The Dynamics of Complex Systems

Randomisation Tests and Surrogate Data
Phase Space Reconstruction
Recurrence Quantification Analysis

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How to study interaction-dominant systems

As you know in a **coupled system** the time evolution of one variable depends on other variables of the system. This implies that one variable contains information about the other variables (of course depending upon the strength of coupling and maybe the type of interaction)

So given the Lorenz system …

$$dX/dt = \delta \cdot (Y - X)$$

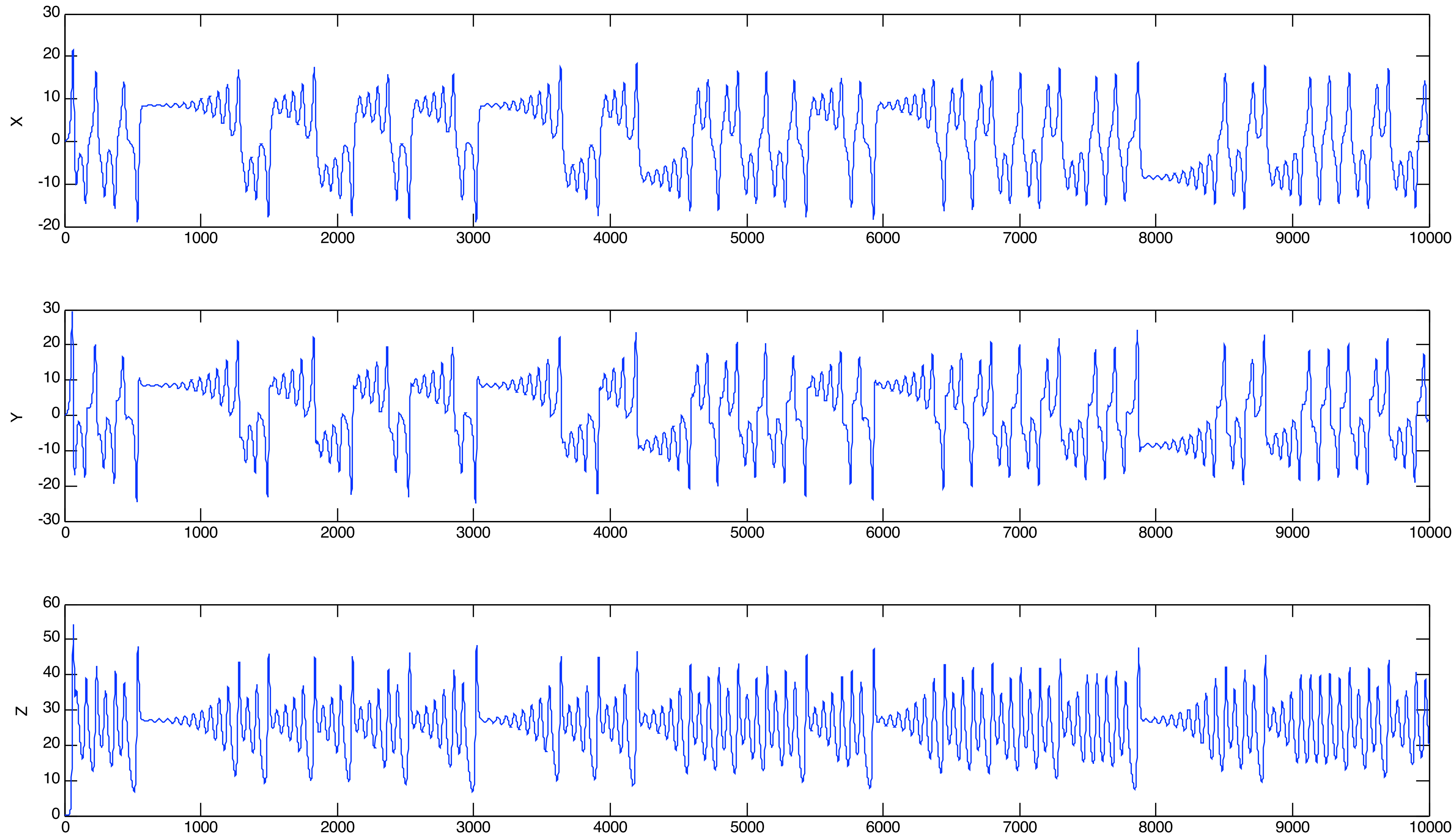
$$dY/dt = r \cdot X - Y - X \cdot Z$$

$$dZ/dt = X \cdot Y - b \cdot Z$$

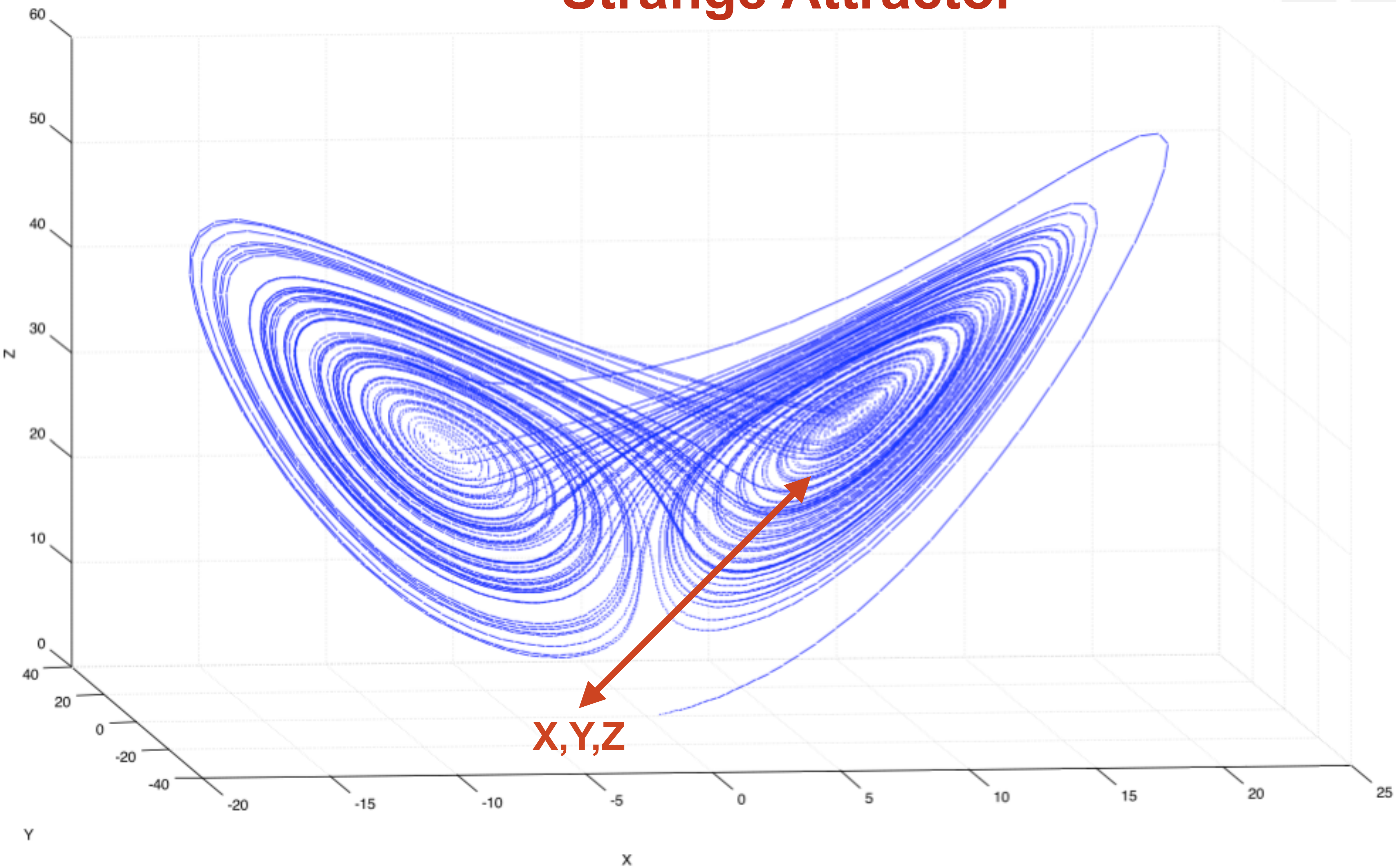
Takens' theorem suggests that we should be able to reconstruct the higly chaotic "butterfly" attractor by just using X(t) [or Y(t) or Z(t)] …

Takens, F. (1981). Detecting strange attractors in turbulence. In D. A. Rand and L.-S. Young (Eds.) Dynamical Systems and Turbulence. *Lecture Notes in Mathematics vol. 898*, 366–381, Springer-Verlag.

Lorenz system – Time series of X, Y and Z

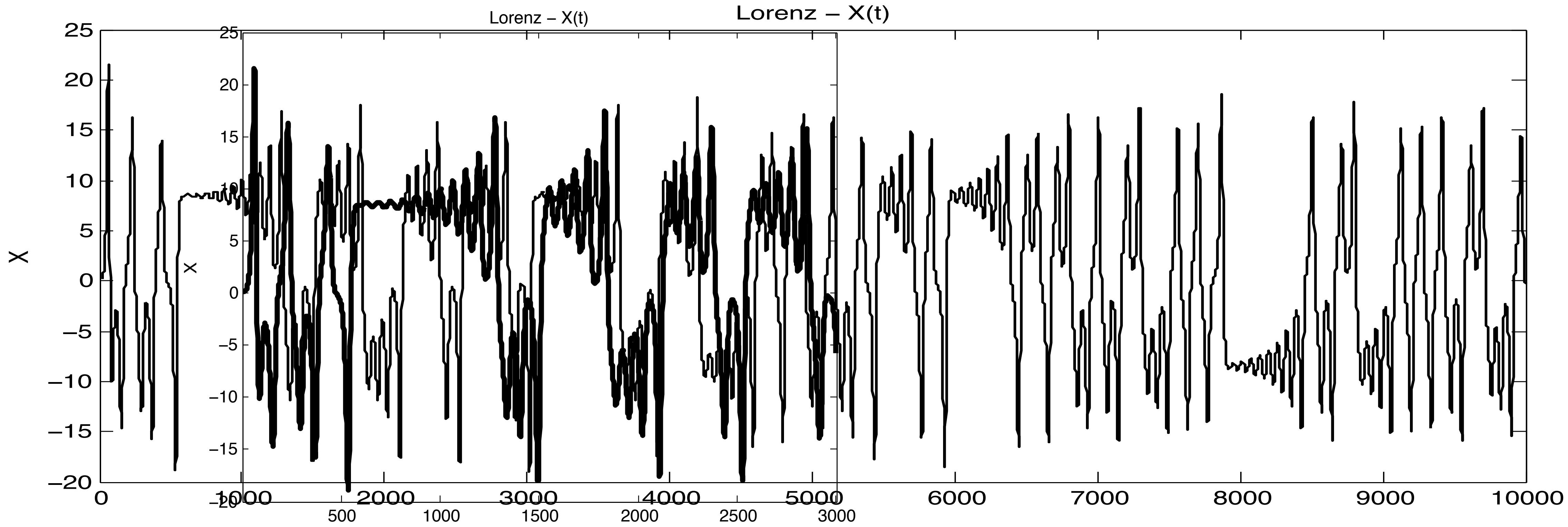


Lorenz system – X,Y,Z State space Strange Attractor



Creating surrogate dimensions using the method of delays

X (t)



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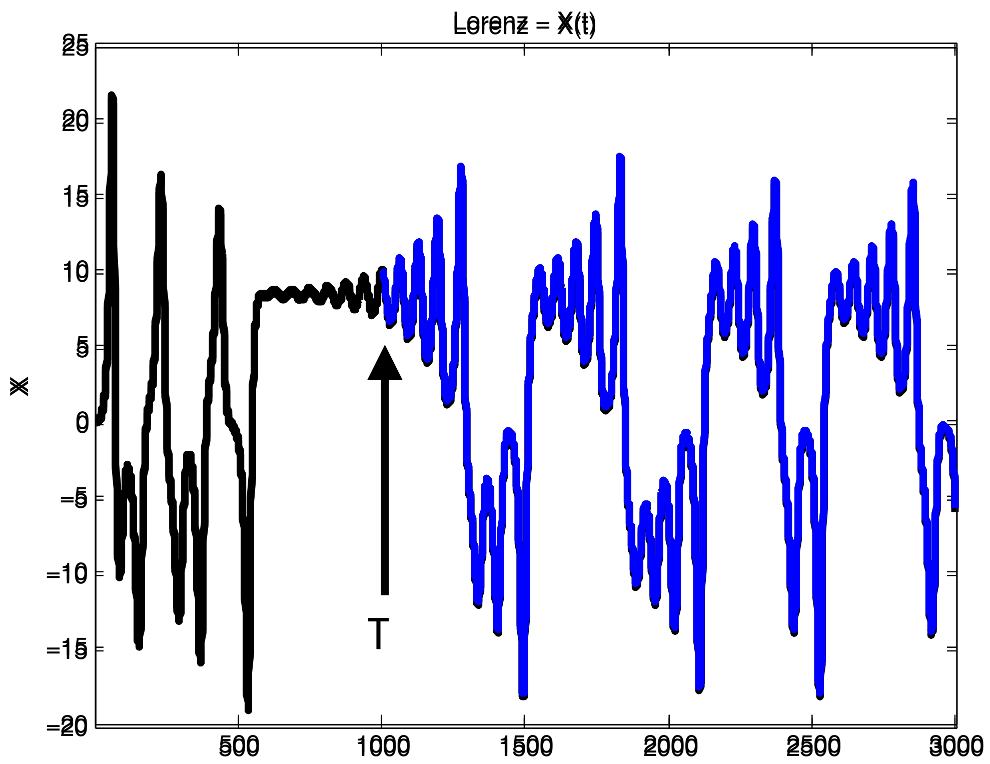
Creating surrogate dimensions using the method of delays

X (t)

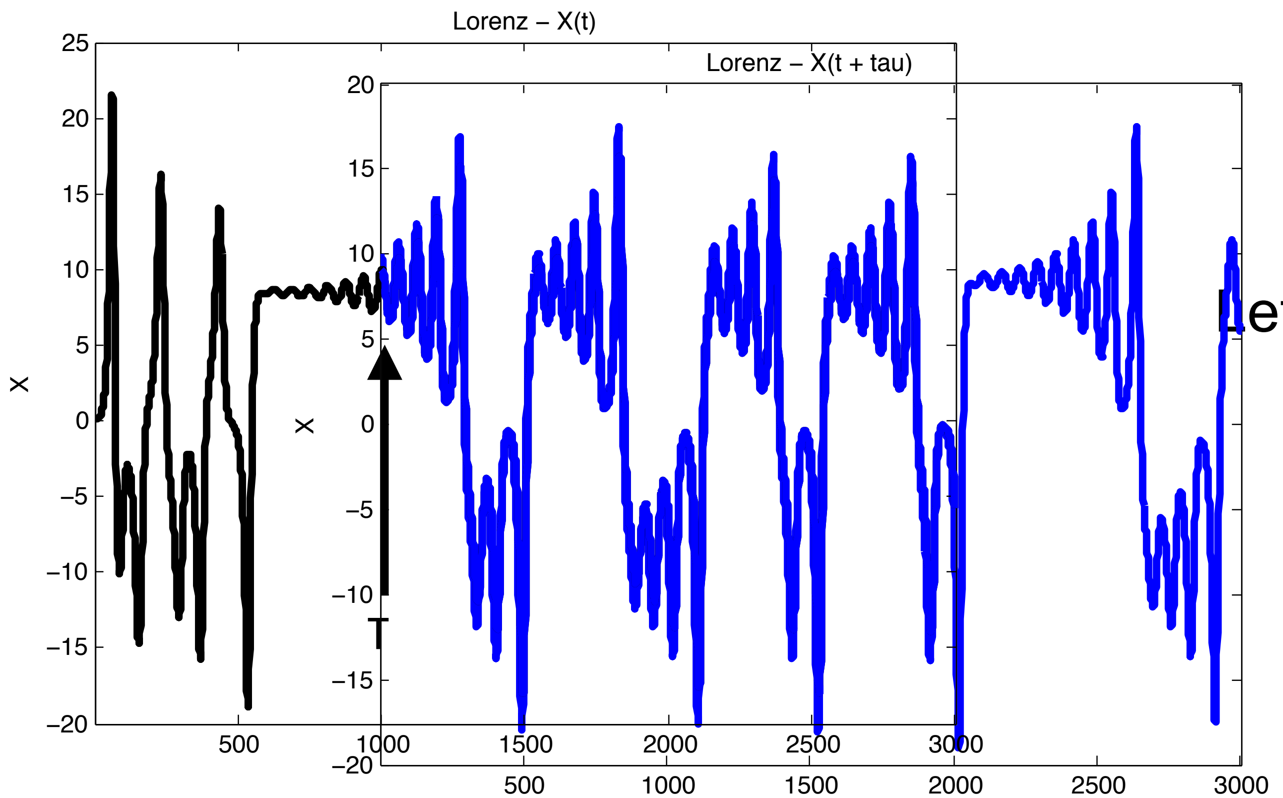


Let's take our embedding delay or lag to be:

T = 1000

Creating surrogate dimensions using the method of delays

X (t)



Let's take our embedding delay or lag to be:

T = 1000

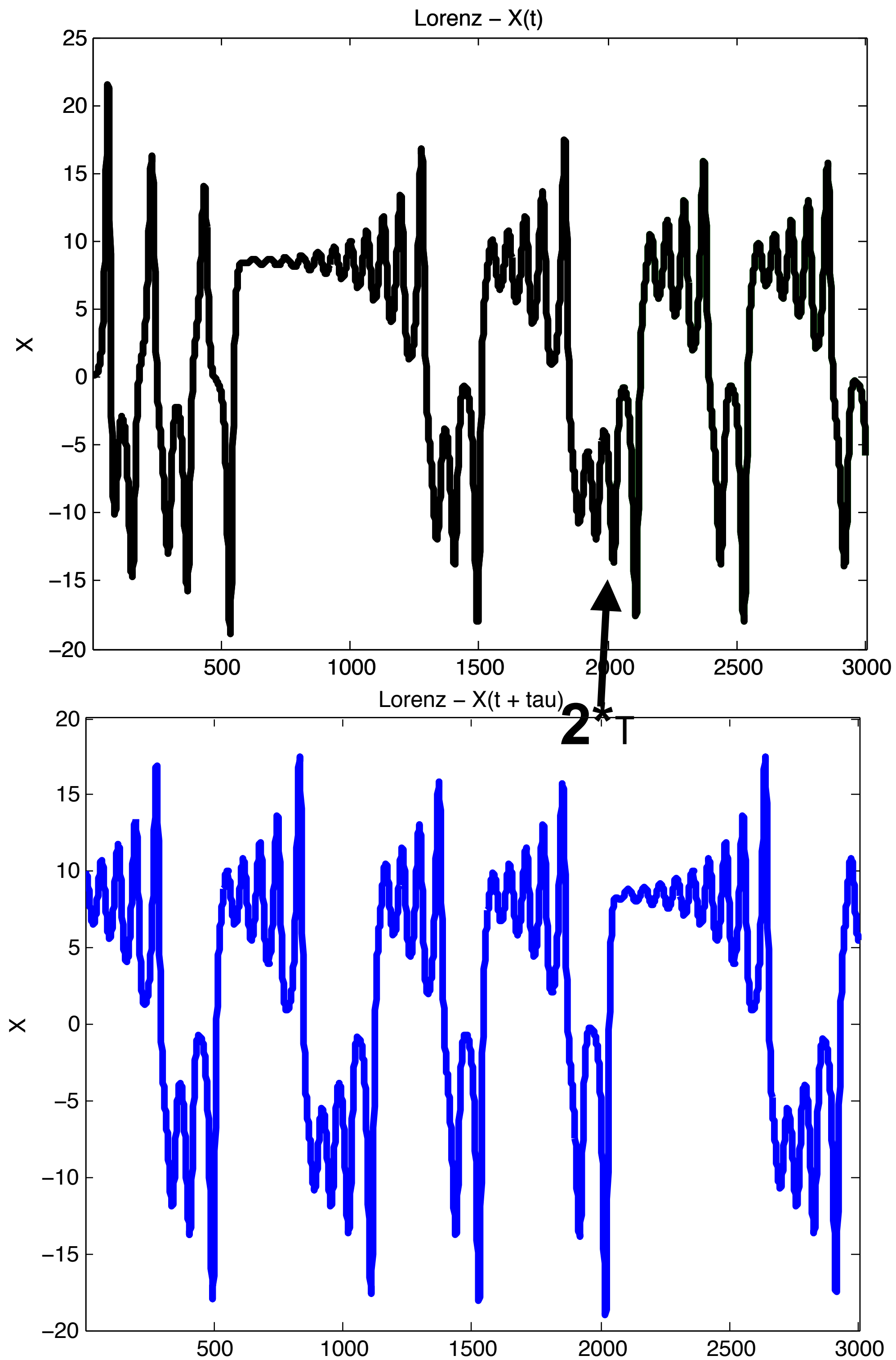
X (t + τ)

Data point 1 + T [X(t) = 1001] becomes data point 1 for this dimension

Creating surrogate dimensions using the method of delays

X (t)

X (t + τ)



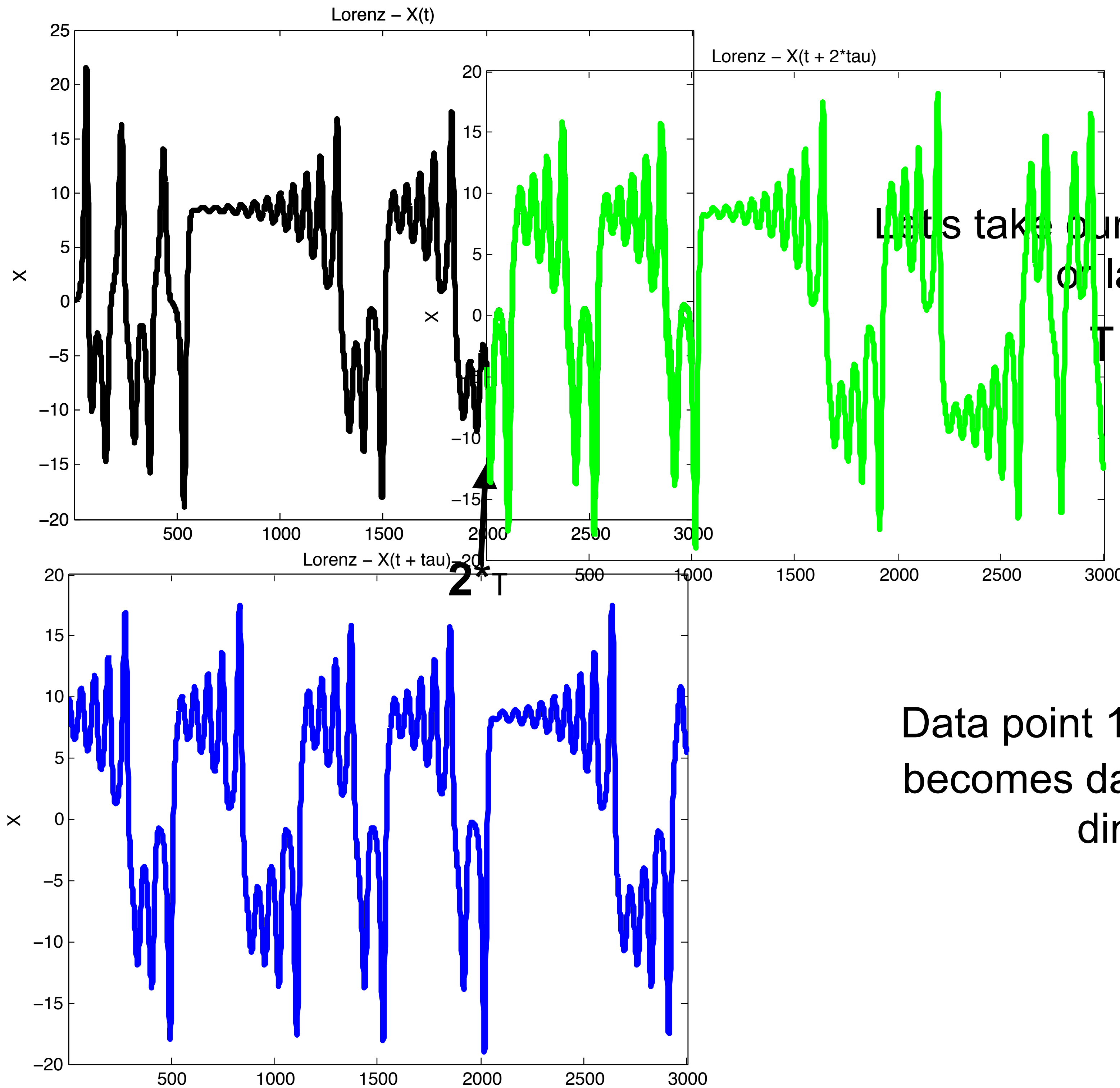
Let's take our embedding delay or lag to be:

T = 1000

Data point 1 + T [X(t) = 1001] becomes data point 1 for this dimension

Creating surrogate dimensions using the method of delays

X (t)



Let's take our embedding delay or lag to be:

τ = 1000

X (t + τ)

Data point 1 + τ [X(t) = 1001] becomes data point 1 for this dimension

X (t + 2*τ)

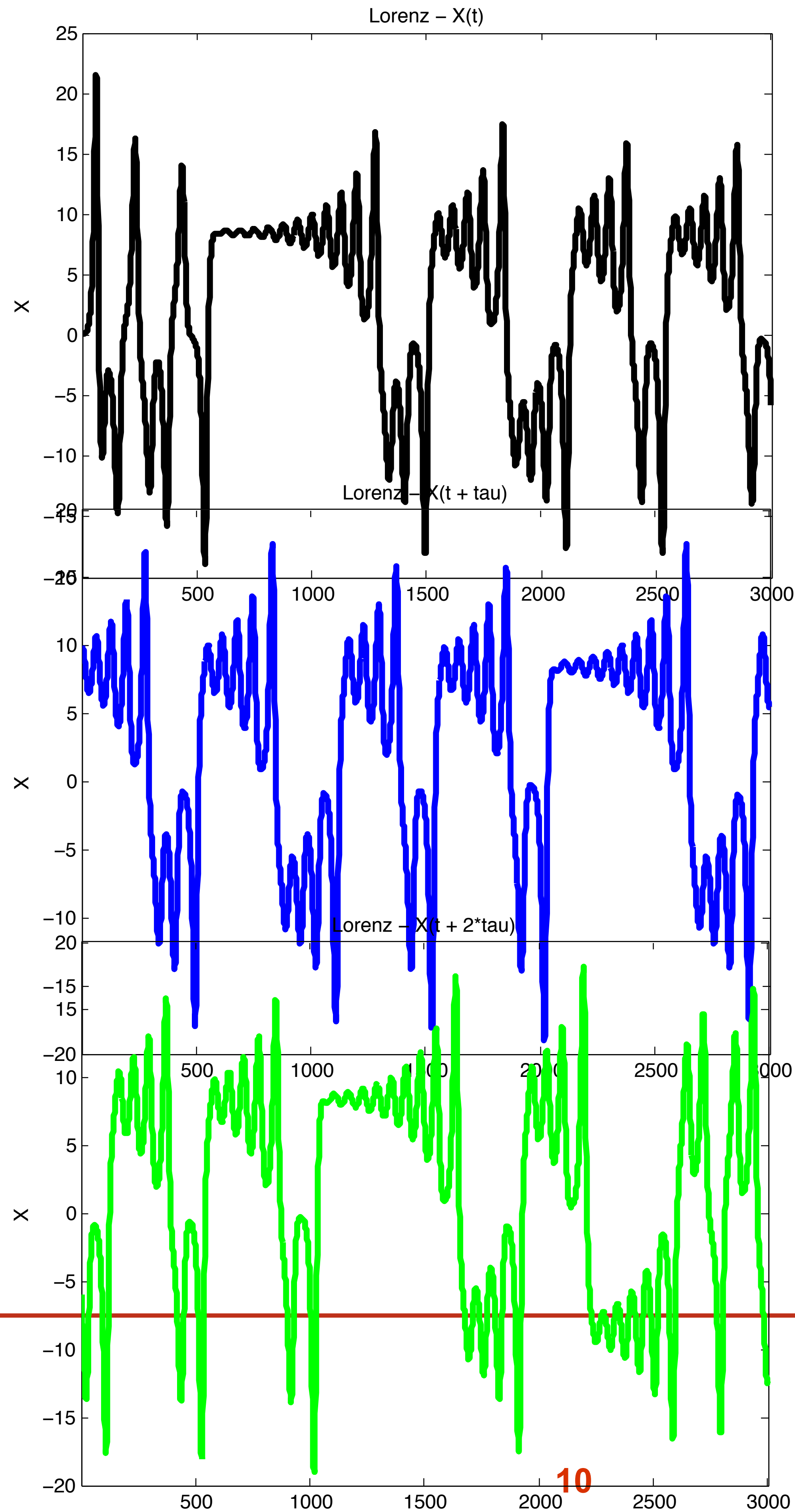
Data point 1 + 2*τ [X(t) = 2001] becomes data point 1 for this dimension

Creating surrogate dimensions using the method of delays

X (t)

X (t + τ)

X (t + 2τ)



The embedding lag reflects the point in the time series at which we are getting **new information** about the system…

In theory any lag can be used, everything is interacting...

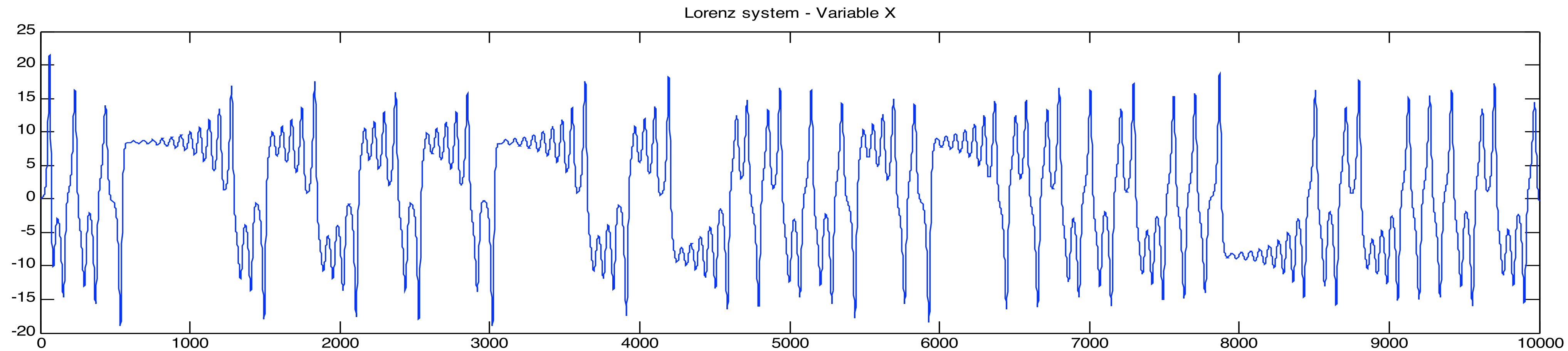
We are looking for the lag which is optimal, gives us maximal new information about the temporal structure in the data…

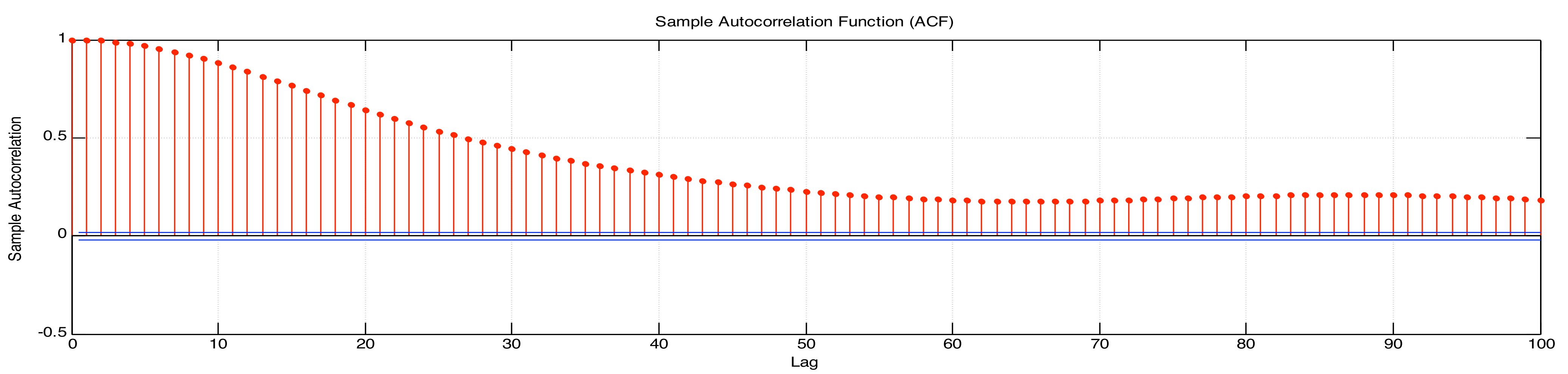
Intuitively:
Where the autocorrelation is zero

We are creating a return plot to examine the systems' state space!

How to determine embedding lag?

- We saw that the autocorrelation function is not very helpful when you are dealing with long range correlations in the data.



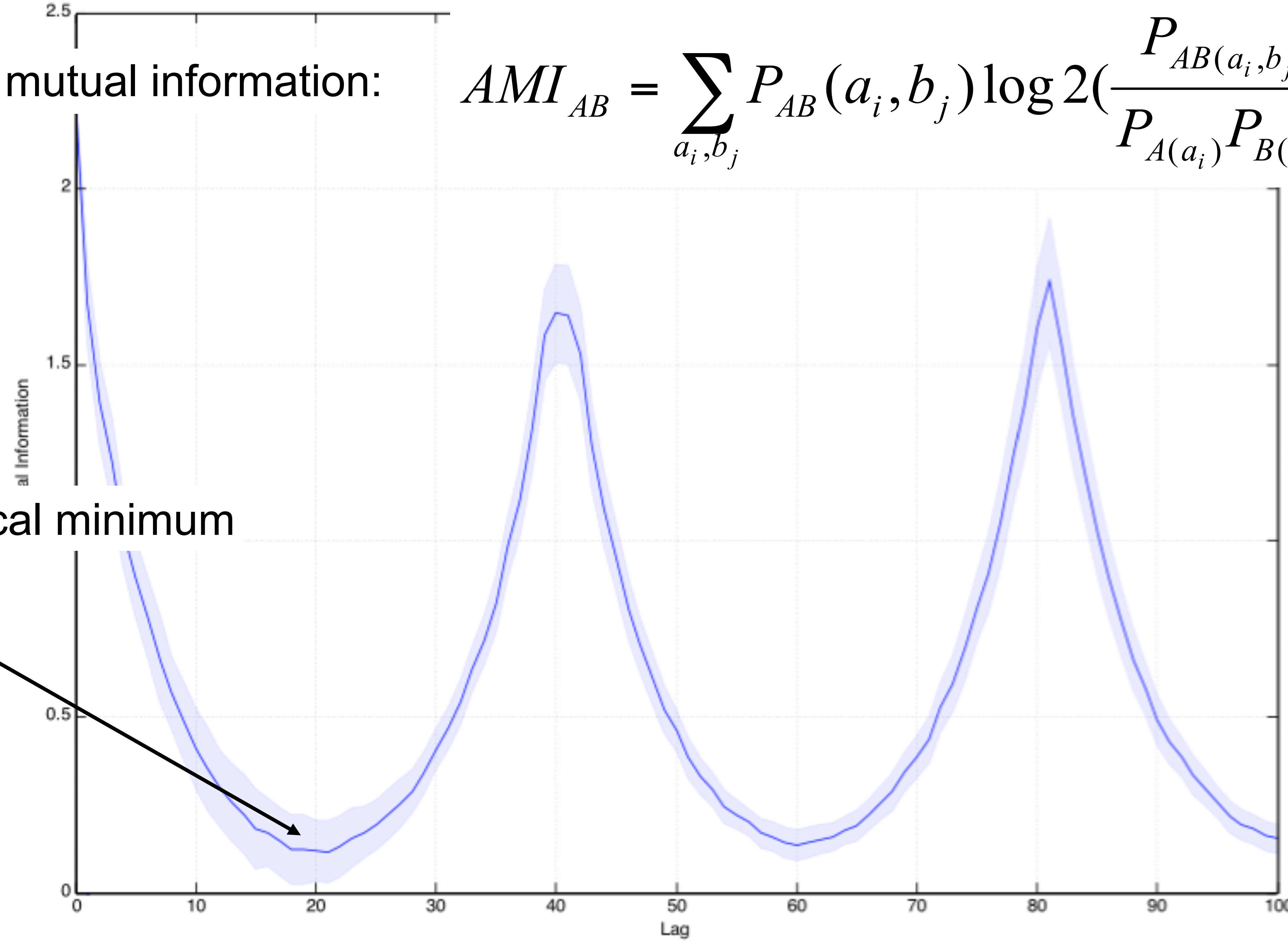


Lorenz system – Determine embedding lag

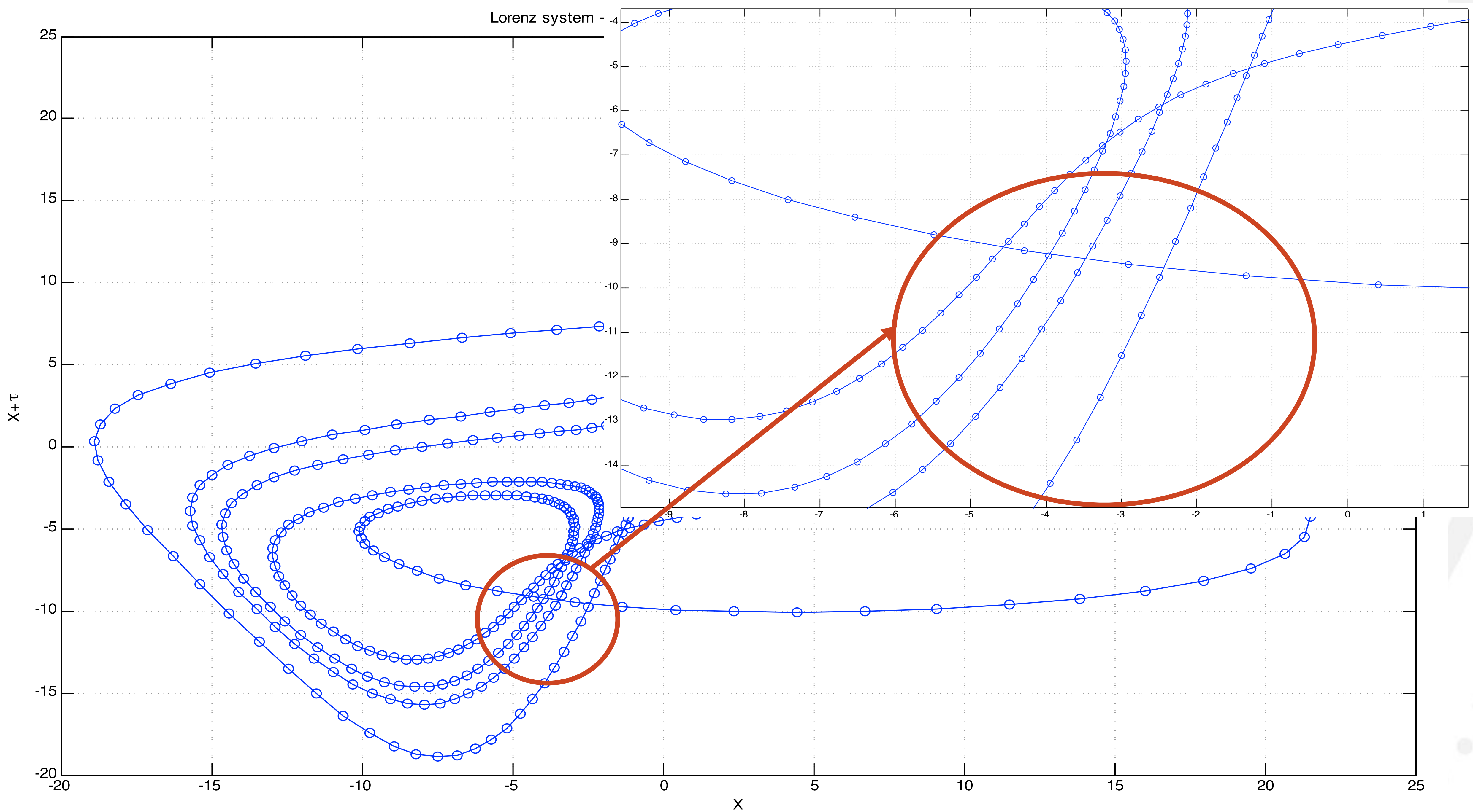
Average mutual information:

$$AMI_{AB} = \sum_{a_i, b_j} P_{AB}(a_i, b_j) \log 2\left(\frac{P_{AB(a_i, b_j)}}{P_{A(a_i)} P_{B(b_j)}}\right)$$

Use first local minimum

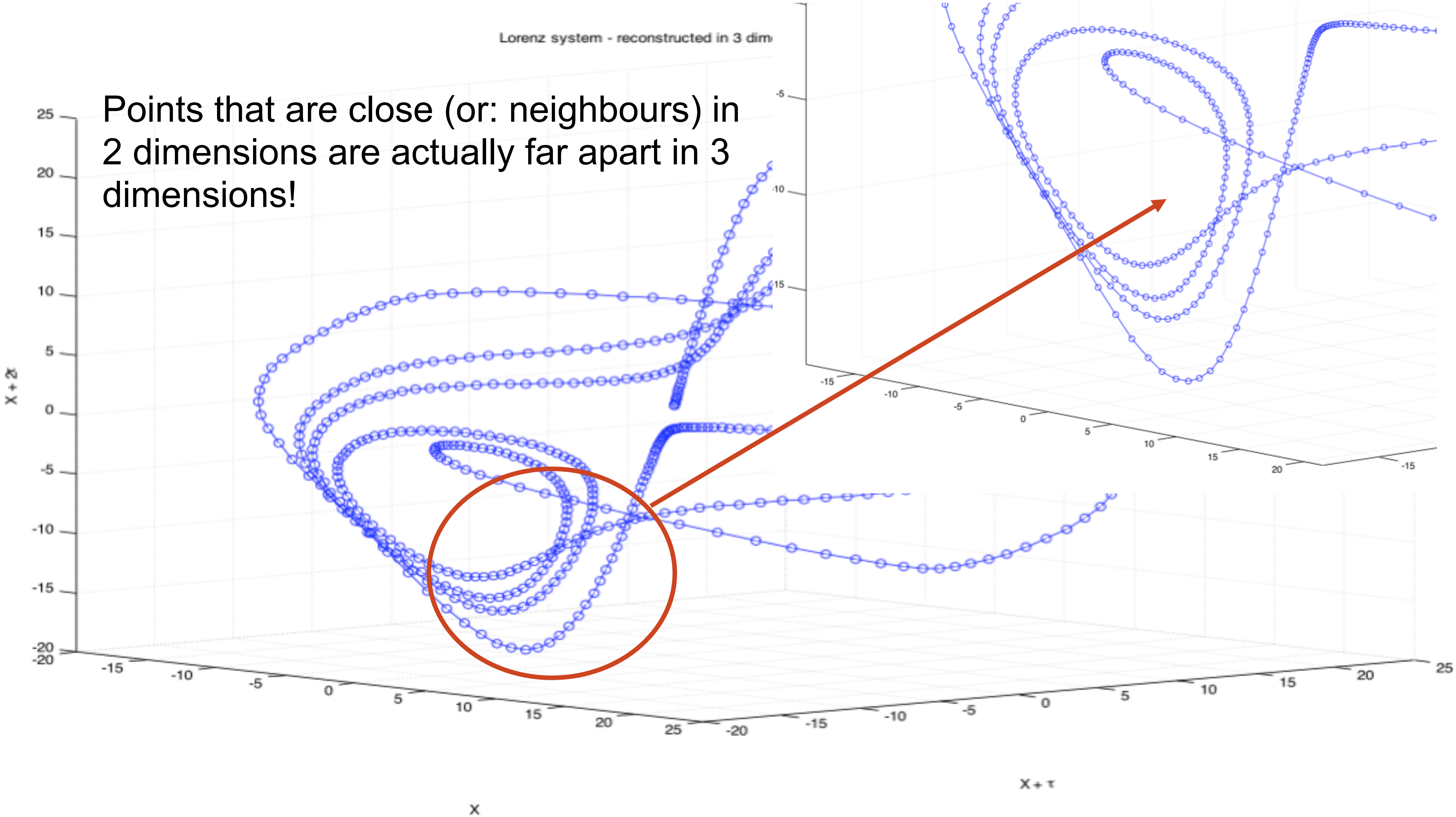


How many dimensions? Determine *embedding dimension* (m)

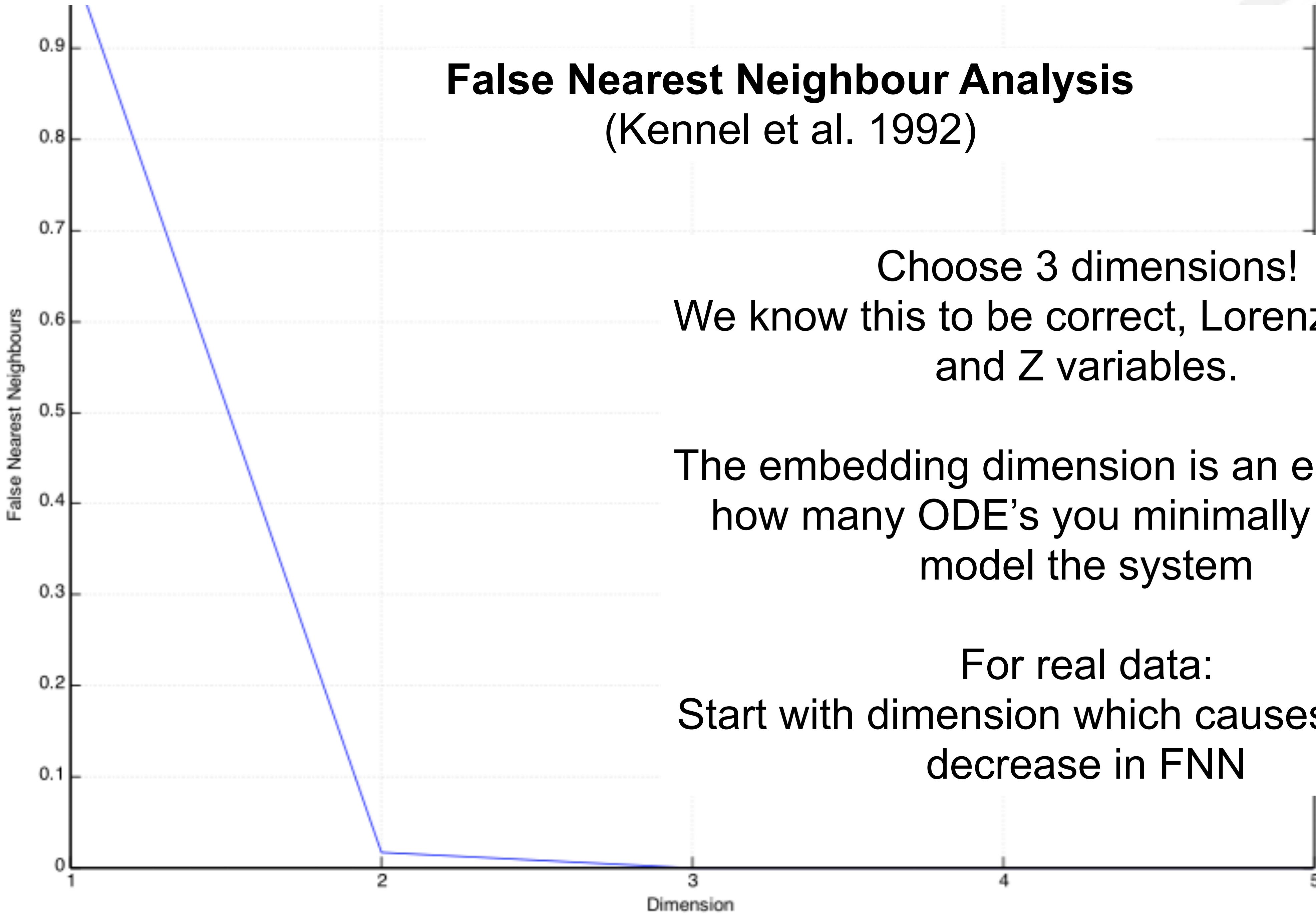


Lorenz system – Determine embedding dimension

Points that are close (or: neighbours) in 2 dimensions are actually far apart in 3 dimensions!



Lorenz system – Determine embedding dimensions



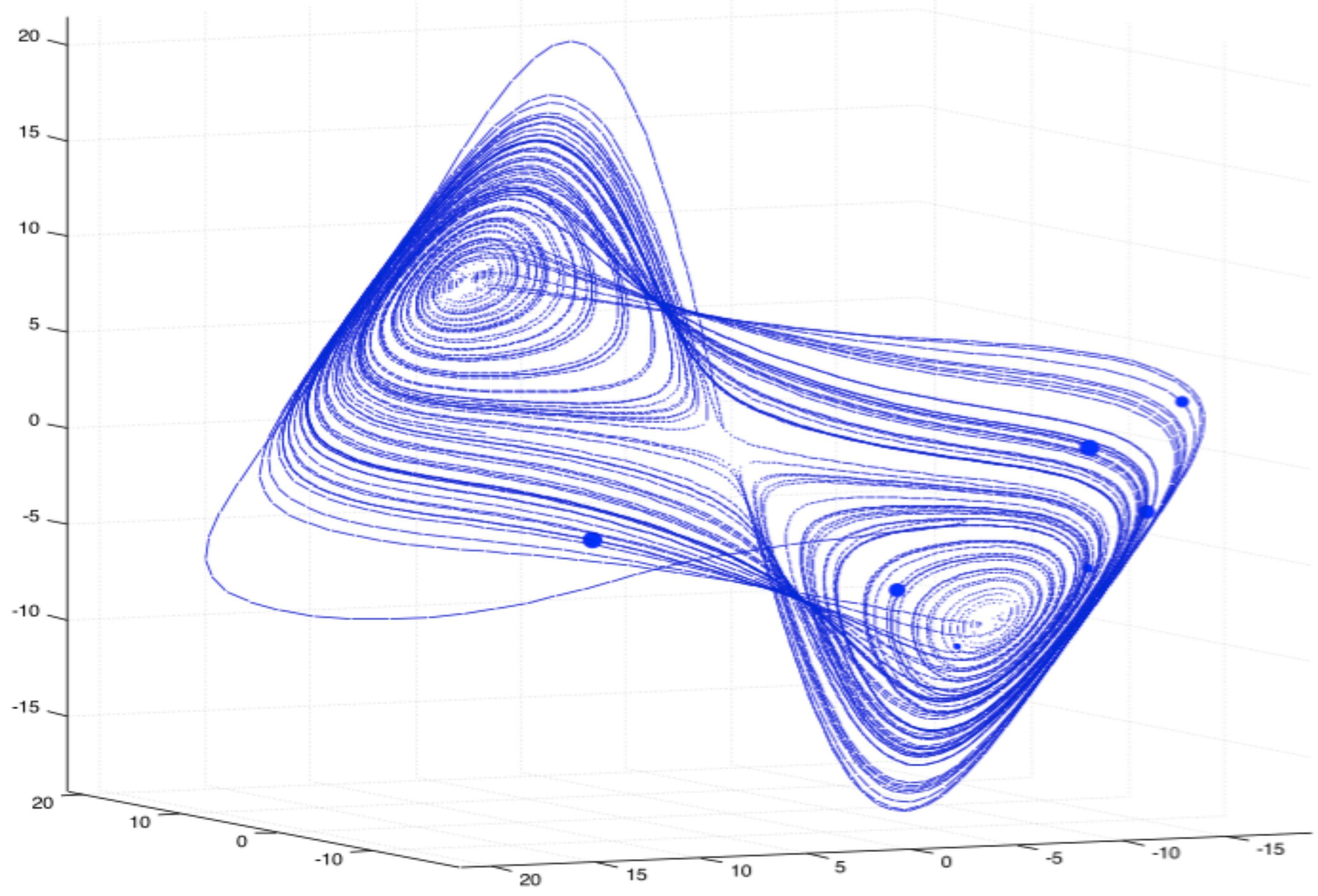
False Nearest Neighbour Analysis
(Kennel et al. 1992)

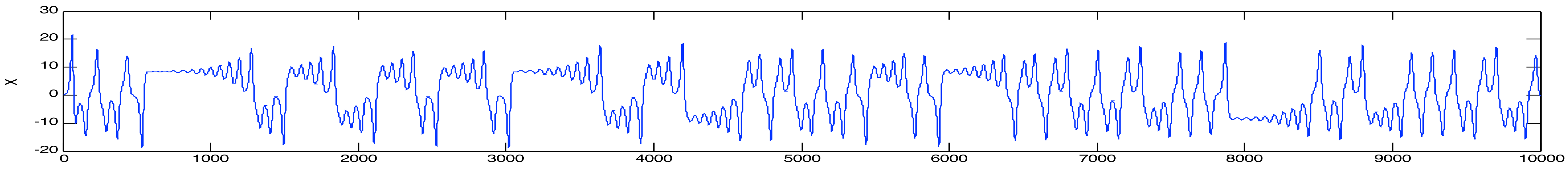
Choose 3 dimensions!
We know this to be correct, Lorenz has X, Y and Z variables.

