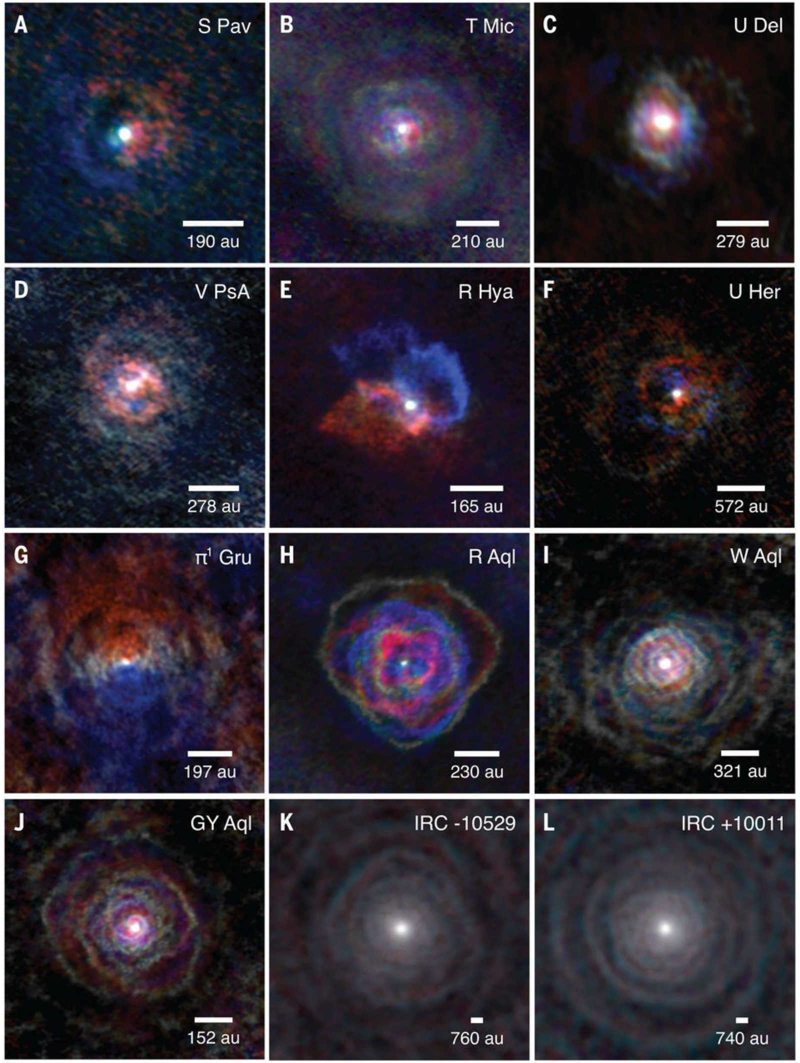
Probabilistic **3D reconstruction** of the **circumstellar environments** of evolved stars

Frederik De Ceuster (KU Leuven, FWO Fellow)

in collaboration with

A. Coenegrachts, J. Malfait, T. Ceulemans, M. Esseldeurs, S. Maes,
T. Konings, T. Danilovich, J. Cockayne, L. Decin, J. Yates, (**You?**)

High-resolution observations revealed **complex morphologies**



ATOMIUM: ALMA Large Program, Decin et al. (2020)

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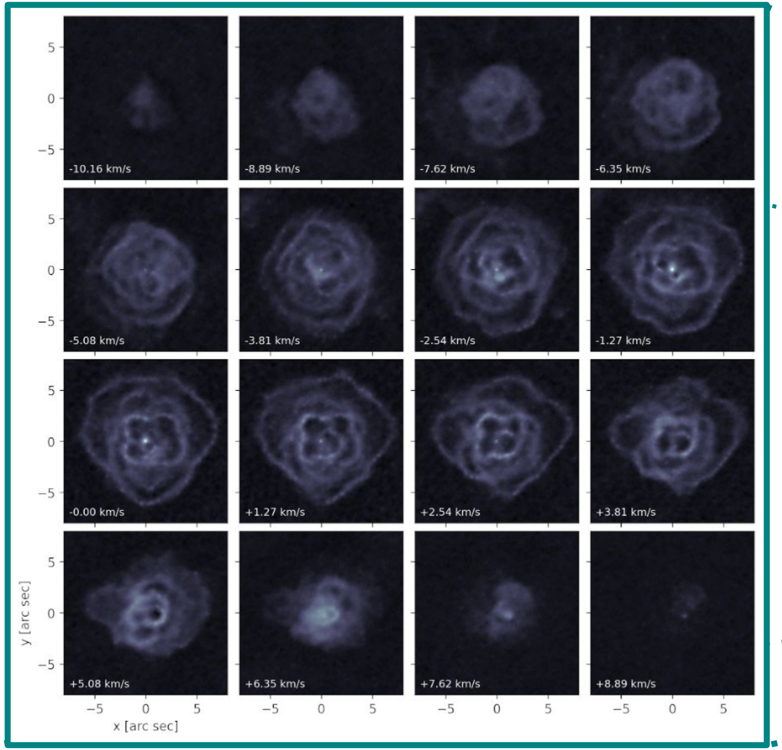
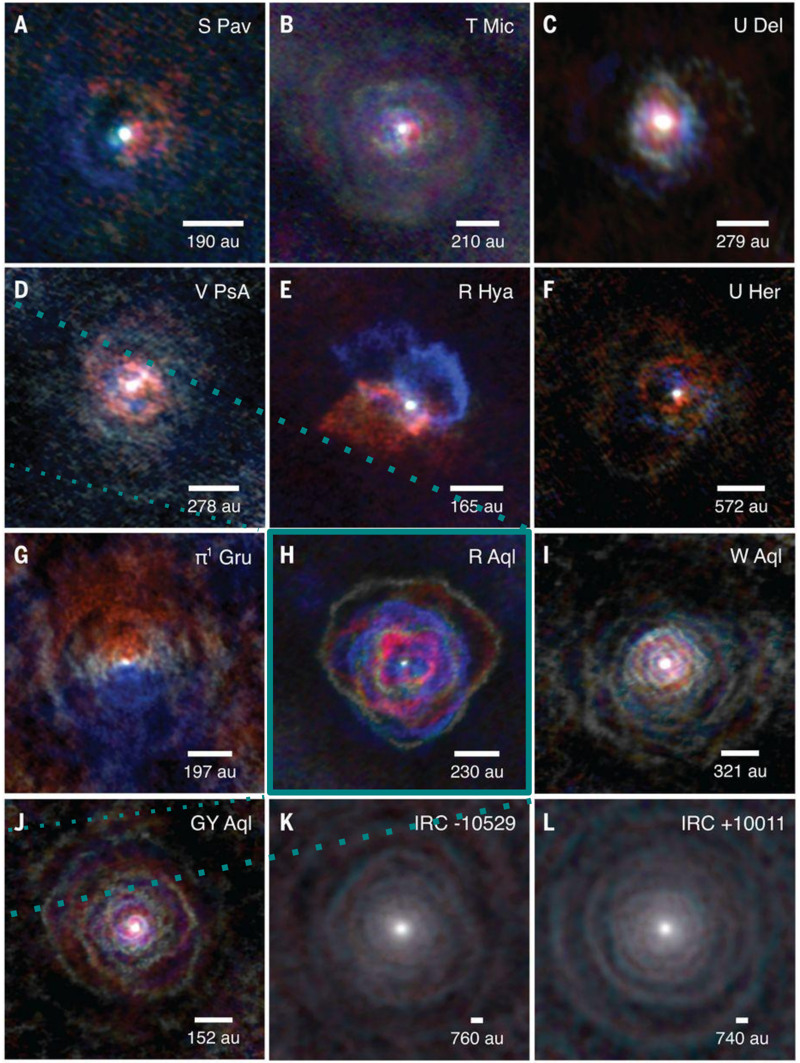
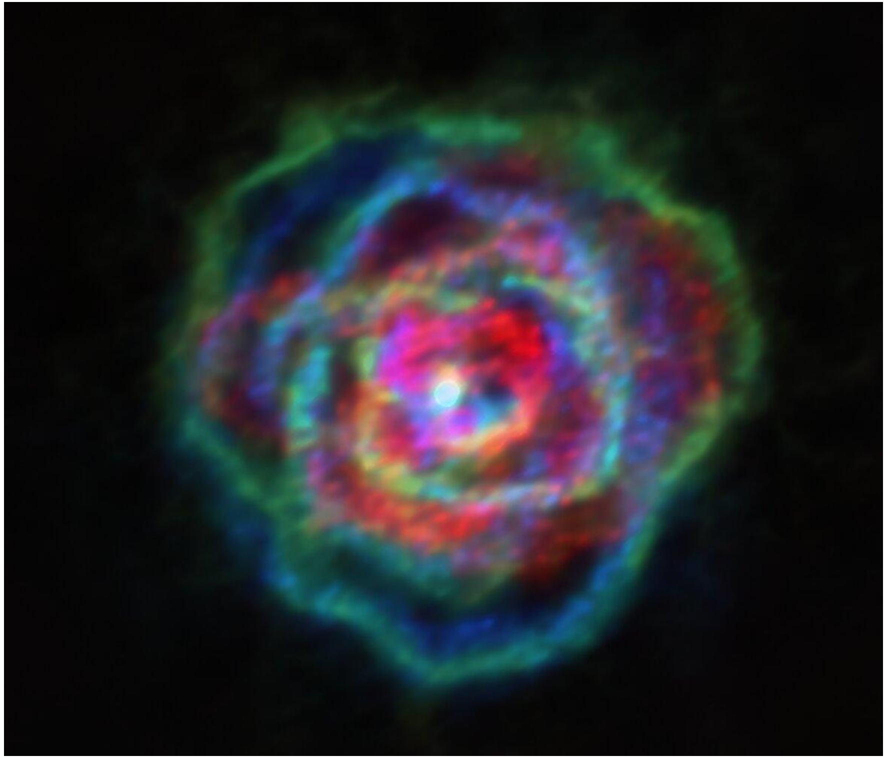




ATOMIUM: ALMA Large Program, Decin et al. (2020)

But, (forward) modelling these observations is **challenging**...



