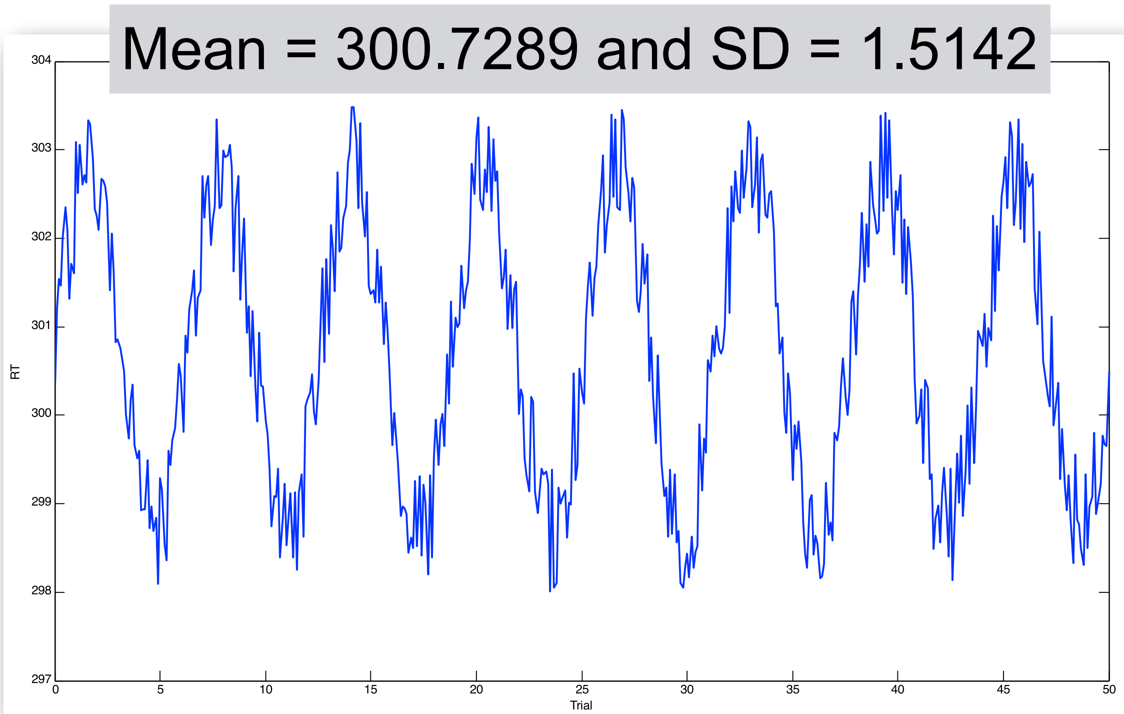
Dynamics of Complex Systems

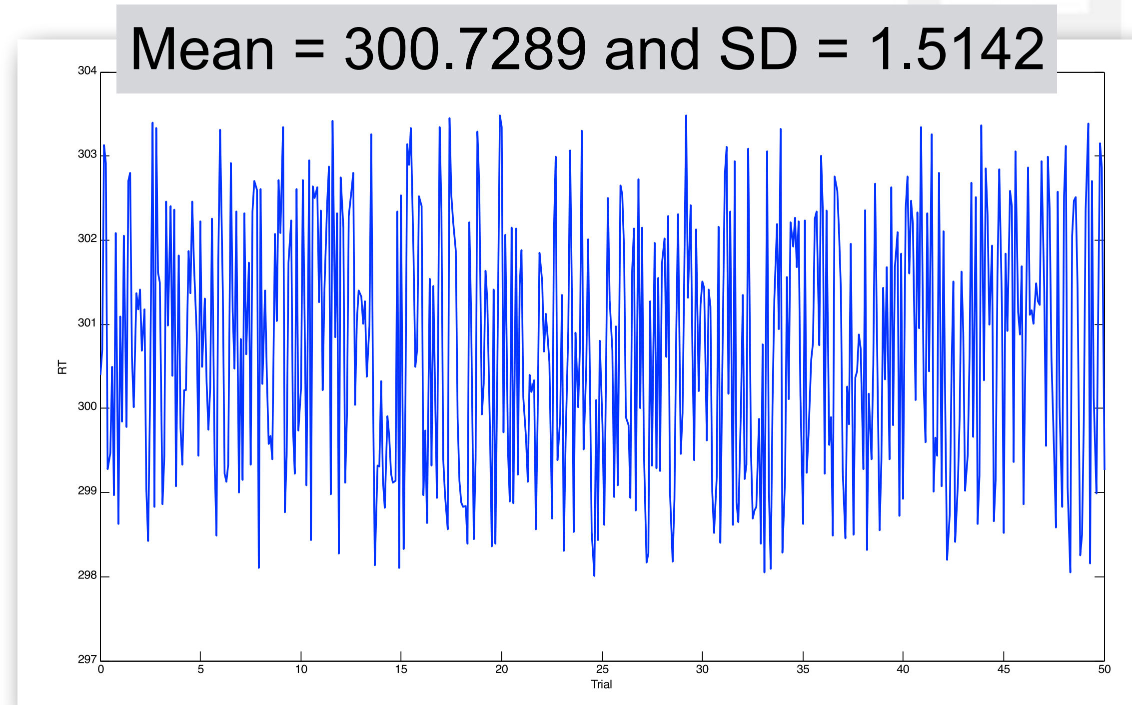
part 1
Temporal Correlations in timeseries
Basic Timeseries Analysis
Scaling Phenomena

