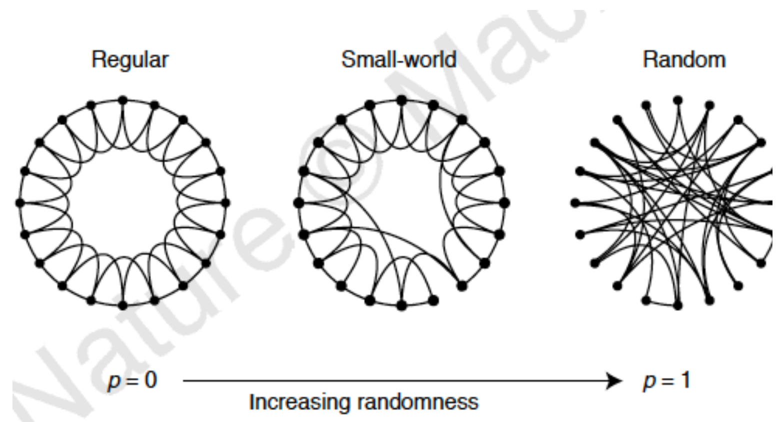
The Dynamics of Complex Systems

Graph theory
Complex Networks

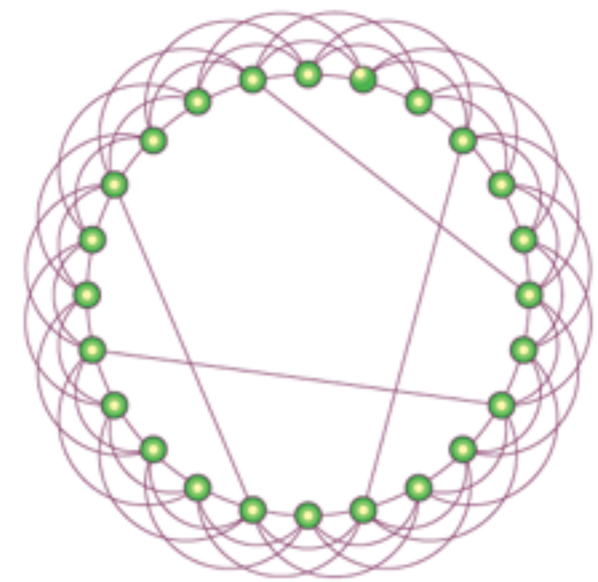
Networks of (Networks of) Complex Systems: Extended Criticality and Biological memory

Radboud University Nijmegen

Complex Networks vs. Psychometric Networks

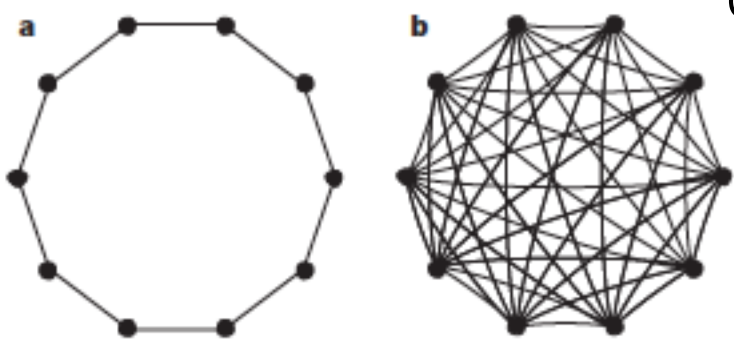


small world



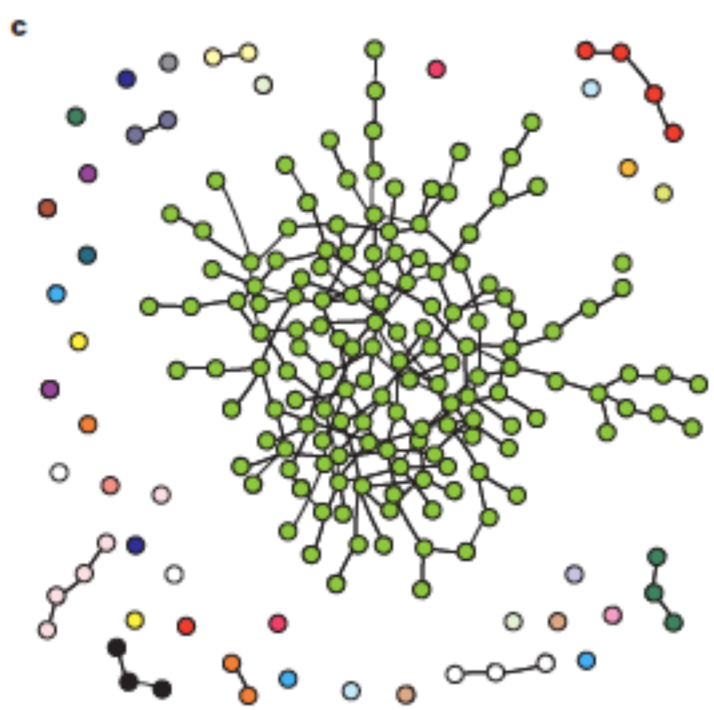
nearest neighbour ring

complete network



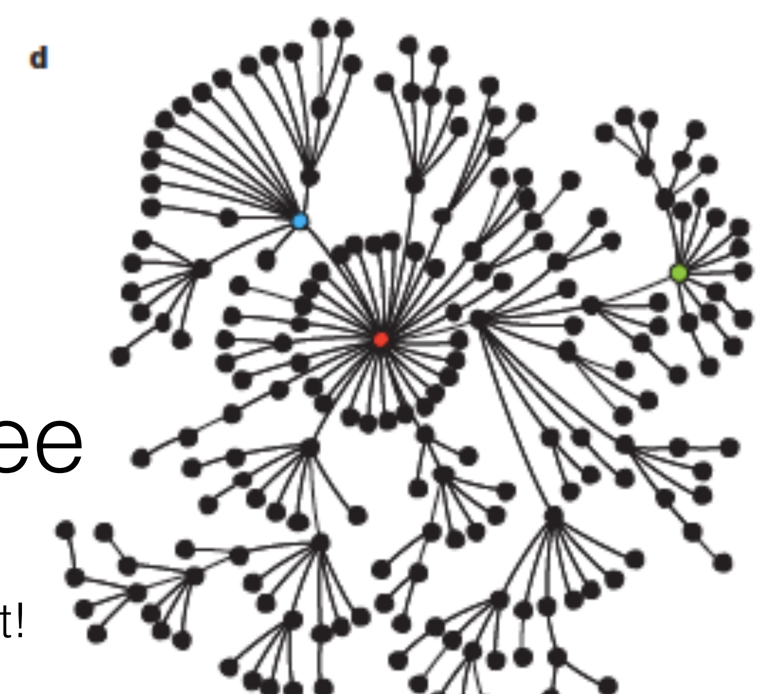
Compositions of Vertices and Edges

Different topologies have different properties: Getting from A to B



random

scale-free



many more topologies and varieties exist!

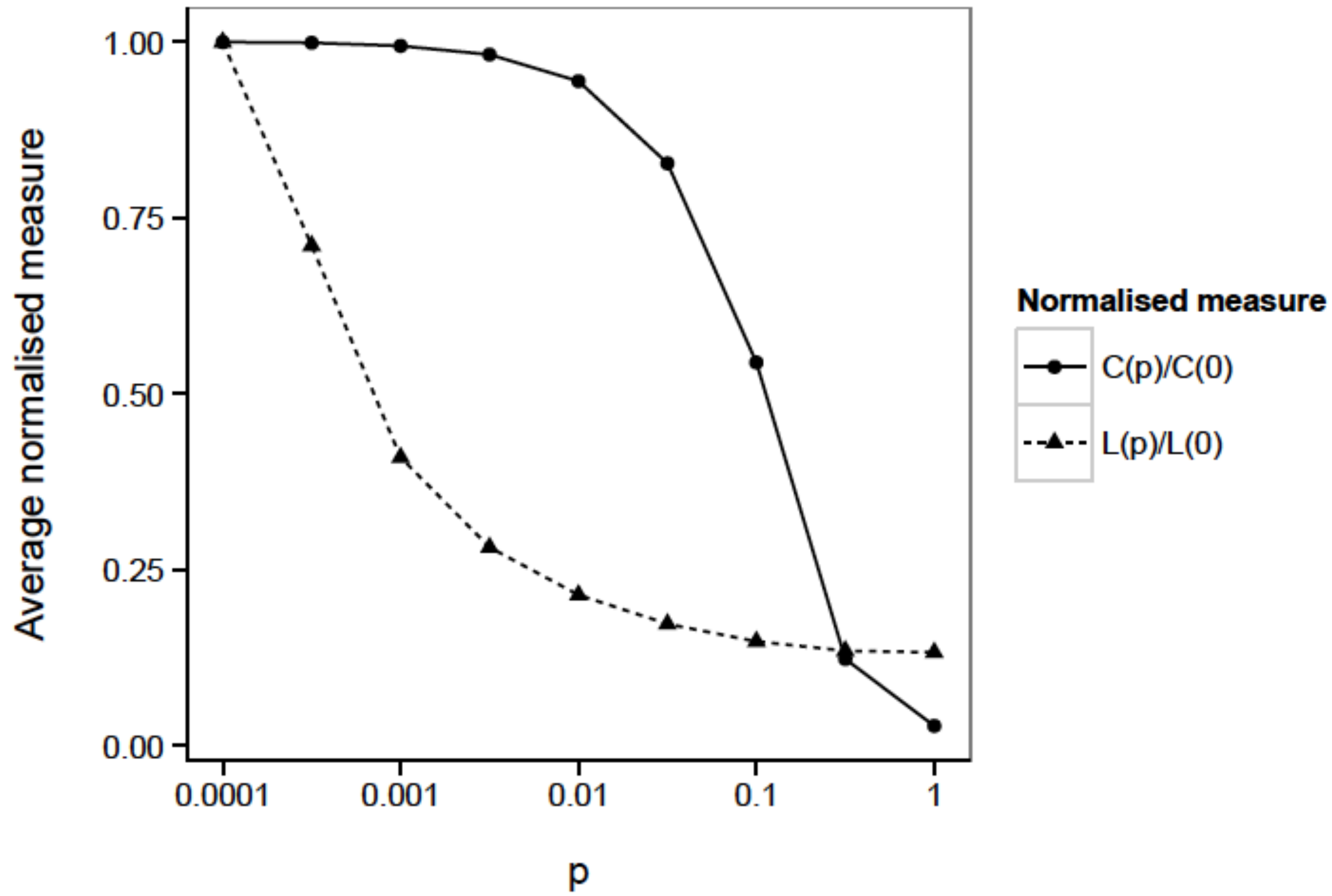
Network / Graph topology: It's a Small World After All

"small-world" test:

Average path length (L)

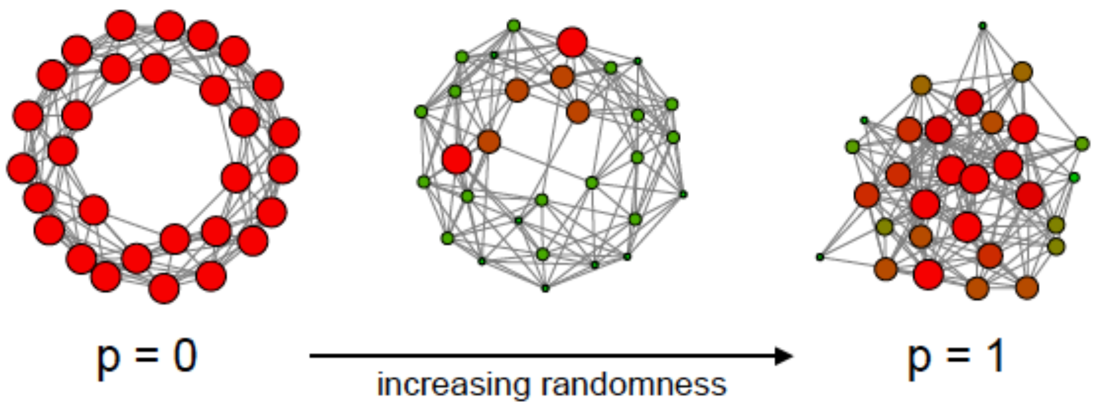
Clustering coefficient (C)

Compare to randomly
rewired version



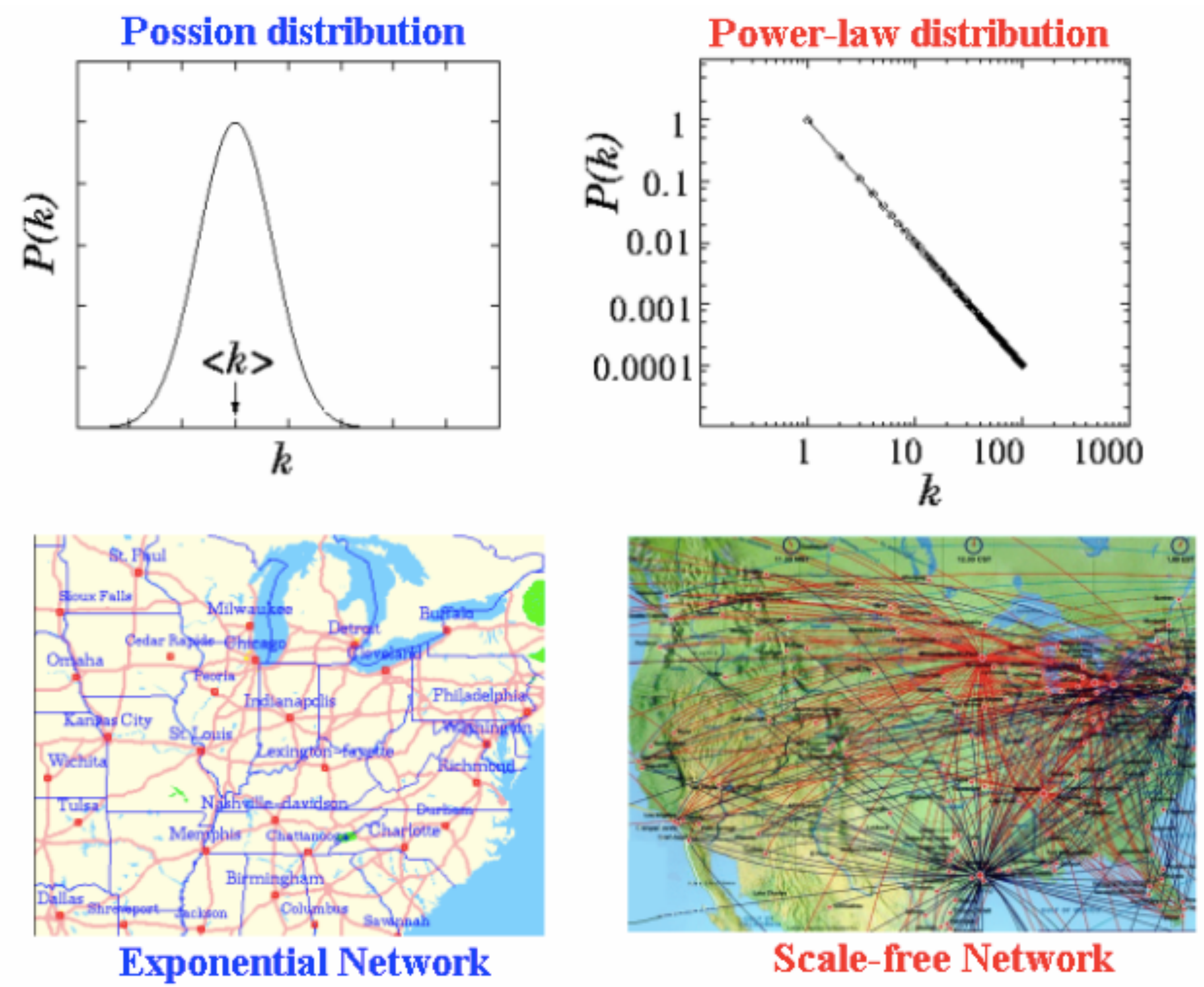
Sound familiar?

In between
fully ordered
&
completely random
=
optimal



Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world'networks. N*ature*, *393*(6684), 440-442.

Network / Graph topology: It's a Scale Free World After All

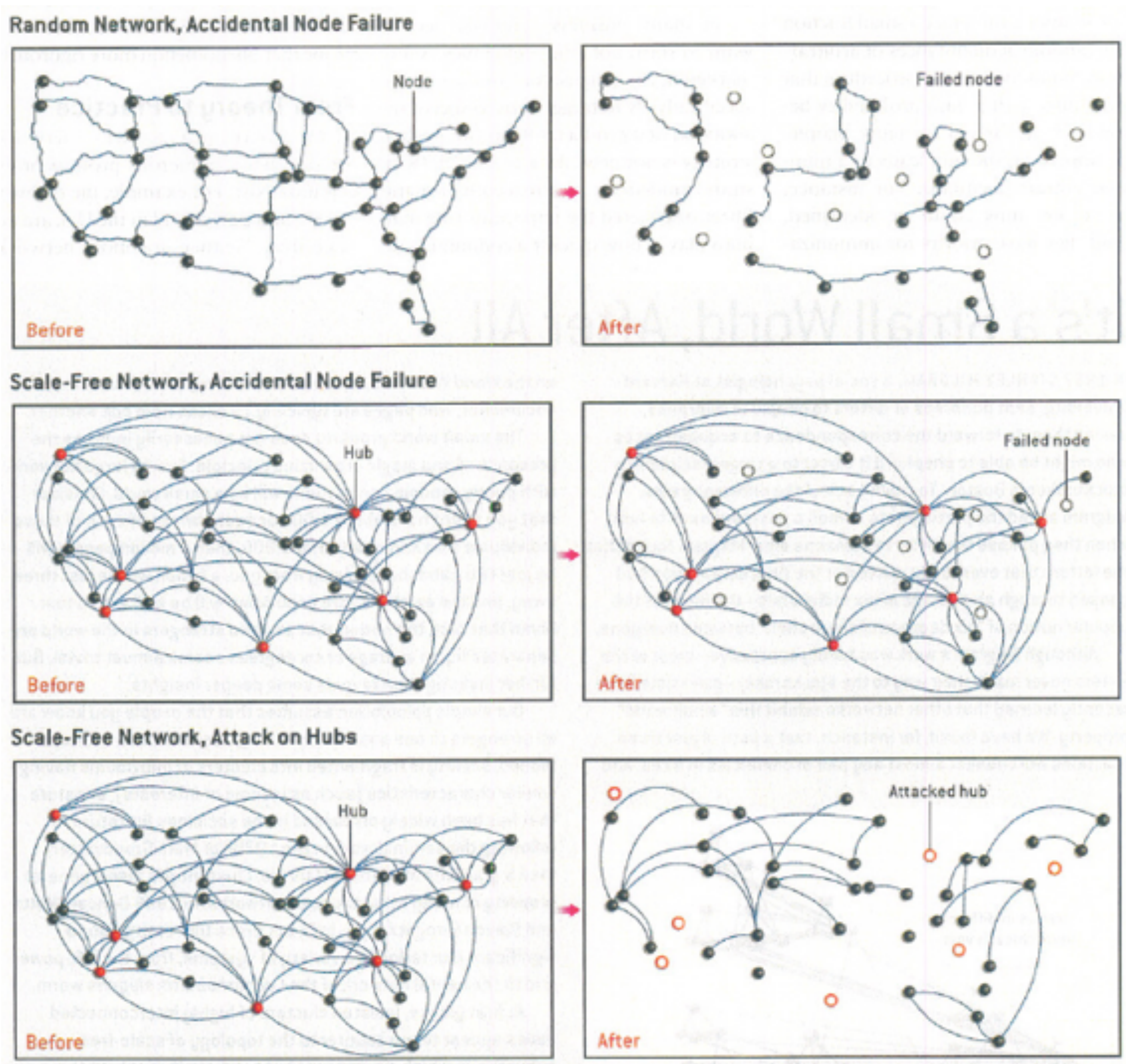


Number of connections a node in the network has: degree (δ)

Scale-free network: degree distribution is a power law!