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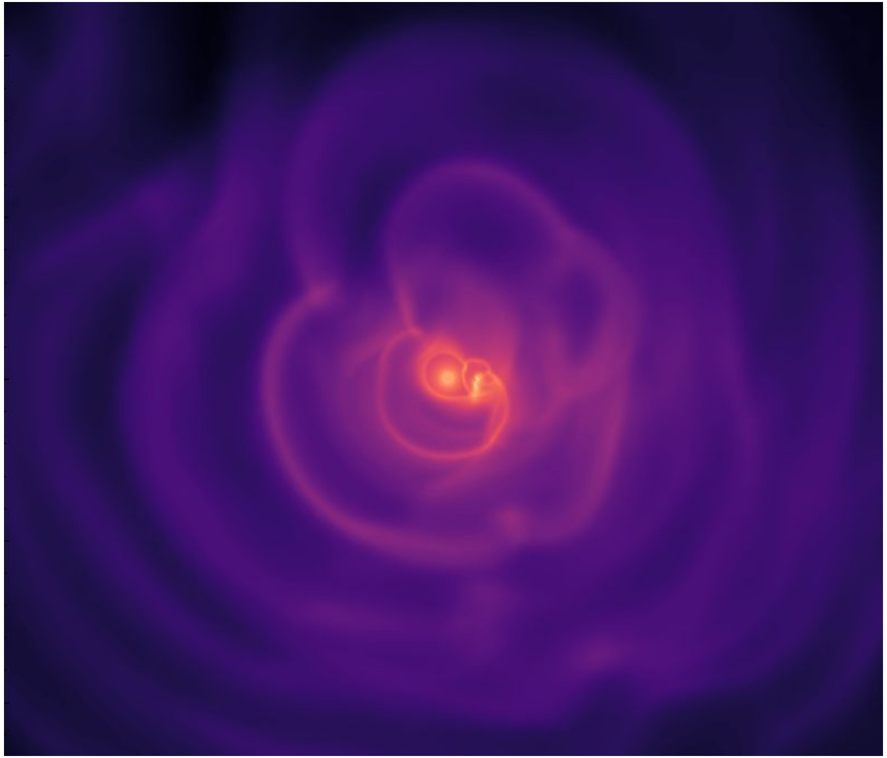
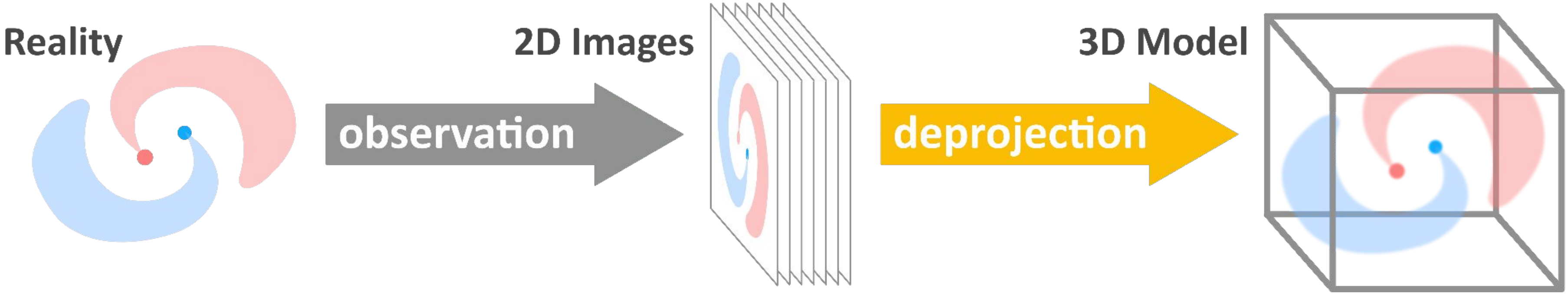


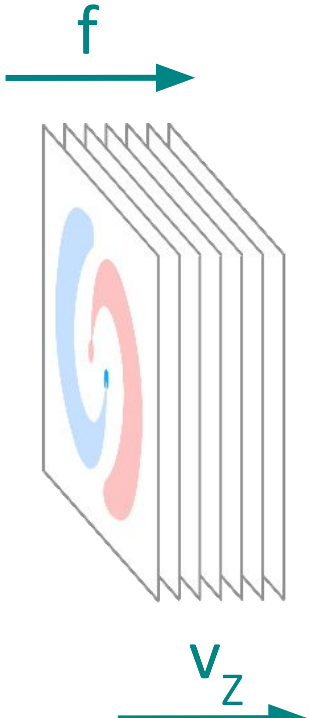
Malfait et al. (2021), Maes et al. (2021), Siess et al. (2022), Esseldeurs et al. (2023), ...

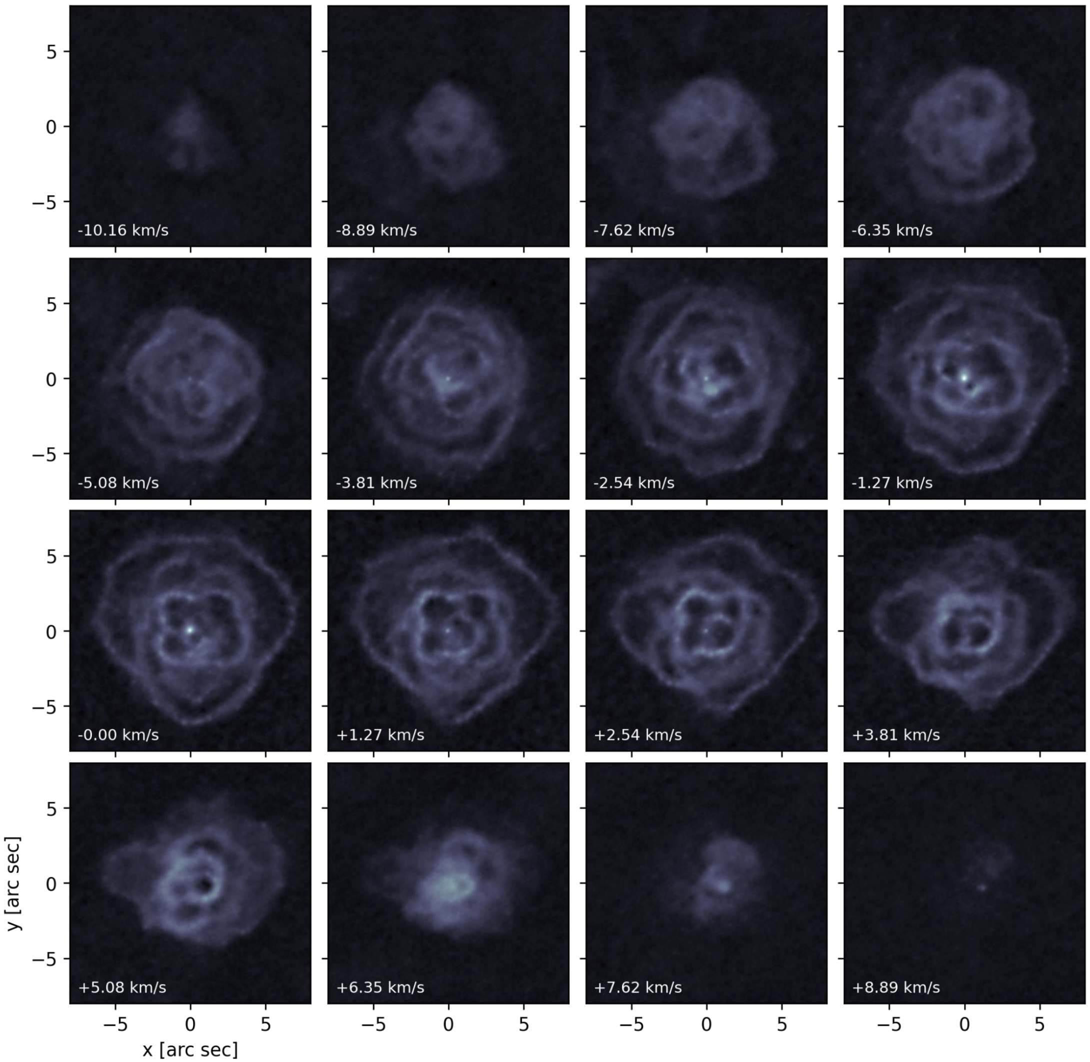
Possible solution: start from the observations, i.e.