The embedding dimension is an estimate of how many ODE's you minimally need to model the system

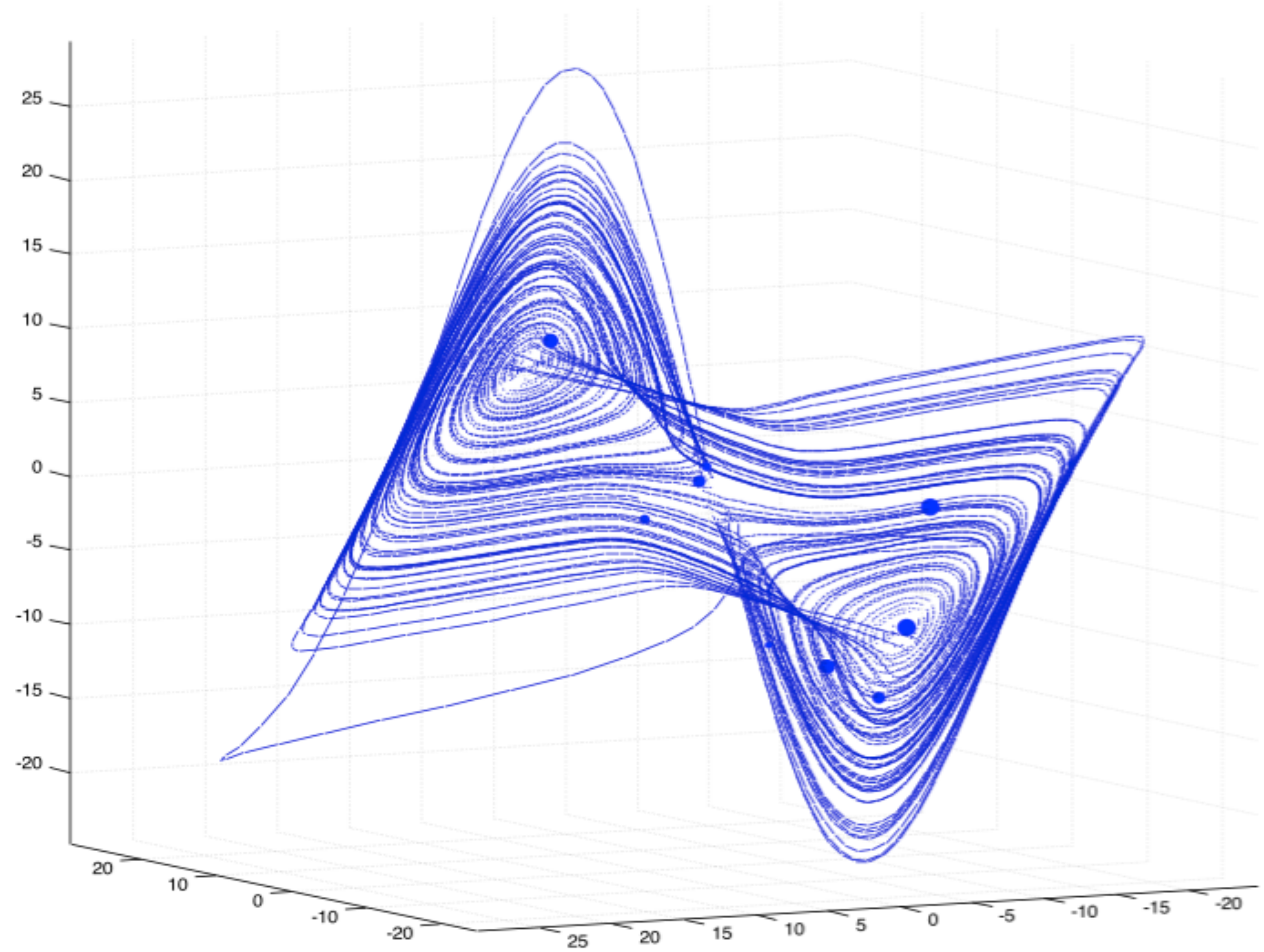
For real data:
Start with dimension which causes greatest decrease in FNN

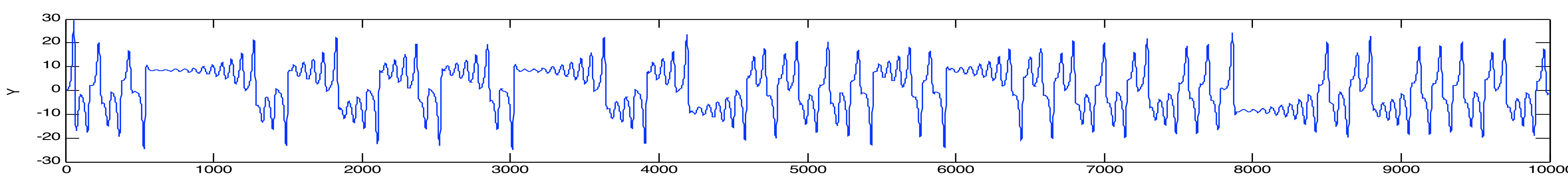
Lorenz system – Reconstruct phase space using X



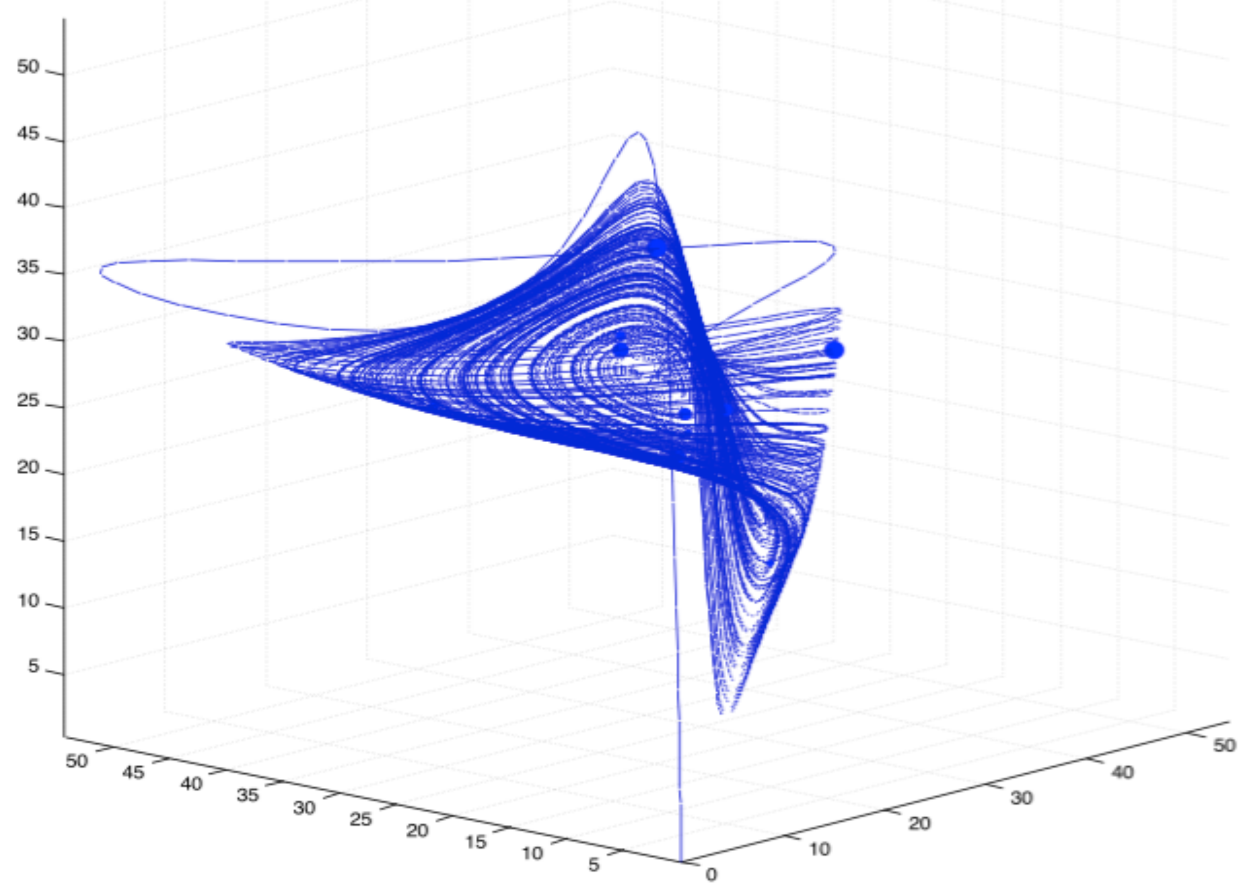


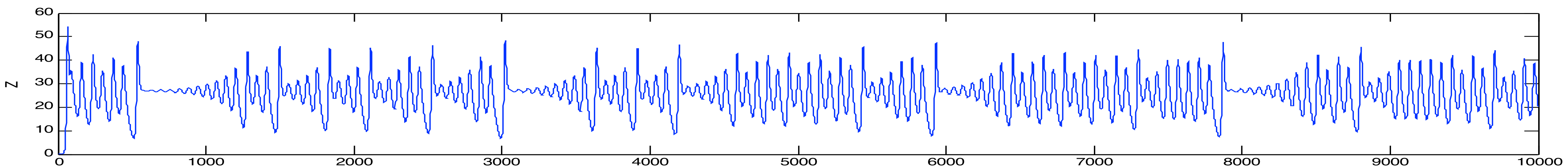
Lorenz system – Reconstruct phase space using Y





Lorenz system – Reconstruct phase space using Z





Isn't that amazing?

- Take a moment to realise what we just did:

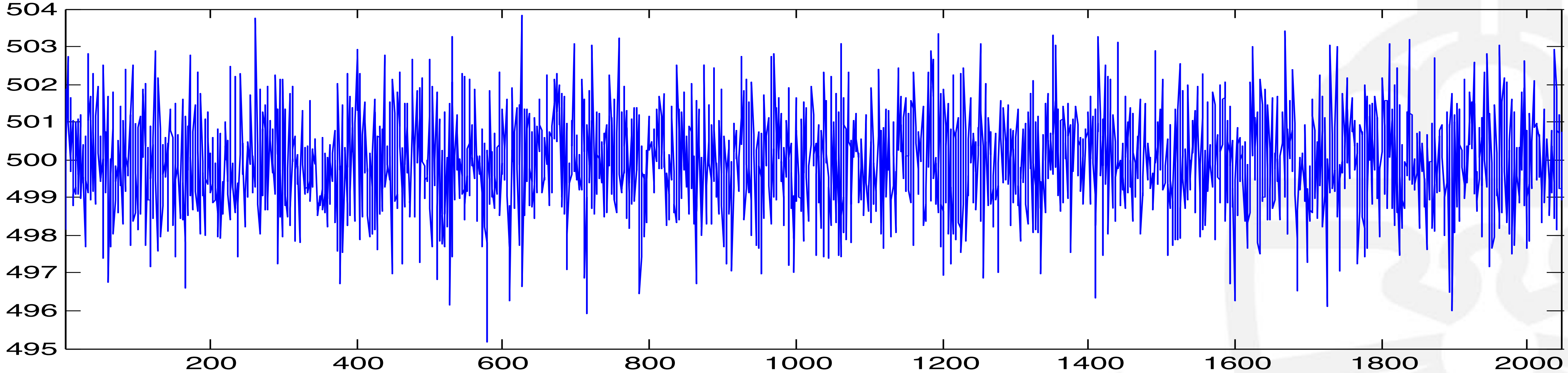
- The state space (defined by X,Y and Z) of a complex, nonlinear chaotic system was reconstructed to a phase space (lag plot) of 3 surrogate dimensions X, $X_{t+\tau}$, $X_{t+2*\tau}$

- **You only need to measure one variable of a system!!**
  … because "everything is interacting"…
  We ***exploit (and need)*** *the dependencies in the data!*

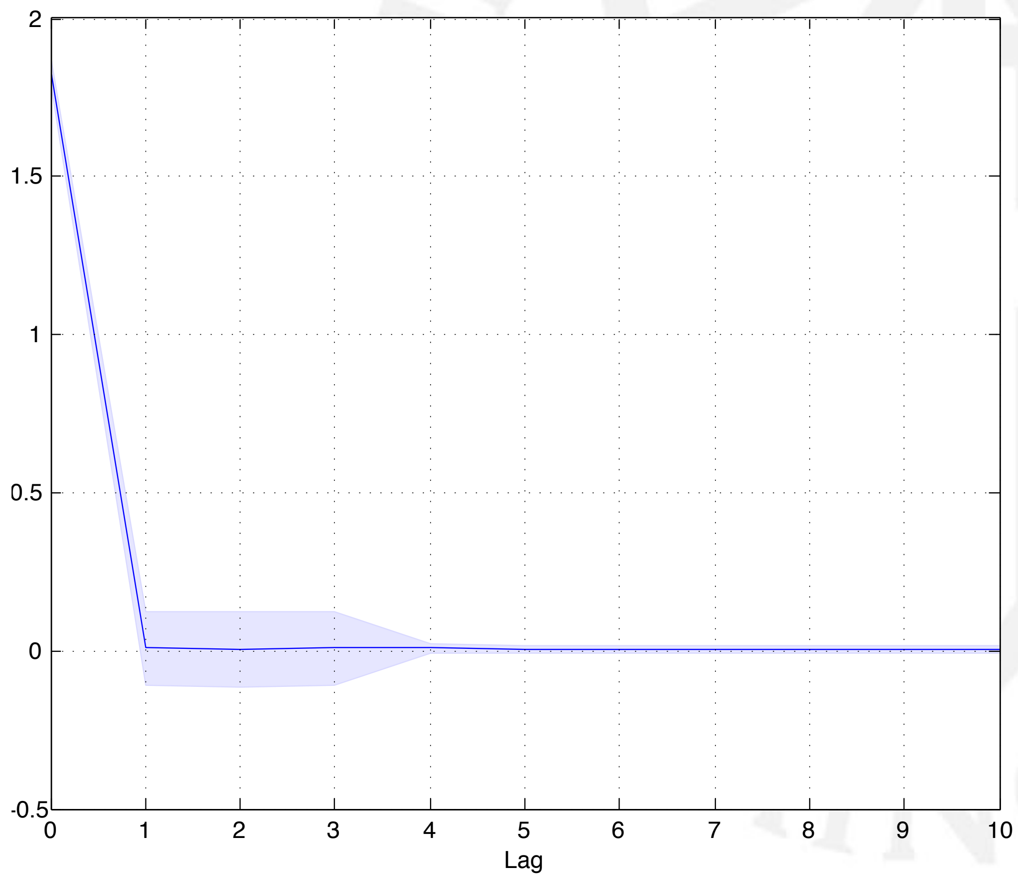
The length of your data set needs to be long enough to create the surrogate dimension. This can be as few as 20 datapoints for categorical data.

- The reconstruction process does not make any assumptions about the data. You can also try to reconstruct a phase space from a random variable. (What will happen?)

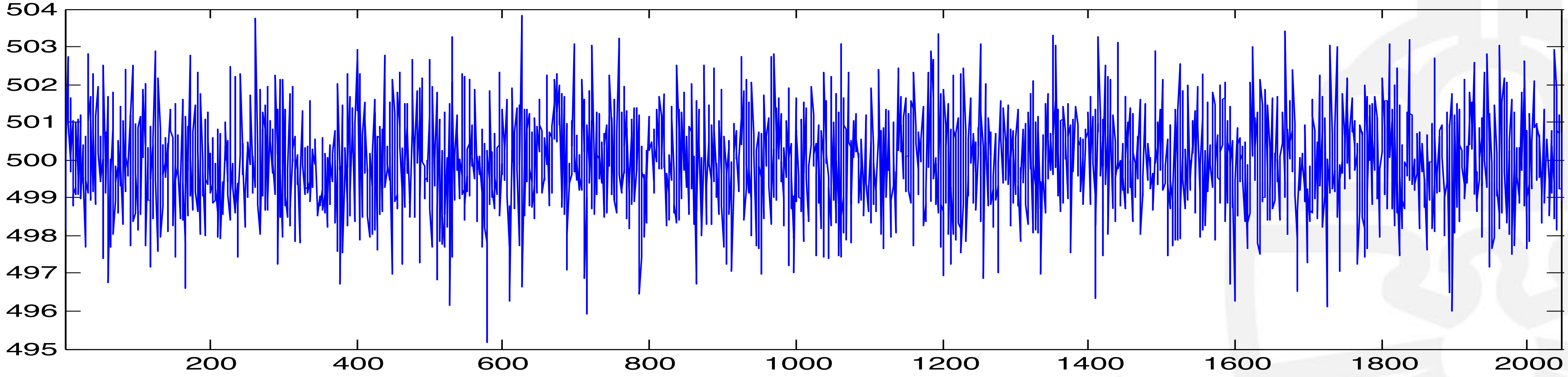
Suppose we have measured a true IID variable



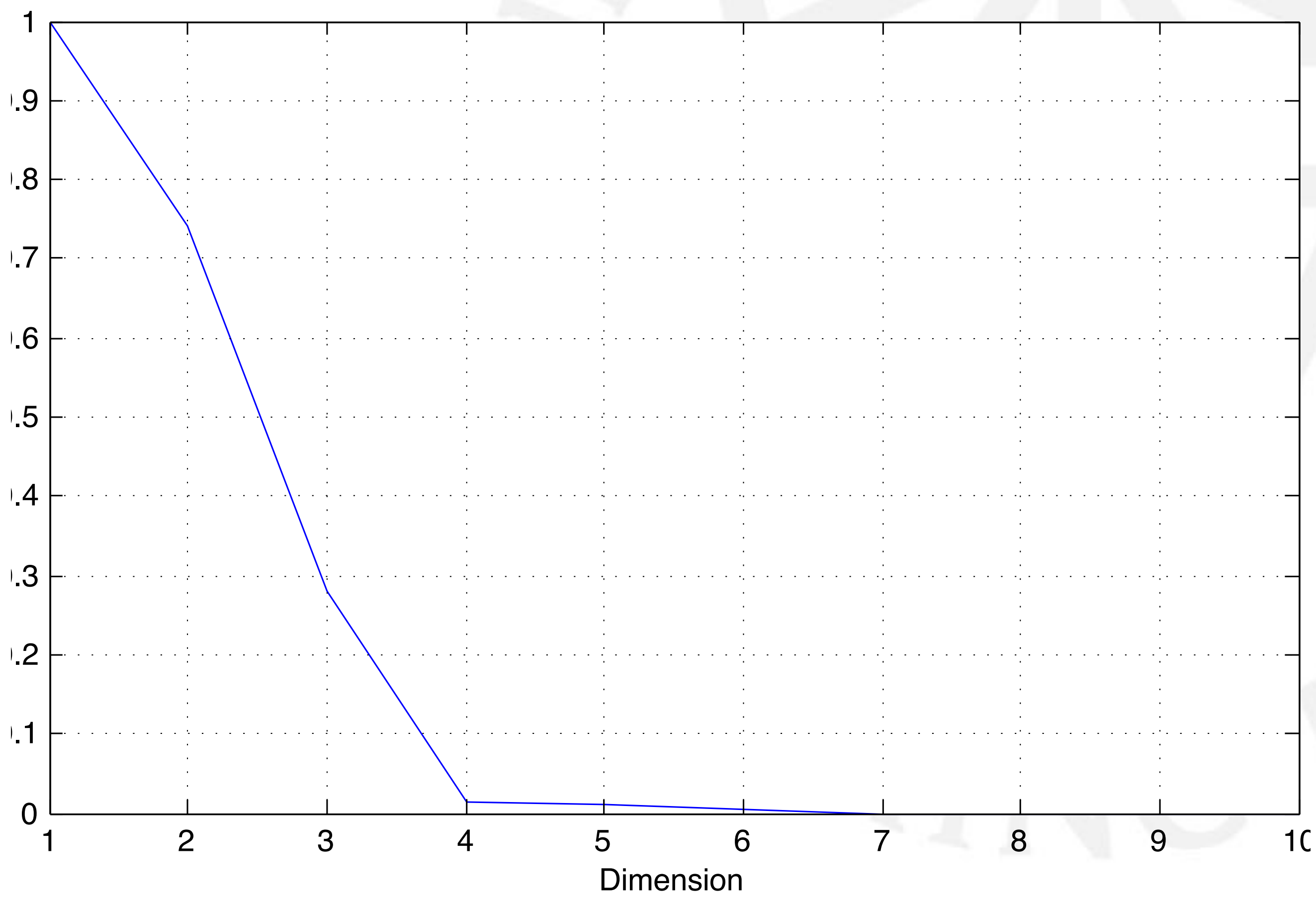
- Determine the embedding lag:
- Lag = 1?



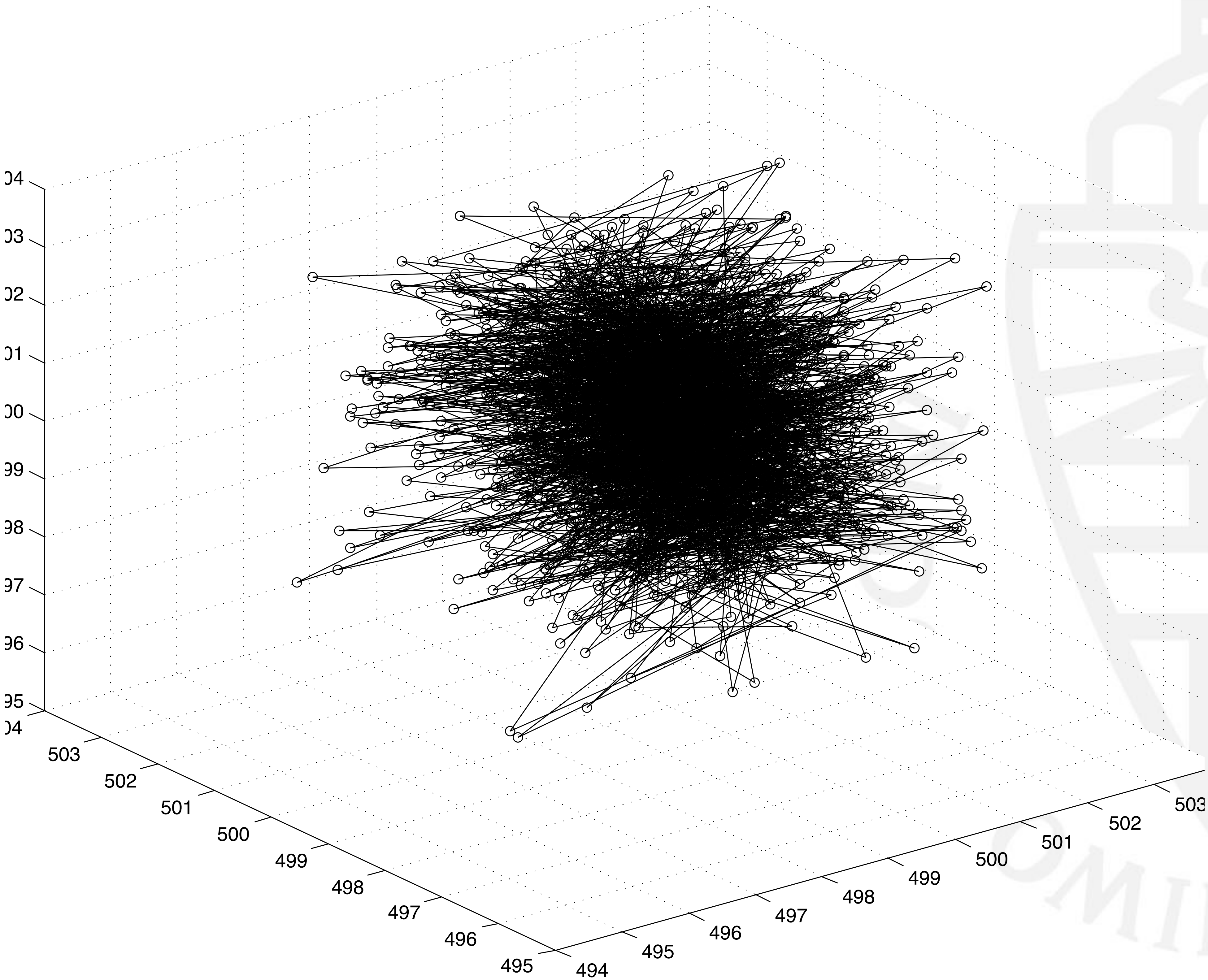
Suppose we have measured a true IID variable



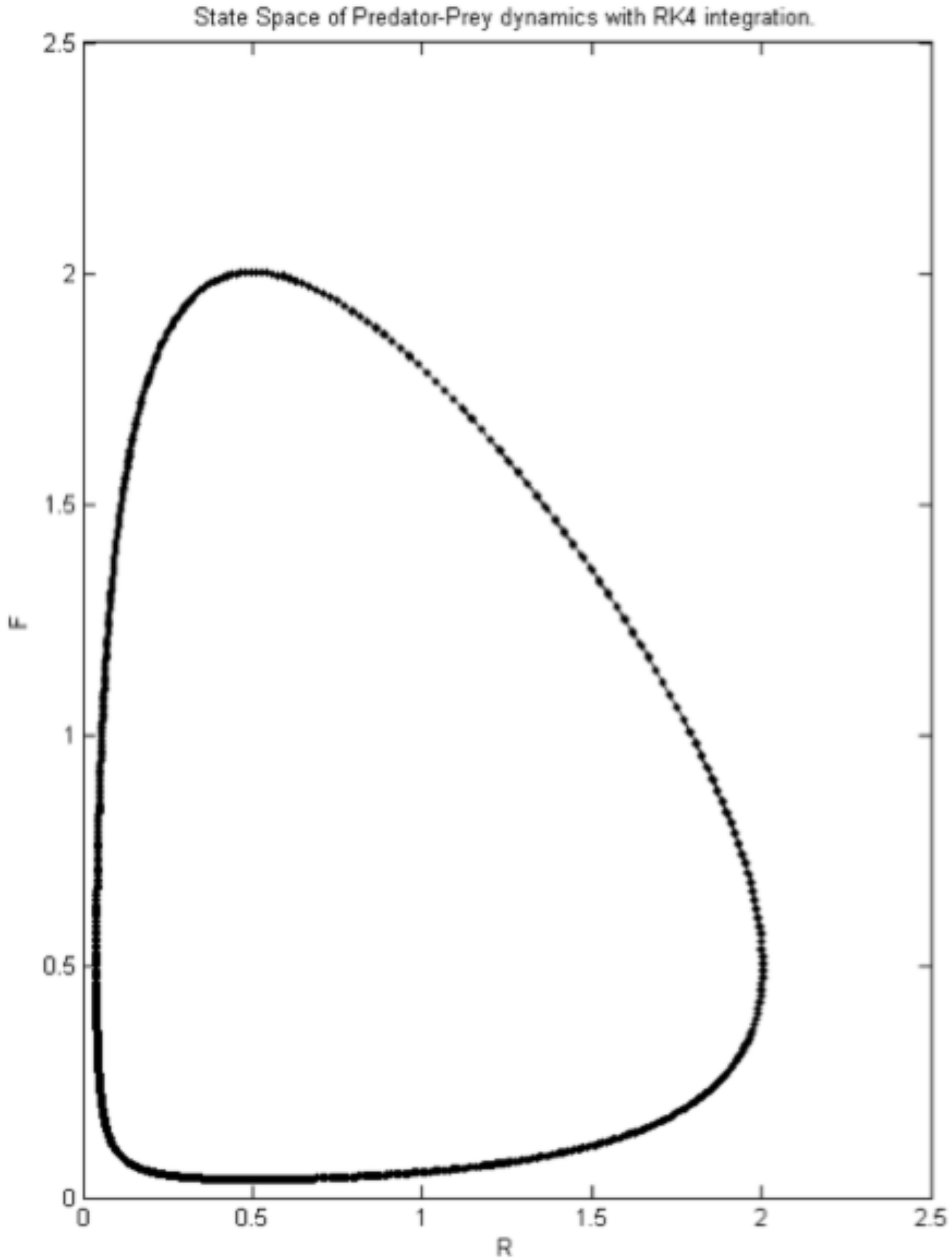
- Determine the embedding dimension:
- Dimension = 4,5,6,7?

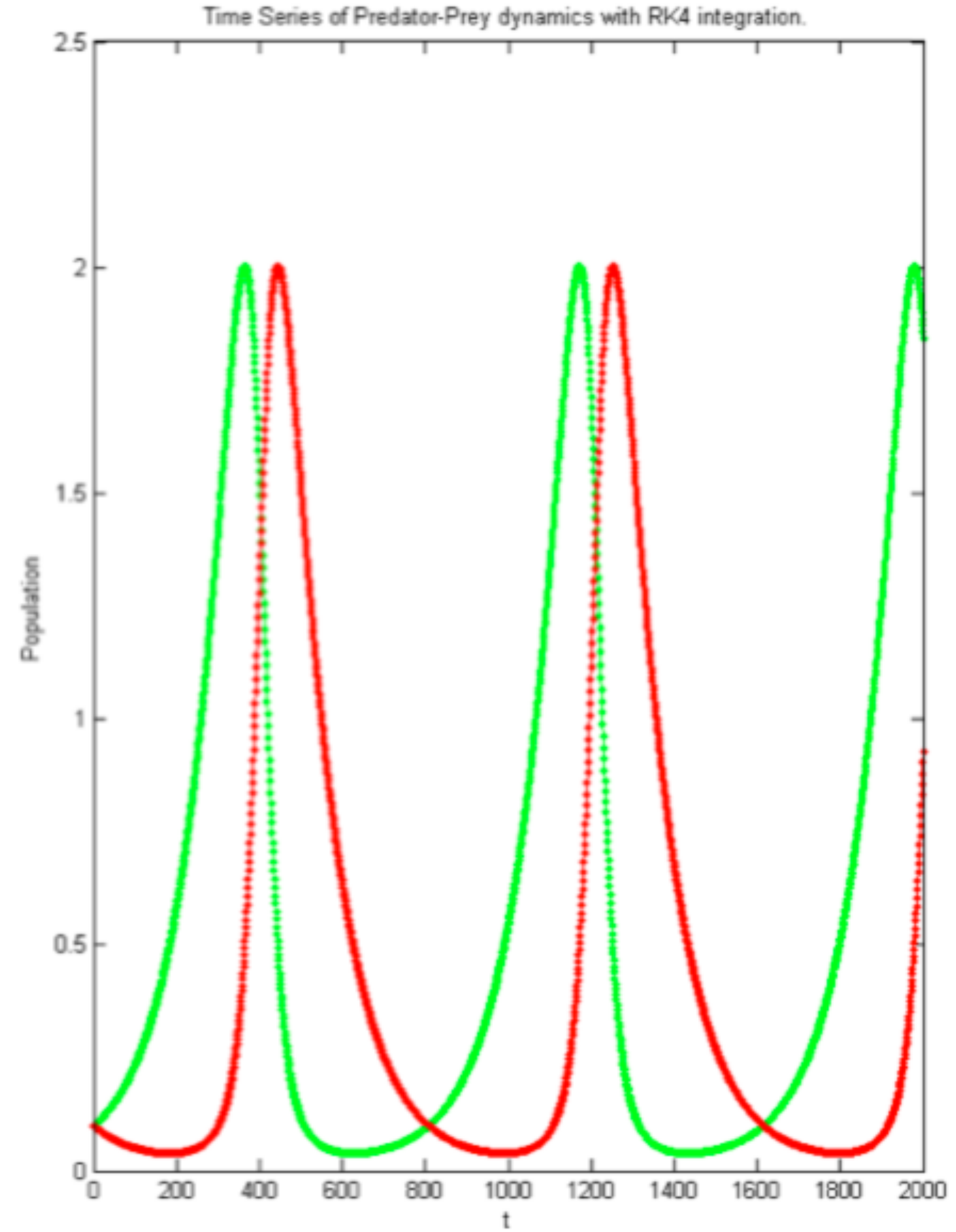


Suppose we have measured a true IID variable



Another familiar example: Predator-Prey dynamics





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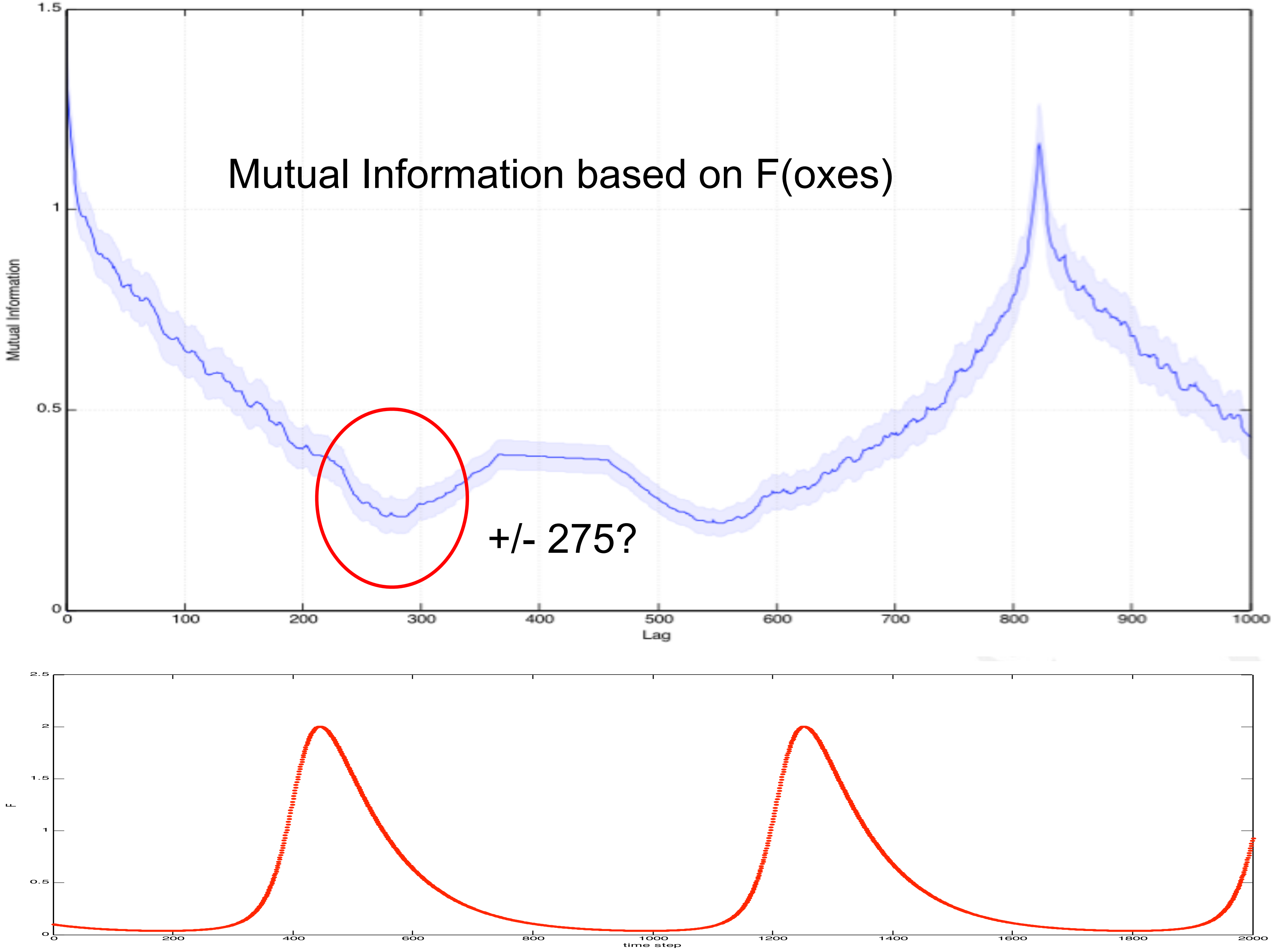

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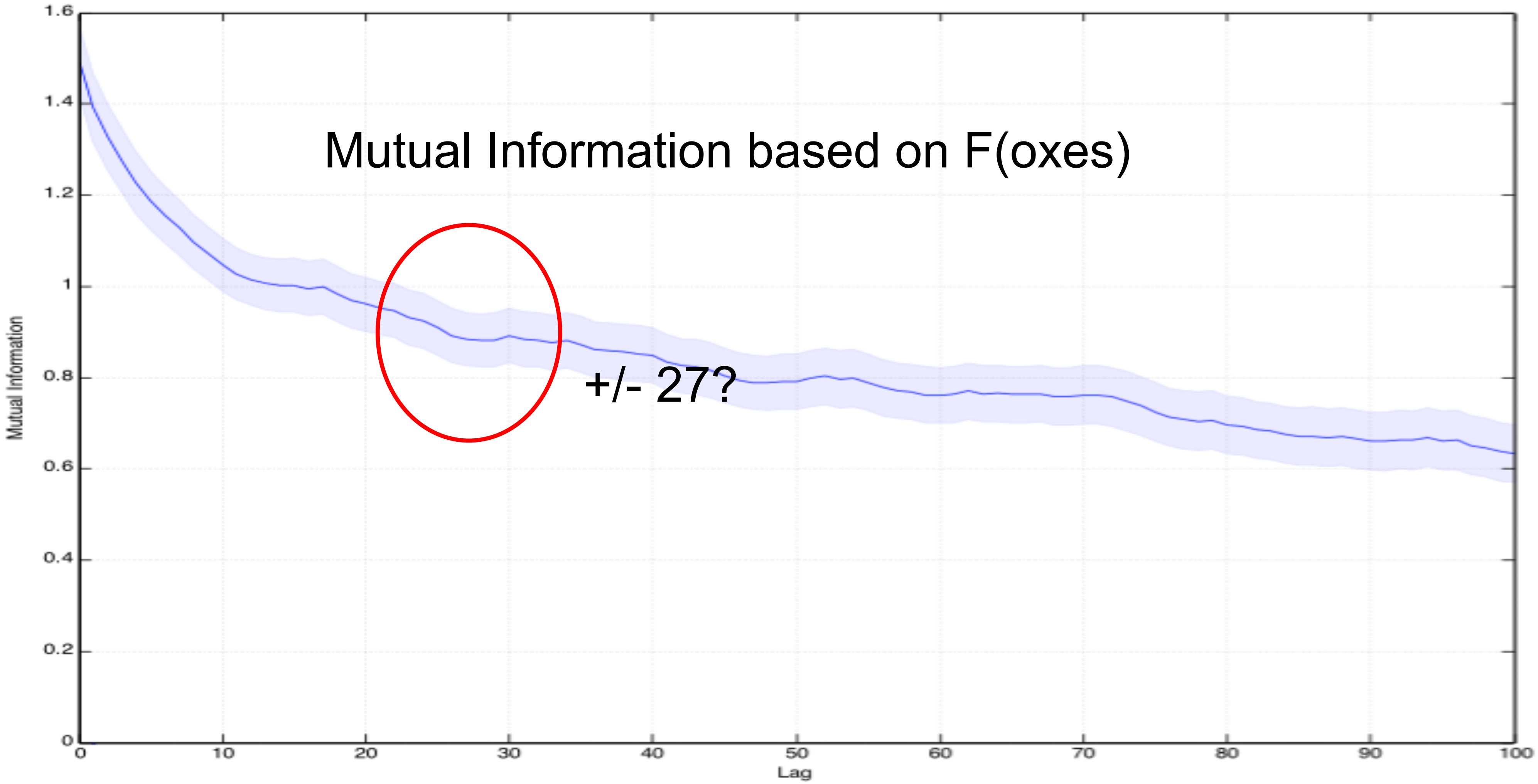

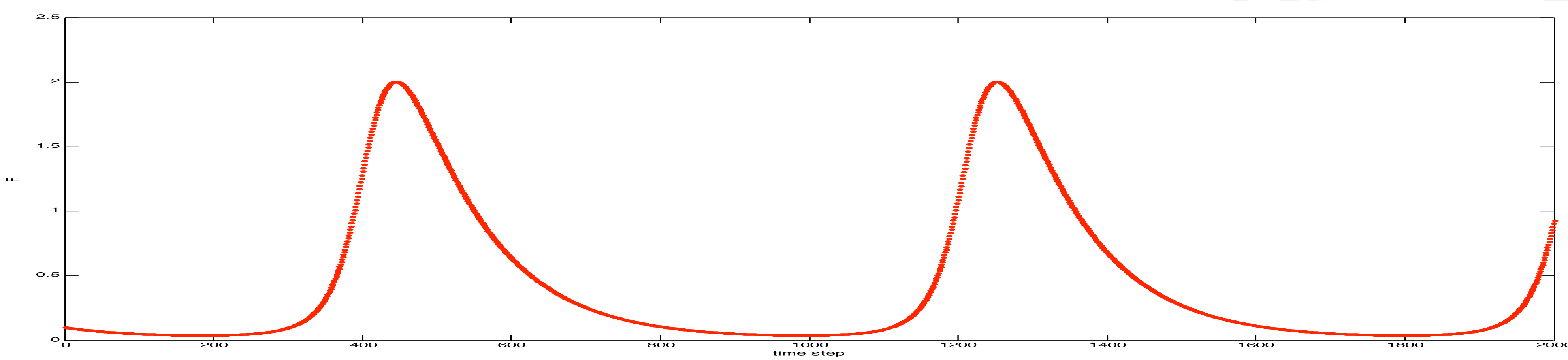

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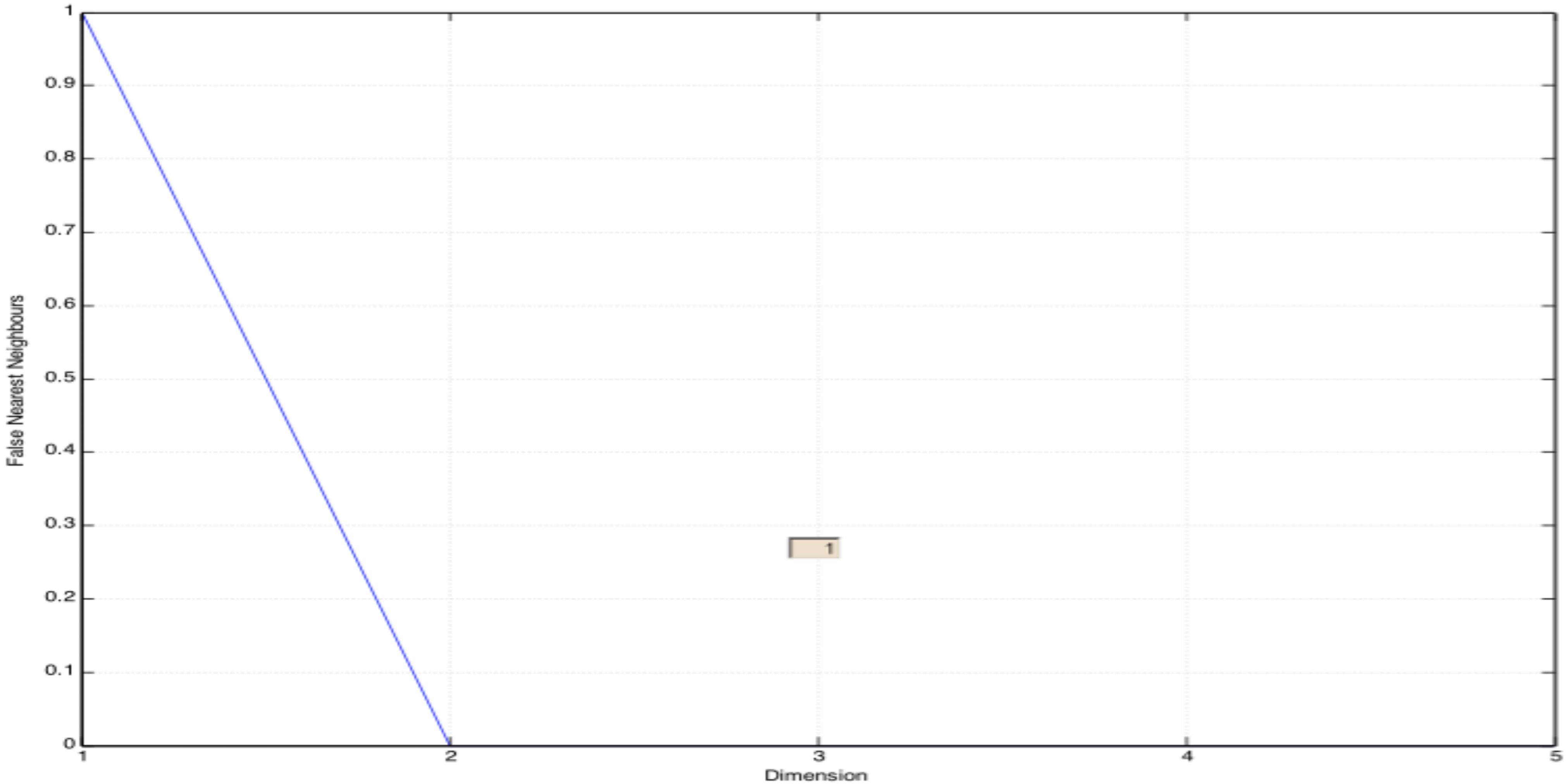
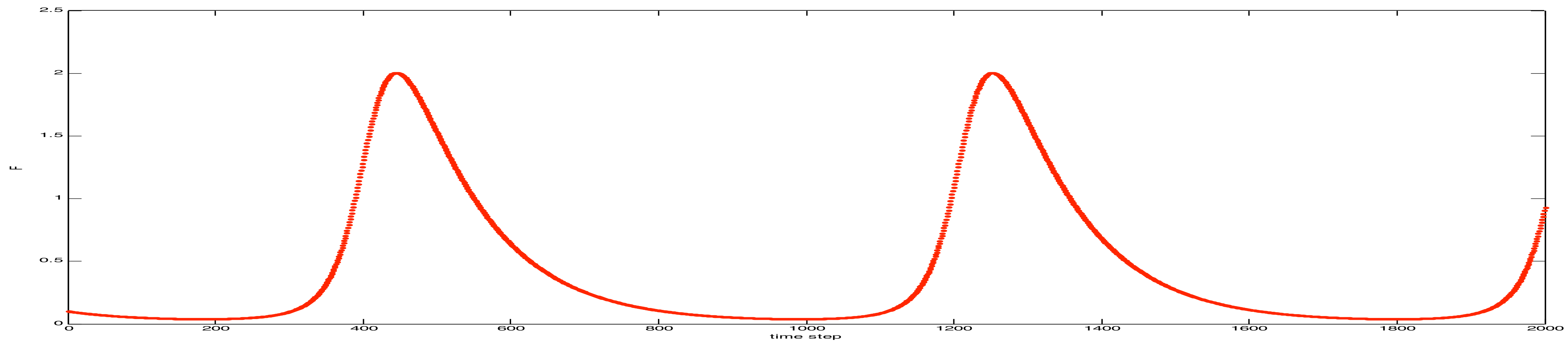
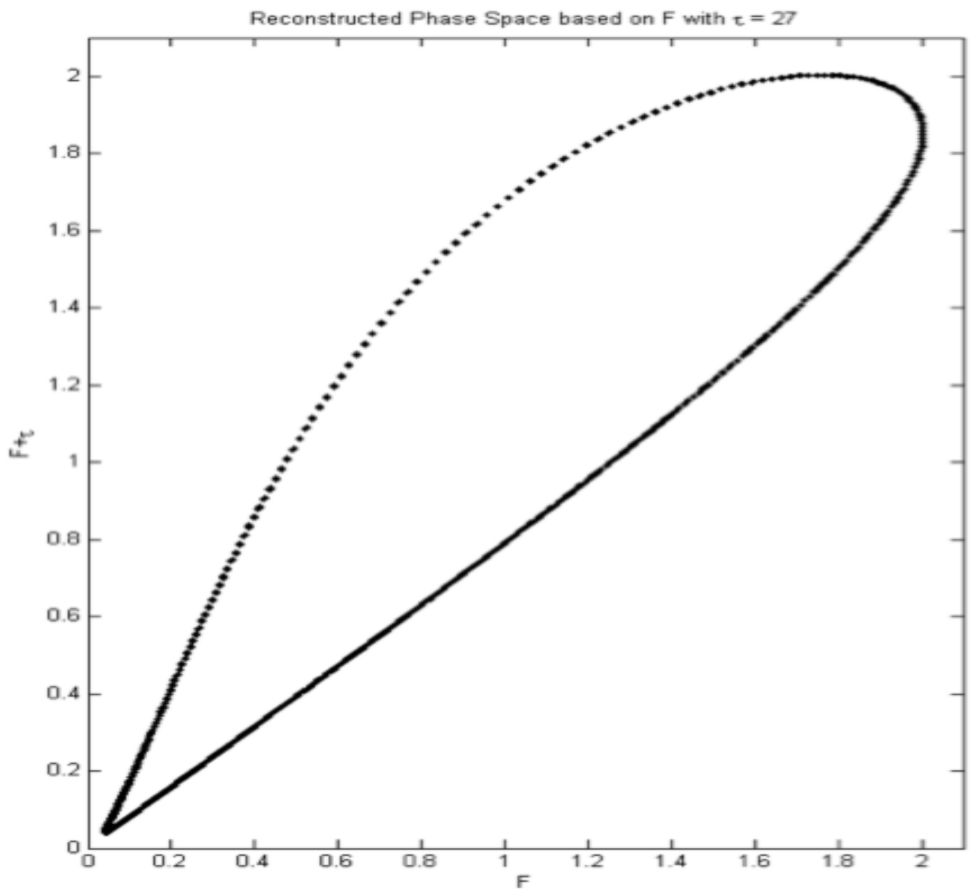
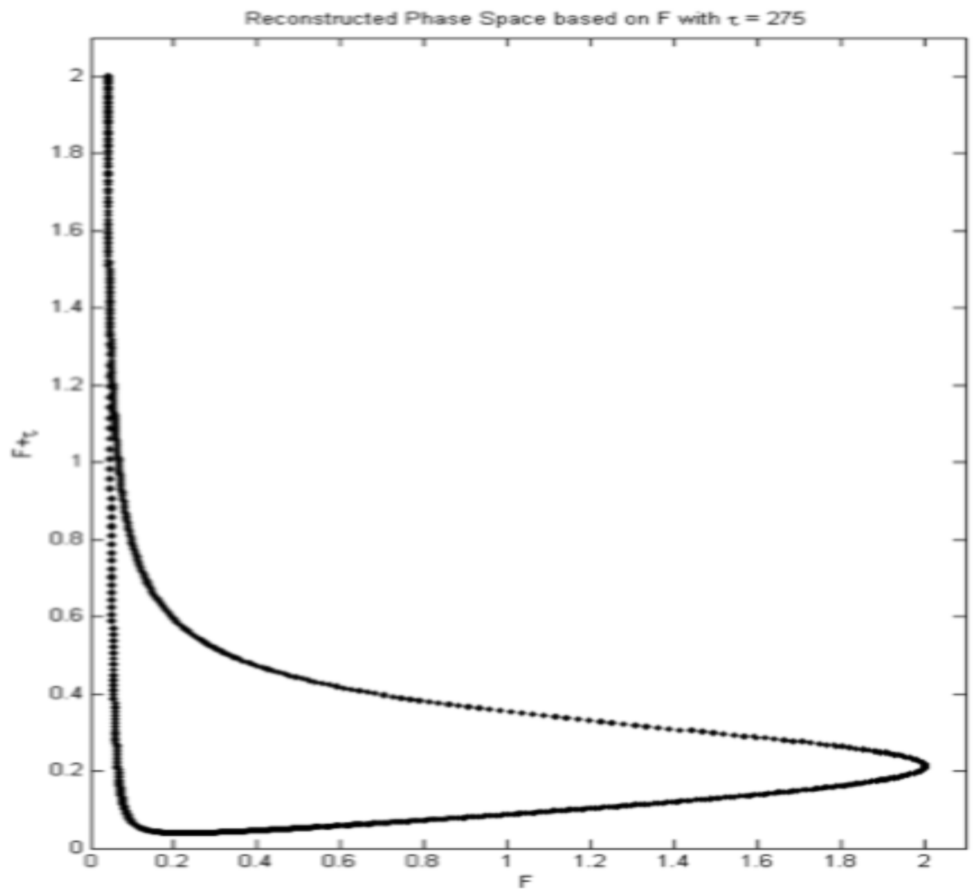


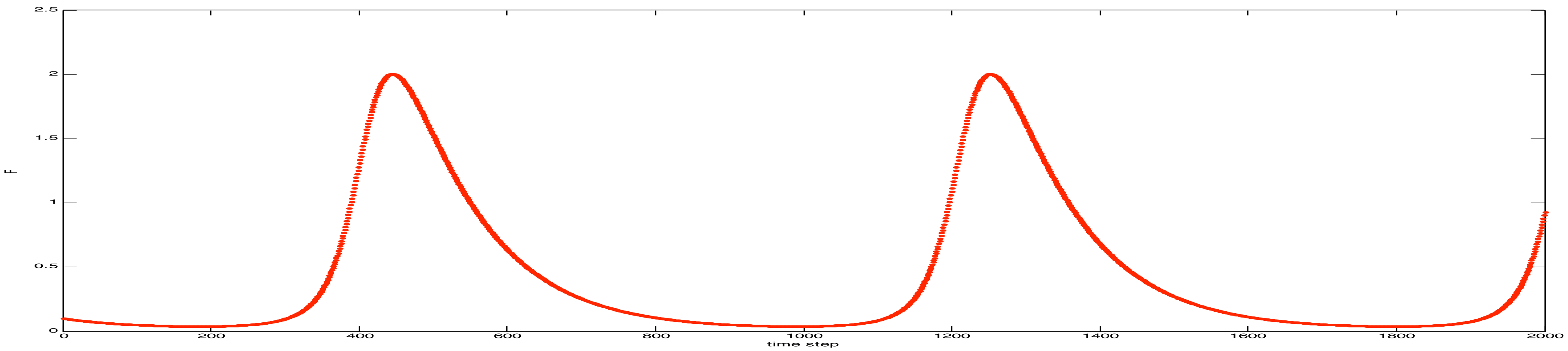


Another familiar example: Predator-Prey dynamics

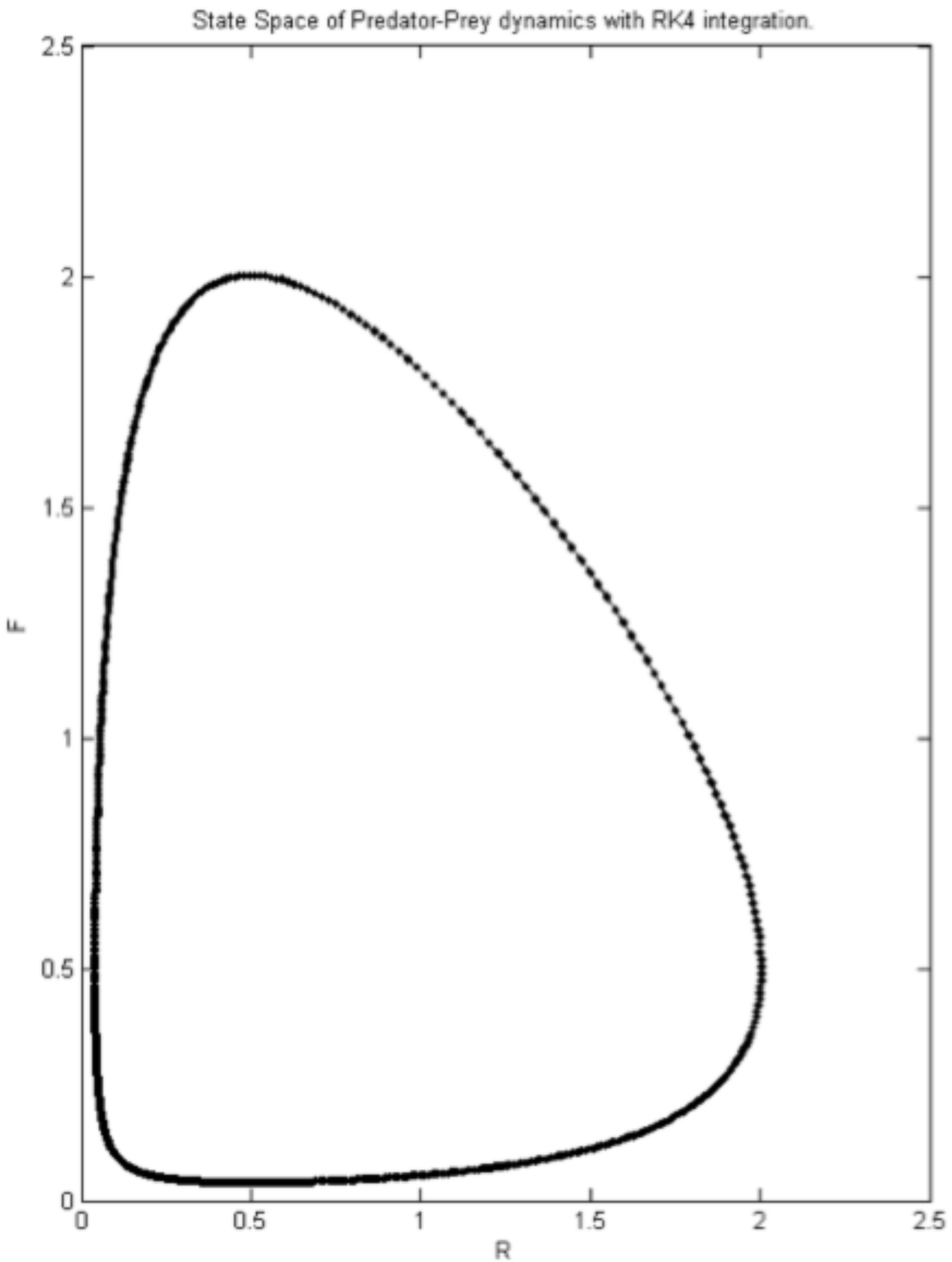
Embedding lag = 27 / 275 Embedding dimensions = 2

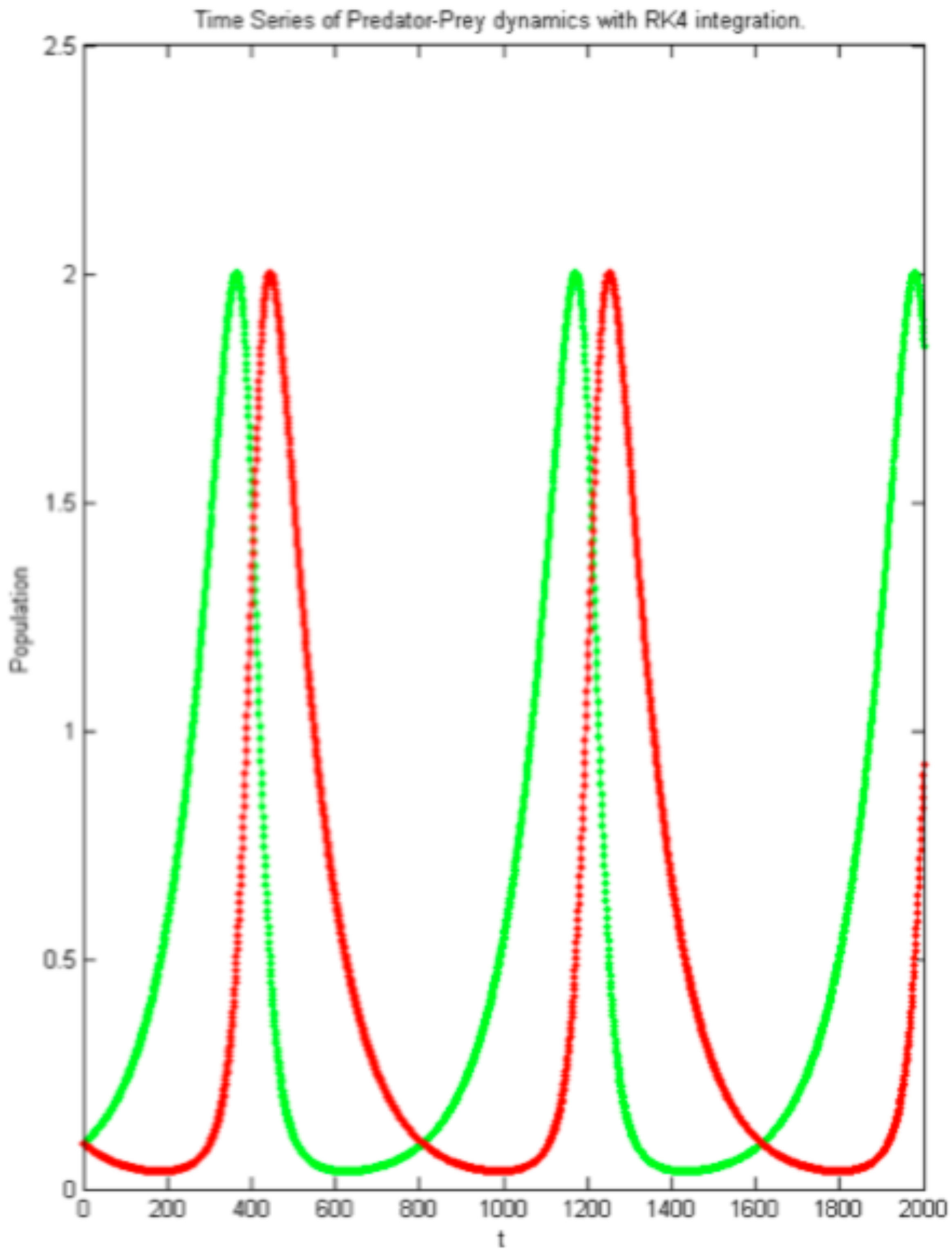






Another familiar example: Predator-Prey dynamics





Not so amazing?

- The reconstructed attractor is *'Topologically equivalent'* not exactly the same!!! (compare to random cloud of points)
  The exact lag is not that important, it is just a way to optimize the reconstruction

- If you are working with 'real' data from psychological experiments you will find that the dimensionality needed to describe the system is usually 10 dimensions or higher… No visual inspection anymore!

- Solution: Quantify the dynamic behaviour of the system in state space in terms of periodicity, randomness, etc. This remains similar to the original dynamics even if the attractor is not reconstructed exactly the same way (the reconstructed attractor is still much more constrained than all the states theoretically possible).

- (Cross) Recurrence Quantification Analysis!

Recurrence Quantification Analysis

- A technique to quantify the dynamical behaviour of the system in its' reconstructed phase space
- Already many applications in physics, biology and psychology
- Many flavours of RQA, today: Auto-Recurrence.
- Next week: *Cross Recurrence Quantification* (analyse if two signals share the same phase space, are synchronised), *Categorical data*, *Order Patterns Recurrence* and *Lagged Recurrence*

Recurrence Quantification Analysis

Consider this poem by Jules Deelder
to be a time series of letters.

Blues on tuesday

Geen geld.
Geen vuur.
Geen speed.

Geen krant.
Geen wonder.
Geen weed.

Geen brood.
Geen tijd.
Geen weet.

