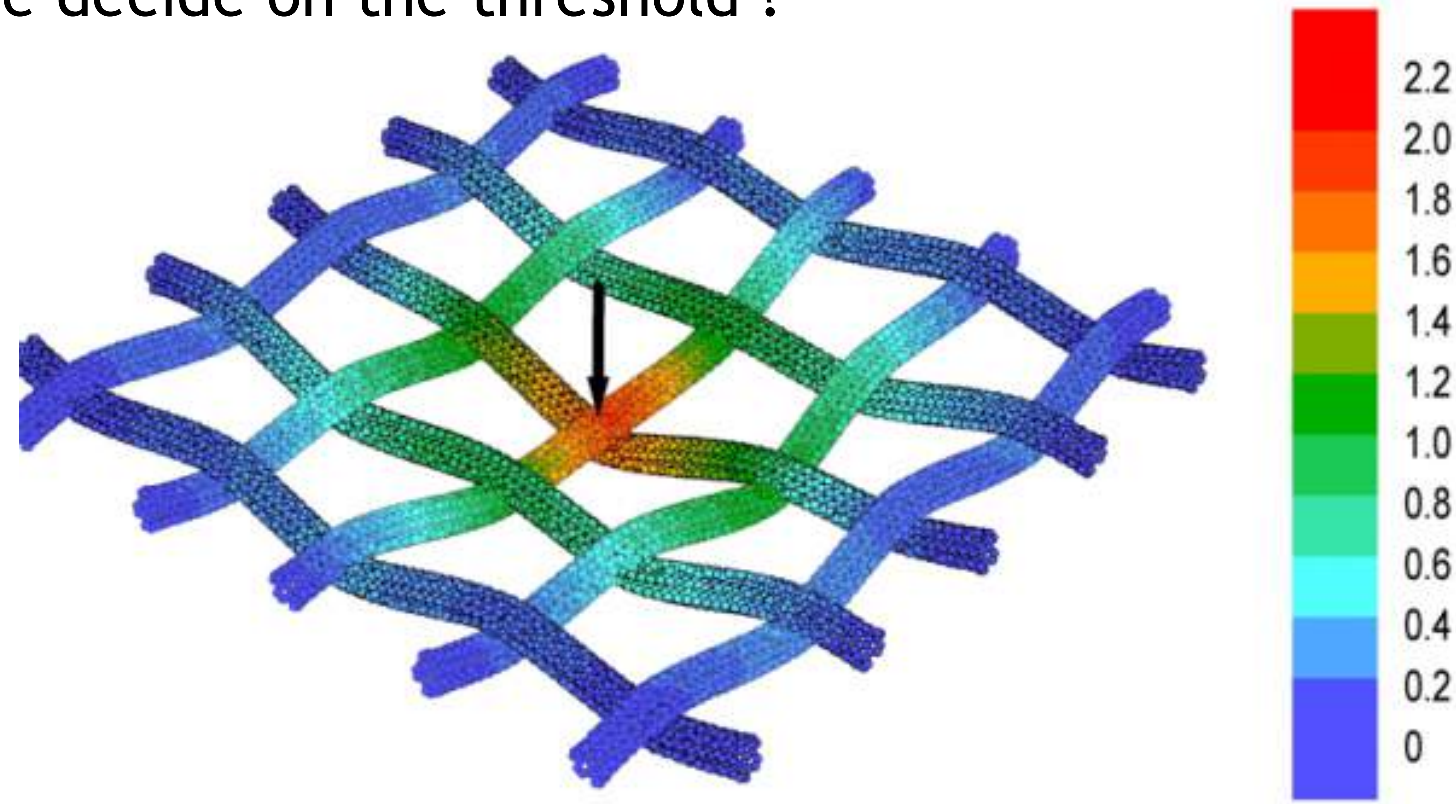
- How do we decide on the threshold ?



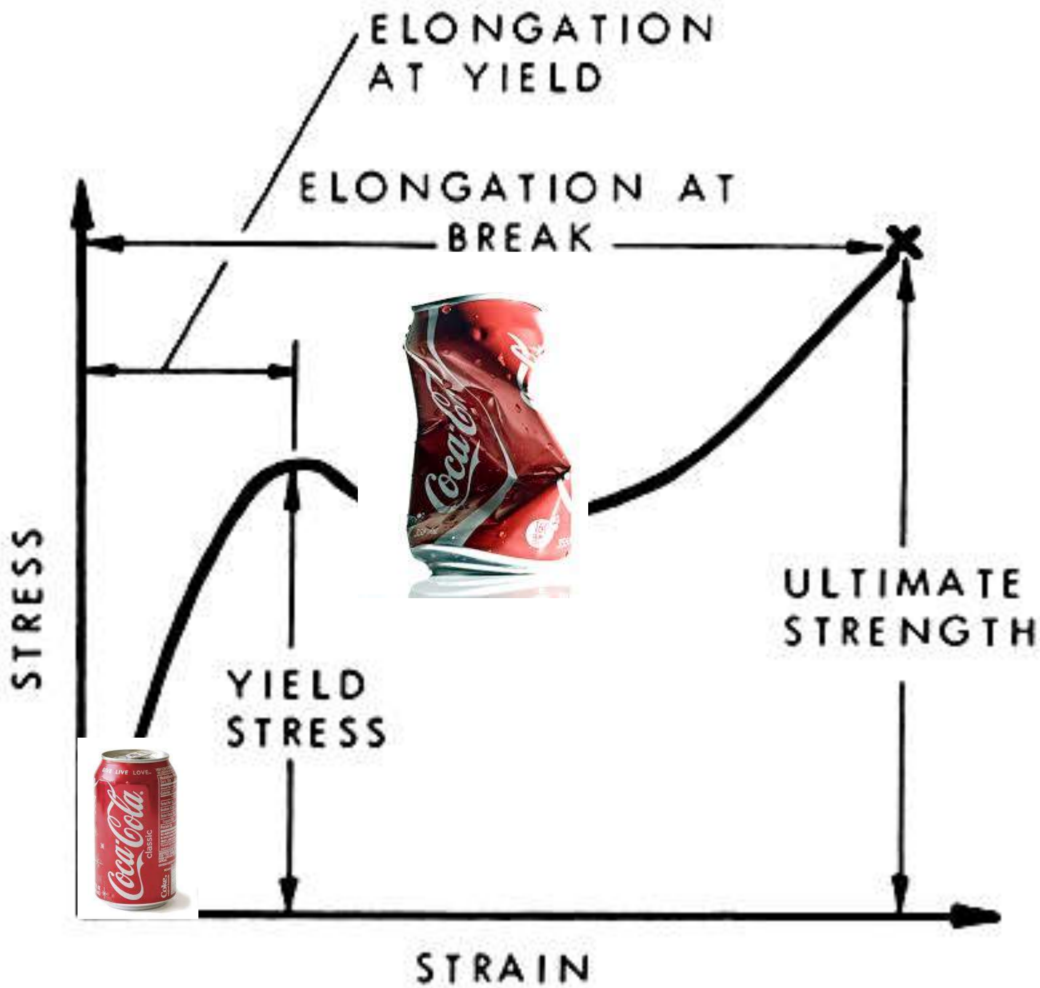
Measuring Plastic Materials

- We use a similar approach to elastic materials
- Except instead of a compression test, we use a tensile test
- We pull on the ends of the object then measure the strain induced

Measuring Plastic Materials

Other Interesting Material Properties

- Plasticity – Change in Reference State

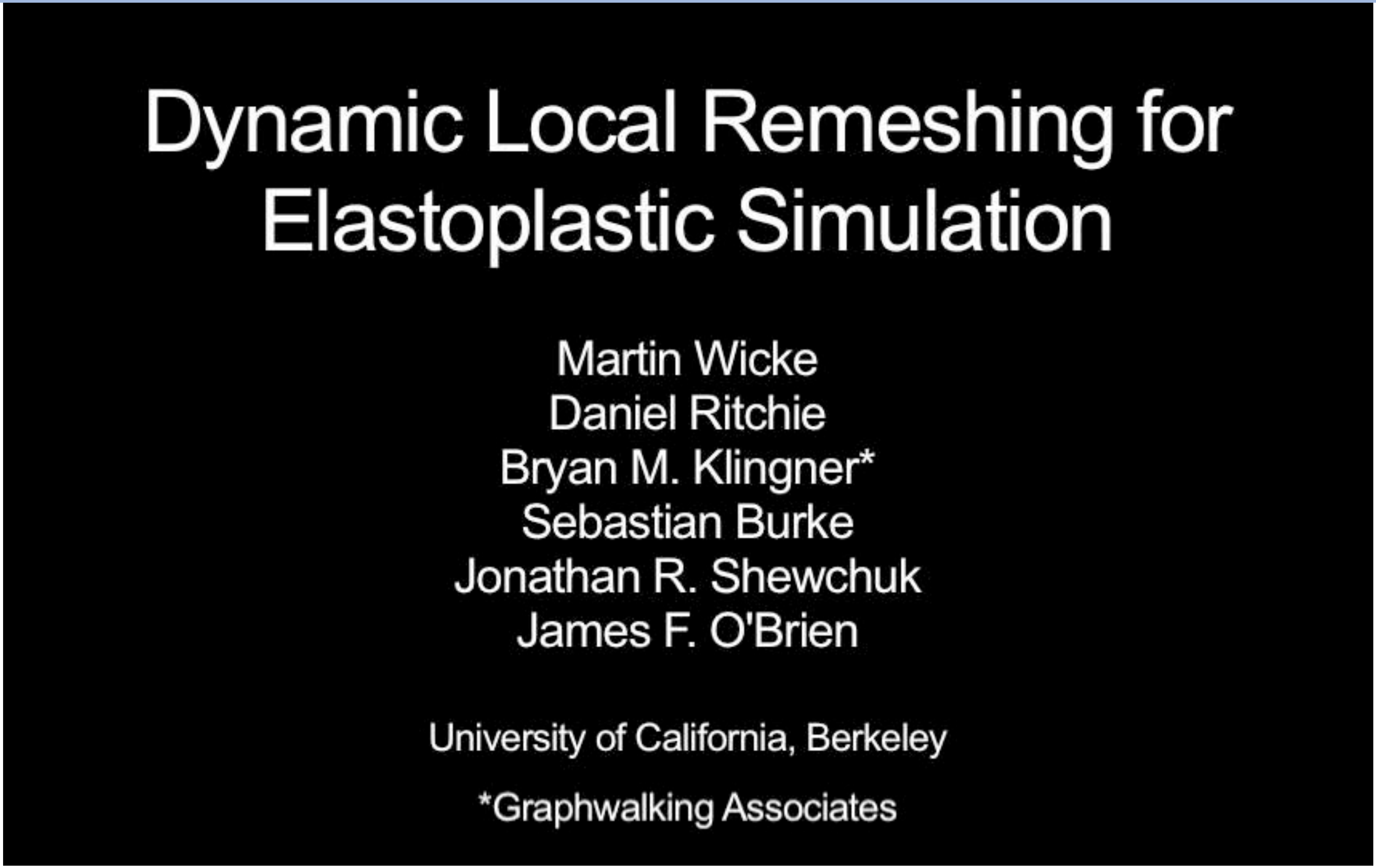


FEM with Plasticity

A Finite Element Method for Animating Large Viscoplastic Flow

Adam W. Bargteil, CMU
Chris Wojtan, Georgia Tech
Jessica K. Hodgins, CMU
Greg Turk, Georgia Tech

Plasticity and Finite Elements

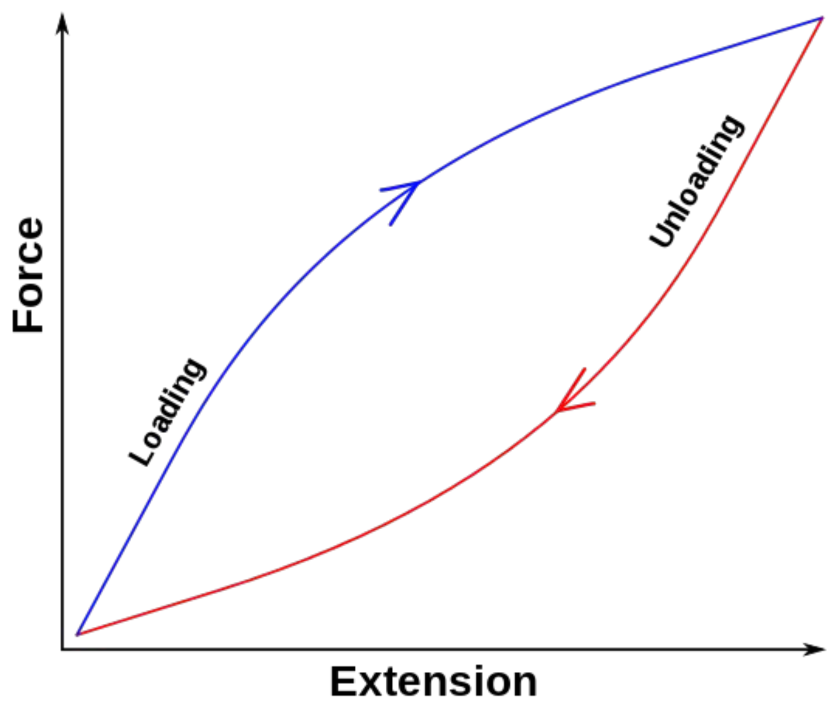


Types of Materials

- There are many types of materials
  - Elastic ← Done
  - Plastic ← Done
  - Hysteresis ← Briefly
  - Composites
  - Cellular Materials
  - Lattice Structures
- Each one has different mechanical properties
- When we fabricate things we exploit these properties to achieve optimal results

Elastic Hysteresis

- The strain of the material does not only depend on its current stress, but also on its history
- Energy is dissipated due to material internal friction



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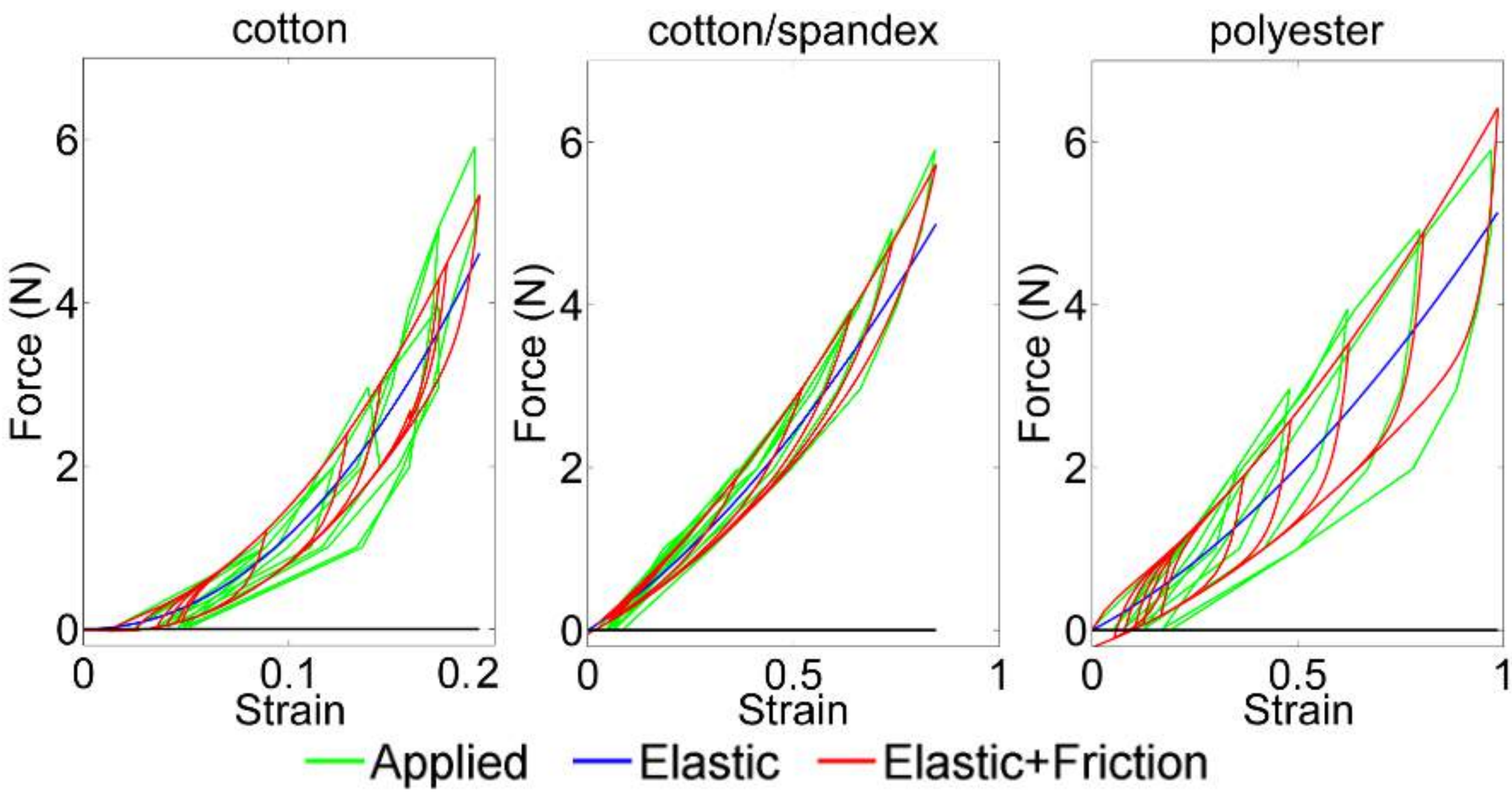


Elastic Hysteresis

Modeling and Estimation of Internal Friction in Cloth

Eder Miguel[1] Rasmus Tamstorf[2] Derek Bradley[3]
Sara C. Schvartzman[1] Bernhard Thomaszewski[3] Bernd Bickel[3]
Wojciech Matusik[4] Steve Marschner[5] Miguel A. Otaduy[1]

[1]URJC Madrid [2]Walt Disney Animation Studios [3]Disney Research Zurich
[4]MIT CSAIL [5]Cornell University