Wang, X. F., & Chen, G. (2003). Complex Networks: small-world, scale-free and beyond. Circuits and Systems Magazine, IEEE, 3(1), 6–20. IEEE. Retrieved May 18, 2011, from http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1228503.

Can complex networks provide a suitable structure?



Scale free networks are resilient to random attacks on nodes or node failures
(cf. internet on 9/11)

when more hub nodes fail though....

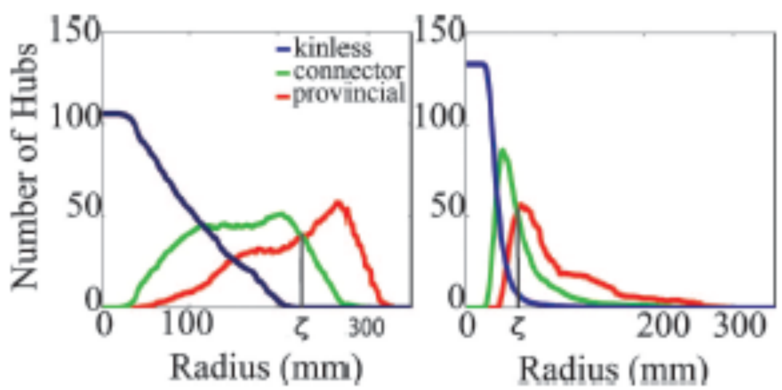
targeted attack!

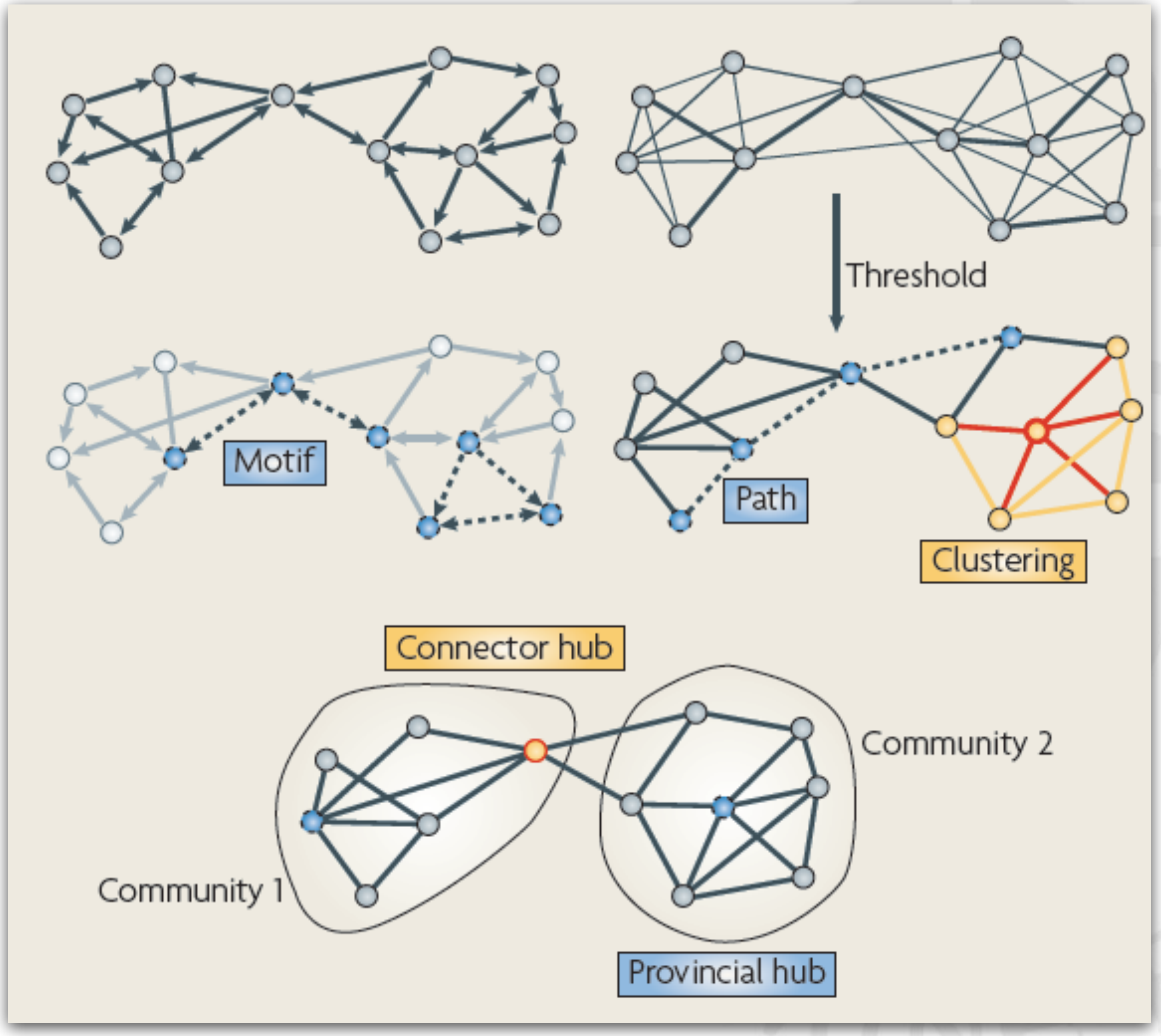
Bonabeau, E., with Albert-László. (2003). Scale-free networks. *Scientific American, 288(5)*, 50–9.

Can complex networks provide a suitable structure?

A brand new zoo of complexity measures!

- Node degree
- Degree distribution
- Assortativity
- Clustering coefficient
- Motifs
- Path length
- Path efficiency
- Connection density or cost
- Hubs
- Centrality
- Robustness
- Modularity



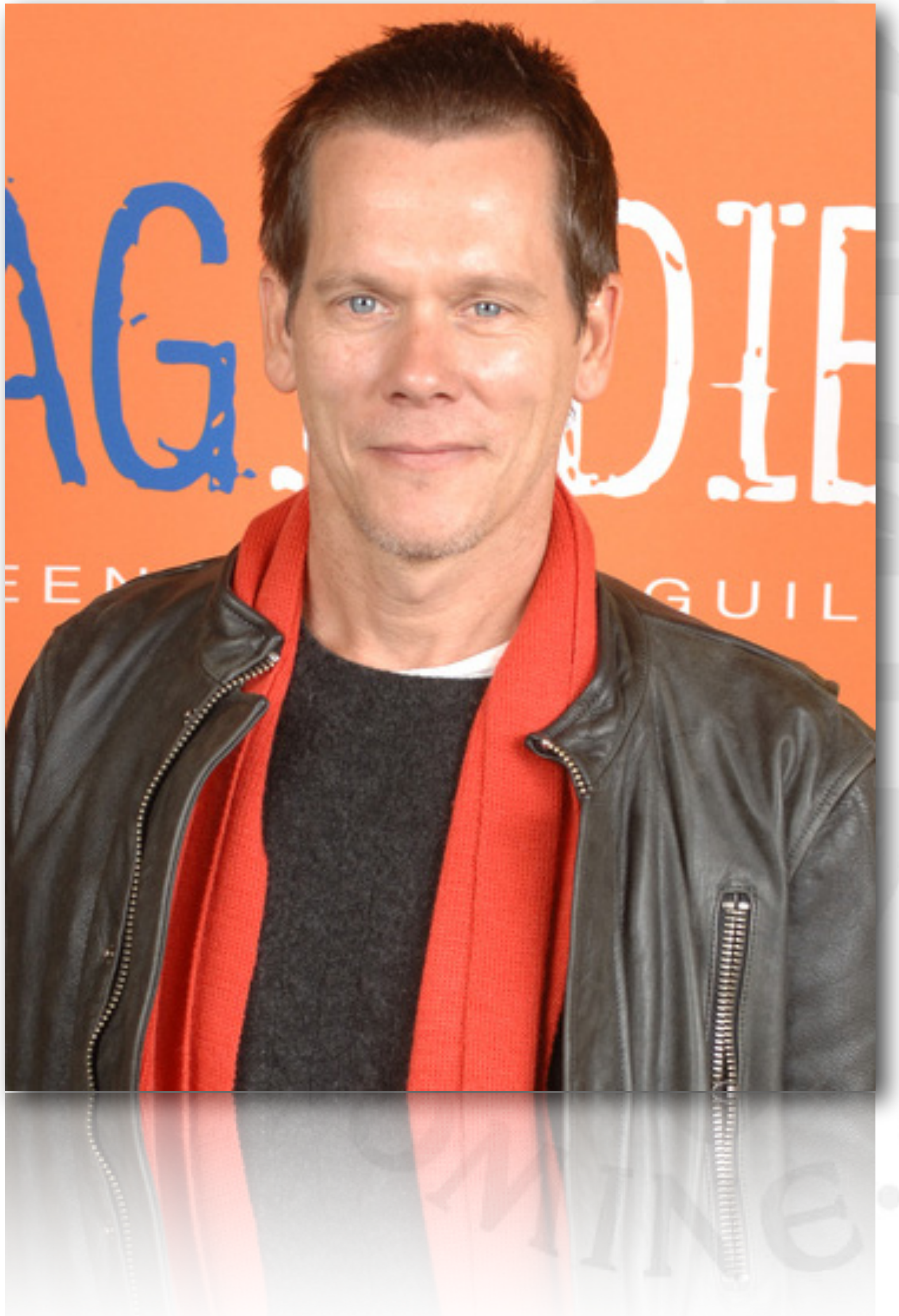


Bullmore, E., & Sporns, O. (2009). Complex brain networks: graph theoretical analysis of structural and functional systems. Nature reviews. Neuroscience, 10(3), 186-98. doi: 10.1038/nrn2575.

Effectiveness / Connectivity: 6 degrees of separation

Kevin Bacon number (Erdős number)

- Node degree
- Degree distribution
- Assortativity
- Clustering coefficient
- Motifs
- Path length
- Path efficiency
- Connection density or cost
- Hubs
- Centrality
- Robustness
- Modularity



Degree of separation
(from Kevin Bacon)

'6 degrees of separation'

http://en.wikipedia.org/wiki/Bacon_number#Bacon_numbers

Effectiveness / Connectivity: 6 degrees of separation

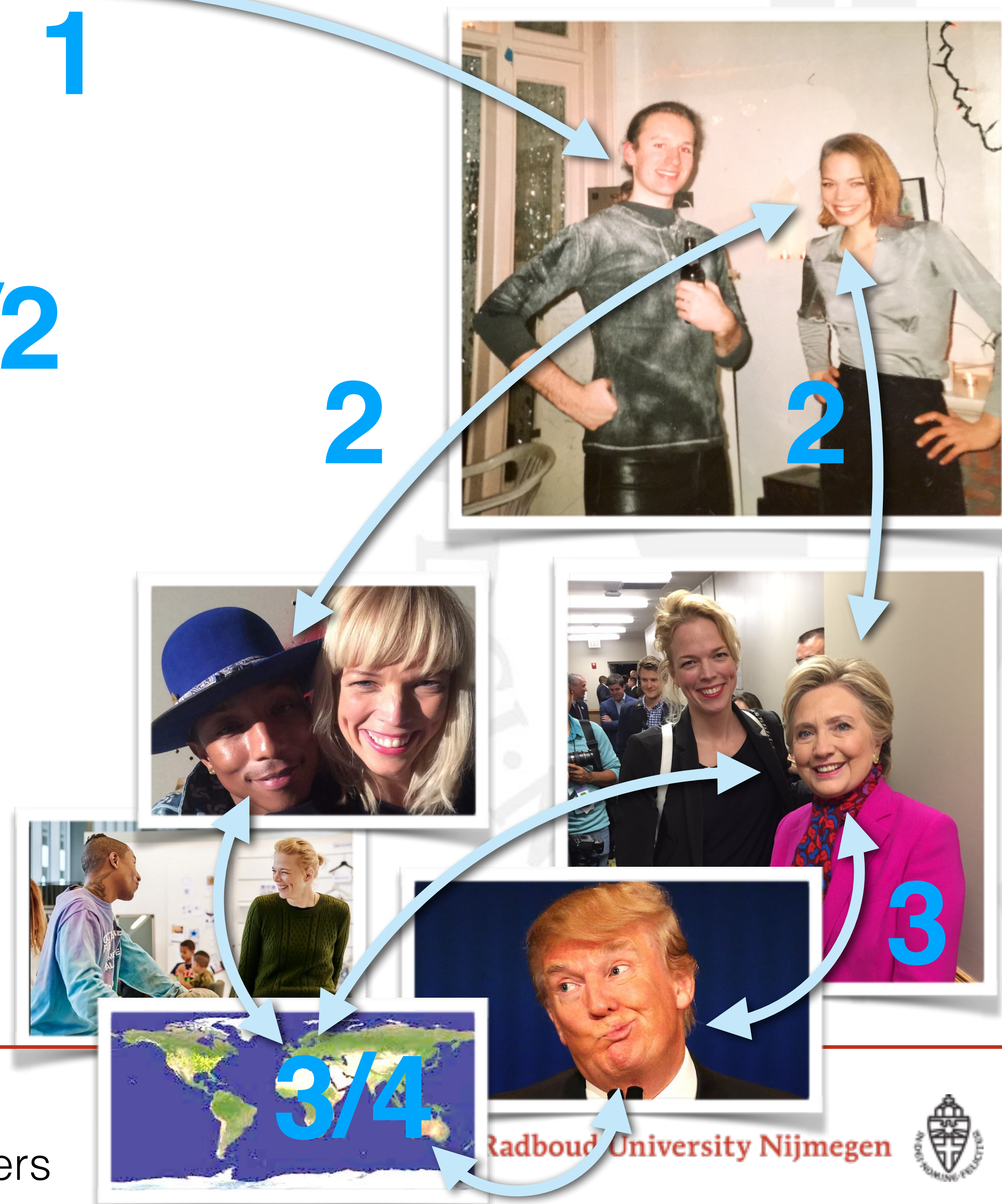
Your degree of separation from:

1

Nobel Laureate **1/2**

Pharrell Williams

Hillary Clinton
Donald Trump

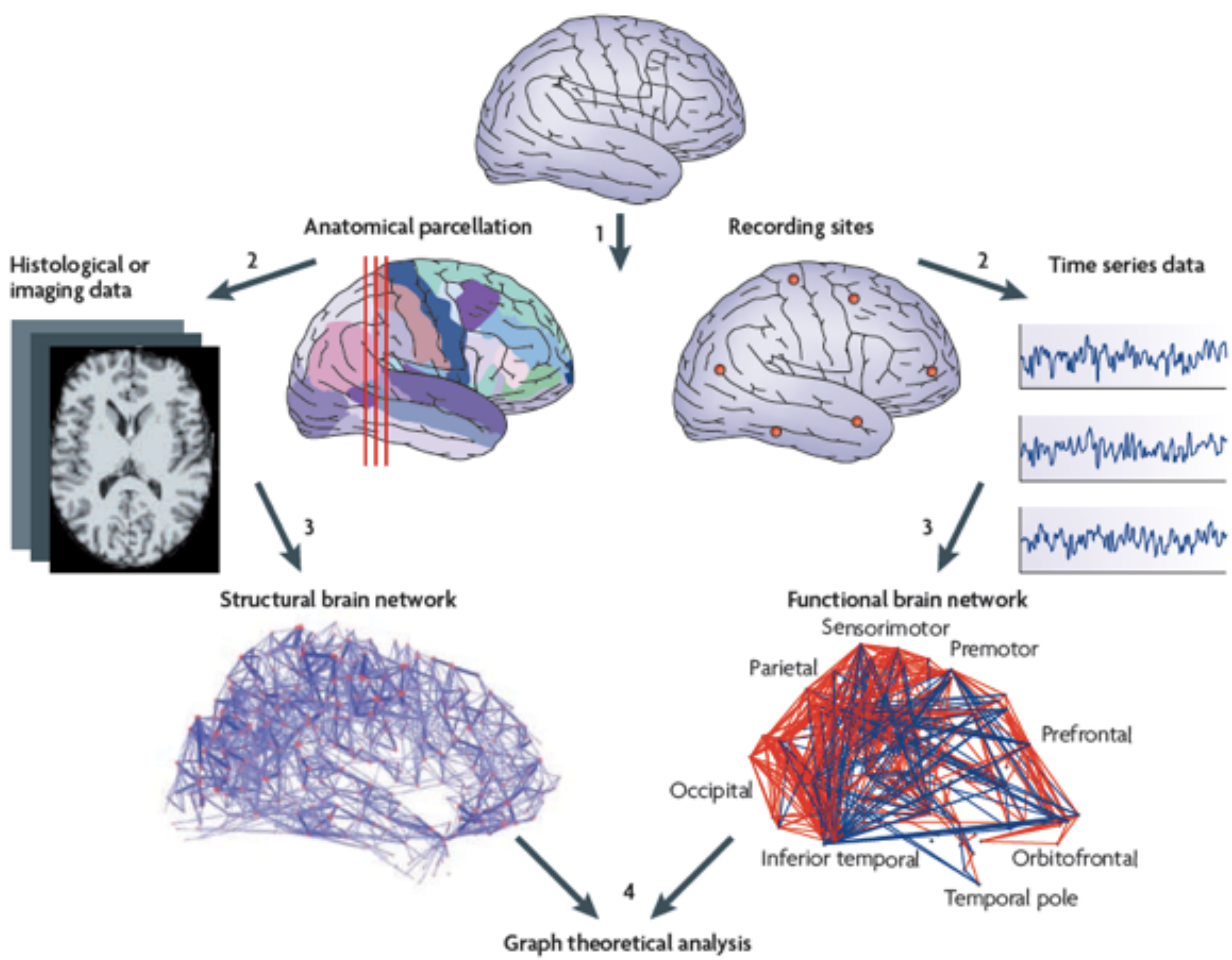


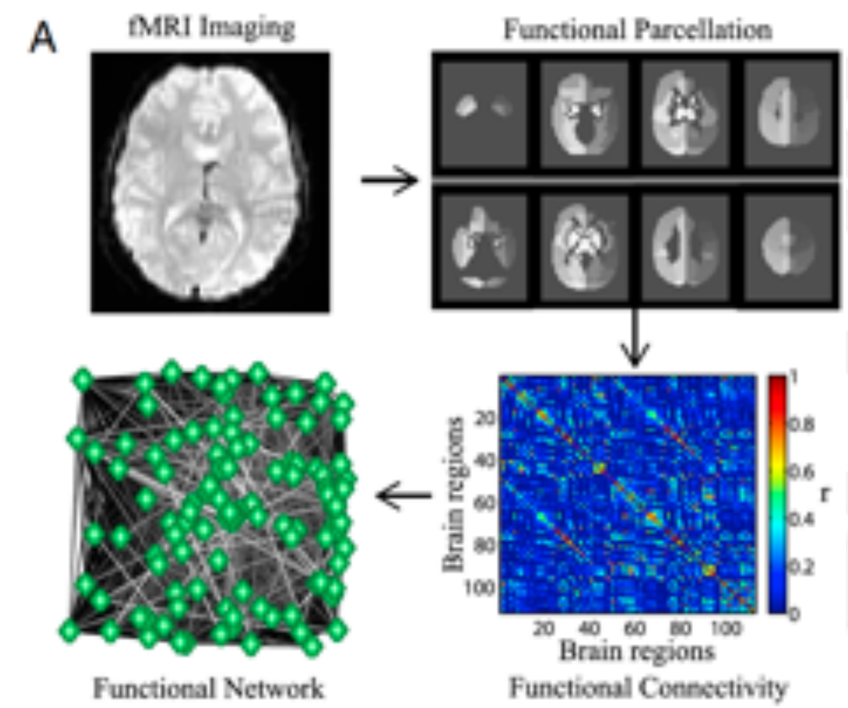
http://en.wikipedia.org/wiki/Bacon_number#Bacon_numbers

Network / Graph topology

Functional vs. Structural networks

Box 1 | Structural and functional brain networks





Bullmore, E., & Sporns, O. (2009). Complex brain networks: graph theoretical analysis of structural and functional systems. Nature reviews. Neuroscience, 10(3), 186-98. doi: 10.1038/nrn2575.