Geen klote.
Geen donder.
Geen reet.

Recurrence Quantification Analysis

Blues on tuesday

Geen geld.
Geen vuur.
Geen speed.

Geen krant.
Geen wonder.
Geen weed.

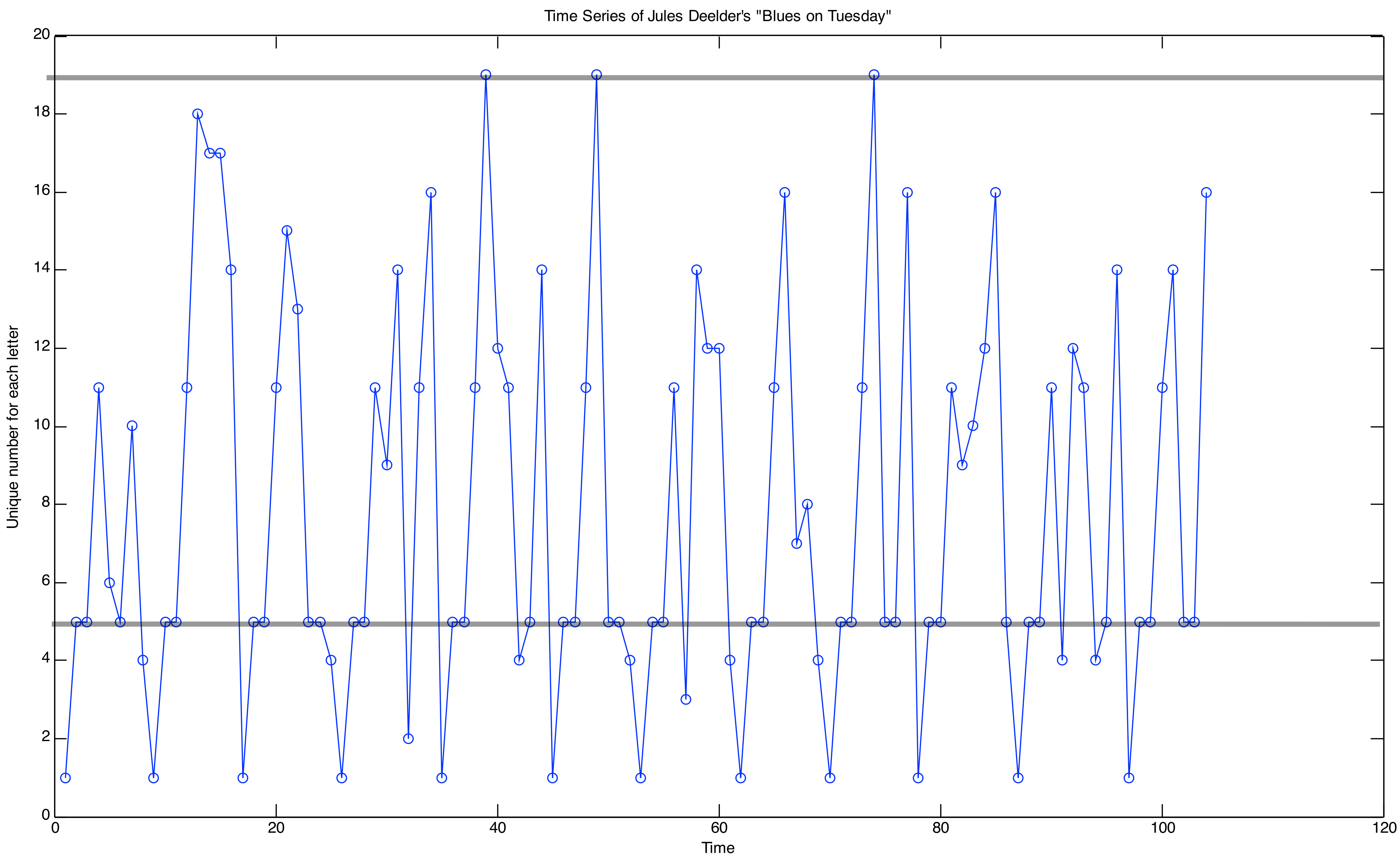
Geen brood.
Geen tijd.
Geen weet.

Geen klote.
Geen donder.
Geen reet.

g = 1
e = 2
e = 2
n = 3
g = 1
e = 2
l = 4
d = 5
g = 1
e = 2
e = 2
n = 3
v = 6
u = 7
u = 7
r = 8
…

Recurrence Quantification Analysis

Blues on tuesday



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Plot a point every time a letter (value) recurs

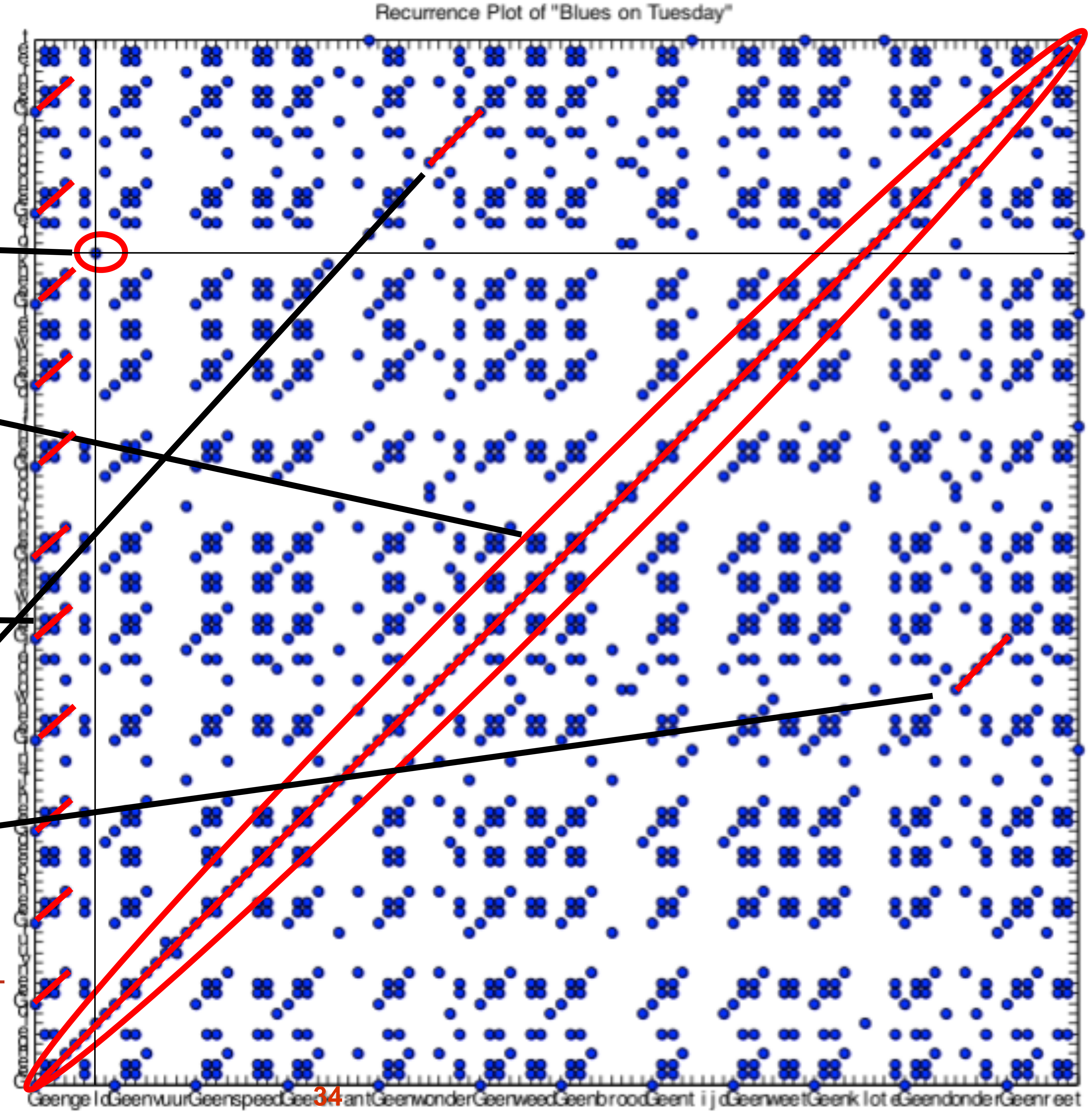
Letter L recurs 1 time

Line of Identity

Plot is symmetrical: Auto-recurrence

Larger recurring patterns form diagonal lines: GEEN

What's this? Recurrence of: ONDER



A Recurrence Plot is a way to quantify the dynamics of a system in its reconstructed phase space

- Look in the reconstructed phase space when a value is recurring
- For continuous data: The value does not have to be exactly the same. Decide on a radius in which it is acceptable to call it recurring
- Many measures can be computed from recurrence plots!

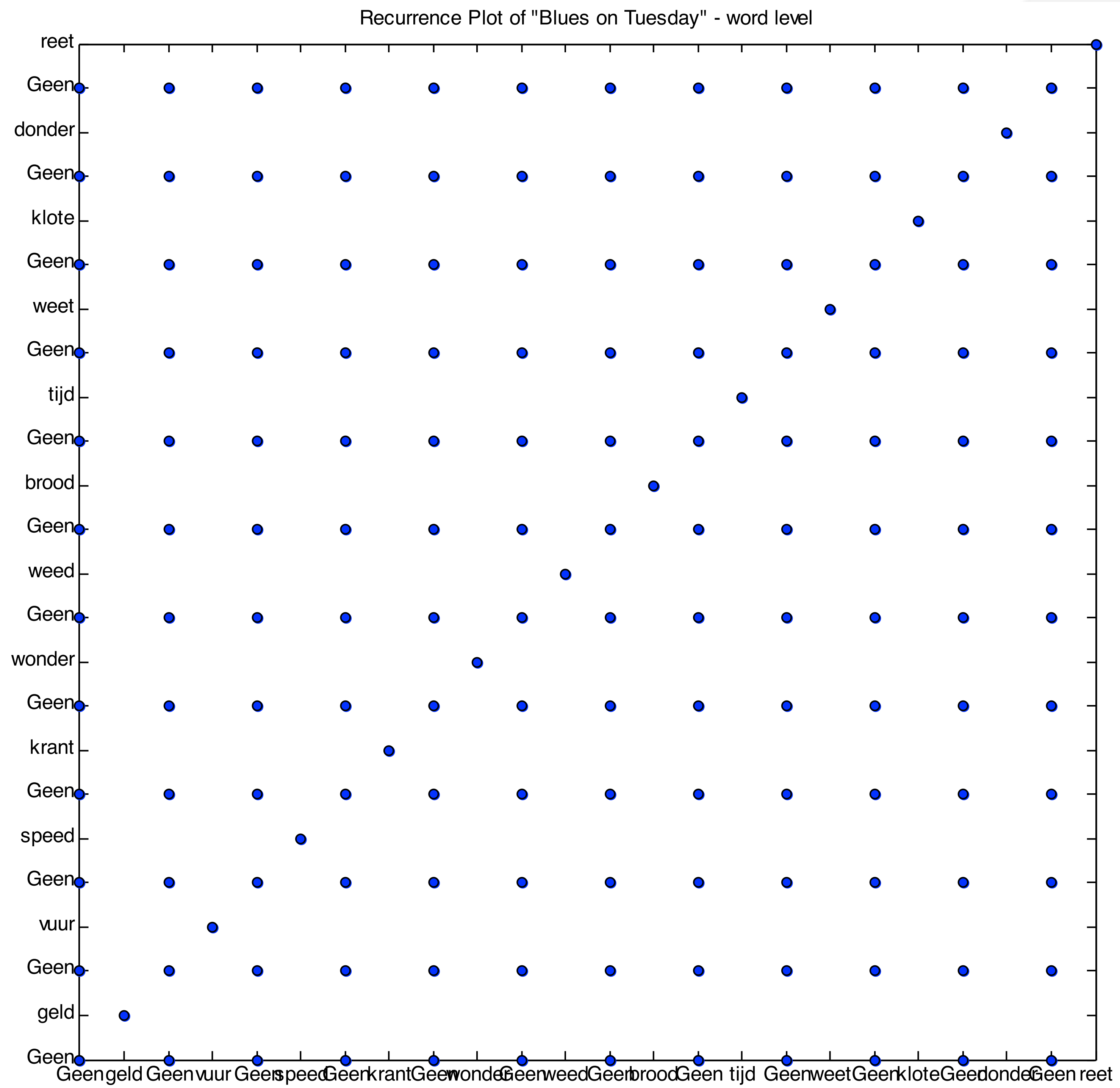
What would a plot of the poem at the word level look like?

Geen geld.
Geen vuur.
Geen speed.

Geen krant.
Geen wonder.
Geen weed.

Geen brood.
Geen tijd.
Geen weet.

Geen klote.
Geen donder.
Geen reet.



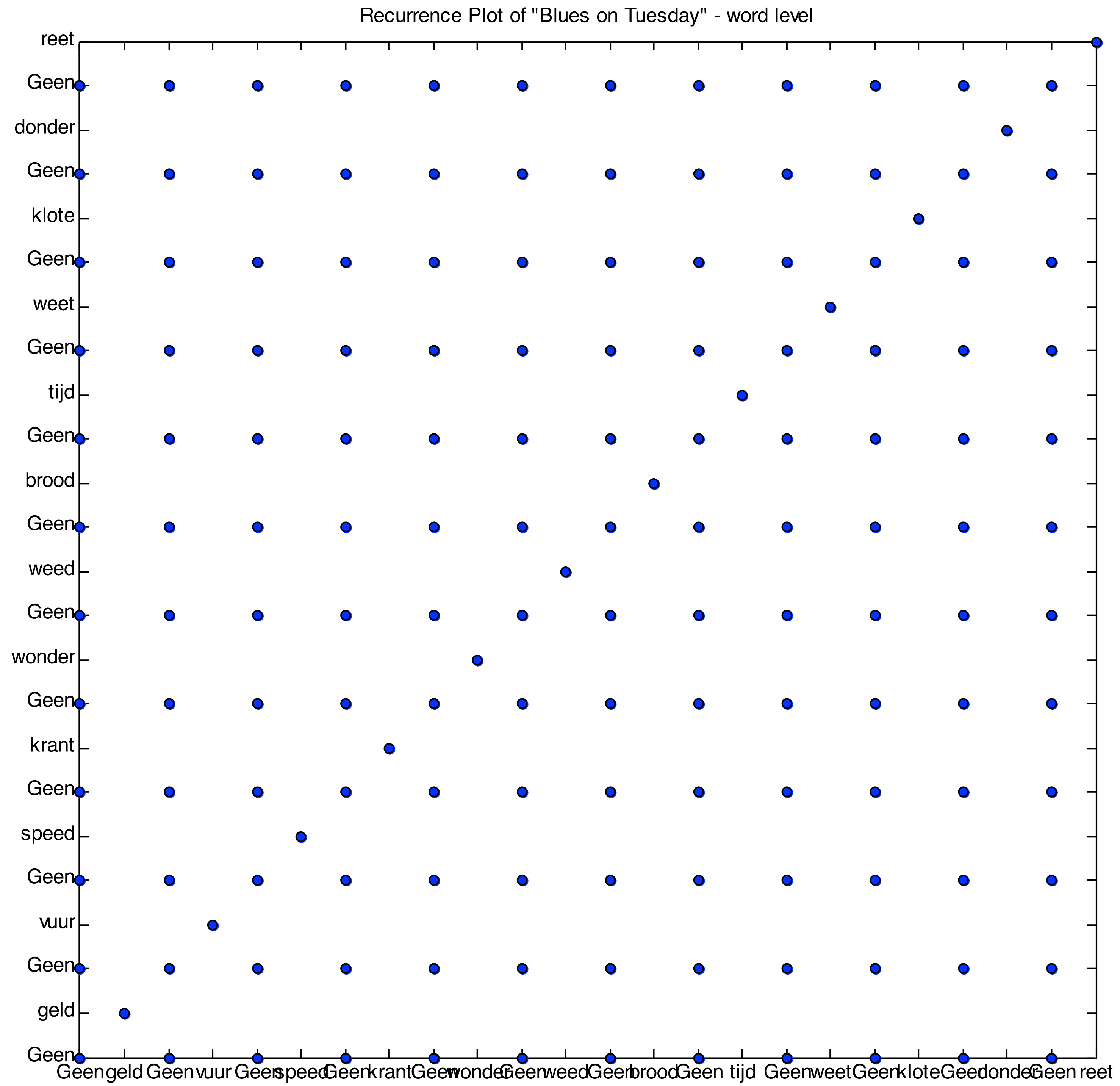
What would a plot of the poem at the word level look like?

Just the word: GEEN

It recurs very regularly

How to quantify this?

Back to Lorenz

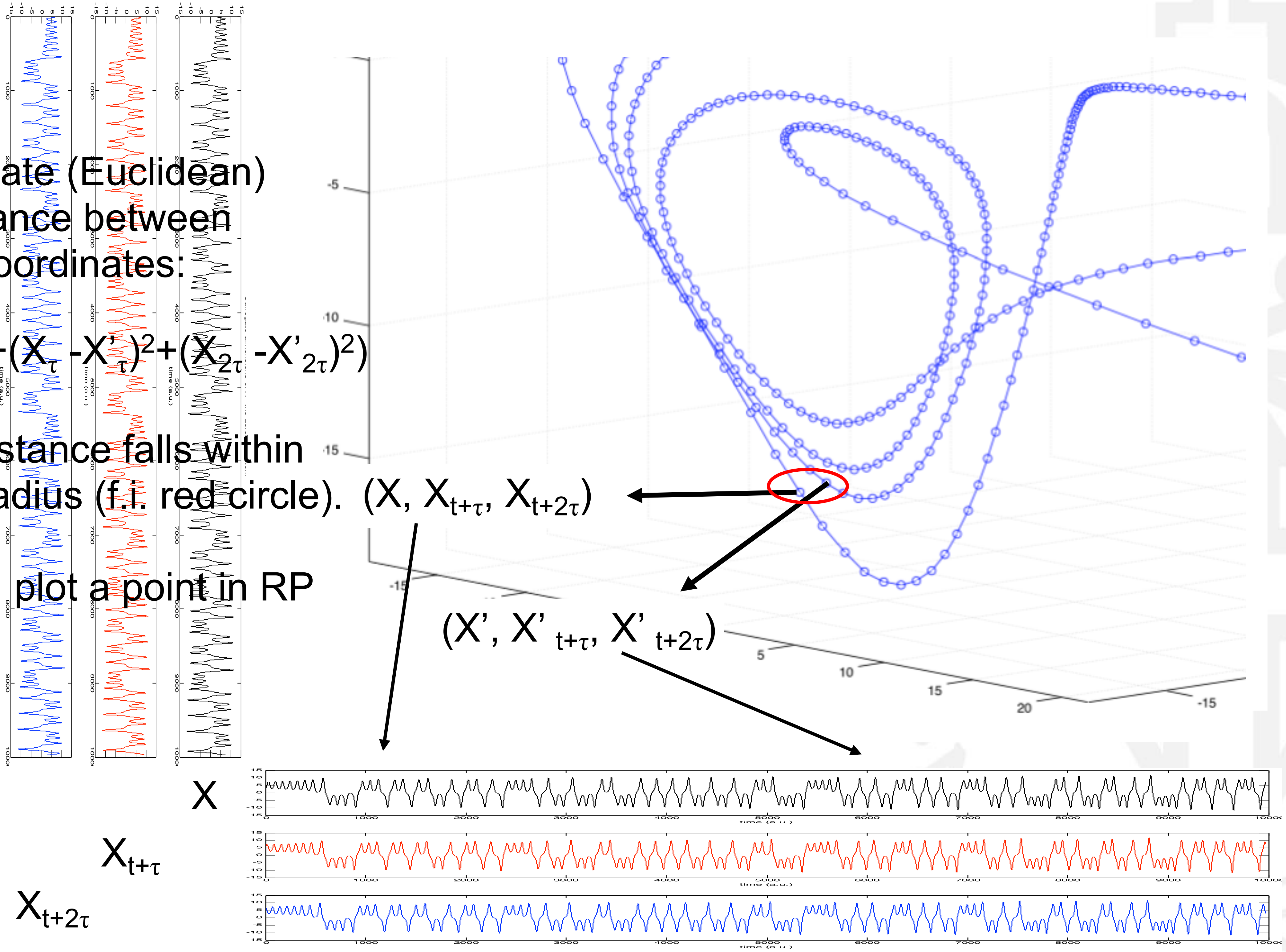


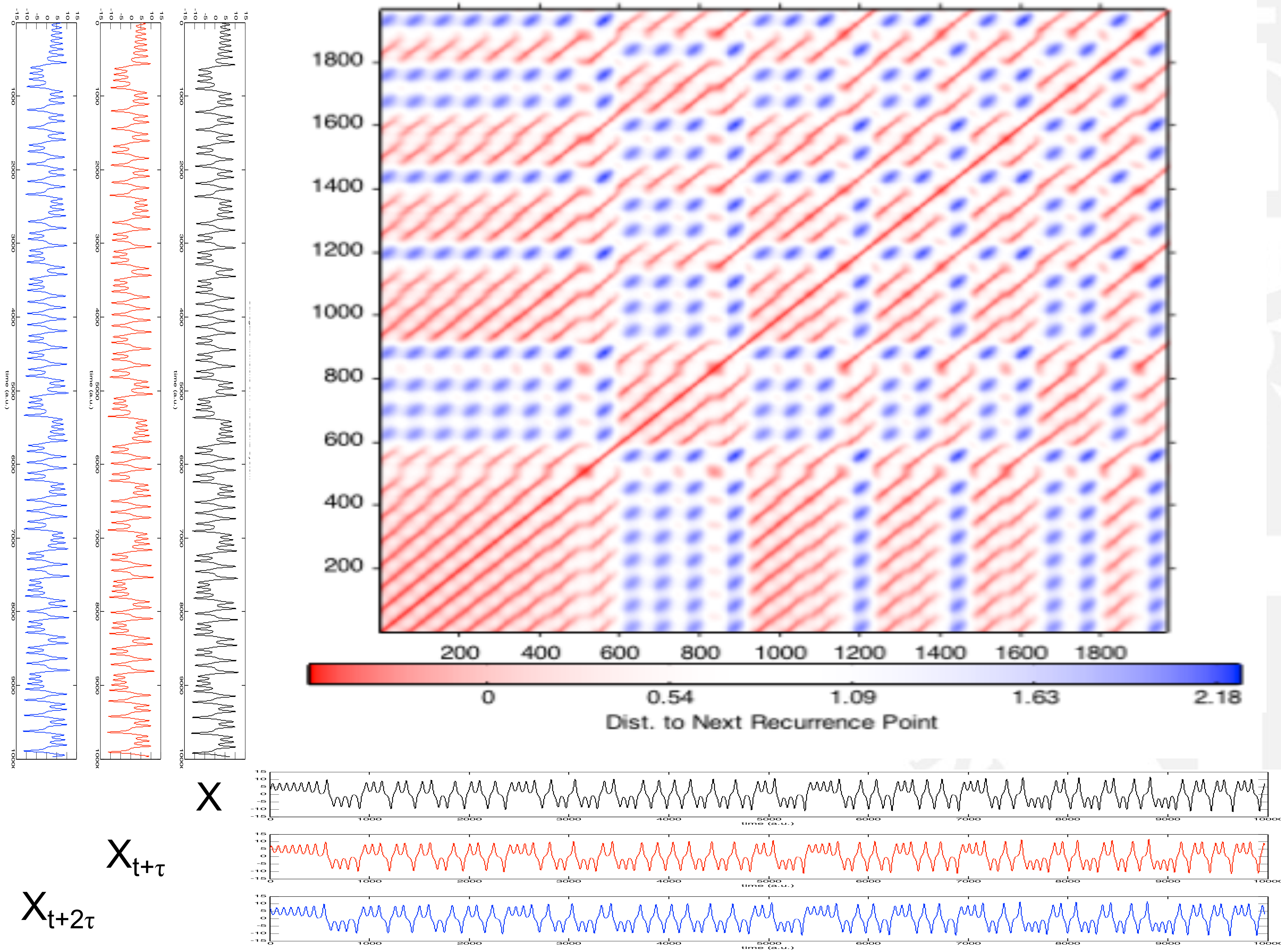
Calculate (Euclidean) distance between coordinates:

$\mathrm{sqrt}((X-X')^2+(X_\tau - X'_\tau)^2+(X_{2\tau} - X'_{2\tau})^2)$

See if distance falls within a certain radius (f.i. red circle).

If it does, plot a point in RP





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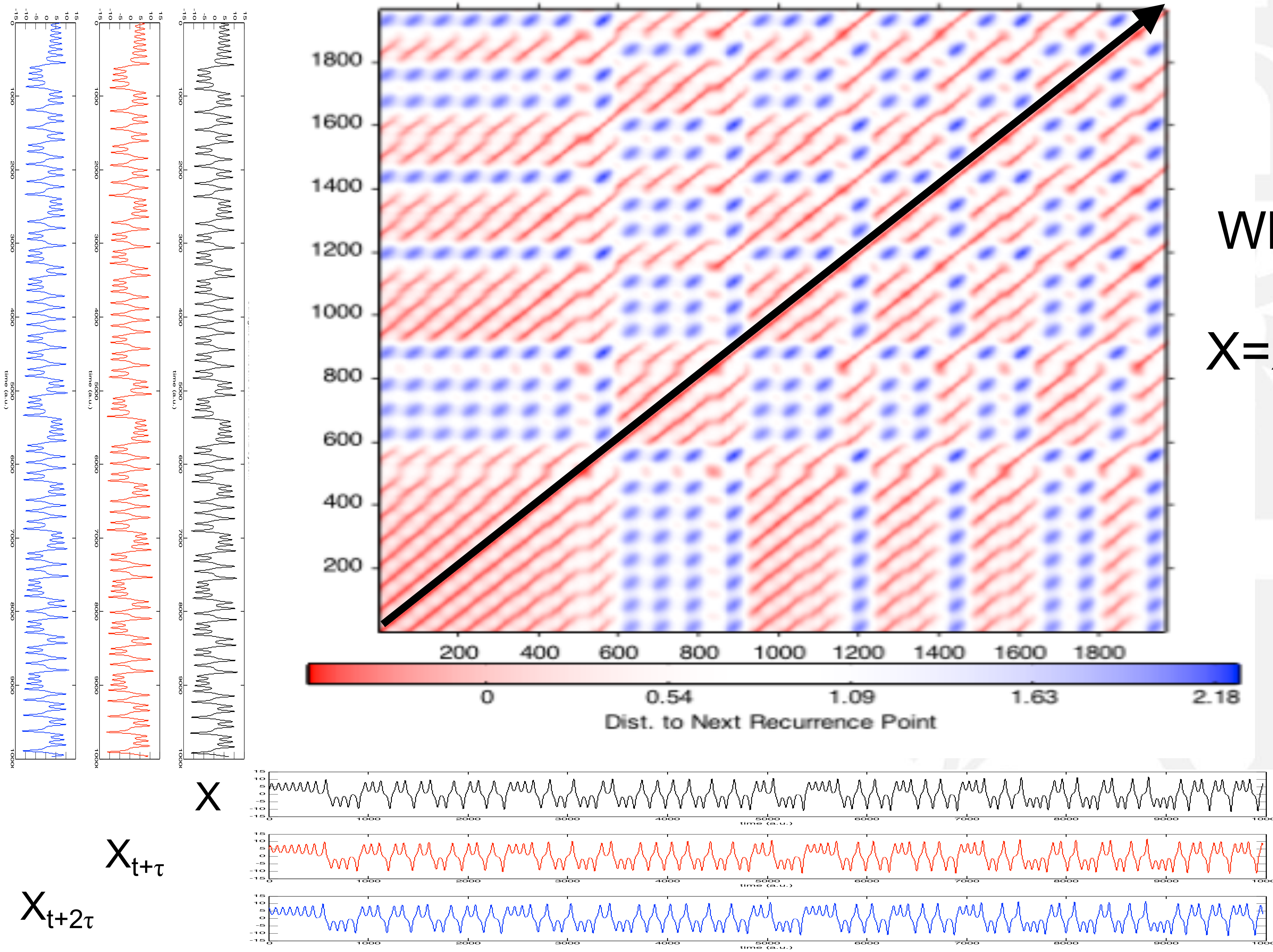
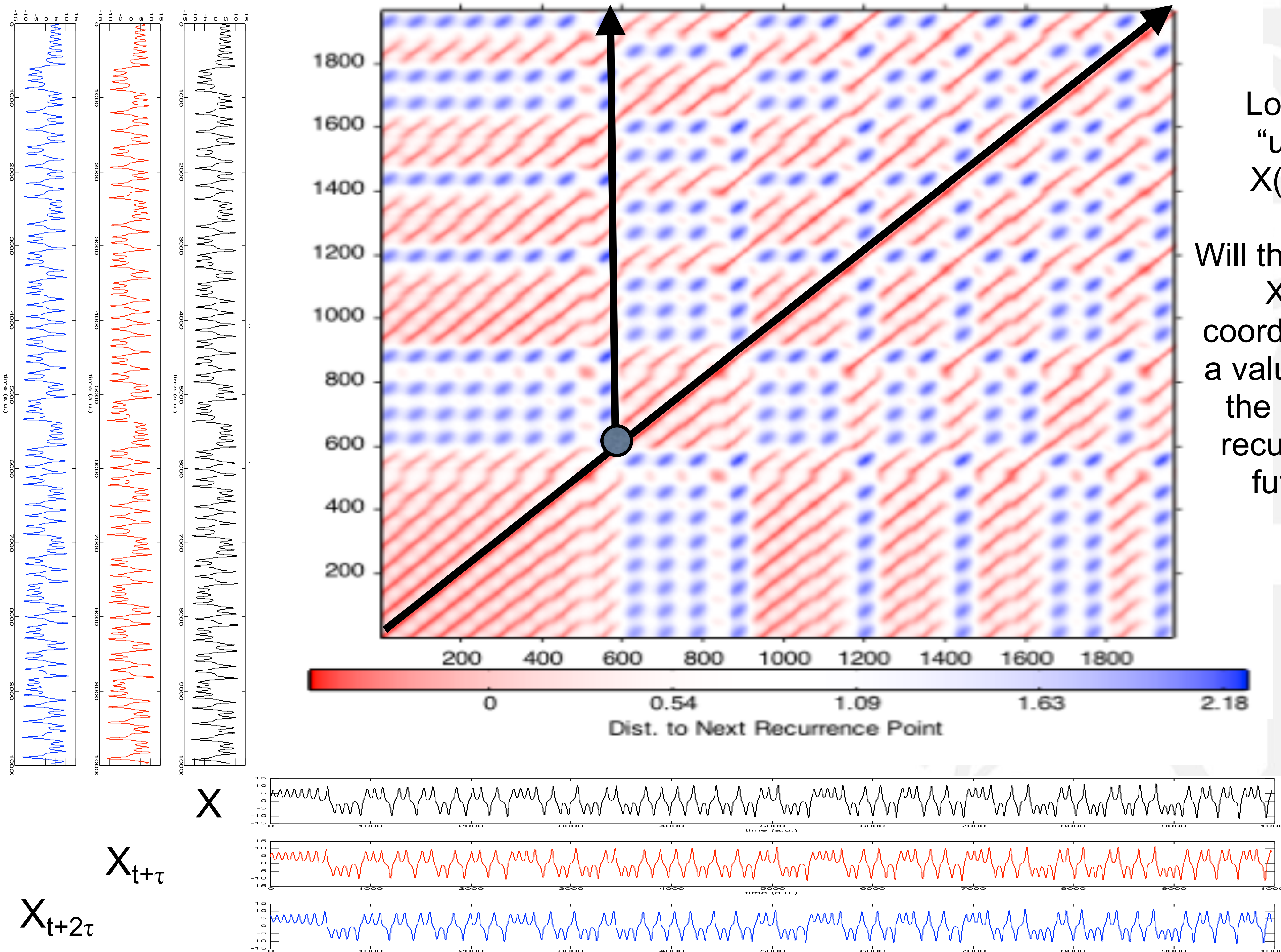


1800
1600
1400
1200
1000
800
600
400
200
200 400 600 800 1000 1200 1400 1600 1800
0 0.54 1.09 1.63 2.18
Dist. to Next Recurrence Point
X
$X_{t+\tau}$
$X_{t+2\tau}$

Where is X=X(t)?



Looking "up" at X(600):

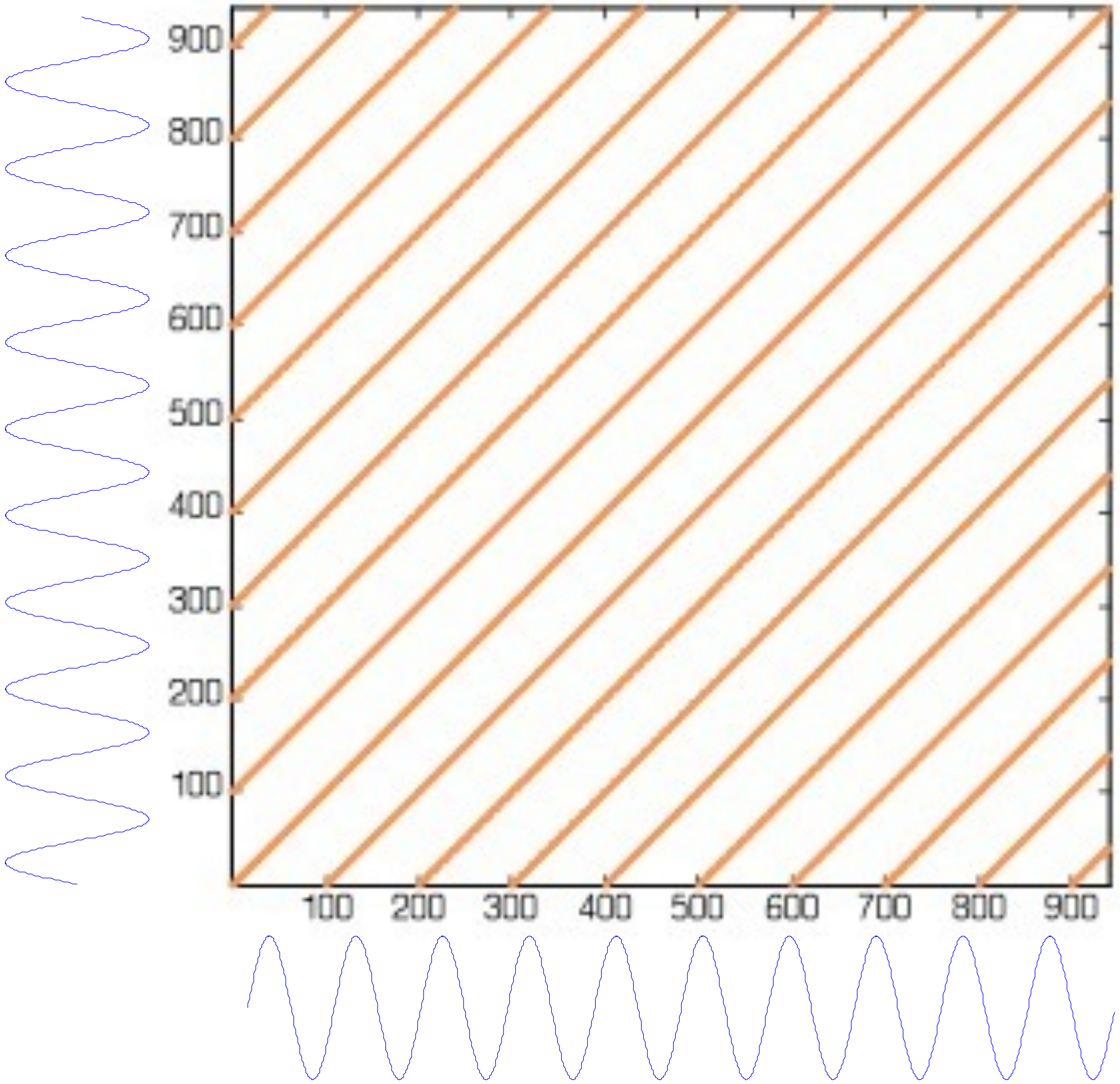
Will the current X,Y,Z coordinate (or a value within the radius) recur in the future?

Quantifying Recurrence

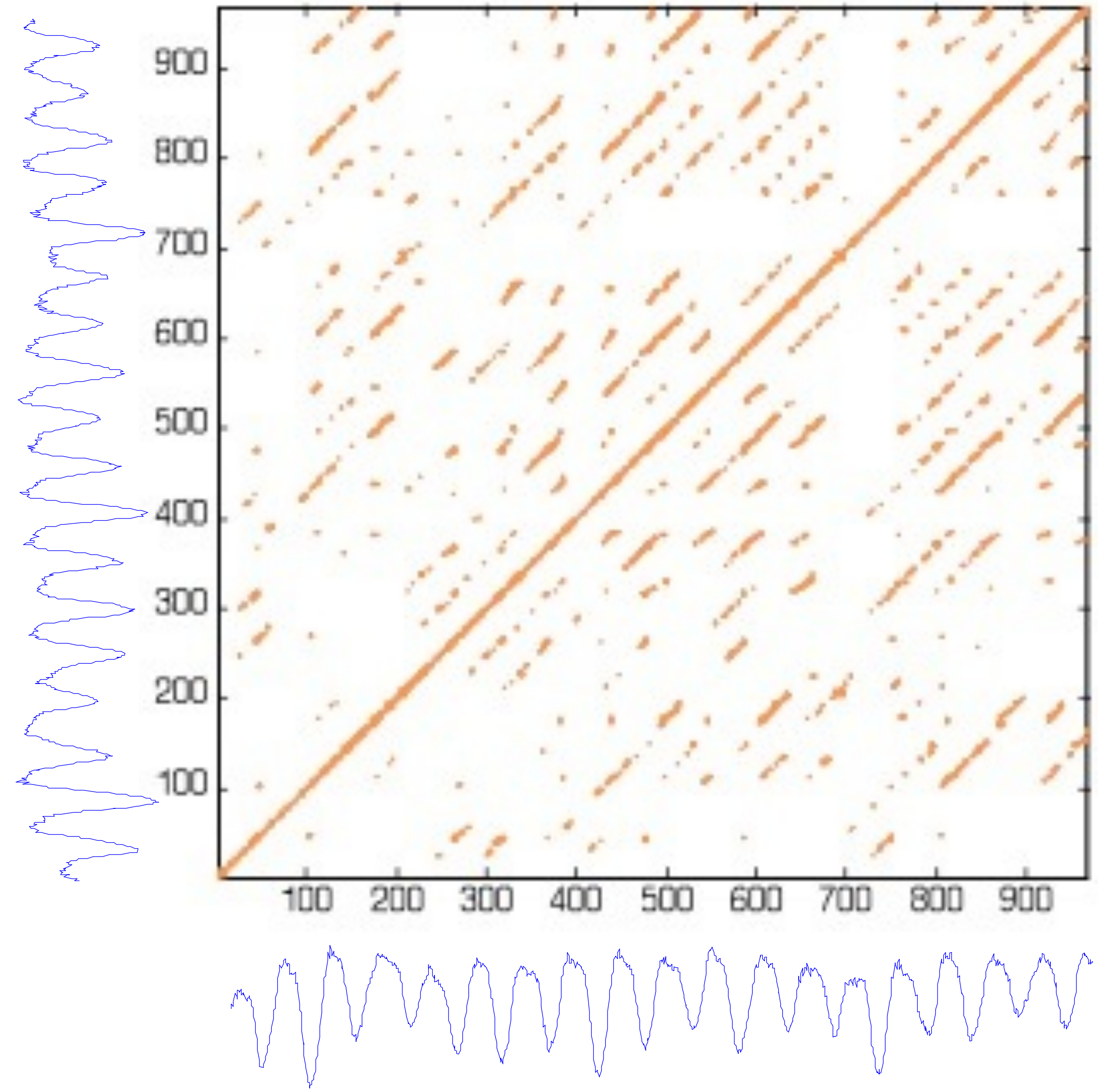
Shockley 2007

$$\%REC = \frac{\text{Number of recurrent points}}{\text{Total number of locations}} \times 100$$

Sine
%REC = 2.9



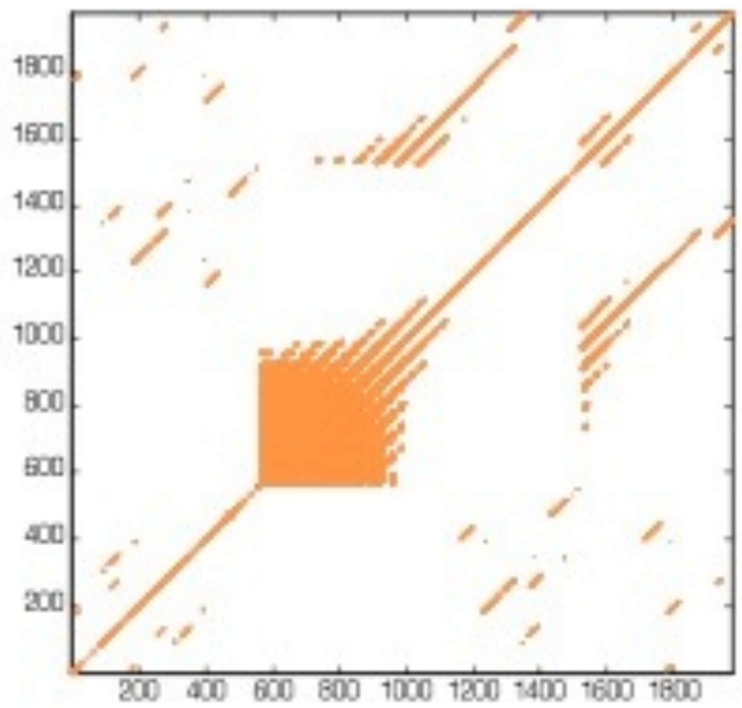
Limb oscillation to a metronome
%REC = .72



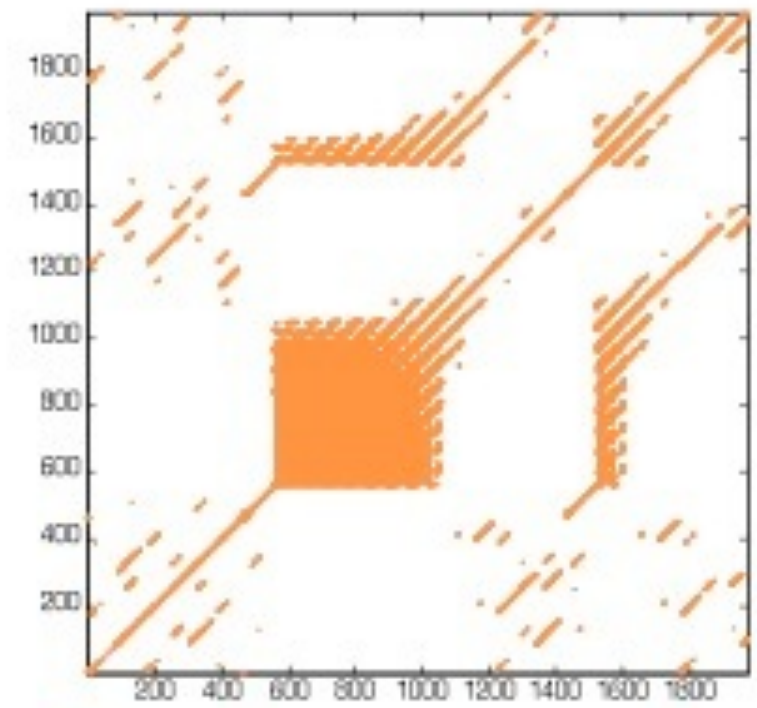
Note that %REC is the number of points in phase space that recur, relative to all possible points that could recur. It is influenced by the radius you choose!
When comparing groups or subjects: keep %REC constant.

Note how the recurrence plot changes with changes in radius

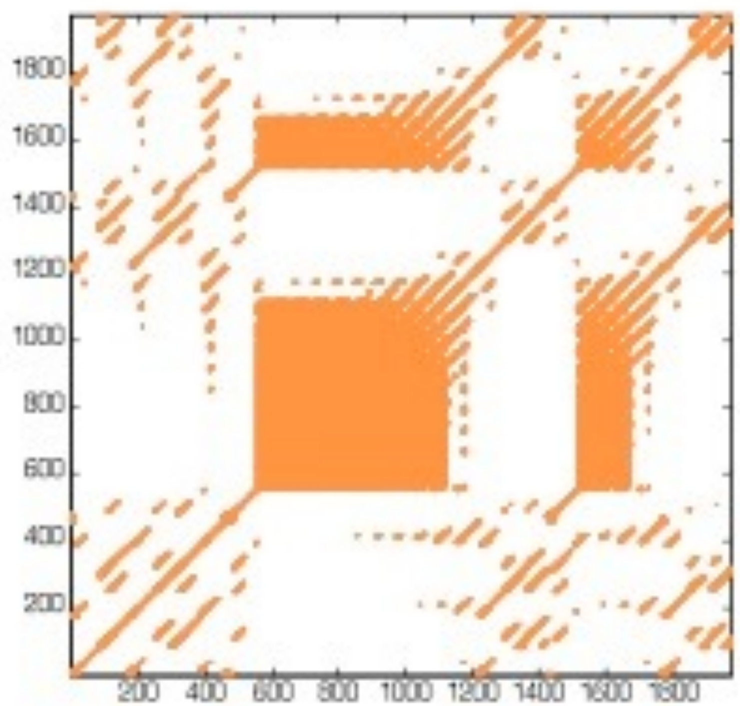
Radius = 3



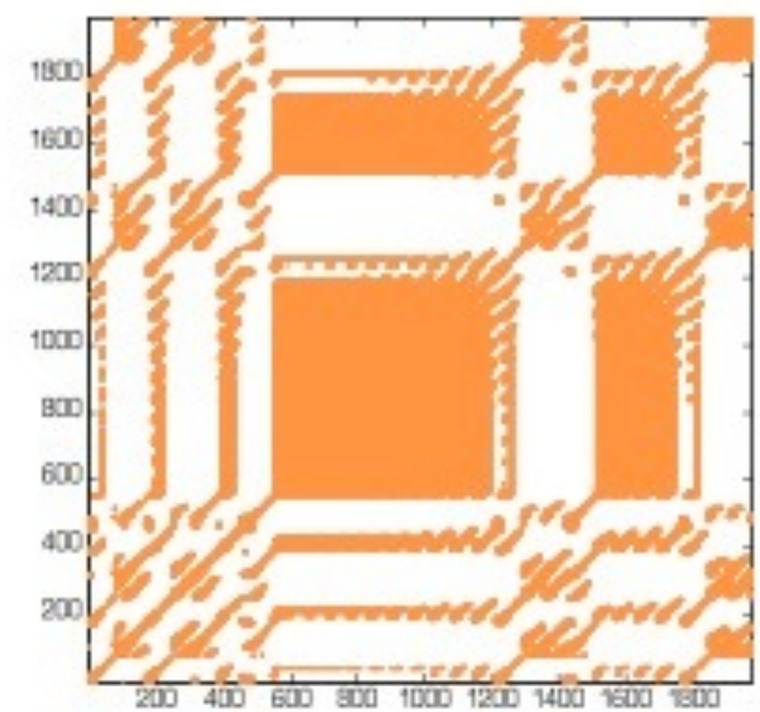
Radius = 5



Radius=10



Radius=20



Shockley 2007

Is there a prescription for picking your radius?

Adapted from Shockley 2007

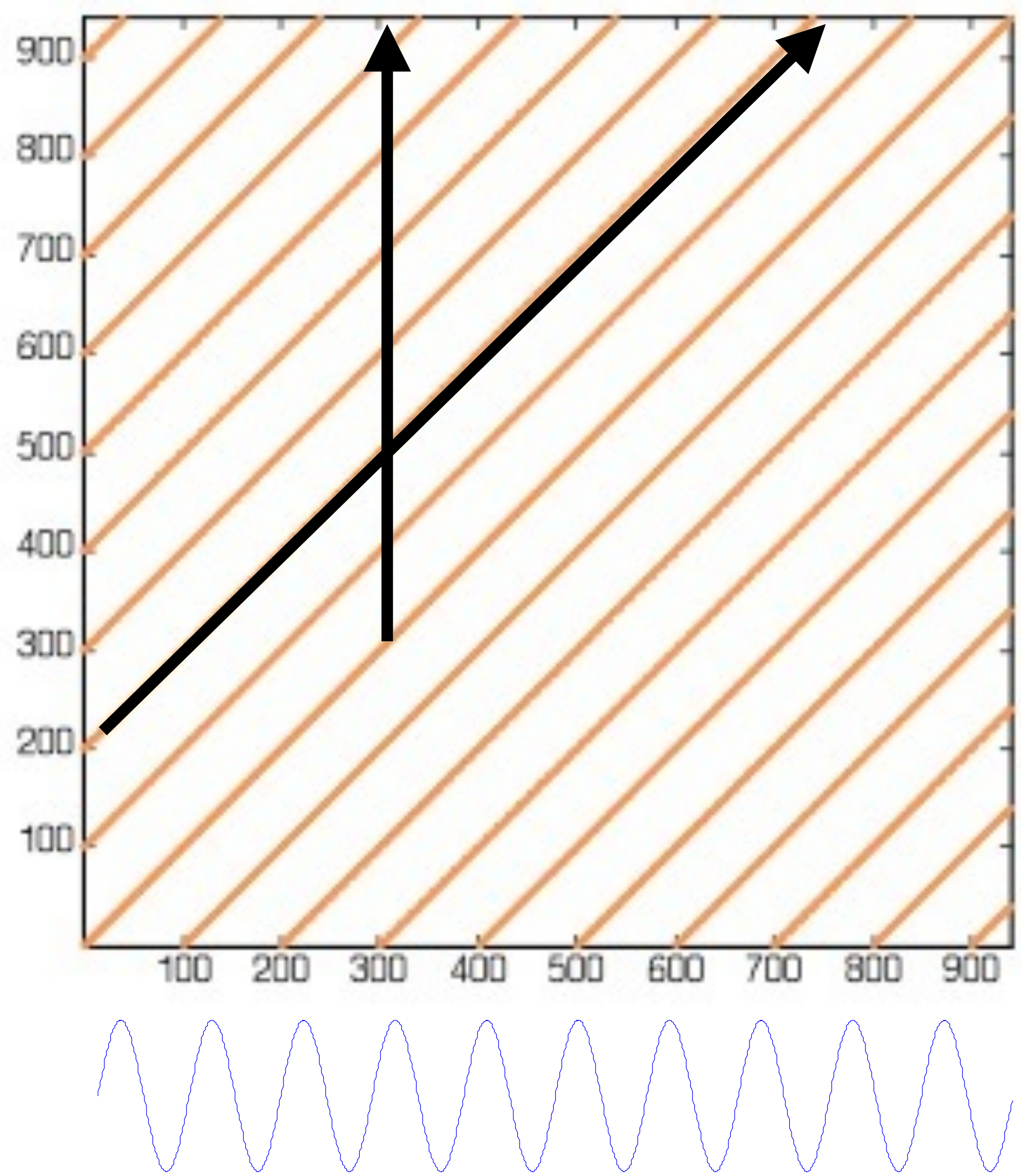
%DETERMINISM

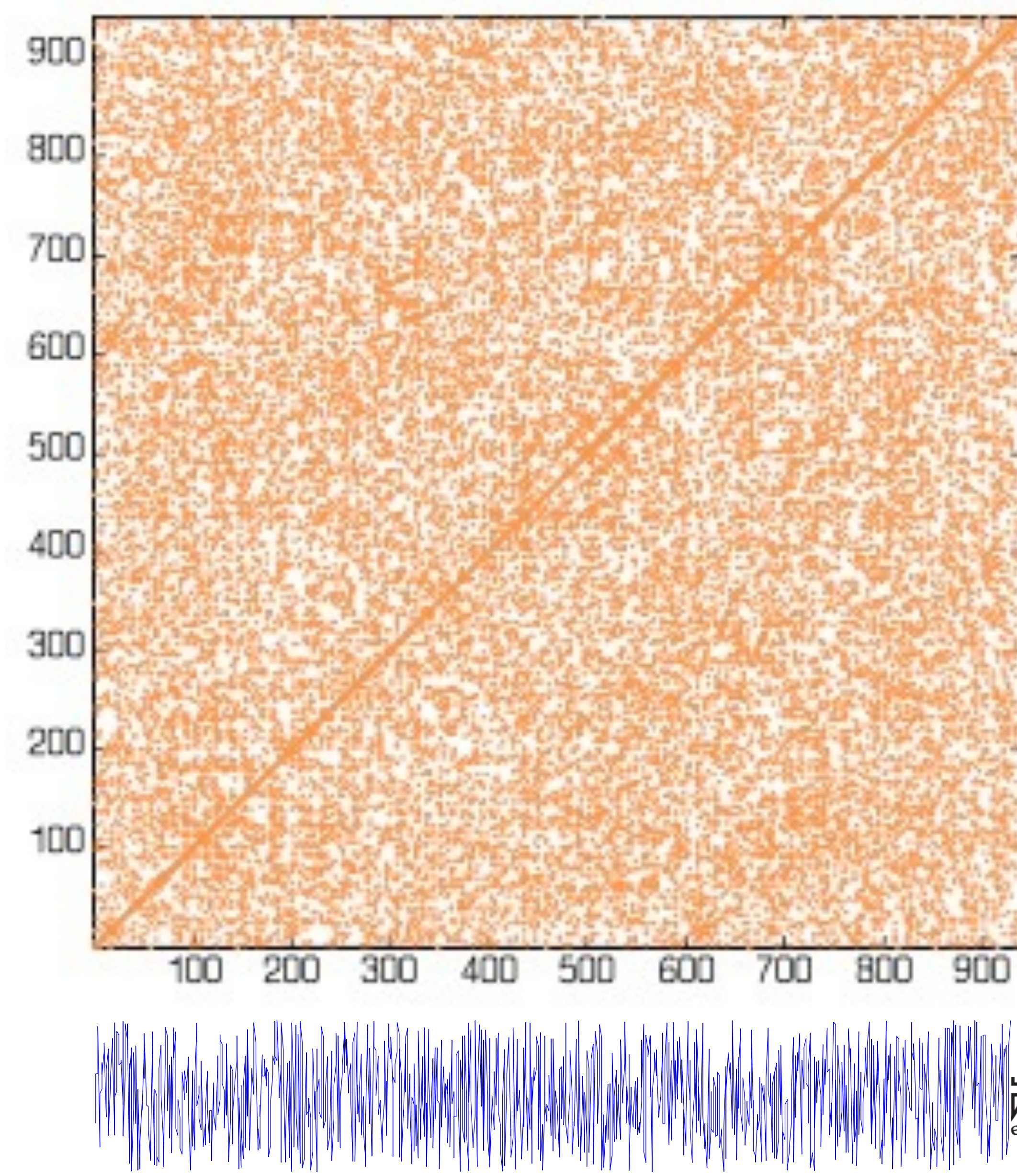
Indexes how “patterned” the data are.

Does the system return to the same region of phase space for a longer period of time?

$$\%DET = \frac{\text{Number of recurrent points forming diagonal line}}{\text{Total recurrent points}} \times 100$$

Sine	White Noise
%REC = 2.9	%REC = 2.9
%DET = 99.8	%DET = 5.4





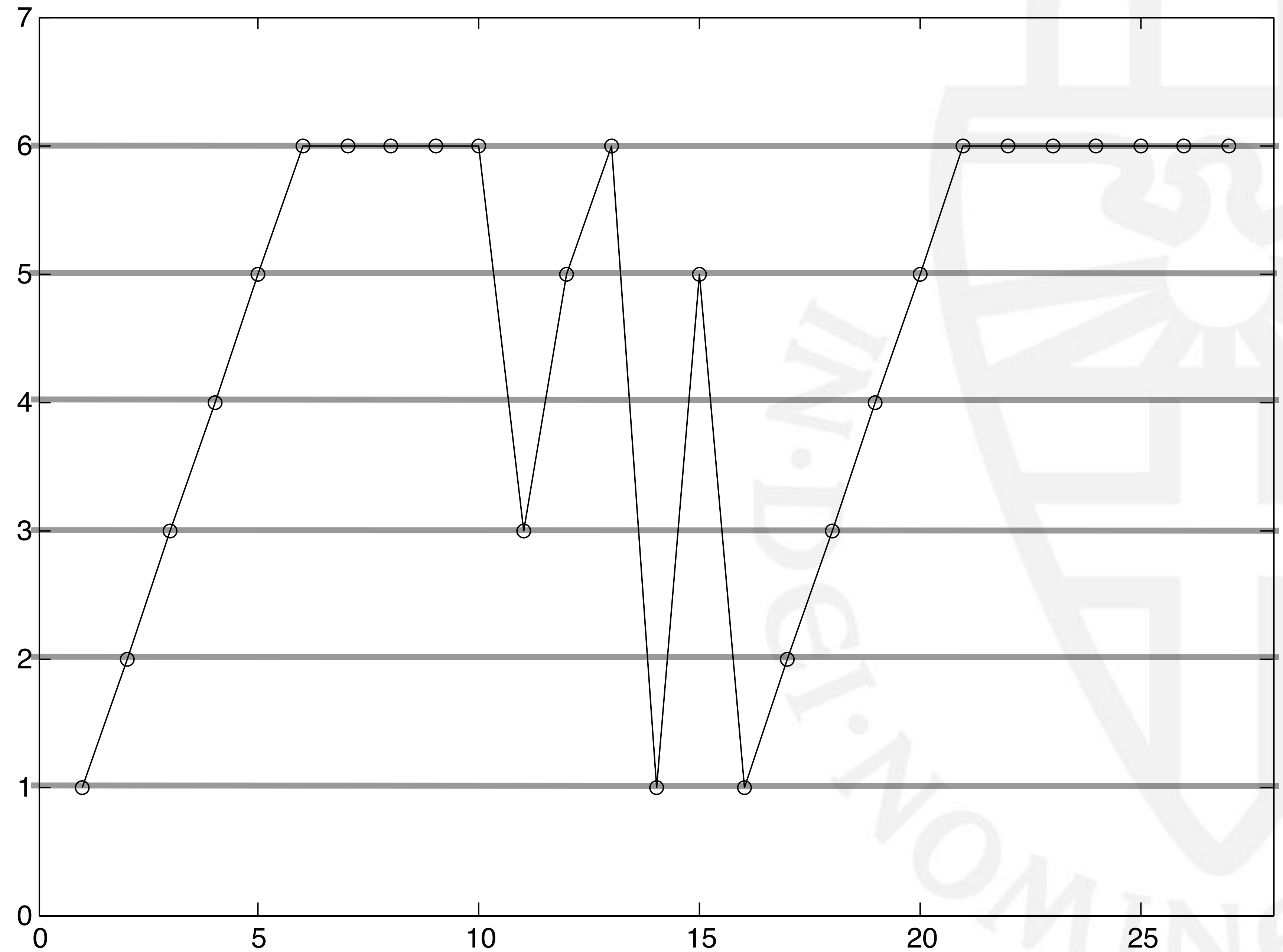
Diagonal lines signify recurring patterns...

not necessarily recurring values!