Universidad Rey Juan Carlos
Disney Research
Massachusetts Institute of Technology
Cornell University

Where we are now

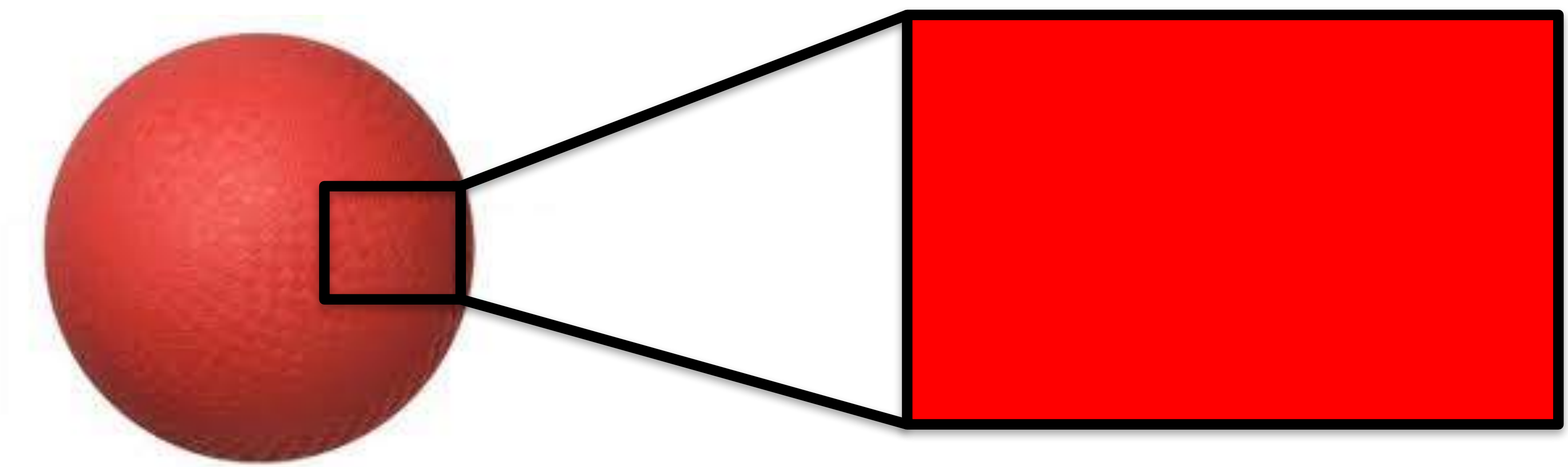
- You have now seen the following
  - Basic equations for continuum mechanics
  - The Finite Element Method
  - Different Material Models
  - How to Measure Parameters
  - How Typical FEM Software works

Additional Reading

- Continuum Mechanics
  - Mase and Mase
- SIGGRAPH Finite Element Method Notes
  - www.femdefo.org
- Nonlinear Continuum Mechanics for Finite Element Analysis
  - Bonet and Wood

Next: Advanced Materials

Simple Materials

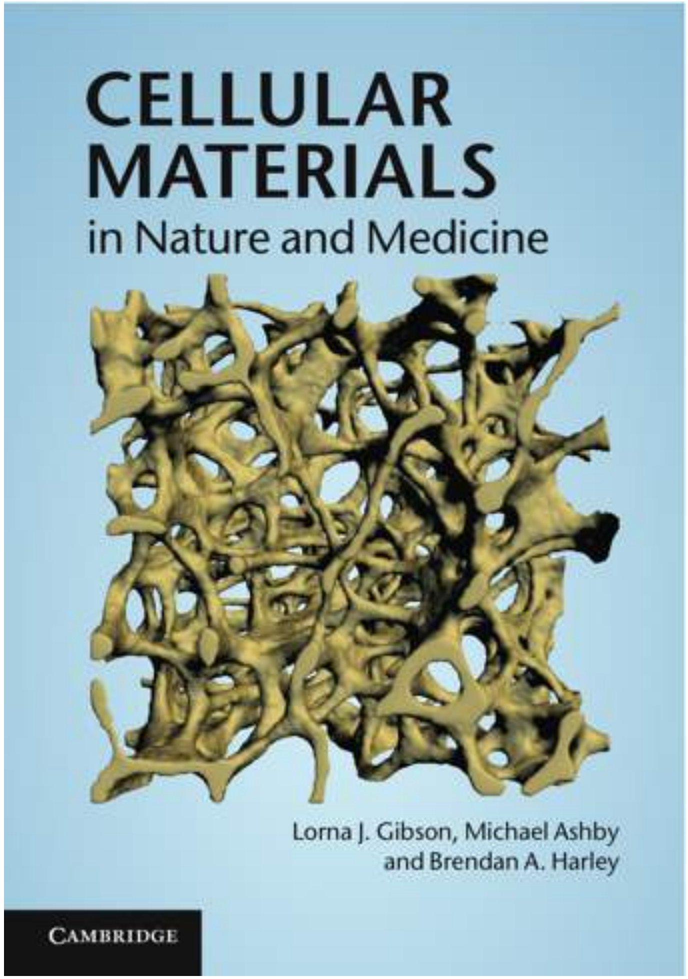


Advanced Materials

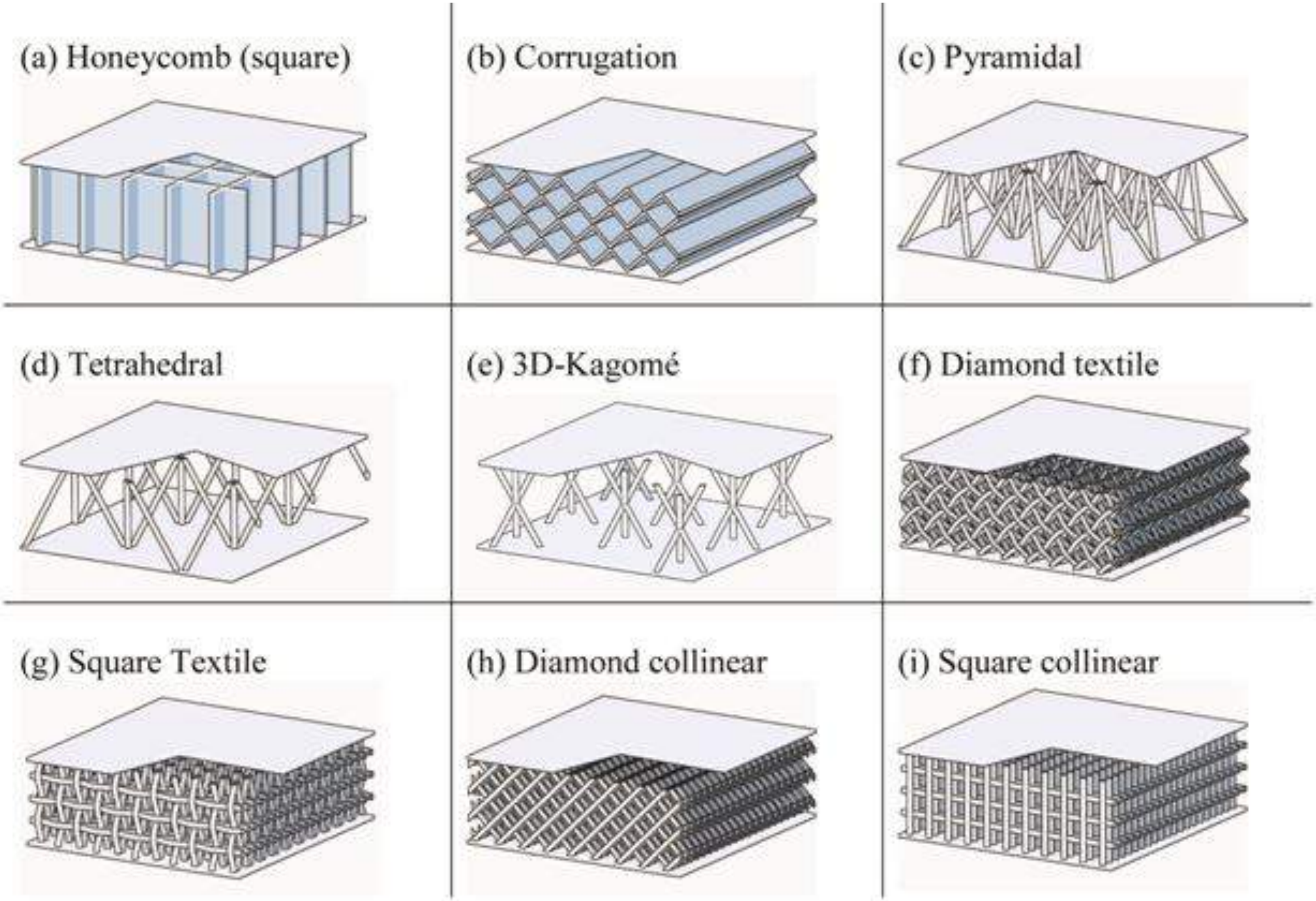
- Cellular materials
  - Metamaterials
- Composite materials
  - Functionally graded materials
- Biomimetic/bio-inspired materials
- Materials with structural hierarchy

Cellular Materials

- Regular
  - Lattice truss structures
- Irregular
  - Foam
    - Open-cell
    - Closed-cell
- Properties governed by:
  - Topology
  - Fraction of cell occupied by material
  - Properties of constituent material

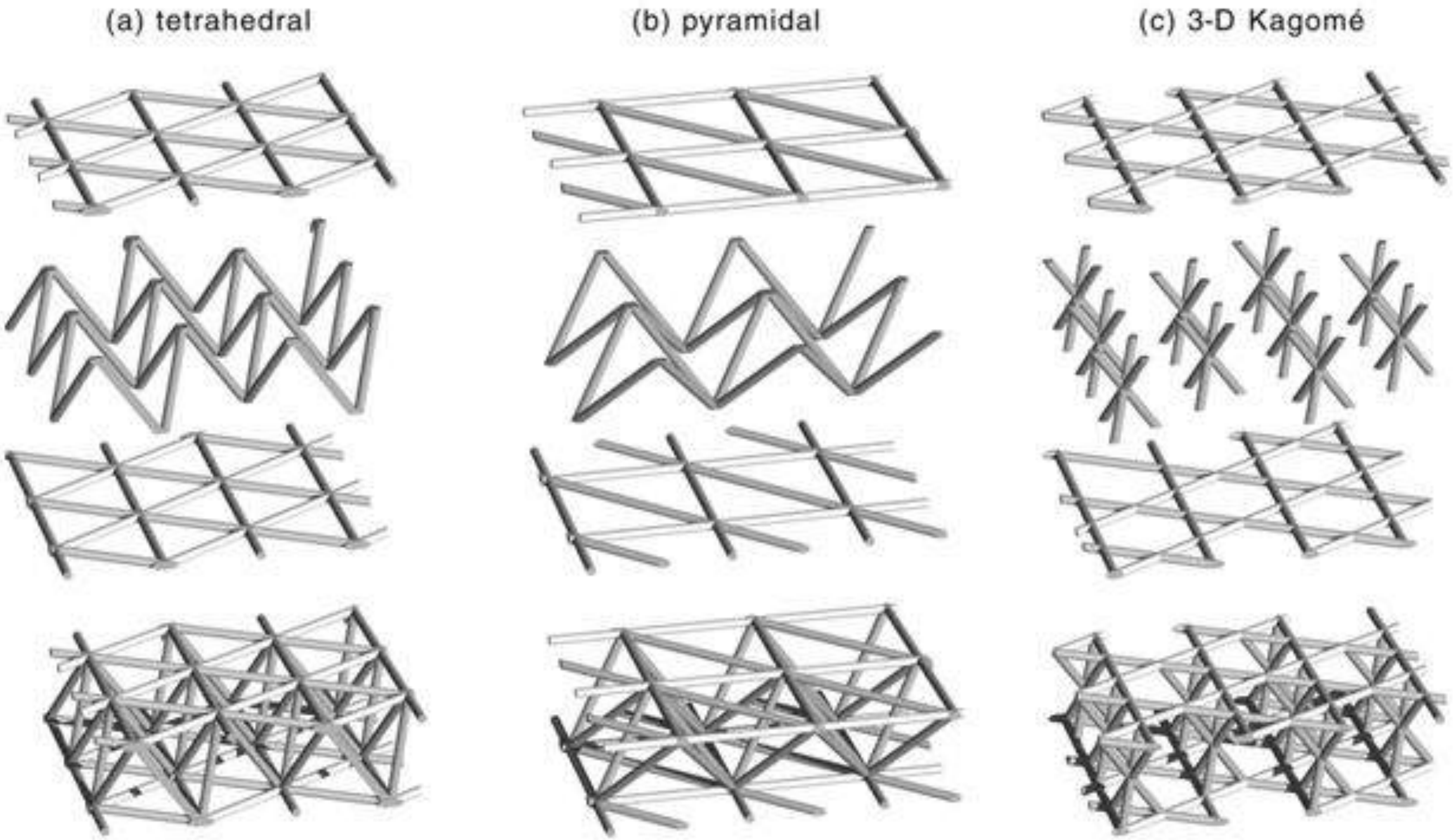


Topologies of Cellular Lattices



http://www.virginia.edu/ms/research/wadley/celluar-materials.html

Topologies of Cellular Lattices



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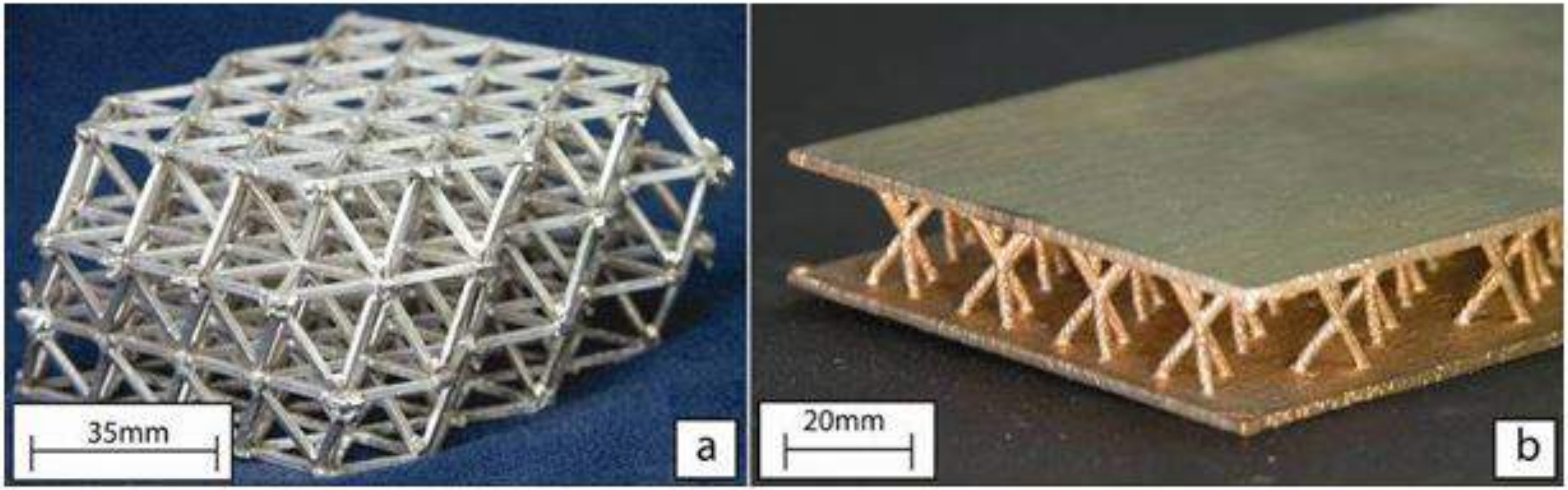