f.hasselman@bsi.ru.nl

Behavioural Science Institute
Radboud University Nijmegen

Rose / Molenaar: "Analyse then Aggregate!"



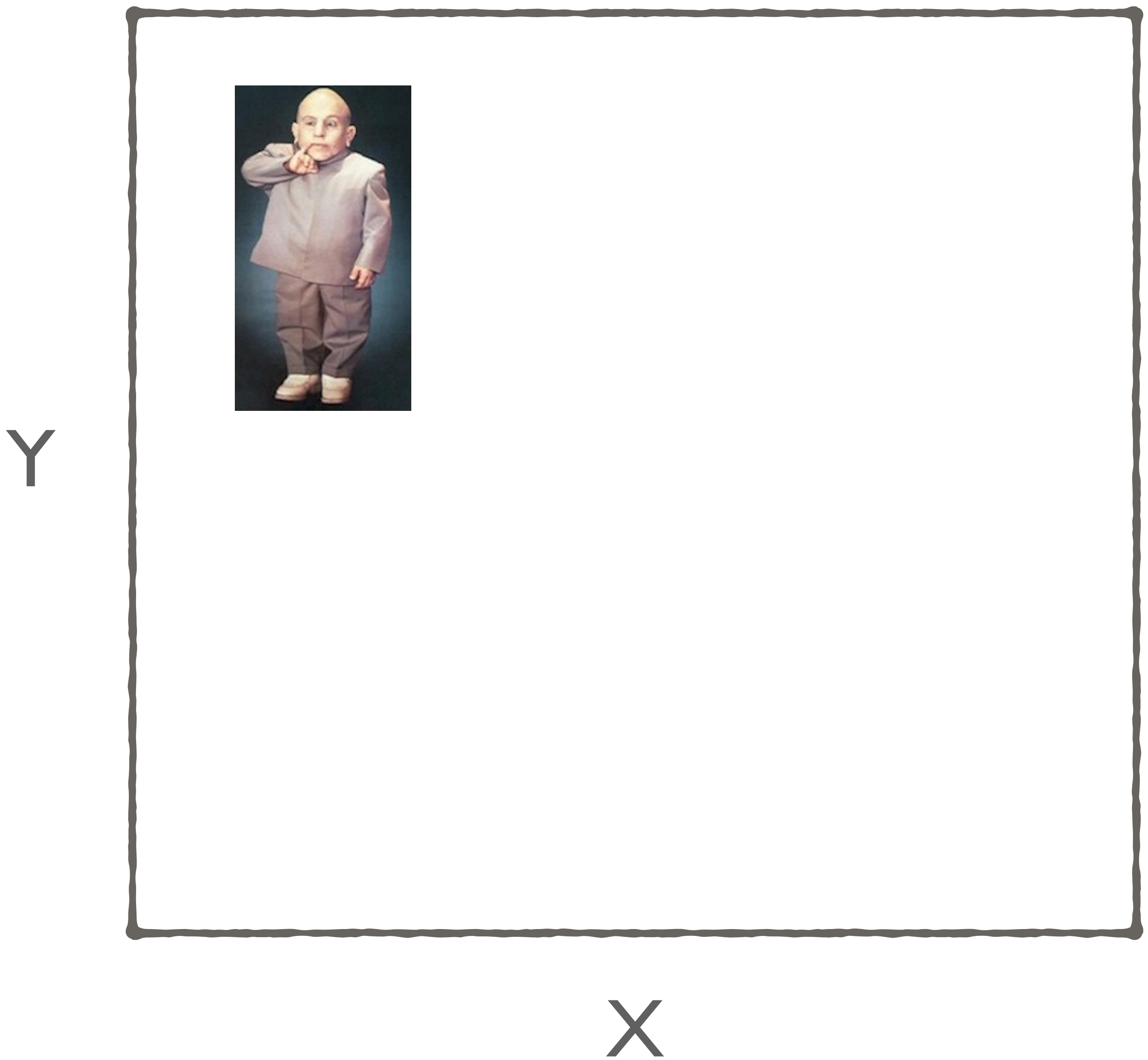


- The first process is very different from the second yet the central tendency measures are the same (MEAN, SD, etc. are equal).

