Report of Zhang Yitang

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Let P be the set of all prime numbers.

Definition 0.1 *$\{h_i, i = 1, \cdots, k\} \subset \mathbb{N}$ is admissible iff*

$$\forall p \in P \exists n \in \mathbb{N}, (P(n) = \prod_i (h_i + n), p) = 1 \ (p \not| h_i + n, i = 1, \cdots, k).$$

Example 0.1 *$\{0, 2\}, \{0, 2, 6\}$ is admissible. $\{h, h+1\}, \{h, h+2, h+4\}$ is not admissible.*

Lemma 0.1 *$\{h_i, i = 1, \cdots k\} \subset \mathbb{N}$ is admissible if*

$$\exists^\infty n, h_i + n \in P, i = 1, \cdots, k.$$

Conjecture 0.1 (Hardy-Littlewood Conjecture) *If $\{h_i, i = 1, \cdots k\} \subset \mathbb{N}$ is admissible, then*

$$\exists^\infty n, h_i + n \in P, i = 1, \cdots, k.$$

Theorem 0.1 *If $\{h_i, i = 1, \cdots k\} \subset \mathbb{N}$ is admissible, then*

$$\mathrm{lrg} k \exists^\infty n \exists^2 i, h_i + n \in P, i = 1, \cdots, k.$$

Proof. Use Lemma 0.2. □

Definition 0.2 *Define an arithmetic function* $\theta(x) = \begin{cases} \ln x, & x \in P, \\ 0, & x \notin P. \end{cases}$

Lemma 0.2 *For large x,*

$$\exists n \sim x (x \leq n < 2x), \sum_i \theta(n + h_i) > \ln 3x.$$

Proof. Let $f(n) \geq 0$. $S_1 = \sum_{n \sim x} f(n), S_1 = \sum_{n \sim x} (\sum_i \theta(n + h_i)) f(n)$. Only need prove

$$S_2 - S_1 \ln 3x > 0.$$

Let $D = x^c, c \in (0, \frac{1}{2})$. Choose $f(n) = \lambda(n)^2$, where

$$\lambda(n) = \sum_{d|P(n)} \mu(d)g(d), g(d) = \begin{cases} \frac{1}{(k+l)!}(\ln \frac{D}{d})^{k+l}, & d < D, \\ 0, & d \geq D, \end{cases} l > 0.$$

GPY gave $S_1 \sim T_1 x, S_2 \sim T_2 x \ln x + O(E)$,

$$T_1 = \sum_{d_0 d_1 d_2} \frac{\mu(d_1 d_2) g(d_0 d_1 d_2)}{d_0 d_1 d_2} g(d_0 d_1) g(d_0 d_2), T_2 = \cdots .$$

□