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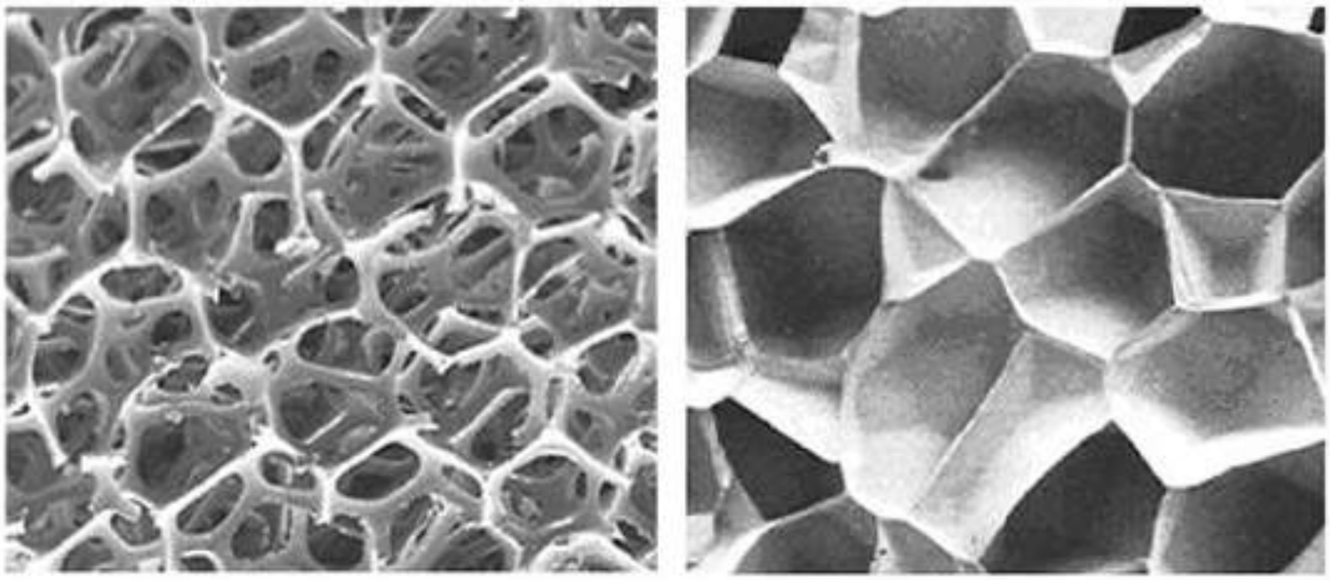
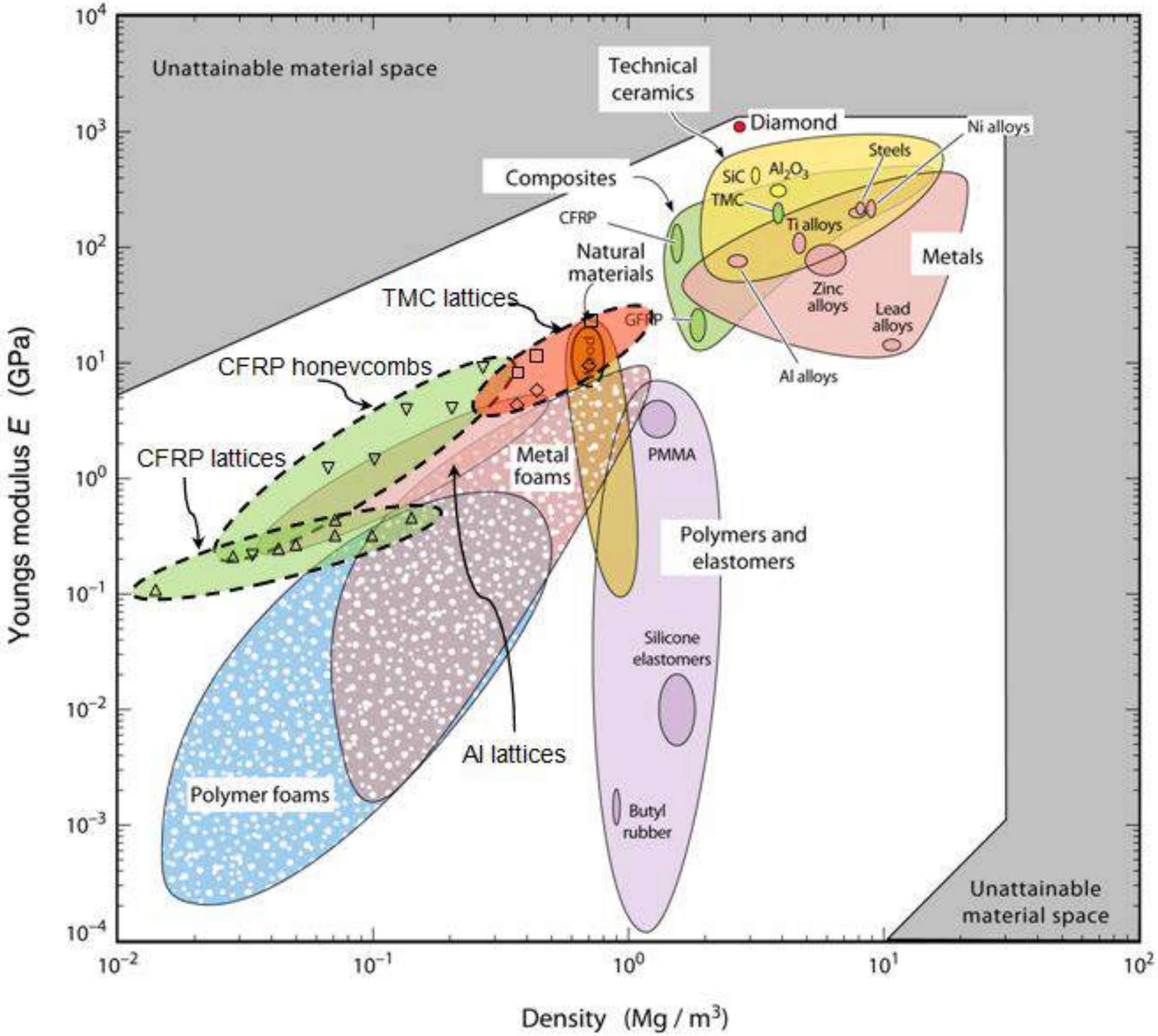
Solid Foams

- Open-cell foams (reticulated foams)
  - Lighter, softer
- Closed-cell foams
  - Heavier, harder



Mechanical Properties



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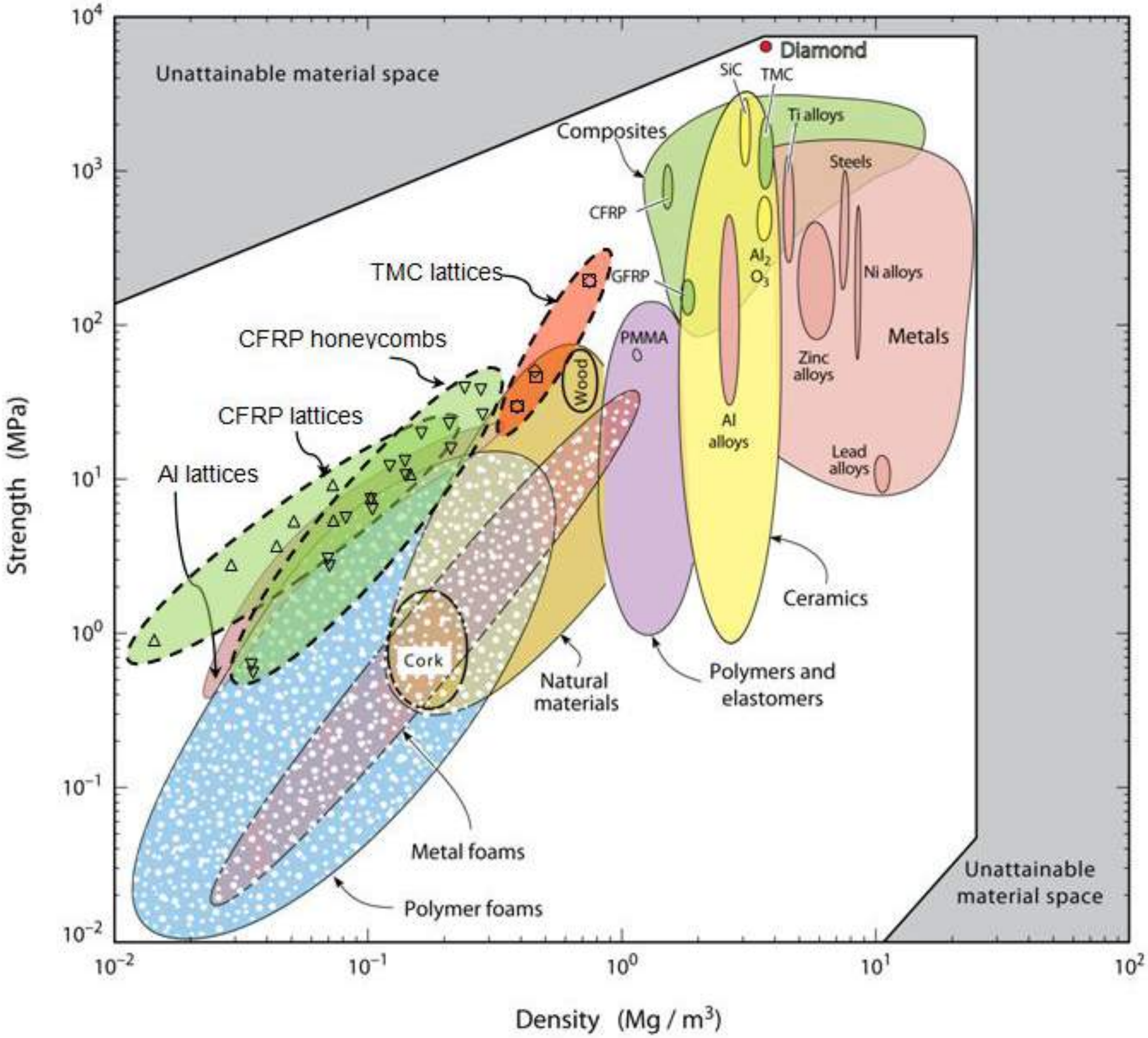


http://www.virginia.edu/ms/research/wadley/celluar-materials.html

Applications

http://www.virginia.edu/ms/research/wadley/celluar-materials.html

Applications

CETEX System 3, a PEI thermoplastic-core sandwich material used in Airbus A340-500/600 aircraft

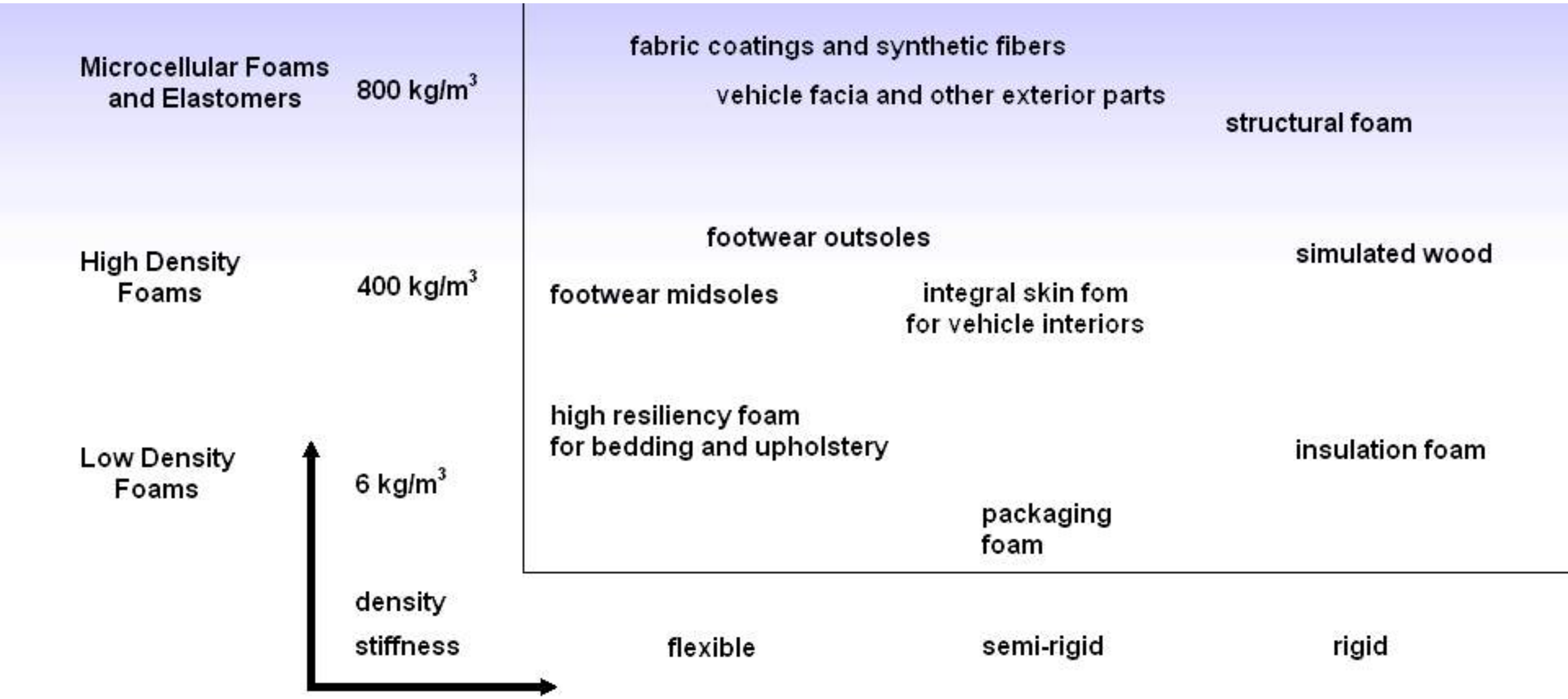
Nomex honeycomb in ATEC 212 SOLO

A Nomex honeycomb core was used to build this boat (NEB)

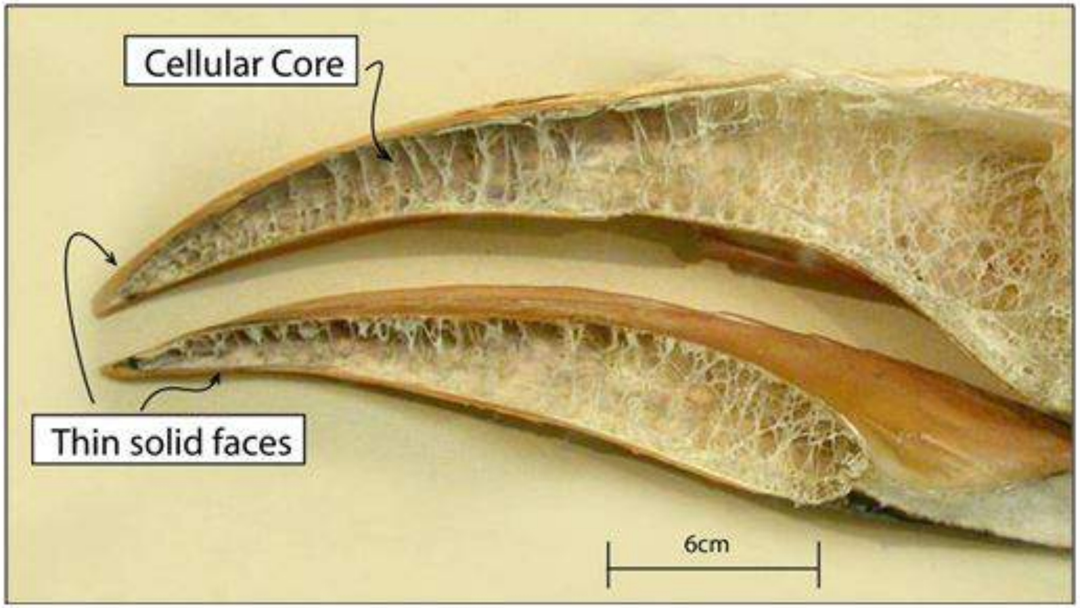
Nomex honeycomb cores

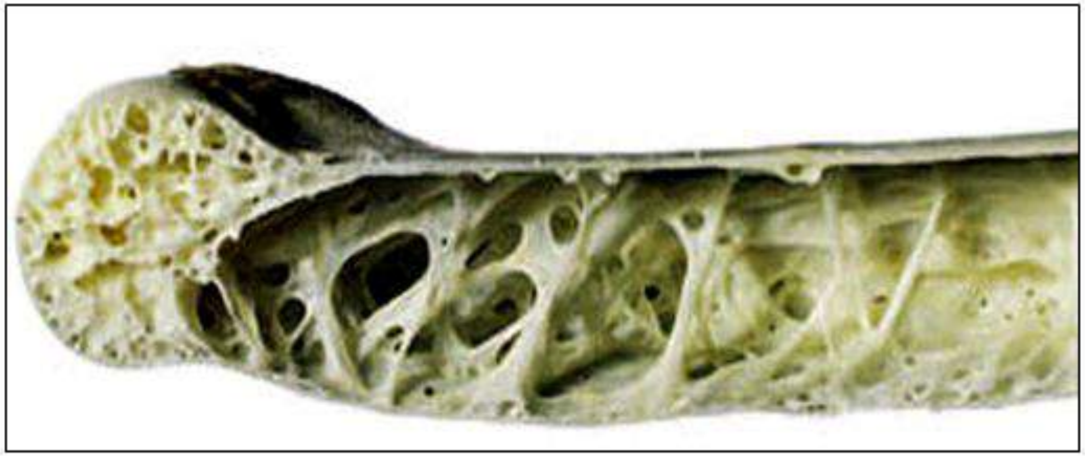
http://www.virginia.edu/ms/research/wadley/celluar-materials.html

Applications



Cellular Materials in Nature





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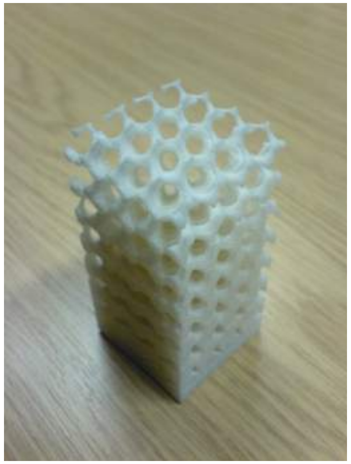
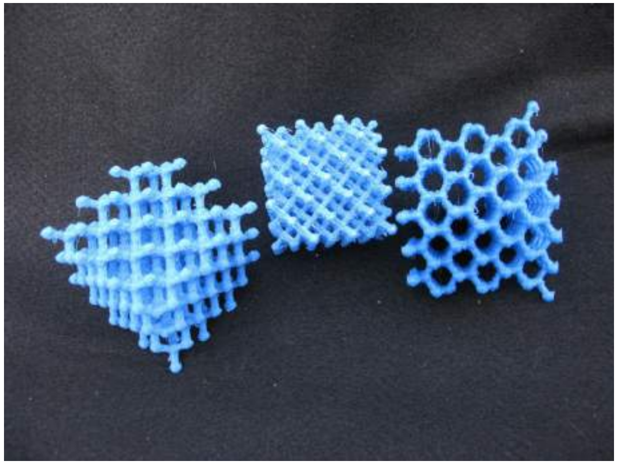
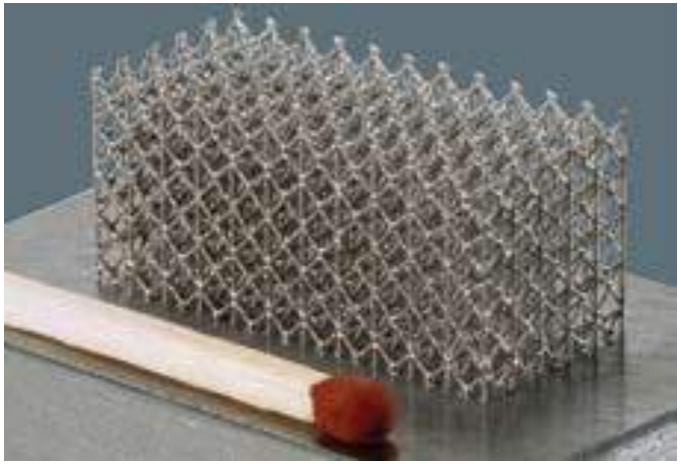
3D Printing Cellular Materials