inverse modelling / 3D reconstruction / deprojection



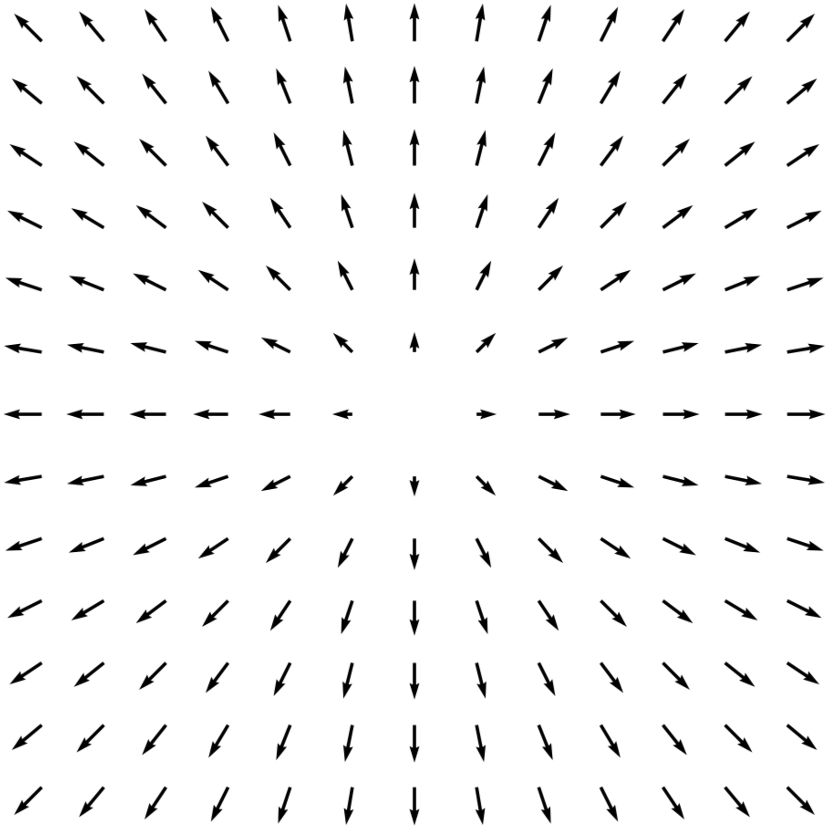
Spectral line **observations**





Deprojection as **we** know it

Assume a spherically symmetric **velocity** field



Guélin et al. (2018)
Montargès et al. (2019)
Coenegrachts et al. (2023)

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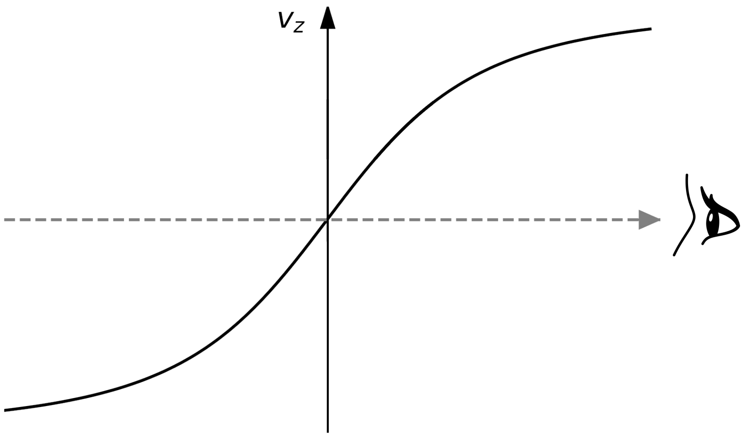
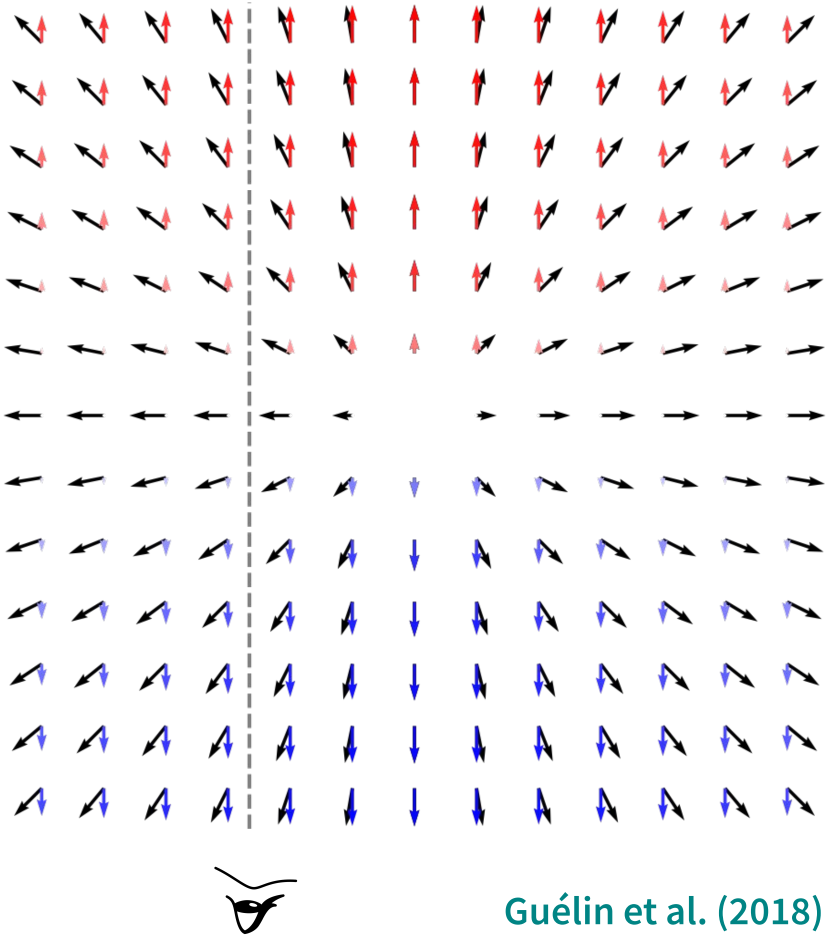


➔ **Monotonic** projection along the line-of-sight

➔ Each velocity / frequency corresponds to a **unique** position along the line-of-sight



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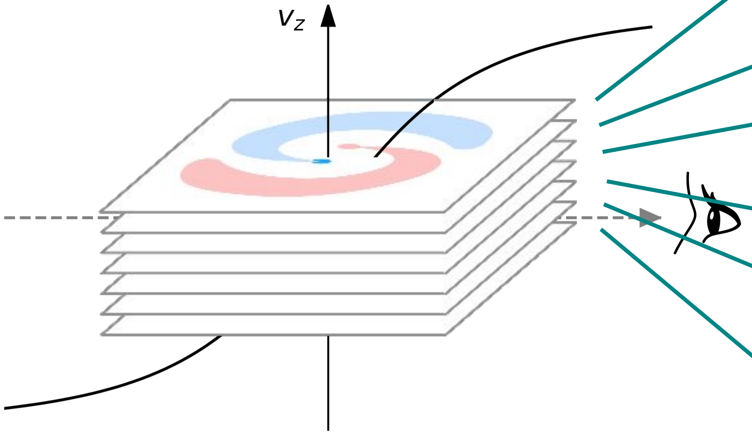
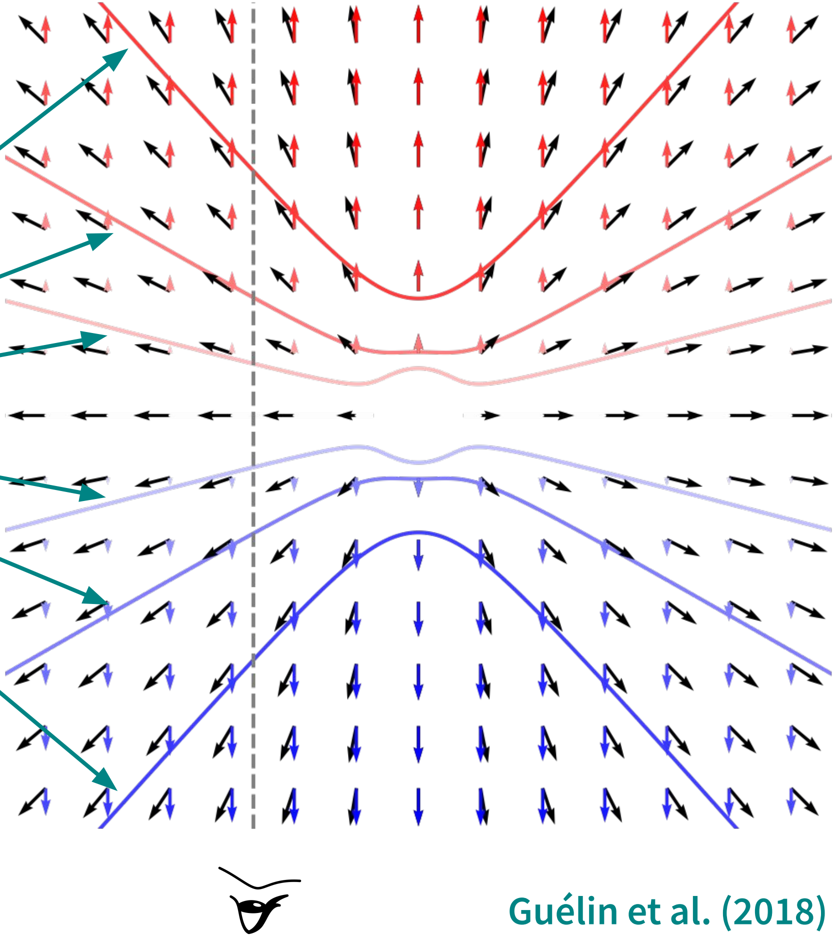




- ➔ **Monotonic** projection along the line-of-sight
- ➔ Each velocity / frequency corresponds to a **unique** position along the line-of-sight
- ➔ Each channel map corresponds to a **unique** contour of constant velocity

Guélin et al. (2018)
Montargès et al. (2019)
Coenegrachts et al. (2023)

Nog invoegen!