Network / Graph topology

Functional vs. Structural networks

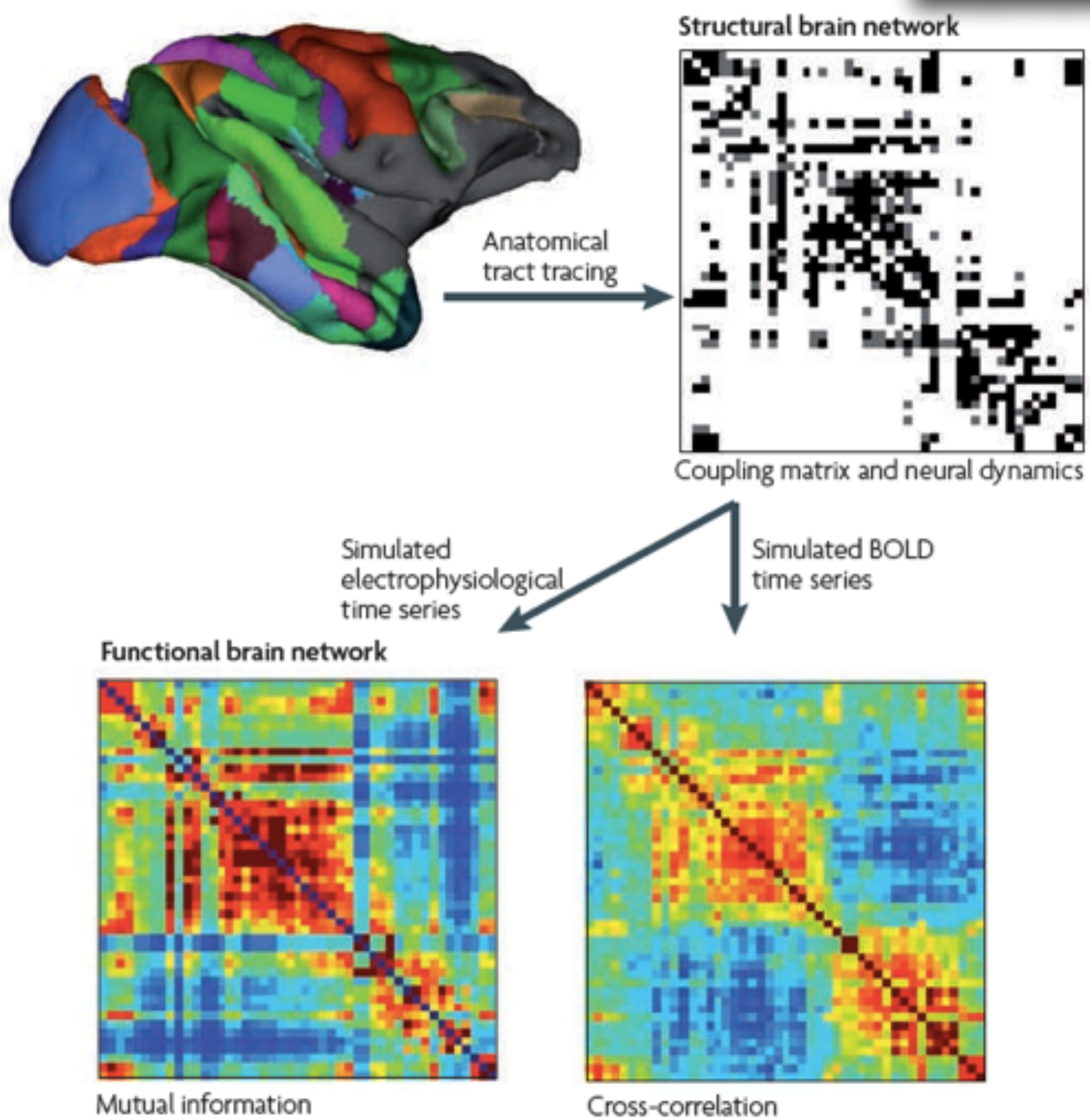


Figure 1 | **Computational modelling of structural and functional brain networks.**

adjacency matrix
similarity matrix
coupling matrix

Compare:
Recurrence matrix

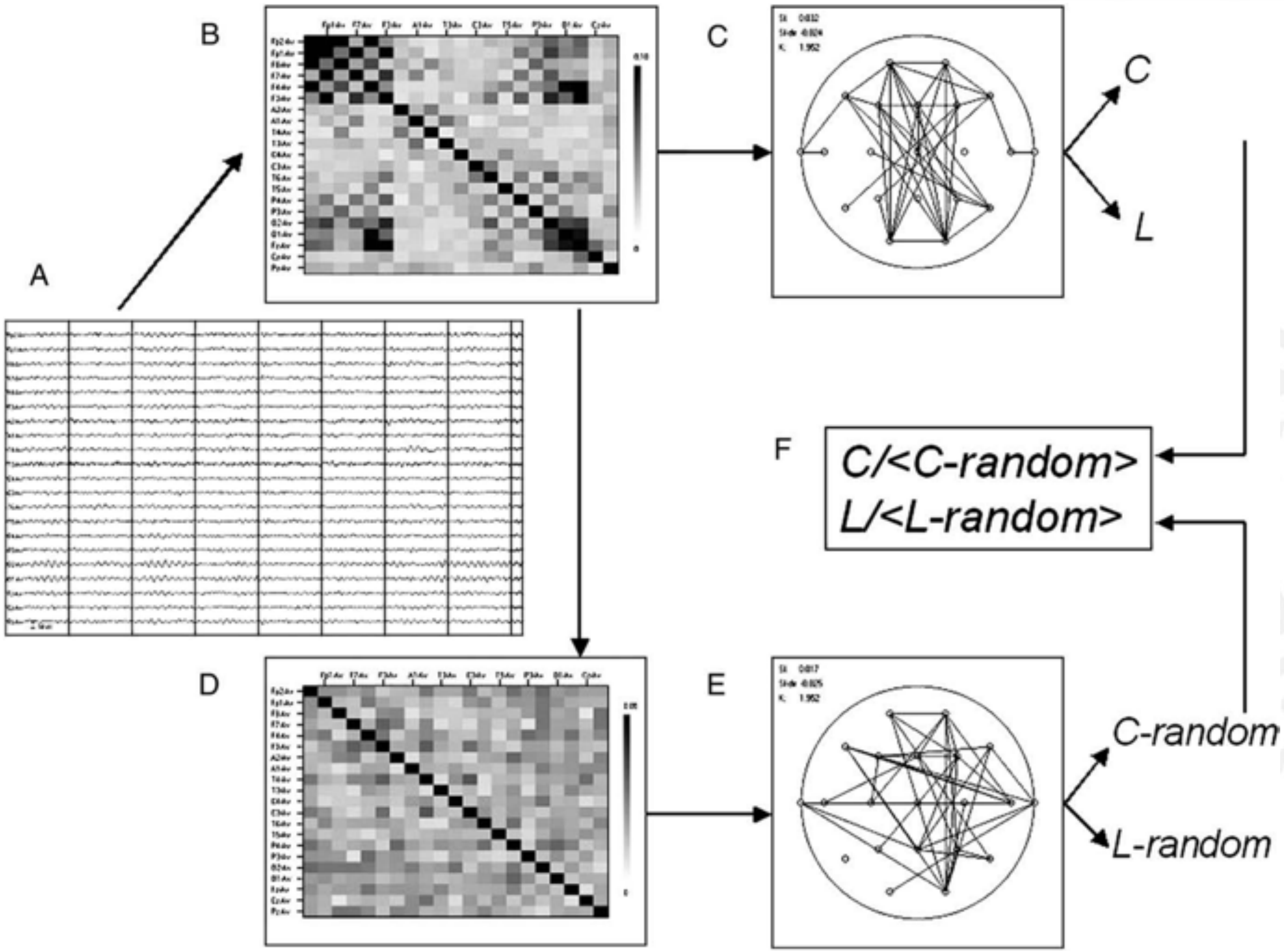
"neural dynamics"

Compare:
Surrogate data generation from recurrence matrix

Bullmore, E., & Sporns, O. (2009). Complex brain networks: graph theoretical analysis of structural and functional systems. Nature reviews. Neuroscience, 10(3), 186-98. doi: 10.1038/nrn2575.

Network / Graph topology

How to get the matrices



Adjacency matrix and weighted graph can be extracted from resting state recordings:

5 min. eyes closed

find 4096 samples without artefacts

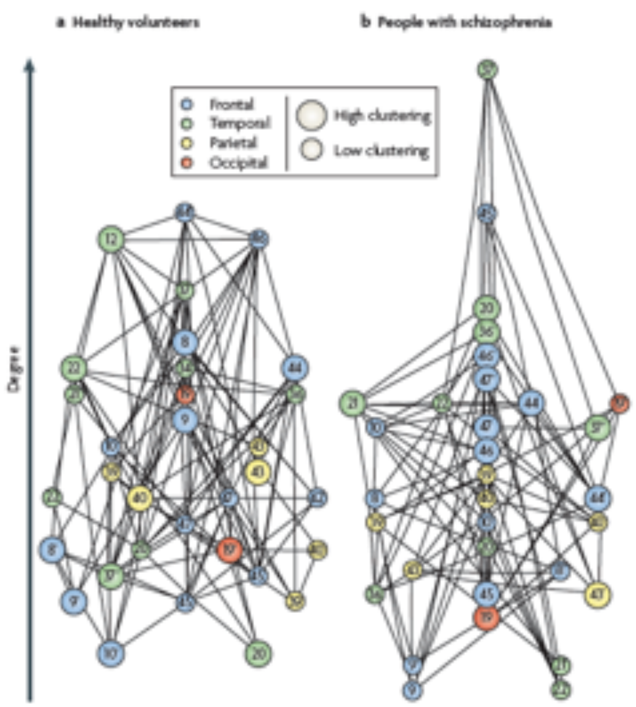
That's about 6-7 seconds!

Stam, C. J. (2010). Characterization of anatomical and functional connectivity in the brain: A complex networks perspective. International journal of psychophysiology : official journal of the International Organization of Psychophysiology, 77(3), 186-194. Elsevier B.V. doi: 10.1016/j.ijpsycho.2010.06.024.

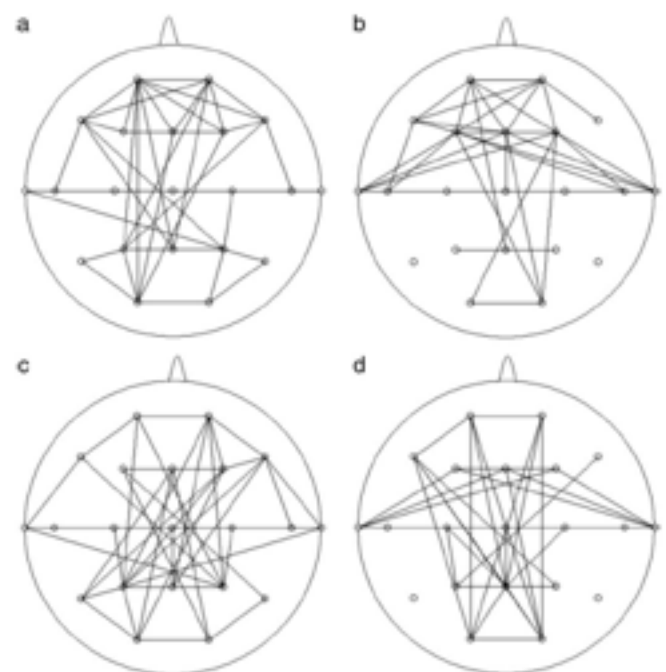
Network / Graph topology

Pathology studies

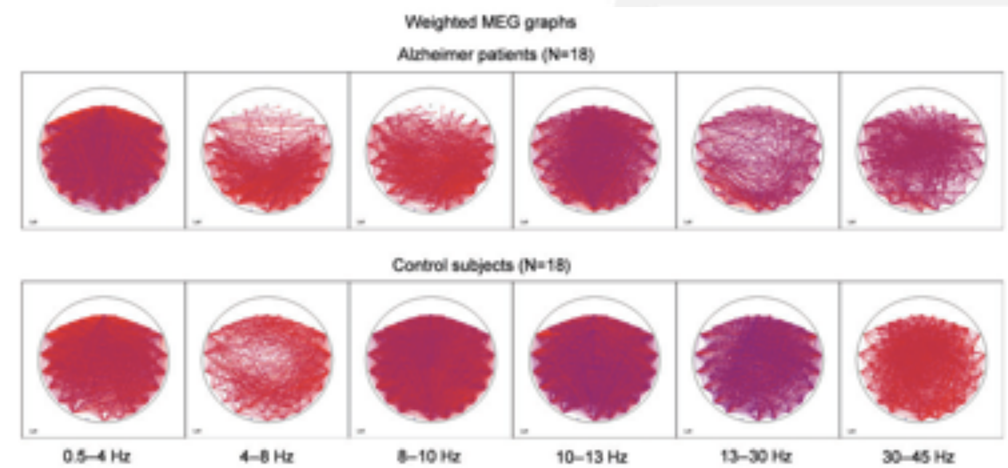
Based on a few samples we can distinguish healthy subjects from patients:



Schizophrenia



Absence seizure



Alzheimer's disease: Targeted attack on hubs!

Parkinson's

Major depressive disorder

Epilepsy

brain tumor patients

Bartolomei, F., Bosma, I., Klein, M., Baayen, J. C., Reijneveld, J. C., Postma, T. J., et al. (2006). Disturbed functional connectivity in brain tumour patients: evaluation by graph analysis of synchronization matrices. Clinical neurophysiology, 117(9), 2039-49. doi: 10.1016/j.clinph.2006.05.018.

Ponten, S. C., Douw, L., Bartolomei, F., Reijneveld, J. C., & Stam, C. J. (2009). Indications for network regularization during absence seizures: weighted and unweighted graph theoretical analyses. Experimental neurology, 217(1), 197-204. Elsevier Inc. doi: 10.1016/j.expneurol.2009.02.001.

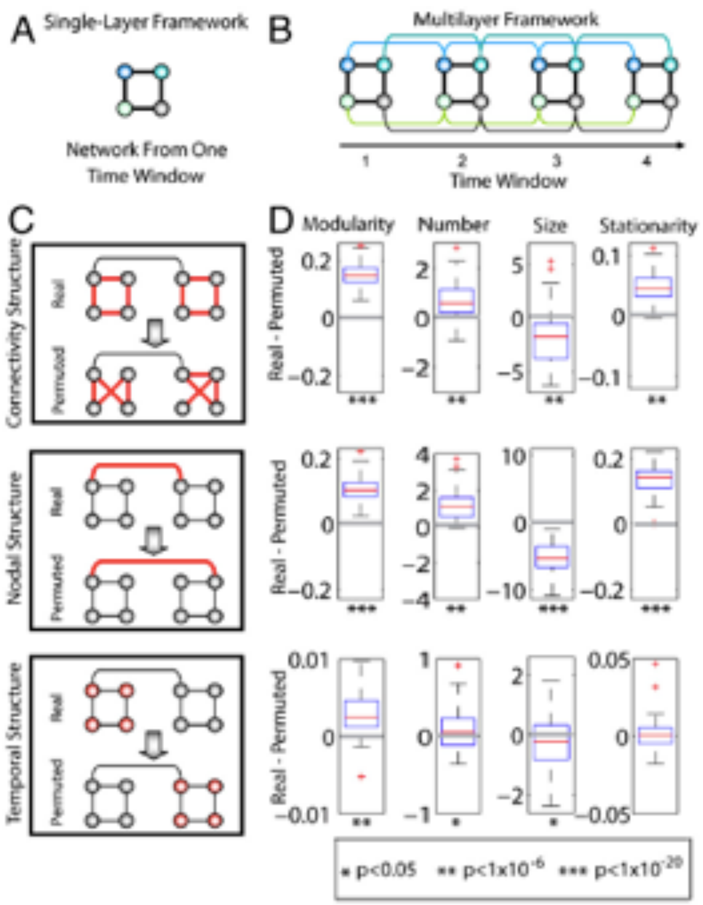
Stam, C. J., Haan, W. de, Daffertshofer, a, Jones, B. F., Manshanden, I., Cappellen van Walsum, a M. van, et al. (2009). Graph theoretical analysis of magnetoencephalographic functional connectivity in Alzheimer's disease. Brain : a journal of neurology, 132(Pt 1), 213-24. doi: 10.1093/brain/awn262.

Stam, C. J. (2010). Use of magnetoencephalography (MEG) to study functional brain networks in neurodegenerative disorders. Journal of the neurological sciences, 289(1-2), 128-34. Elsevier B.V. doi: 10.1016/j.jns.2009.08.028.

Network / Graph topology

Cognition studies

Systematic dynamic reconfiguration / topology differences during / correlated with, performance / characteristics



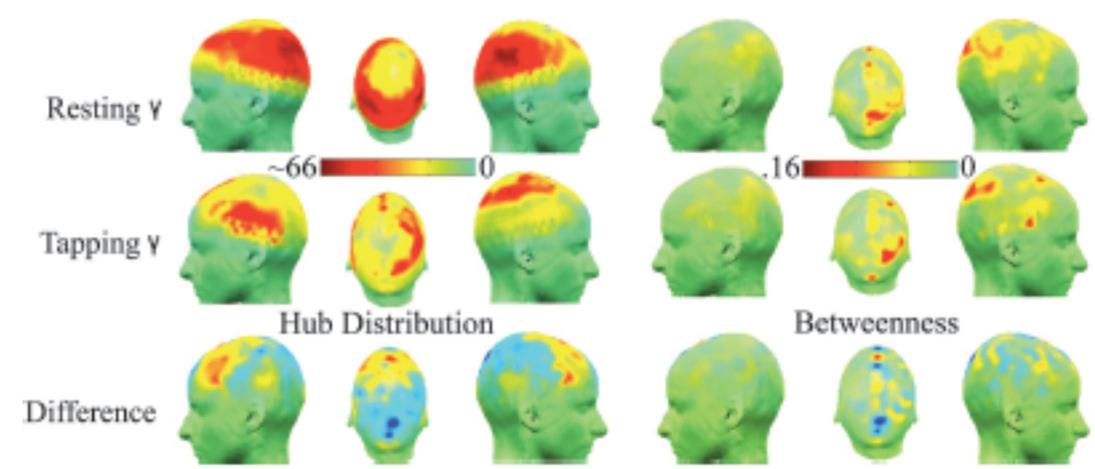
Cognitive abilities

Gender

Age

Learning

Tapping



Bassett, D. S., Meyer-Lindenberg, A., Achard, S., Duke, T., & Bullmore, E. (2006). Adaptive reconfiguration of fractal small-world human brain functional networks. Proceedings of the National Academy of Sciences of the United States of America, 103(51), 19518-23. doi: 10.1073/pnas.0606005103.