- Many structures can be printed using FDM
- Closed-cell foams are difficult to print



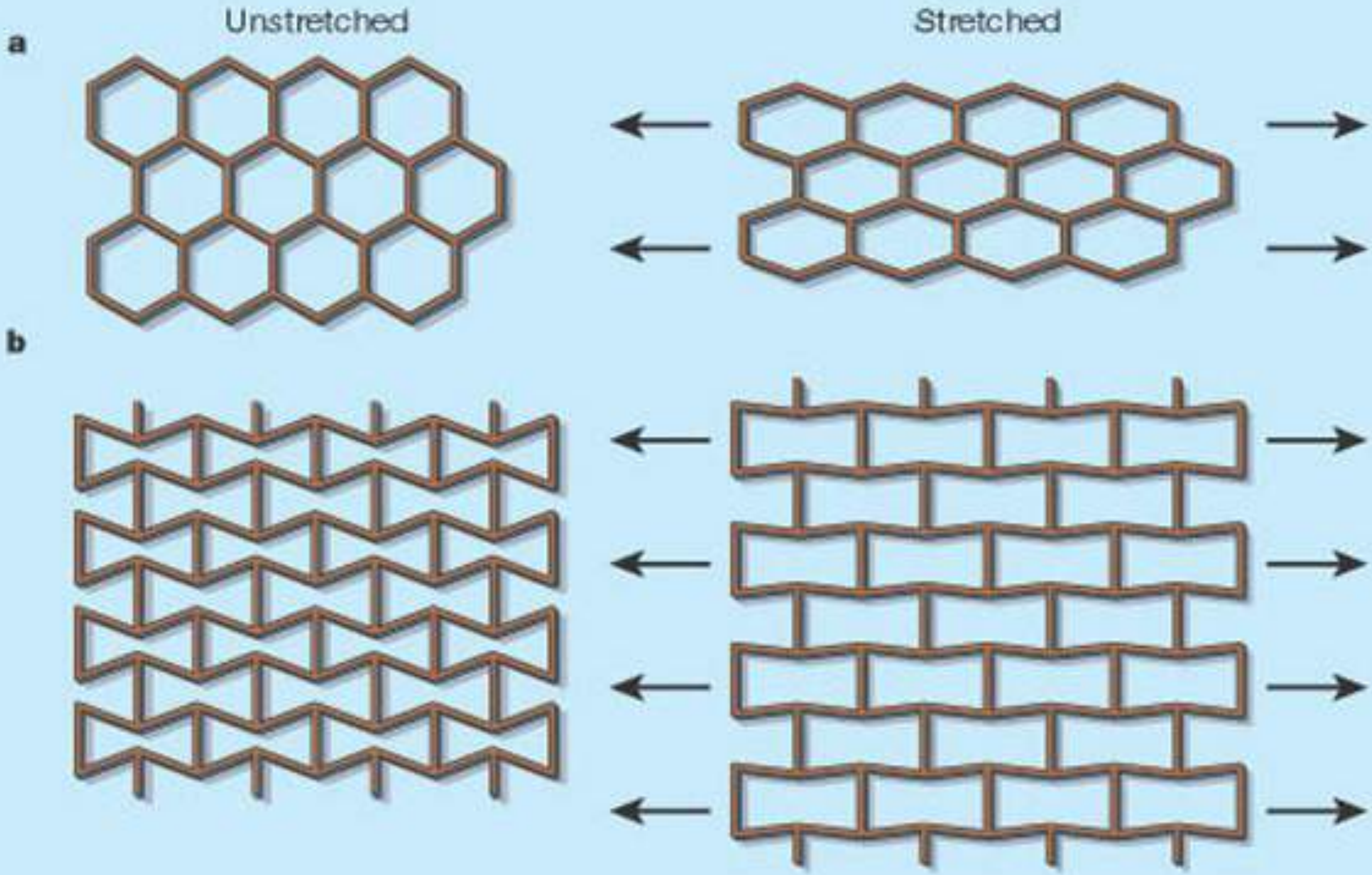




Mechanical Metamaterials

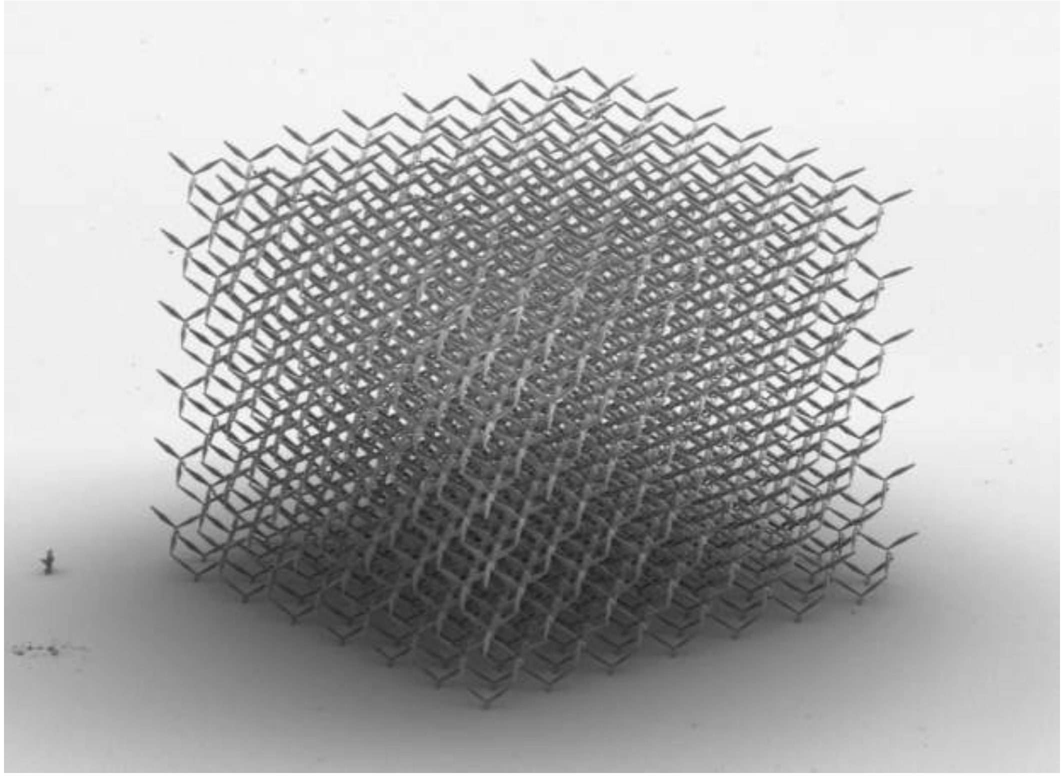
- Periodic cellular structures made of polymers, ceramics, or metals
- Mechanical properties can be designed to have values which cannot be found in nature

Negative Poisson's Ratio

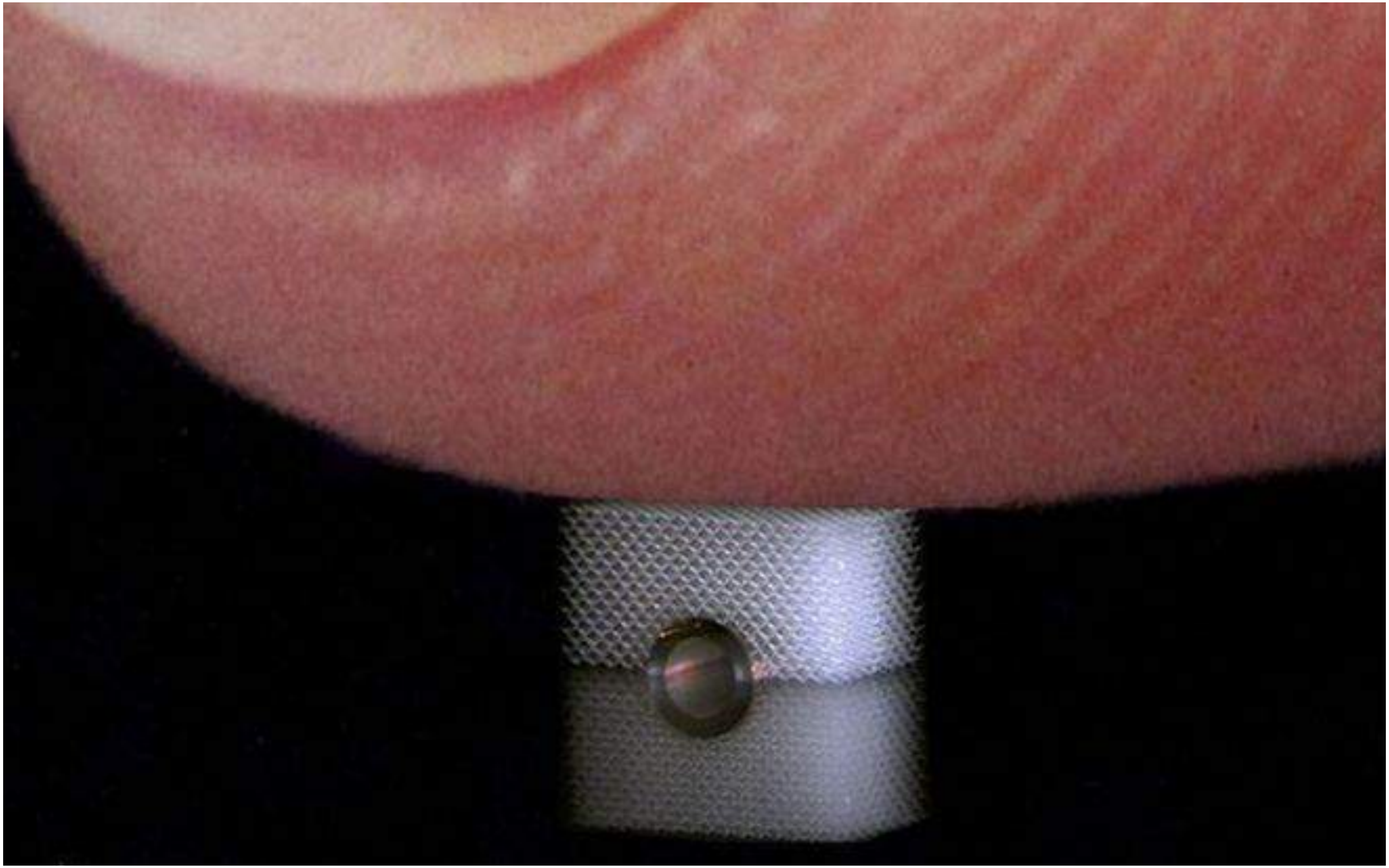


Pentamode Metamaterials (Meta-fluids)

- Solid that behaves like a fluid
- Hard to compress, easy to deform



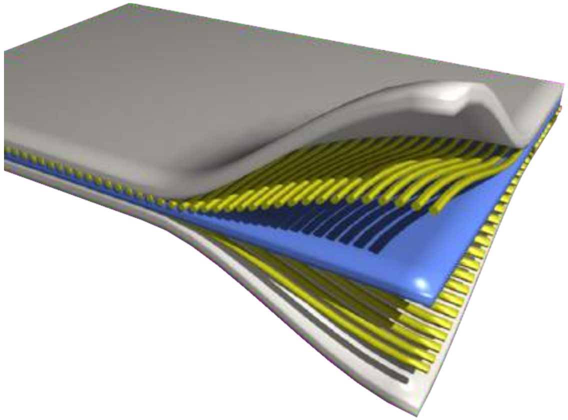
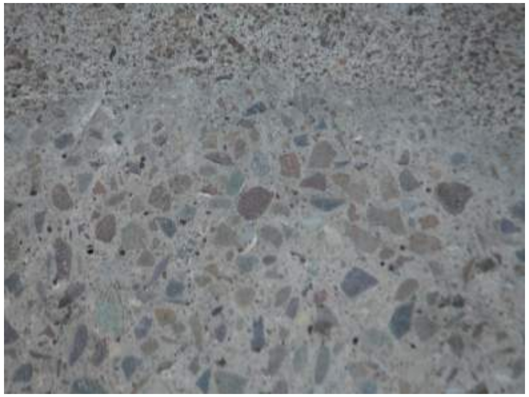
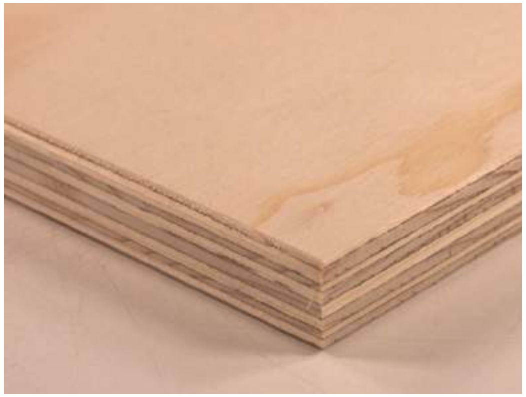
Interesting Uses: Mechanical Cloaking Device



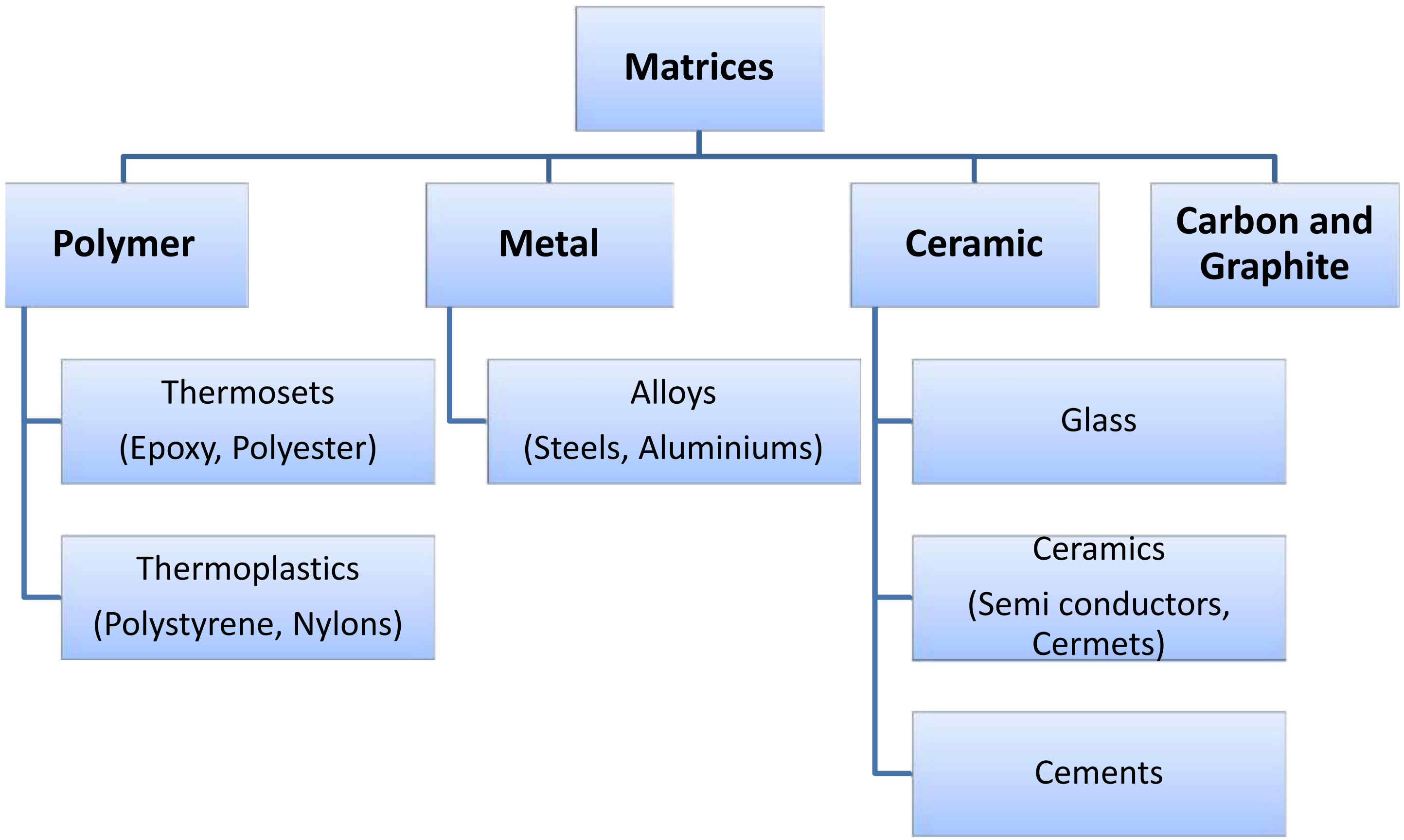
http://www.nature.com/articles/ncomms5130