NaCl around **IK Tauri**

Deprojection as **we** know it

Issues:

- **False assumption** of assigning an entire channel map to a single contour
- Difficult to **incorporate more physics** or **chemistry**
- **Strong assumptions** on the velocity structure
- Difficult to **combine** different observations
- Difficult to deal with **uncertainties**

Deprojection as **others** know it

Drawing inspiration from solar/plasma physics, machine learning, and medical imaging

Bayesian inversion, compressed sensing, …

Asensio Ramos et al. (2007)

Asensio Ramos & de la Cruz Rodríguez (2015)

…

Reviews

del Toro Iniesta & Ruiz Cobo (2016)

de la Cruz Rodríguez & van Noort (2017)

Recent developments

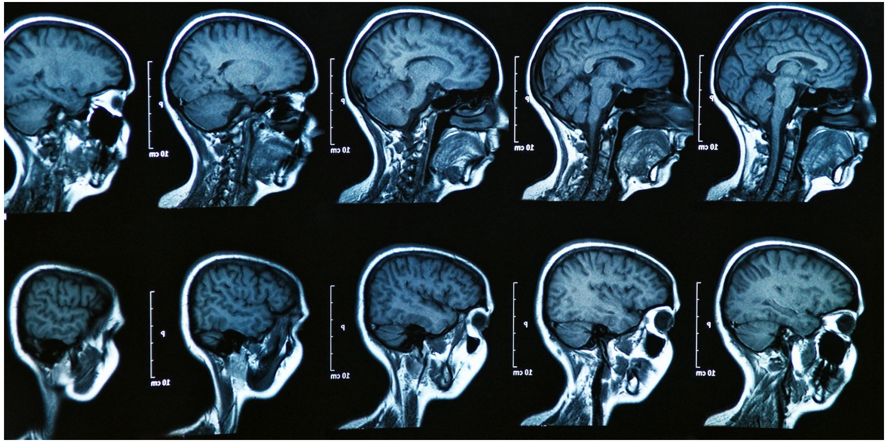
Asensio Ramos et al. (2022)

Díaz Baso et al. (2022)

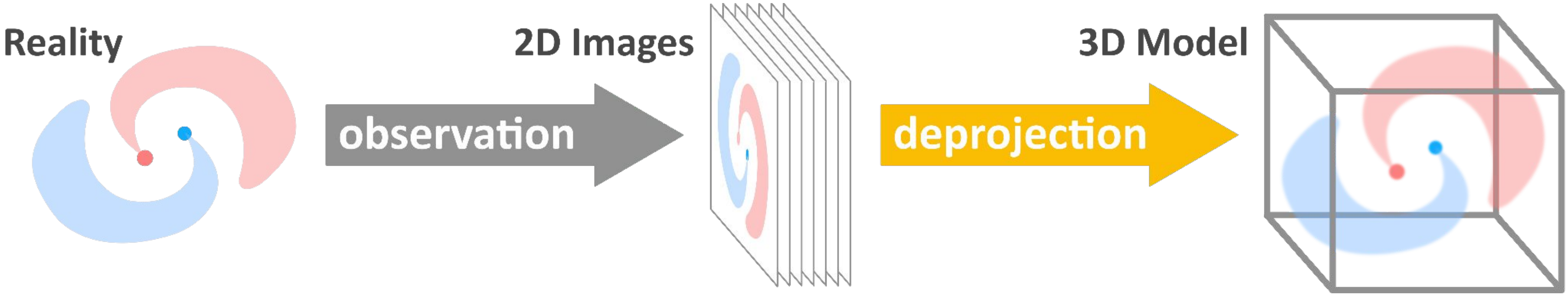
Štepán et al. (2022)

Vicente Arévalo et al. (2022)

Visual deprojection — Balakrishnan et al. (2019)



Probabilistic 3D reconstruction



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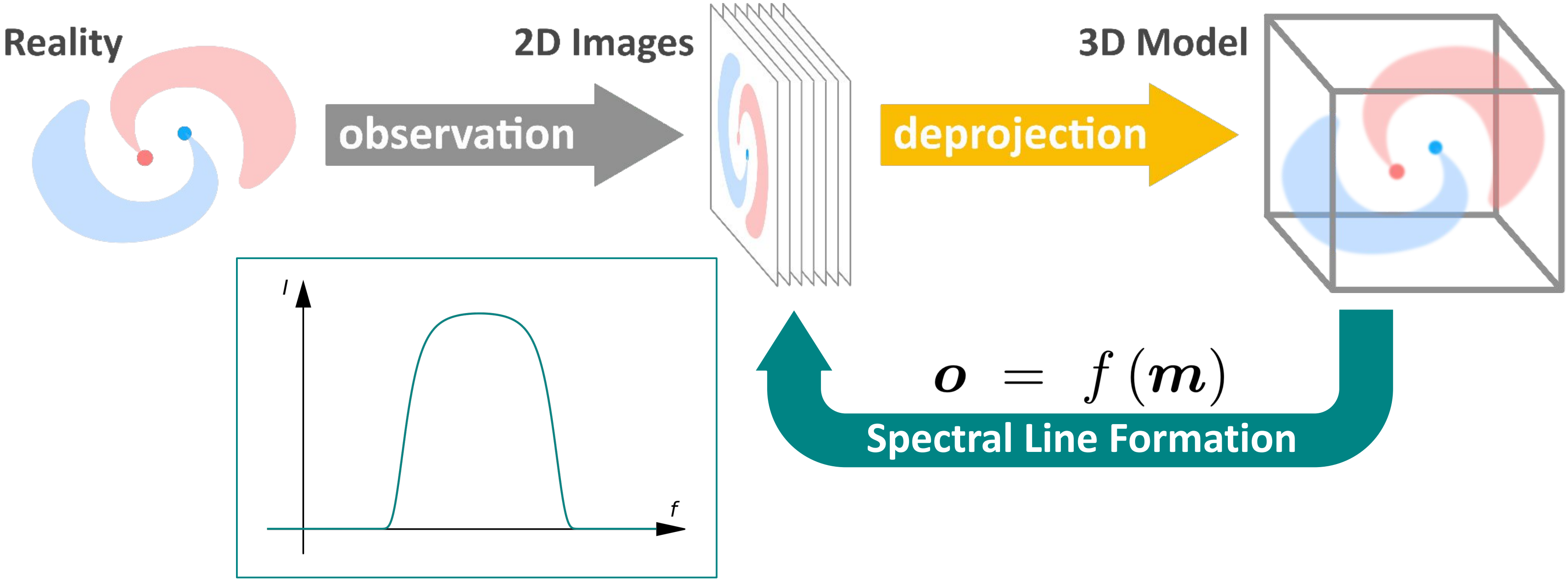

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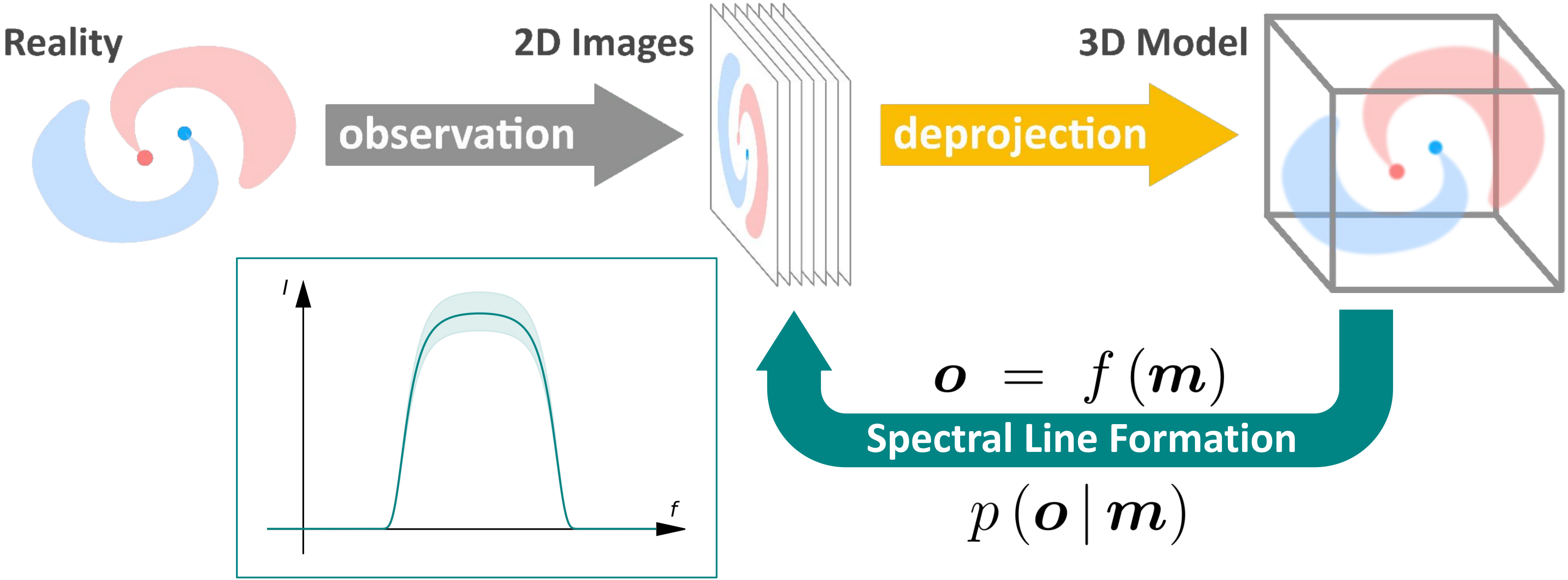