Bassett, D. S., Wymbs, N. F., Porter, M. a, Mucha, P. J., Carlson, J. M., & Grafton, S. T. (2011). Dynamic reconfiguration of human brain networks during learning. Proceedings of the National Academy of Sciences. doi: 10.1073/pnas.1018985108.

Douw, L., Schoonheim, M. M., Landi, D., Meer, M. L. van der, Geurts, J. J. G., Reijneveld, J. C., et al. (2011). Cognition is related to resting-state small-world network topology: an magnetoencephalographic study. Neuroscience, 175, 169-77. Elsevier Inc. doi: 10.1016/j.neuroscience.2010.11.039.

Network / Graph topology - Social networks

Sex	ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
F	1					+				−	−		+								+	−			
M	2	−								+						+				+			−	−	
F	3						+	−			−		+				+					−			
F	4										−		+	+			+			−	−				
F	5	+									−			+	+					−	−				
F	6	−		+									+			−	+						−		
M	7		+								−	+								−		−		+	
F	8				+		−							+				−		+					−
M	9		+					+				+		−		−									−
M	10		+					−				−				+				+		−			
M	11		+								−					+		+		−	−				
F	12	+						−			−					−	+				+				
F	13				+								+				+			−	−	−			
F	14					+	−	+		−											+		−		
M	15							+			−				−					+				+	−
F	16				+						−		+								+		−	−	
M	17				−							+								+	−	−		+	
M	18							−				+								−		−	+		+
M	19		−									+	−			+					+	−			
F	20						−			−			+		−		+			+					
F	21	−	−		+								+							−	+				
M	22						−			−		+					−	+							+
M	23	−						+				+						−		+		−			
M	24											+						+			−	−	+	−	
	+	2	4	1	4	2	1	4	0	1	0	8	8	3	1	4	6	3	0	7	6	0	2	3	2
	−	4	2	0	1	0	4	4	0	4	9	1	1	1	2	3	1	2	0	7	6	10	4	3	3

Figure 9.6: Data on the three most liked or disliked classmates.

http://www.distributed-systems.net/gtcn/

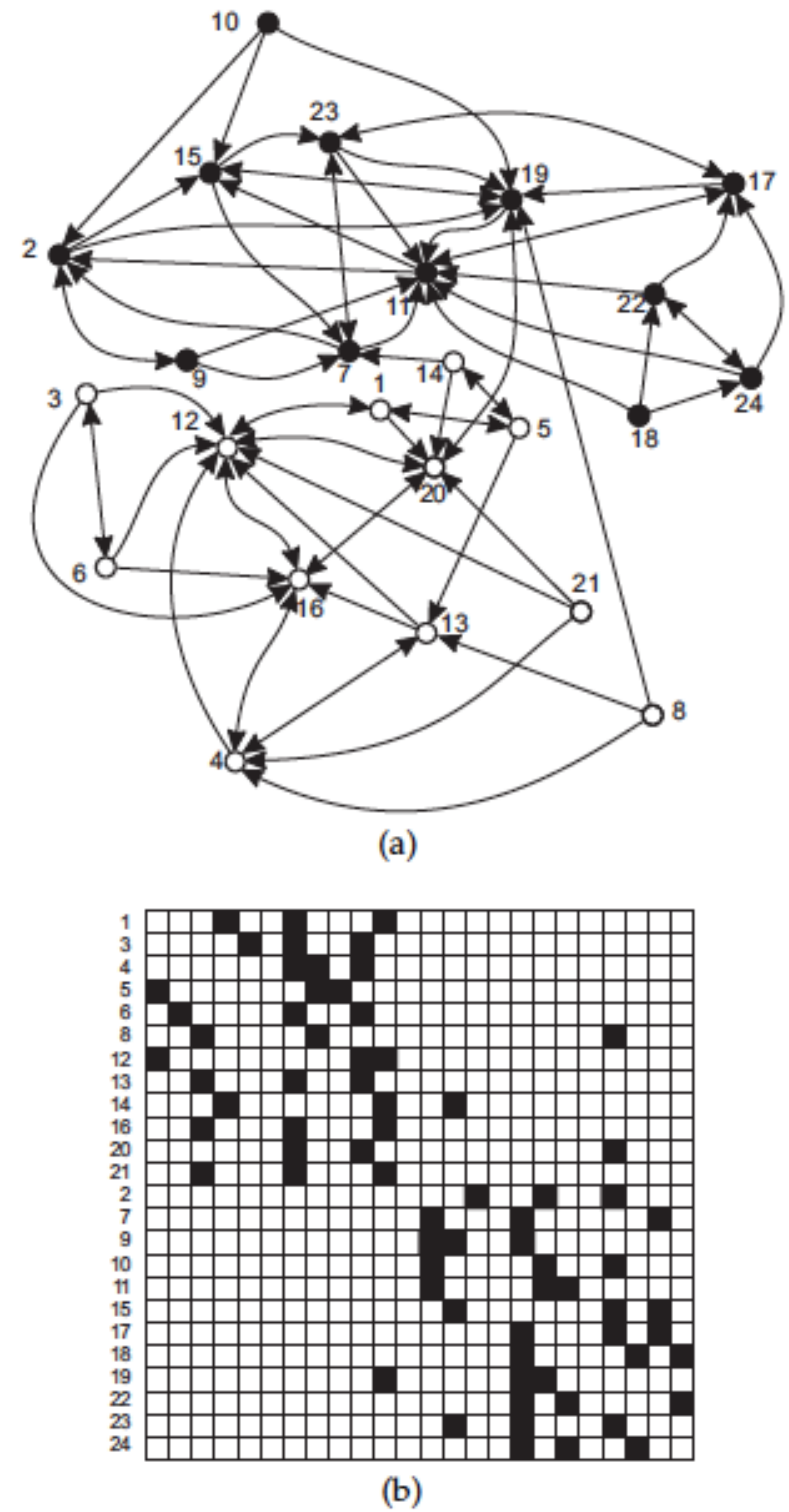


Figure 9.7: (a) The sociogram for positive nominations represented as a directec graph. Boys are represented by black-colored vertices; girls by white-colored ver

Network / Graph topology - Social networks

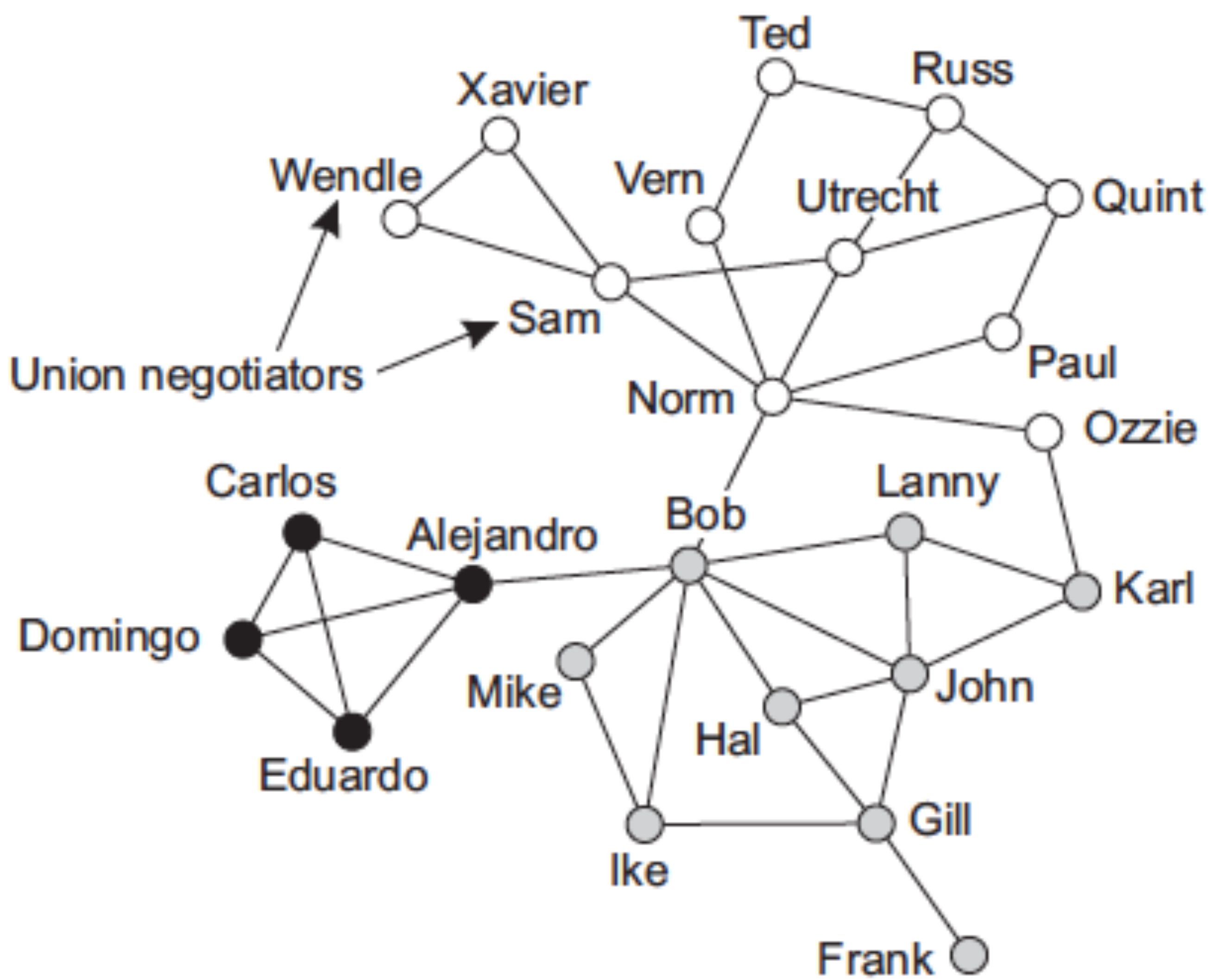


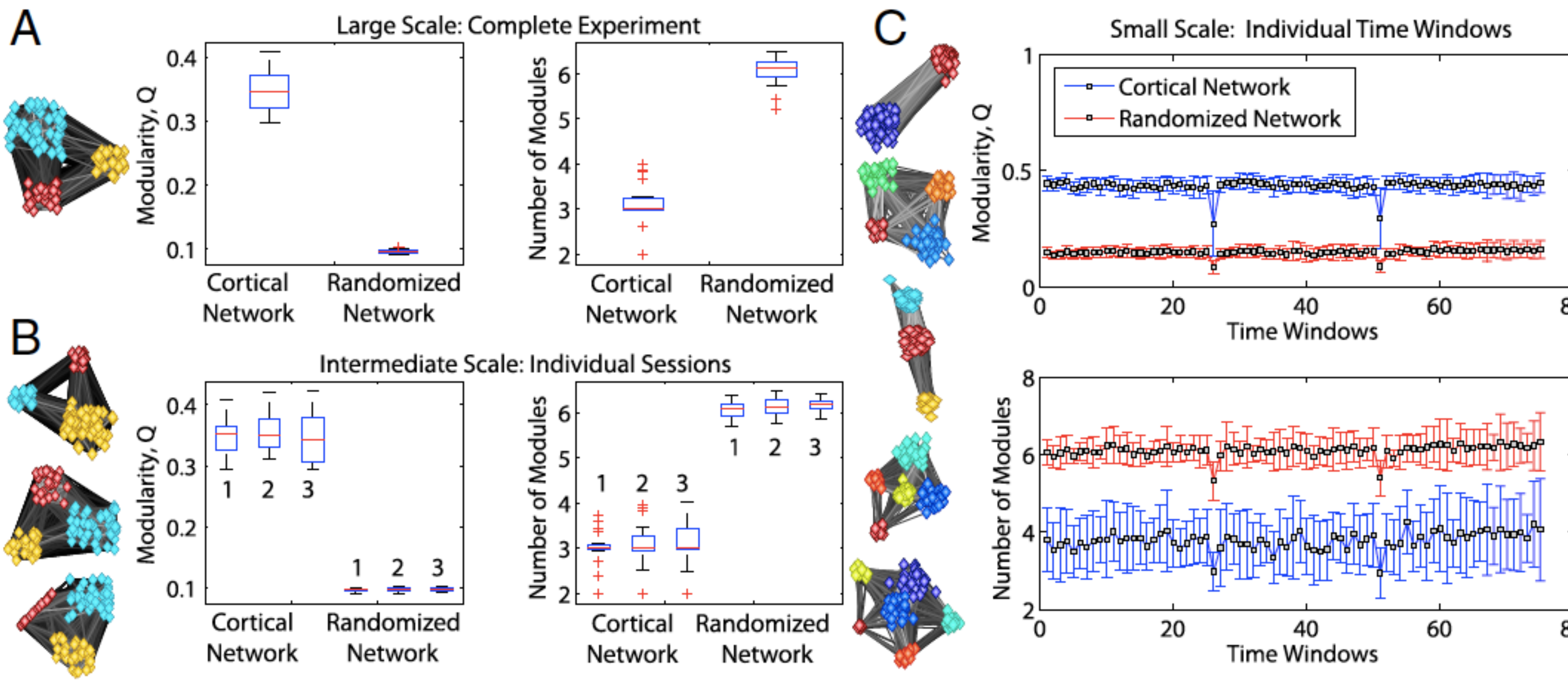
Figure 9.1: The relationship between workers on strike in a wood-processing firm.

http://www.distributed-systems.net/gtcn/

Can complex networks provide a suitable structure?

Multi-scale analysis

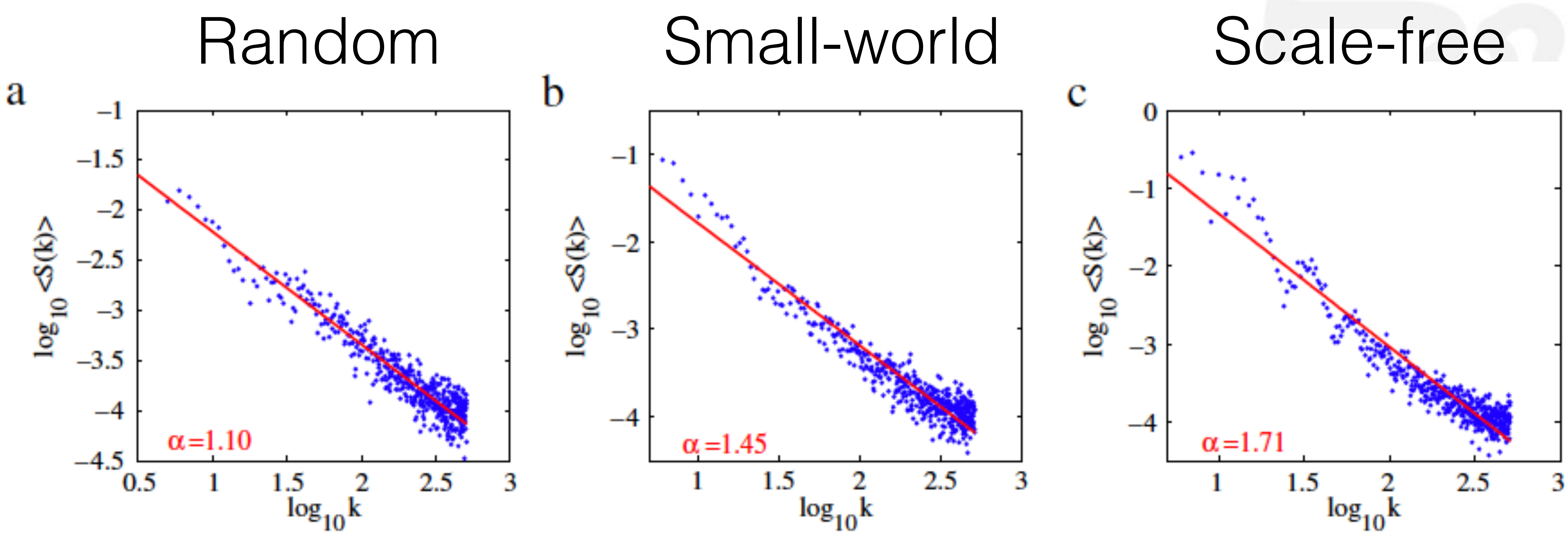
see also: Wijnants, M. L., Cox, R. F. A., Hasselman, F., Bosman, A. M. T., & Van Orden, G. (2012). A Trade-Off Study Revealing Nested Timescales of Constraint. *Frontiers in Fractal Physiology*, *3*(May), 1-15. doi:10.3389/fphys.2012.00116



Bassett, D. S., Wymbs, N. F., Porter, M. a., Mucha, P. J., Carlson, J. M., & Grafton, S. T. (2011). Dynamic reconfiguration of human brain networks during learning. *Proceedings of the National Academy of Sciences*, 1-36. doi:10.1073/pnas.1018985108