Composite Materials

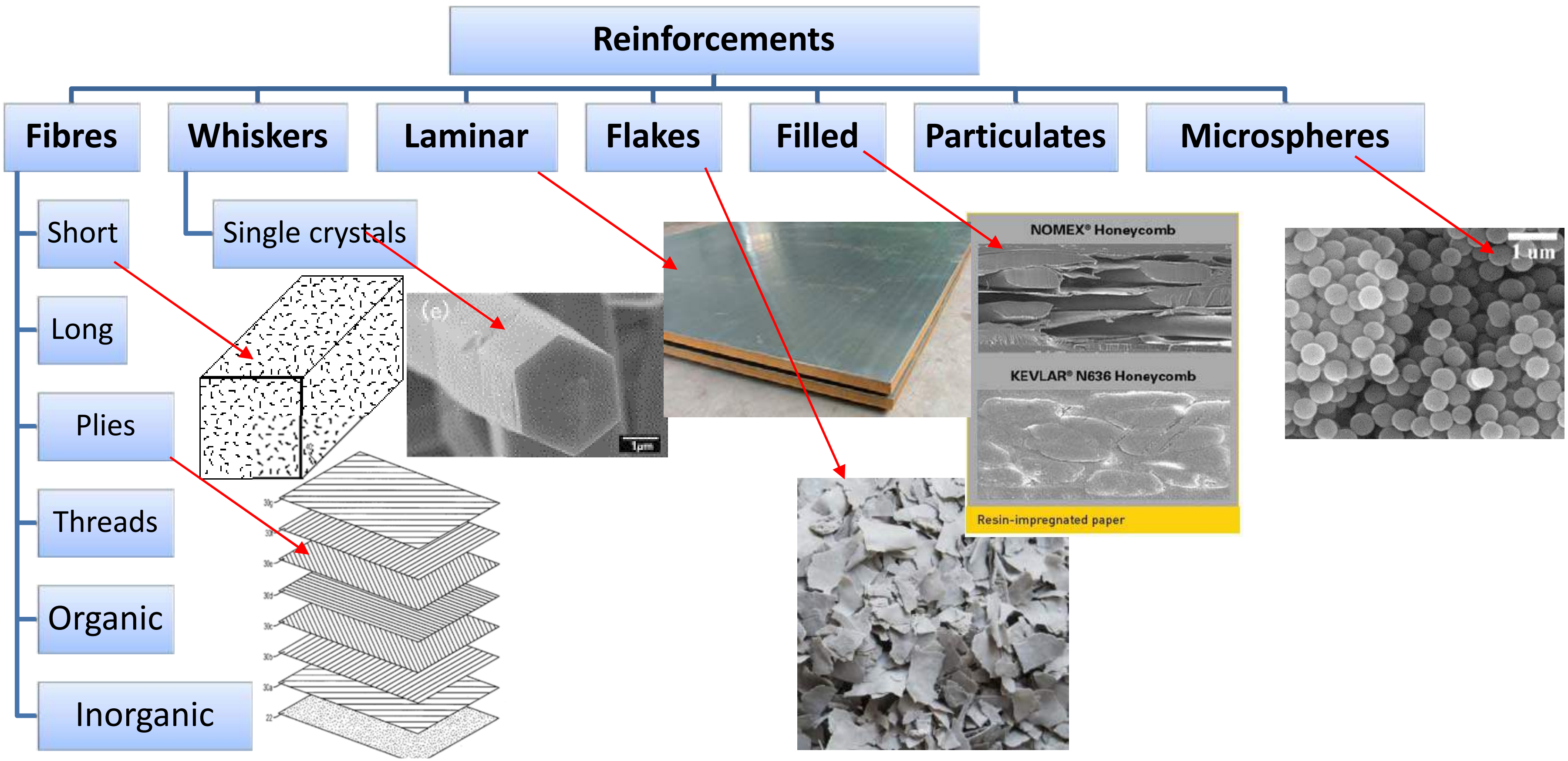
- Made from two or more constituent materials
  - At least one **matrix** and one **reinforcement material** e.g., polymer + fiber

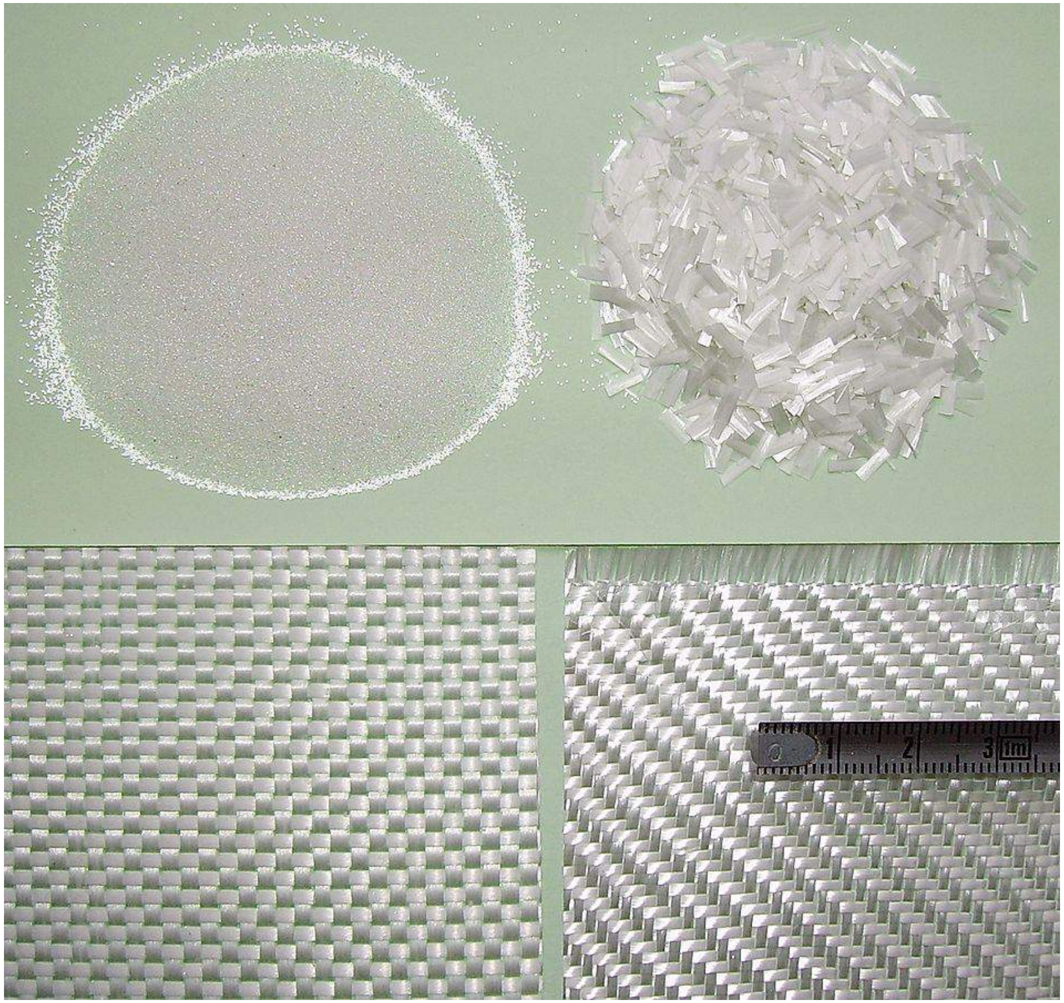
Matrices


Matrices
Polymer
Metal
Ceramic
Carbon and Graphite
Thermosets
(Epoxy, Polyester)
Thermoplastics
(Polystyrene, Nylons)
Alloys
(Steels, Aluminiums)
Glass
Ceramics
(Semi conductors, Cermets)
Cements

Reinforcements



Reinforcements



Why Composite Materials

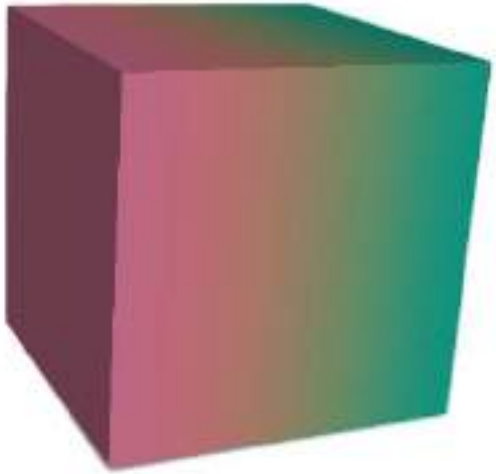
- Advantages
  - Lower density (20 to 40%)
  - Higher directional mechanical properties
  - Strength (ratio of material strength to density)
    - 4 times greater than that of steel and aluminum
  - Higher fatigue endurance
  - Higher toughness than ceramics and glasses
  - Versatility and tailoring by design
  - Easy to machine
  - Can combine other properties (damping, corrosion)
  - Cost

Why Not Composite Materials

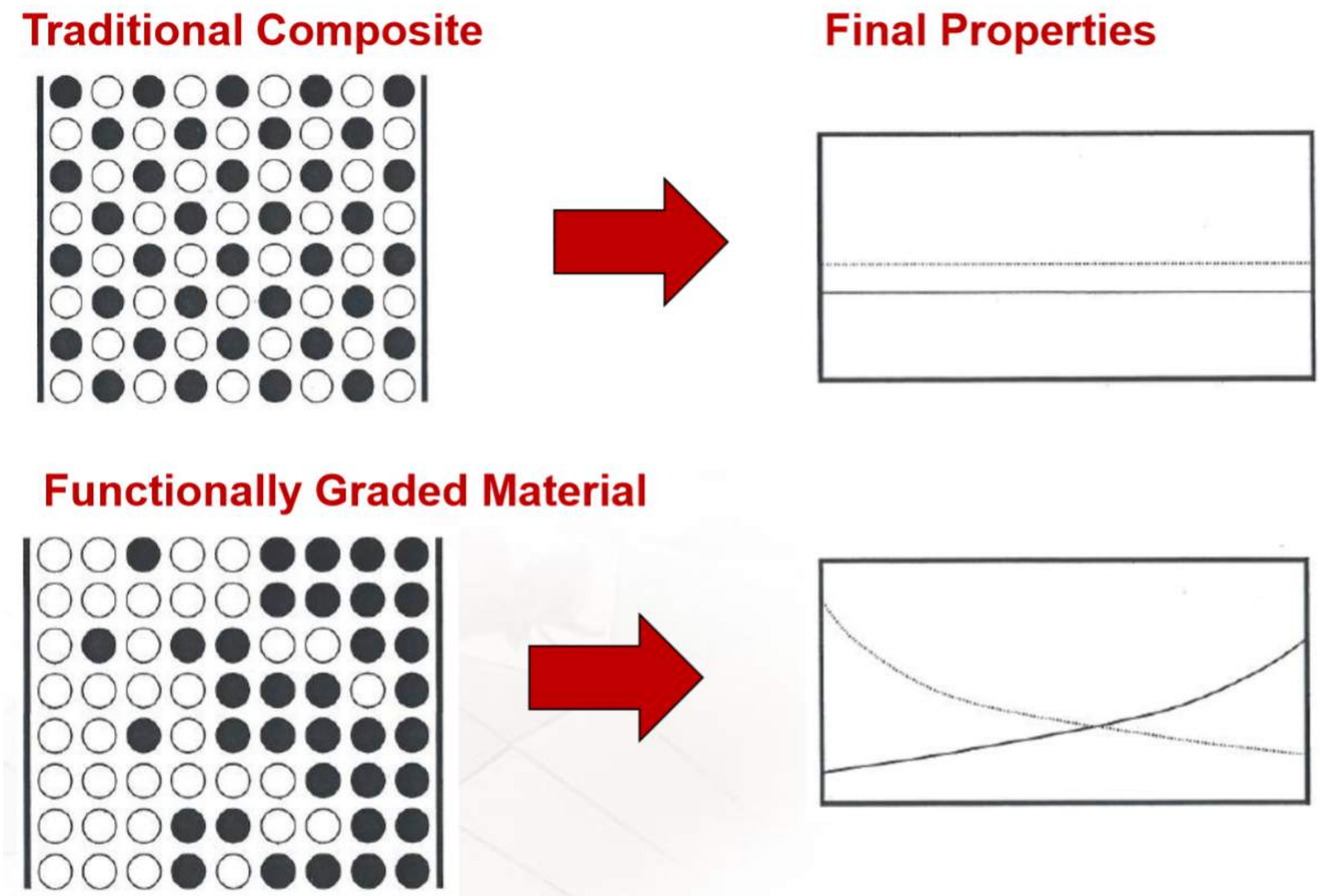
- Disadvantages
  - Not often environmentally friendly
  - Low recyclability
  - Can be damaged
  - Anisotropic properties
  - Matrix degrades
  - Low reusability

Functionally Graded Materials (FGMs)

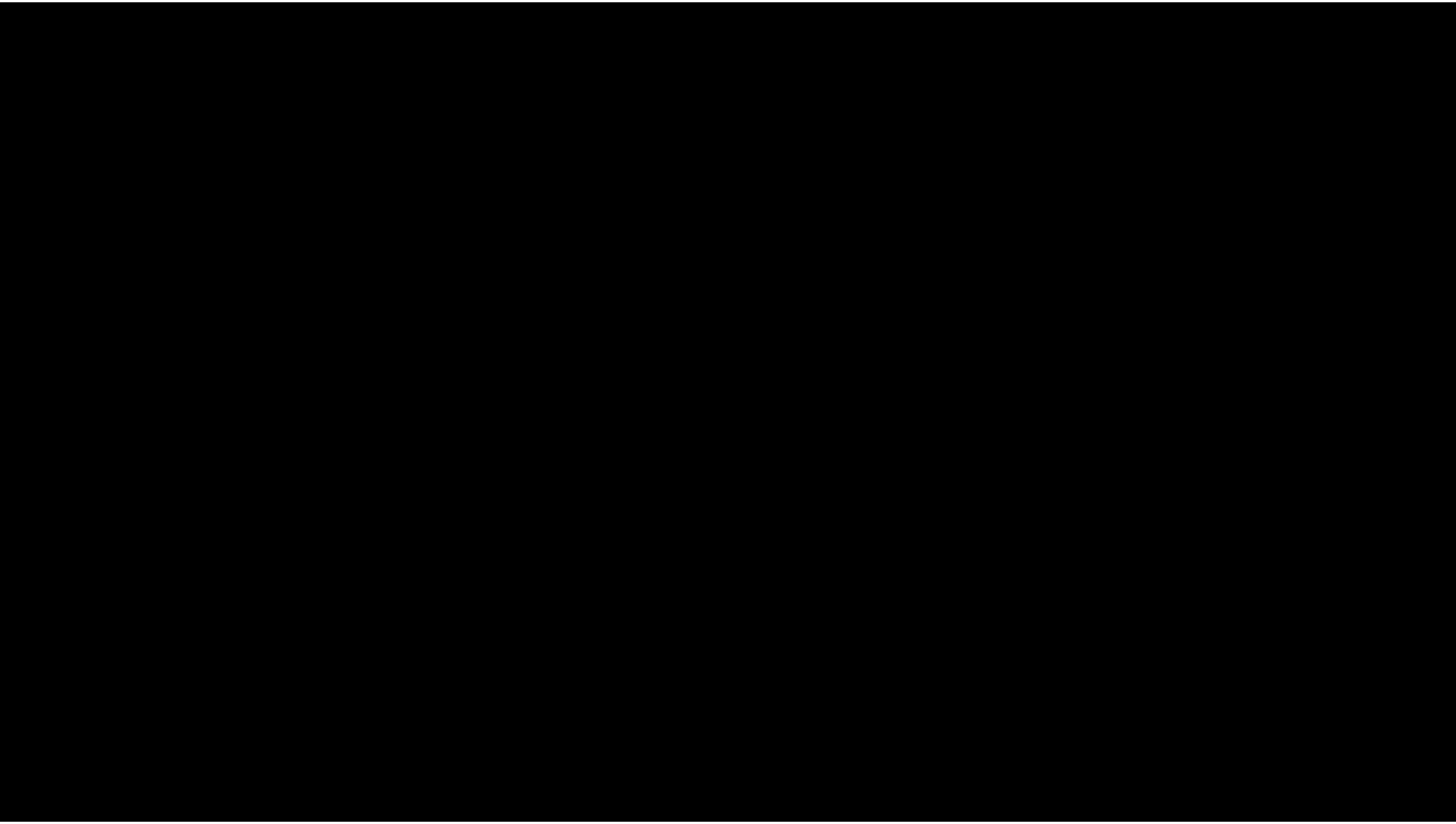
- A special case of composite materials
- Composition and structure of the constituent materials can gradually change



Functionally Graded Materials (FGMs)



Uses: Jumping Robots



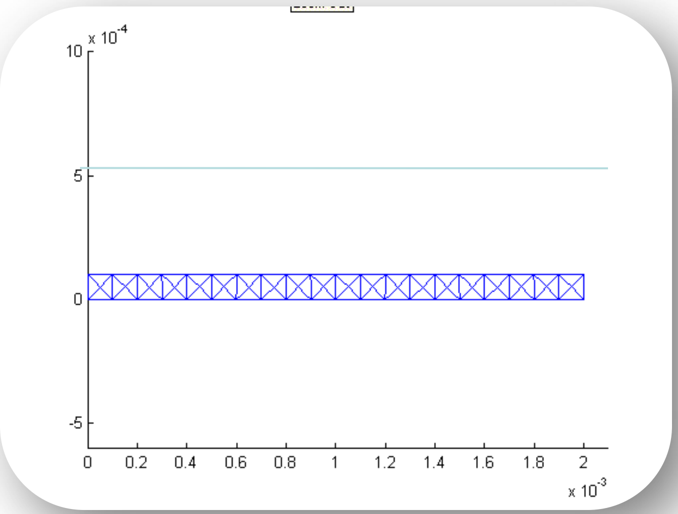
https://www.youtube.com/watch?v=JhdosqStbO4

FGM Origin

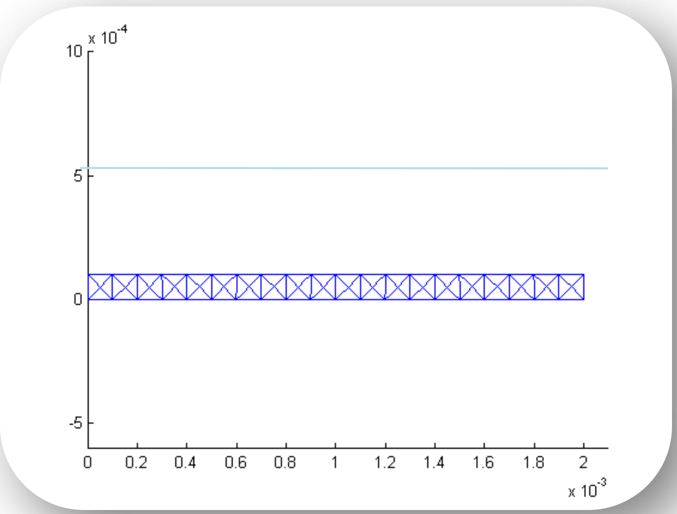
- The “first” FGM developed in Japan in 1984-85
- Many FGM materials have existed for decades
- Some FGM also occur naturally
  - Bones and teeth
  - Seashells