FDC, T. Ceulemans, J. Cockayne, L. Decin, and J. Yates (2023)

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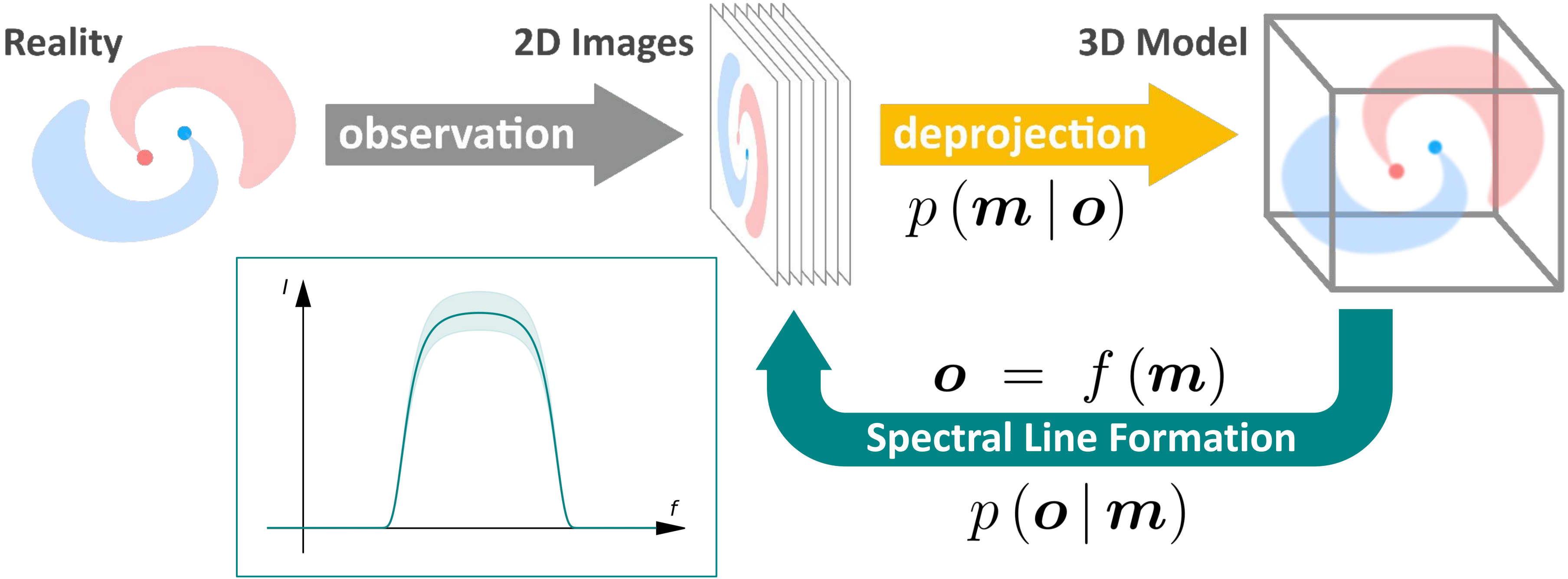

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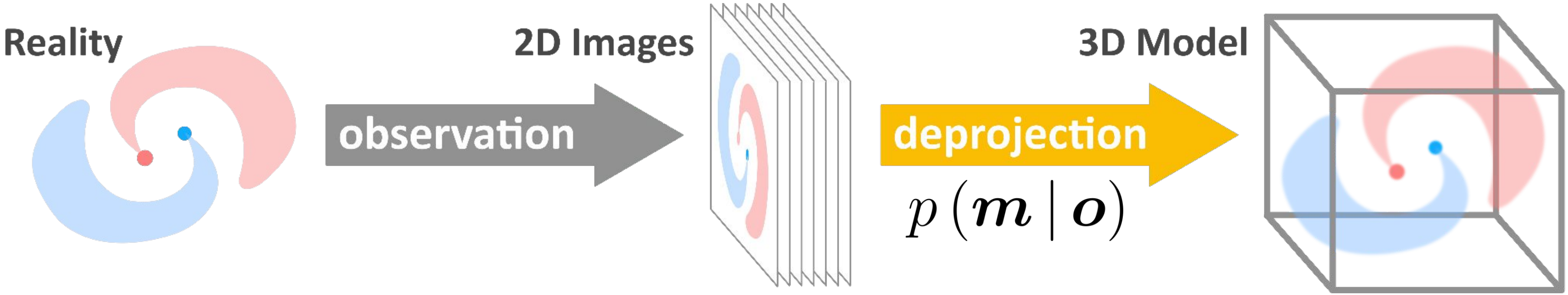

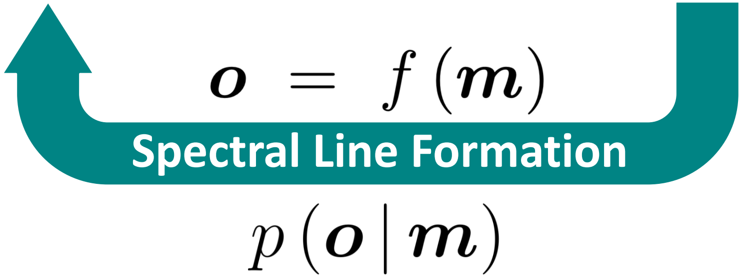

Bayes' rule

$$p(\boldsymbol{m}\,|\,\boldsymbol{o}) = \frac{p(\boldsymbol{o}\,|\,\boldsymbol{m})\,p(\boldsymbol{m})}{p(\boldsymbol{o})}$$

Lucy (1974); Asensio Ramos et al. (2007); Stuart (2010)

Probabilistic 3D reconstruction

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Reconstruct $\boldsymbol{m}$ by maximising the posterior, or, equivalently, by minimising the negative log posterior

$$-\log p(\boldsymbol{m} \,|\, \boldsymbol{o}) \;=\; -\log p(\boldsymbol{o} \,|\, \boldsymbol{m}) \;-\; \log p(\boldsymbol{m})$$

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$$\mathcal{L}_{\text{tot}}\left(\boldsymbol{m}, \boldsymbol{o}\right) \;=\; \mathcal{L}_{\text{rep}}\left(f\left(\boldsymbol{m}\right), \boldsymbol{o}\right) \;+\; \mathcal{L}_{\text{reg}}\left(\boldsymbol{m}\right)$$

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$$\downarrow \qquad \downarrow \qquad \downarrow$$

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Mean square **reproduction loss** implies a Gaussian **likelihood**

$$\mathcal{L}_{\text{rep}}\left(f\left(\boldsymbol{m}\right),\boldsymbol{o}\right) = \|f\left(\boldsymbol{m}\right) - \boldsymbol{o}\|^2 \implies p\left(\boldsymbol{o}\,|\,\boldsymbol{m}\right) = \mathcal{N}\left(\mathbf{o},\boldsymbol{\Sigma}\right)$$

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The **regularisation loss** / **prior** represents our *a priori* assumptions about the model

FDC, et al. (*in prep.*)

Probabilistic 3D reconstruction

Our ***a priori*** **assumptions** about the model

Bayes' rule

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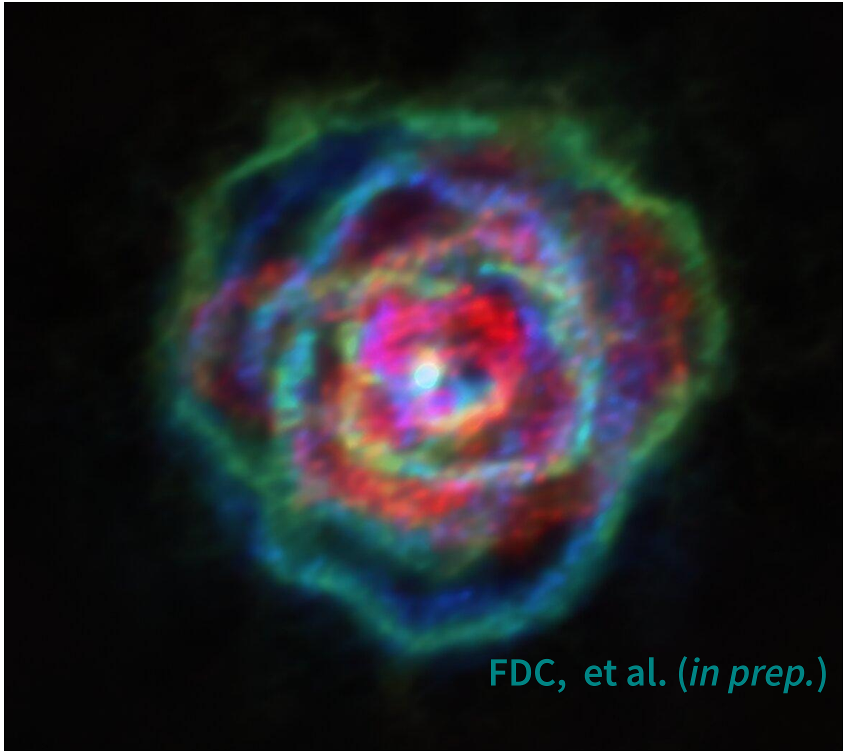

