The spectral signature of circumstellar molecules: the slab model

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February 24, 2012

1 Introduction

Cool, evolved stars such as the Mira variables often show spectral absorption and/or emission features which can be traced back to molecules in the outer regions of the (pulsationally) levitated atmosphere. Since full hydrodynamical modelling of these types of stars is computationally extremely expensive, more ad hoc, simplified models have to be constructed in order to analyse and interpret the plethora in observed spectra. The most basic of these approximations is called the *"slab" model*. In a slab model, the object is decomposed in two parts: (1) the stellar photosphere, most often represented by a black-body radiator with temperature $T_\star$ [K], and (2) a slab/layer of molecular material with column density cd [#molecules cm^{-2}] and temperature T_{lyr} [K]. The molecular properties are introduced via the wavelength-dependent cross section of the particles in the slab.

In this assignment, you will explore the vast range in infrared molecular spectra that can result from such a simplified geometrical set-up. Scientific results have been obtained with this approach in the recent past, e.g. Cami et al. 2000, and Matsuura et al. 2002.

2 The radiative transfer equation in the slab model

The equation of radiative transfer can be written as:

$$\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu, \tag{1}$$

with I_ν the emerging intensity, τ_ν the optical depth of the medium and S_ν the source function. The formal solution of this equation is:

$$I_\nu(\tau_\nu) = I_\nu^0 e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu-\tau'_\nu)} S_\nu(\tau'_\nu) d\tau'_\nu, \tag{2}$$

where $I_\nu^0 = I_\nu(\tau_\nu = 0)$ denotes the background intensity. Under the assumption of a medium in LTE with a constant source function S_ν (isothermal), the analytical solution becomes:

$$I_\nu(\tau_\nu) = I_\nu^0 e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}). \tag{3}$$

Turn this equation into the correct equation for a slab model consisting of a black-body photosphere at $T_\star$ as background source, and a thermally radiating molecular slab with temperature T_{lyr} and optical depth $\tau_{lyr} = cd \times \sigma_\nu$, where σ_ν is the cross section [cm^2] per molecule.

These cross sections depend on the temperature of the molecule (**why?**), and for the purpose of this assignment, we provide these, pre-calculated, for the H_2O molecule at the following web address:

http://www.ster.kuleuven.be/~tijl/StellarWinds/CrossSections/

The equation you obtained determines the *intensity* as a function of frequency or wavelength. However, most observations only yield the *flux*, i.e. the intensity integrated over the solid angle of the object on the sky, as a function of frequency or wavelength.

Adjust your equation so that it allows you to compute the flux of a slab model. This implies that you will have to multiply the intensity of the individual components with their solid angle on the sky (e.g in Steradian if that is the unit you used in your black-body description.). This of course requires the introduction of the stellar angular diameter $\theta_\star$, and the layer angular diameter θ_{lyr}, both of which are usually expressed in milli-arcseconds or mas.

3 Generating synthetic spectra

With the equation derived in the previous section, we can now generate synthetic spectra for different parameter sets. Since IR observations often have insufficient spectral resolution to resolve the individual lines, molecules are in these spectra seen as absorption "bands". This means you will have to downgrade the resolution of your synthetic spectra to a resolution of $\lambda/\Delta\lambda \sim 1000$. This can be done by convolving your synthetic spectrum with a gaussian of width $\Delta\lambda$. You can compute $\Delta\lambda$ for the central wavelength of the wavelength range you want to study.

Start by making an infrared spectrum of a 3000 K star with a 1500 K molecular slab at a column density of 1×10^{18}cm^{-2}. Assume that the slab has the same angular diameter as the star. Are the water features observed in emission or in absorption? What happens if you increase the size of the slab, and similarly, if you increase its temperature or column density? Can you distinguish between these alterations based on the resulting spectrum?

4 Qualitative analysis of an observed spectrum

The ISO-SWS spectra of the red supergiant μ Cephei ($T_{eff} = 3600K$) are shown in Fig. 1. With respect to the reference atmosphere model, we observe absorption around

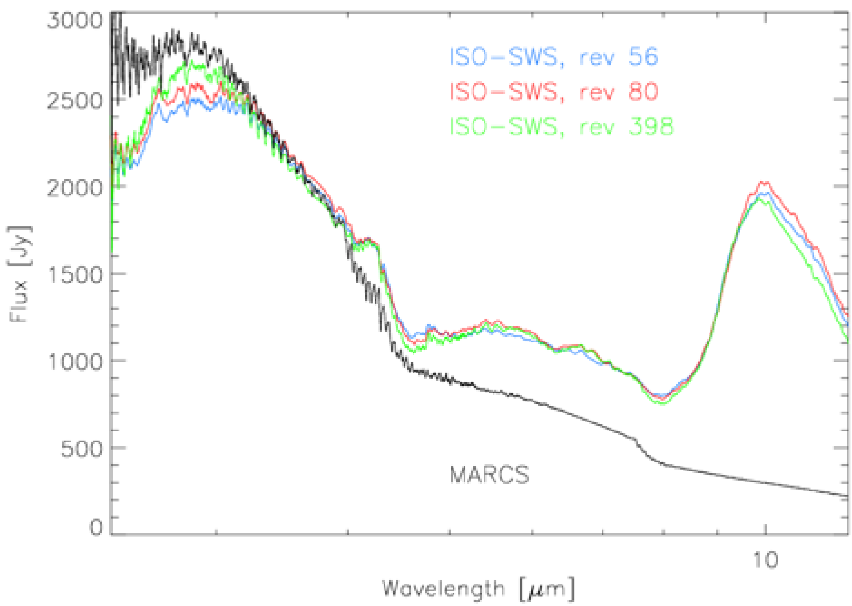

Figure 1: The ISO-SWS spectra of red supergiant μ Cephei at 3 different dates, and a comparison atmosphere model computed with the MARCS code. The strong emission feature at $10\,\mu$m is due to silicate dust in the stellar wind.

$3\,\mu$m, and broad emission from $4\,\mu$m onwards. These effects are believed to be (partially) due to a water layer surrounding the star. The strong feature at $10\,\mu$m is due to silicate dust in the stellar wind.

Perform some slab model simulations to get a qualitative idea of the water layer around μ Cephei. How does its temperature compare to that of the star? Is it very extended? Is the column density high?

5 Improvements

Some improvements to the slab model as used here can be made without adding too much complexity. Which ones can you think of? (Your list doesn't have to be exhaustive).

References

Cami, J. et al., 2000, Astronomy and Astrophysics, Vol. 360, p.562-574

Matsuura et al., 2002, Astronomy and Astrophysics, Vol. 383, p.972-986