FGM Motivation

- FGMs allow better customization and tailoring of materials for specific tasks



Stiffer at clamped end

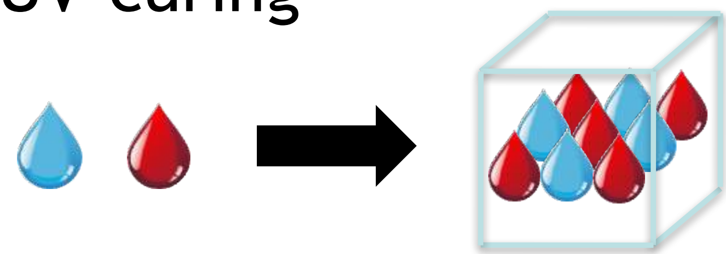


Softer at clamped end

More variety in material selection for engineering design

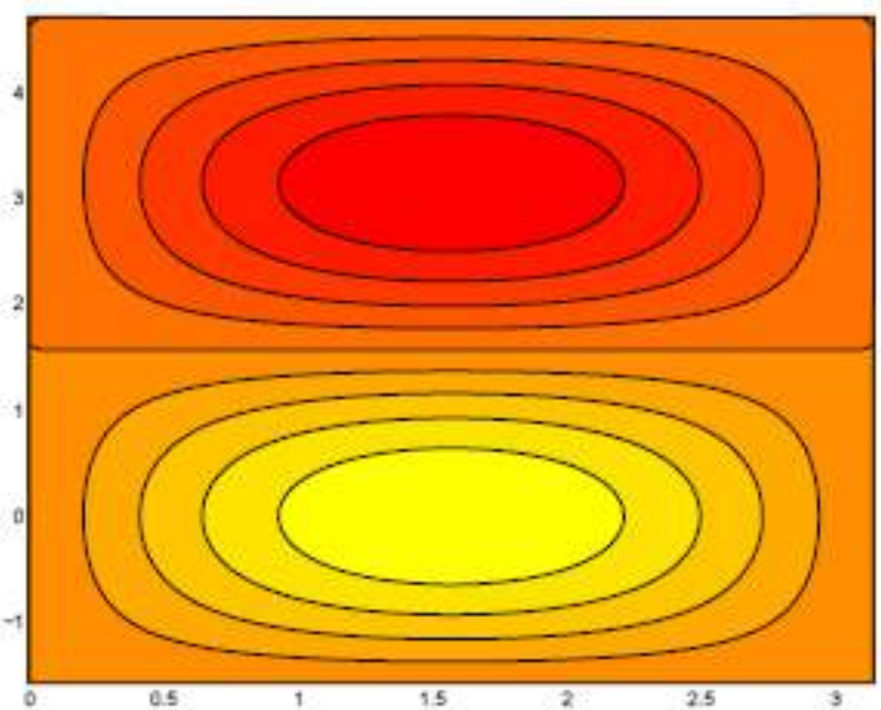
3D Printing FGMs

- FGMs can be printed using inkjet-based 3D printers
  - Mix before UV-curing

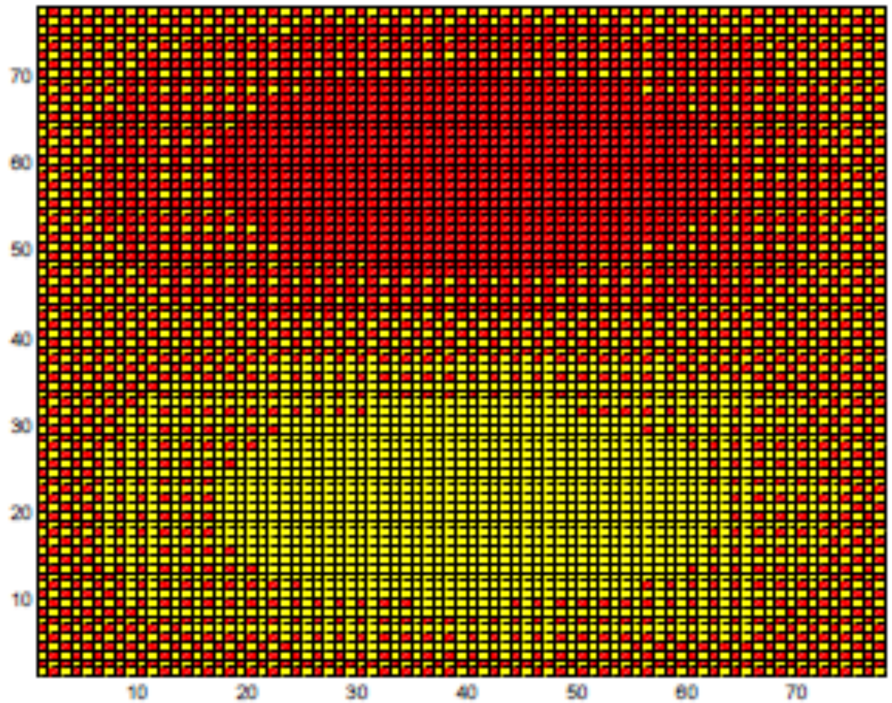


3D Printing FGMs

- FGMs can be printed using inkjet-based 3D printers
  - Input volume is dithered

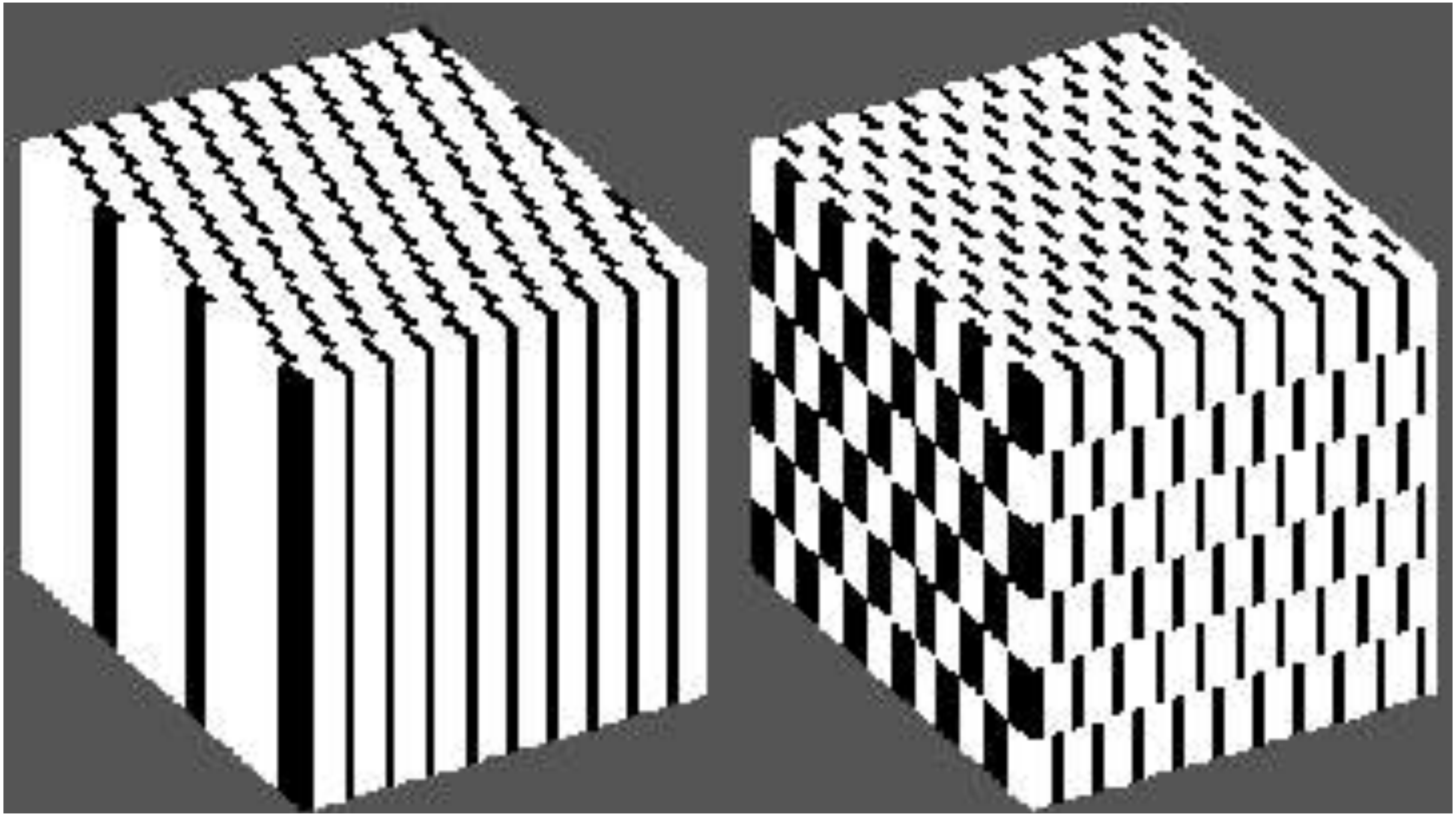


Input Volume



Halftoned Volume

Halftoning in 3D

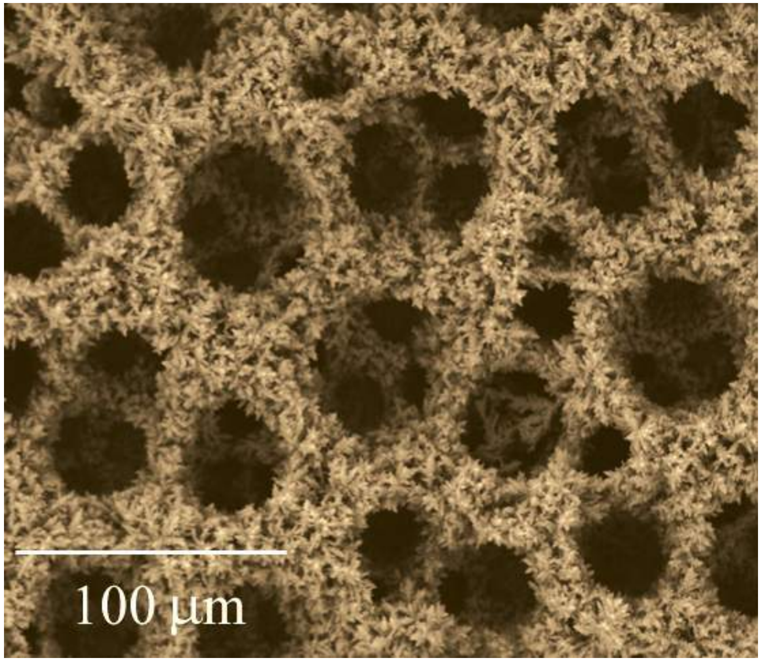


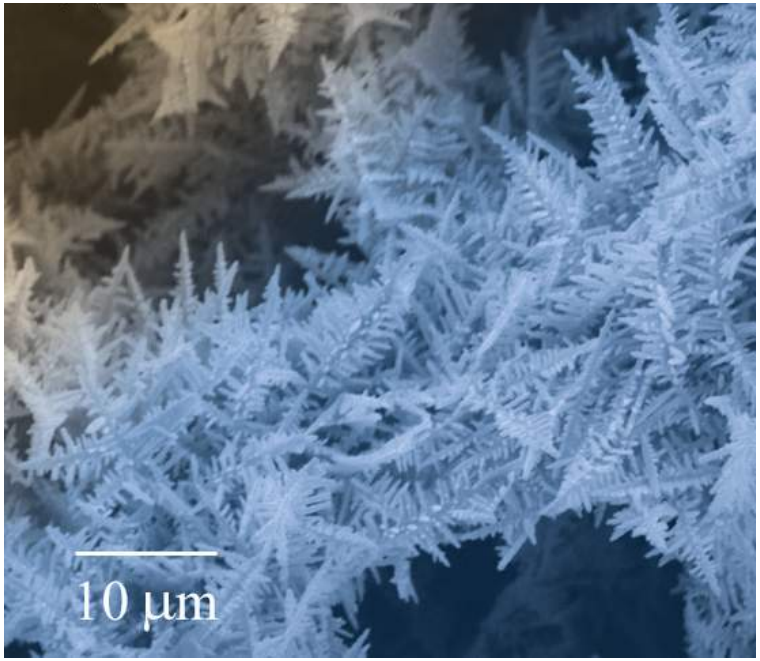
Applications: Aerospace

- Ceramic-metal FGMs are particularly suited for thermal barriers in space vehicles
  - Metal side can be bolted onto the airframe rather than bonded as are the ceramic tiles used in the Orbiter

Applications: Fuel Cells

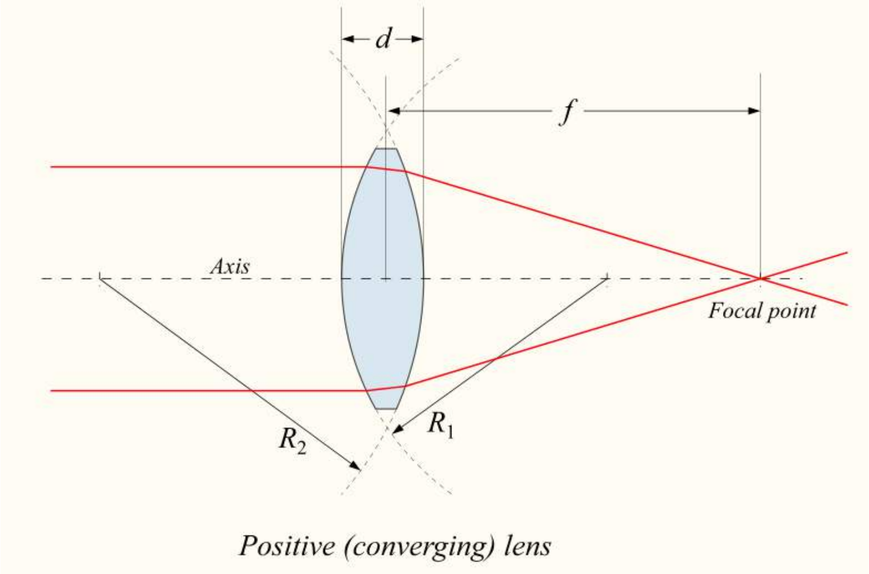
- Creating a porosity gradient in the electrodes
  - the efficiency of the reaction can be maximized



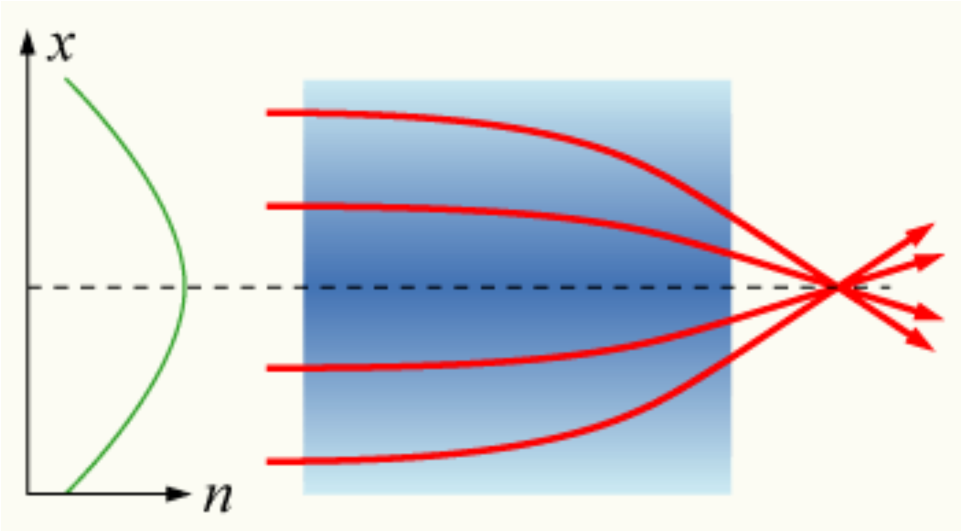


Applications: GRIN Optics

- GRIN = Graded Refractive Index



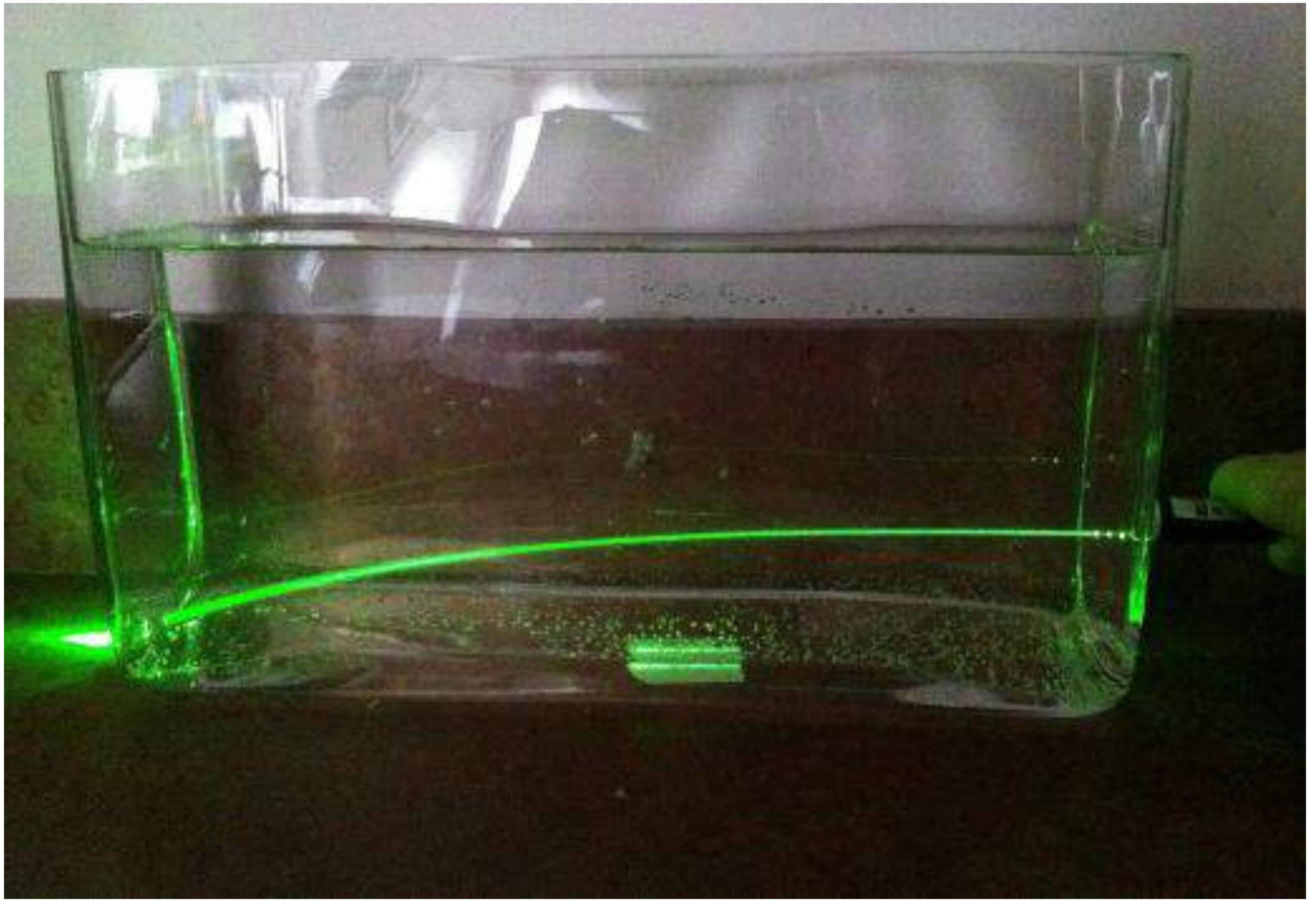
Traditional Lens



GRIN Lens

Applications: GRIN Optics

- GRIN = Graded Refractive Index



http://www.laurawaller.com/opticsfun/sugarGRINlens.htm

FGMs: Advantages and Challenges

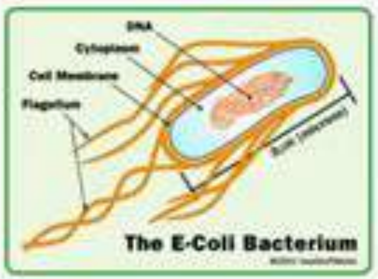
- Advantages
  - Multiple functions
    - benefits of different materials e.g., ceramics and metals
  - Control of deformation, dynamic response, wear, corrosion
  - Design for different complex environments
  - Removing stress concentrations
- Challenges:
  - Mass production
  - Quality control
  - Cost