Let’s take radius / threshold = 0

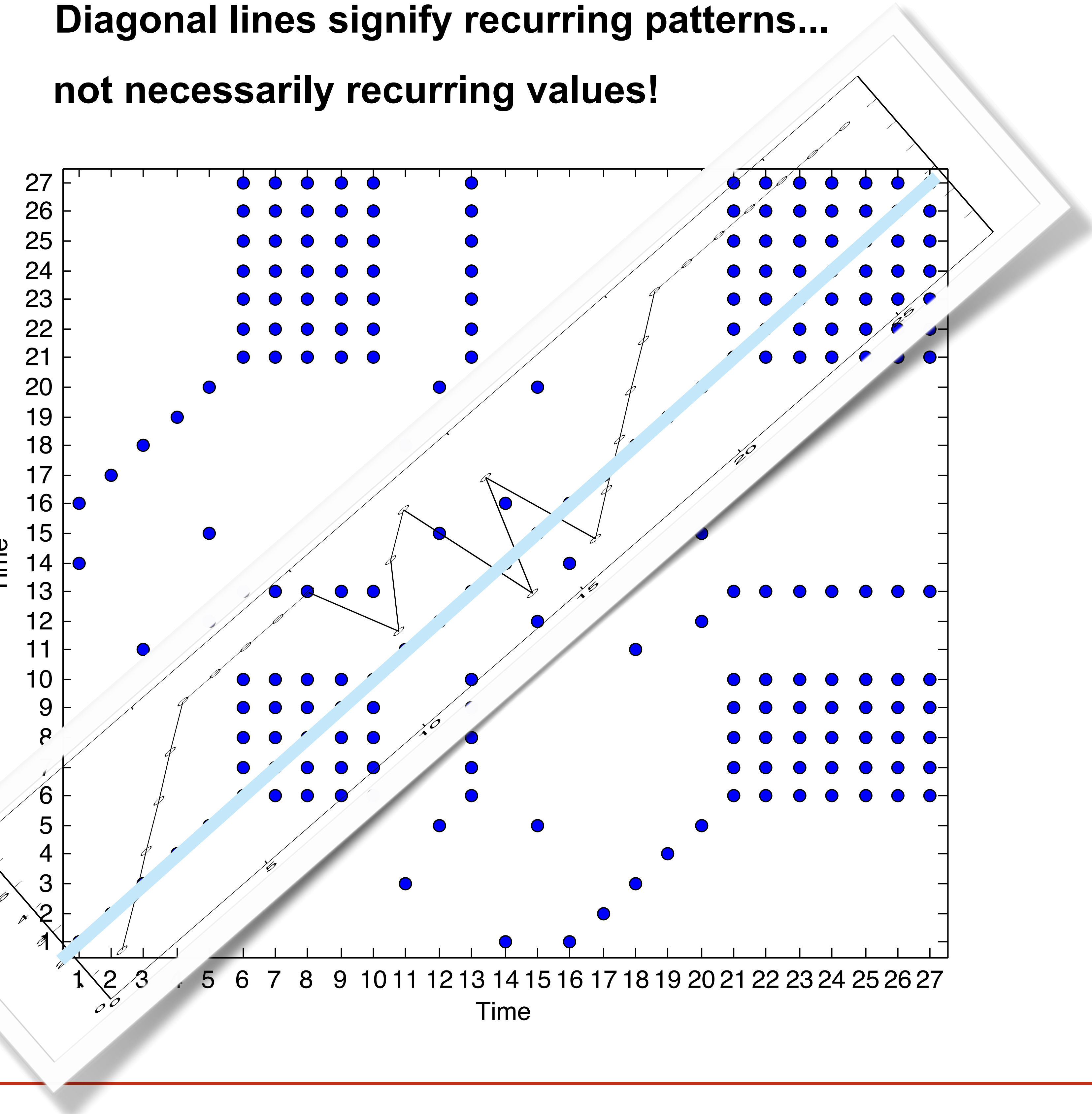
Looking at exact recurrence

1 dimensional state space (no embedding, just “raw” recurrence)



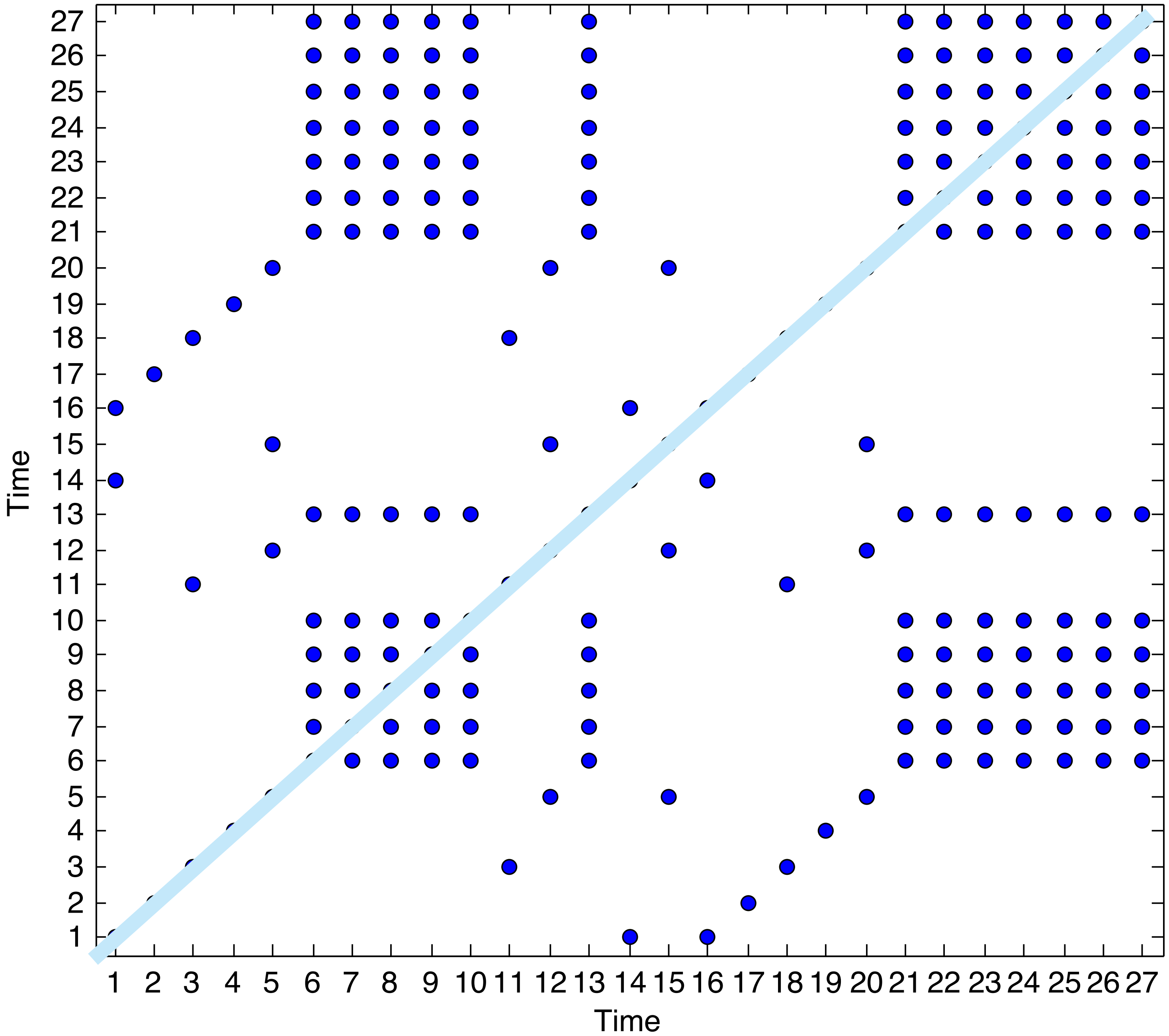
Diagonal lines signify recurring patterns...

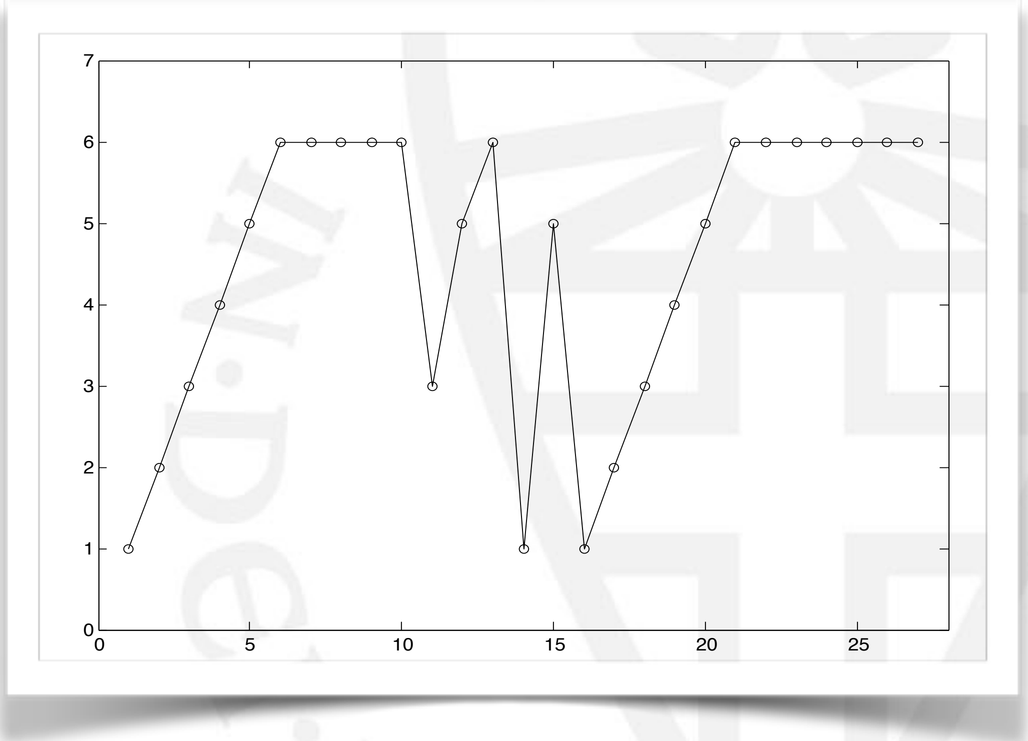
not necessarily recurring values!



Diagonal lines signify recurring patterns...

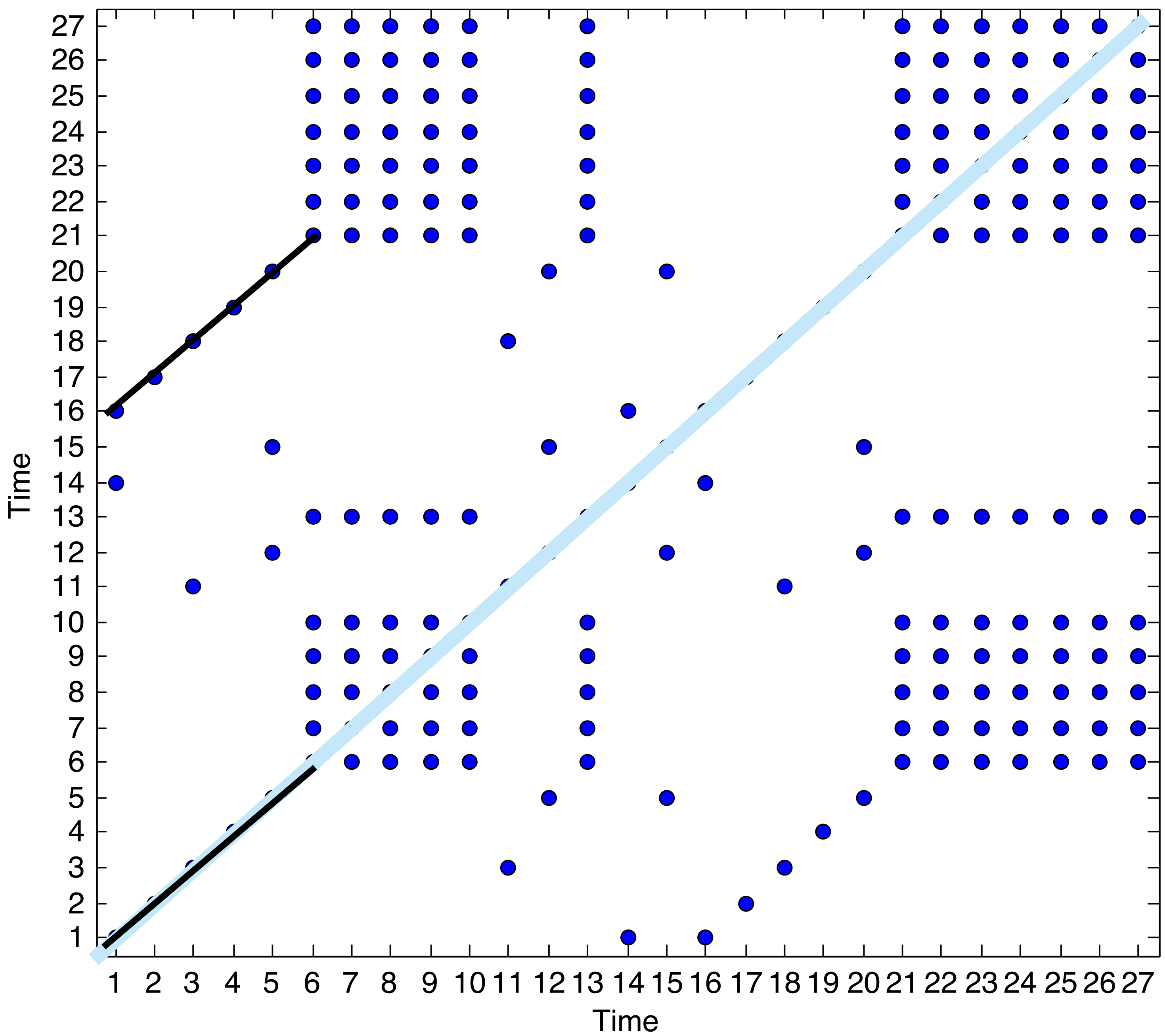
not necessarily recurring values!



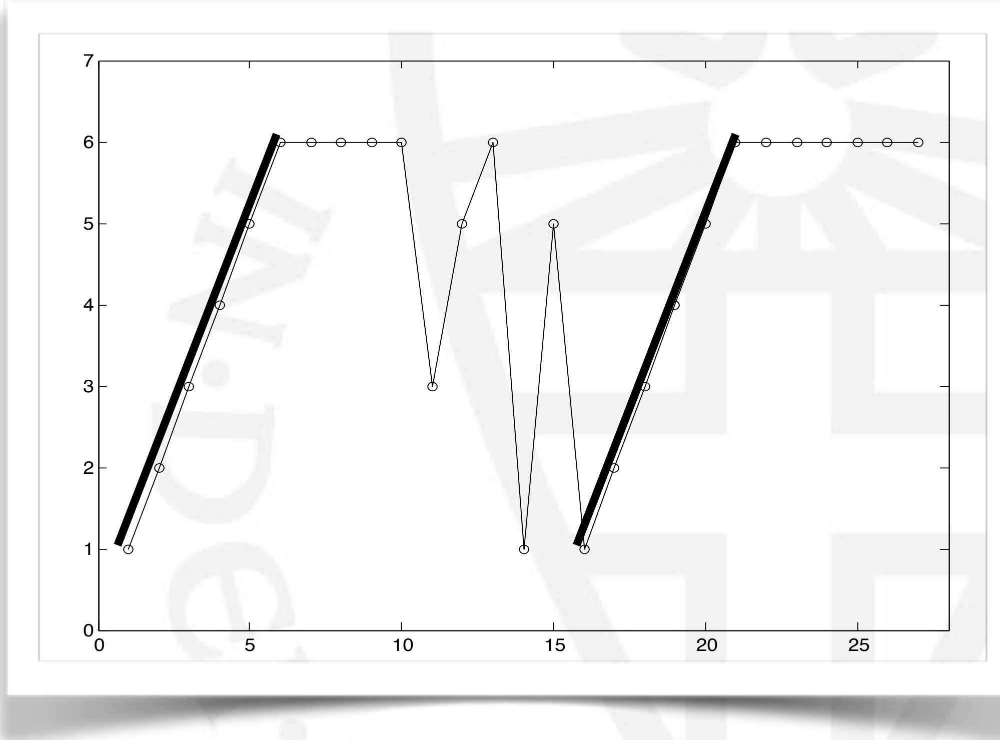


Diagonal lines signify recurring patterns...

not necessarily recurring values!

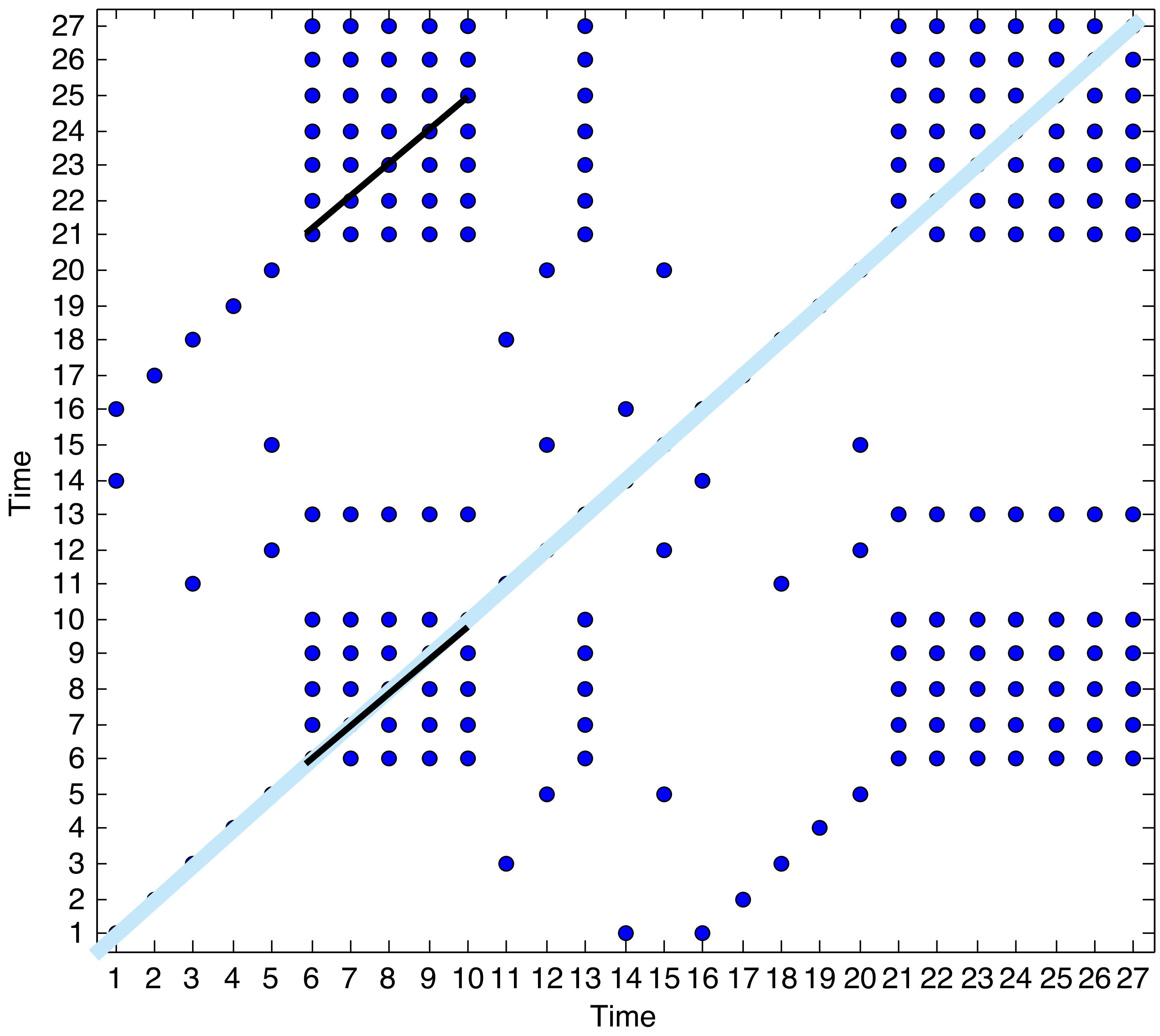


1-2-3-4-5-6 at t=1 recurs at t=16



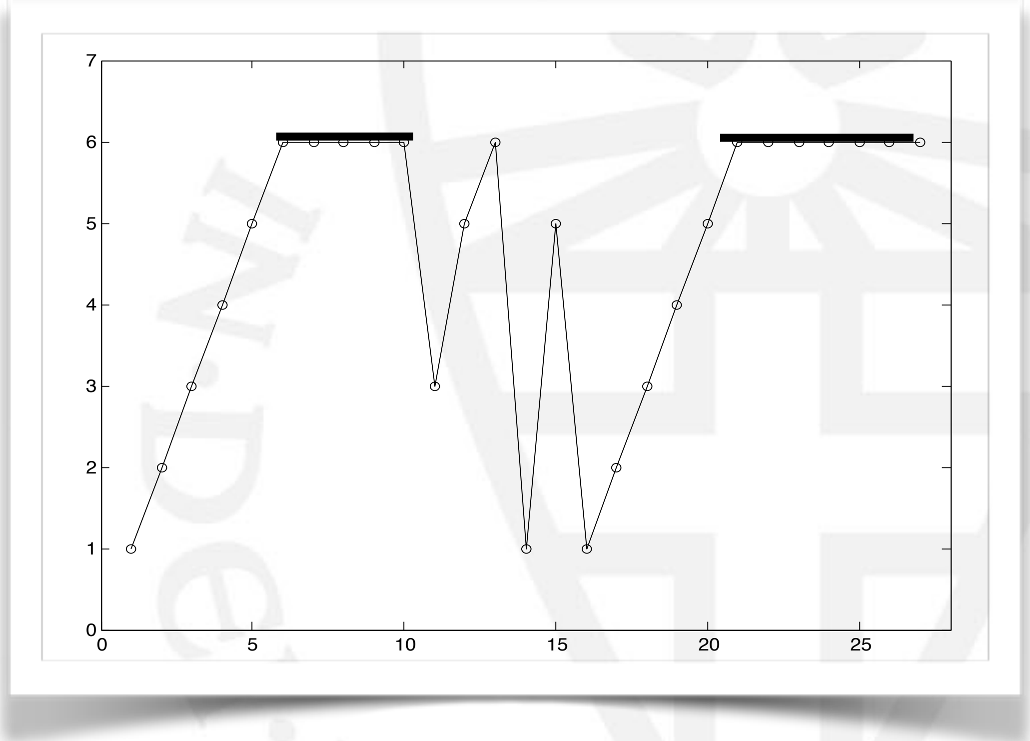
Diagonal lines signify recurring patterns...

not necessarily recurring values!



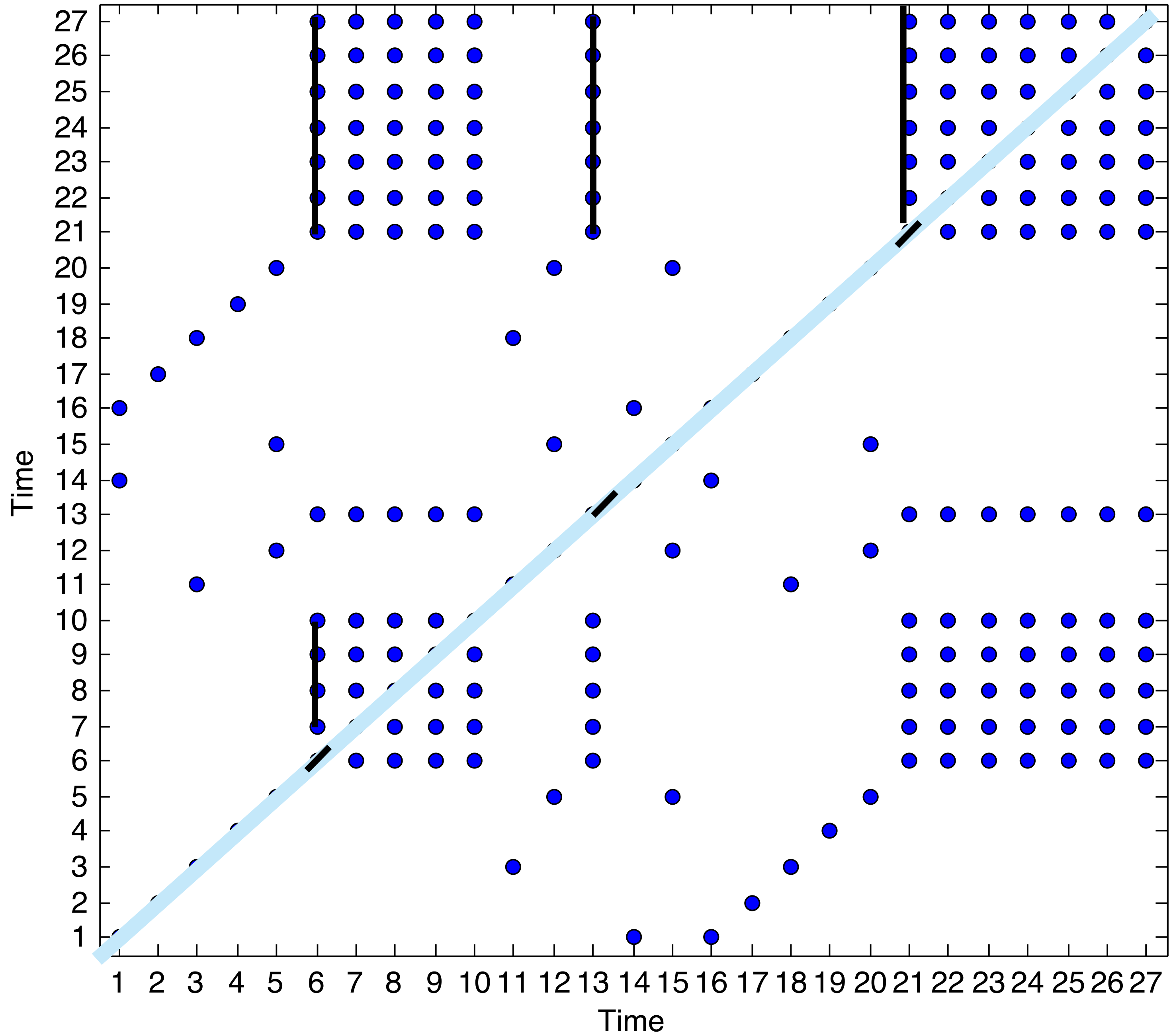
1-2-3-4-5-6 at t=1 *recurs* at t=16

6-6-6-6-6 at t=6 *recurs* at t=21



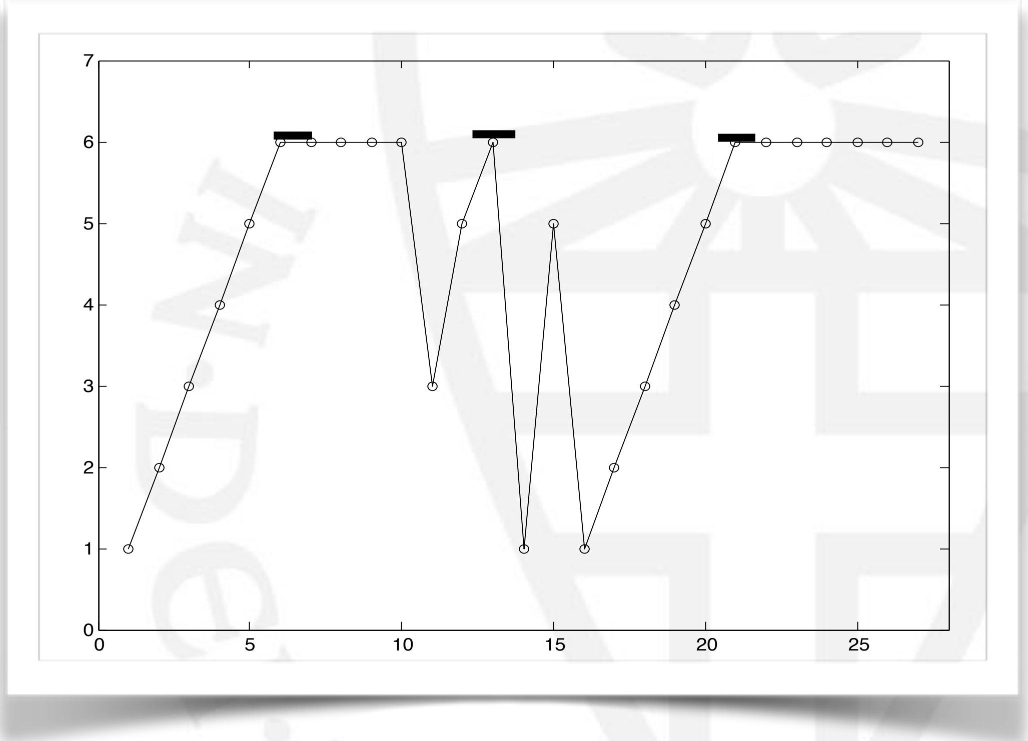
Diagonal lines signify recurring patterns...

not necessarily recurring values!



1-2-3-4-5-6 at t=1 *recurs* at t=16

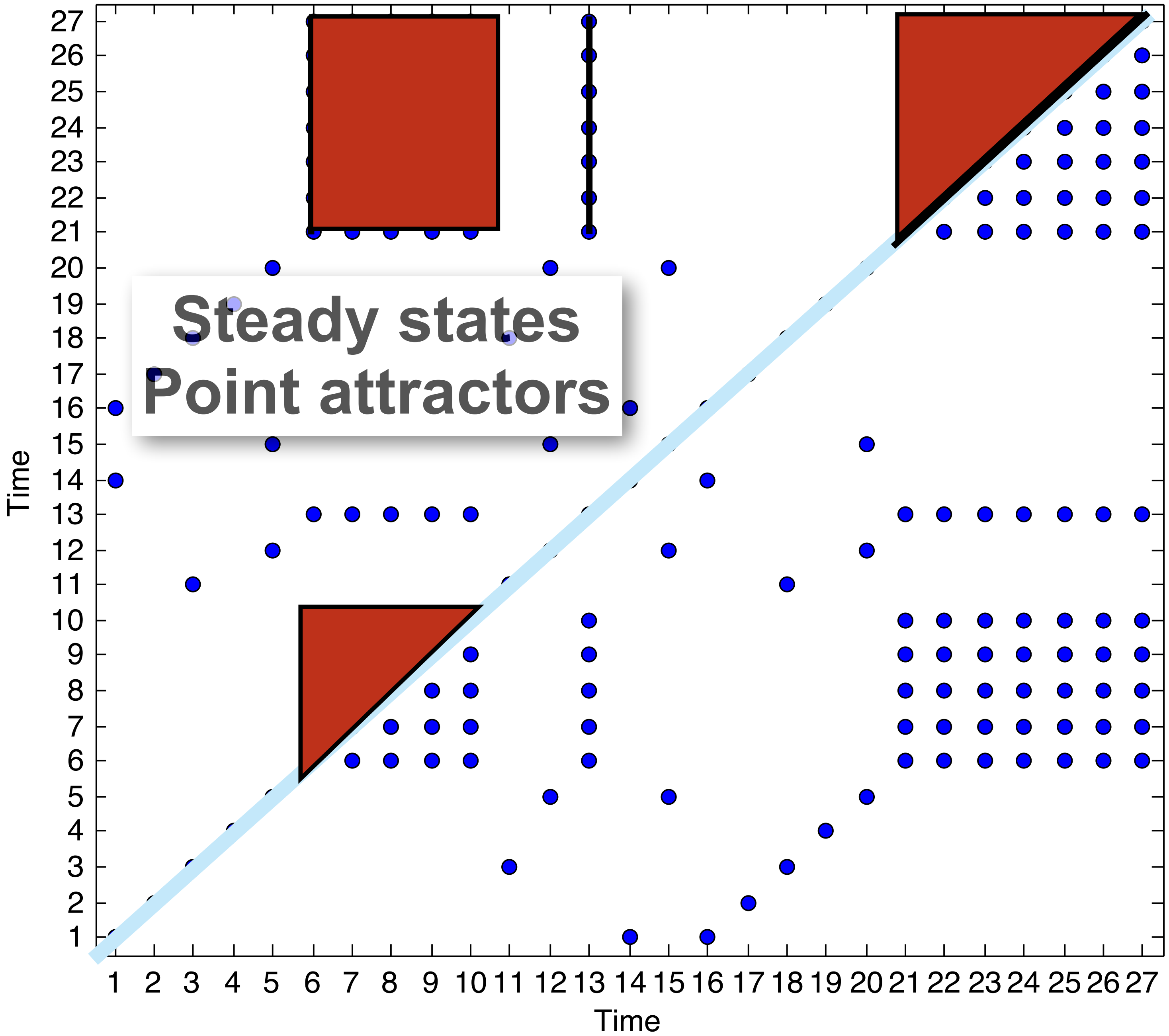
6-6-6-6-6 at t=6 *recurs* at t=21



Once the value 6 has occurred
The system will get "trapped"
in displaying 6 again in the future
(*Laminarity* = %points on vertical line, *Trapping Time* = aver. vertical line length)

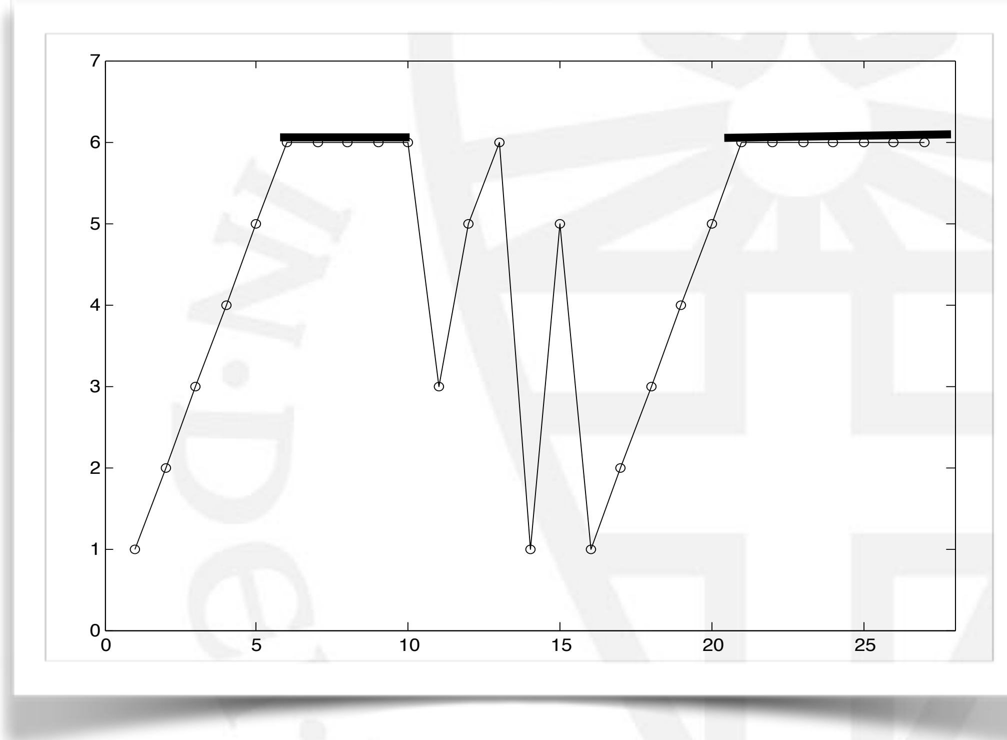
Diagonal lines signify recurring patterns...

not necessarily recurring values!



1-2-3-4-5-6 at t=1 *recurs* at t=16

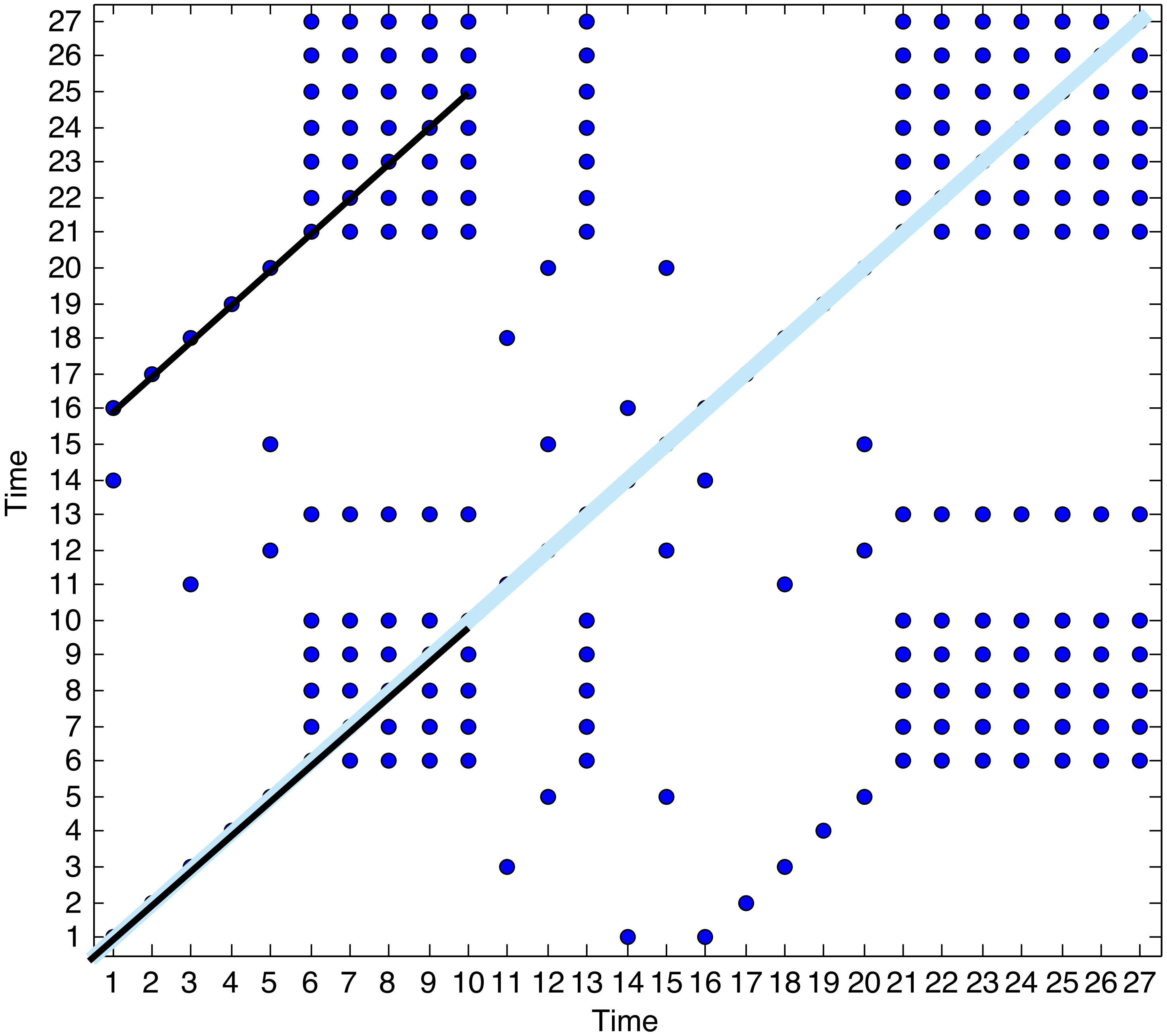
6-6-6-6-6 at t=6 *recurs* at t=21



Once the value 6 has occurred
The system will get "trapped"
in displaying 6 again in the future
(*Laminarity* = %points on vertical line, *Trapping Time* = aver. vertical line length)

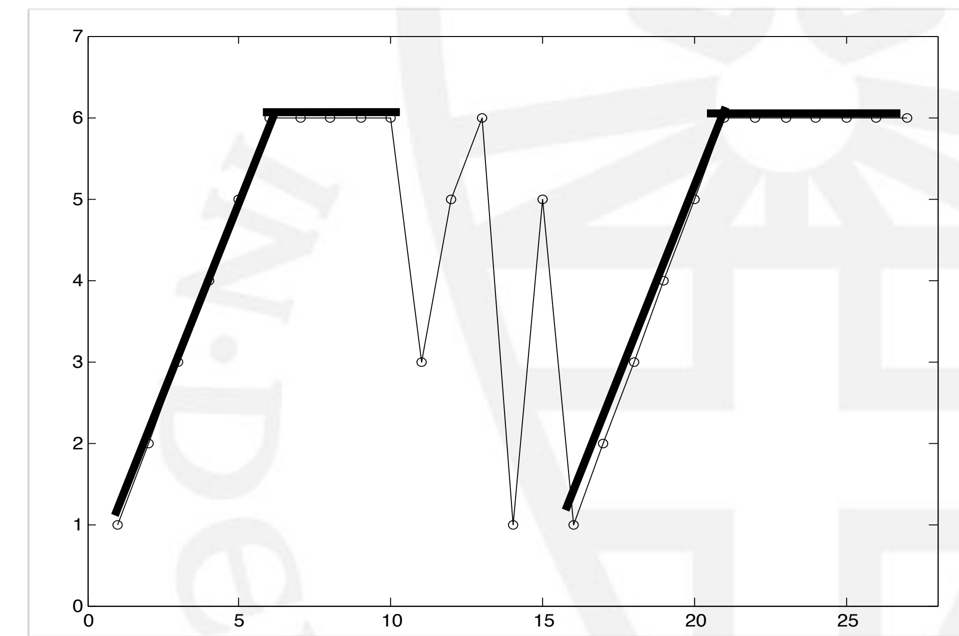
Diagonal lines signify recurring temporal patterns...

not necessarily recurring values!



1-2-3-4-5-6 at t=1 *recurs* at t=16

6-6-6-6-6 at t=6 *recurs* at t=21



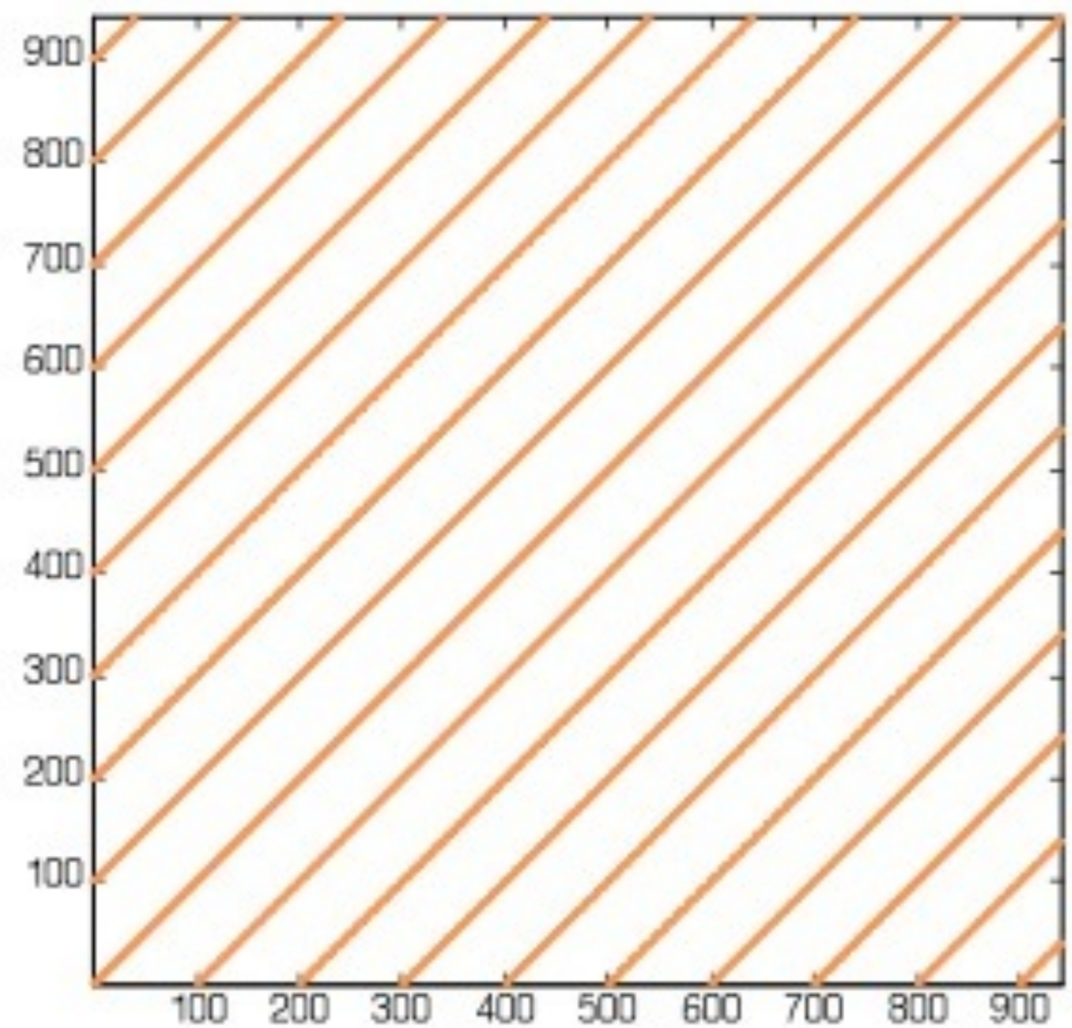
1-2-3-4-5-6-6-6-6-6 at t=1 *recurs* at t=16

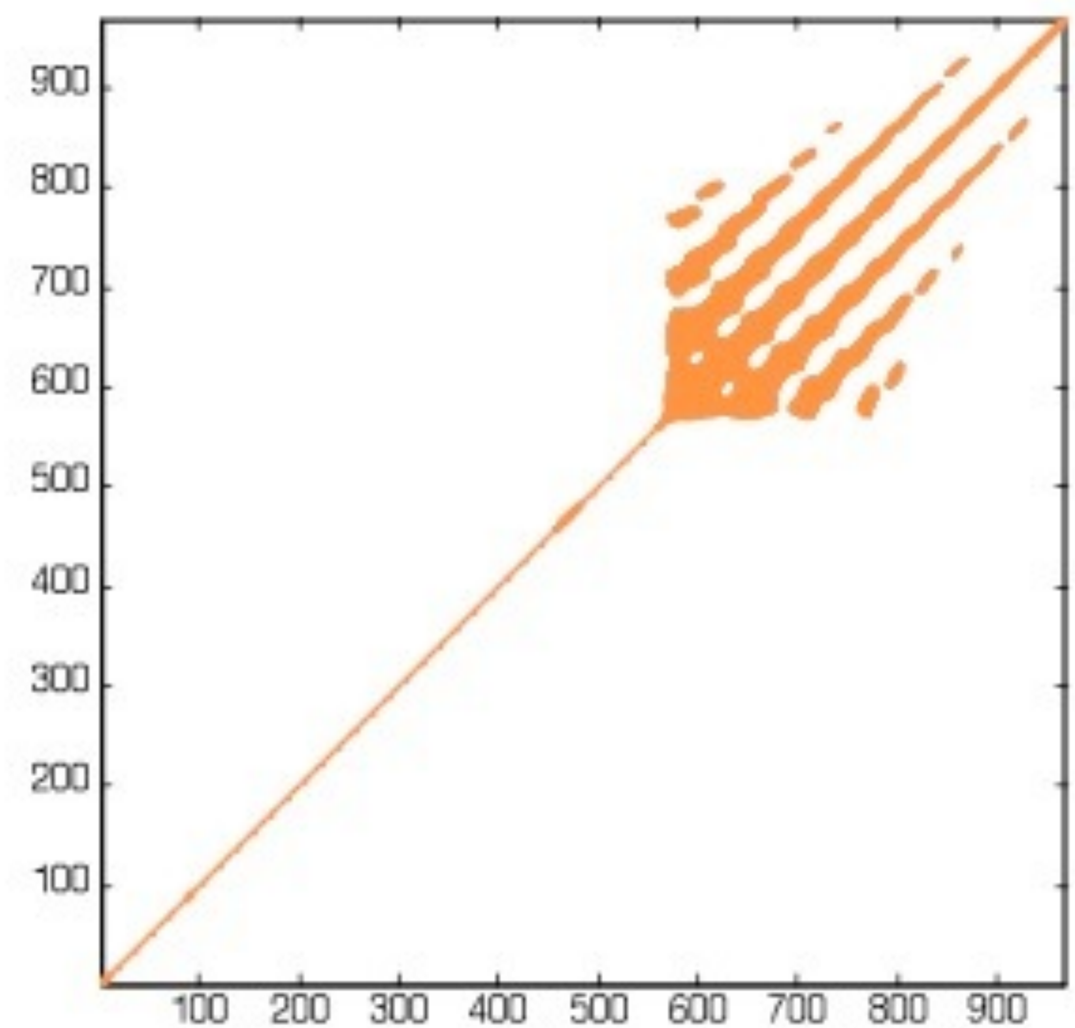
MAXLINE

How long the system can maintain a recurring pattern ~ “Stability”

MAXLINE = The longest sequence of recurring points

Sine	Lorenz
%REC = 2.9	%REC = 2.9
MAXLINE = 938	MAXLINE = 410





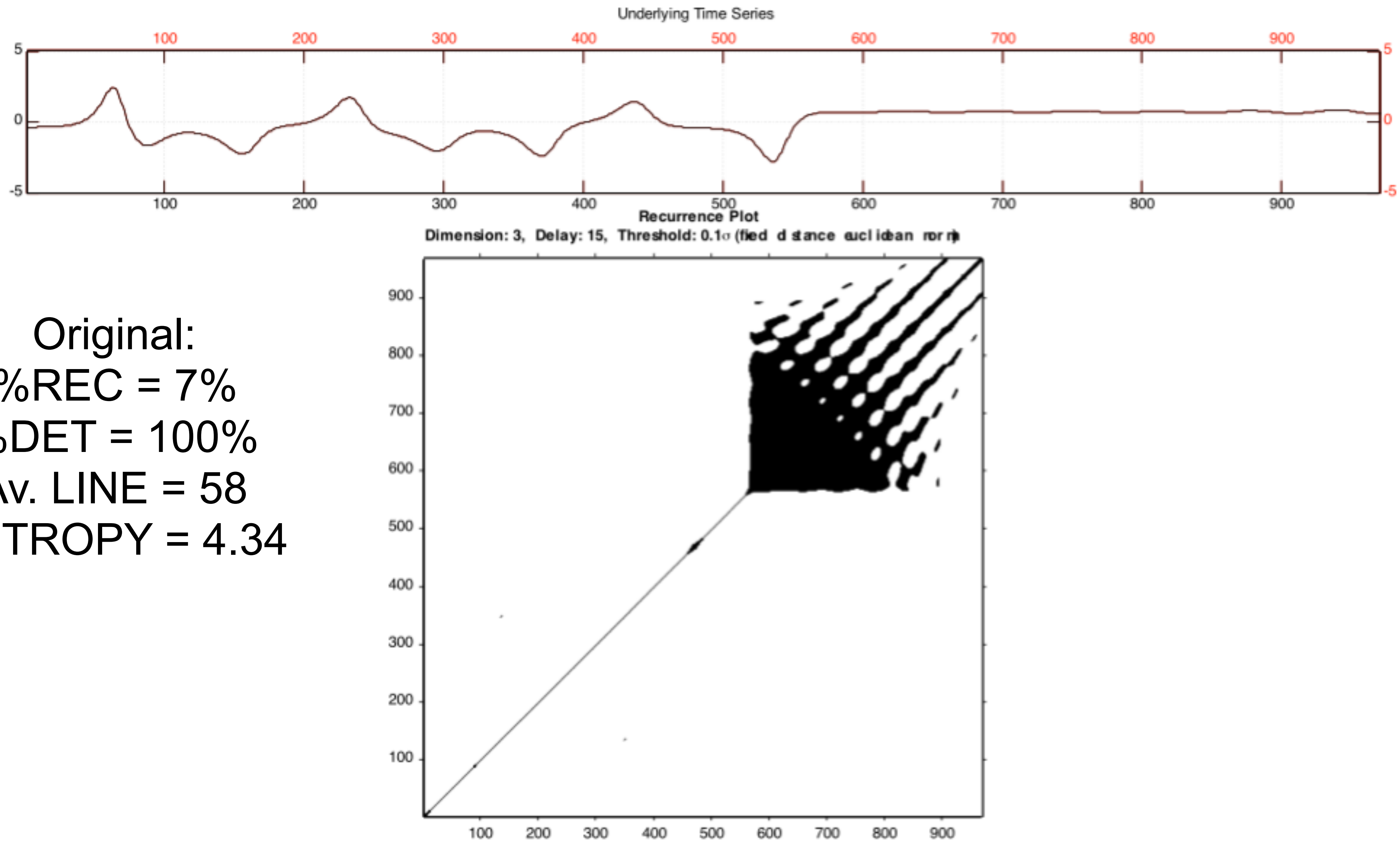
1/maxline = Divergence (Thought to be an estimate of largest Lyapunov exponent)

RQA measures

- %REC or RR (recurrence rate)
- %DET (is the data from a deterministic process or random?)
- MAXLINE (maximal diagonal line length)
- DIV (divergence, 1/maxline, suggested estimate of largest Lyapunov exponent)
- Average LINE (average diagonal line length)
- ENTROPY (complexity of deterministic structure)
- TREND (is the data stationary?)
- %LAM (laminarity, points on vertical lines, connected to Laminar phases)
- TT (Trapping Time, average length of vertical lines: How long the system stays in a specific state)

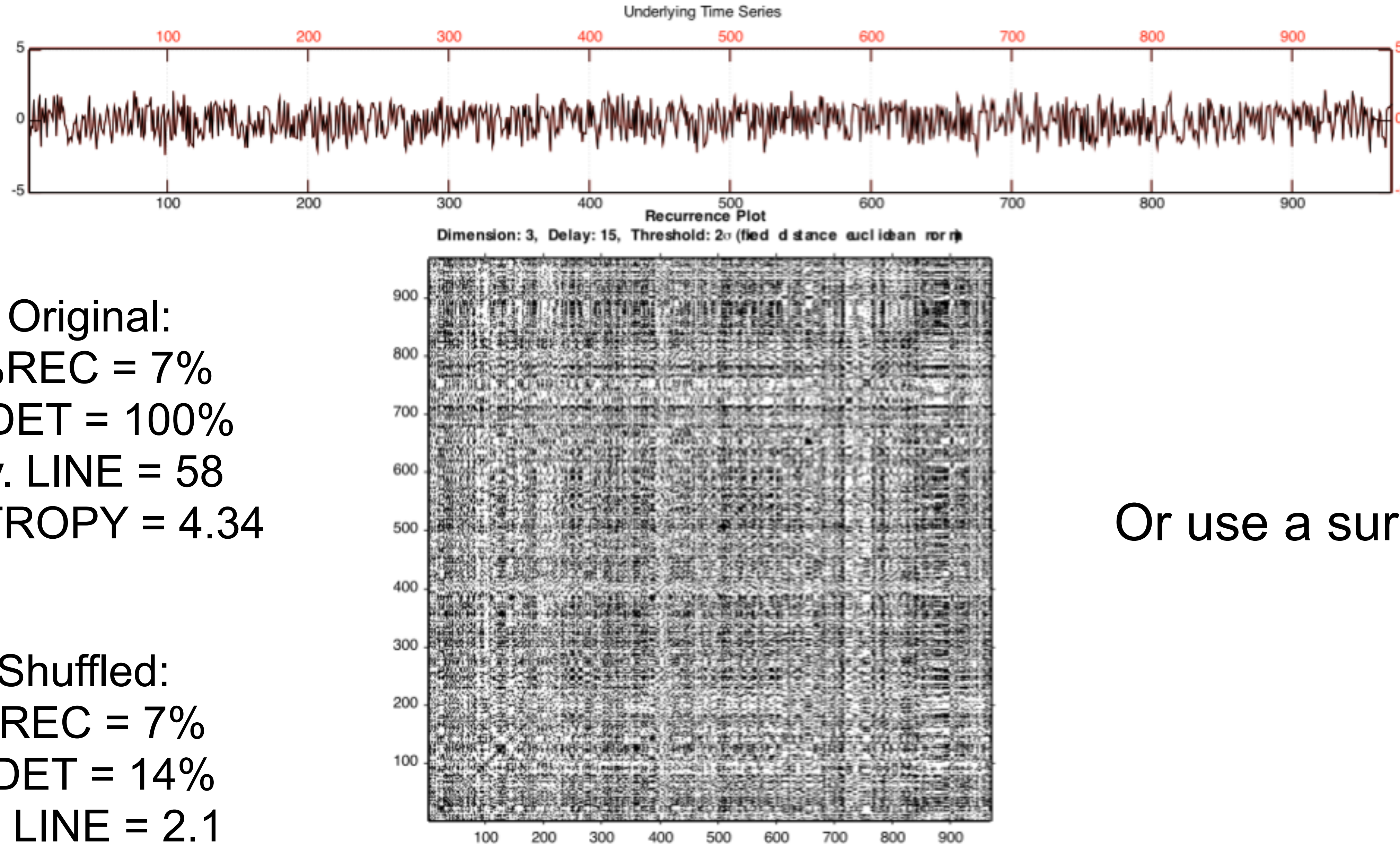
- Create your own…

How to decide these values have meaning?