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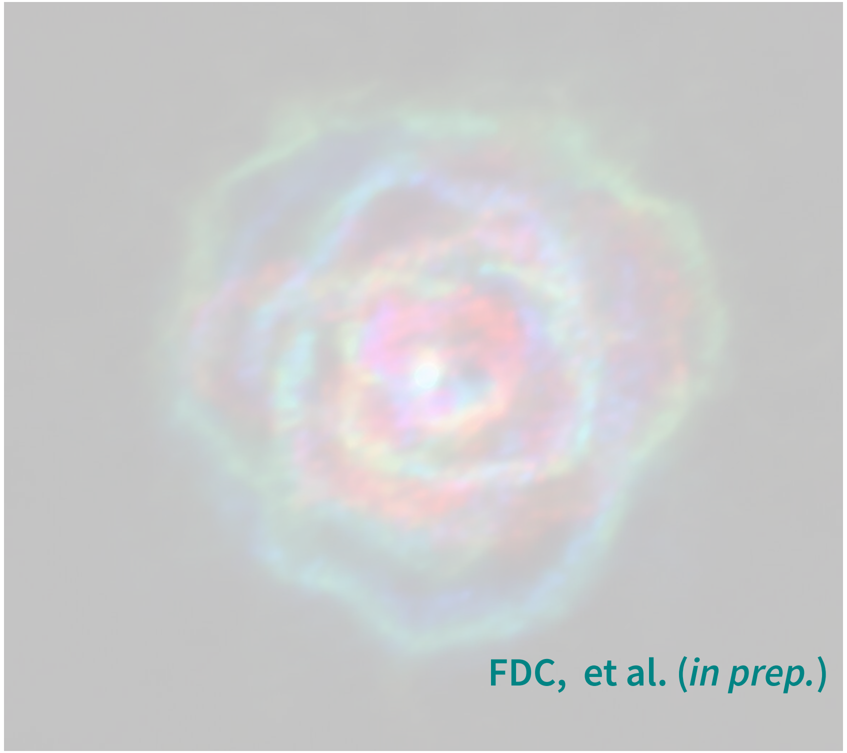

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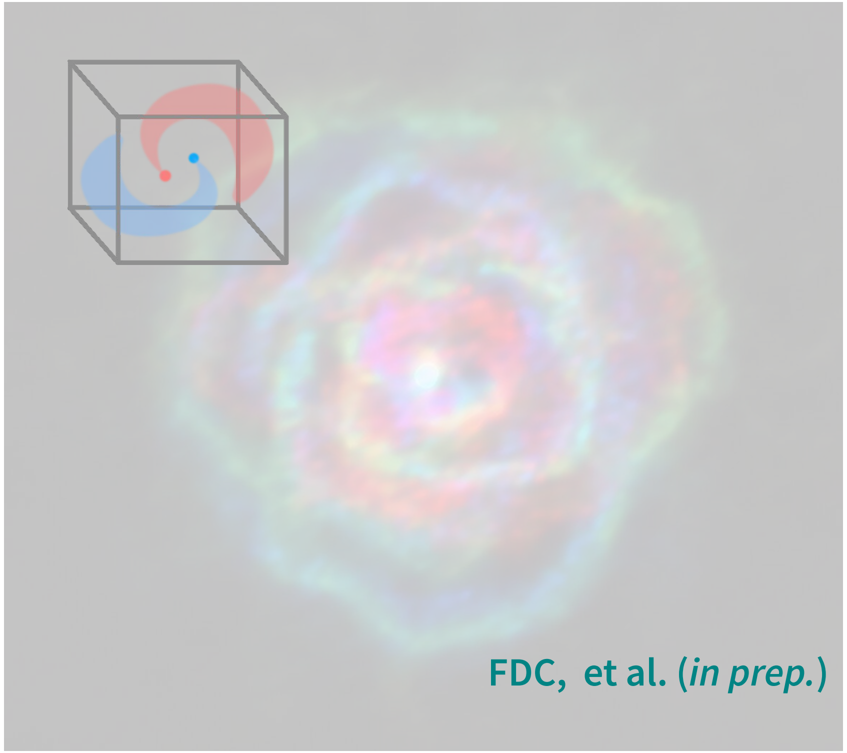
Our *a priori* **assumptions** about the model

The model

$$\boldsymbol{m} = \left\{\rho(\boldsymbol{x}),\, \boldsymbol{v}(\boldsymbol{x}),\, T(\boldsymbol{x})\right\}$$



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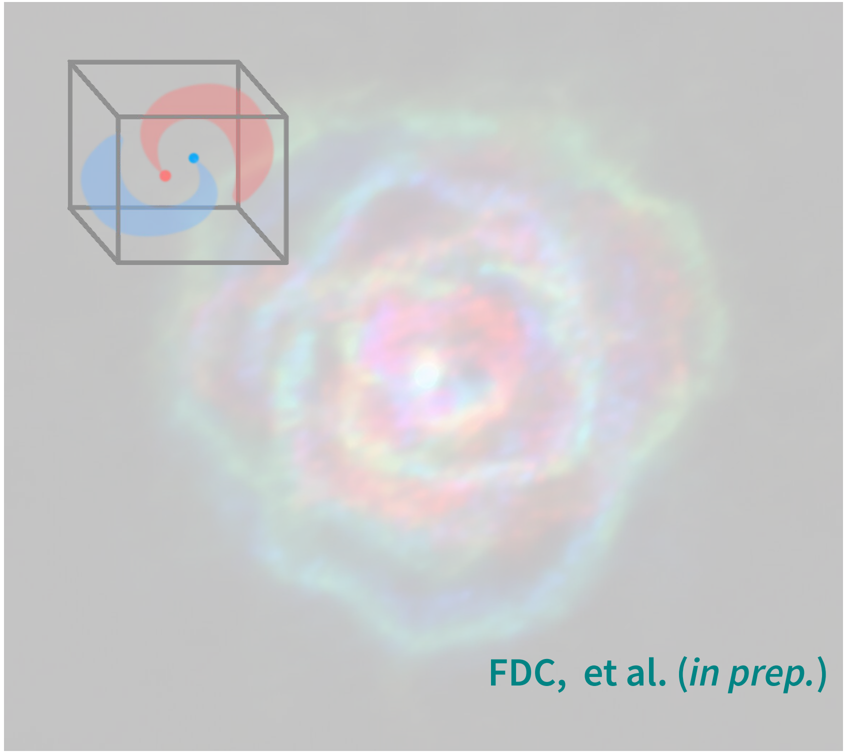

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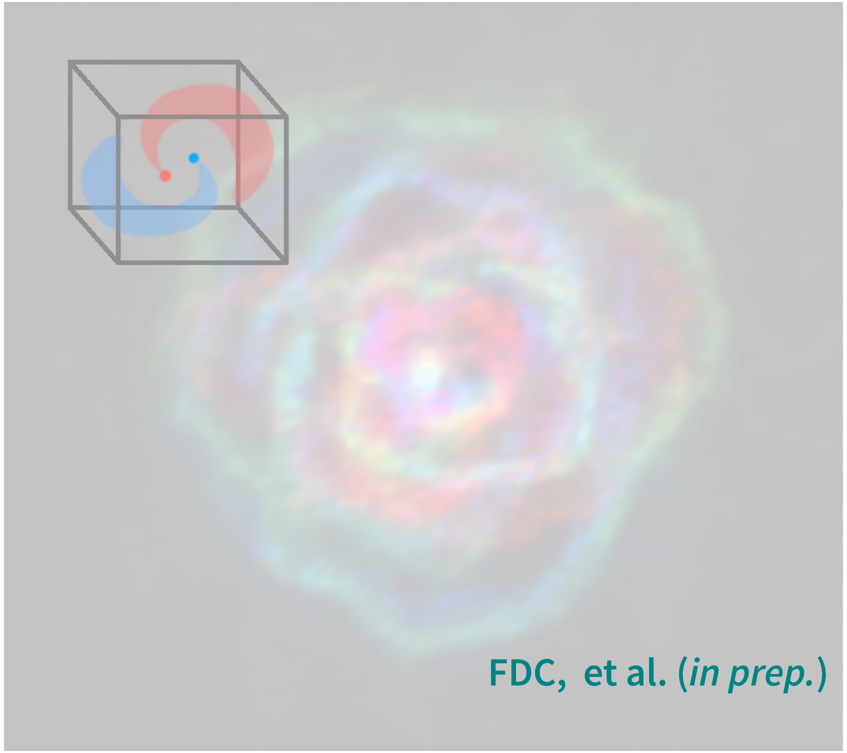
Our *a priori* assumptions