Network / Graph topology

1/f$^{\alpha}$ Noise in Spectral Fluctuations of Complex Networks

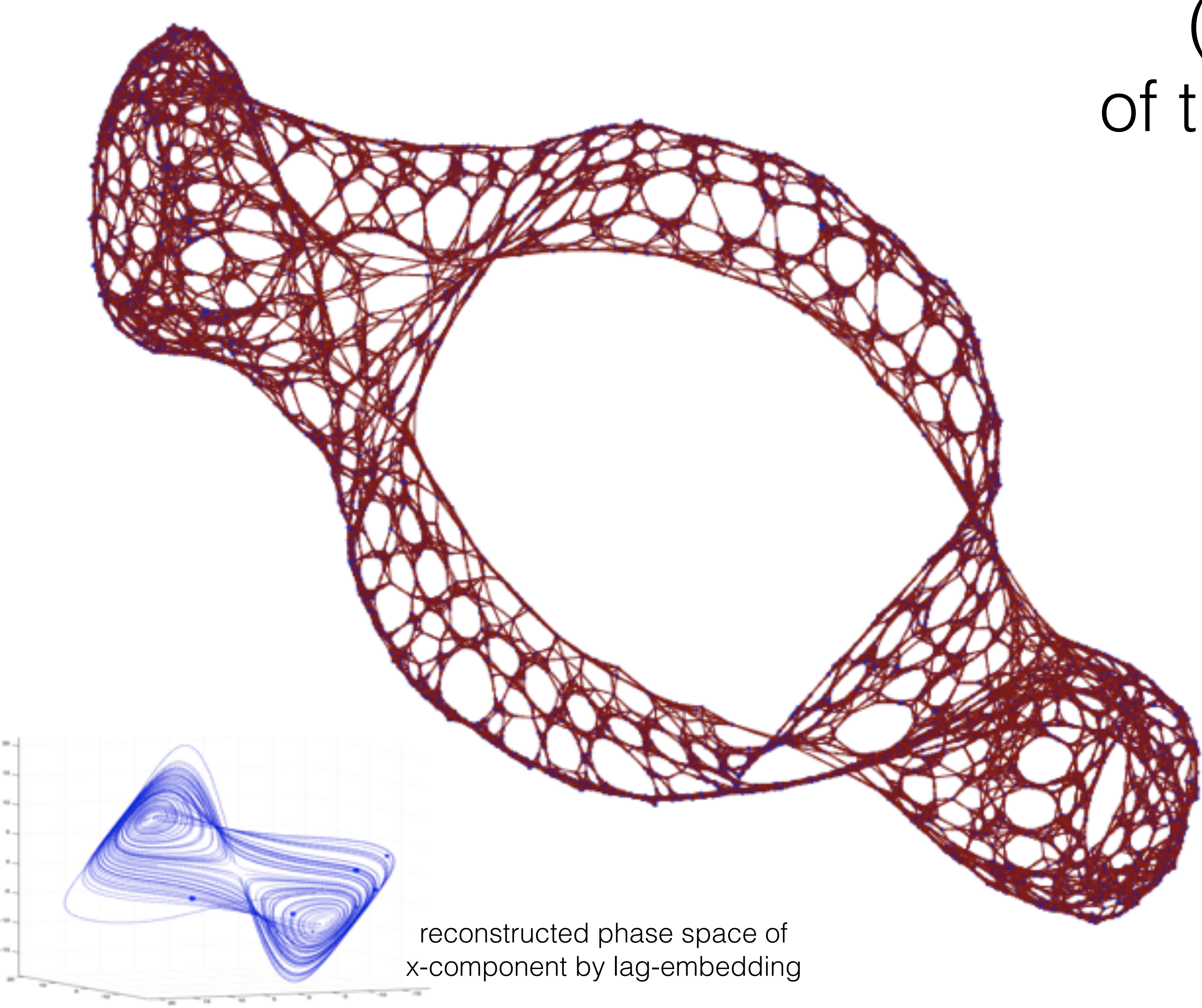


Spectrum of nearest-neighbour spacing
in the adjacency matrix

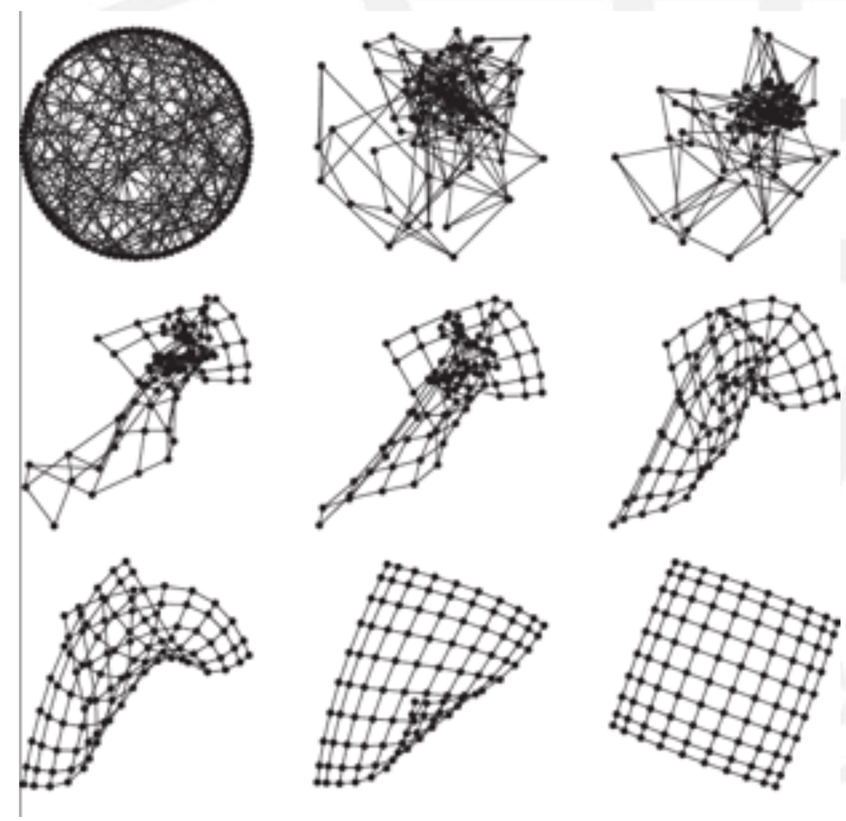
Ye, B., Li, H.-jun, & Ma, X.-ping. (2010). 1/Fα Noise in Spectral Fluctuations of Complex Networks. Physica A: Statistical Mechanics and its Applications, 389(22), 5328-5331. Elsevier B.V. doi: 10.1016/j.physa.2010.07.023.

Network / Graph topology

Network representation
(spring embedding)
of the x-component of the
Lorenz-system



reconstructed phase space of
x-component by lag-embedding

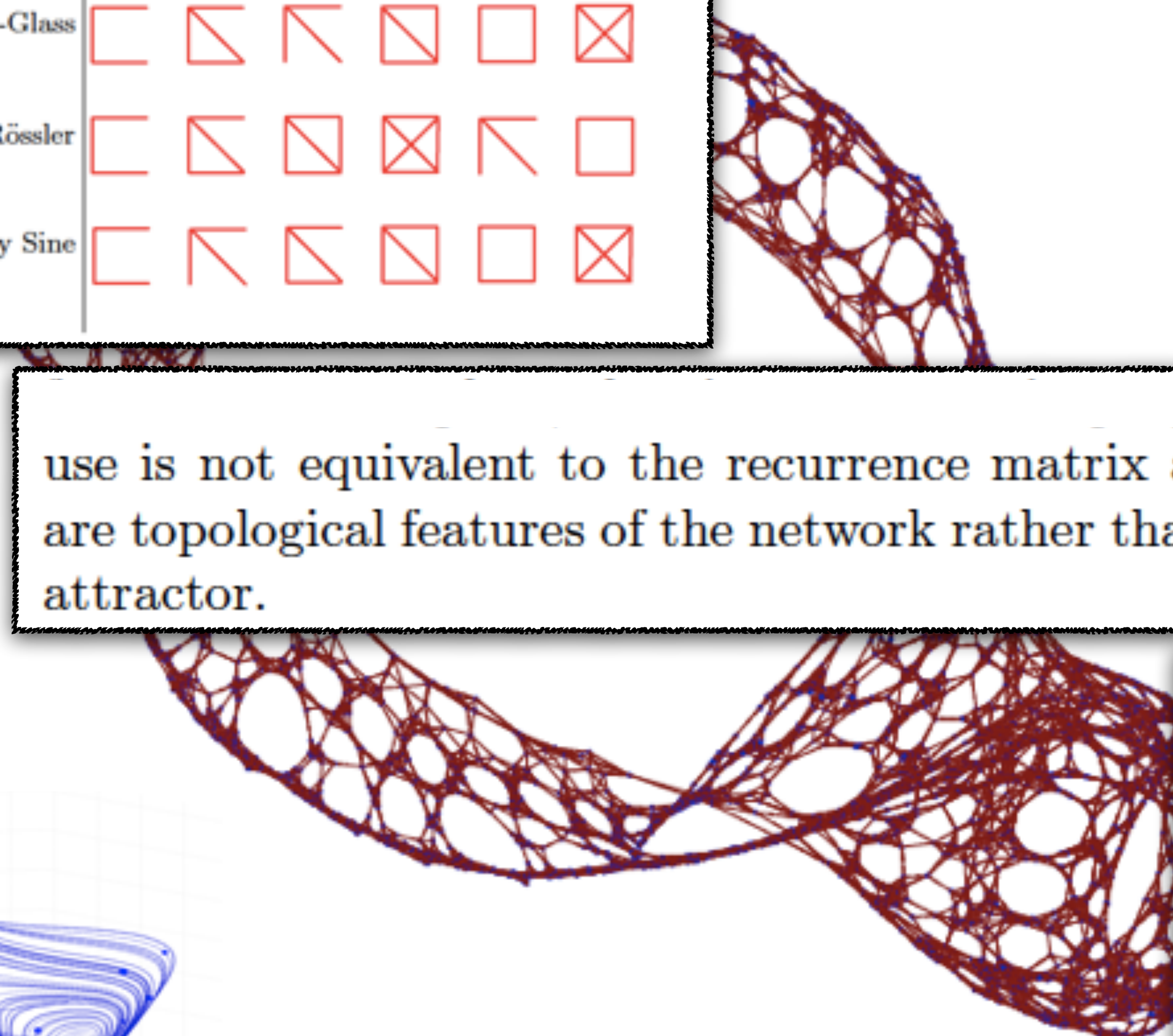


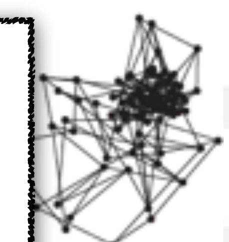
Small, M., Zhang, J., & Xu, X. (2009). Transforming time series into complex networks. Complex Sciences, 2078–2089.

Network / Graph topology

Network representation (spring embedding) of the x-component of the Lorenz-system

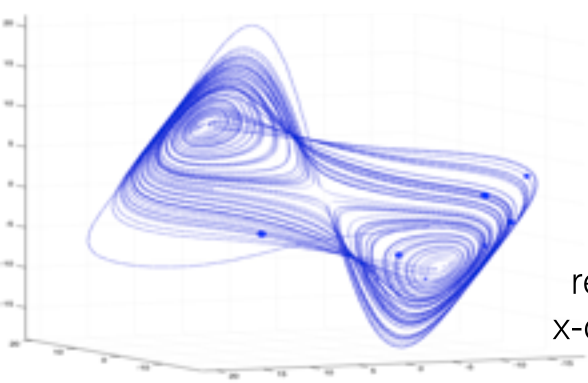
Data Source	Motif frequency
Chaotic Lorenz Chaotic Rössler Chaotic Chua's circuit	
Hyper-chaotic Mackey-Glass	
Periodic Rössler	
Noisy Sine	





The adjacency matrix we use is not equivalent to the recurrence matrix and the properties we examine are topological features of the network rather than the temporal structure of the attractor.

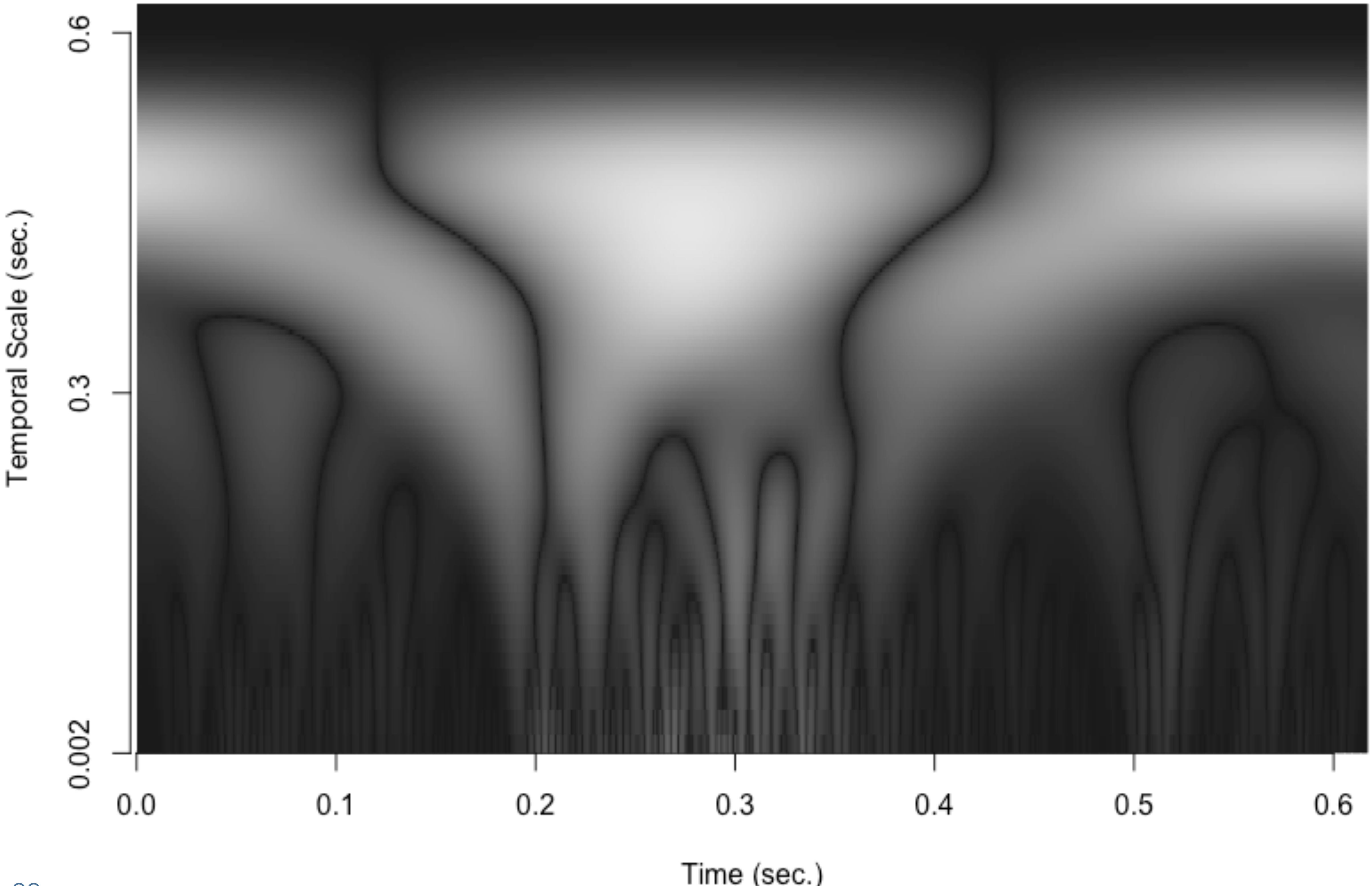
Data Source	Motif frequency
Chaotic logistic map Chaotic Hénon map Chaotic Ikeda map	
Hyper-chaotic folded towel map Hyper-chaotic generalised Hénon map	
White noise Fractal noise	



reconstructed phase space of x-component by lag-embedding

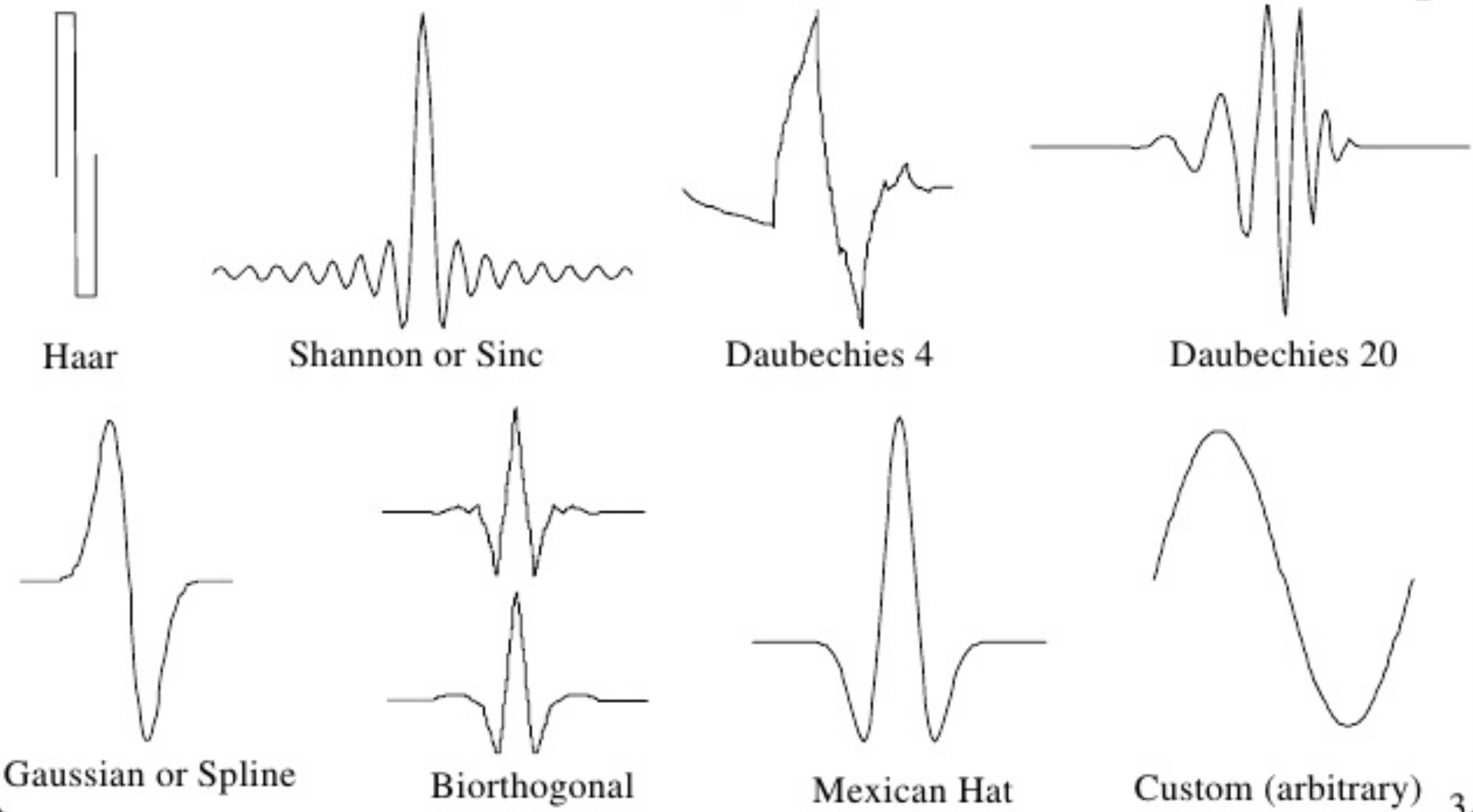
Small, M., Zhang, J., & Xu, X. (2009). Transforming time series into complex networks. Complex Sciences, 2078–2089.