Biomimetic/Bio-inspired Materials

Plant: the energy reservoir
www.gardeningoncloud9.com

Spider silk: tough materials
www.tehrantimes.com

Bird: the natural airplane
http://www.guidetobelize.info



Flagellum: the mechanical motor
http://creationrevolution.co

Lotus leaf: hydrophobic surface
http://sustainabledesignupdate.com

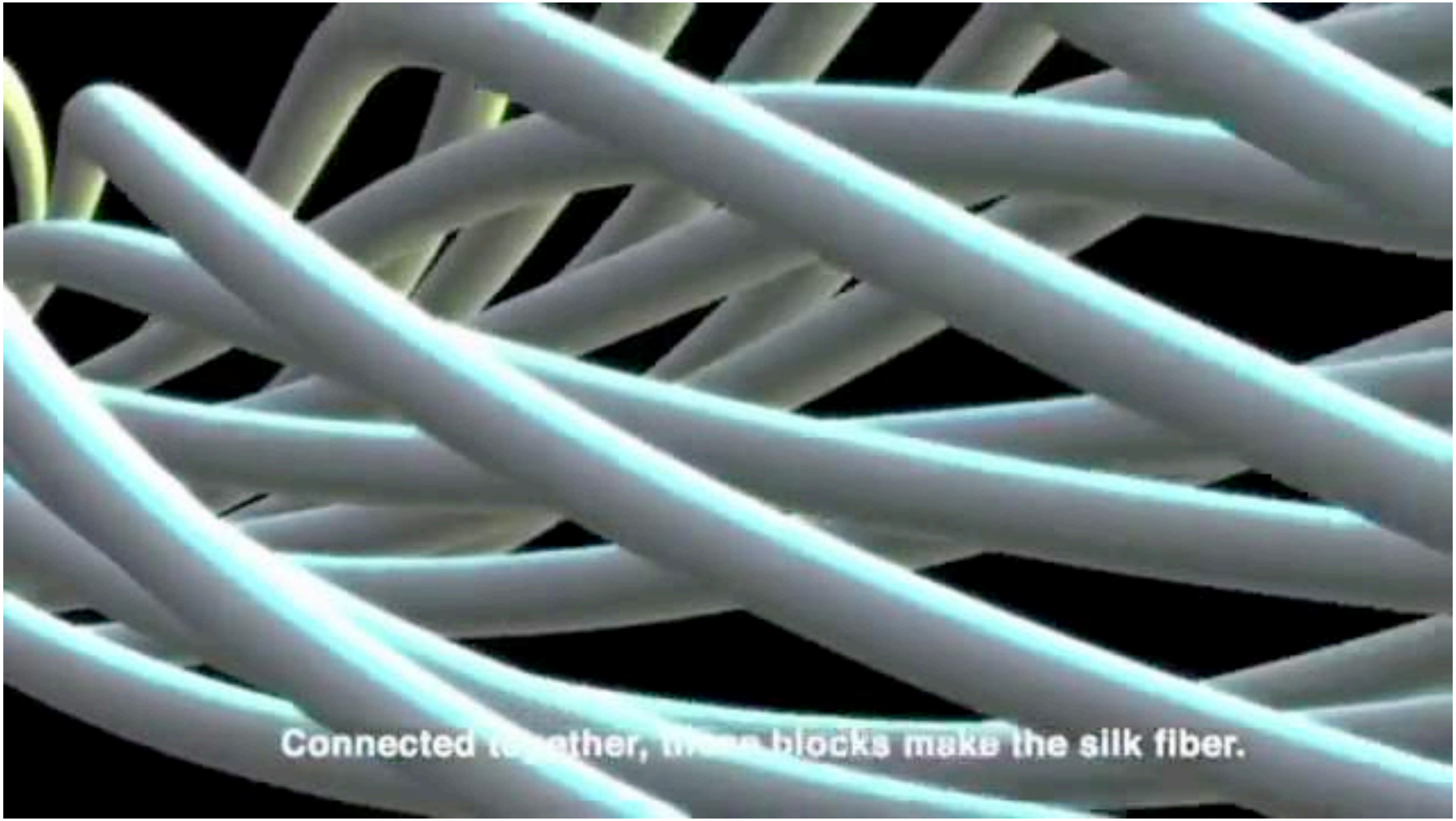
Eye: nature's best camera
www.photoshopstar.com

Brain: the super computer
www.healthguide.howstuffworks.com

Termites mound the natural cooler
www.animals.howstuffworks.com

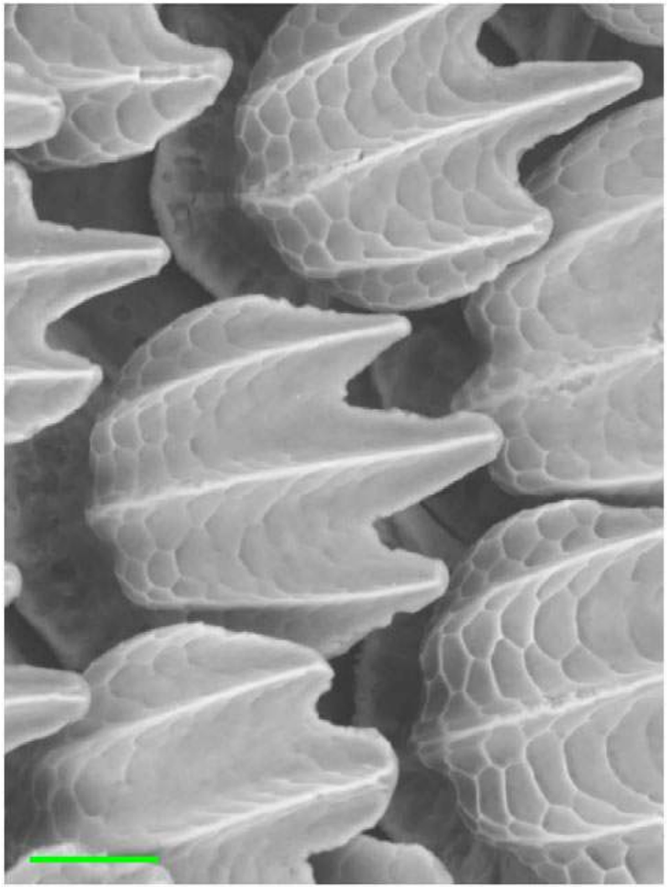
Dolphins the best ship
www.ashnature.org

Biomimetic Materials: Spider Silk



Biomimetic Materials: Swimming Faster

- The structure of shark skin reduces drag in the water leading to more energy efficient locomotion



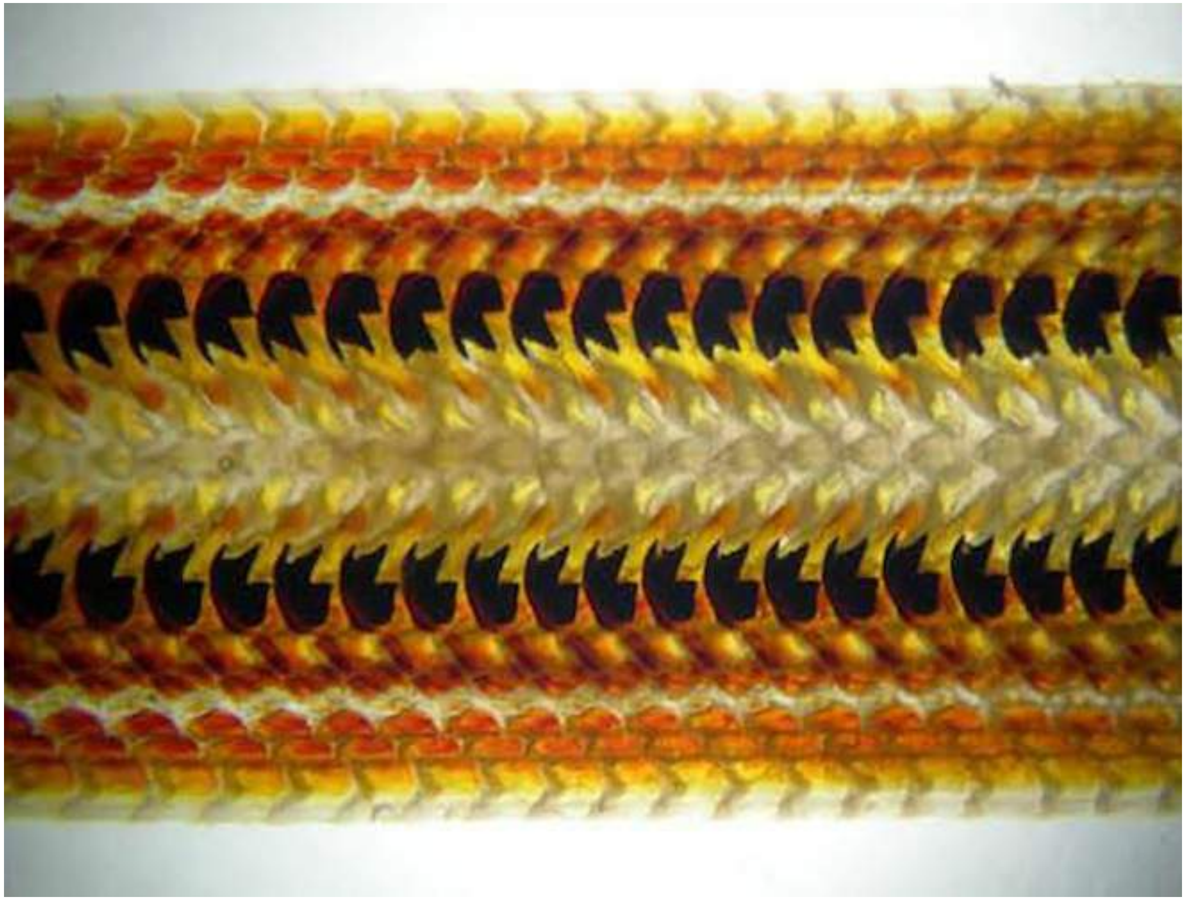
Denticles in shark skin

Biomimetic Materials: Swimming Faster

- This was big news for the 2008 Summer Olympics

Biomimetic Materials: Strong Materials

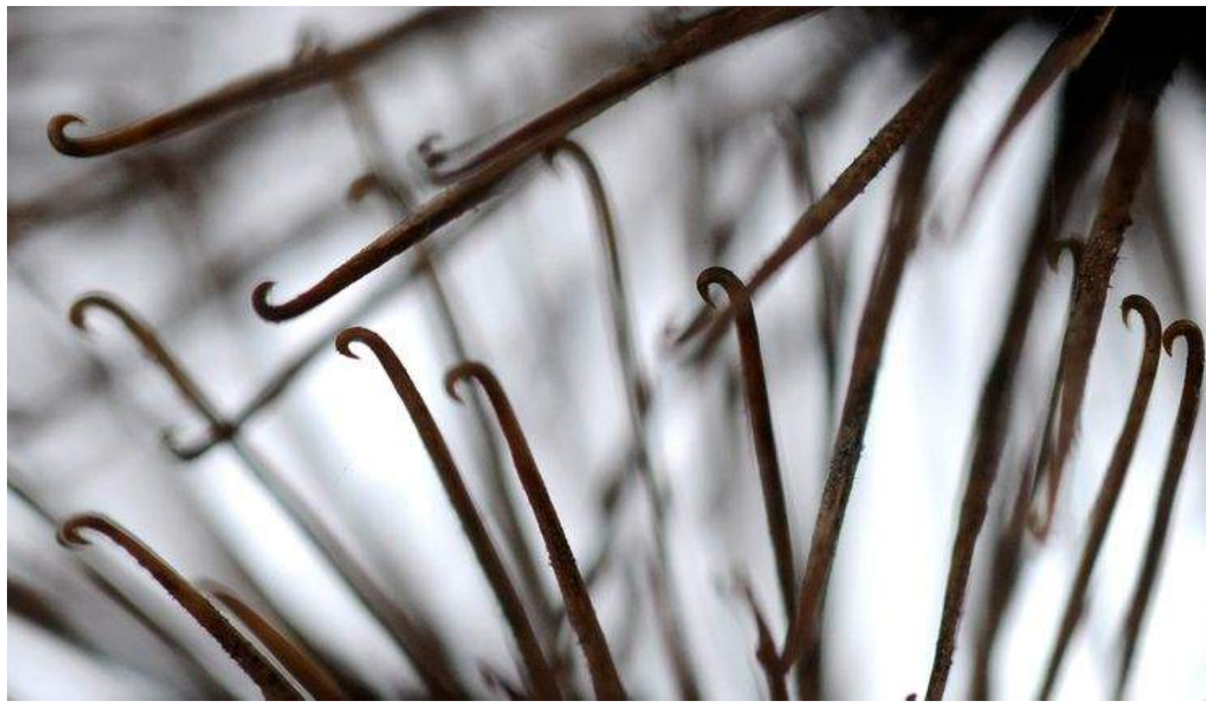
- Mollusc Teeth



yle Gordon, Joester group, Materials Science and Engineering, Northwestern University