- **Regularity**

  minimise $\|\nabla\rho(\boldsymbol{x})\|^2$



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- **Symmetry**

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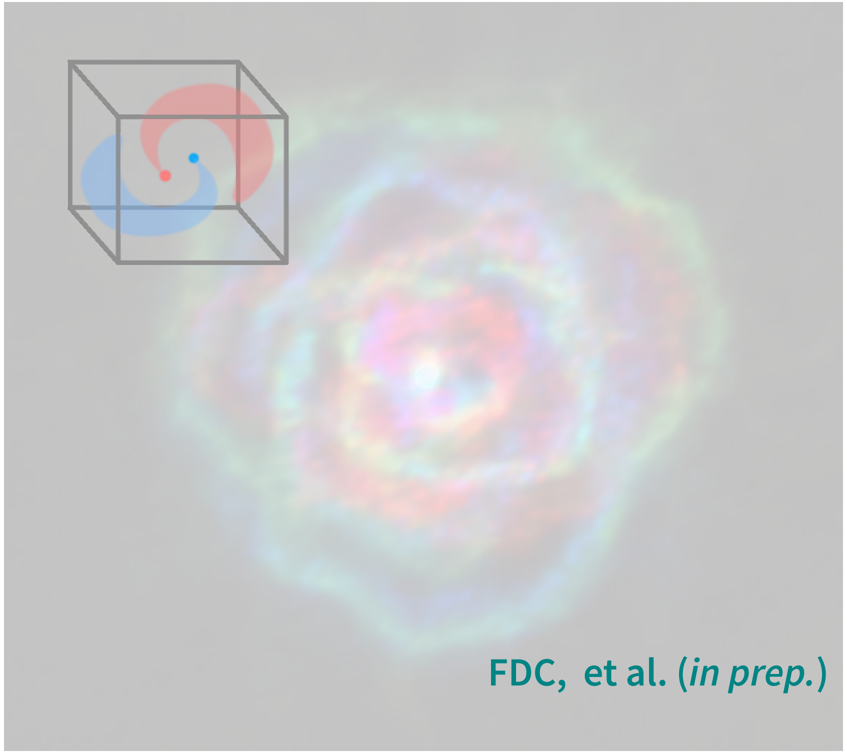

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- **Hydrodynamic steady state**

$$\partial_t \rho = \partial_t \boldsymbol{v} = \partial_t T = 0$$

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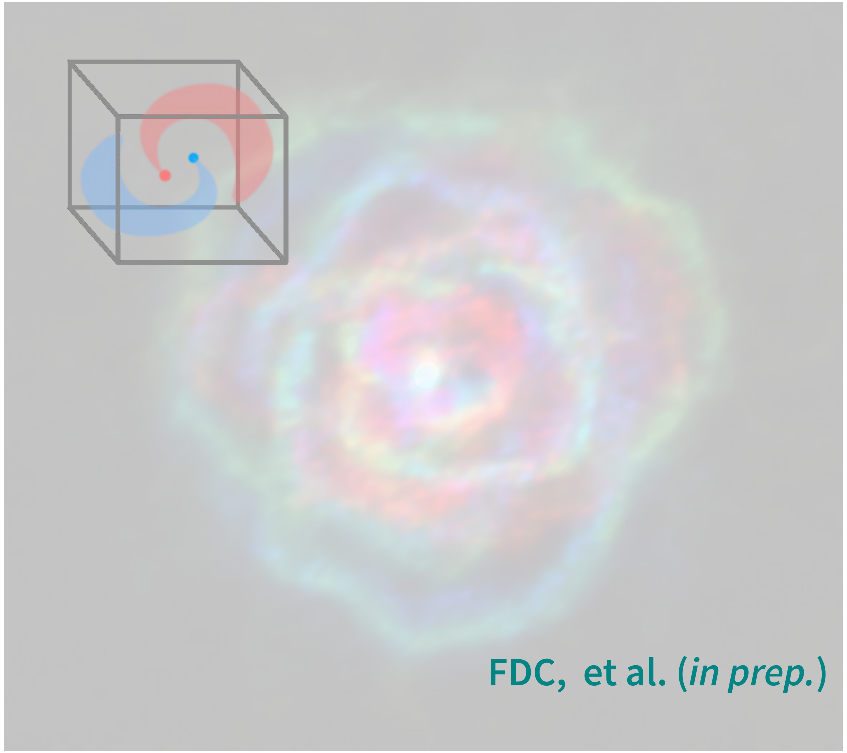

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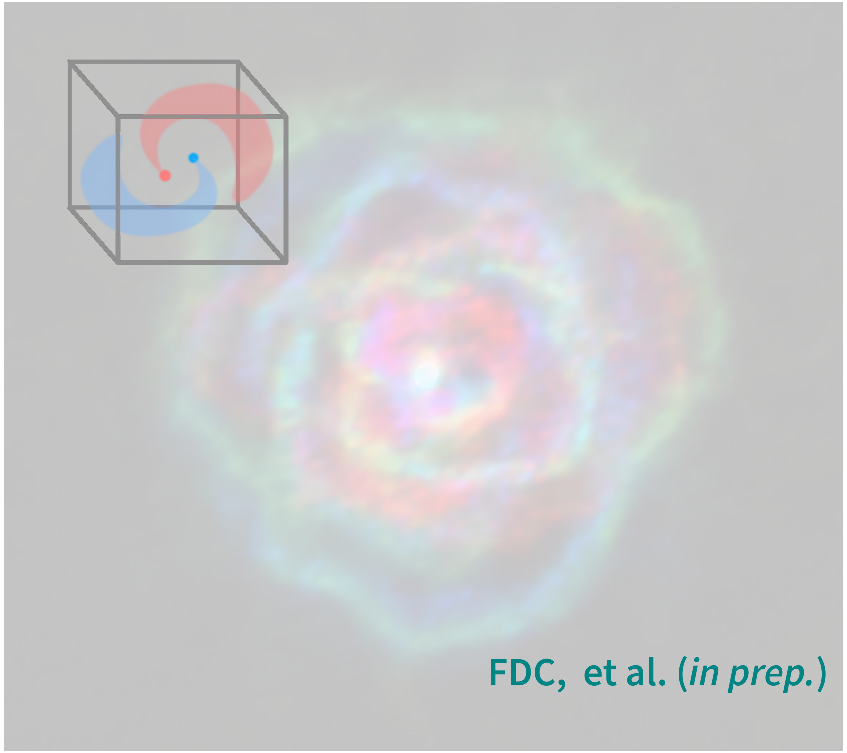

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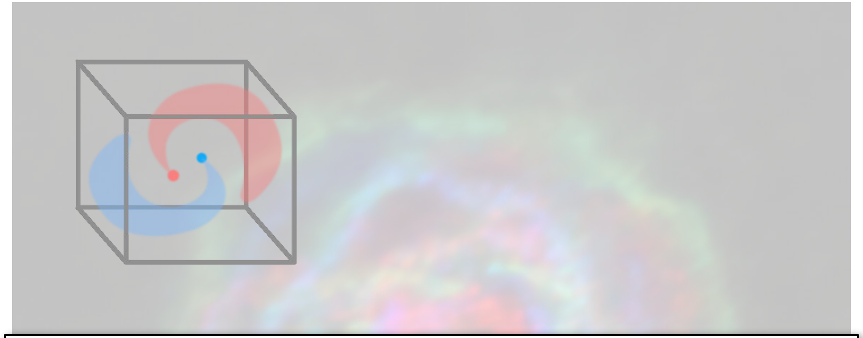

Euler equations

$$\cancel{\frac{\partial\rho}{\partial t}} + \nabla\cdot\left(\rho\,\boldsymbol{v}\right) = 0$$

$$\cancel{\frac{\partial\boldsymbol{v}}{\partial t}} + \left(\boldsymbol{v}\cdot\nabla\right)\boldsymbol{v} + \frac{1}{\rho}\nabla P + \nabla\Phi = 0$$

$$\cancel{\frac{\partial E}{\partial t}} + \nabla\cdot\left(\left(E+P\right)\boldsymbol{v}\right) + \Lambda = 0$$

Probabilistic 3D reconstruction

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$$\begin{aligned}
\mathcal{L}_{\text{rep}}\left(f(\boldsymbol{m}), \boldsymbol{o}\right) \;&=\; \quad \left\| f(\boldsymbol{m}) \;-\; \boldsymbol{o} \right\|^2 \\
\mathcal{L}_{\text{reg}}\left(\boldsymbol{m}\right) \;&=\; \quad w_{\text{reg}}\; \mathcal{L}'_{\text{reg}}\; \left(\boldsymbol{m}\right) \\
&\quad + w_{\text{sym}}\, \mathcal{L}_{\text{sym}}\left(\boldsymbol{m}\right) \\
&\quad + w_{\text{hyd}}\; \mathcal{L}_{\text{hyd}}\; \left(\boldsymbol{m}\right)
\end{aligned}$$