Scaleogram - Continuous Wavelet Transform



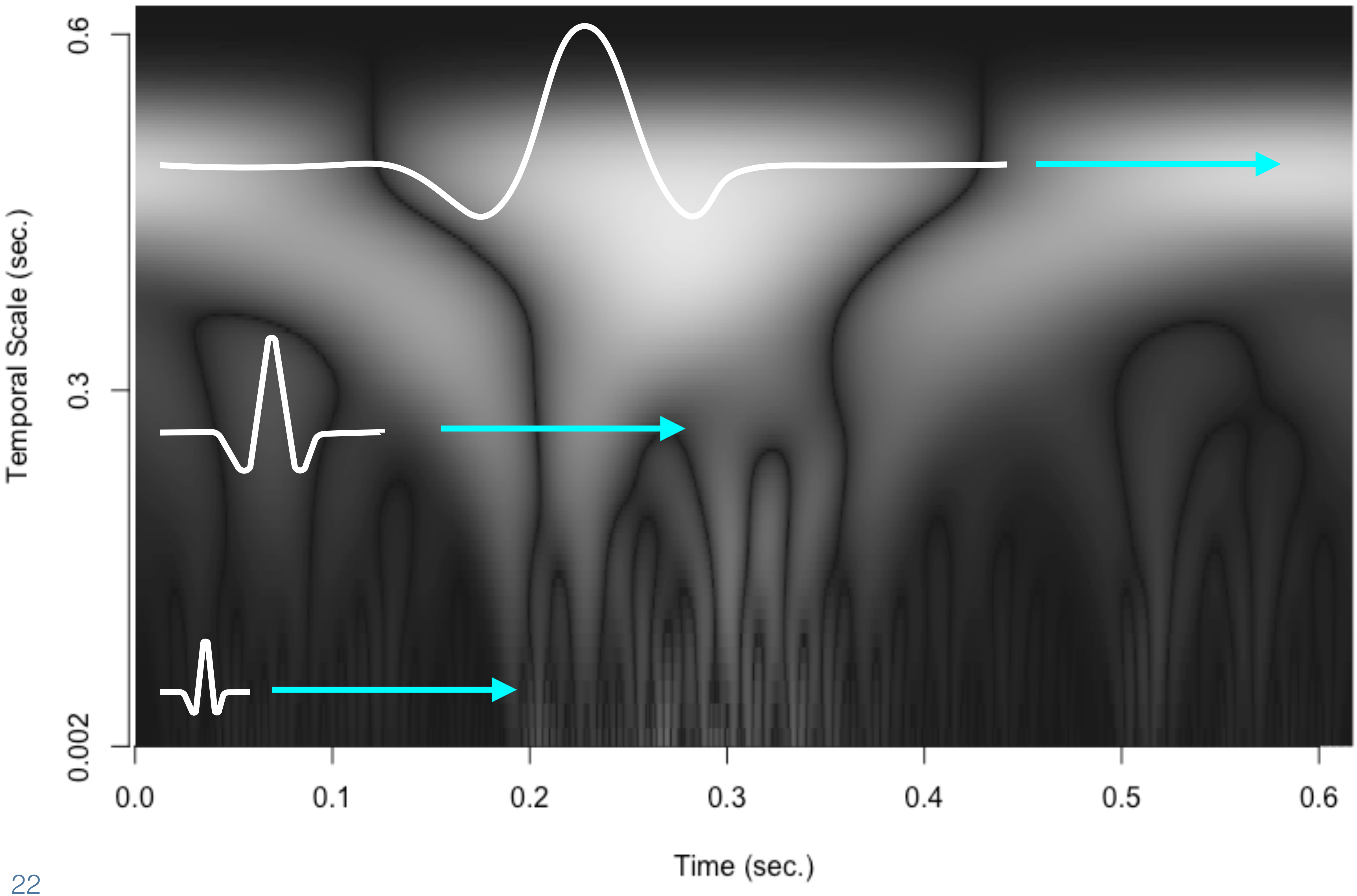
EXAMPLES OF WAVELETS

- WAVE for Frequency, LET indicates Compact Support.
- Jargon Alert*: Compact Support = having start & stop time
- Some more localized in time, some more localized in freq.

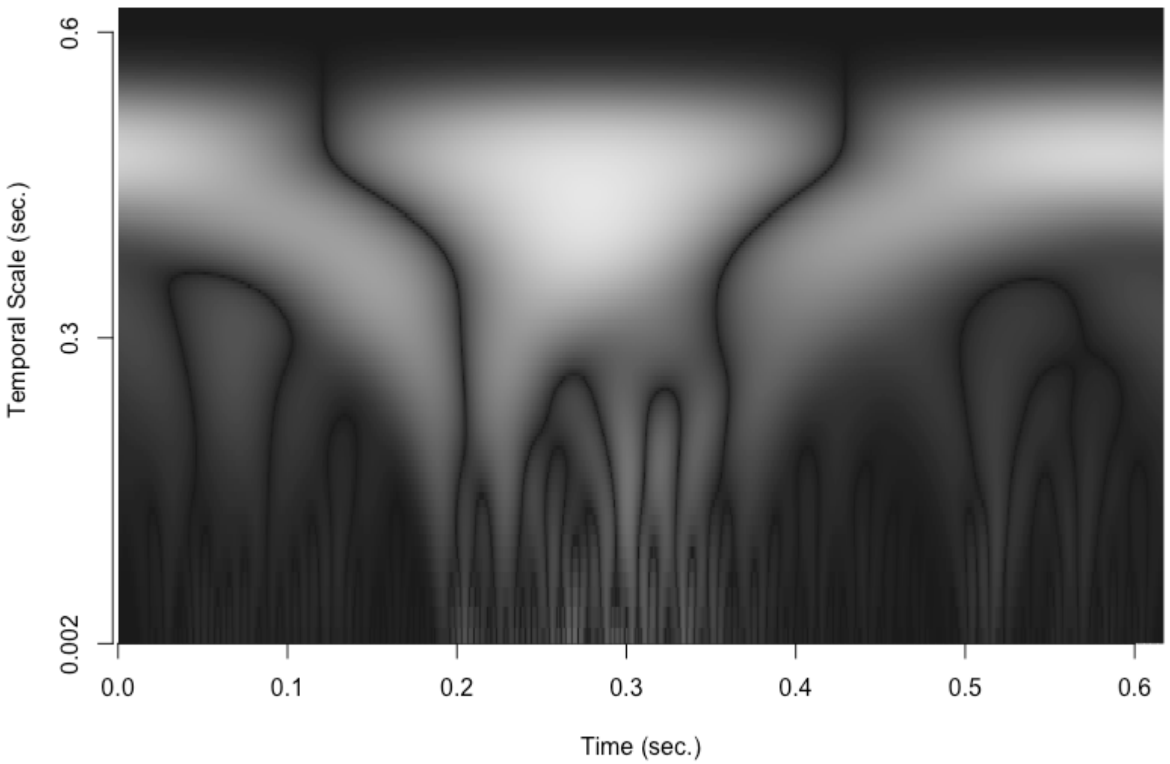


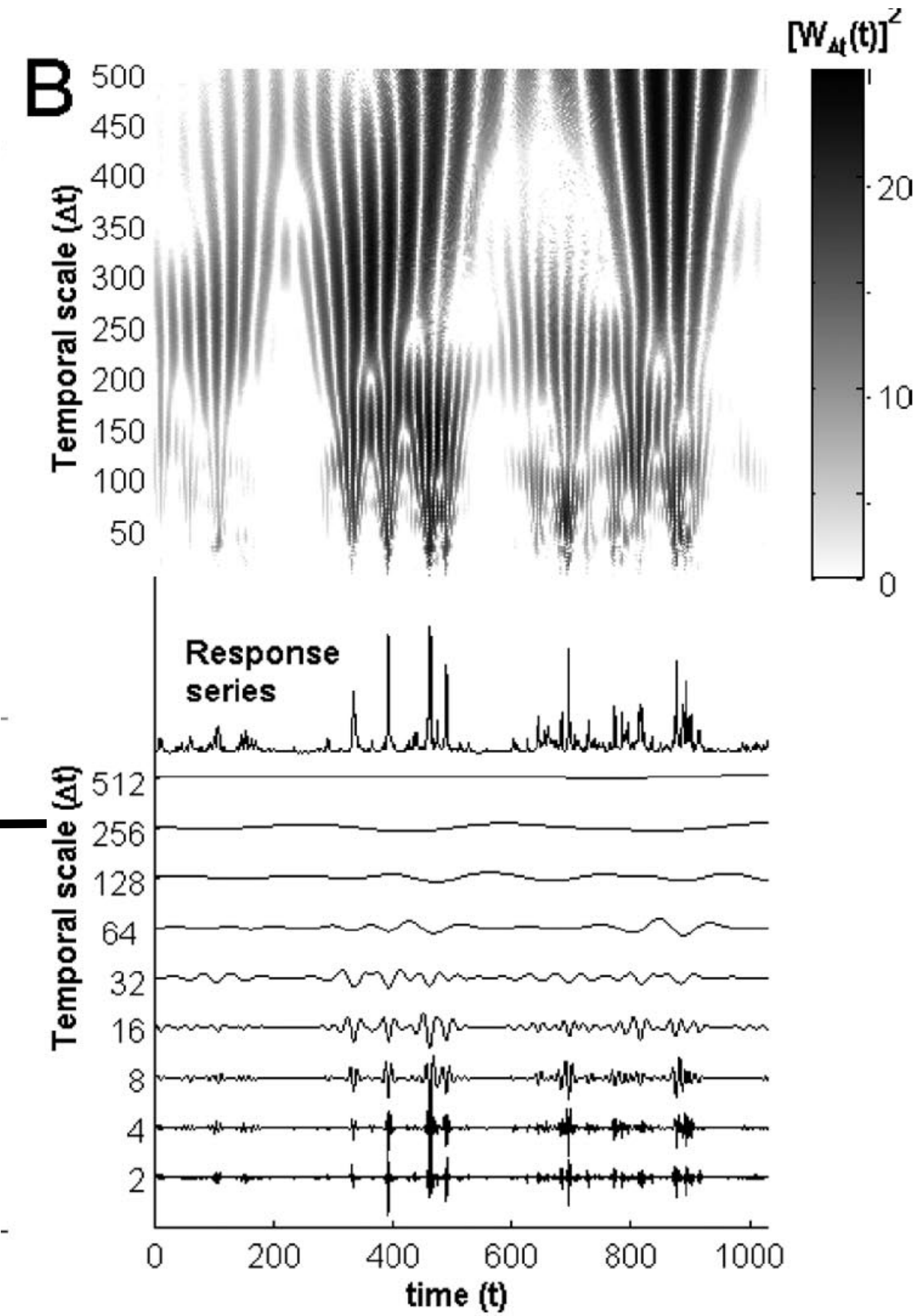
 www.ConceptualWavelets.com 877-845-6459

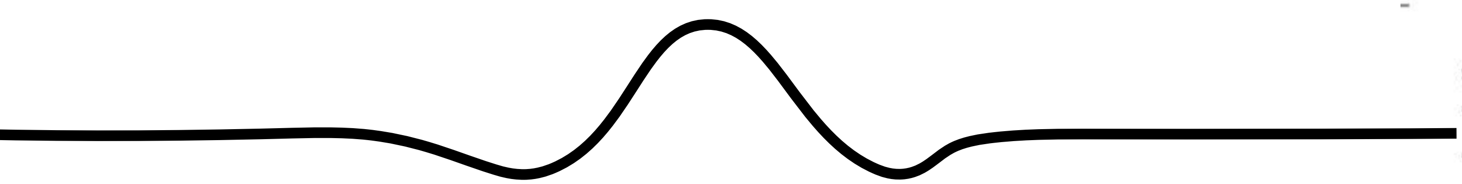
Scaleogram - Continuous Wavelet Transform

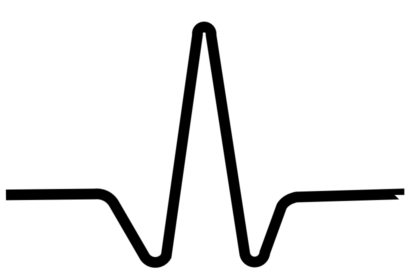


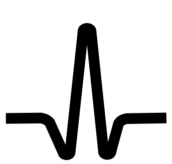
Scaleogram - Continuous Wavelet Transform