Biomimetic Materials: Velcro

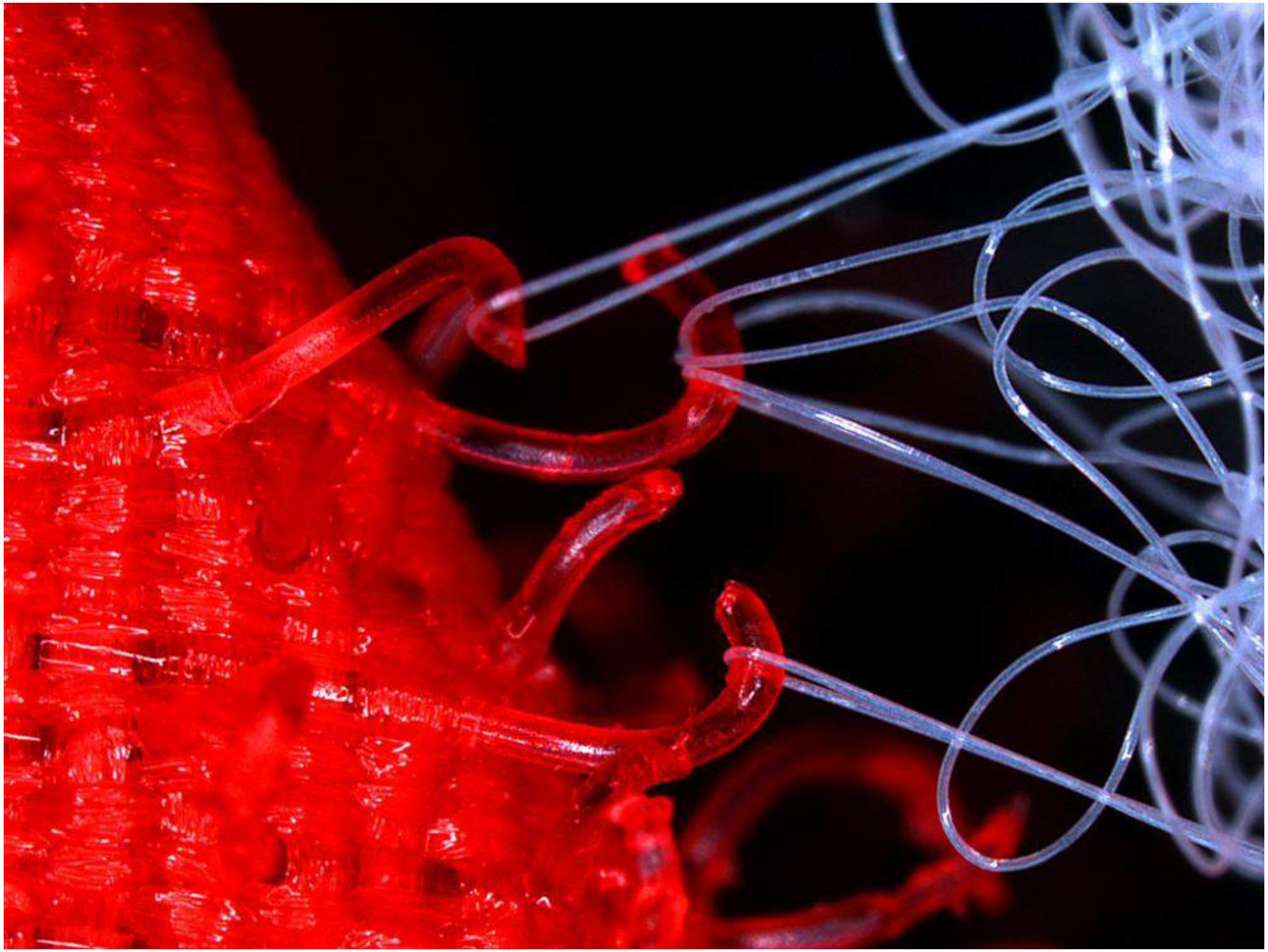
- Inspired by Burrs



Biomimetic Materials: Velcro

- Detachable adhesive

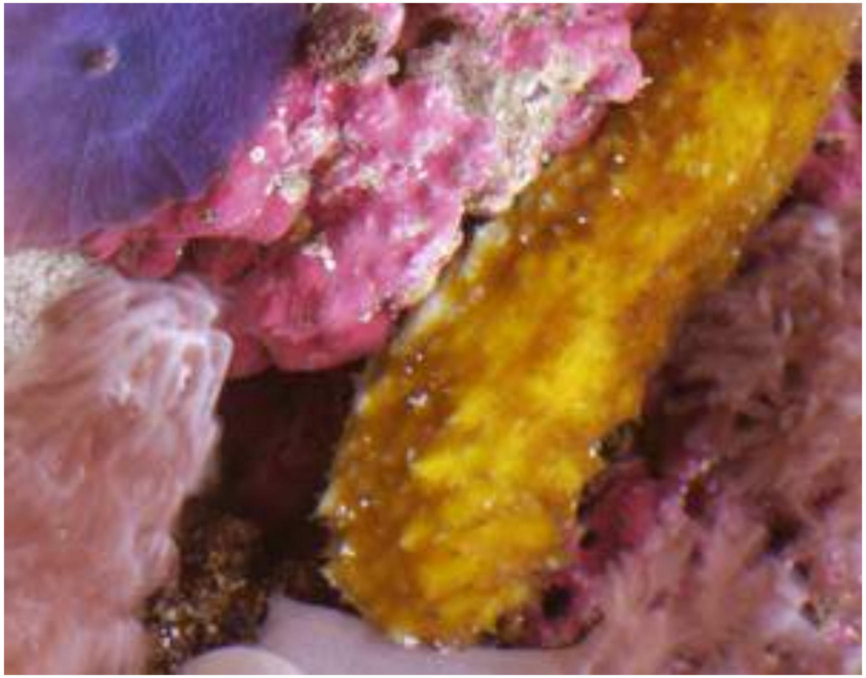
Biomimetic Materials: Velcro



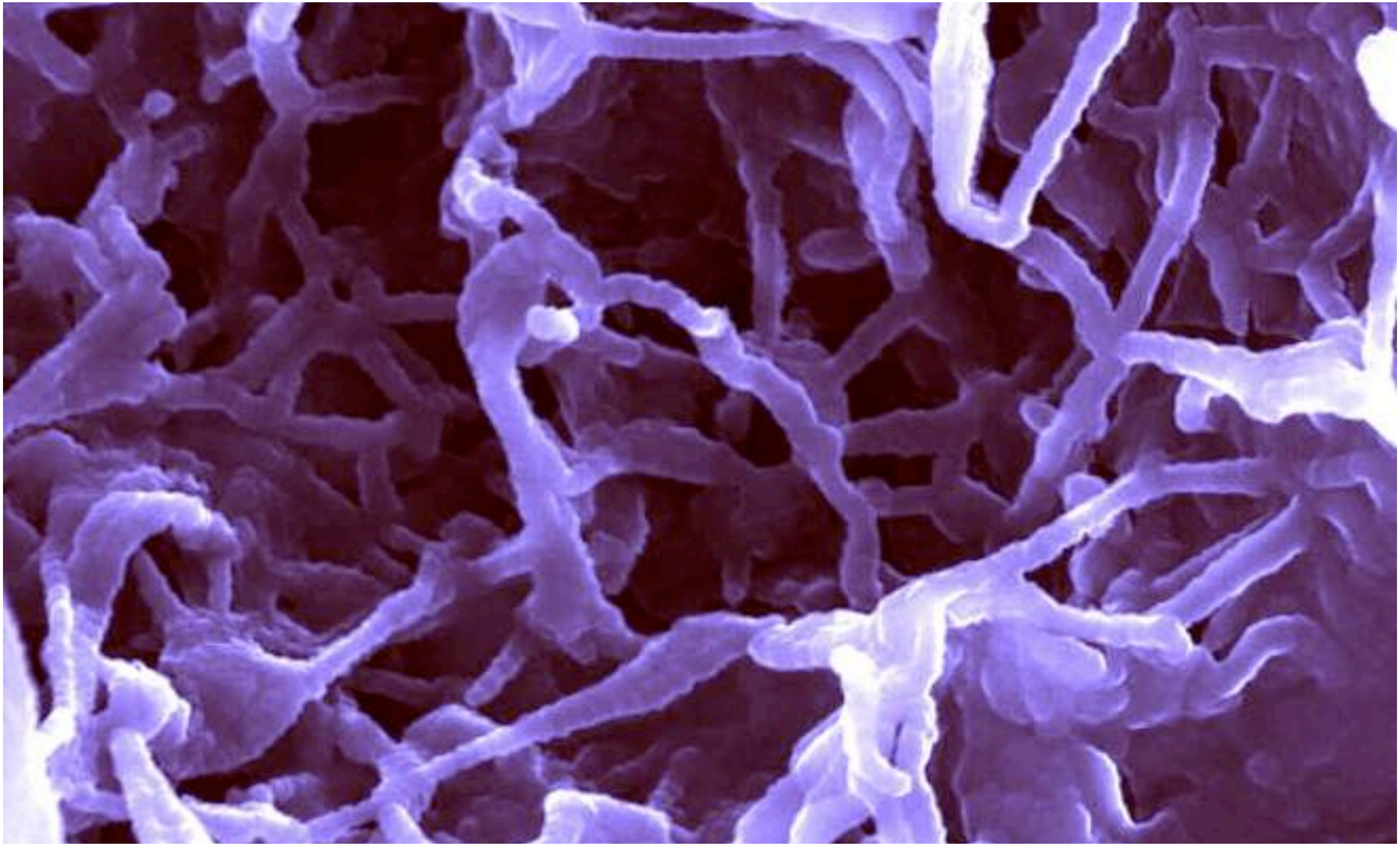
Biomimetic Materials: Velcro in Action

Materials with Varying Stiffness

- Inspired by sea cucumbers which can alter the stiffness of their dermis (outer skin layer)
- Change the structure of collagen fibers embedded in low stiffness matrix

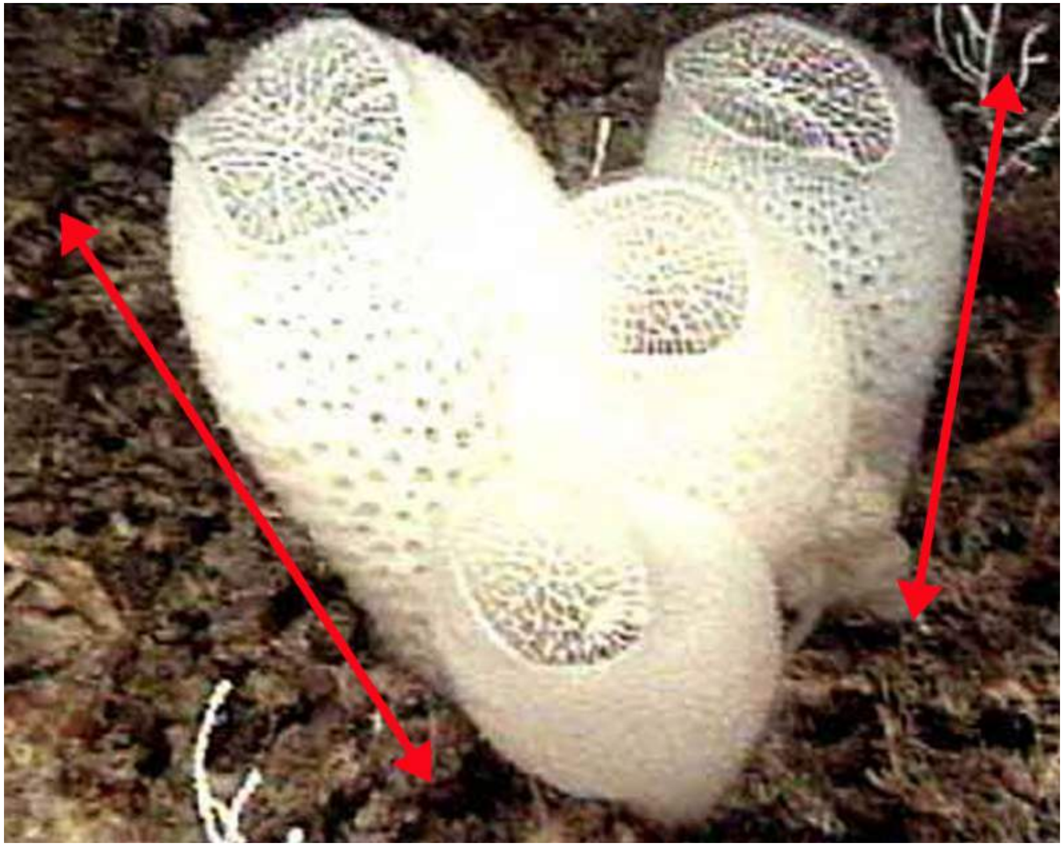


Materials with Varying Stiffness



Transparent Construction Materials

- Inspired by skeletons of undersea sponges made of glass and Venus Flower Basket



Transparent Construction Materials

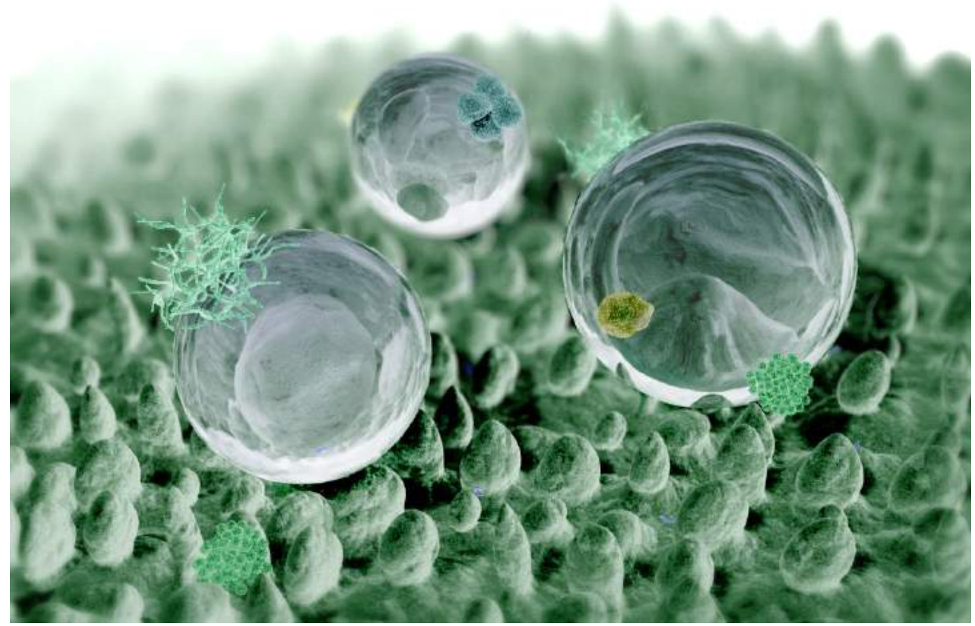
- Glass sponge

Mussel Superglue

- Mussels can stay attached to rocks in very strong tides
- They emit a slime that forms a thread-like, ultra-strong, water resistant adhesive on contact with water

Hydrophobic Materials: Lotus Leaf

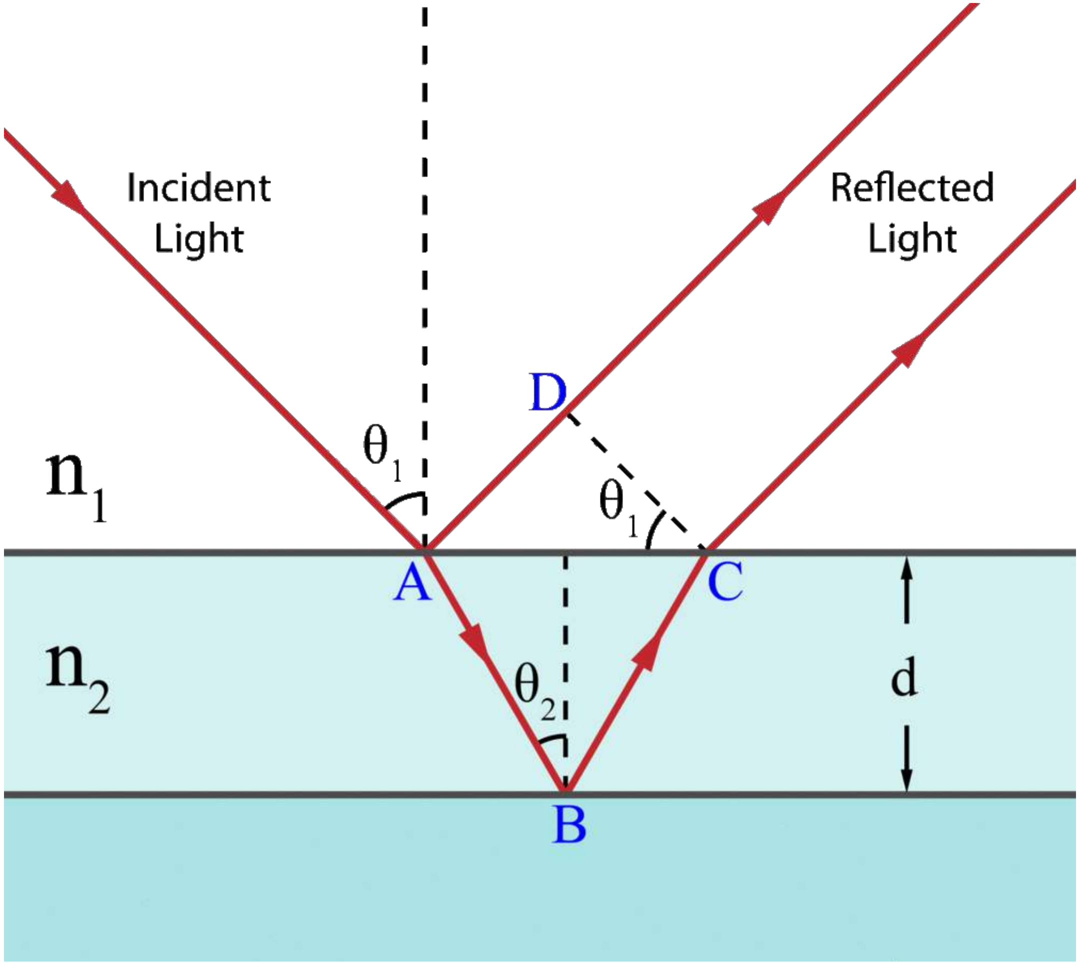
- Lotus leaves have a bumpy structure that causes water to bead and roll off

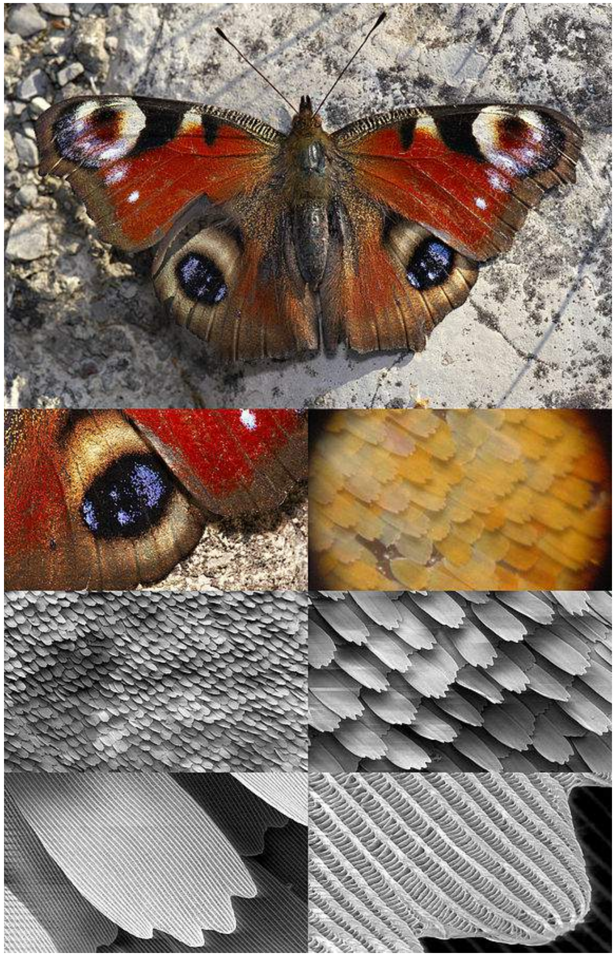


http://en.wikipedia.org/wiki/Lotus_effect

Hydrophobic Materials

Structural Coloration





Materials with Structural Hierarchy

- Both man-made and natural
- Structure at more than one scale
- Structural hierarchy can result in improved mechanical properties
- Examples:
  - Bones
  - Livers