`p3droslo`: Probabilistic 3D reconstruction of spectral line observations

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Goal: Find the model that minimises the loss

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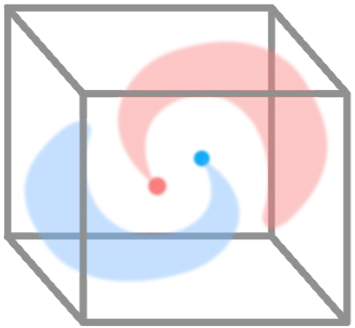


Goal: Find the model that minimises the loss

Model:

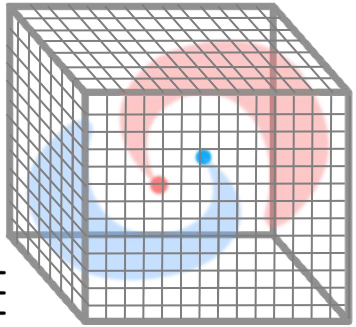
$$\boldsymbol{m} = \{\rho(\boldsymbol{x}),\ \boldsymbol{v}(\boldsymbol{x}),\ T(\boldsymbol{x})\}$$

`p3droslo`: Probabilistic 3D reconstruction of spectral line observations

Goal: Find the model that minimises the loss

Model: Variables as PyTorch tensors

$$\boldsymbol{m} = \{\rho(\boldsymbol{x}_d),\ \boldsymbol{v}(\boldsymbol{x}_d),\ T(\boldsymbol{x}_d)\}$$

$\boldsymbol{x}_d \in$ 

PyTorch

Paszke et al. (2017, 2019)

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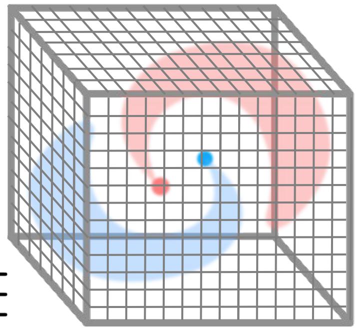
Algorithm: Stochastic Gradient Descent (SGD)

$\boldsymbol{x}_d \in$ 

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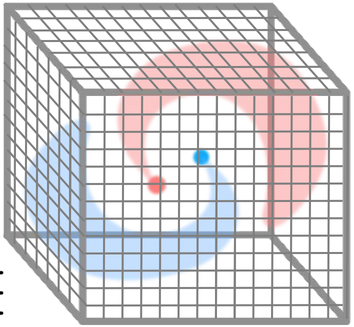
$\boldsymbol{x}_d \in$ 

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Algorithm: Stochastic Gradient Descent (SGD)

- Spectral Line Formation: $\boldsymbol{o} = f(\boldsymbol{m})$

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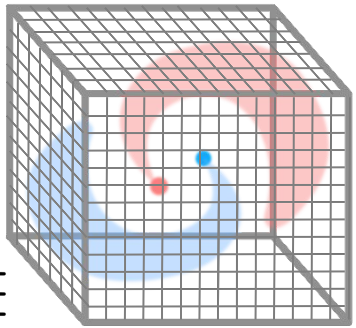
$\boldsymbol{x}_d \in$ 

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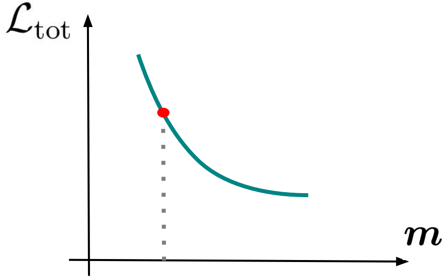
$$\boldsymbol{m} = \{\rho(\boldsymbol{x}_d),\ \boldsymbol{v}(\boldsymbol{x}_d),\ T(\boldsymbol{x}_d)\}$$

PyTorch

Paszke et al. (2017, 2019)

Algorithm: Stochastic Gradient Descent (SGD)

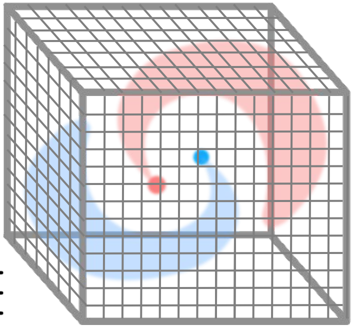
- Spectral Line Formation: $\boldsymbol{o} = f(\boldsymbol{m})$
- Compute losses



`p3droslo`: Probabilistic 3D reconstruction of spectral line observations

Goal: Find the model that minimises the loss

$\boldsymbol{x}_d \in$



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Paszke et al. (2017, 2019)

Algorithm: Stochastic Gradient Descent (SGD)

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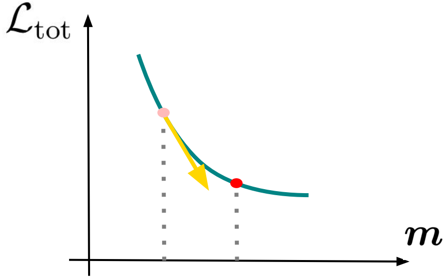
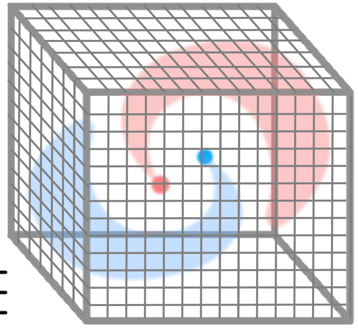


`p3droslo`: Probabilistic 3D reconstruction of spectral line observations



$\boldsymbol{x}_d \in$

Goal: Find the model that minimises the loss

Model: Variables as PyTorch tensors

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Paszke et al. (2017, 2019)

Algorithm: Stochastic Gradient Descent (SGD)

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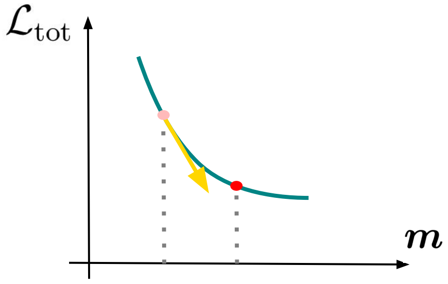
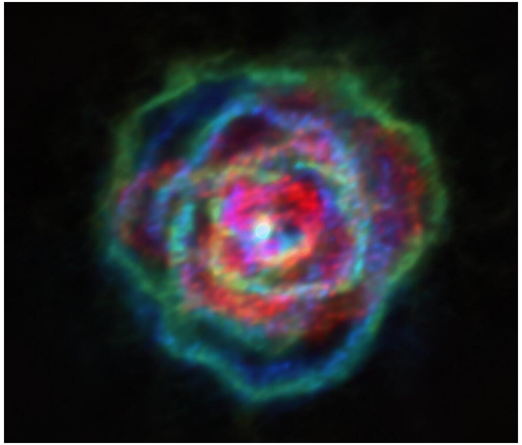
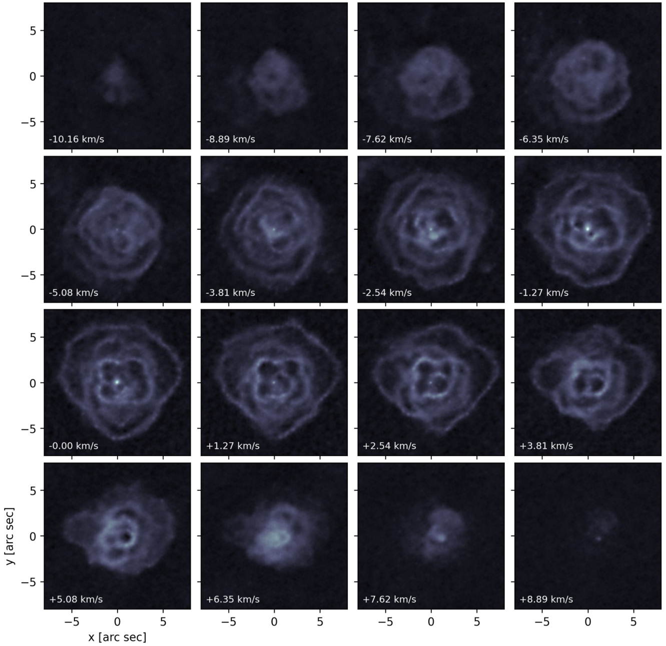
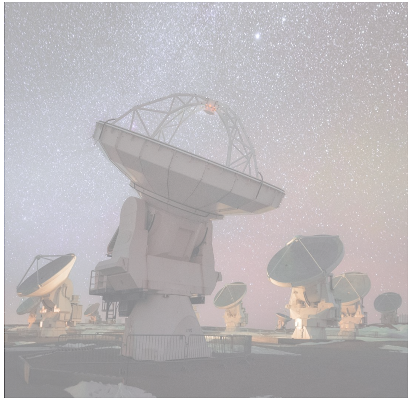


Example **R Aquilae**





`p3droslo`: Probabilistic 3D reconstruction of spectral line observations



Future work

- **Learn hyperparameters** from simulations
- Implement **non-LTE line radiative transfer**
- Model observations in the **visibility domain** (*uv*-plane)
- …
- (Your project?)

More info:

freddeceuster.github.io/p3droslo

Get in touch!

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