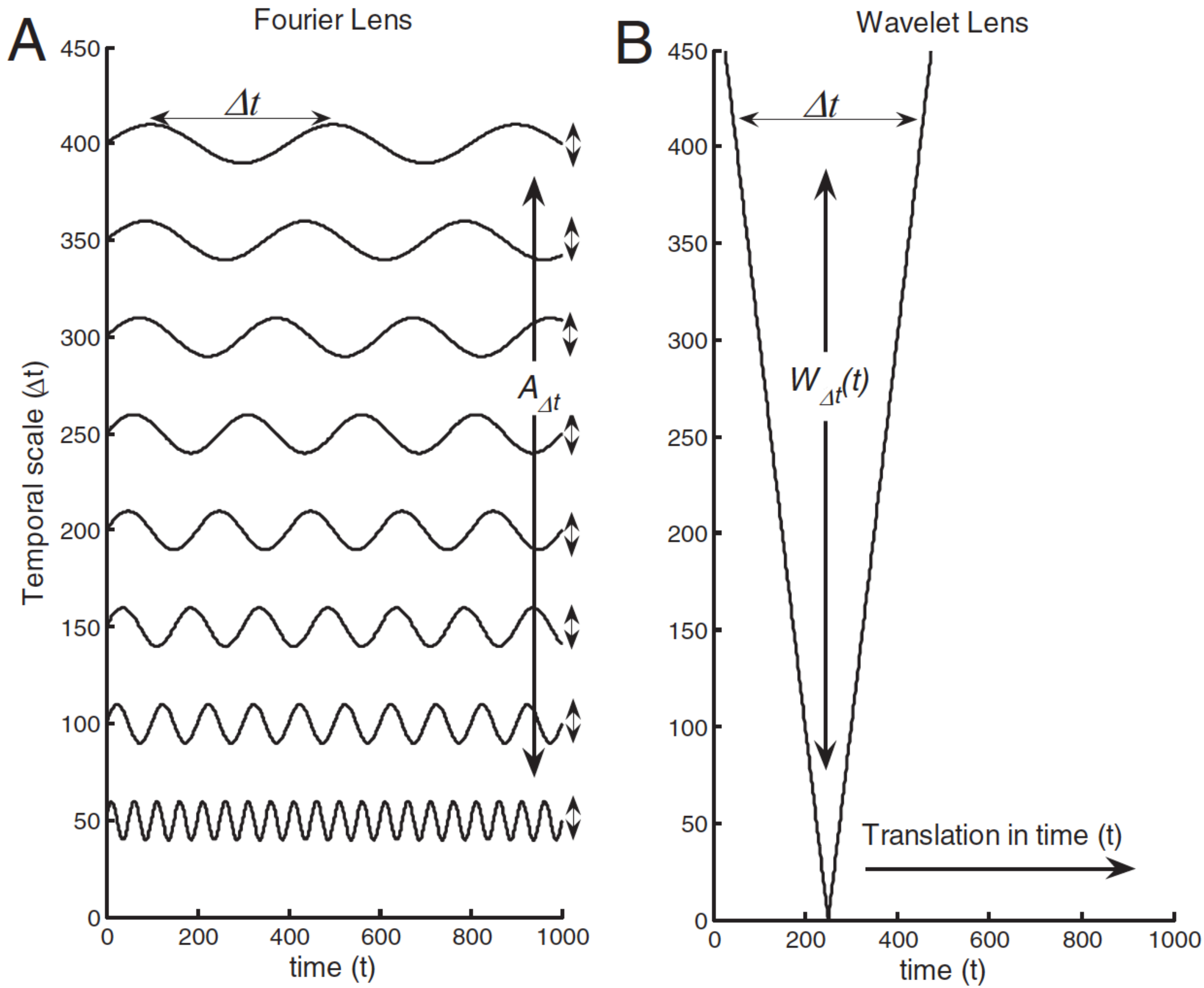






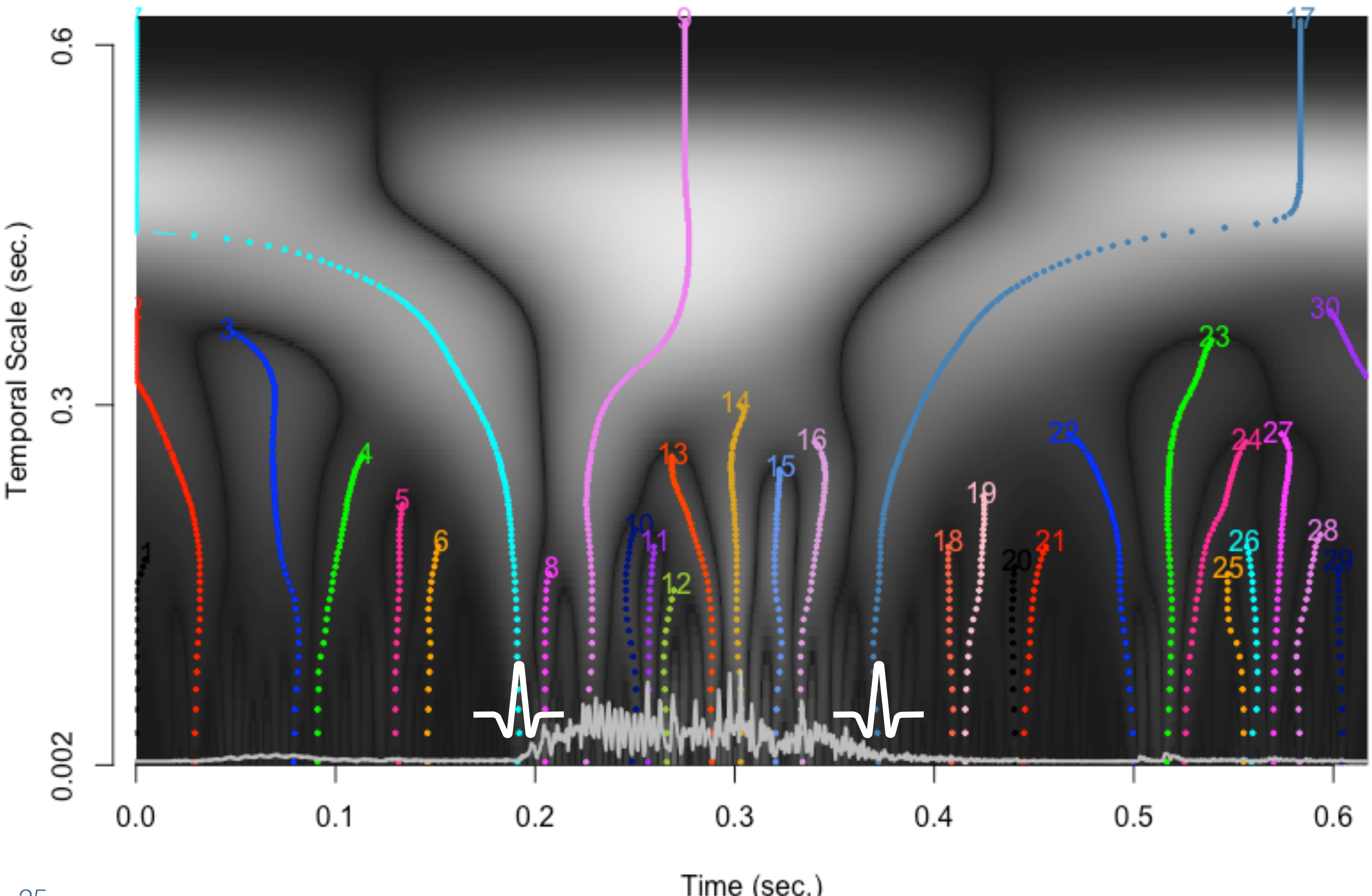




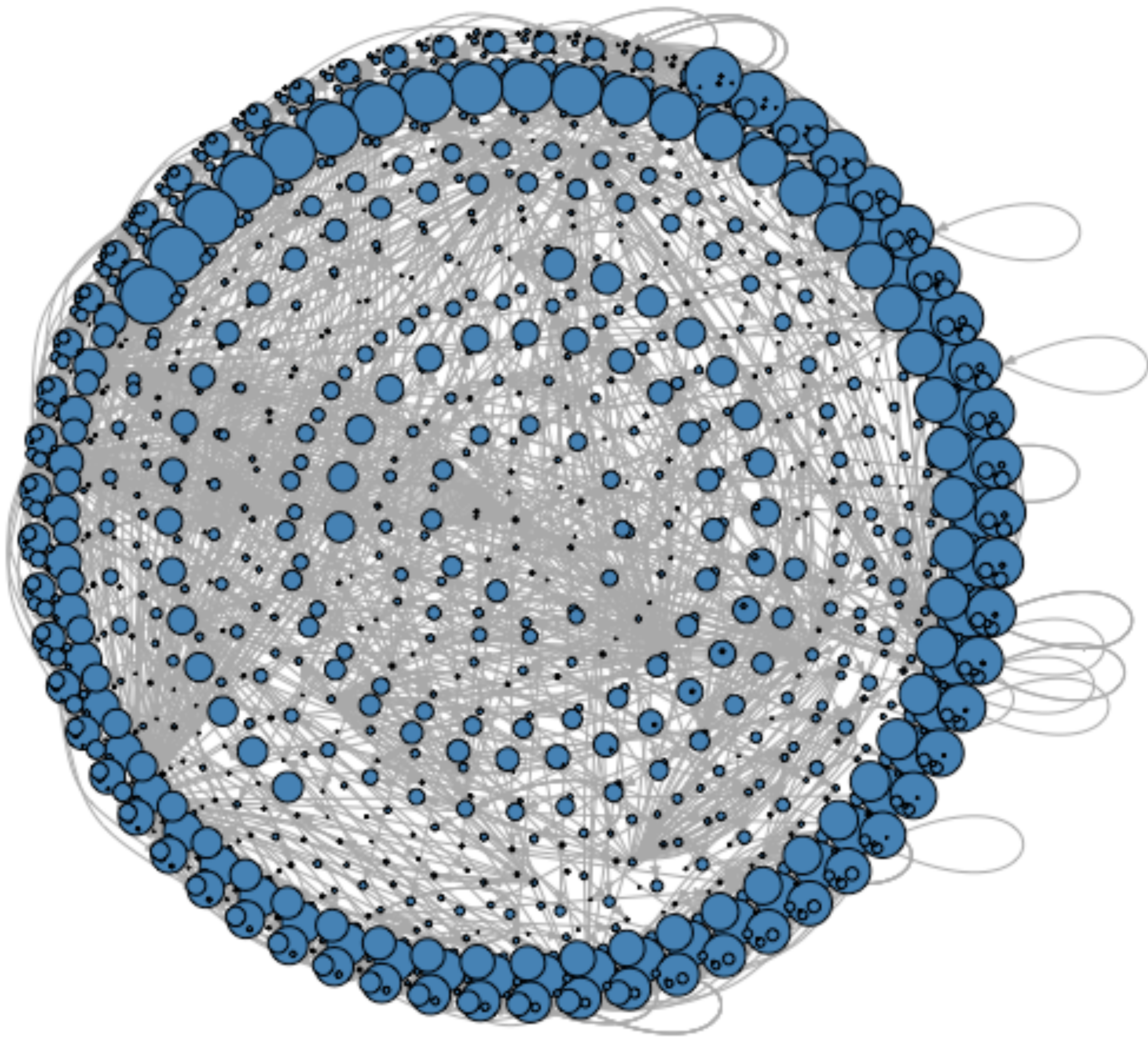

A
Fourier Lens
Δt
A_Δt
Temporal scale (Δt)
time (t)
0
50
100
150
200
250
300
350
400
450
0
200
400
600
800
1000
B
Wavelet Lens
Δt
W_Δt(t)
Translation in time (t)
time (t)
0
50
100
150
200
250
300
350
400
450
0
200
400
600
800
1000

Self-Affine Resonance

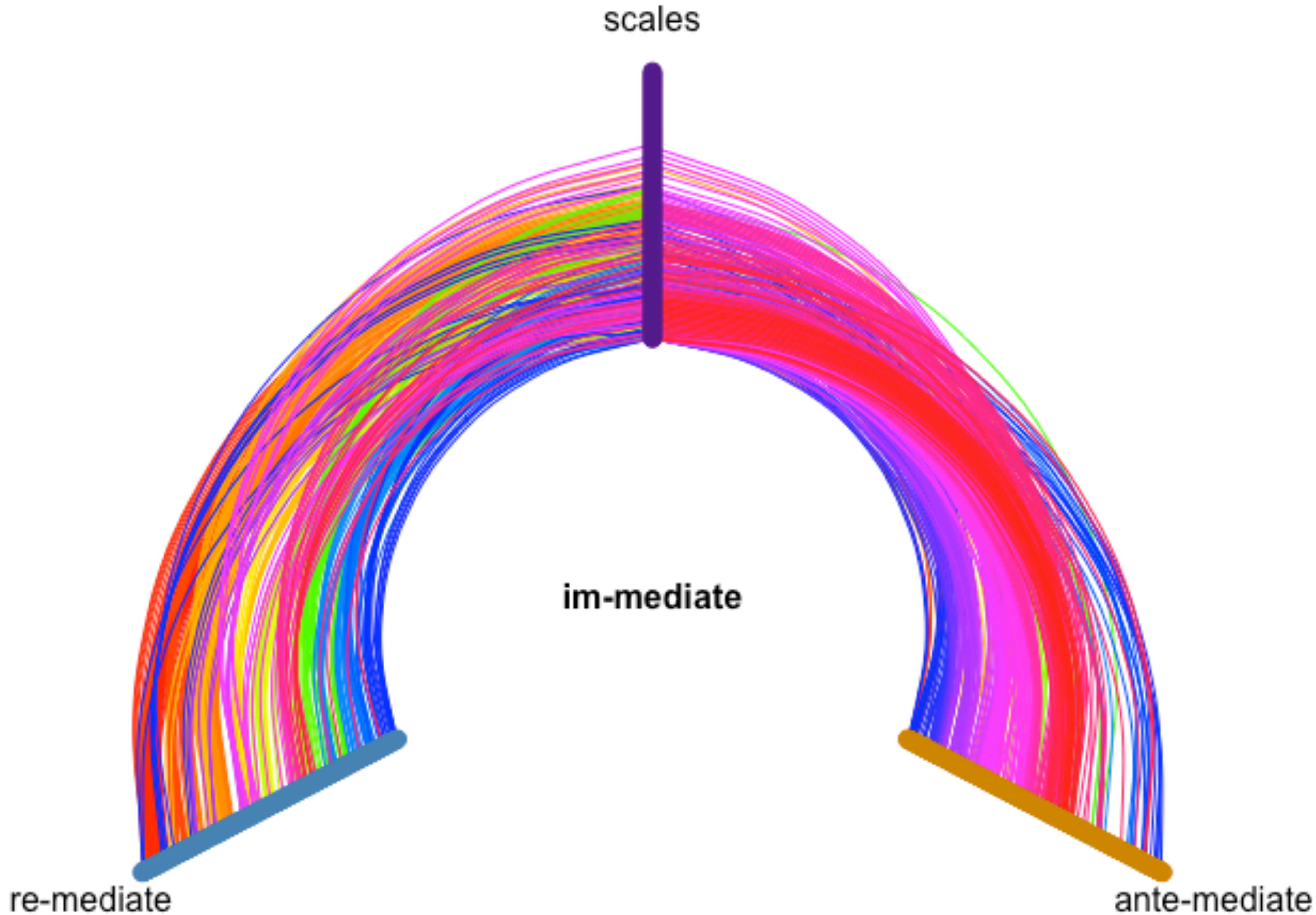
Mannigfaltigkeit der unmittelbaren (Sinnes-) Erlebnisse"
-Einstein (1954)



Network - Singularity Skeleton



Network - Singularity Skeleton



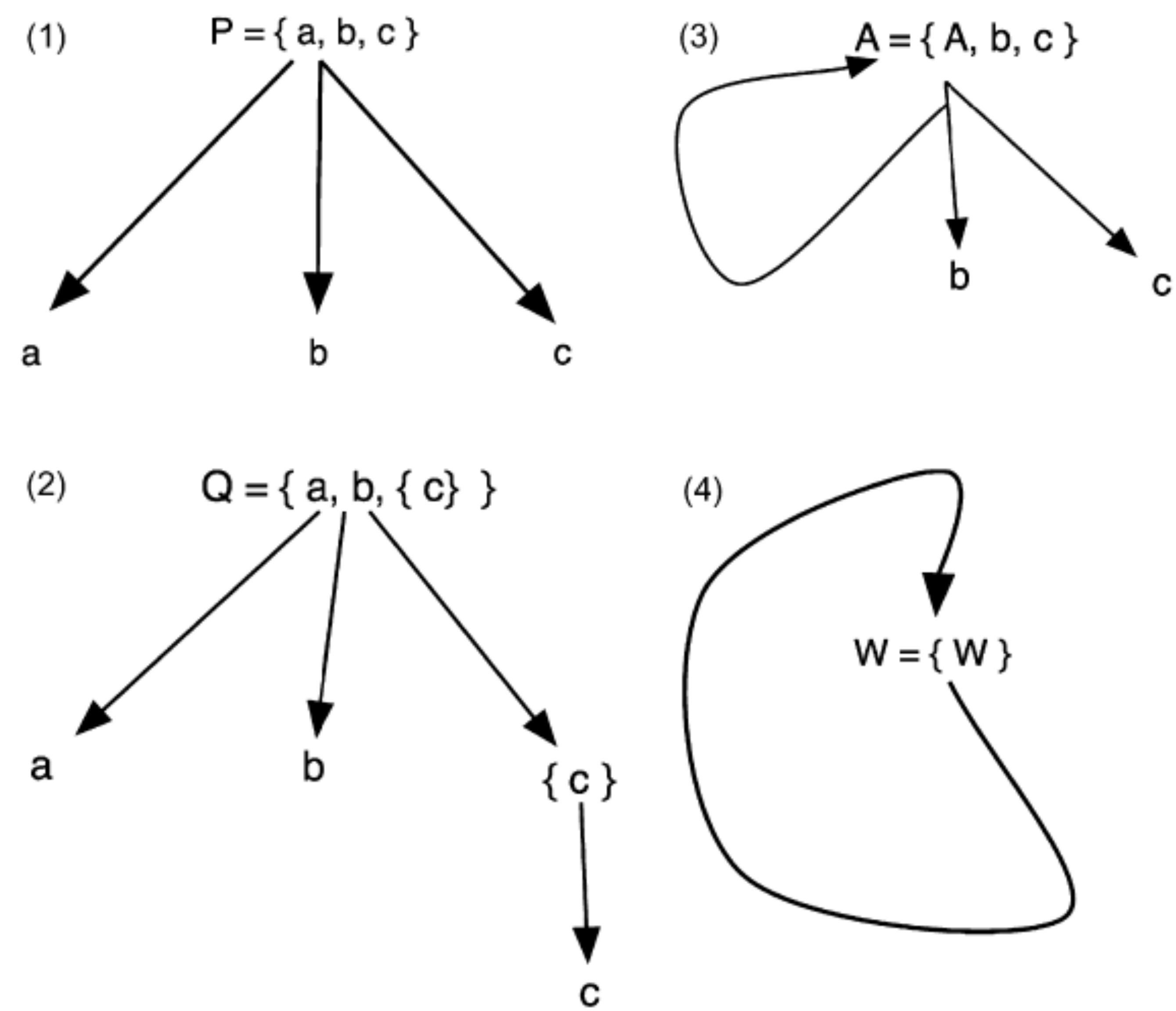
reproduction of similarity by analogy

Complex structure:
Hyperset theory + Graph theory = Hyperset Graphs

Aczel's Anti-Foundation Axiom (1988)
(hyperset theory, circular causality, complexity analysis)

Non well-founded sets:
The definition of a set contains the set itself

A. Chemero, M.T. Turvey / BioSystems 91 (2008) 320–330



Aczel, P. (1988). *Non-well-founded sets. Foreword by Jon Barwise*. CSLI Lecture Notes, 14.

Complex structure:
Hyperset theory + Graph theory = Hyperset Graphs

Impredicative loop

Hyperset loop

Rosen's definition of a living system

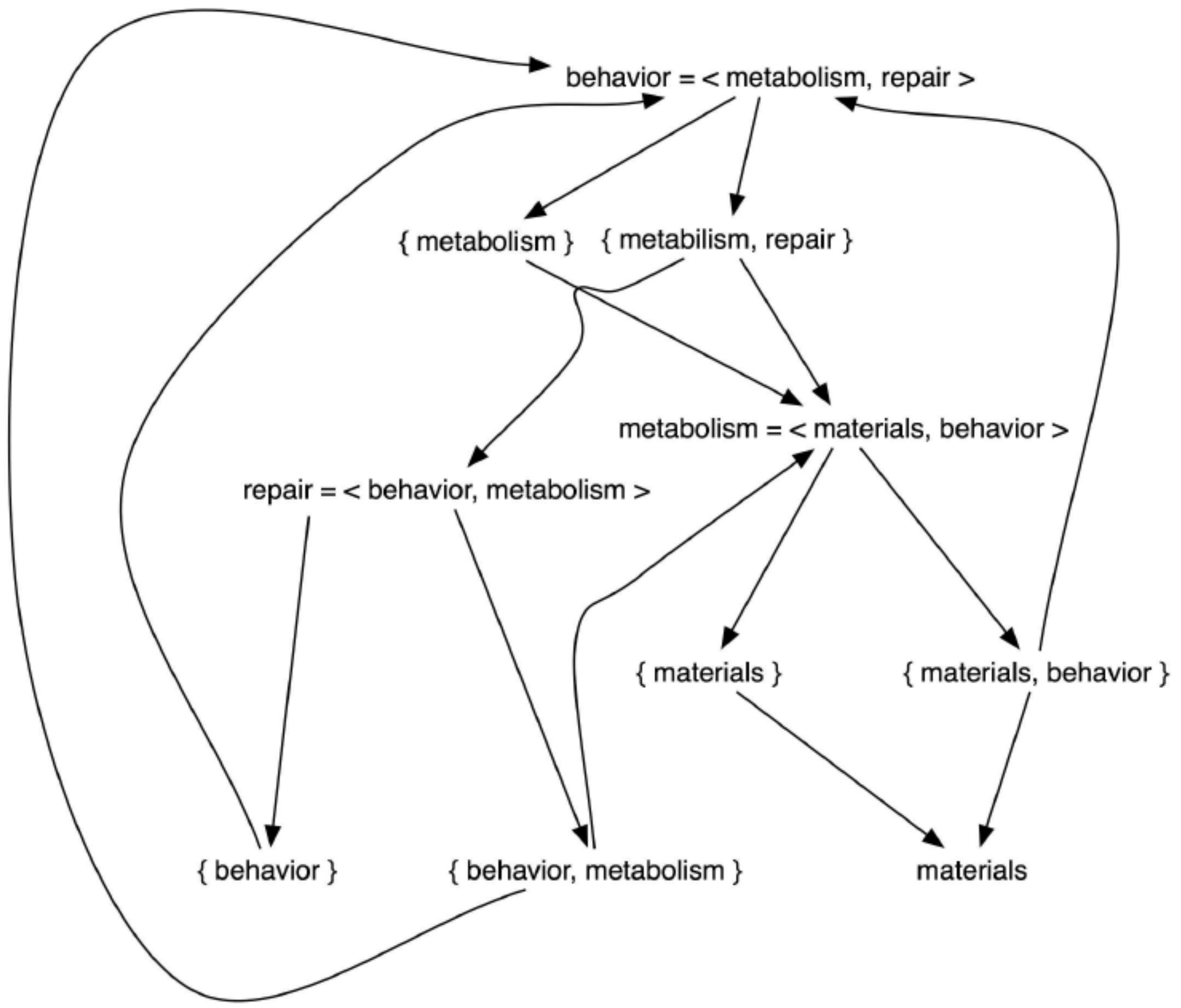


Fig. 6. Hyperset diagram of Rosen's metabolism-repair system. Functions are represented as ordered pairs containing their domain and range. So f (a) = b is represented as f = < a, b >.

Chemero, A., & Turvey, M. T. (2010). Is Life Computable? *Advances in modeling adaptive and cognitive systems*, 29-37.

Complex networks: Do our current models suffice?

Neural network models:
Energy/potential functions for end-states
Recurrent networks

$$\xi_i := sign(\sum_j w_{ij}\xi_j) \quad H = -\tfrac{1}{2}\sum_{ij} w_{ij} S_i S_j$$

$$\Rightarrow w_{ij} \propto \xi_i \xi_j$$

Self-organizing maps

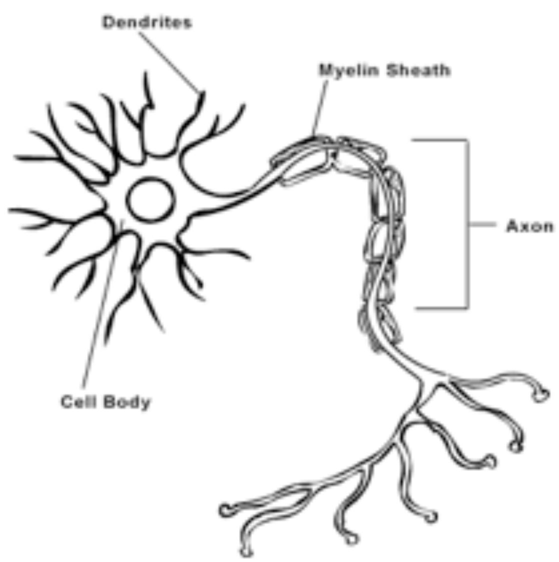
Two-neuron models:
adaptive resonance

Continuous single-neuron models:
dynamic field theory

Ising-spin models:
global, local and external forces

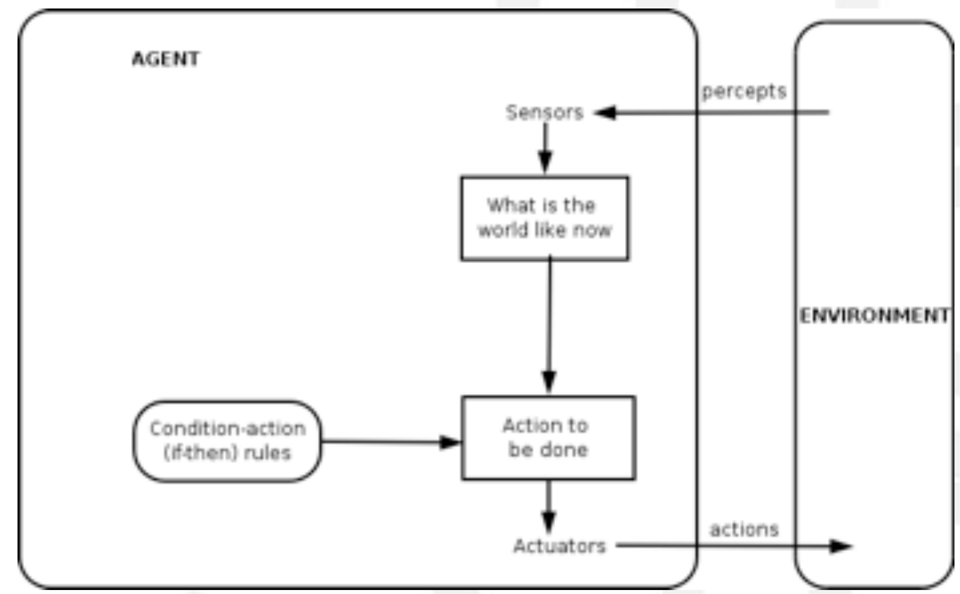
$$H_i = -\frac{J}{2} \sum_{\substack{neighbor\\ atoms}} S_i S_{neighbor}$$