Original:
%REC = 7%
%DET = 100%
Av. LINE = 58
ENTROPY = 4.34

How to decide these values have meaning?

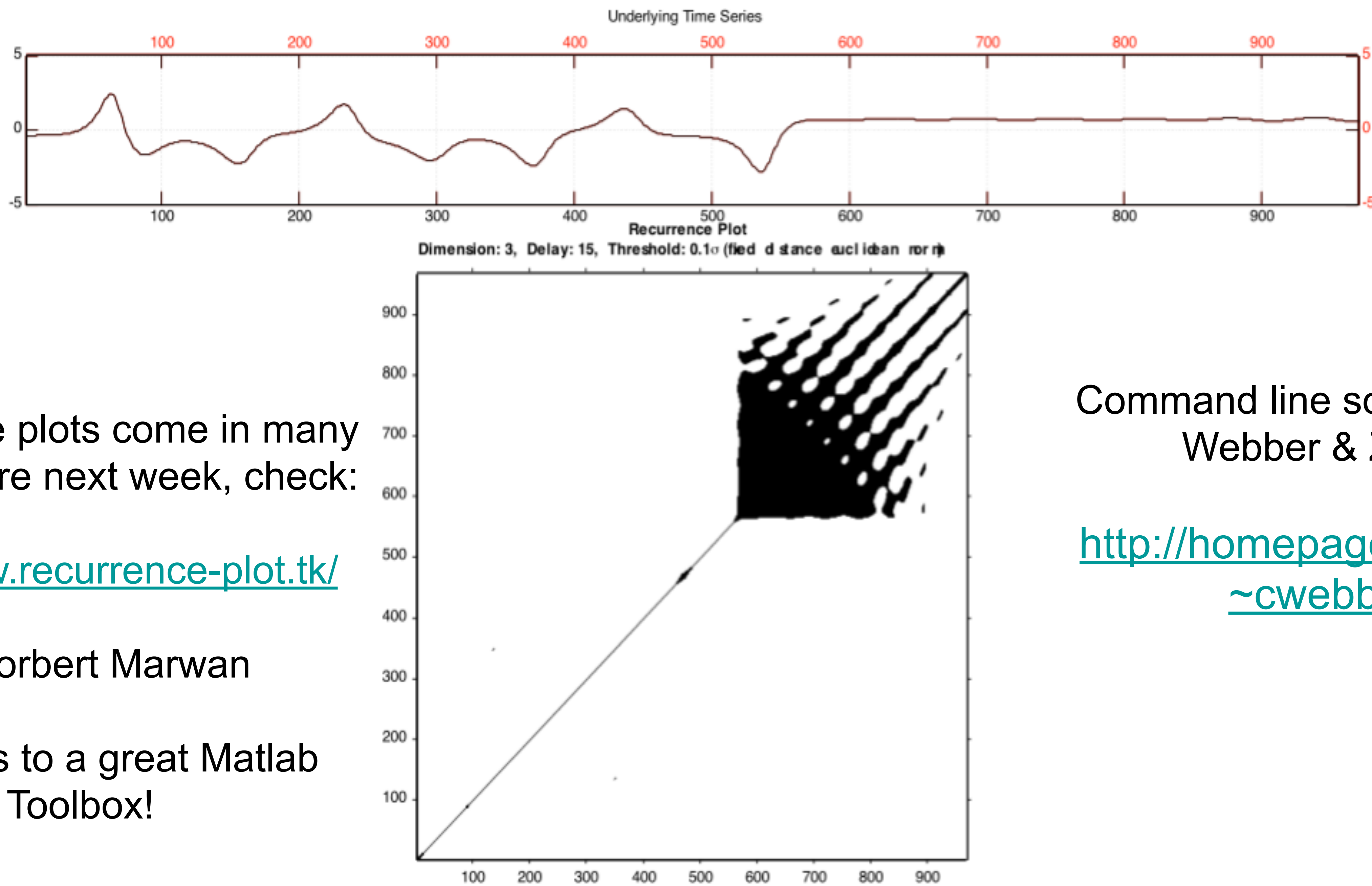


Original:
%REC = 7%
%DET = 100%
Av. LINE = 58
ENTROPY = 4.34

Shuffled:
%REC = 7%
%DET = 14%
Av. LINE = 2.1
ENTROPY = 0.25

Or use a surrogate

Recurrence Plots - Software



Recurrence plots come in many flavors, more next week, check:

http://www.recurrence-plot.tk/

By Norbert Marwan

Also links to a great Matlab Toolbox!

Command line software from Webber & Zbilut:

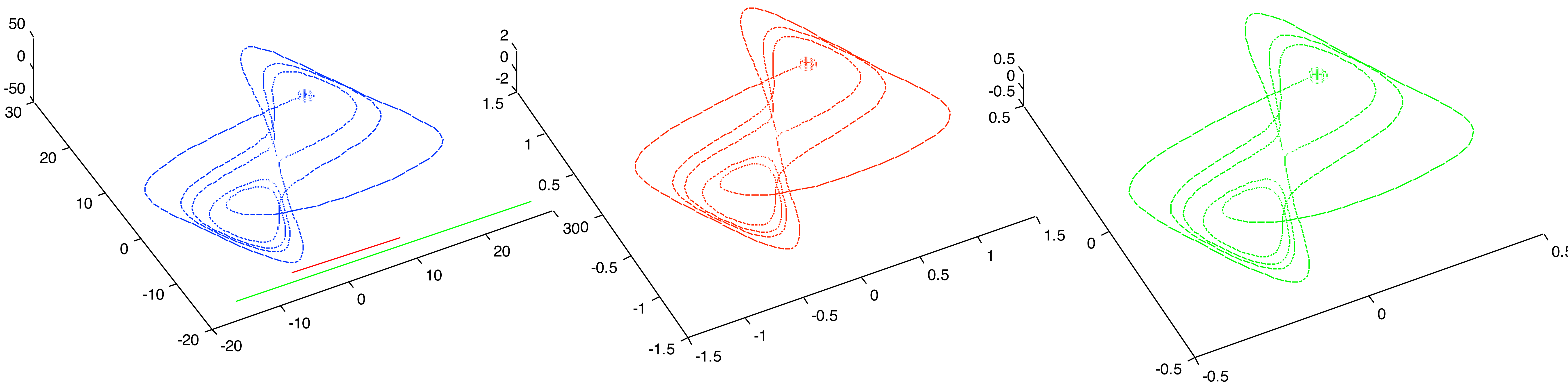
http://homepages.luc.edu/~cwebber/

Data Considerations

Generally it is a good idea to re-scale your data relative to either the mean or maximum distance separating points in reconstructed phase space.

This way data is scaled to itself which allows comparisons across data sets.

No Rescale | Mean Distance Rescale | Maximum Distance Rescale



Maximum distance re-scaling recommended

Webber, C.L., Jr., & Zbilut, J.P. (2005). Recurrence quantification analysis of nonlinear dynamical systems. In: *Tutorials in contemporary nonlinear methods for the behavioral sciences*, (Chapter 2, pp. 26-94), M.A. Riley, G. Van Orden, eds. Retrieved June 5, 2007 http://www.nsf.gov/sbe/bcs/pac/nmbs/nmbs.pdf

General Recipe for Recurrence Quantification with toolbox:

- Decide which **lag** to use:
  Calculate the Average Mutual Information for a range of lags (*mi*).
  Take the lag where AMI reaches its first minimum. This is the lag at which least is known about X(t+τ) given X(t), so we can create surrogate dimensions which give most new information about the system.

- Decide which **embedding dimension** to use:
  Calculate how many False Nearest Neighbours you loose by adding a dimension (*fnn*). Take the embedding dimension with the lowest % of nearest neighbours (or start with the dimension which gives the greatest decrease of neighbours).

- Decide which type of **rescaling** you want to use:
  Plot your timeseries: Lots of outliers? Use Mean Distance. Otherwise: Max Distance.
  Calculate the max distance in reconstructed phasespace using *pss,*(after lag and embedding are kown) divide by this value.
  Note: Marwan's toolbox will automatically normalize your data unless you tell it not to.

- Decide which **radius / threshold** to use:
  Marwan's toobox allows you to show unthresholded (no radius) plots, decide based on those plots which 'threshold' to use.

- Run **RQA** (*crp,* or *crqa*) with these parameters!

- Compare to shuffled data (*shuffle*)

“Concerns”:

Recurrence values **will** change with changes in the parameters

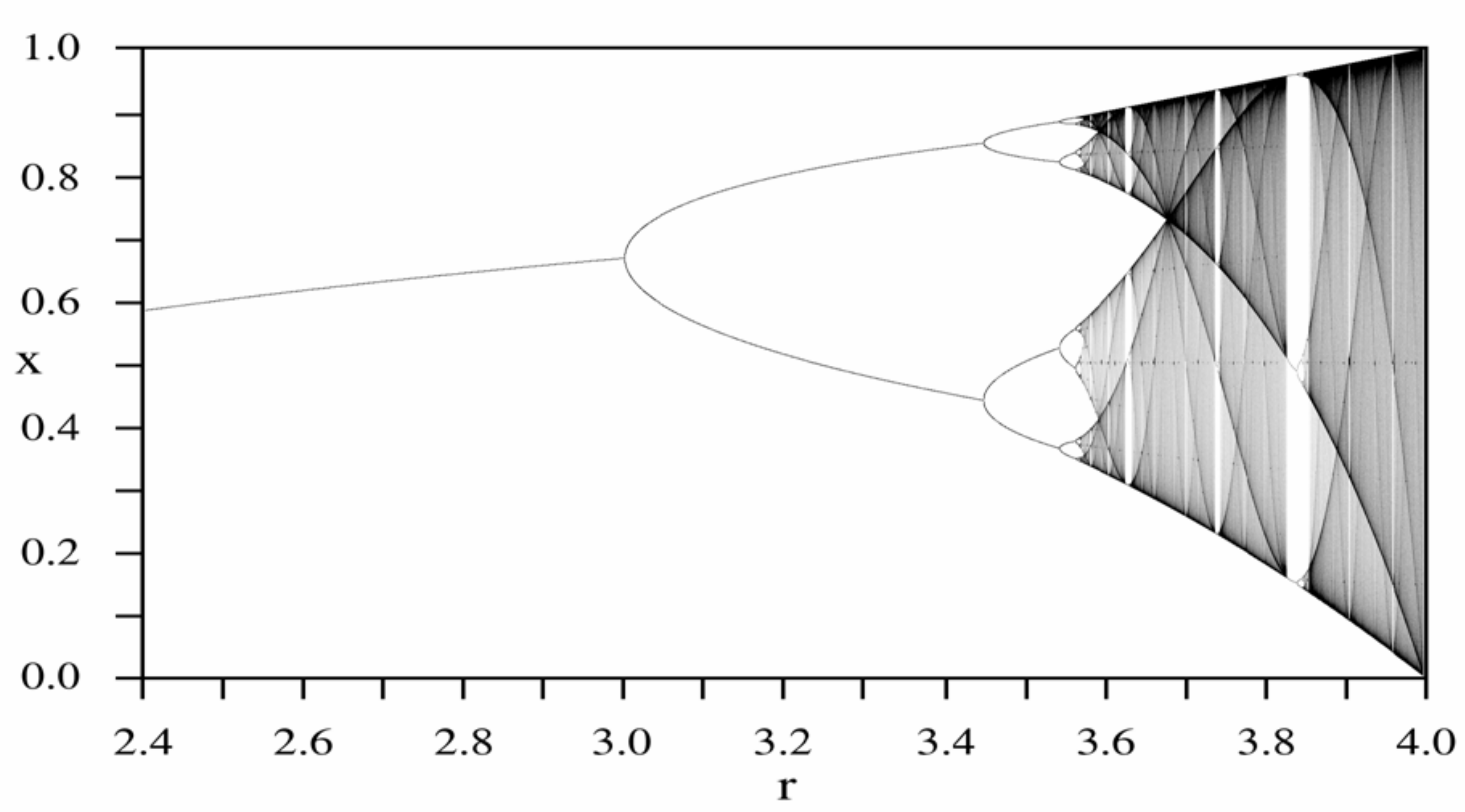
The safest bet for behavioral data:

- Do recurrence calculations with one set of parameters for all of your data sets.

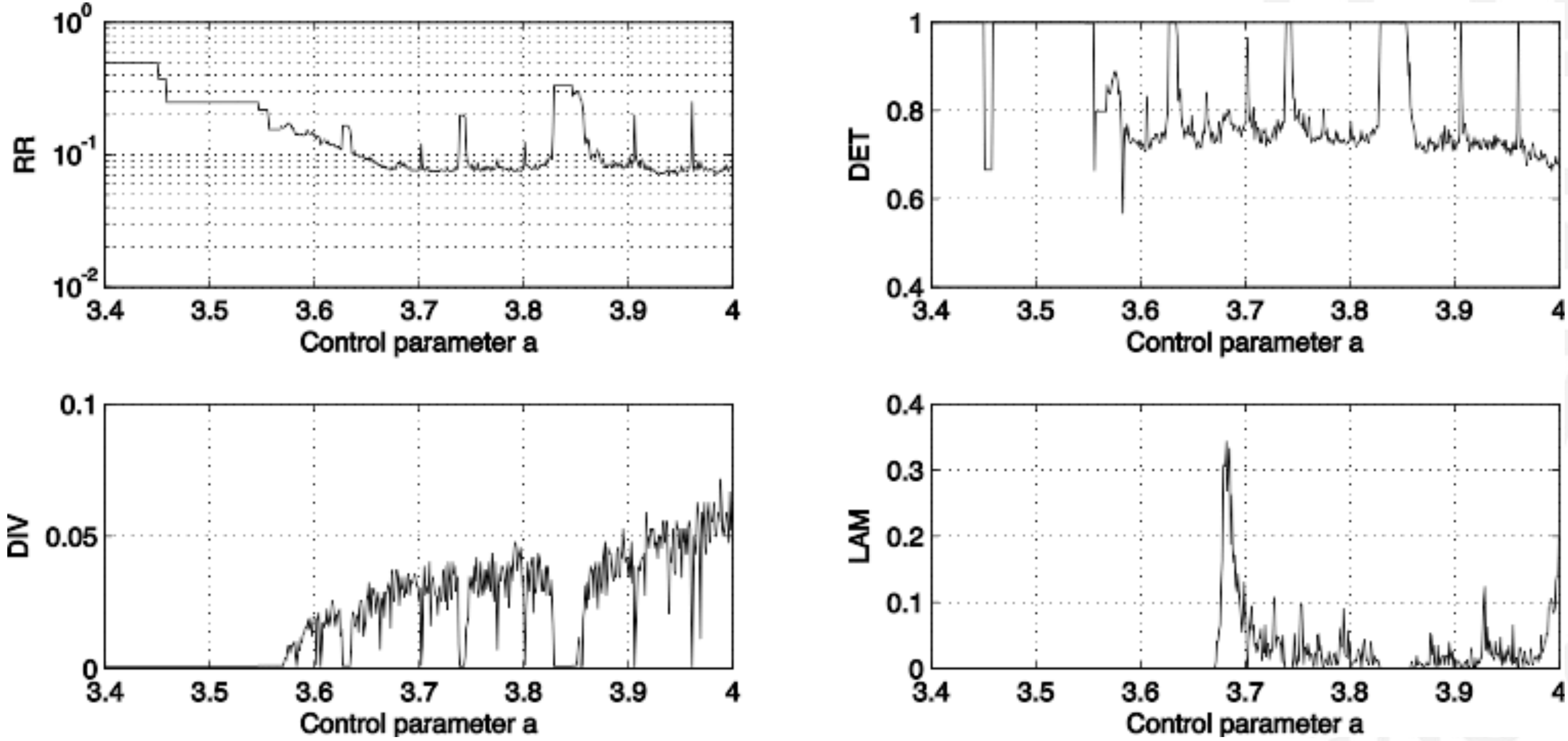
- Then, do this again with another set of parameters and make sure the overall results pattern the same way.

- Then, you can be sure that your results are not artefacts of your parameter selection

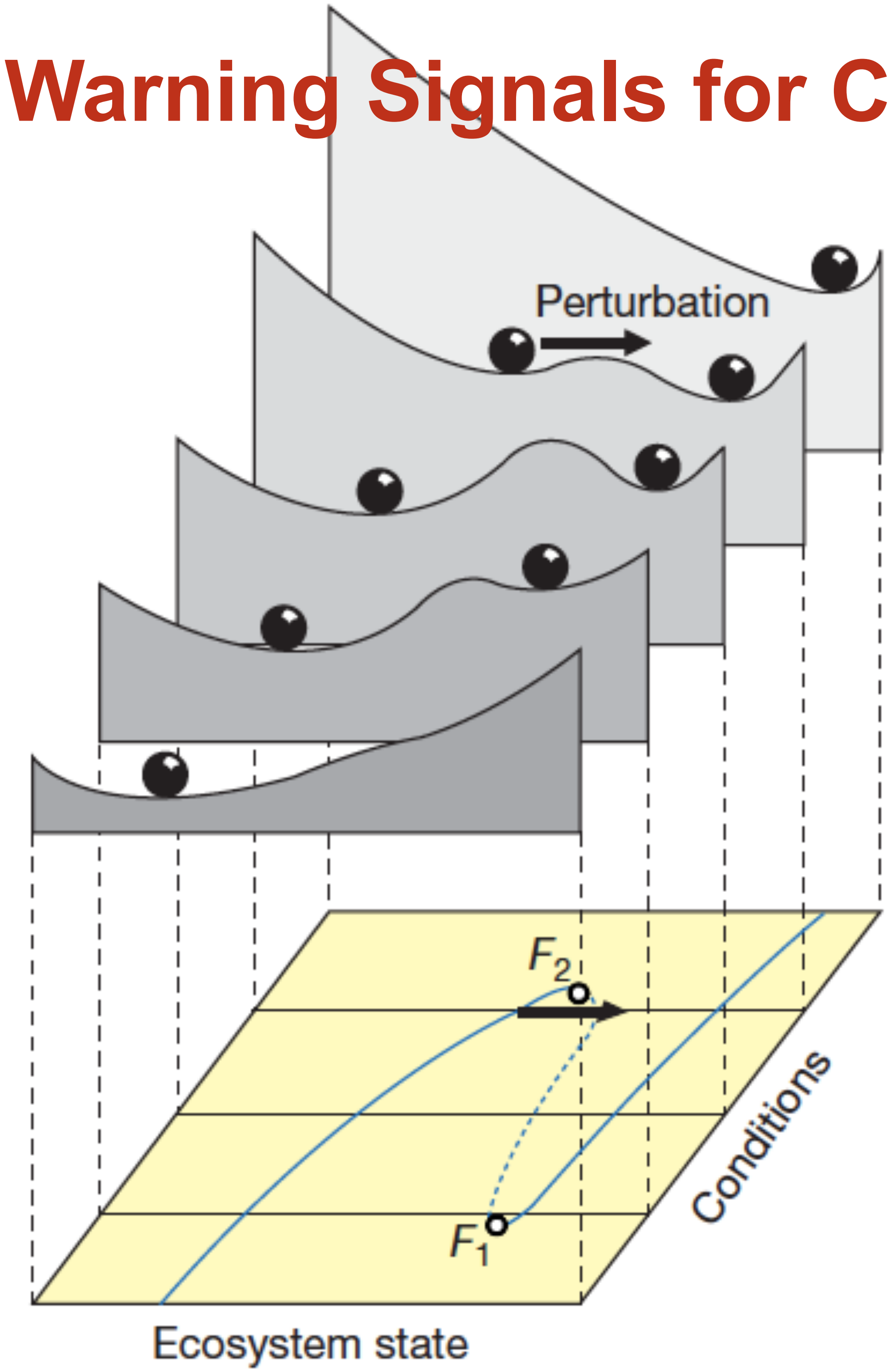
Logistic map – Transitions revealed by lagged RQA

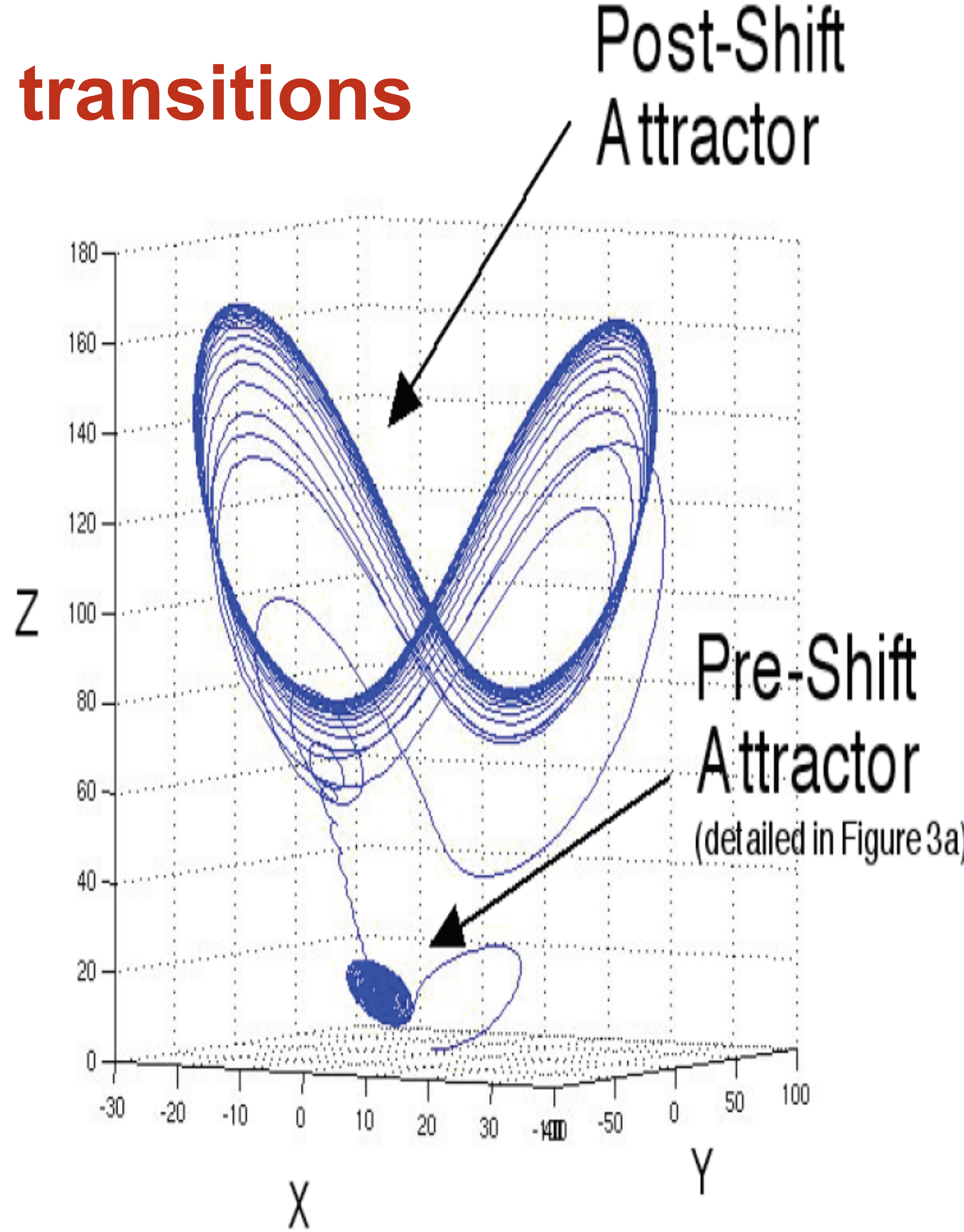


Logistic map – Transitions revealed by lagged RQA



Warning Signals for Critical transitions





Scheffer, M., Carpenter, S., Foley, J. a, Folke, C., & Walker, B. (2001). Catastrophic shifts in ecosystems. Nature, 413(6856), 591-6. doi:10.1038/35098000

Warning Signals for Critical transitions

- Increase in variance (Fluctuation), autocorrelation (FD), critical “slowing down”

- Increase in Entropy... in physics entropy is **a measure of disorder** (2nd law of thermodynamics), or a loss of structure in a system. Everything in the universe is moving towards a maximum entropy state, which means all structure is lost, all energy in the universe is distributed homogeneously

- Entropy is also used in information theory: **the amount of information needed to describe a message / data stream.** If much information is needed, the message is disorderly, random, not patterned -> High entropy.

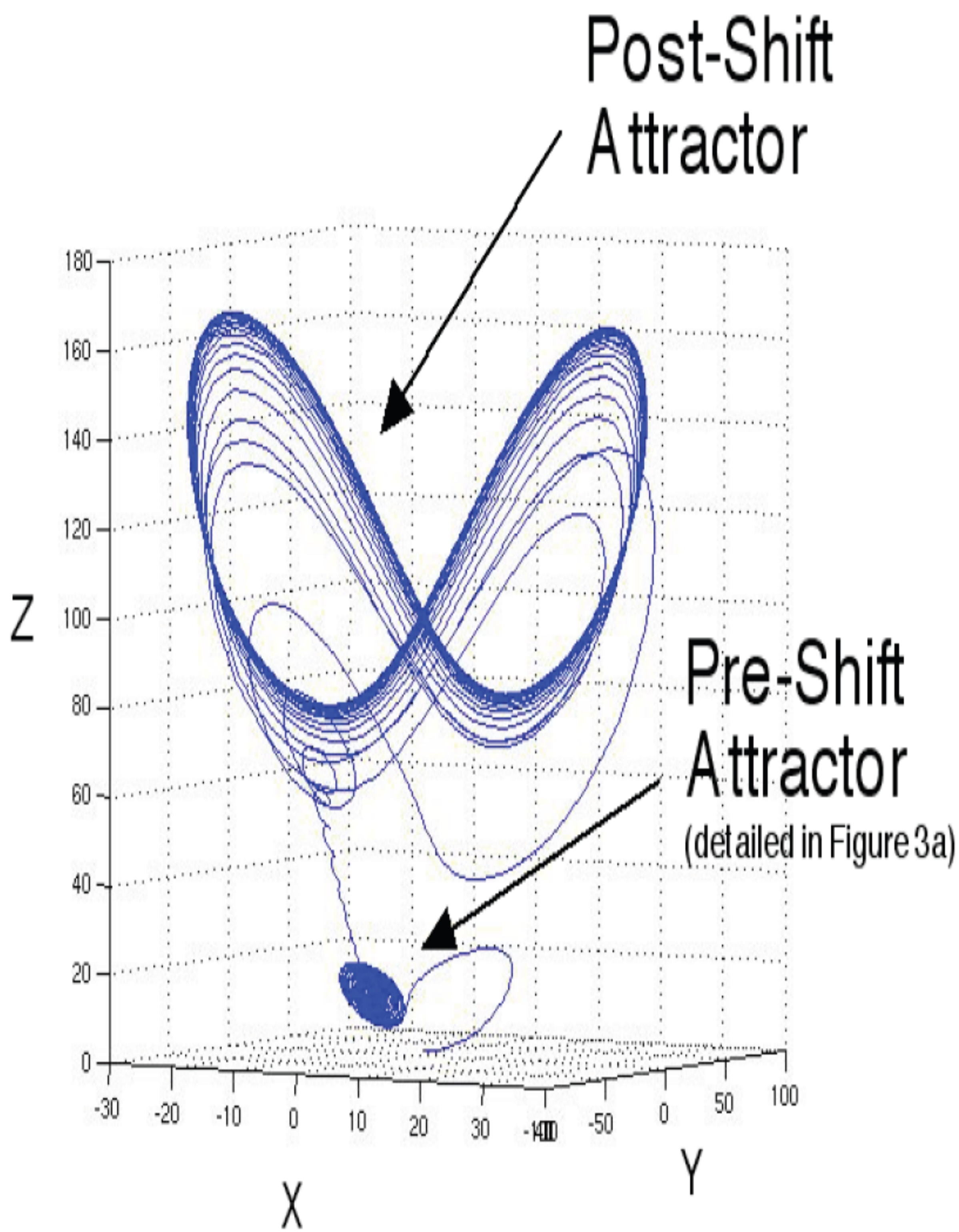
- Entropy in RQA: **the entropy in the distribution of line lengths in the Recurrence Plot.** If the entropy is high there are a lot of different line lengths, pointing towards less stable dynamics in reconstructed phase space. If entropy is low, the system is more deterministic, there are few very stable recurring patterns.

1. If we can reconstruct the state space of a complex dynamical system from one observable dimension....

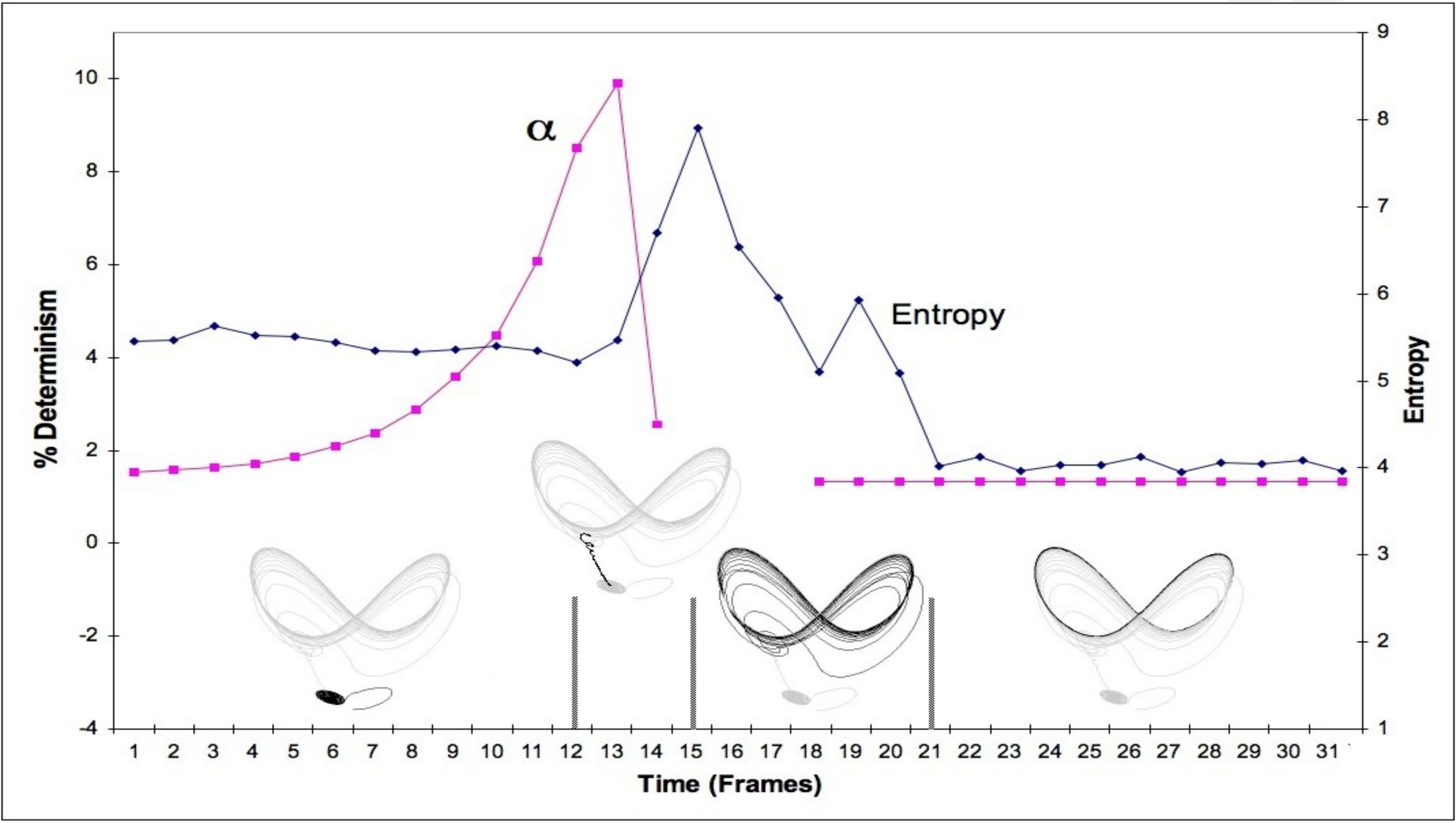
2. If we can quantify the attractor dynamics in this state space...

3. Direct measurements of physical observables in humans should tell us something about the the dynamics of the unobservable cognitive system

4. Could we predict insight in problem solving from a phase transition in phase space reconstructed from hand movements?

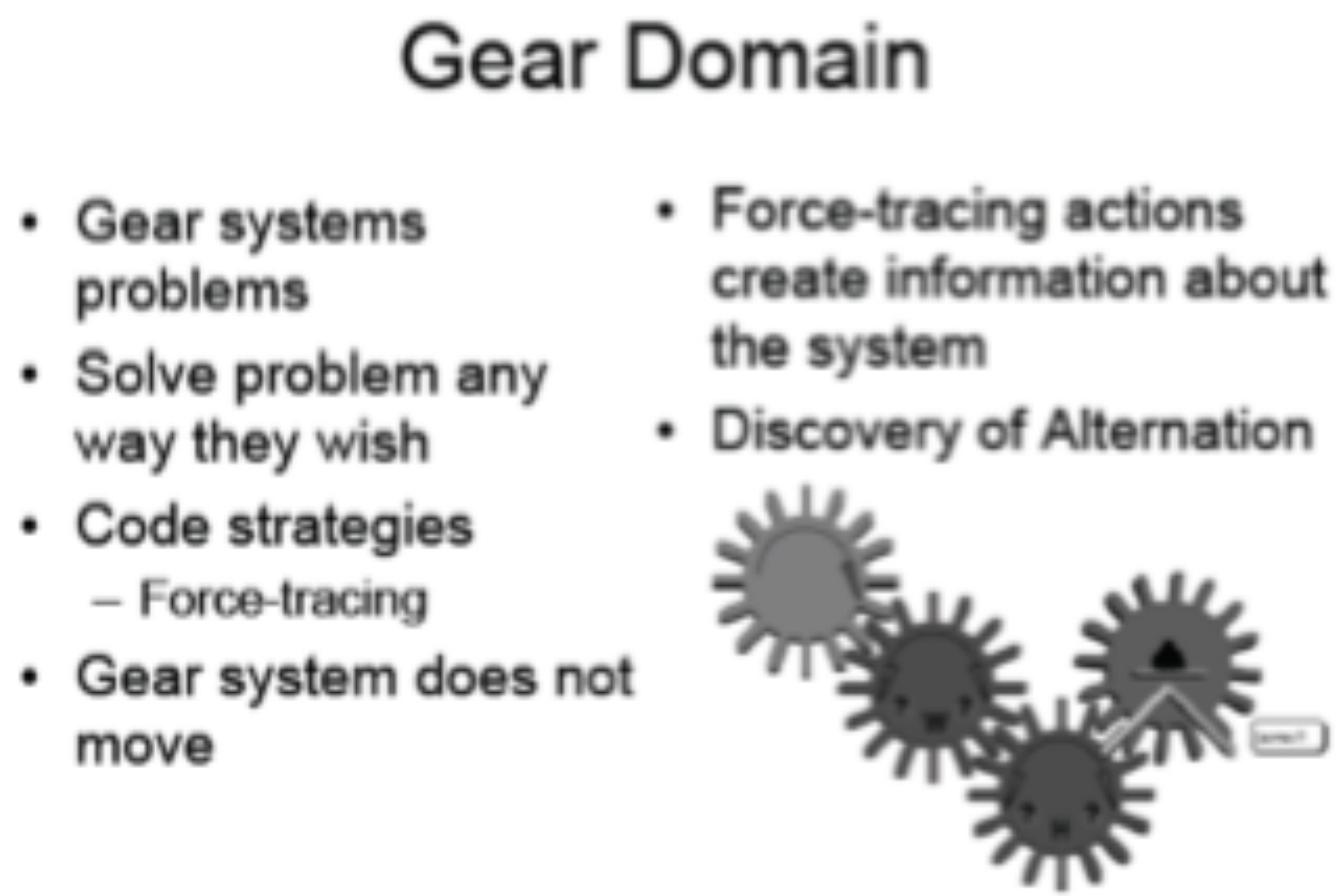


Lorenz system – Transitions in phase space



Insight as a phase transition

- Stephen, D.G., Dixon, J.A., & Isenhower, R.W. (2009). Dynamics of representational change: Entropy, action, and cognition. JEP: HPP.



Insight as a phase transition

Optotrak

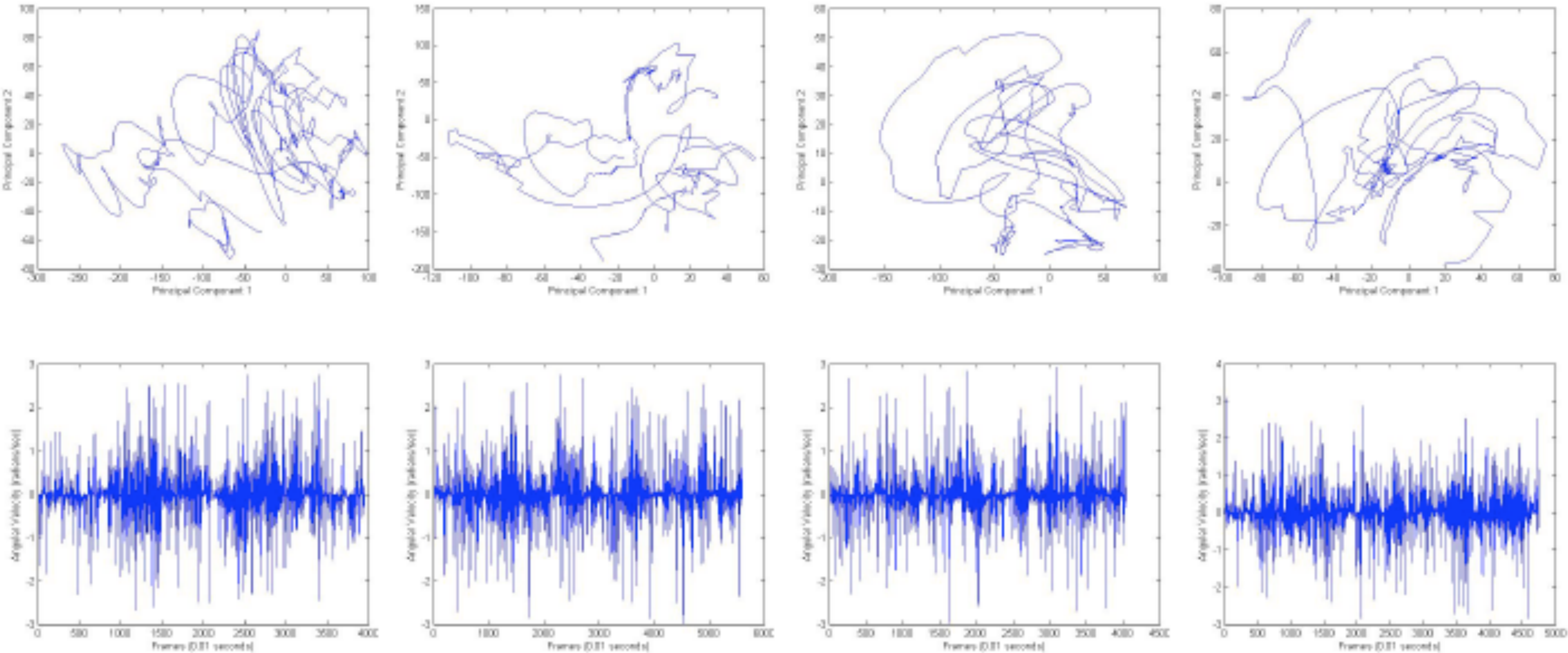
100 Hz sampling rate, 4 markers Velcro-ed to forefinger

Markers emit infrared light

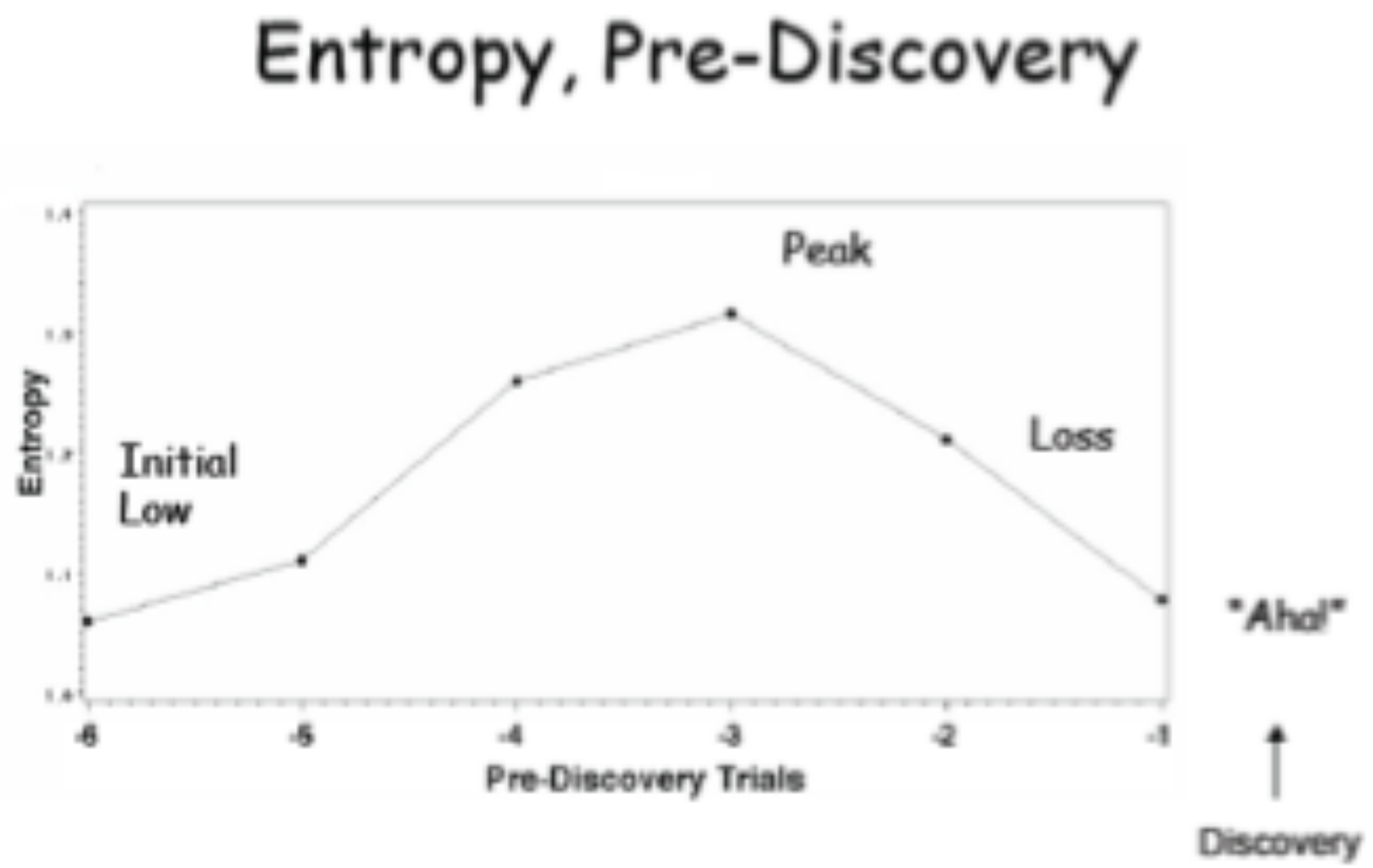
2 markers for
left camera

2 markers for
right camera

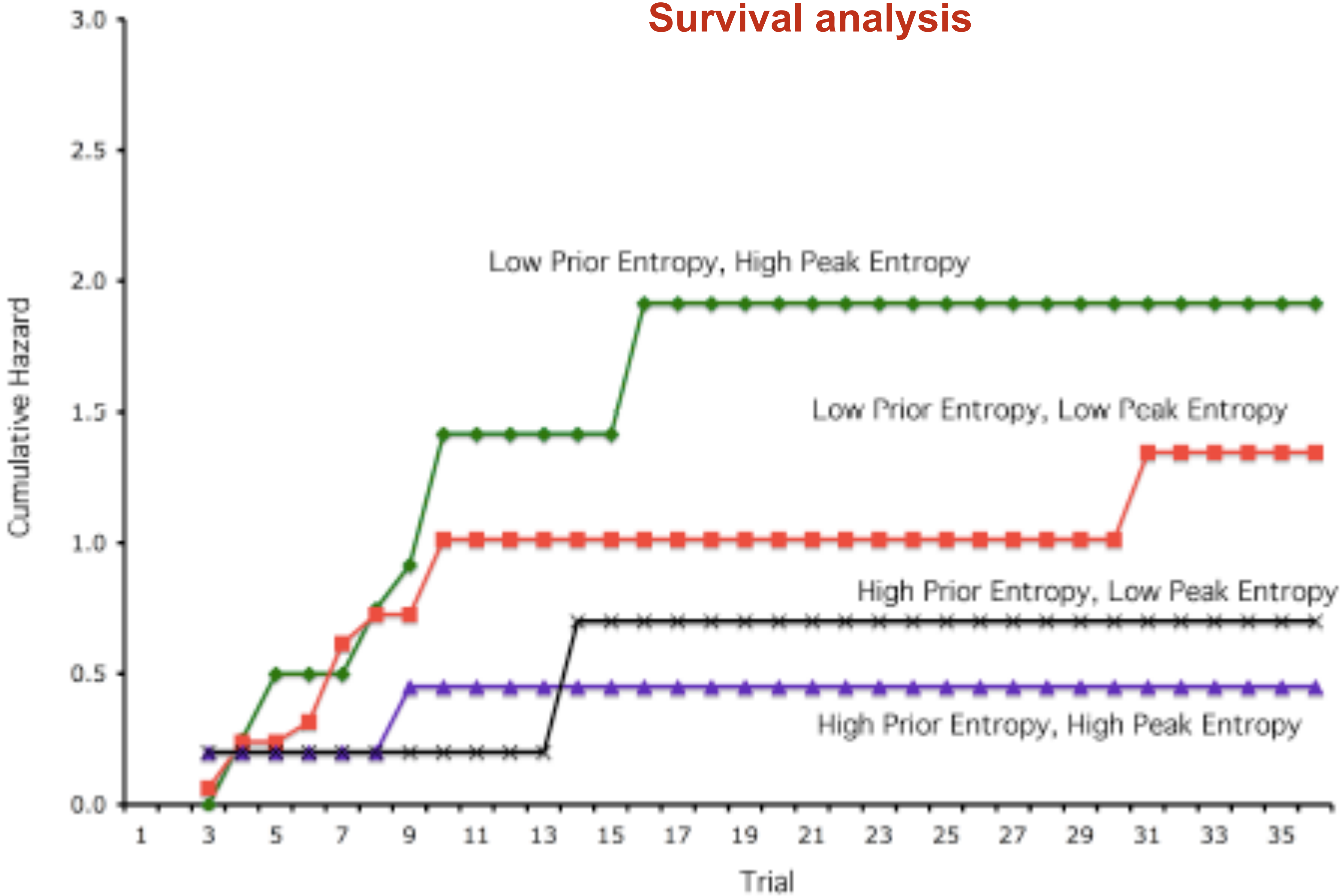
Angular velocity of finger movements



Insight as a phase transition



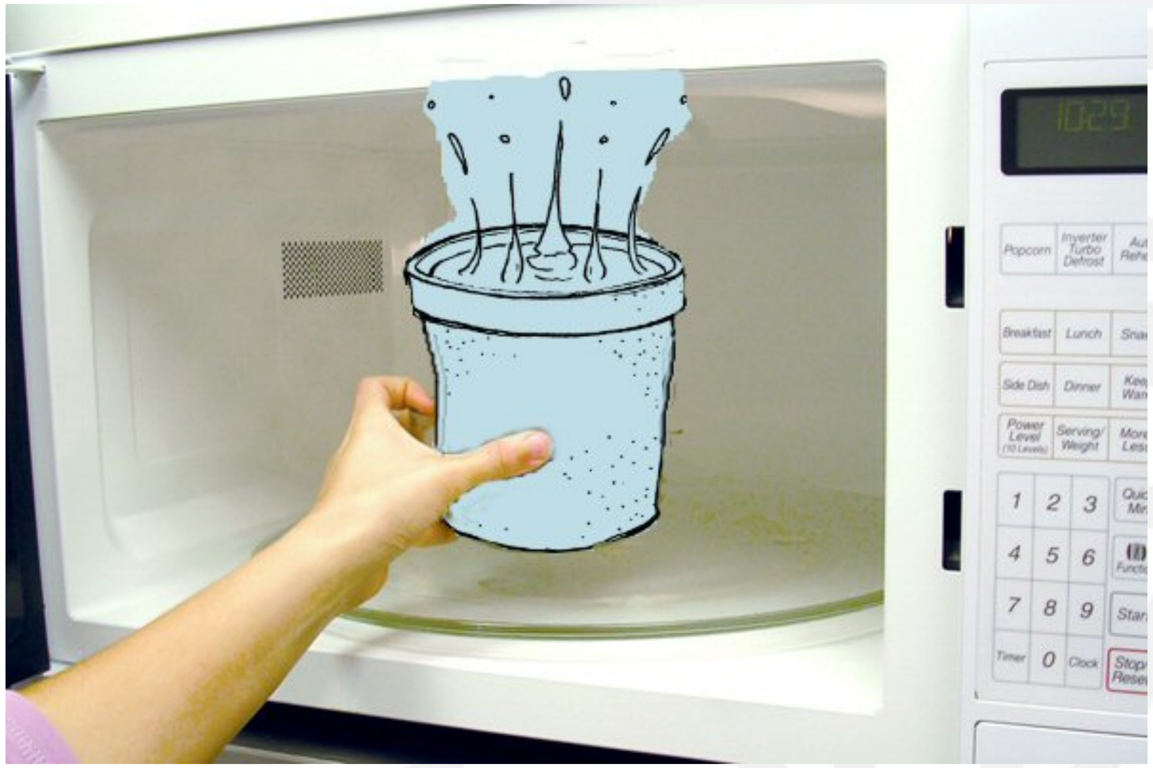
Survival analysis

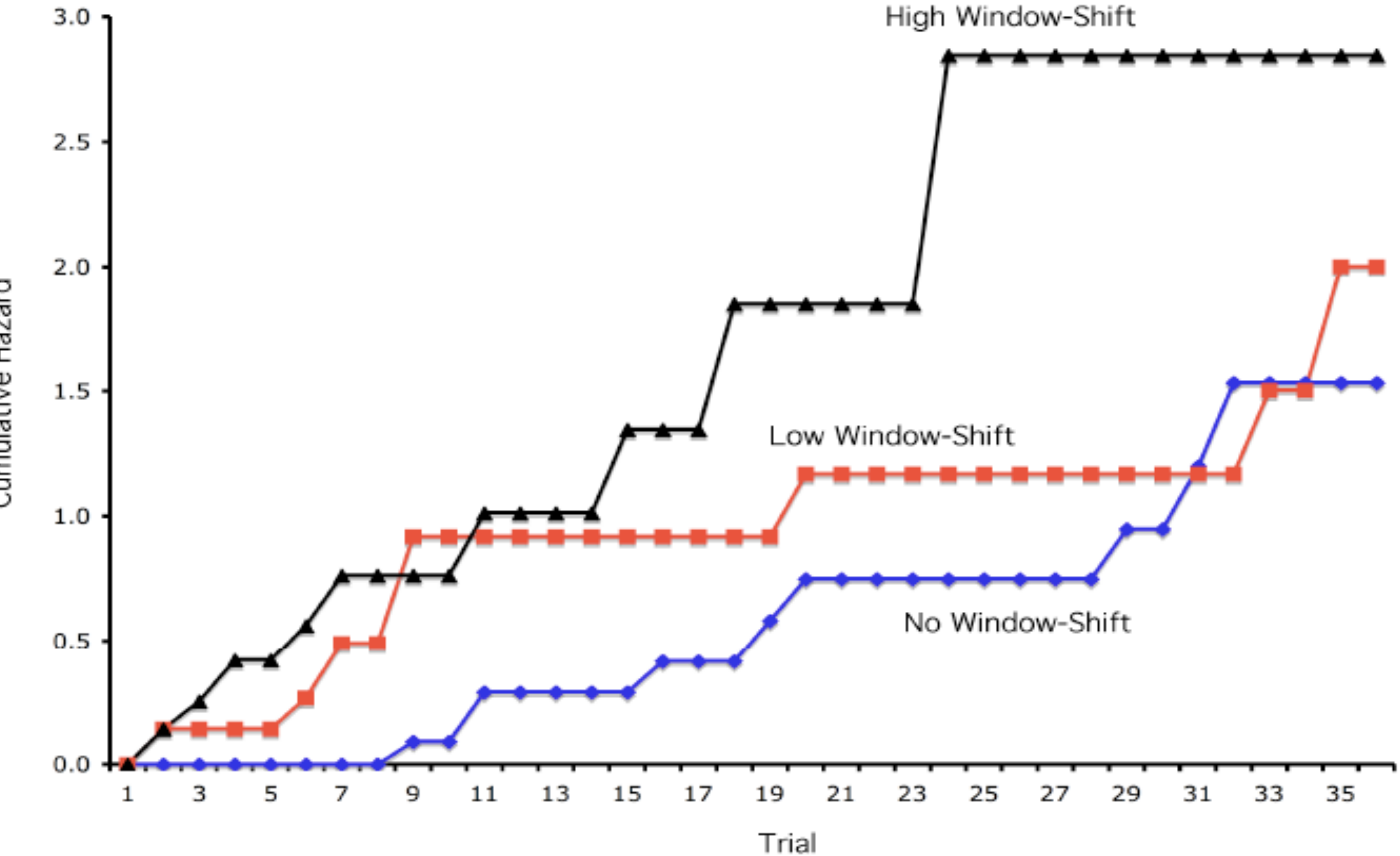


1. Assumption: Noise / Entropy drives the structural change

2. Hypothesis: Increase noise, this will lead to an earlier discovery of the rule

3. Additional condition: increase noise by making the gear problems shift position on the screen




High Window-Shift
Low Window-Shift
No Window-Shift
Cumulative Hazard
Trial
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35

The Dynamics of Complex Systems

Cross-Recurrence Quantification Analysis and other flavours of RP's

Fred Hasselman
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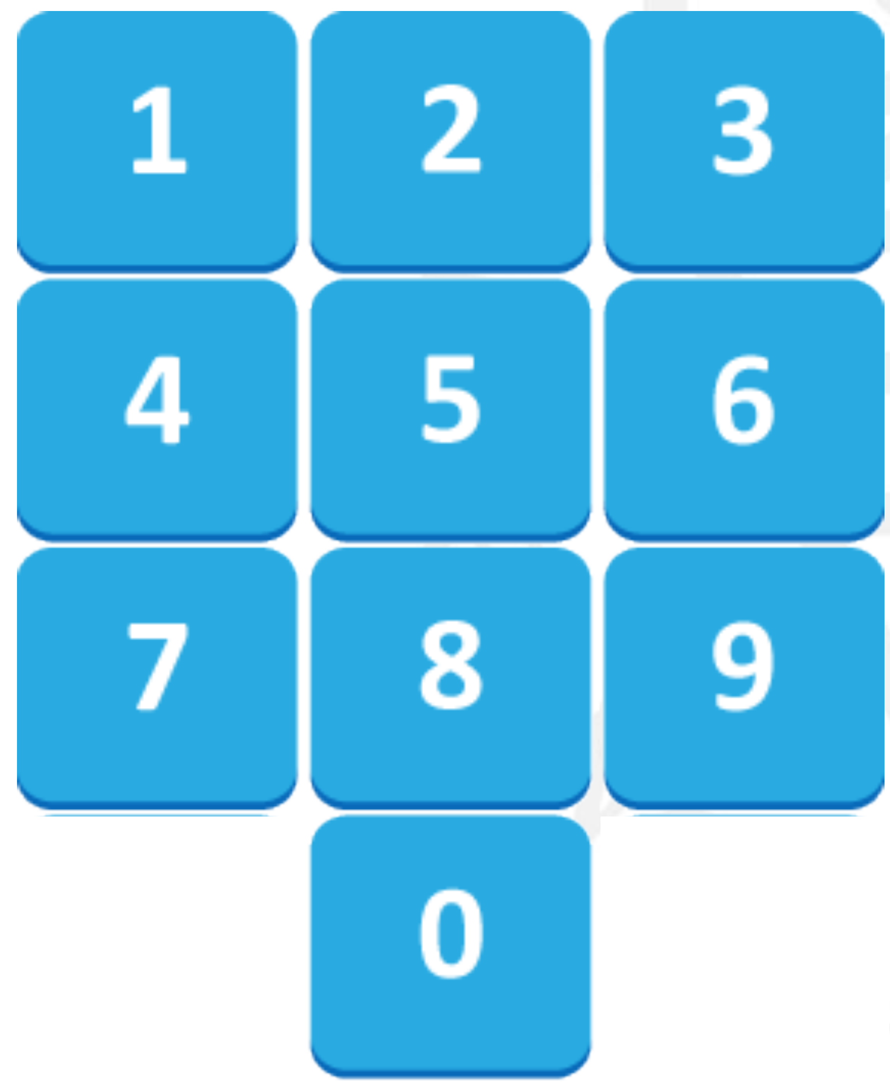
Behavioural Science Institute
Radboud University Nijmegen

Many Applications of RQA

Oomens, W., Maes, J. H., Hasselman, F., & Egger, J. I. (2015). A time series approach to random number generation: using recurrence quantification analysis to capture executive behavior. *Frontiers in Human Neuroscience*, *9*

Executive control:

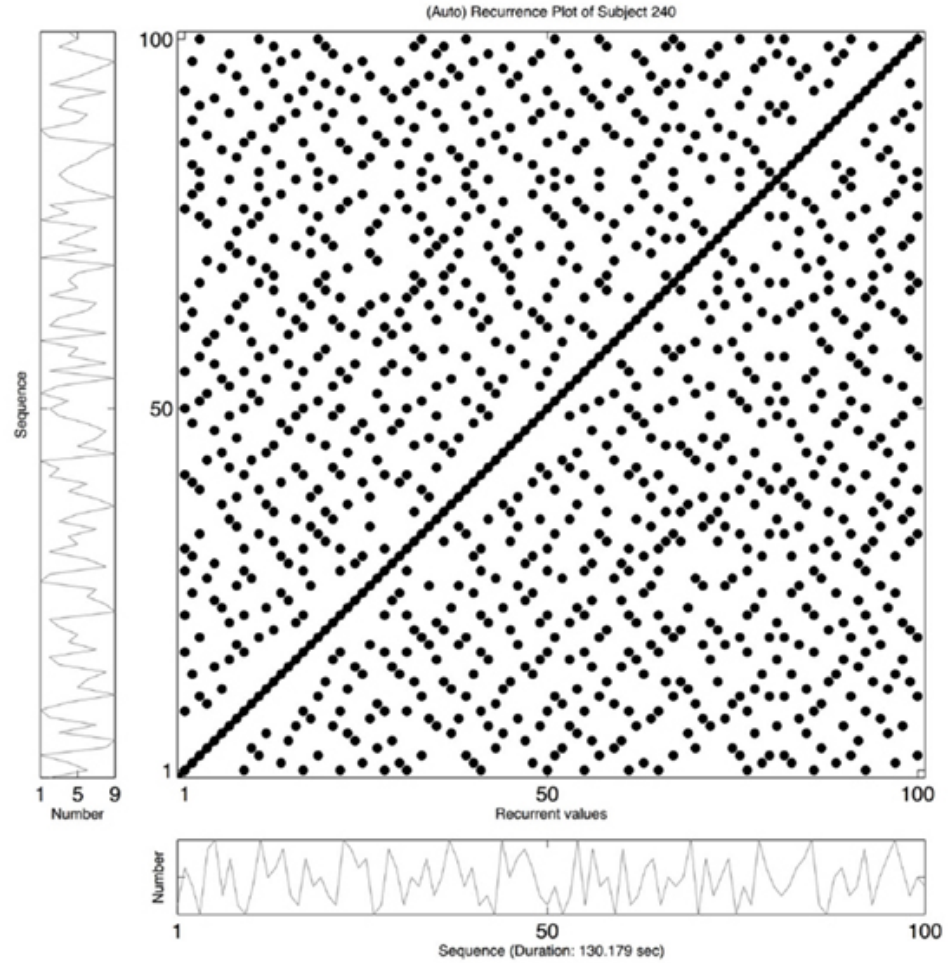
"be as random as you can"



A

RQA measures:

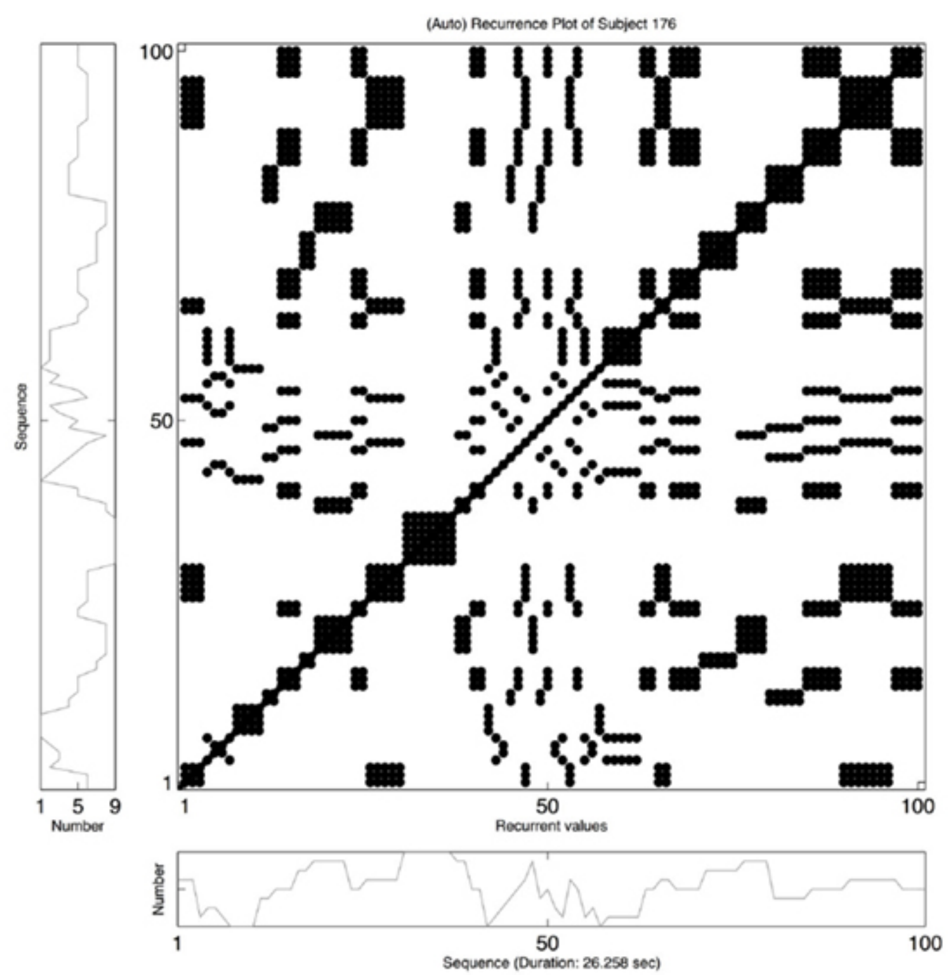
REC = 0.105
DET = 0.185
Lmn = 2.04
Lmx = 3
ENT = 0.176
LAM = 0
Vmn = NaN
Vmx = 0



C

RQA measures:

REC = 0.138
DET = 0.654
Lmn = 2.65
Lmx = 7
ENT = 1.1
LAM = 0.82
Vmn = 3.3
Vmx = 7



N=181

	Updating	Inhibition of prepotent responses	Output inhibition	Undefined
Redundancy	0.792			
RNG2		0.859		
RG median	−0.785			
RG mean	−0.586			
Coupon	0.830			
Adjacency		0.874		
TPI		−0.844		
Runs		0.478		0.769
RNG		0.874		
Phi 2			0.876	
Phi 3			0.811	
Phi 4			0.691	
Phi 6	0.423	−0.569		
Phi 5	0.445		0.462	0.521
Phi 7	0.631			
RG mode	−0.475			
Eigenvalues	3.409	3.844	2.729	1.201
% of variance	21.304	24.026	17.059	7.508

Output is sorted by size and a cut-off value of 0.4 was used.

