Binary Arithmetic

Addition

Base Cases:

0	0	1	1
+ 0	+ 1	+ 0	+ 1
0	1	1	10

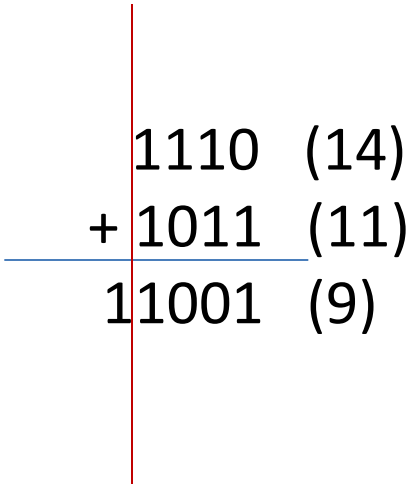
Addition

```
  1011   (11)
+ 0010   (2)
------
  1101   (13)
```

```
  1001   (9)
+ 0010   (2)
------
  1011   (11)
```

- Number of bits in input must be the same
- Number of bits in output will be the same as input.

Overflow



Data does not fit in the space provided due to finite storage

Exercises

$$\begin{array}{r} 0011 \\ +\ \ 10 \\ \hline \end{array}$$

$$\begin{array}{r} 1101 \\ +\ 0110 \\ \hline \end{array}$$

$$\begin{array}{r} 1011 \\ +\ 1101 \\ \hline \end{array}$$

Subtraction

Base Cases:

0	1	1	0
- 0	- 1	- 0	- 1
0	0	1	1

Borrowing

```
  1000
- 0001
------
```

```
  10001000
- 00001001
---------
```

Exercises

$$\begin{array}{r} 1011 \\ -\ 0010 \\ \hline \end{array}$$

$$\begin{array}{r} 1101 \\ -\ 0110 \\ \hline \end{array}$$

$$\begin{array}{r} 1000 \\ -\ 0011 \\ \hline \end{array}$$

Multiplication

Base Cases:

0	1	1	0
x 0	x 1	x 0	x 1
0	1	0	0

Multiplication

```
      1011
x     0011
------------
      1011
     1011
    0000
   0000
------------
  00100001
```

Multiplication

Output size is double the size of the input.

If input size is 4 bits, output size is potentially 8 bits.

Signed Addition

	unsigned	2's complement
1011	(11)	(-5)
+ 0010	(2)	(2)
1101	(13)	(-3)

- Process works for both Unsigned and Signed addition!

Signed Subtraction

$$A - B = A + (-B)$$

Change to an Addition Problem!

Exercises

$$\begin{array}{r} 1011 \\ -\ 0010 \\ \hline \end{array}$$

$$\begin{array}{r} 1101 \\ -\ 0110 \\ \hline \end{array}$$

$$\begin{array}{r} 1000 \\ -\ 0011 \\ \hline \end{array}$$

Overflow?

$$\begin{array}{r} 0110 \\ +\ 0010 \\ \hline \end{array}$$

$$\begin{array}{r} 1011 \\ +\ 1010 \\ \hline \end{array}$$

$$\begin{array}{r} 1101 \\ +\ 1110 \\ \hline \end{array}$$

$$\begin{array}{r} 1111 \\ +\ 0001 \\ \hline \end{array}$$

$$\begin{array}{r} 0101 \\ +\ 0010 \\ \hline \end{array}$$

$$\begin{array}{r} 0111 \\ +\ 1000 \\ \hline \end{array}$$

Signed Multiplication

1011
x 0011

Sign Extension

0000 0101 = 0101

How to extend a negative number?

Signed Multiplication using Sign Extension

1111 1011
x 0000 0011

Signed Multiplication using positive absolute value

1) Convert all inputs to POSITIVE
2) Multiply
3) Convert back to NEGATIVE if necessary

$$\begin{array}{r} 1011 \\ \times\ 0011 \\ \hline \end{array}$$