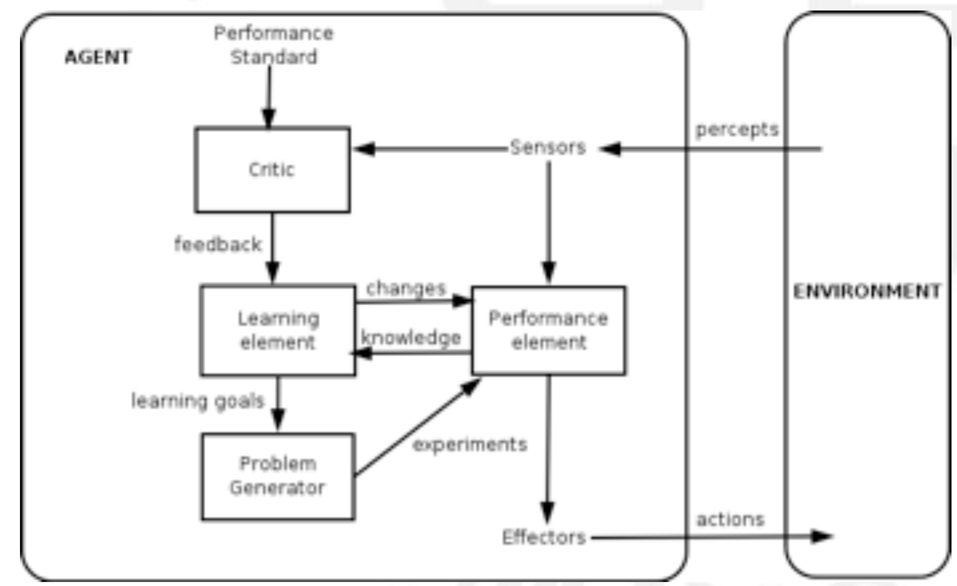
$$I_i = \sqrt{\sum_j^N \left(\frac{s_j}{d_{ij}^2}\right)^2}$$



$$\boxed{\tau\dot{u}(x,t) = -u(x,t) + I(x,t) + g_{\text{intra-field}}[u(x');x'] + h + q\xi(x,t)}$$

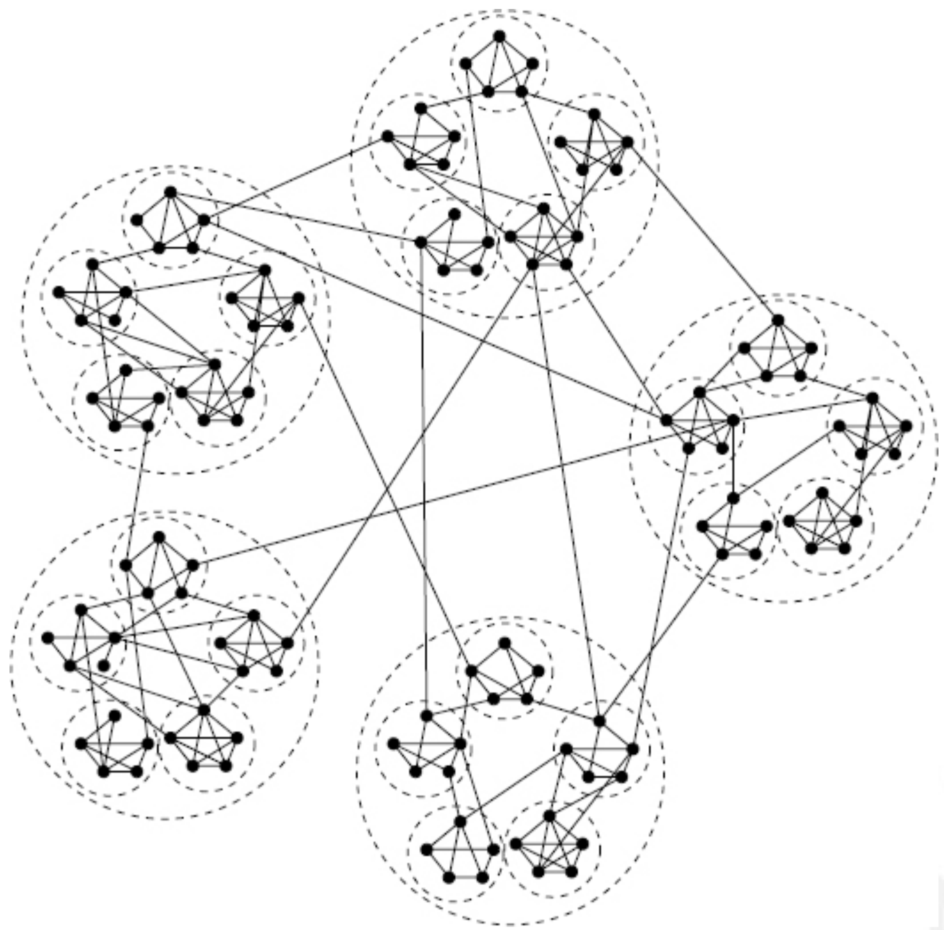
Agent-based models

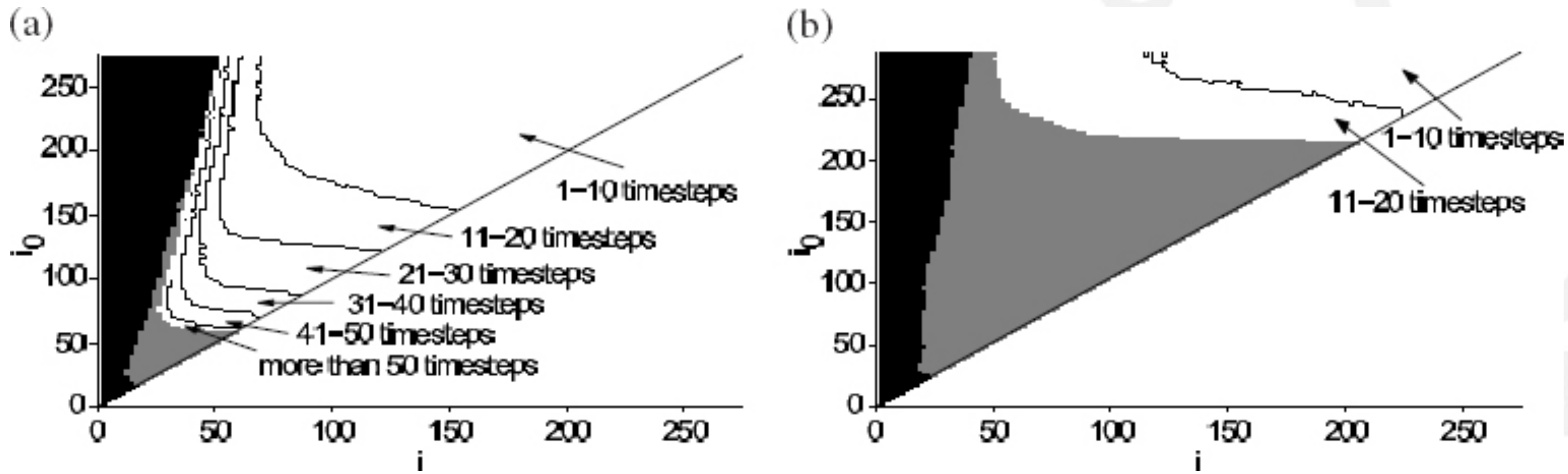




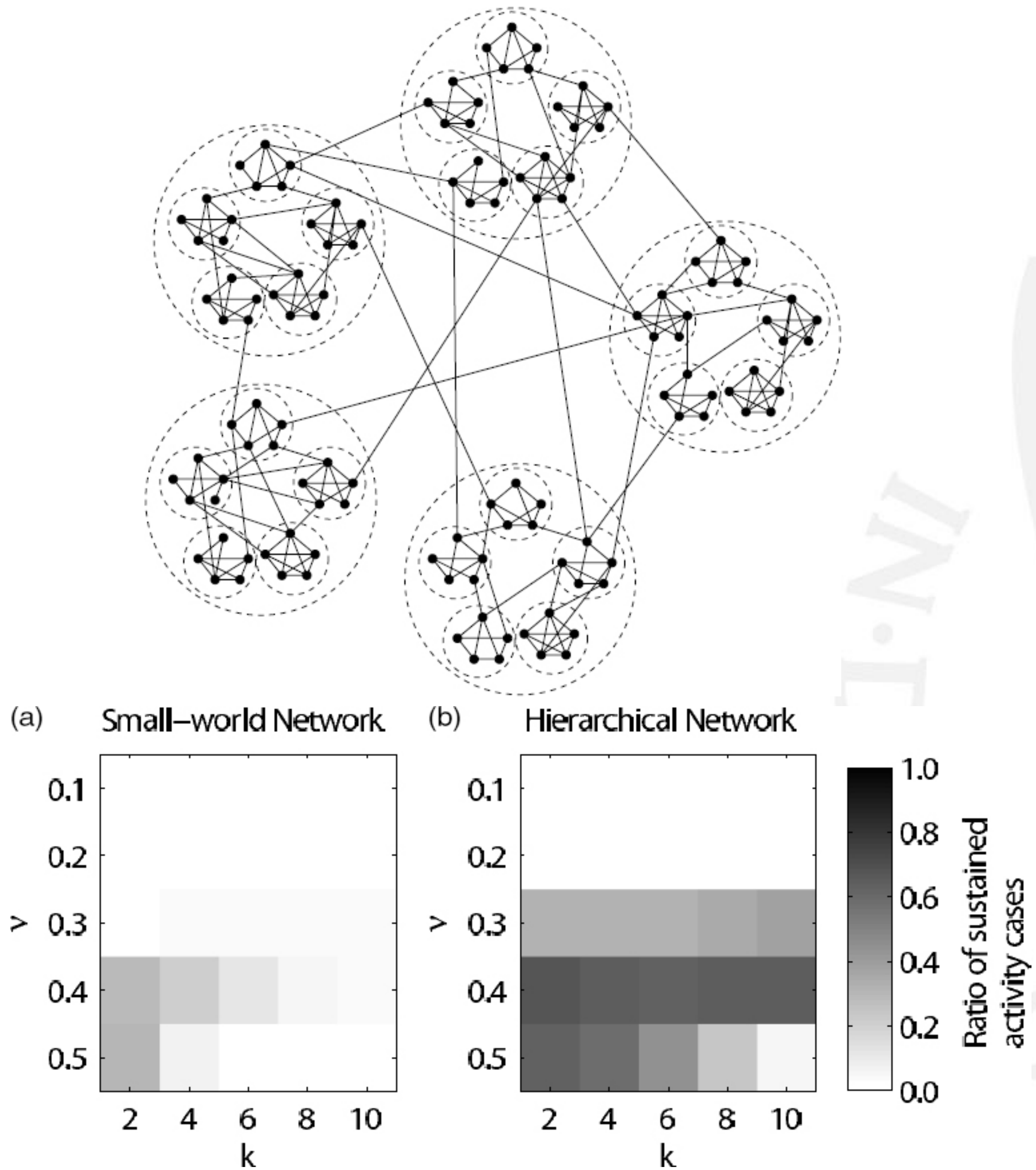
Evolving agent-based models

Extended (stretched) criticality = Hierarchical topology

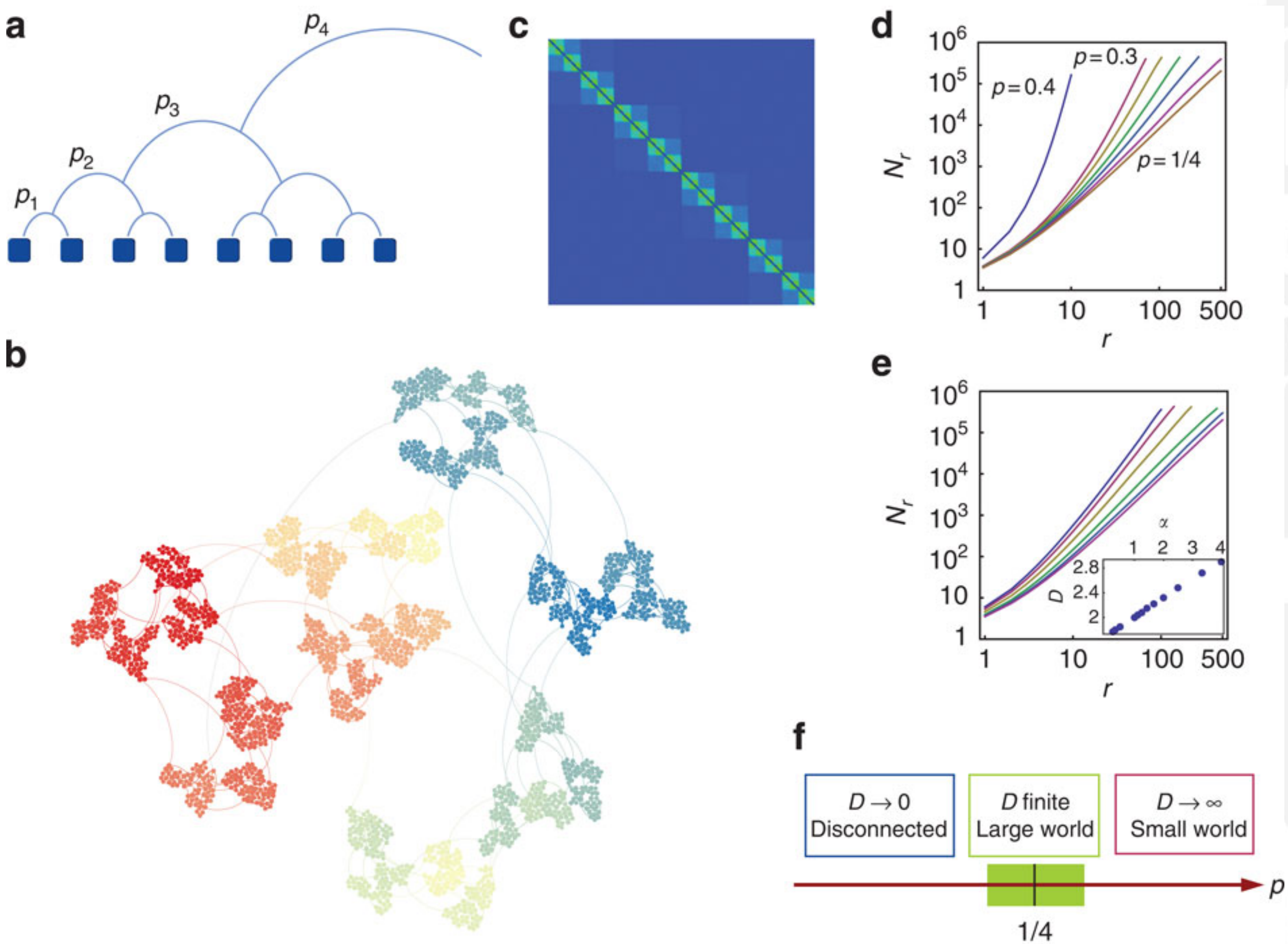




Extended (stretched) criticality = Hierarchical topology



Extended (stretched) criticality = Hierarchical topology



The wish list: Holism and Emergence

- **Composition principle**: How do parts of the system relate to wholes?

  "*A physical theory is holistic if and only if it is* ***impossible in principle****, for a set of local agents each having access to a single subsystem only, to infer the global properties of a system as assigned in the theory (which can be inferred by global measurements), by using the resource basis available to the agents." (Epistemological criterion - Seevinck, 2004).*

  "It was shown that all theories on a state space using ***a Cartesian product*** *to combine sub-system state spaces,* ***such as classical physics and Bohmian mechanics, <u>are not holistic</u>*** *in both the supervenience and epistemological approach. The reason for this is that the Boolean algebra structure of the global properties is determined by the Boolean algebra structures of the local ones.* ***The orthodox interpretation of quantum mechanics, however, was found to instantiate holism.****" (Seevinck, 2004).*

- **Conjecture:** *Only a holistic system can set the stage for strong emergent properties and patterns arising from dynamic interactions (e.g. time evolution of the system) - Work in progress: same analysis for strong emergence*

Seevinck, M. P. (2004). Holism, physical theories and quantum mechanics. [doi: DOI: 10.1016/j.shpsb.2004.08.001]. *Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics, 35*(4), 693-712.

Some additional evidence?
No impredicative loops for the Lorenz system!

Our interpretation: system defined on a classical state space, cannot be a holistic system, hence no strong emergent patterns / properties!

(Hyper)set graphs of some physical systems and their numerical implementations

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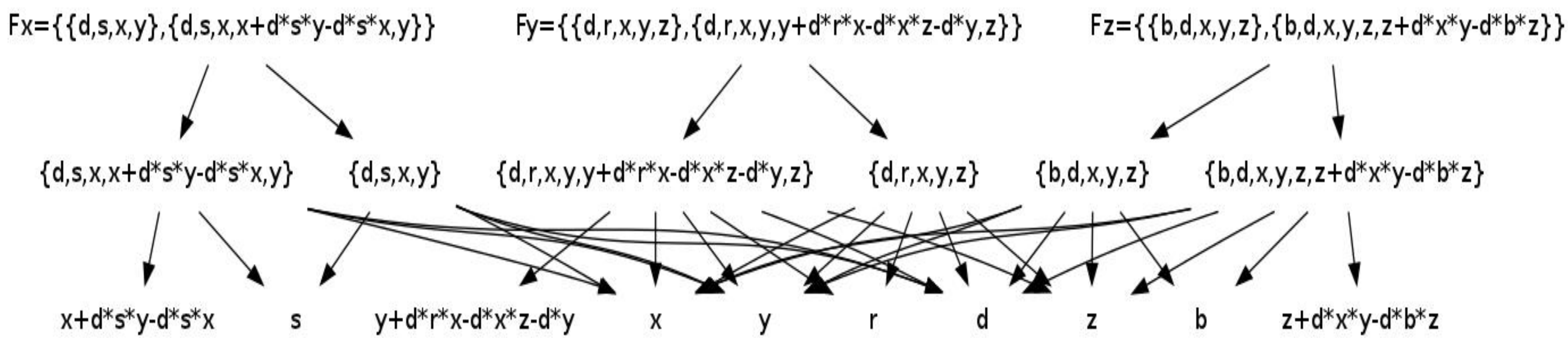
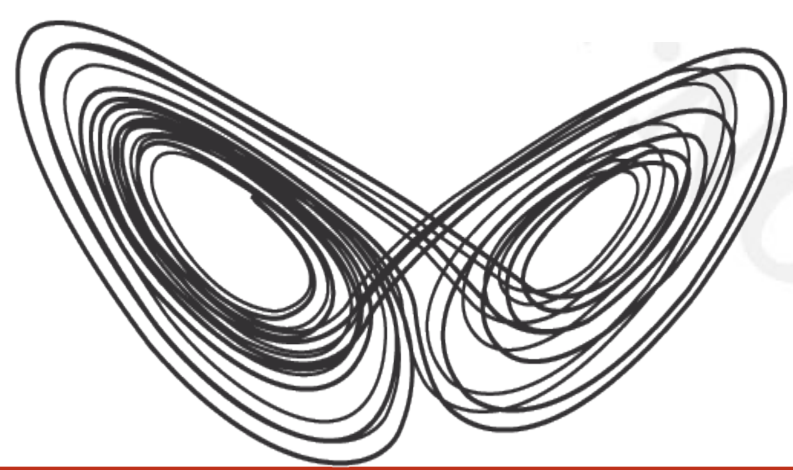


dx/dt=σ(y – x)
dy/dt=x(ρ – z) – y
dz/dt=xy – βz



“Life is a fractal in Hilbert space.”

- Rudy Rucker, 1987

Rucker, R. (1987). *Mind Tools: The Five Levels of Mathematical Reality*.
Boston : Houghton Mifflin.