N=242

	Updating	Inhibition of prepotent responses	Output inhibition	Undefined
Redundancy	0.782			0.432
RNG2	0.713	0.478		
RG median	−0.674			−0.486
RG mean	−0.652		−0.461	
Coupon	0.630			0.515
Adjacency		0.885		
TPI		−0.828		
Runs		0.791		
RNG	0.593	0.645		
Phi 2			0.879	
Phi 3			0.719	0.455
Phi 4			0.570	0.556
Phi 6				0.803
Phi 5				0.637
Phi 7				0.634
RG mode				−0.546
Eigenvalues	3.200	2.817	2.392	3.047
% of variance	19.998	17.607	14.949	19.045

Output is sorted by size and a cut-off value of 0.4 was used.

Component Measures!

Phi n ~ prob of lag n “sameness” = recurrence

Coupon / Adjacency / TPI ~ attempts at RQA measures

N=181

	Updating	Inhibition of prepotent responses	Output inhibition	Undefined
Redundancy	0.792			
RNG2		0.859		
RG median	−0.785			
RG mean	−0.586			
Coupon	0.830			
Adjacency		0.874		
TPI		−0.844		
Runs		0.478		0.769
RNG		0.874		
Phi 2			0.876	
Phi 3			0.811	
Phi 4			0.691	
Phi 6	0.423	−0.569		
Phi 5	0.445		0.462	0.521
Phi 7	0.631			
RG mode	−0.475			
Eigenvalues	3.409	3.844	2.729	1.201
% of variance	21.304	24.026	17.059	7.508

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Adjacency		0.885		
TPI		−0.828		
Runs		0.791		
RNG	0.593	0.645		
Phi 2			0.879	
Phi 3			0.719	0.455
Phi 4			0.570	0.556
Phi 6				0.803
Phi 5				0.637
Phi 7				0.634
RG mode				−0.546
Eigenvalues	3.200	2.817	2.392	3.047
% of variance	19.998	17.607	14.949	19.045

Output is sorted by size and a cut-off value of 0.4 was used.

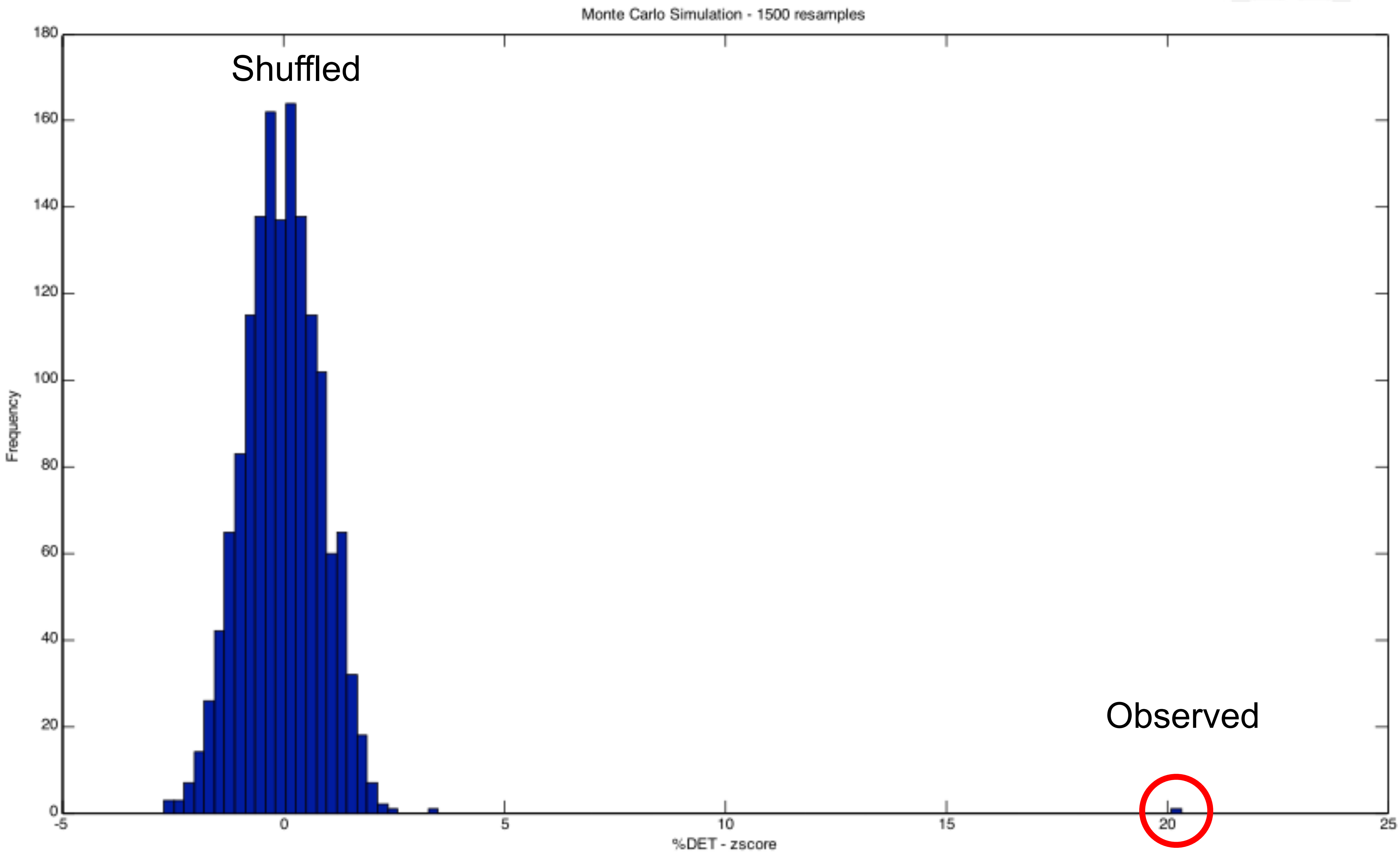
	Inhibition of prepotent responses	Updating
Averaged diagonal	0.963	
Longest diagonal	0.922	
Determinism	0.917	
Entropy	0.839	
Laminarity		0.918
Trapping time		0.878
Recurrence rate		0.486
Eigenvalues	3.487	1.857
% of variance	49.818	26.523

Output is sorted by size and a cut-off value of 0.4 was used.

	Inhibition of prepotent responses	Updating
Averaged diagonal	0.957	
Entropy	0.937	
Longest diagonal	0.852	
Determinism	0.730	
Laminarity		0.861
Trapping time		0.765
Recurrence rate		0.712
Eigenvalues	3.086	1.948
% of variance	44.085	27.830

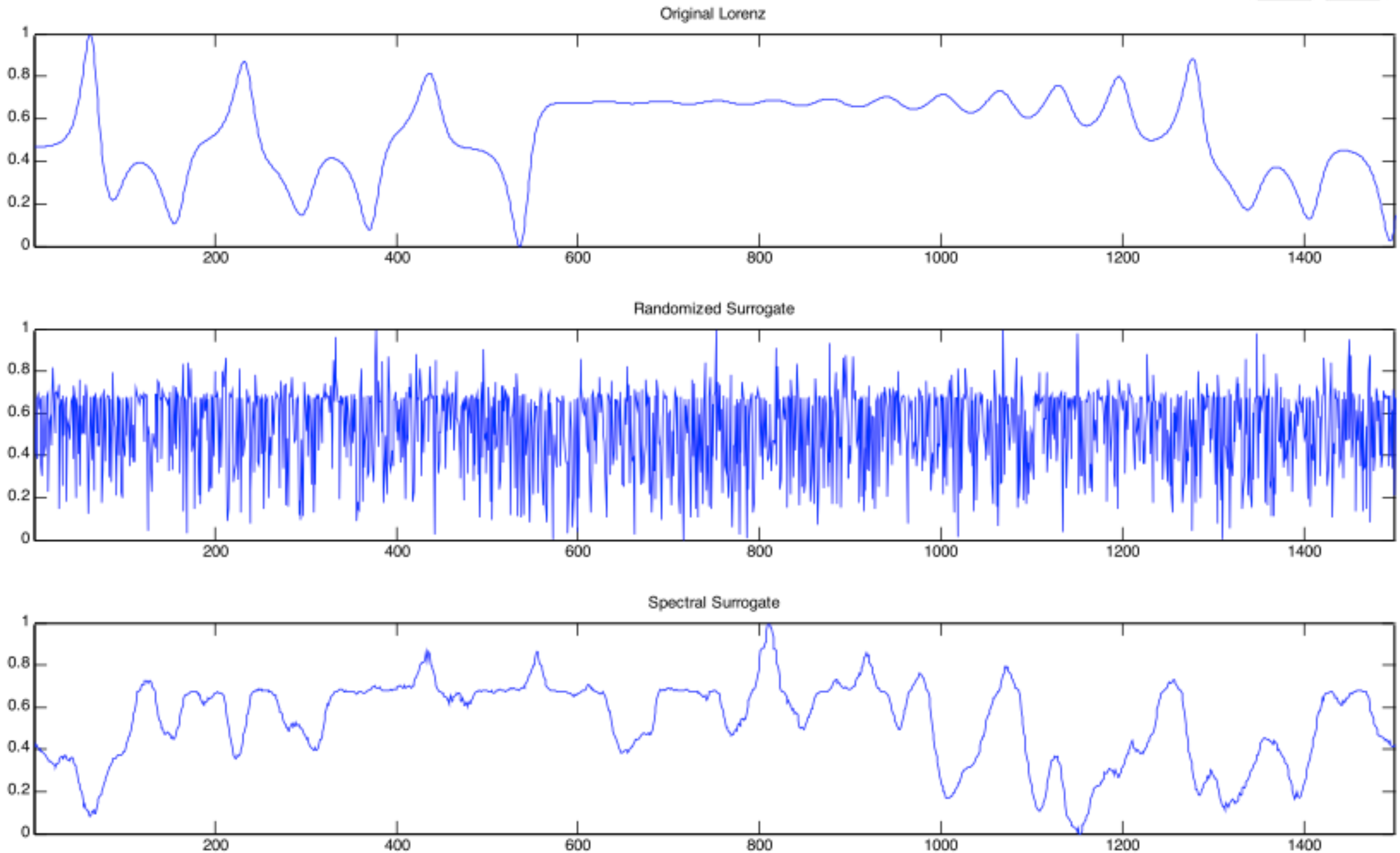
Output is sorted by size and a cut-off value of 0.4 was used.

Random process vs. randomising …

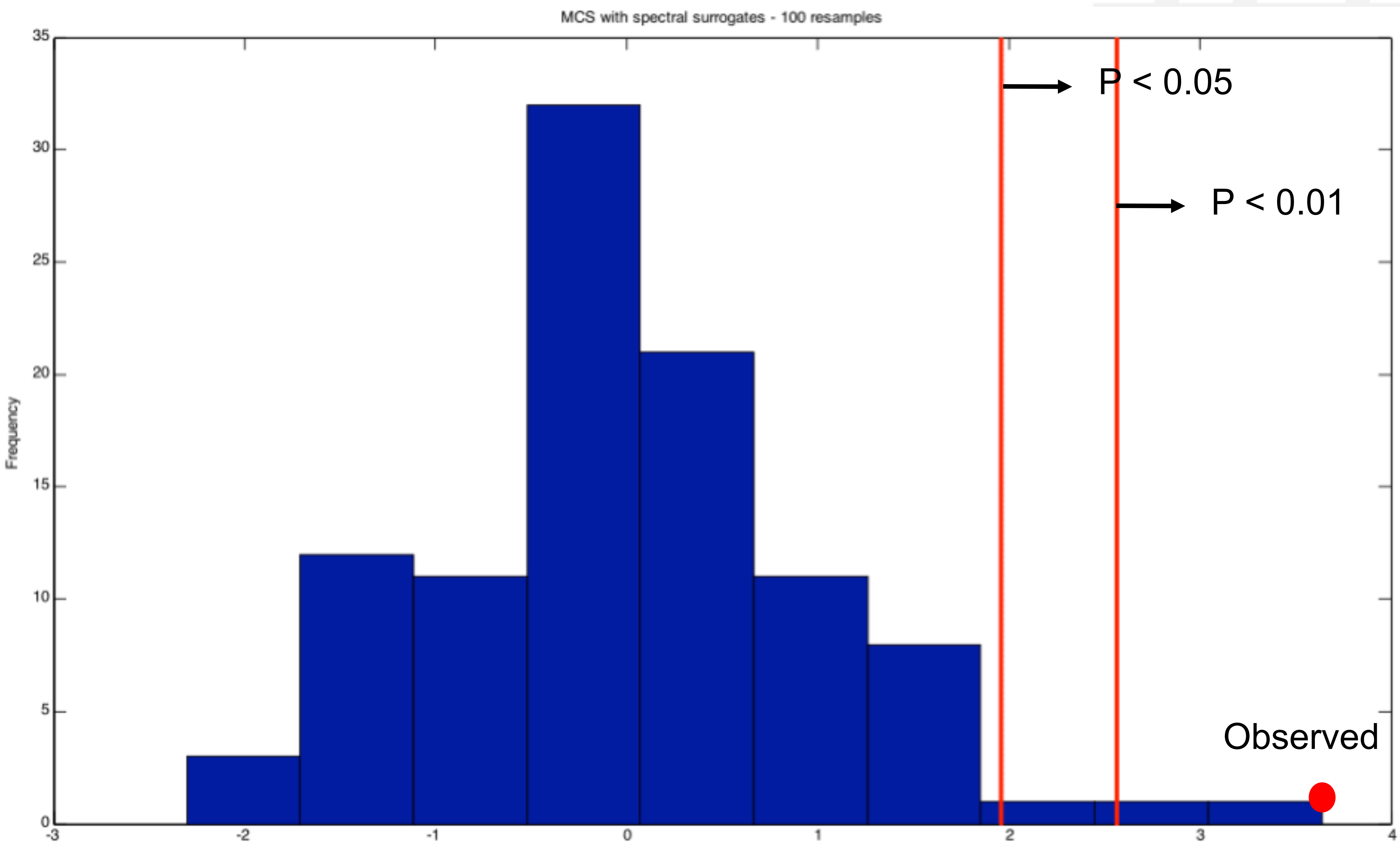


Behavioural Science Institute
Radboud University Nijmegen

About doing the Shuffle…



Spectral surrogates



Surrogates for synchronisation hypotheses

- So-called twin surrogates generated based on the Recurrence plot! Used for synchronisation studies.

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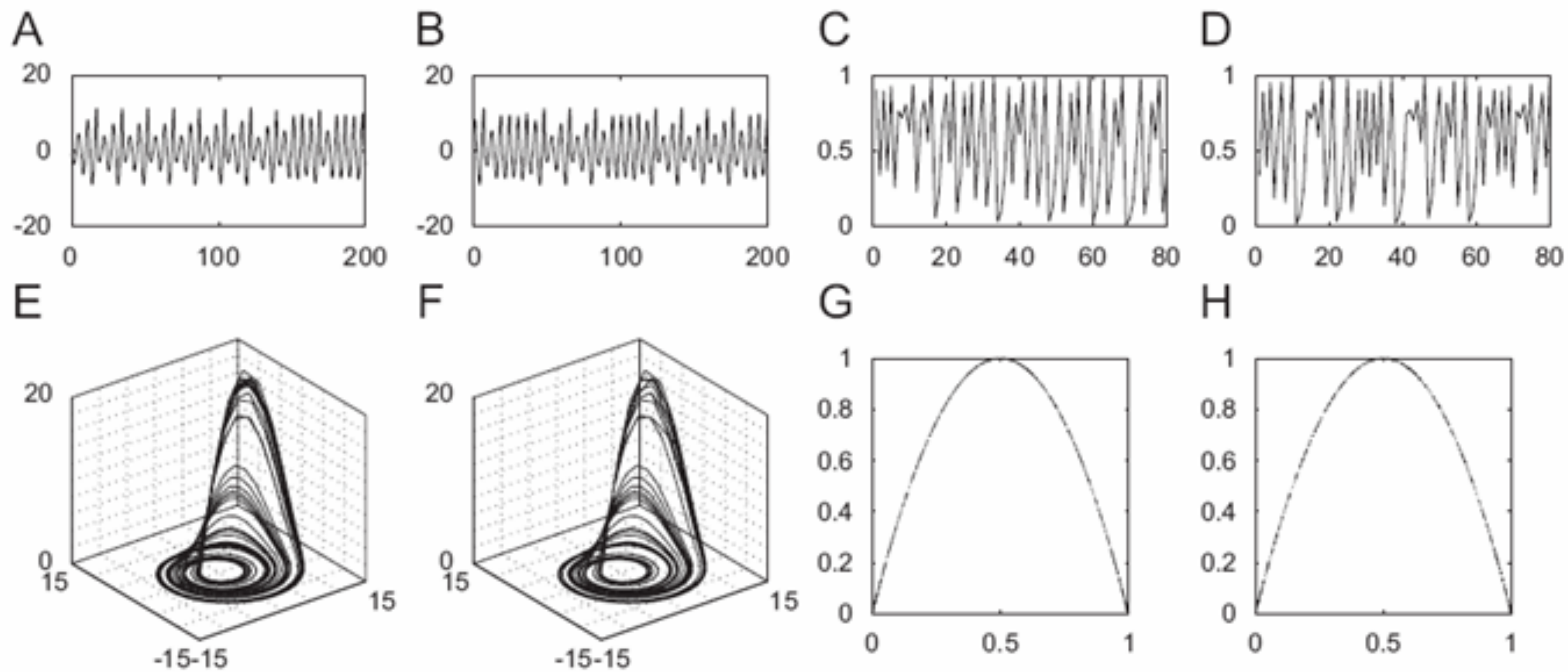


Fig. 46. Segment of the x-component of the (A) original Rössler system, Eqs. (A.5), for $a = 0.15$, $b = 0.2$ and $c = 10$ and (B) of one twin surrogate (TS) of the Rössler system. Trajectory of the (C) Rössler system in phase space and (D) of one TS of the Rössler system. Segment of the trajectory of the (E) logistic map, Eq. (A.3), for $a = 4.0$ and (F) of one TS of the logistic map. Phase portrait (G) of the logistic map and (H) of one TS of the logistic map (modified after [163]).

Flavours of Recurrence Plots

- **Cross Recurrence Analysis**
- RP’s of Categorical data
- Lagged RQA
- Order Patterns RQA, Joint RQA, Close Returns Plot

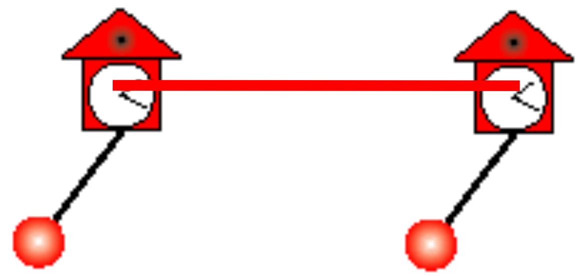
Cross Recurrence Analysis

- Instead of analysing if a system re-visits locations in reconstructed phase space: Analyse if two systems share locations in phase space
- Cross recurrence analysis tells you something about synchronization or coupling of systems in time.
- Same strategy as autorecurrence: Reconstruct phase space and see if points between two trajectories are adjecent

Synchronisation: Huygens' Clocks

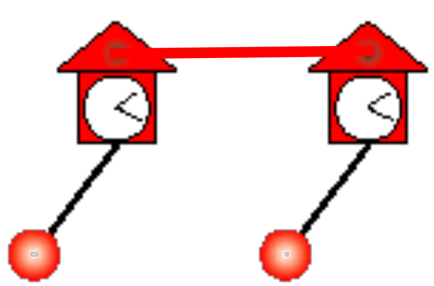
Synchronisation is a science in itself…

For now just note that synchronisation occurs in systems which are coupled in some way

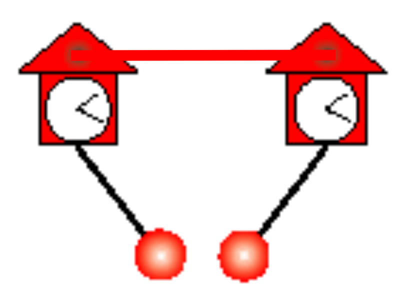


1629-1695

In-Phase



Anti-Phase



CRQA plots
What do these guys have in common?

CRQA plots
Blake vs. Deelder

Auguries of Innocence

To see a World

in a Grain of Sand

And a Heaven

in a Wild Flower,

Hold Infinity

in the palm of your hand

And Eternity

in an hour.

A = 1
B = 2
D = 3
E = 4
F = 5
G = 6
H = 7
I = 8
J = 9
K = 10
L = 11
M = 12
N = 13
O = 14
P = 15
R = 16
S = 17
T = 18
U = 19
V = 20
W = 21
Y = 22

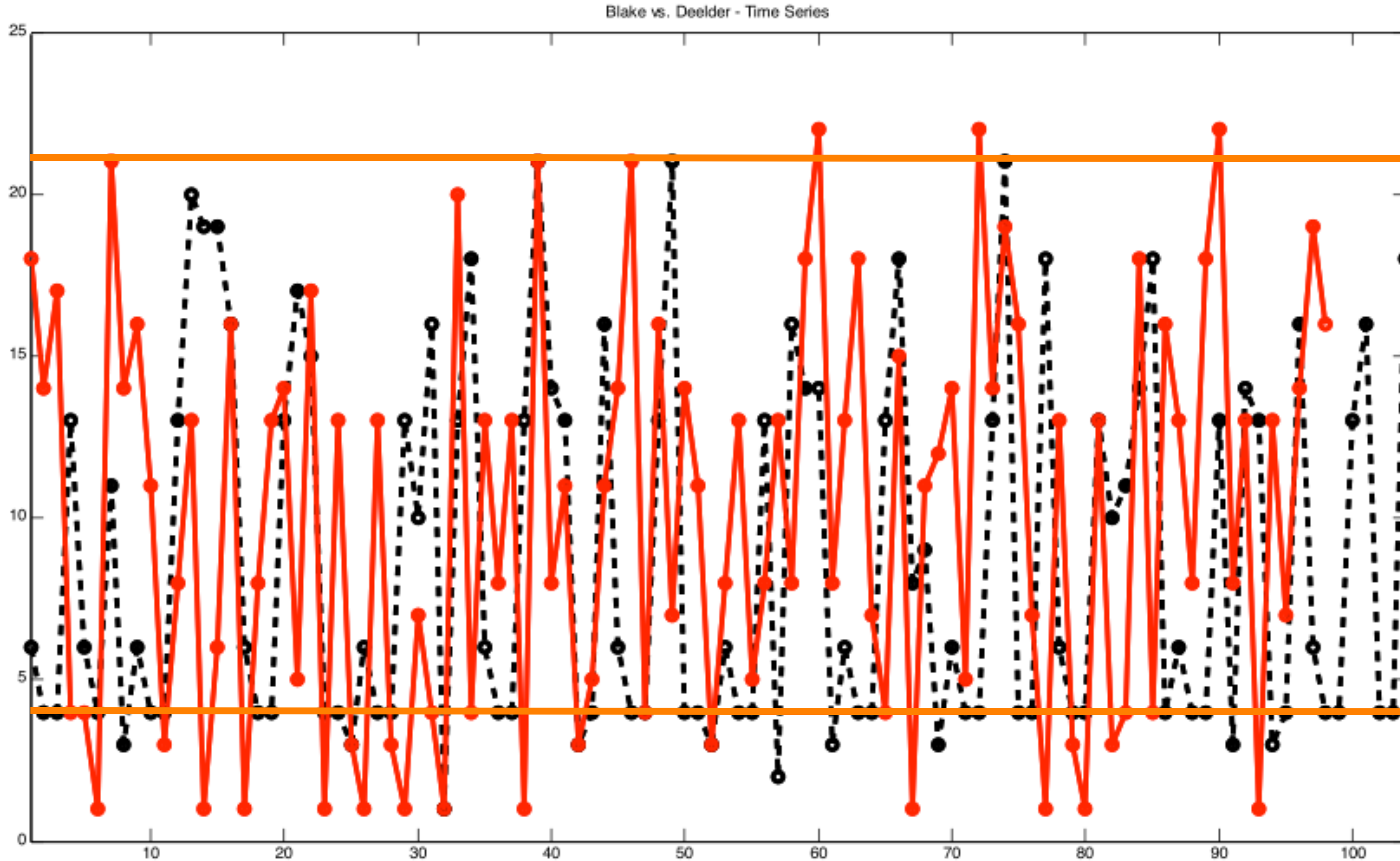
Blues on tuesday

Geen geld.
Geen vuur.
Geen speed.

Geen krant.
Geen wonder.
Geen weed.

Geen brood.
Geen tijd.
Geen weet.

Geen klote.
Geen donder.
Geen reet.



Blake vs. Deelder - Time Series
25
20
15
10
5
0
10
20
30
40
50
60
70
80
90
100

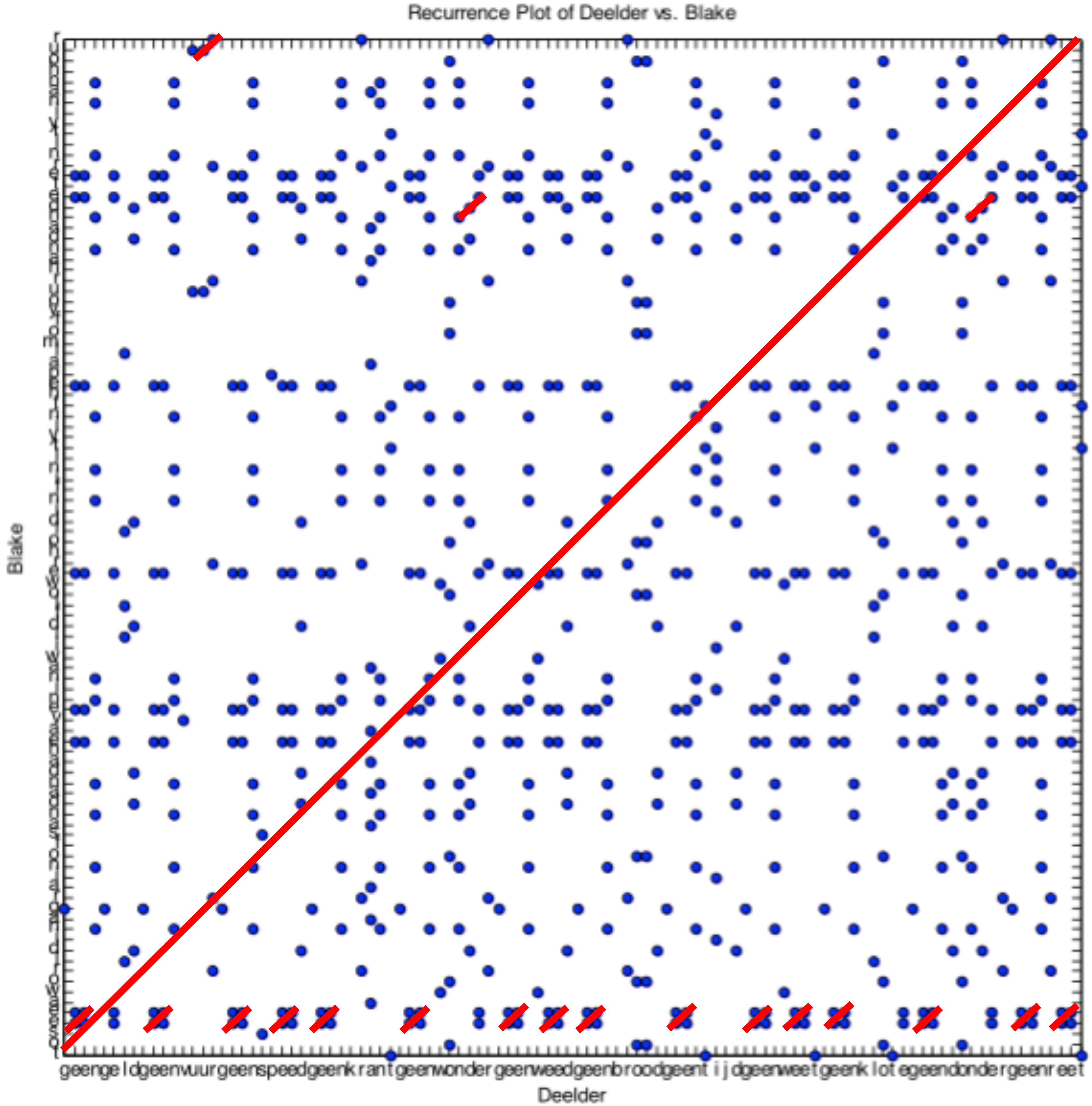
No symmetry in the plot!

Line of Incidence (LOI) =
Line of Synchronisation (LOS),
X(t) = Y(t)

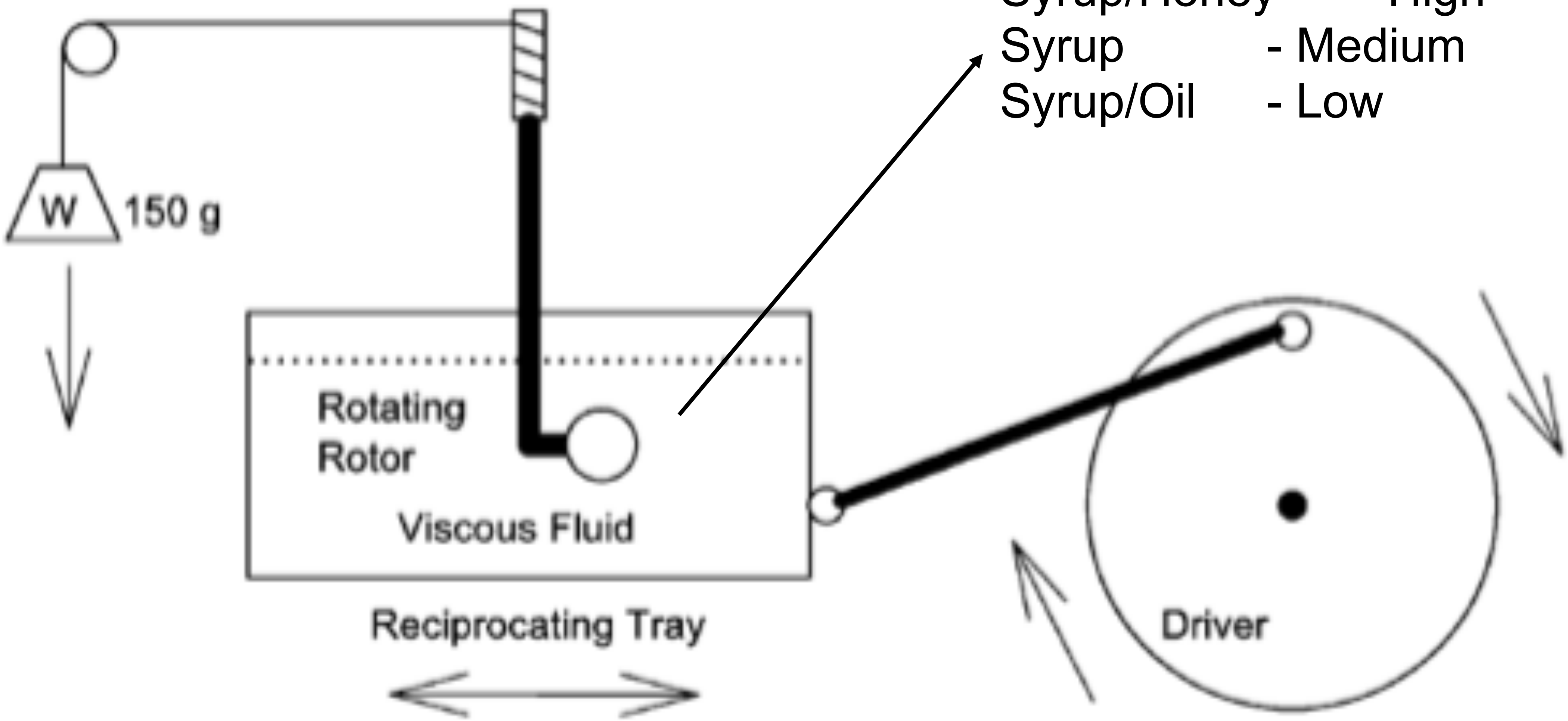
Blake and Deelder
don't synchronise very often
just a few bigrams: **ee - ur**

And the odd trigram:
nde

They are'nt really coupled
of course



Cross RQA of Coupled Oscillators

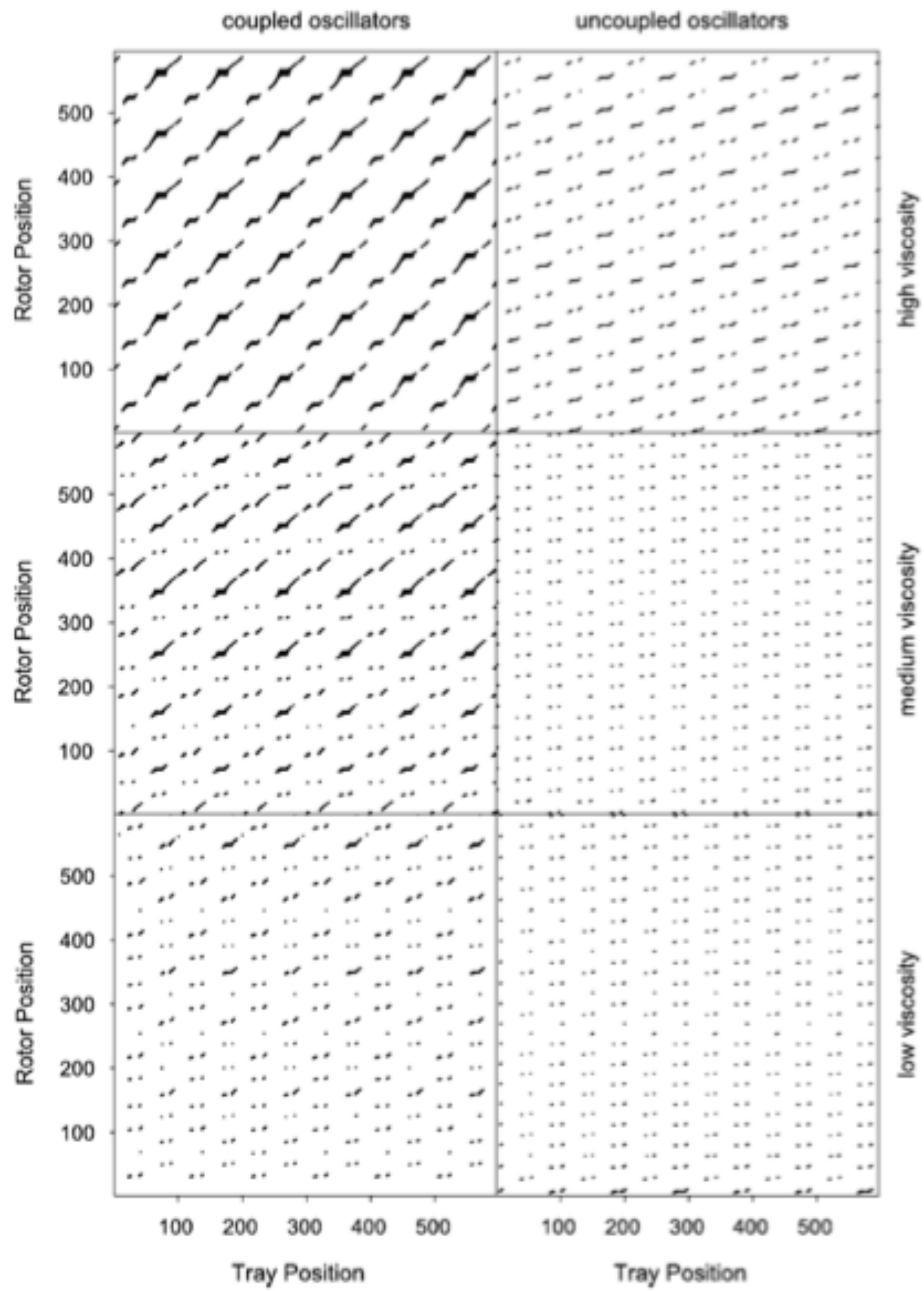


Syrup/Honey - High
Syrup - Medium
Syrup/Oil - Low

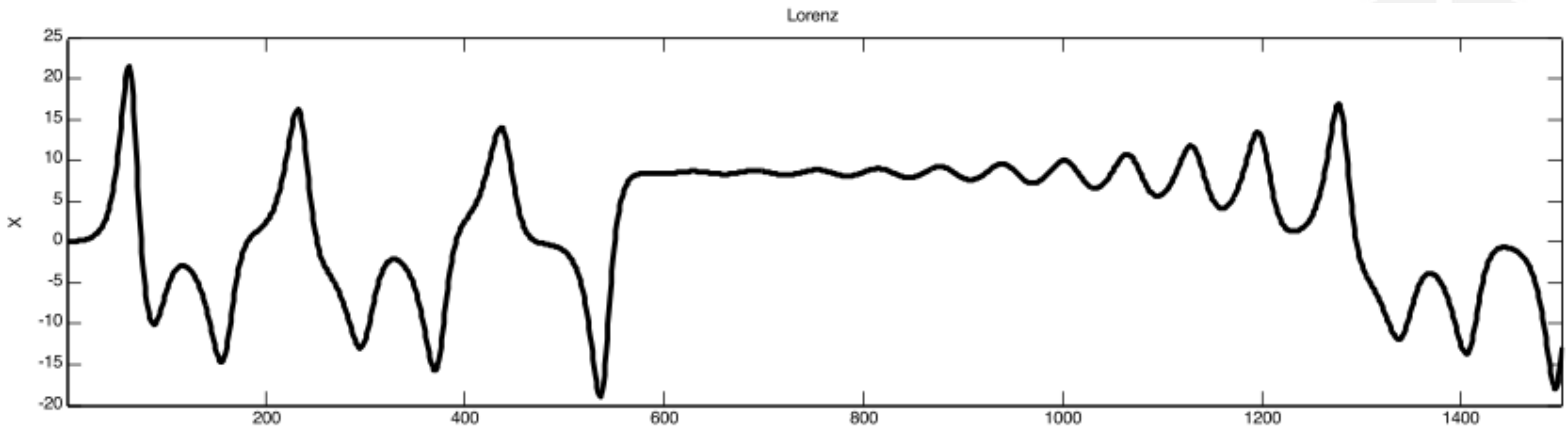
Kevin Shockley, Matthew Butwill, Joseph P. Zbilut, Charles L. Webber Jr. Cross recurrence quantification of coupled oscillators. *Physics Letters A 305* (2002) 59–69

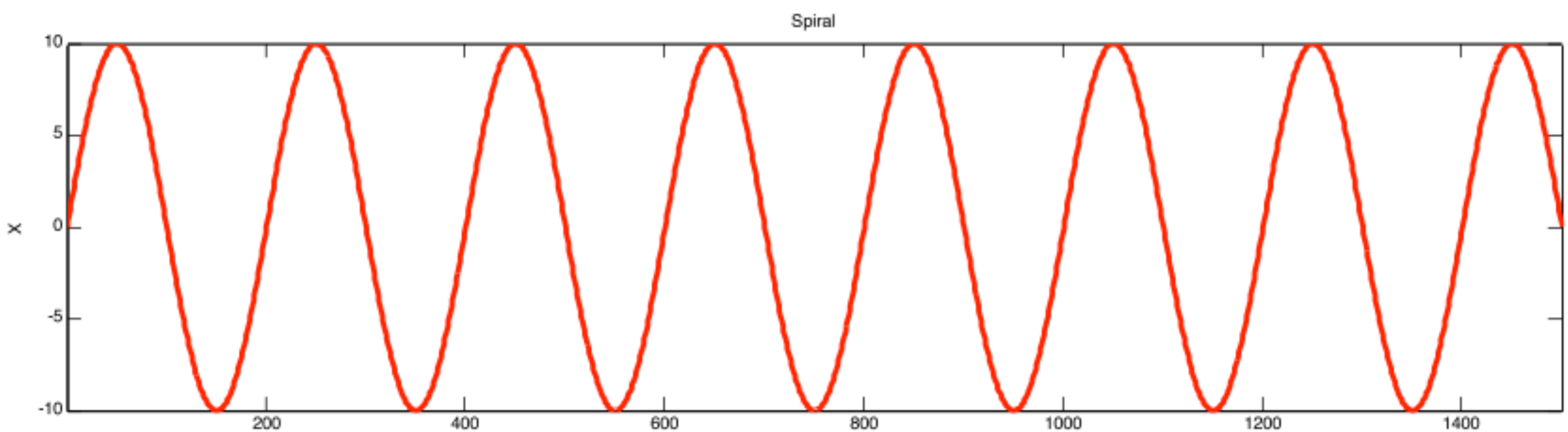
CRQA plots

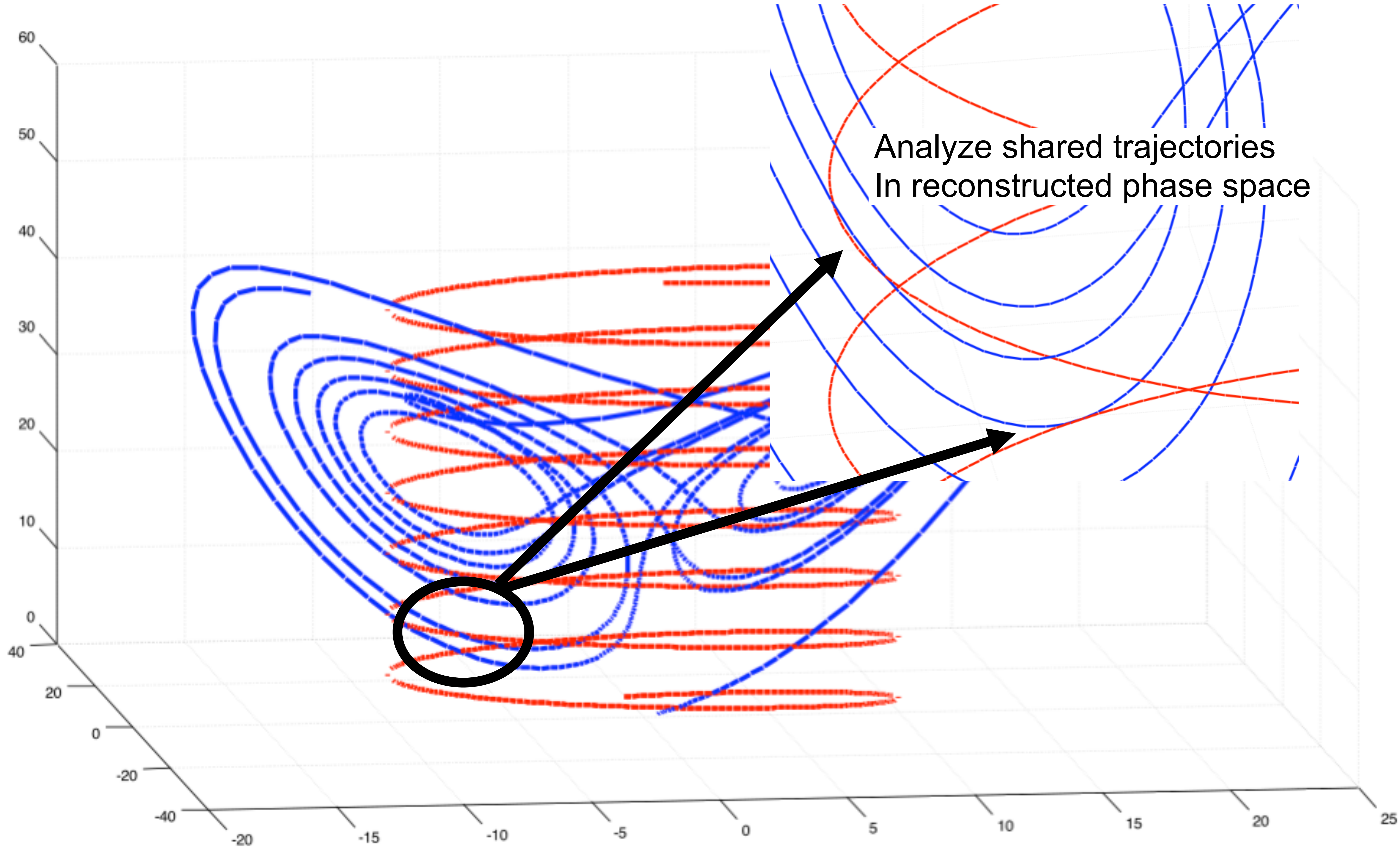
- Strength of coupling was detected by CRQA (not by FFT)
- More recurrent points More determinism
- Note that this coupling is “weak”…and CRQA is still able to detect it



Time Series of X – Lorenz and Spiral






Analyze shared trajectories
In reconstructed phase space
60
50
40
30
20
10
0
40
20
0
-20
-40
-20
-15
-10
-5
0
5
10
15
20
25

Within radius / threshold = shared trajectory

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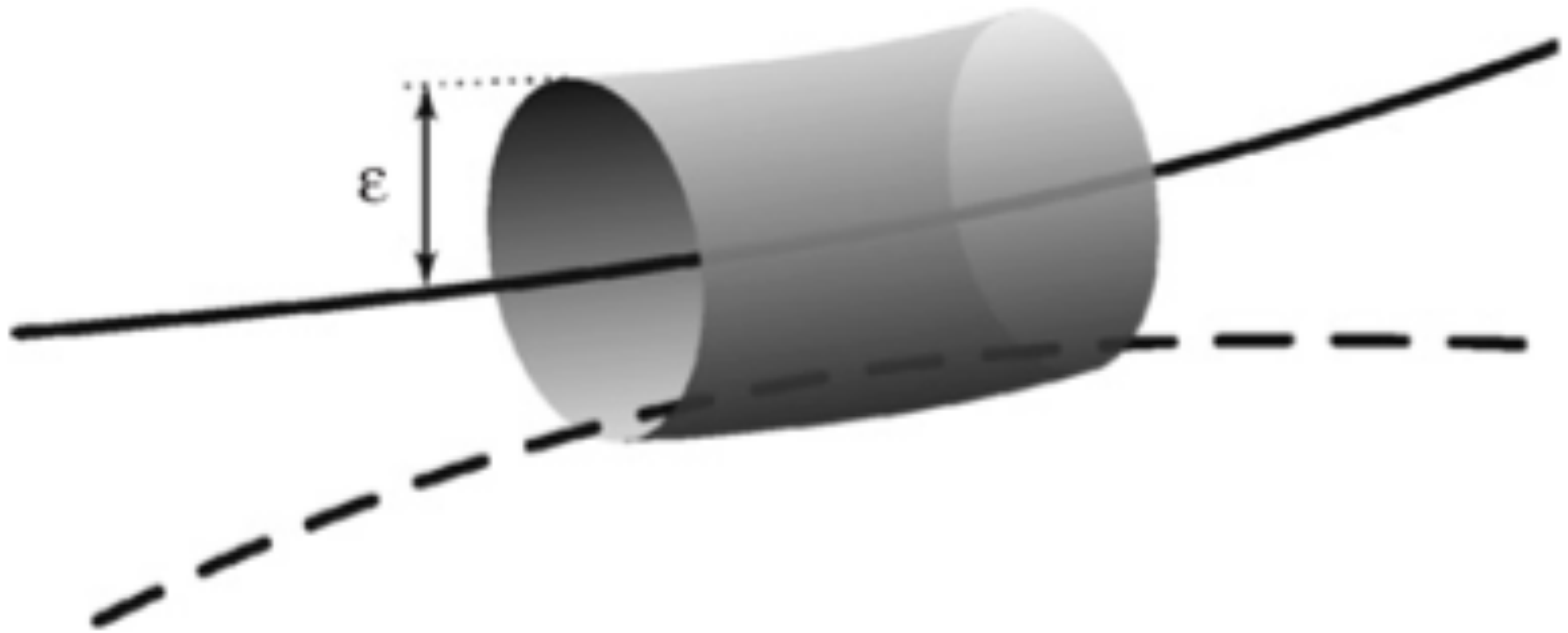
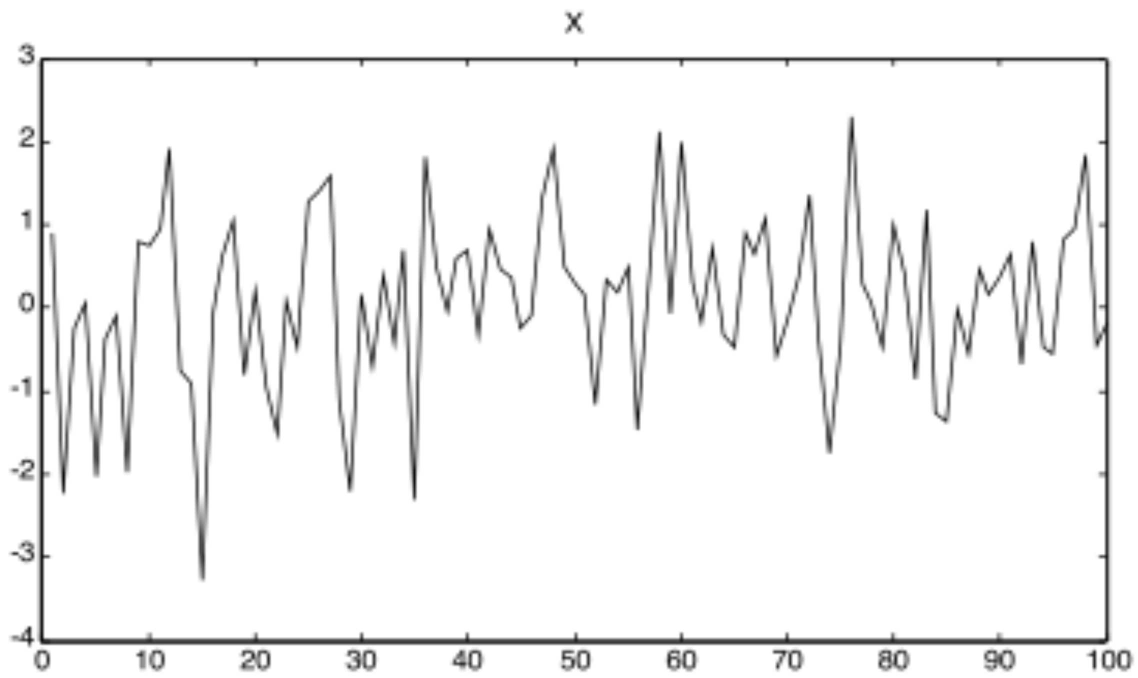
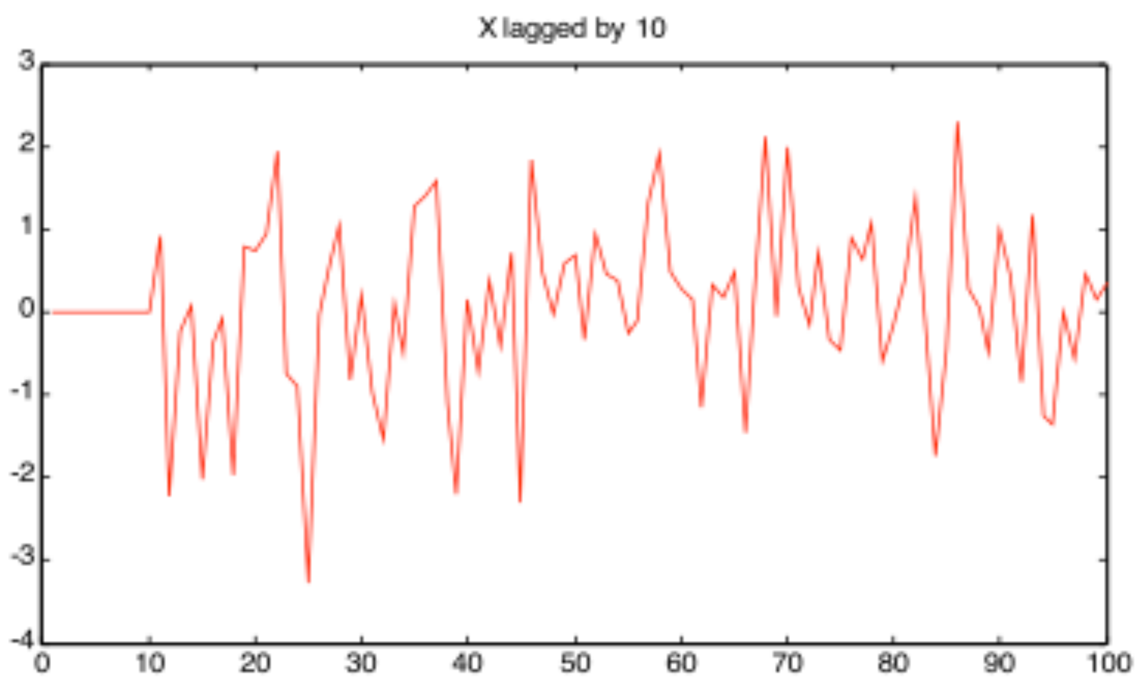
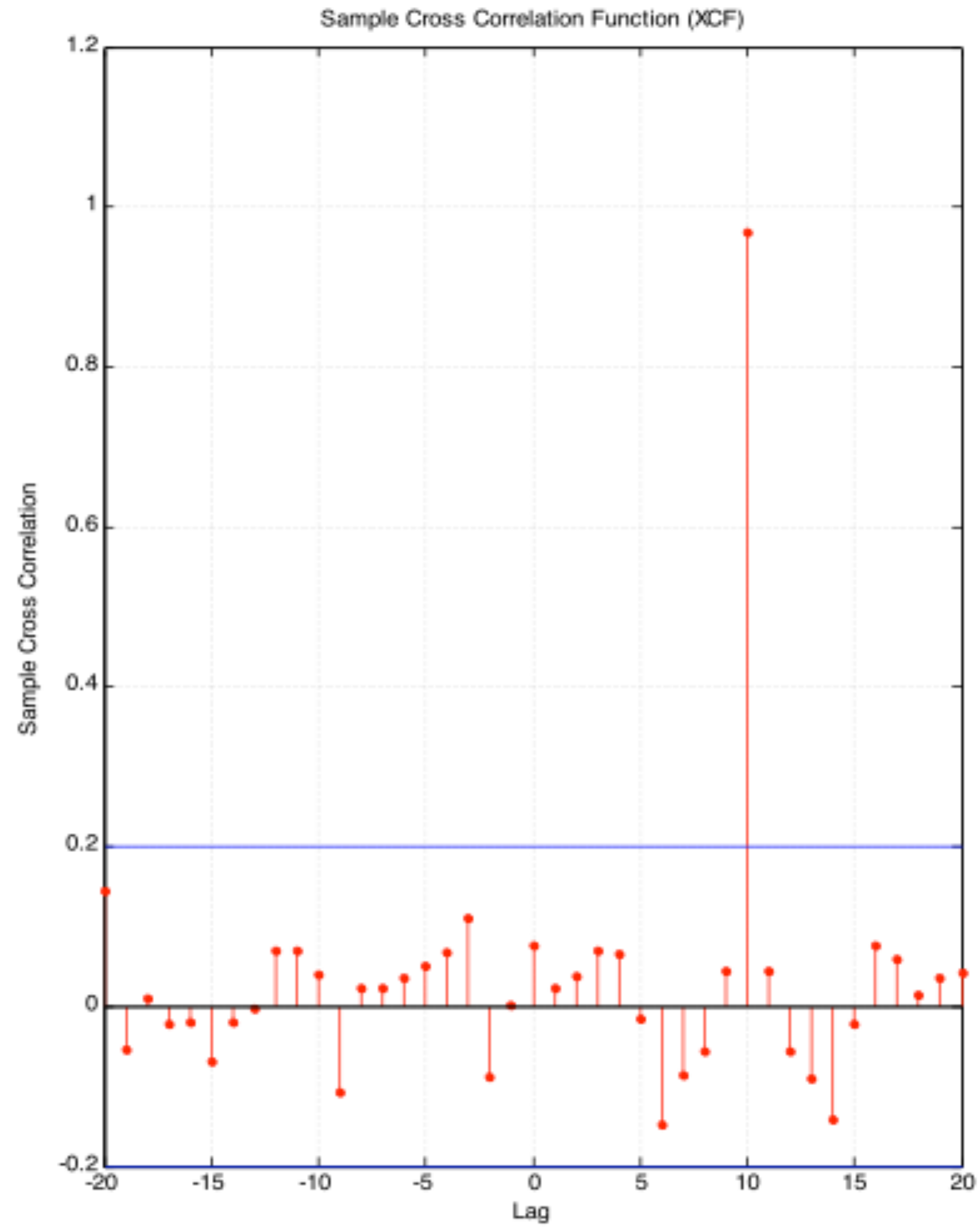


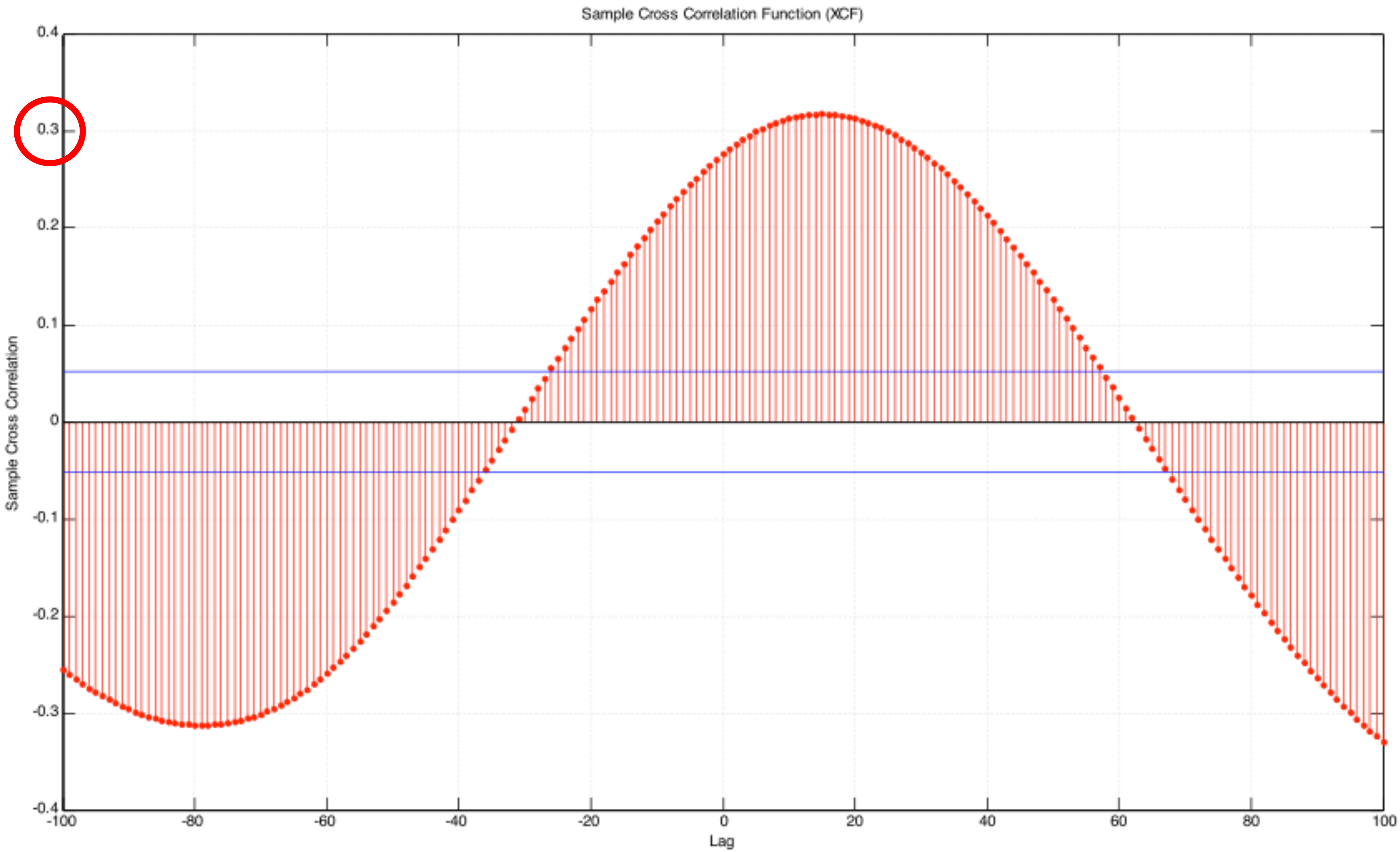
Intuitive notion of synchronisation – Cross Correlation



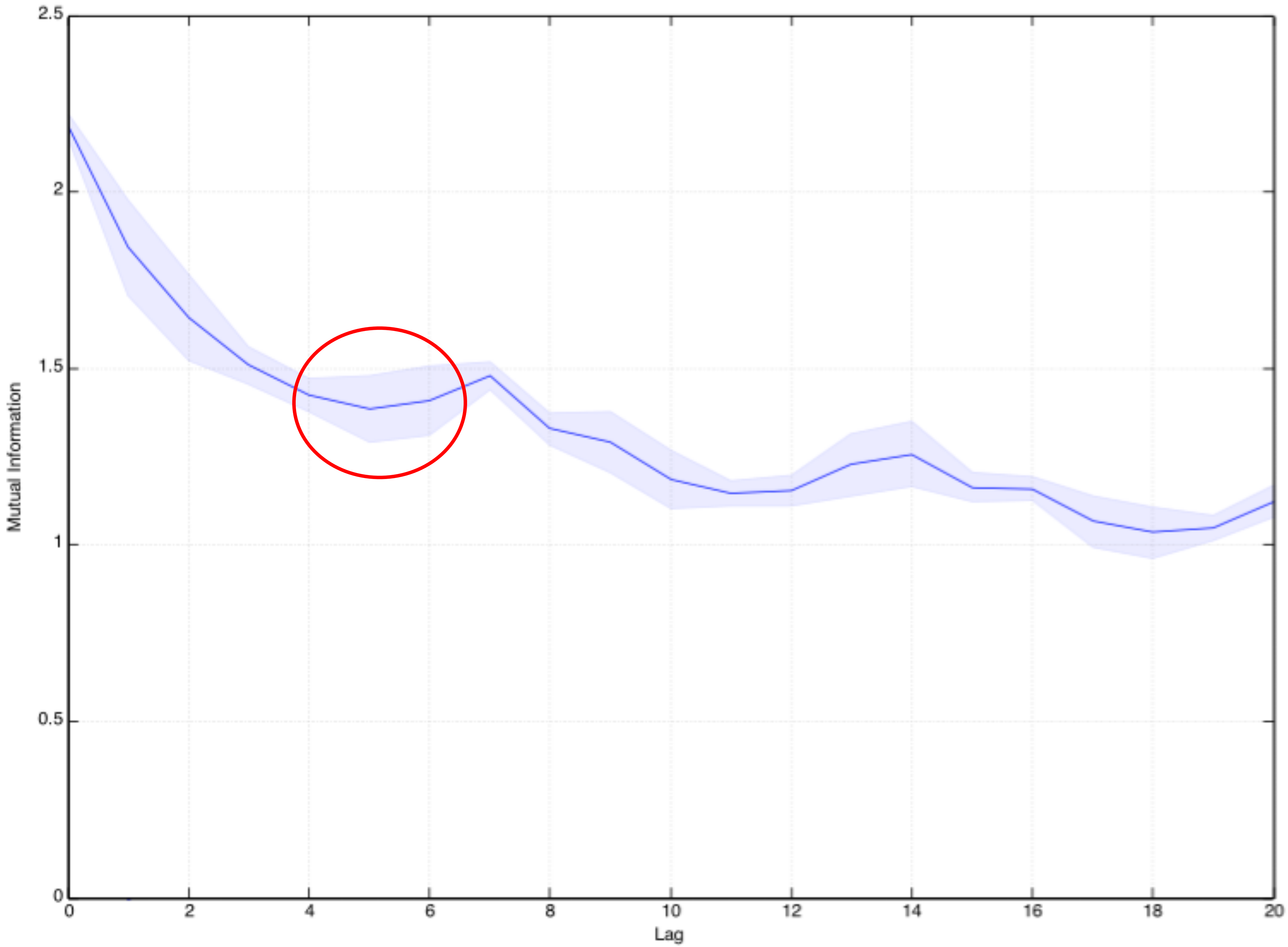


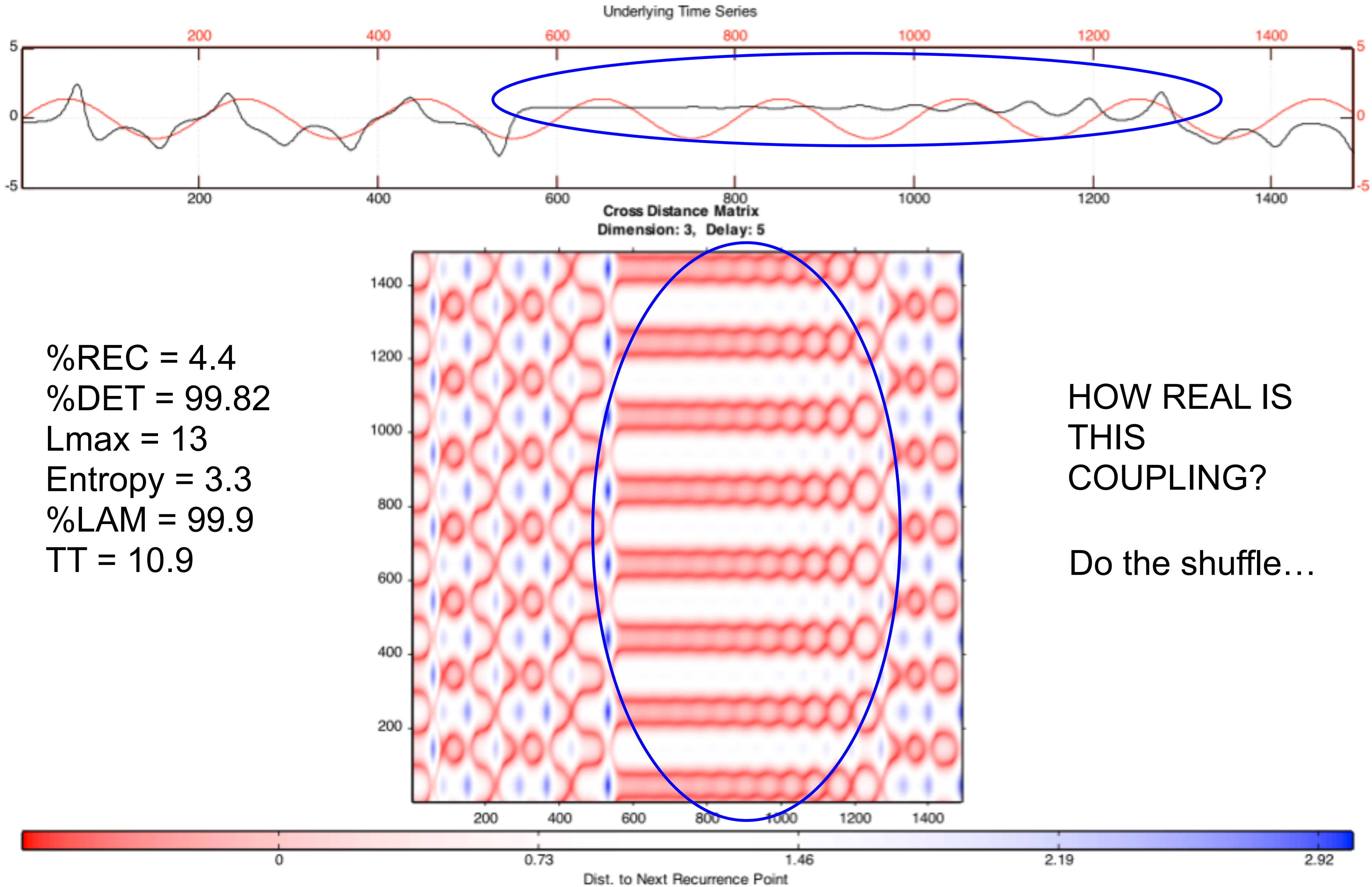


Intuitive notion of synchronisation – Cross Correlation



Lorenz and Spiral – Mutual Information

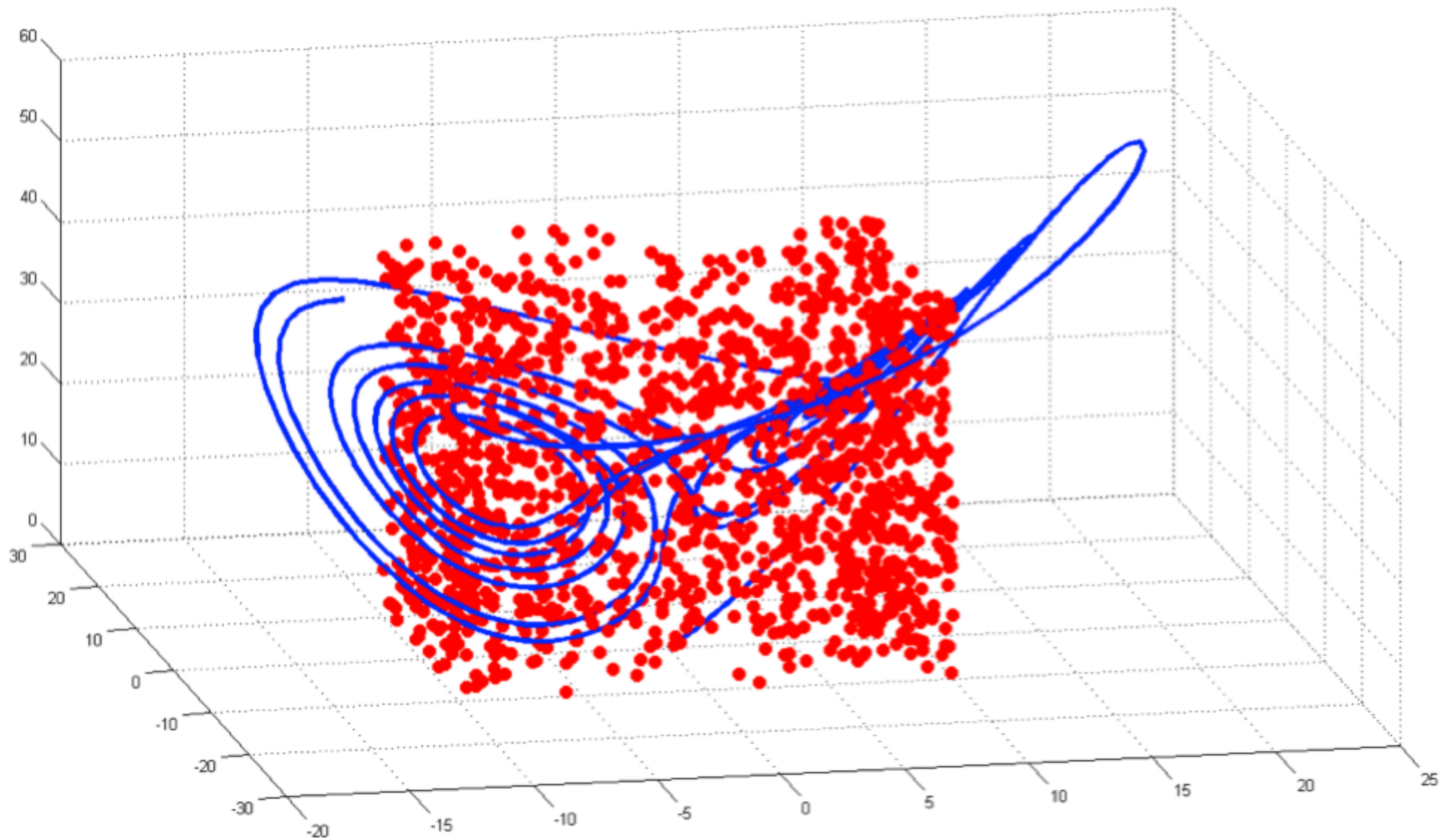


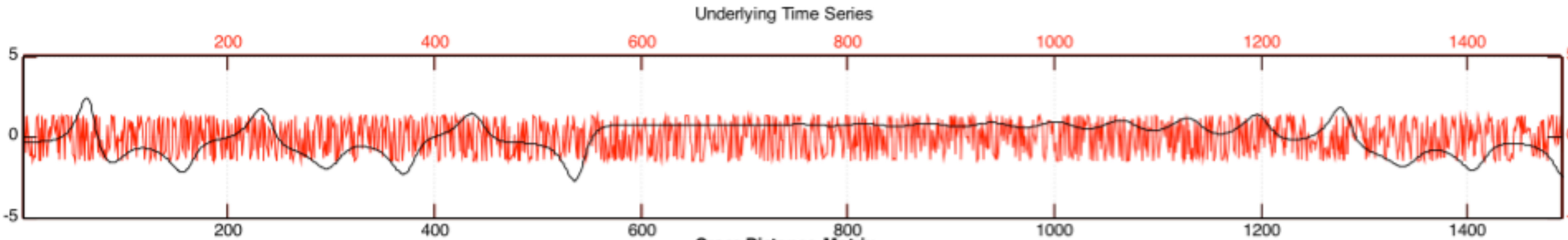


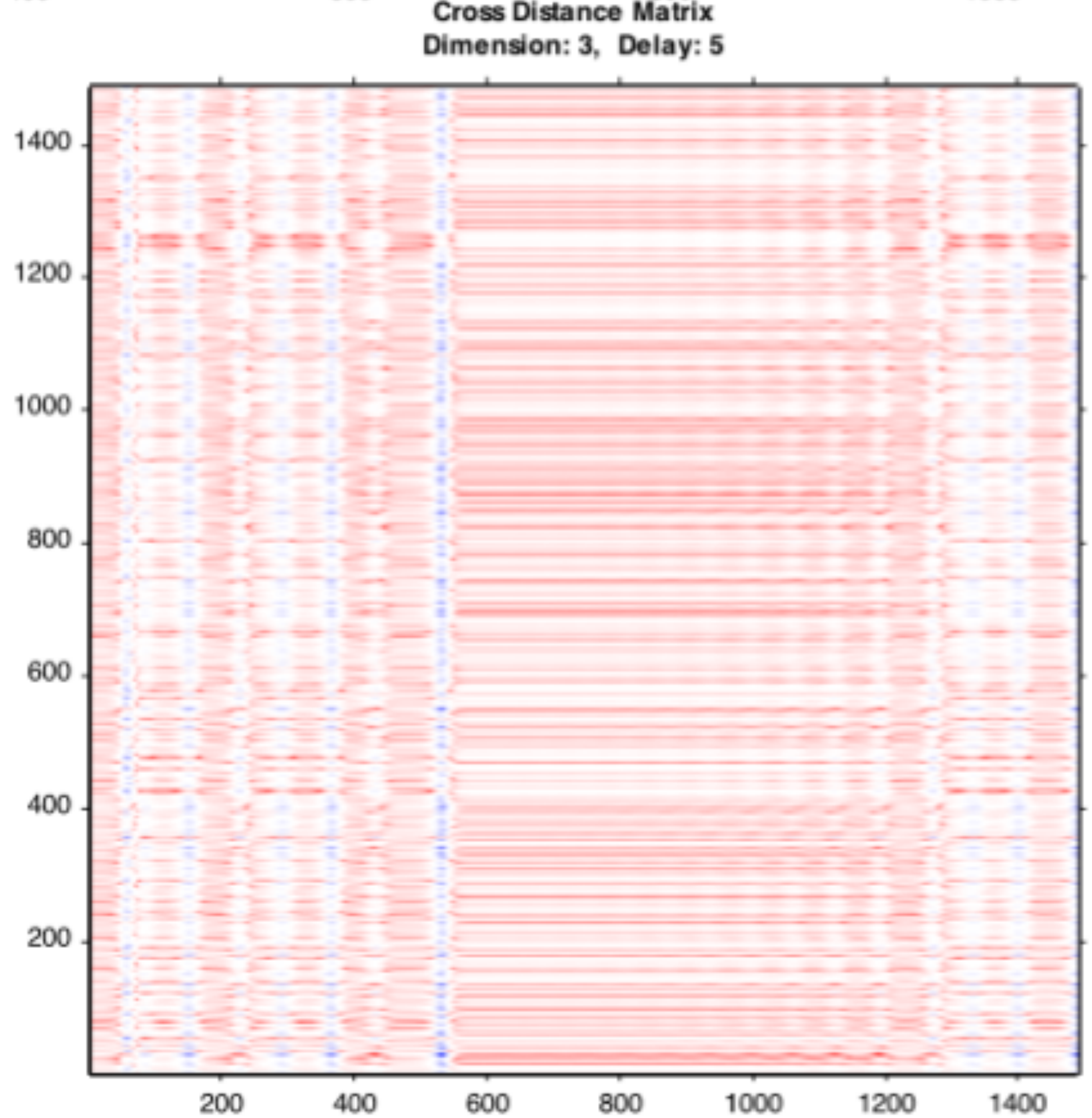
%REC = 4.4
%DET = 99.82
Lmax = 13
Entropy = 3.3
%LAM = 99.9
TT = 10.9

HOW REAL IS THIS COUPLING?

Do the shuffle…


60
50
40
30
20
10
0
30
20
10
0
-10
-20
-30
-20
-15
-10
-5
0
5
10
15
20
25





Shuffled:

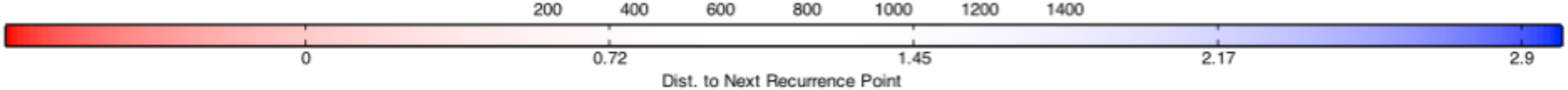
%REC = 4.4
%DET = 8.7
Lmax = 2
Entropy = 0.28
%LAM = 8.5
TT = 2.1

Threshold: 0.745

Unshuffled:

%REC = 4.4
%DET = 99.82
Lmax = 13
Entropy = 3.3
%LAM = 99.9
TT = 10.9

Threshold: 0.5



Data considerations
(Using just the unit scale will usually do)

- Decide on **max** or **mean** scaling for both streams:
- Max distance -> unit scale $\mathbf{X}_{unit} = (\mathbf{X}\text{-min}(\mathbf{X})) / (\max(\mathbf{X})\text{-min}(\mathbf{X}))$

  Scale of 0-1.
  Calculate *[max mean]=pss(*$\mathbf{X}_{unit}$*,m,d)* and divide by max
  In Marwans toolbox always add: *'nonormalize' to crqa / crp*

- Mean distance -> z-score $\mathbf{X}_{z} = (\mathbf{X}\text{-mean}(\mathbf{X})) / \text{std}(\mathbf{X})$

  Z-score scale.

  Calculate *[max mean]=pss(*$\mathbf{X}_{unit}$*,m,d)* and divide by mean
  In Marwans toolbox add: *'nonormalize' to crqa / crp*

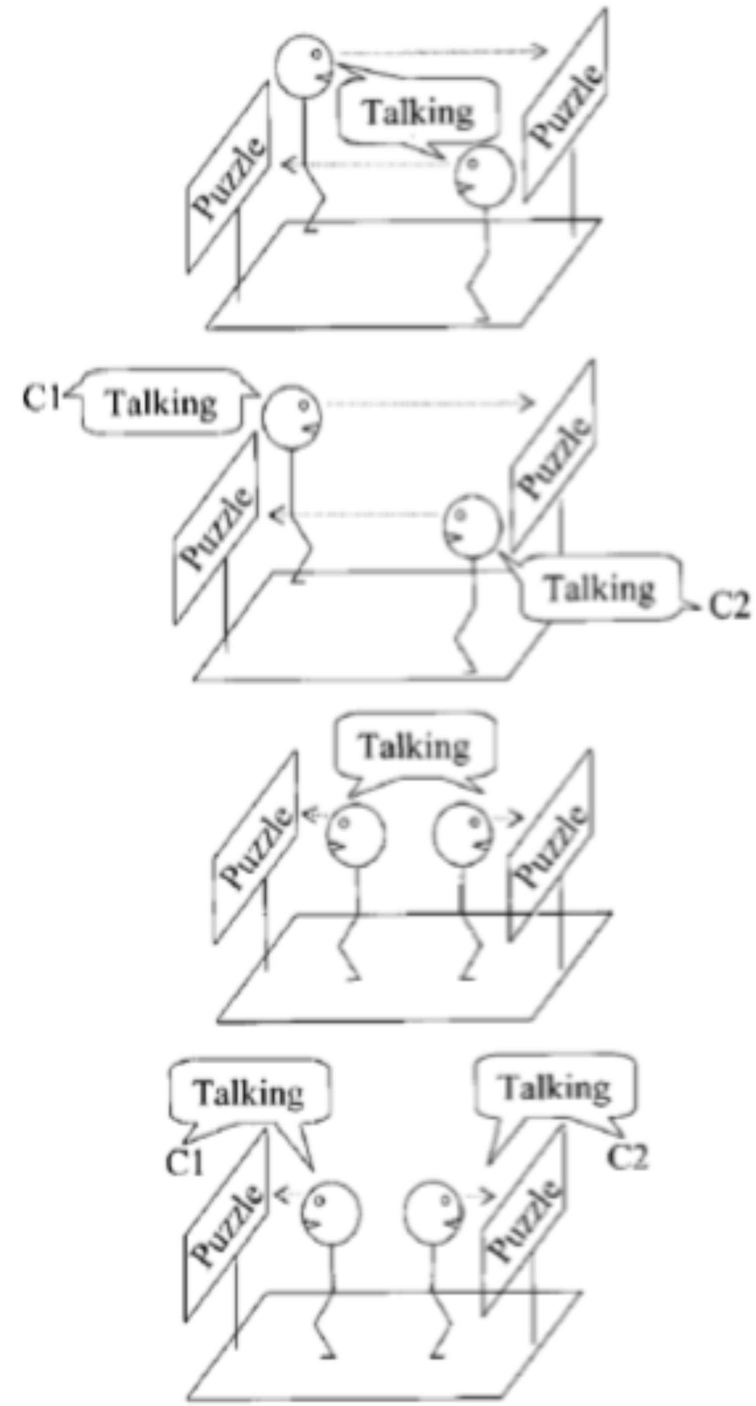
Some Applications

- Coupling of postural sway through communication
- Coupling of language development between infant and caretaker
- Coupling of eye movements to communication

Coupling of postural sway through communication

- Postural sway measured by force plate

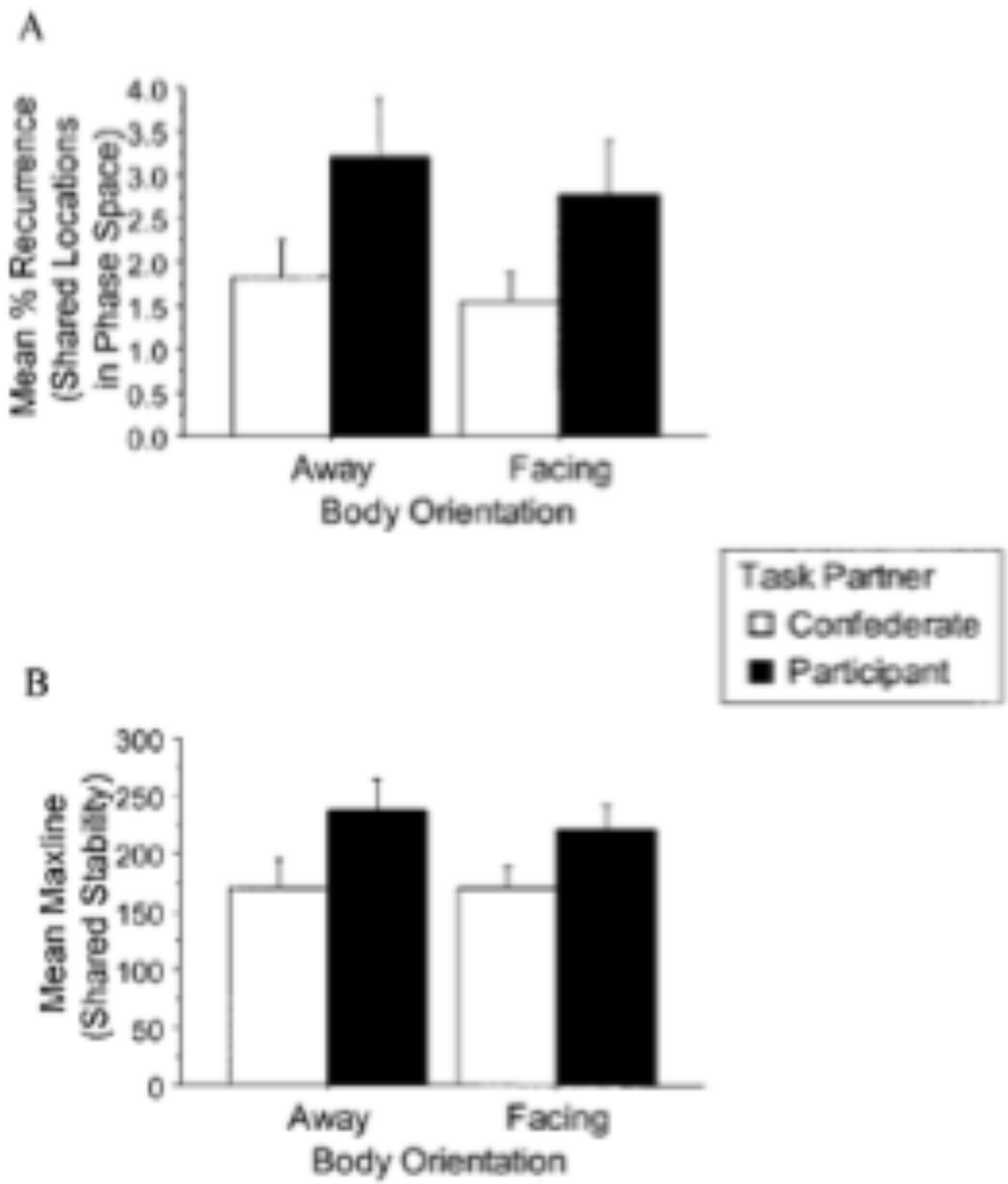
- Level of direct communication manipulated by talking directly or to confederate / visibility



Shockley, K., Santana, M-V., Fowler, C. (2003). Mutual Interpersonal Postural Constraints Are Involved in Cooperative Conversation. *Journal of Experimental Psychology: Human Perception and Performance, 29*, 326-323.

Coupling of postural sway through communication

Speech can be a “coupling tool” for coordination of previously autonomous bodies



Shockley, K., Santana, M-V., Fowler, C. (2003). Mutual Interpersonal Postural Constraints Are Involved in Cooperative Conversation. *Journal of Experimental Psychology: Human Perception and Performance, 29*, 326-323.

Coupling of language development between infant and caretaker

Dale, R., & Spivey, M.J. (2006). Unraveling the dyad: Using recurrence analysis to explore patterns of syntactic coordination between children and caregivers in conversation. *Language Learning, 56(3)*, 391–430

Rick Dale has introduced some interesting applications of Recurrence Analysis:

- CRQA on categorical/nominal data
- “LOS”-profile, as a measure of who’s leading and who’s trailing

Categorical (C)RQA:

- The RP’s of the poems are an example of recurrence plots on categorical data. The recurring values represent an arbitrary category.

- Dale examined transcriptions of conversations between children and caregivers (CHILDES). The unit of analysis was syntactic structure

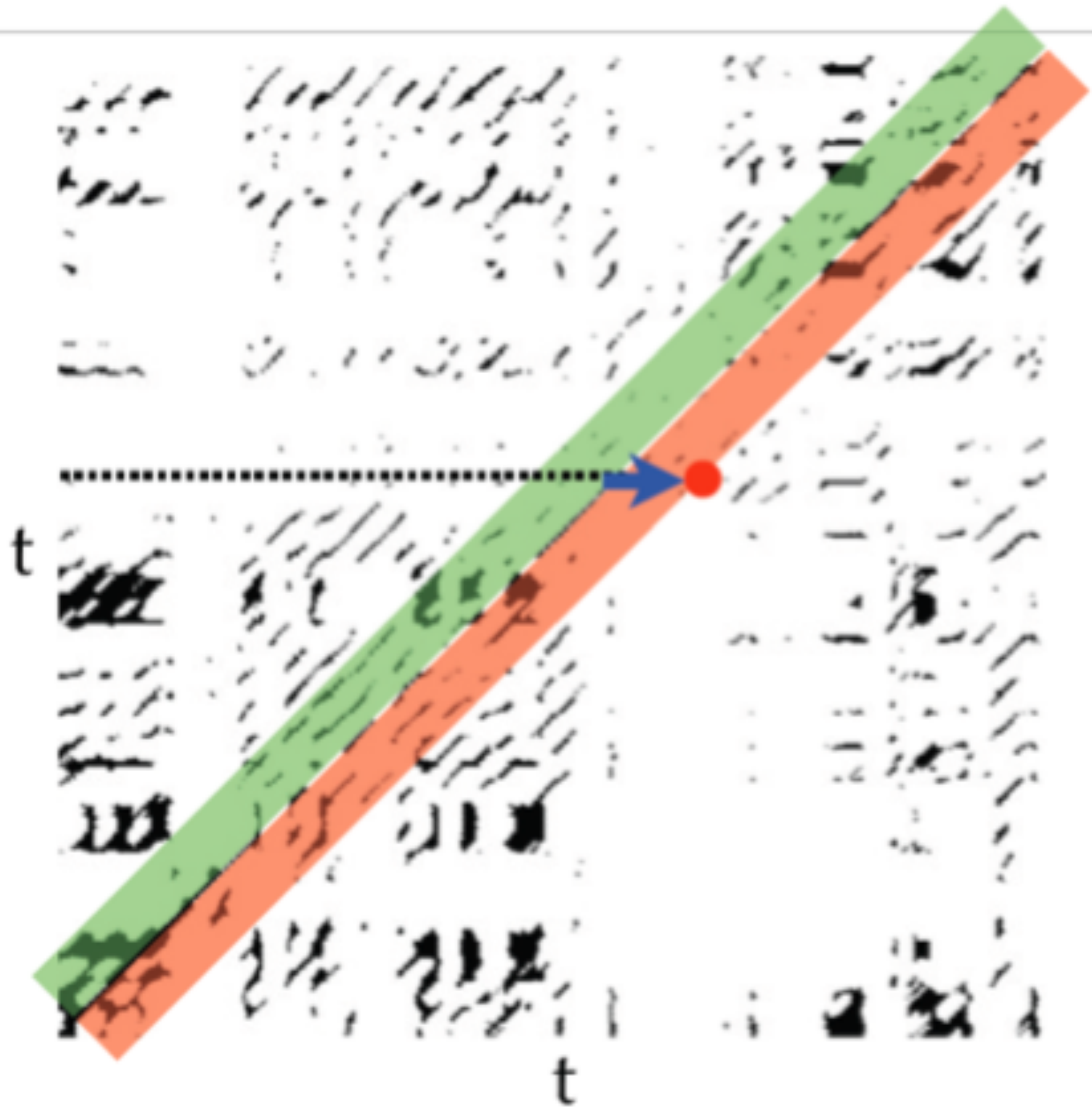
The RQA parameters become extremely simple, no need for estimation:

Lag = 1, Embedding = 1, Threshold / Radius = 0

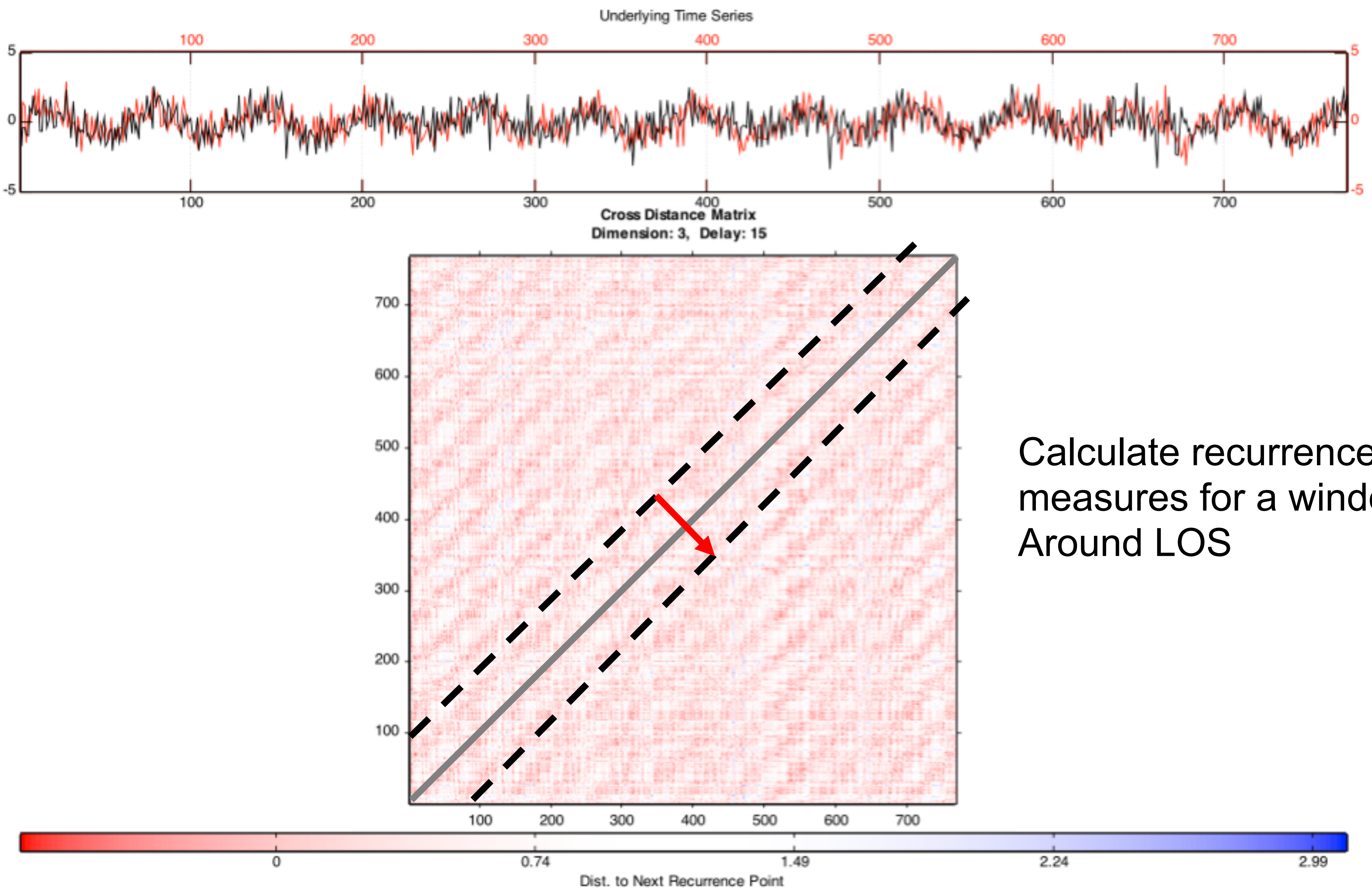
Who leads?

Time Series
On Y-axis
leads at red dot:

The category- / word- /
syntactic- / pattern first
occurred there,
in the X-axis series
it occurred later

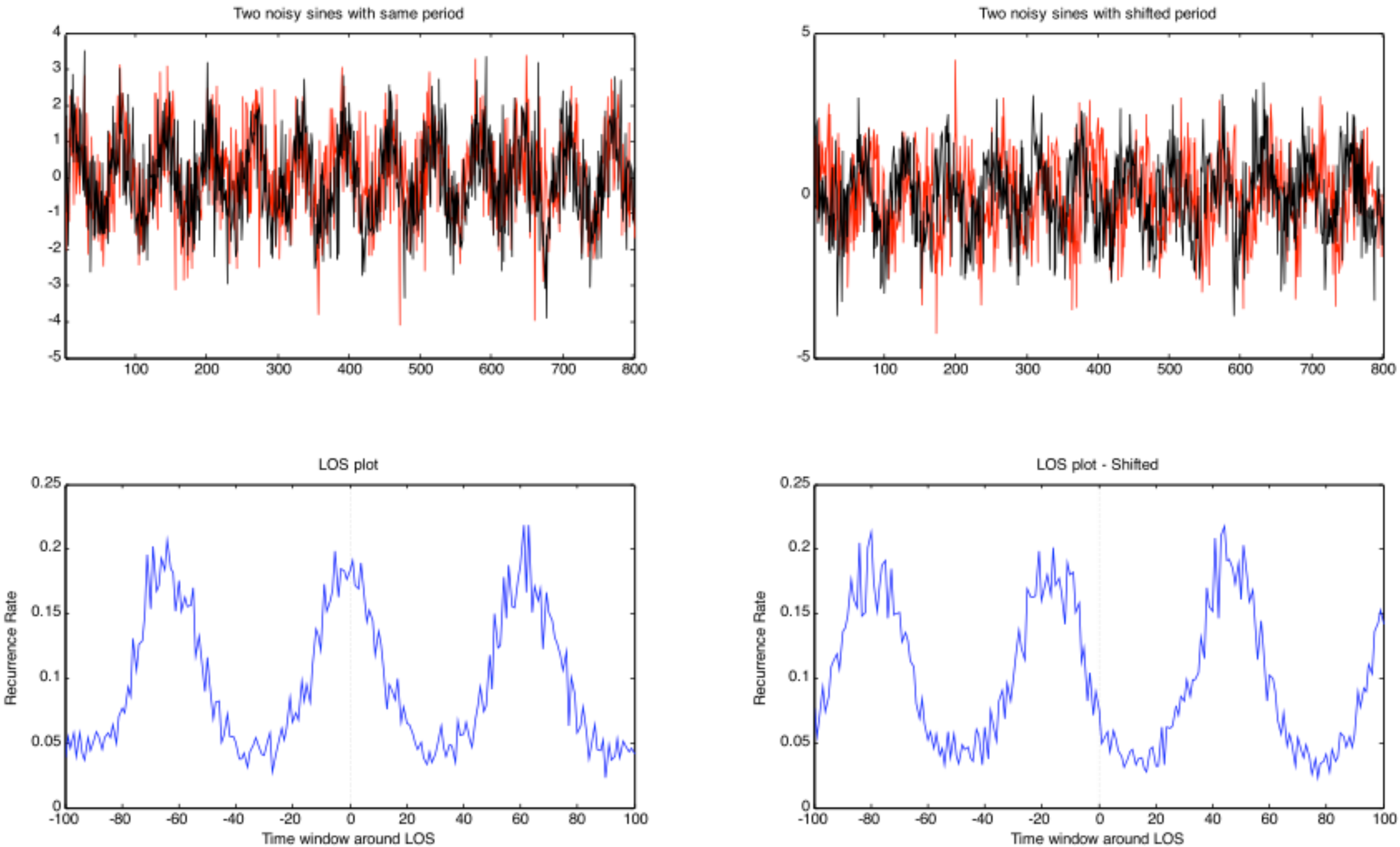


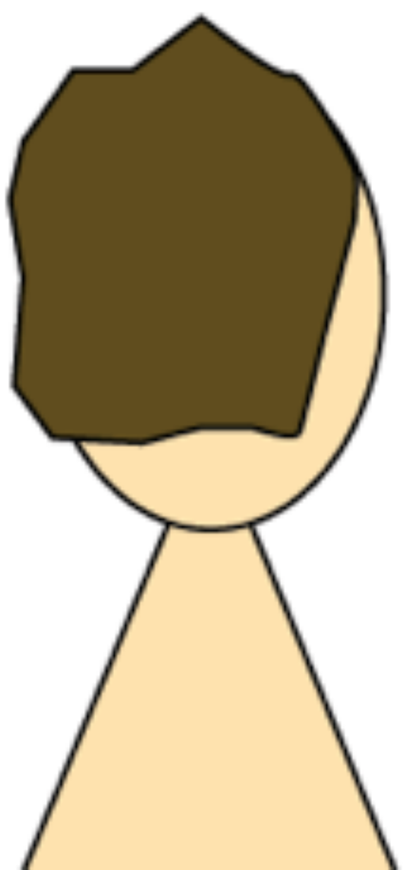
Diagonal Recurrence Profile



Calculate recurrence measures for a window Around LOS

Diagonal Recurrence Profile



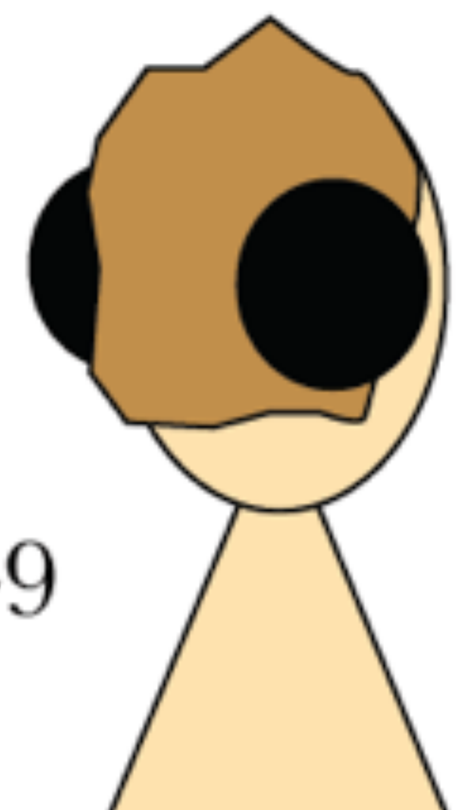


eye movement patterns...

1, 2, 2, 2, 2, 4, 4, 5, ...

“blah blah blah...”

N = 4



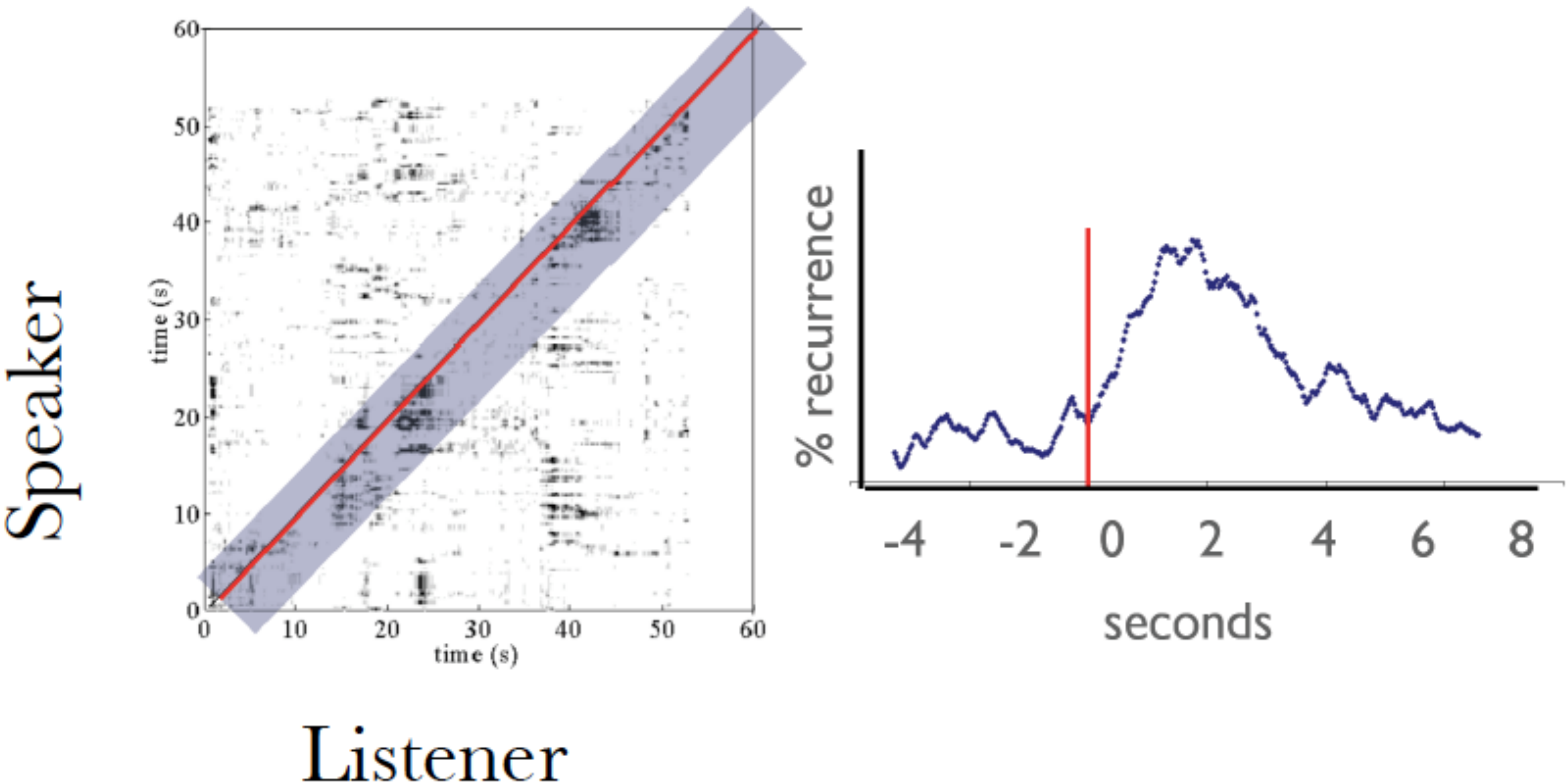
eye movement patterns...
1, 1, 1, 2, 2, 2, 2, 4, ...

"blah blah blah..."

N = 49

Coupling of eye movements to communication

Richardson, D.C., Dale, R., Kirkham, N.Z. (2007). The art of conversation is coordination. *Psychological Science*, *18*, 407-413.

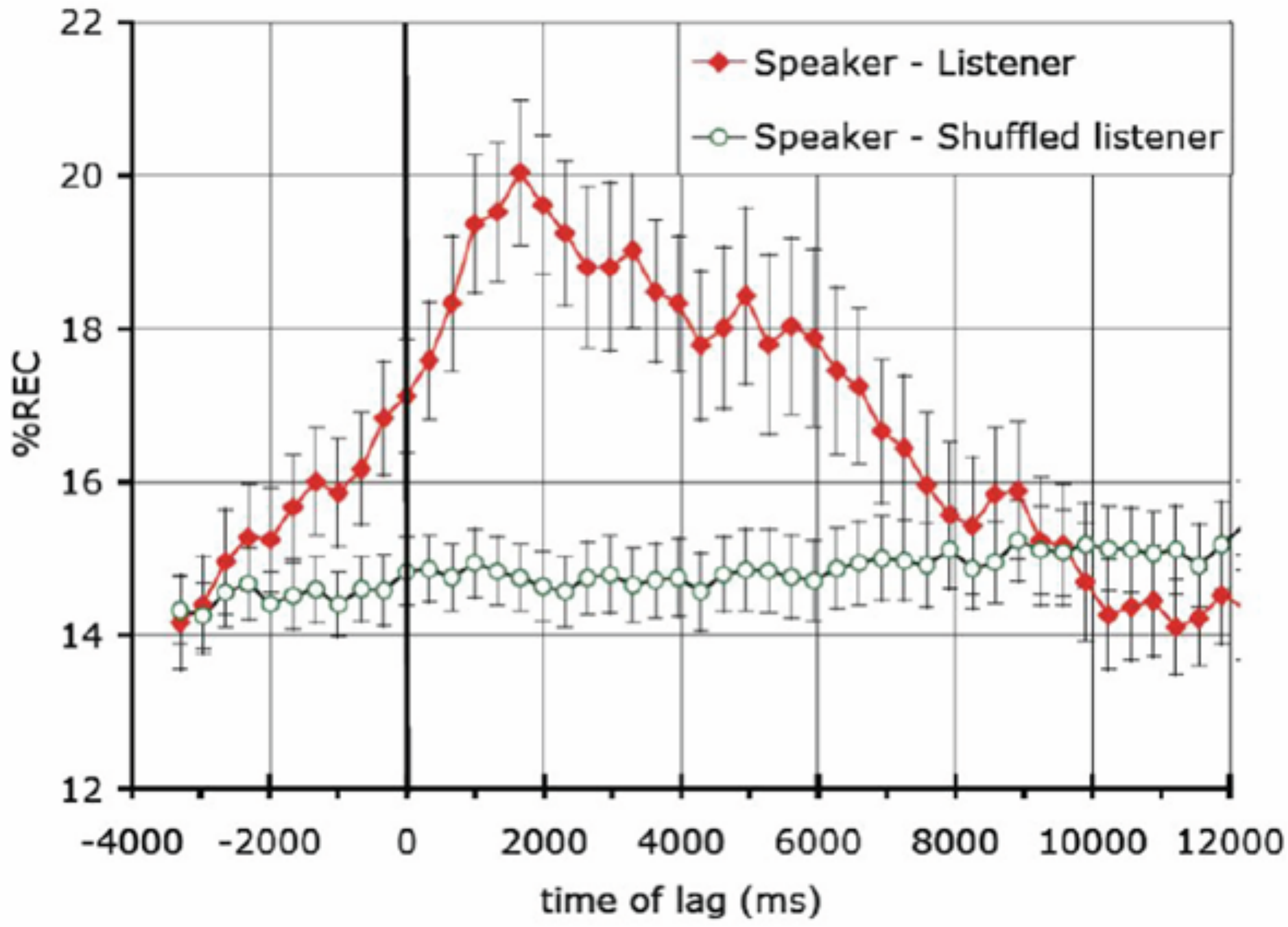


Richardson & Dale, 2005

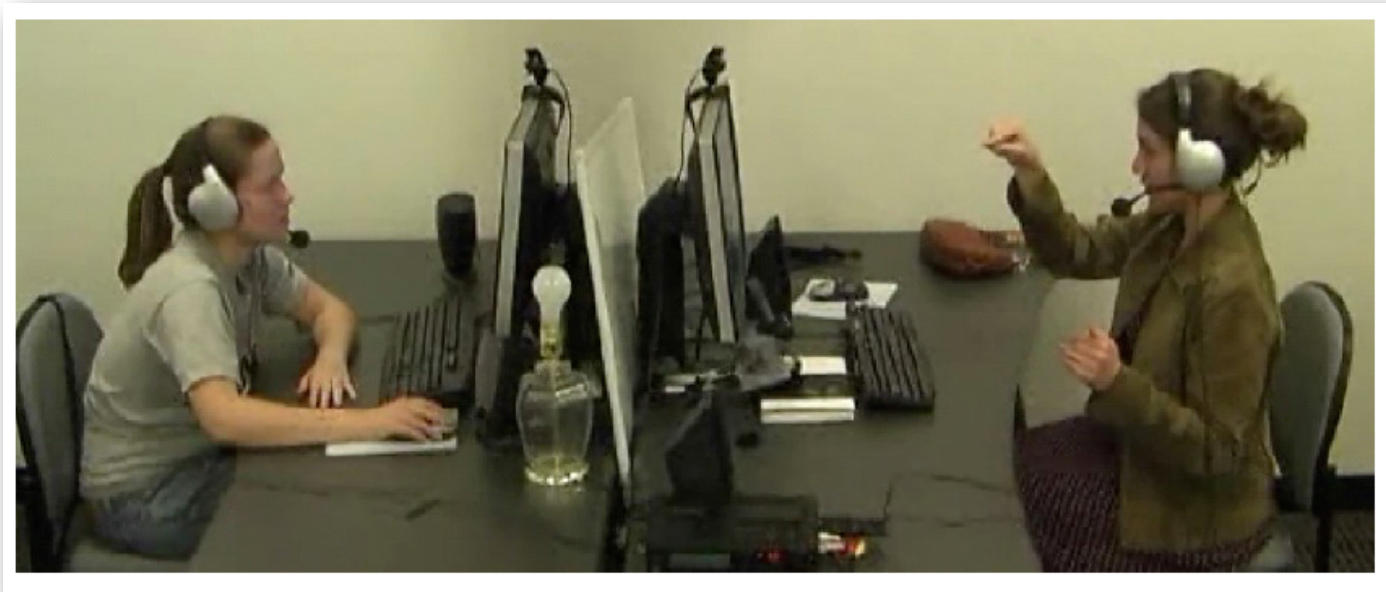
Coupling of eye movements to communication

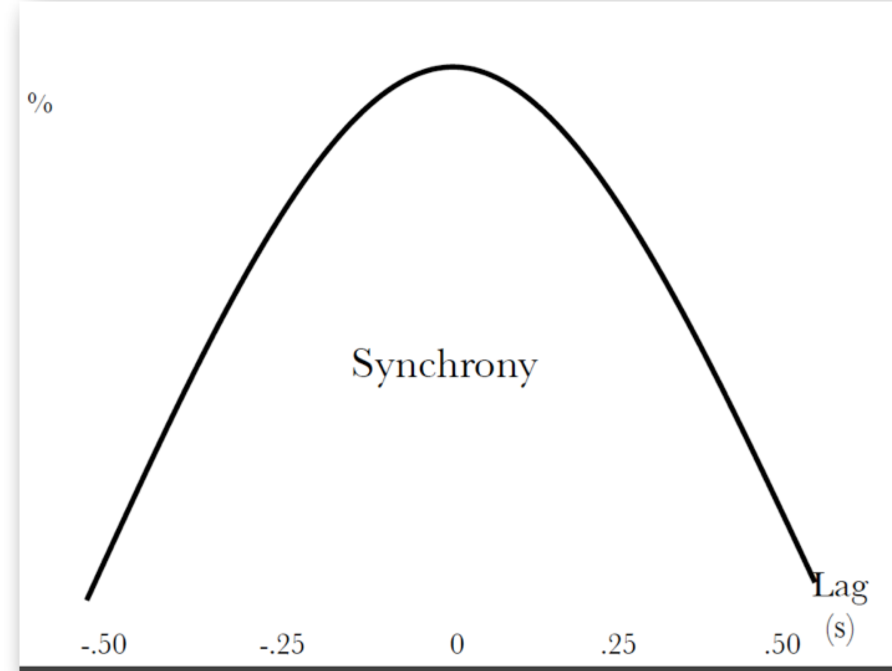
Richardson, D.C., Dale, R., Kirkham, N.Z. (2007). The art of conversation is coordination. *Psychological Science*, *18*, 407-413.

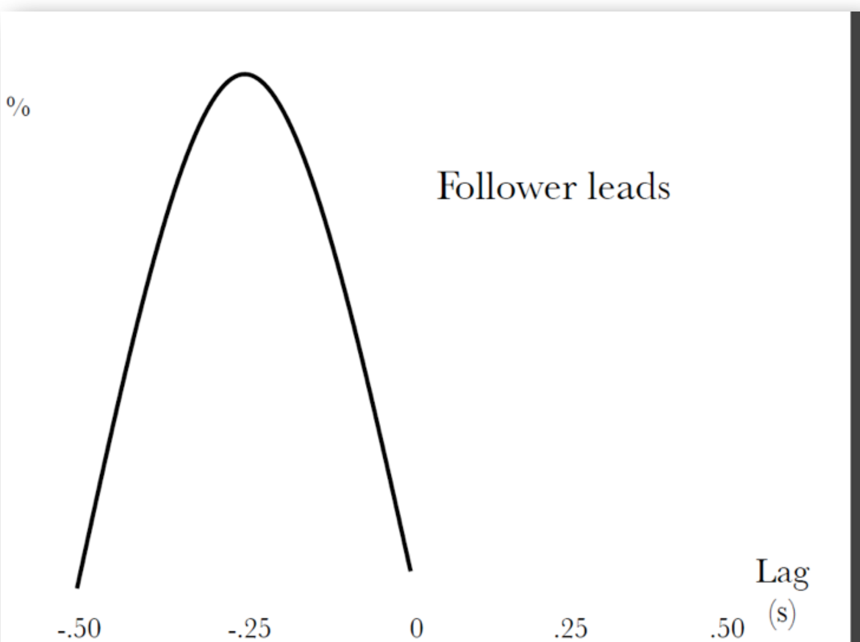
Listeners eye movements are coupled and lagging depending on level of interaction in conversation

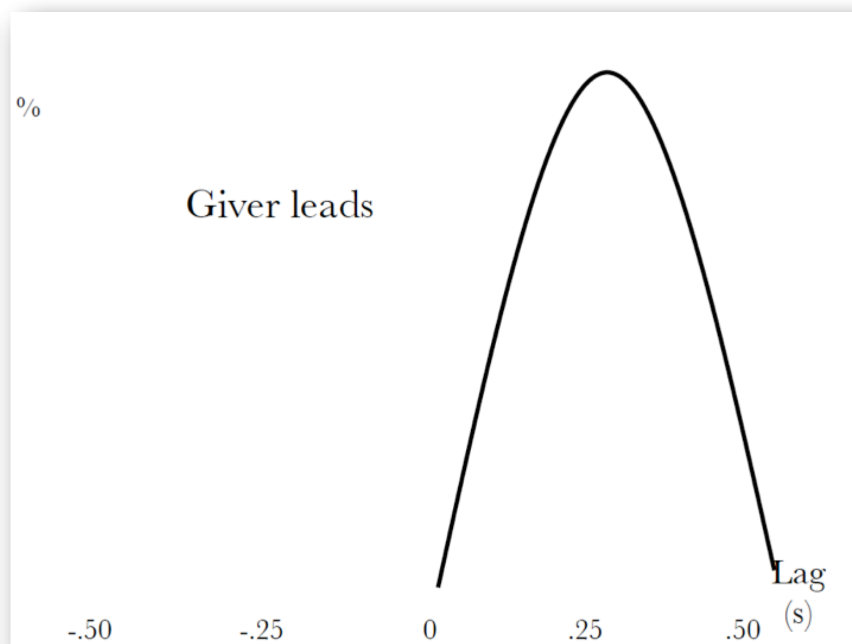


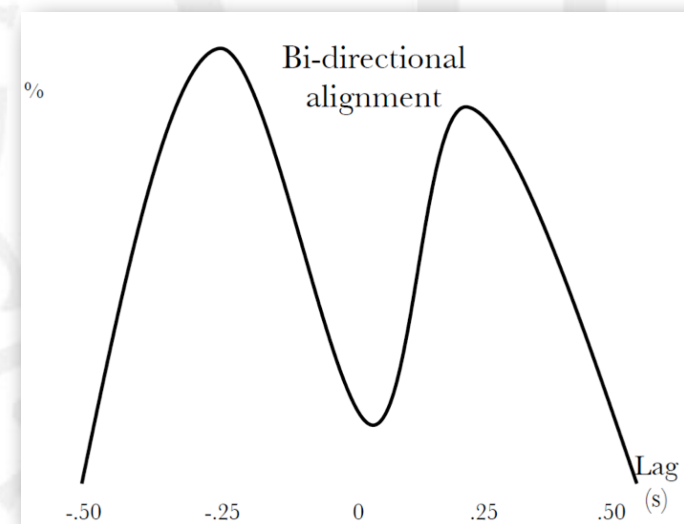
Behavioural Science Institute
Radboud University Nijmegen



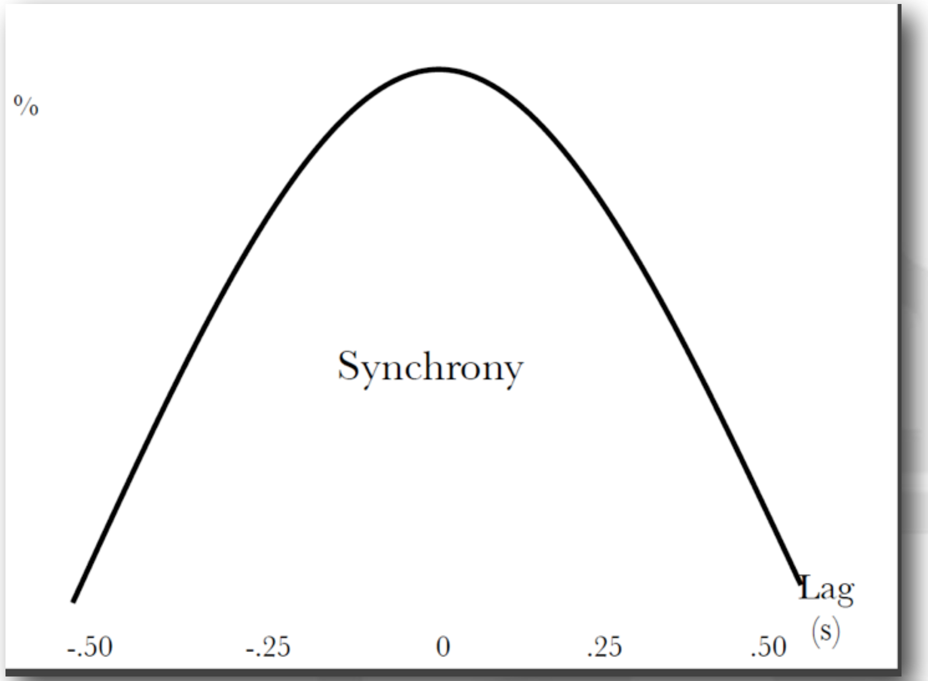


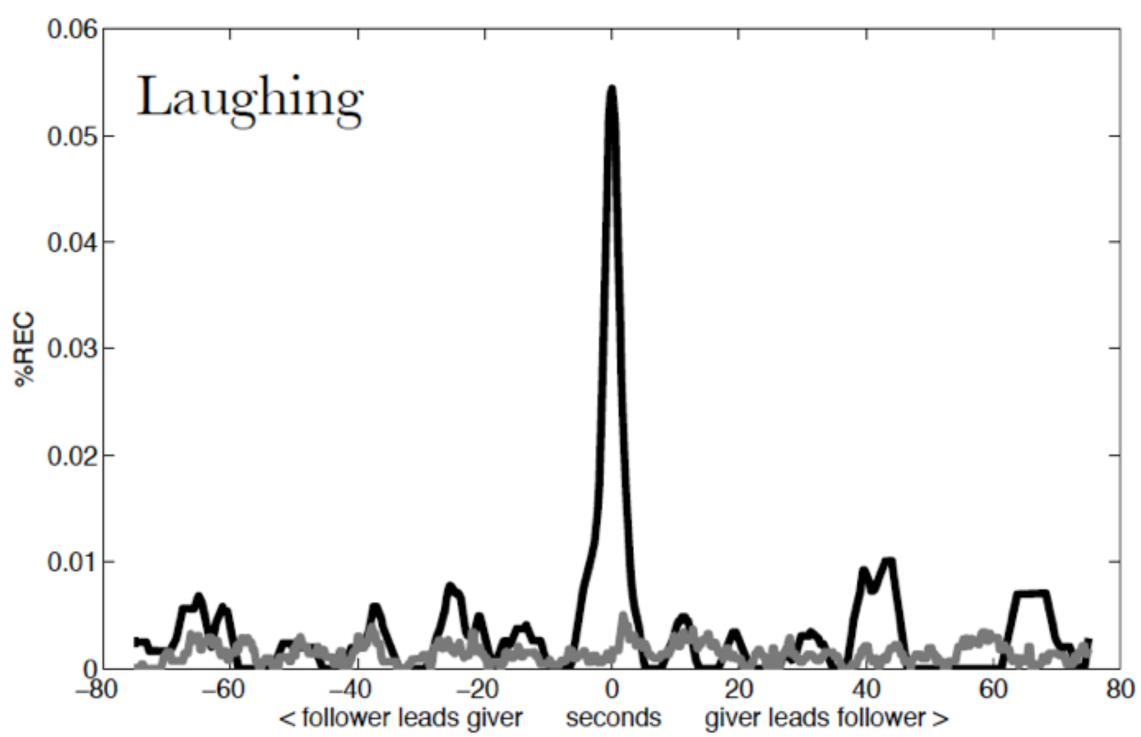






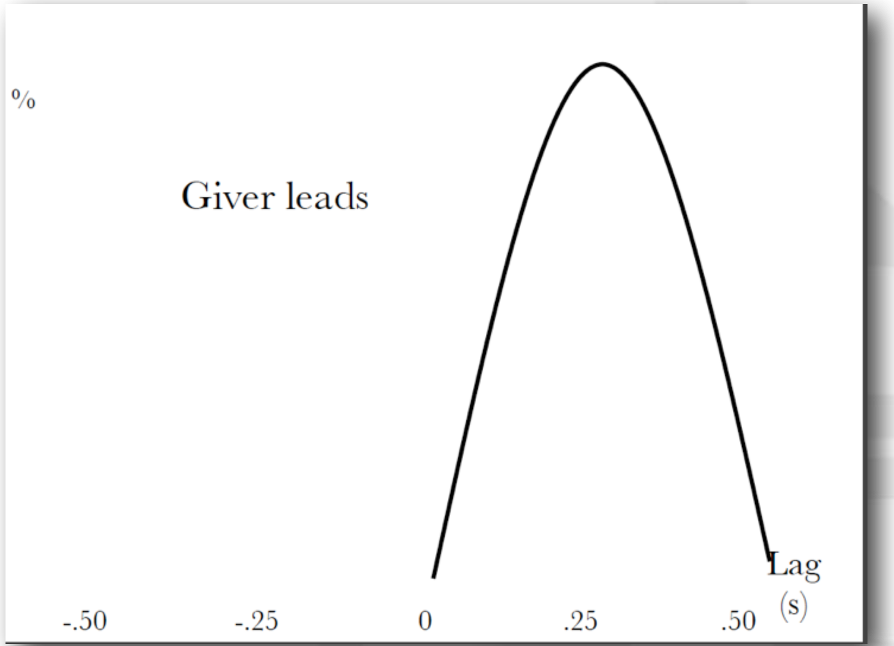
Louwerse, M. M., Dale, R., Bard, E. G., & Jeuniaux, P. (2012). Behavior matching in multimodal communication is synchronized. *Cognitive science*, *36*(8), 1404–26. doi:10.1111/j.1551-6709.2012.01269.x

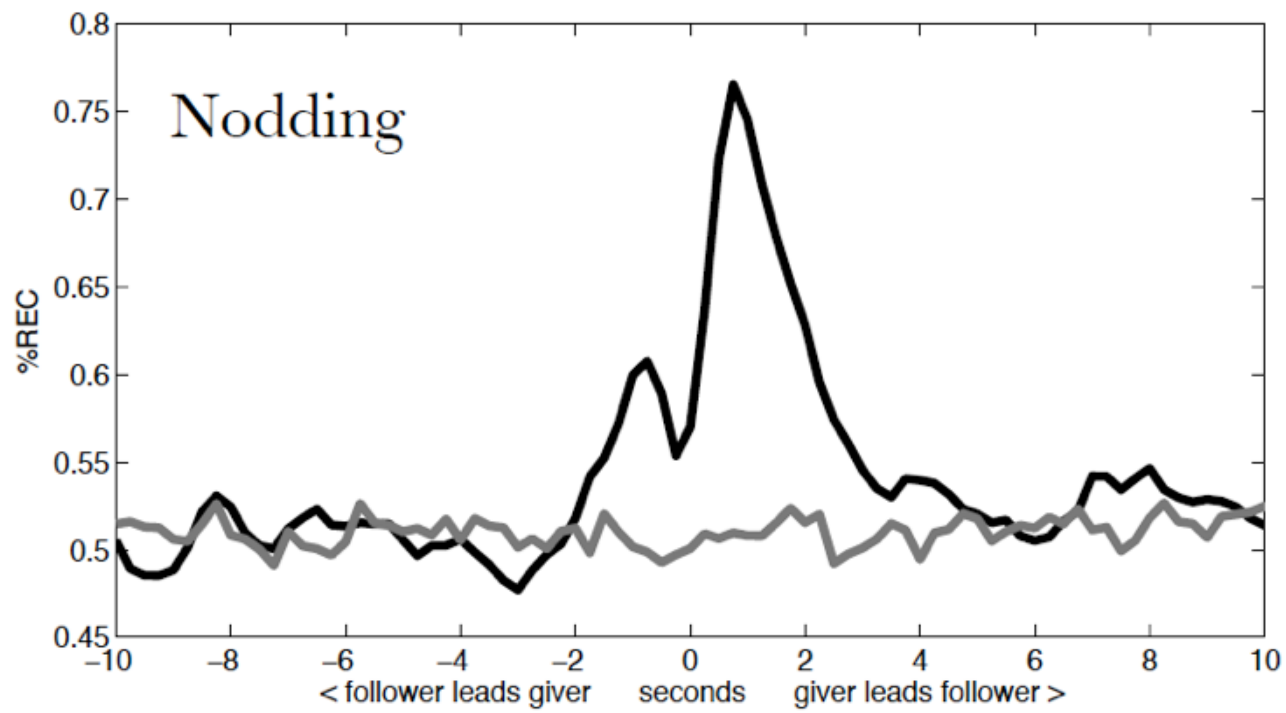




Louwerse, Dale, et al., in prep

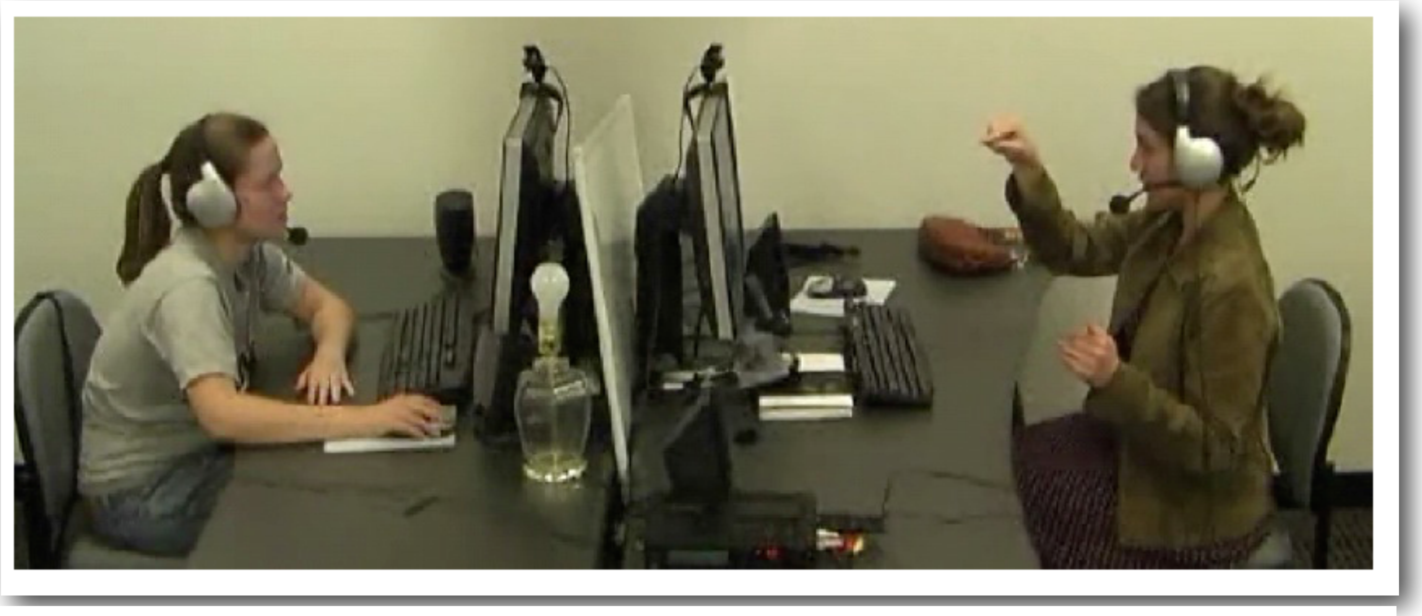
Louwerse, M. M., Dale, R., Bard, E. G., & Jeuniaux, P. (2012). Behavior matching in multimodal communication is synchronized. *Cognitive science*, *36*(8), 1404–26. doi:10.1111/j.1551-6709.2012.01269.x

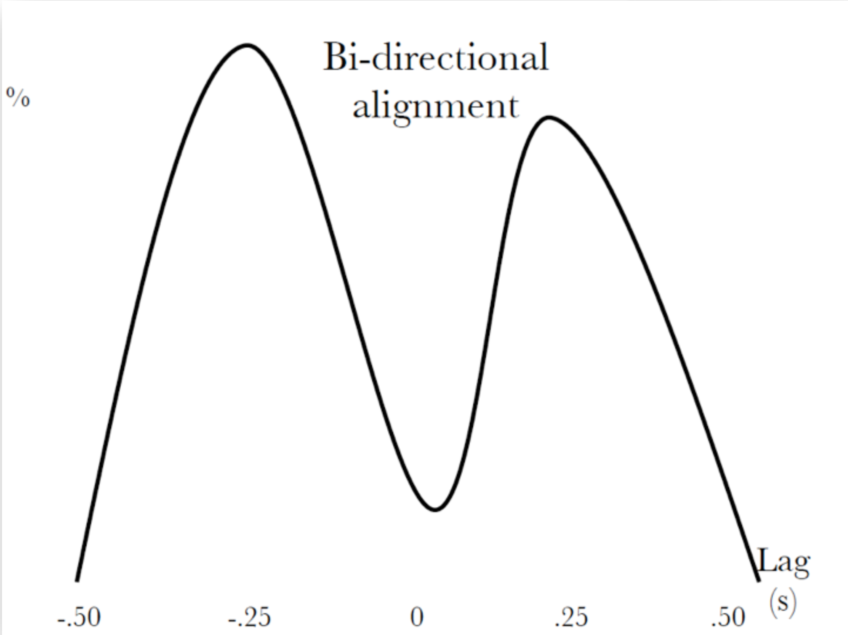


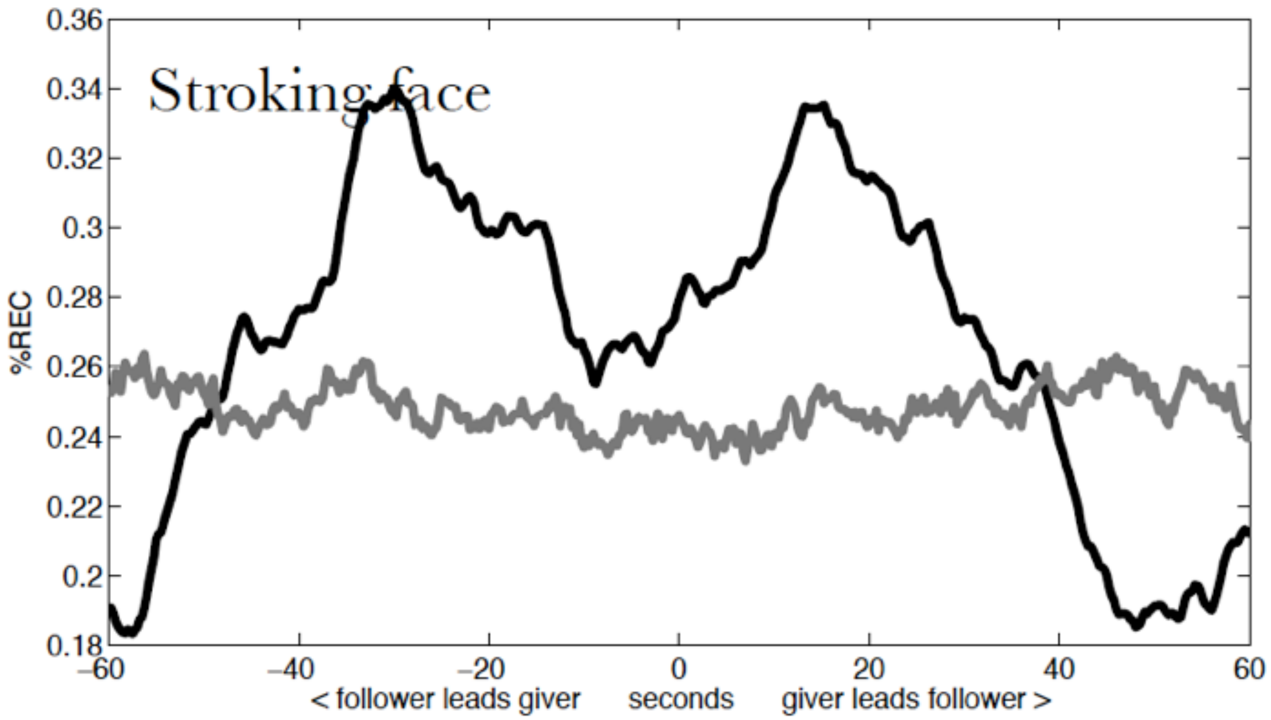


Louwerse, Dale, et al., in prep

Louwerse, M. M., Dale, R., Bard, E. G., & Jeuniaux, P. (2012). Behavior matching in multimodal communication is synchronized. *Cognitive science*, *36*(8), 1404–26. doi:10.1111/j.1551-6709.2012.01269.x

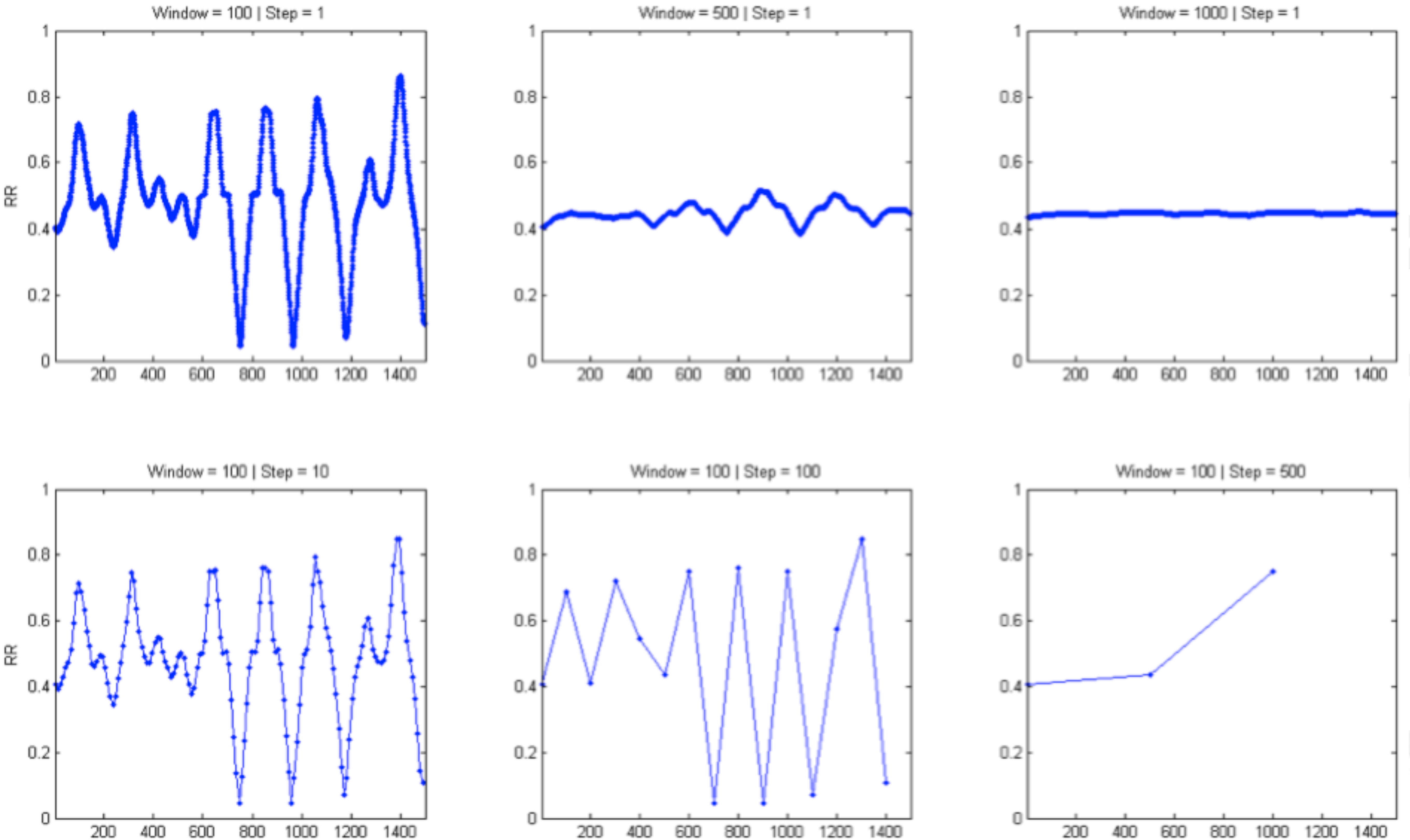




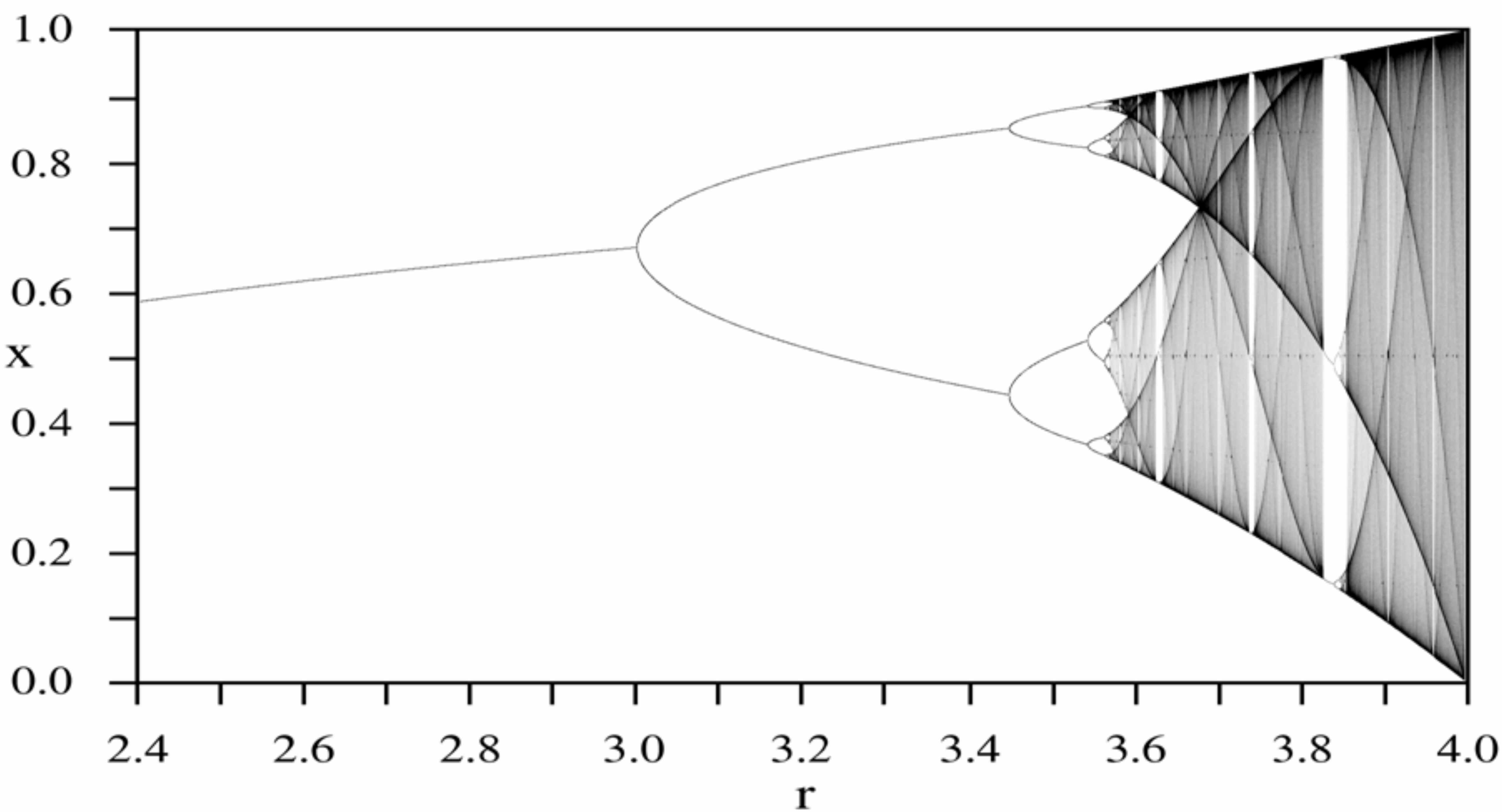


Louwerse, Dale, et al., in prep

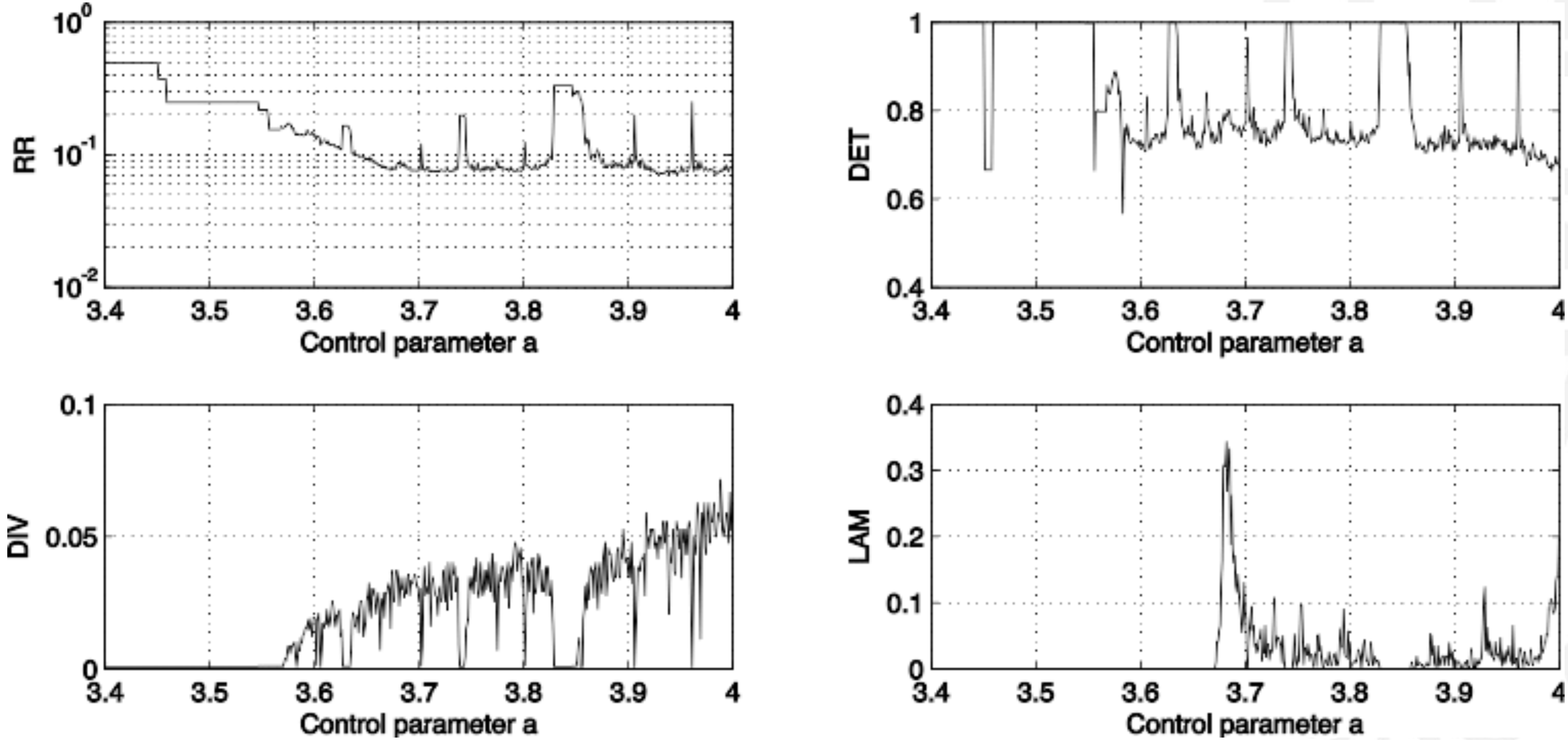
Louwerse, M. M., Dale, R., Bard, E. G., & Jeuniaux, P. (2012). Behavior matching in multimodal communication is synchronized. *Cognitive science*, *36*(8), 1404–26. doi:10.1111/j.1551-6709.2012.01269.x


Window = 100 | Step = 1
Window = 500 | Step = 1
Window = 1000 | Step = 1
Window = 100 | Step = 10
Window = 100 | Step = 100
Window = 100 | Step = 500
RR
0
0.2
0.4
0.6
0.8
1
200
400
600
800
1000
1200
1400

Logistic map – Transitions revealed by lagged RQA



Logistic map – Transitions revealed by lagged RQA



Further reading

- The paper by Marwan et al in Physics Reports tells you everything you wanted to know… and more.
- Paper by Rick Dale in psych. science gives a good explanation of CRQA on categorical data

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Table 2
Comparison of RQA measures based on diagonal (DET, L and L_{max}) and vertical structures (LAM, TT and V_{max}) regarding periodic-chaos/chaos–periodic transitions (PC/CP), chaos–chaos transitions (band merging—BM and inner crisis—IC) and laminar states

Measure	PC/CP transitions	BM and IC	Laminar states
DET	Increases	—	—
L	Increases	—	—
L_{max}	Increases	—	—
LAM	Drops to zero	—	Increases
TT	Drops to zero	Increases	Increases
V_{max}	Drops to zero	Increases	Increases

Order Patterns Recurrence Plot

- Sort of “filter”: not recurrences of values, but order patterns

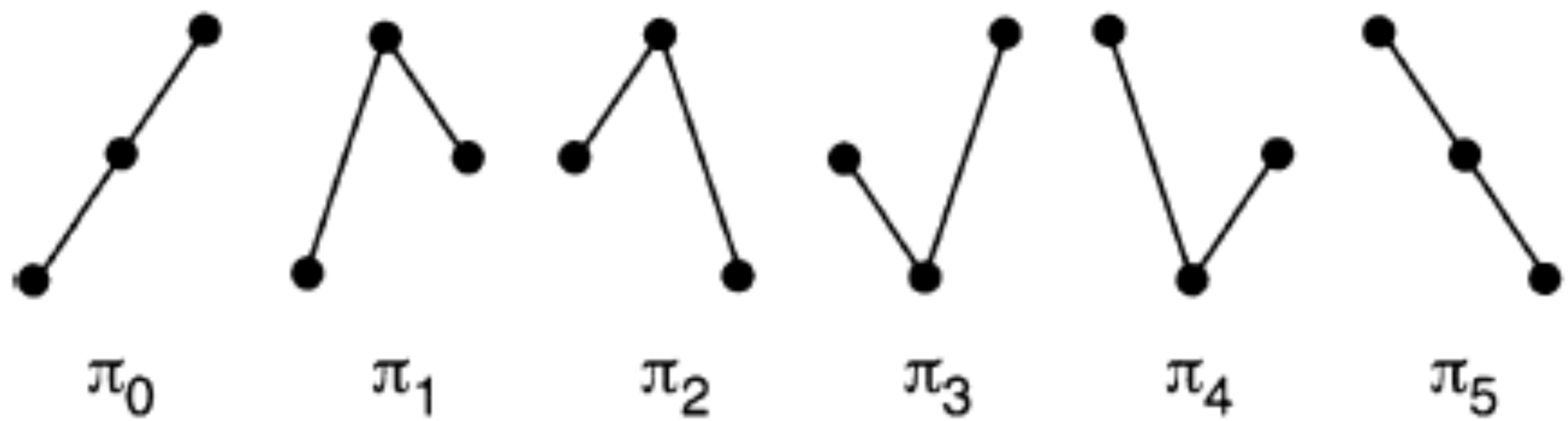
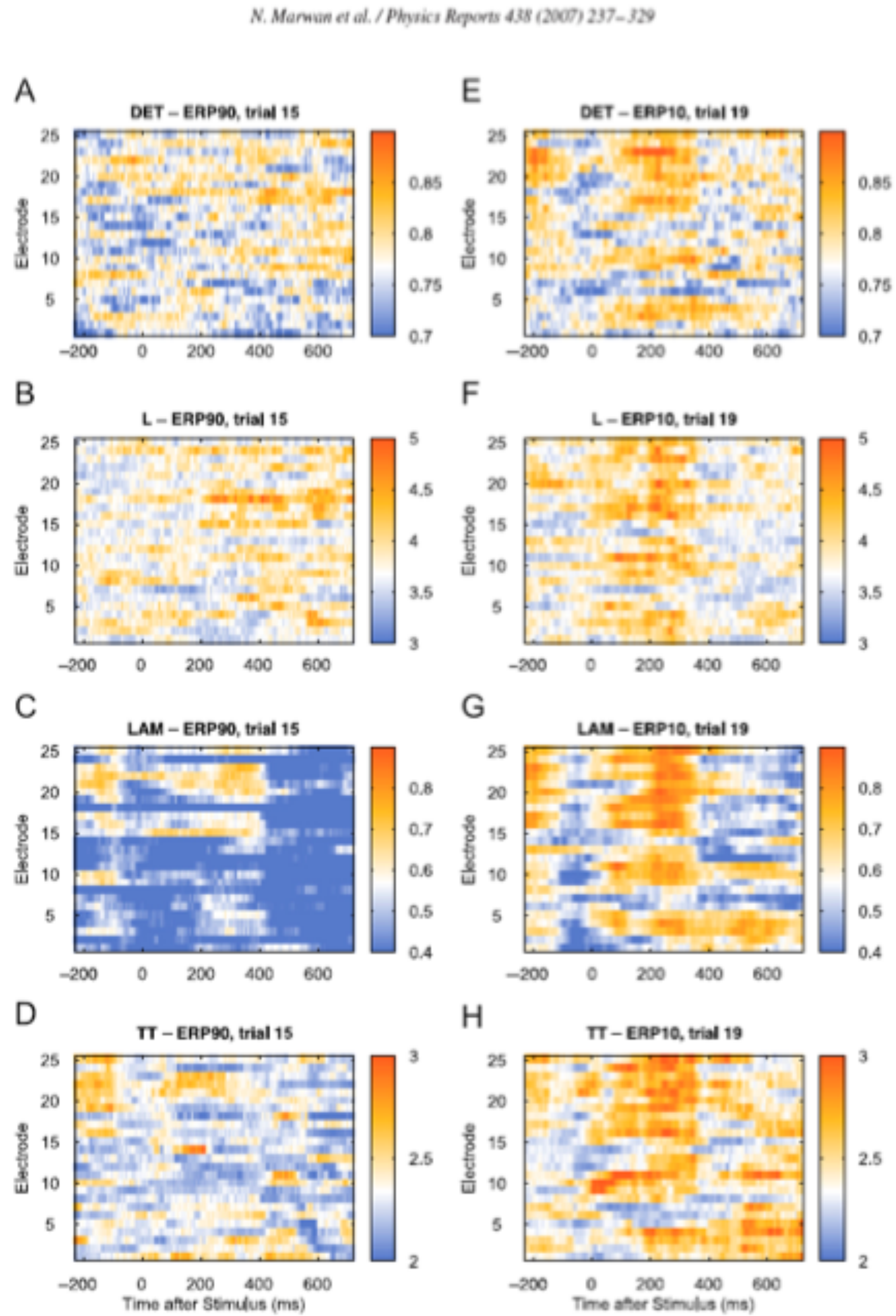


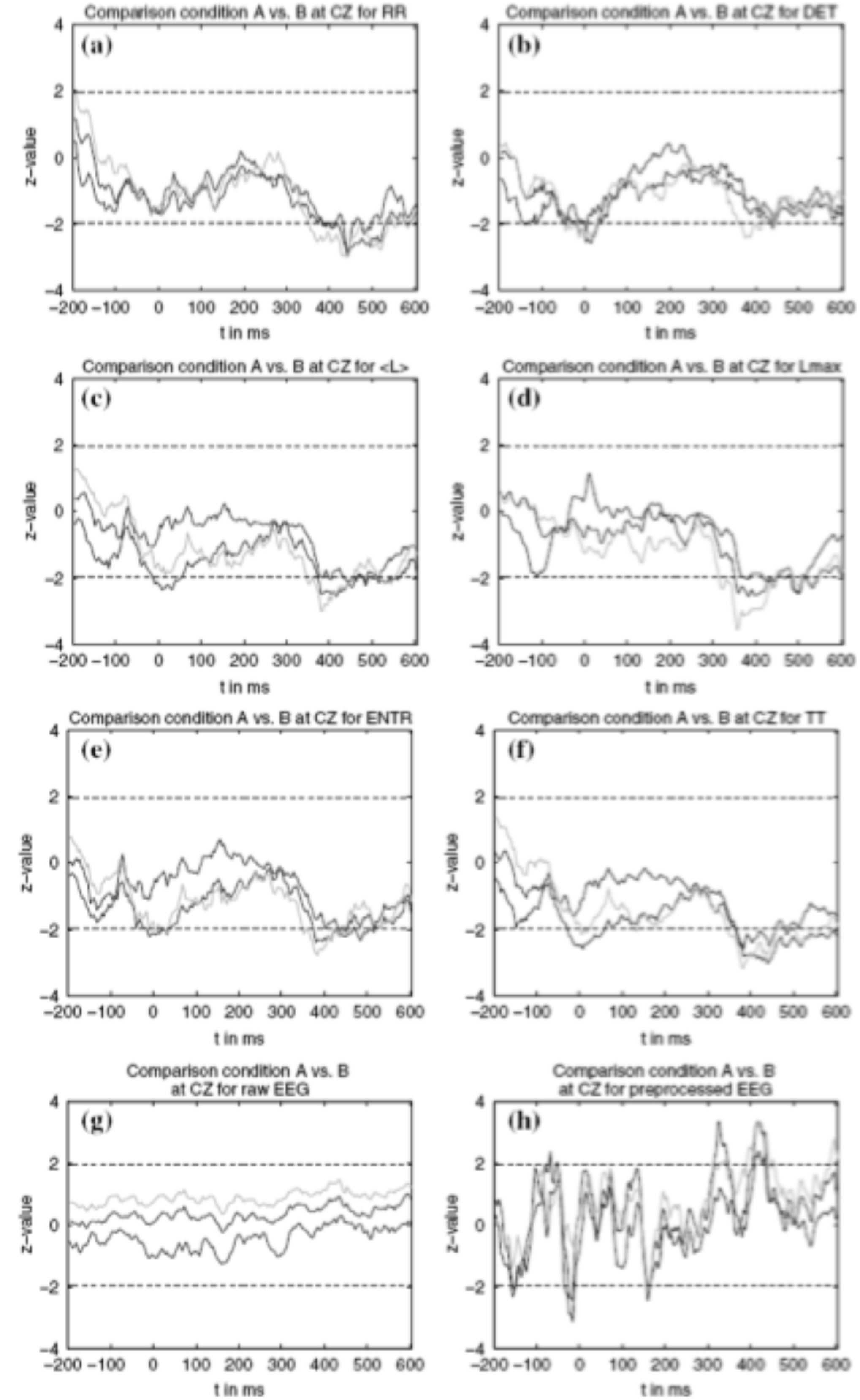
Fig. 2 Order patterns for dimension $d = 3$ (tied ranks $u_i = u_{i+\tau}$ are assumed to be rare)

Order patterns recurrence plots in the analysis of ERP data

Stefan Schinkel · Norbert Marwan · Jürgen Kurths

Schinkel, S., Marwan, N., & Kurths, J. (2007). Order patterns recurrence plots in the analysis of ERP data. *Cognitive neurodynamics*, *1*(4), 317-25. doi:10.1007/s11571-007-9023-z





Schinkel, S., Marwan, N., & Kurths, J. (2009). Brain signal analysis based on recurrences. *Journal of physiology, Paris*, *103*(6), 315-23. Elsevier Ltd. doi:10.1016/j.jphysparis.2009.05.007

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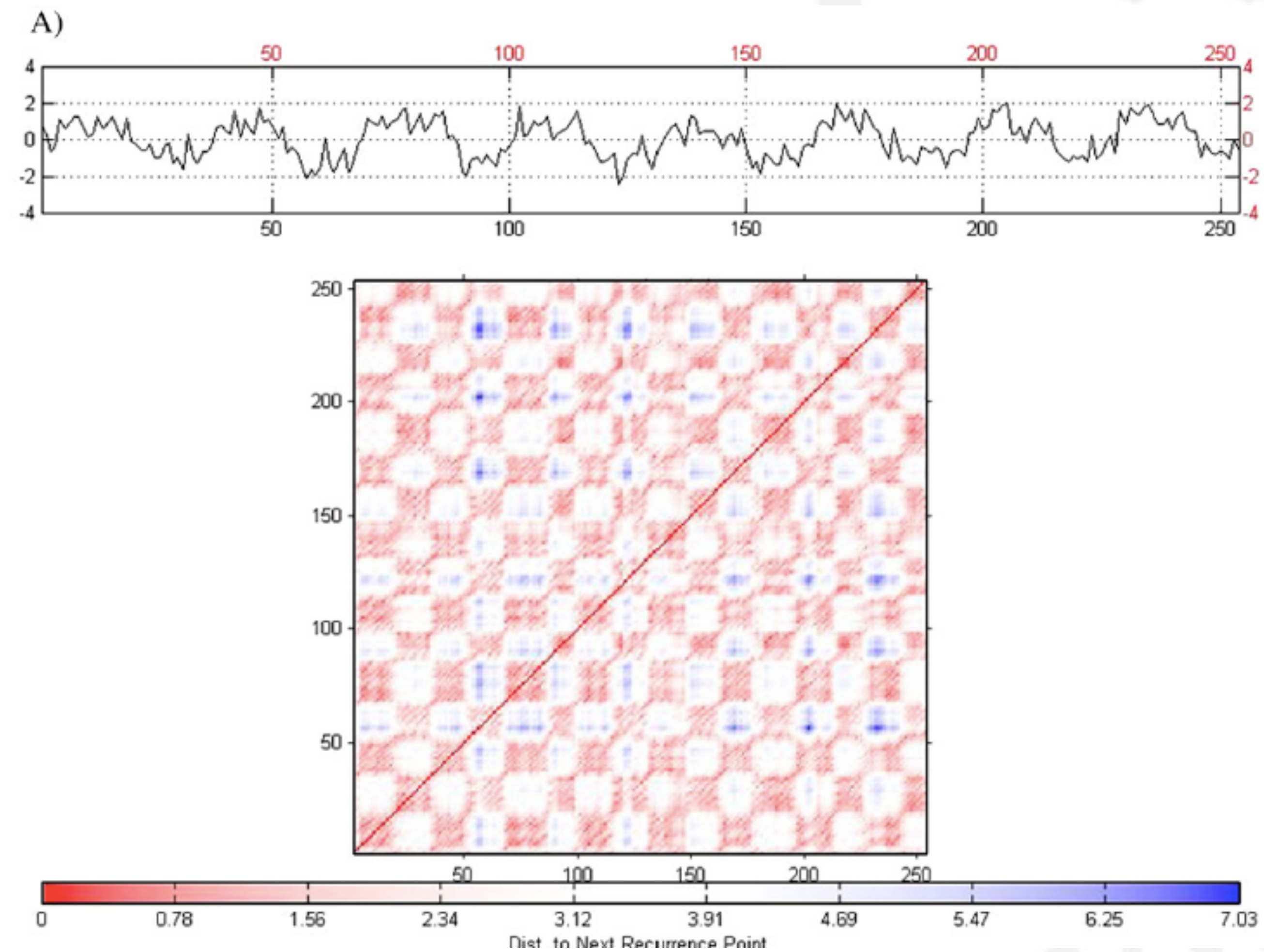
NeuroImage 37 (2007) 489–503

Model-free analysis of brain fMRI data by recurrence quantification

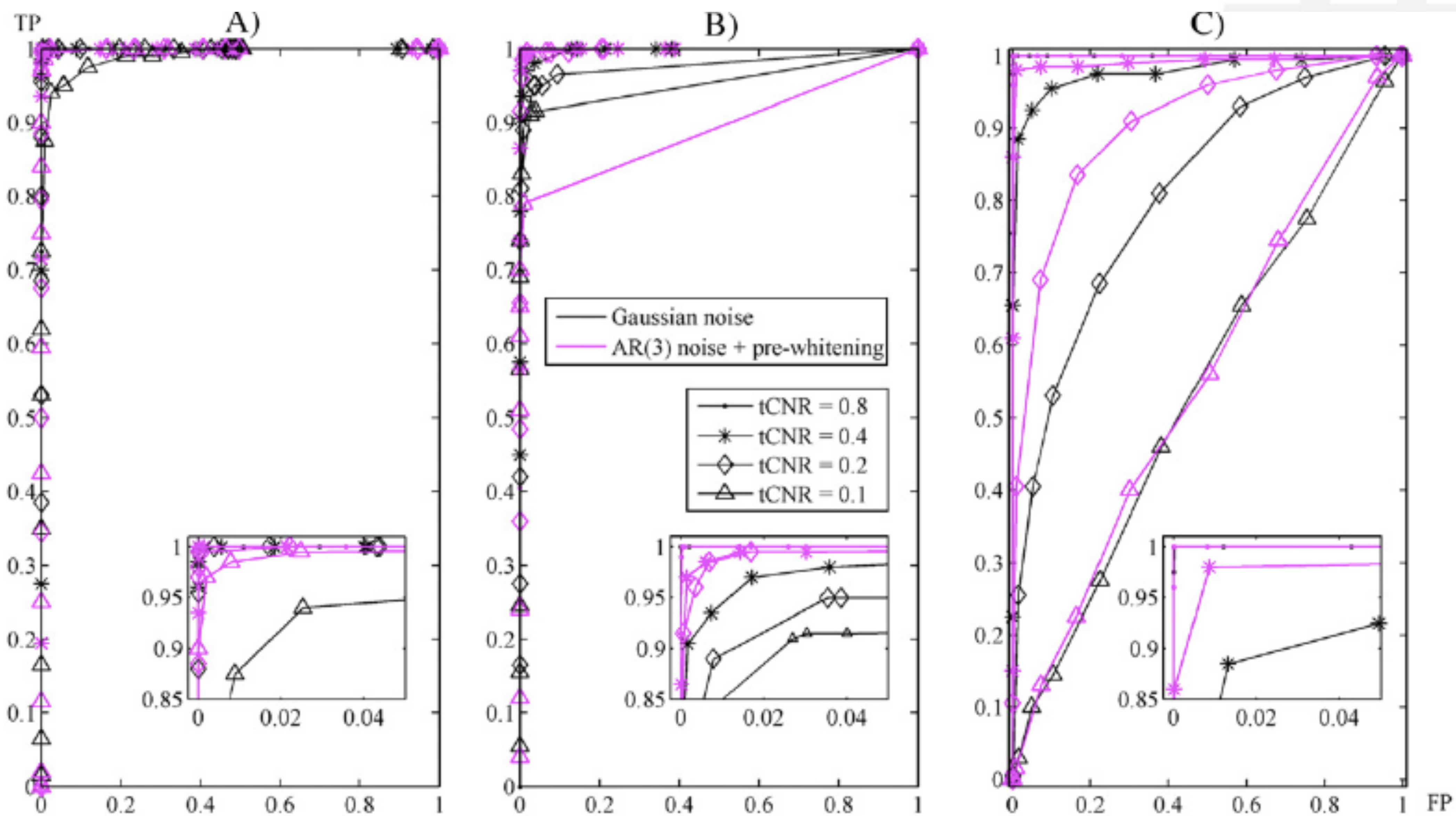
Marta Bianciardi,[a] Paolo Sirabella,[b] Gisela E. Hagberg,[a] Alessandro Giuliani,[c]
Joseph P. Zbilut,[d] and Alfredo Colosimo[b,*]

Comparison of RQA with GLM and P-ICA



Ability of three analyses to distinguish between noises in fMRI signal (ROC analysis)

GLM P-ICA RQA



Bianciardi, M., Sirabella, P., Hagberg, G. E., Giuliani, A., Zbilut, J. P., & Colosimo, A. (2007). Model-free analysis of brain fMRI data by recurrence quantification. *NeuroImage*, *37*(2), 489-503. doi:10.1016/j.neuroimage.2007.05.025

Summary

RQA(TM) - now comes with an errorbar

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Confidence bounds of recurrence-based complexity measures

Stefan Schinkel[a,*], N. Marwan[a,b], O. Dimigen[c], J. Kurths[b,d]

http://www.agnld.uni-potsdam.de/~schinkel

Schinkel, S., Marwan, N., Dimigen, O., & Kurths, J. (2009). Confidence bounds of recurrence-based complexity measures. *Physics Letters A*, *373*(26), 2245-2250. Elsevier B.V. doi:10.1016/j.physleta.2009.04.045