- How can we characterise this difference?
    1. Quantify patterns of dependencies in the data as: deterministic/periodic/stochastic, stable/unstable fluctuations …
    2. For each individual, each measure, in each context of interest
    3. Aggregate (if necessary)

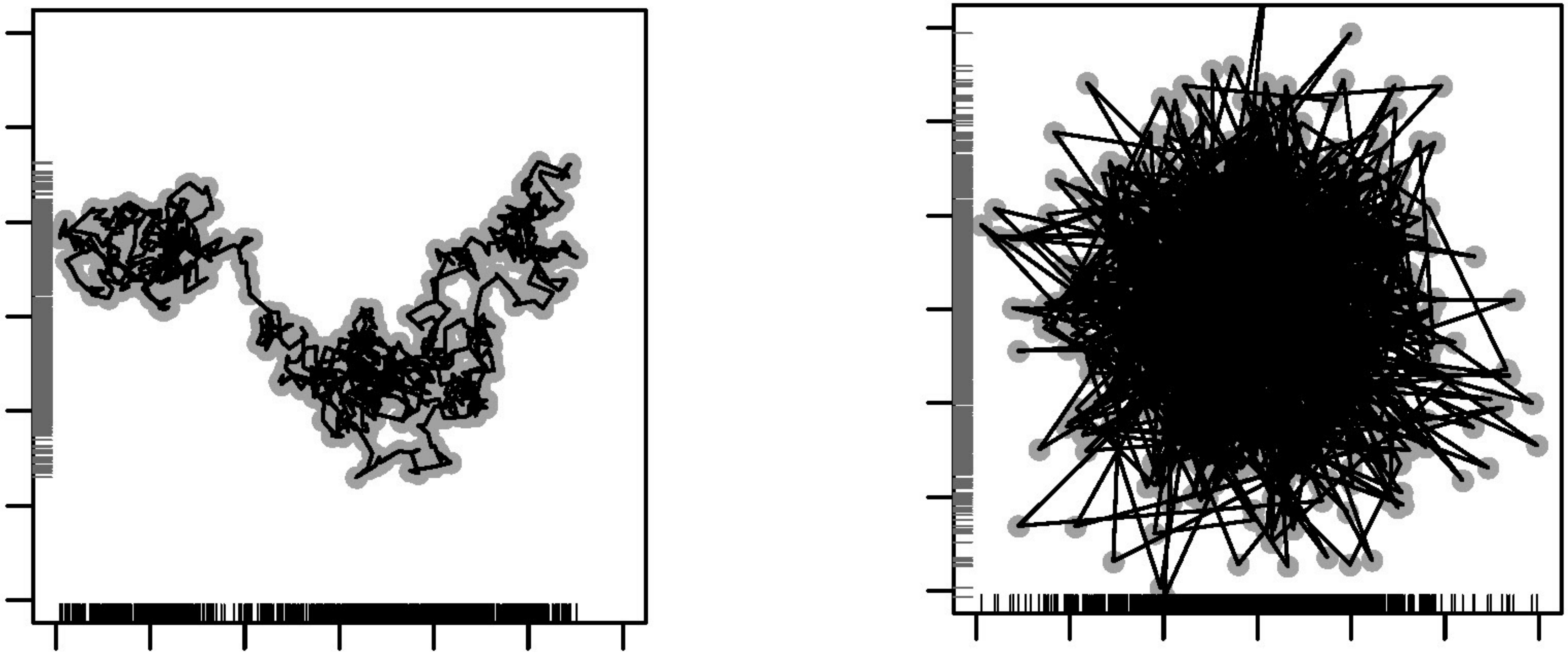
- Basics steps to get step 1. - Correlation functions & AR(fi)MA – Auto Regressive (fractional integrated) Moving Average (Box, G.E.P. and Jenkins, G.M. (1976), *Time Series Analysis: Forecasting and Control*, San Francisco: Holden-Day.)

MINIME SYSTEM

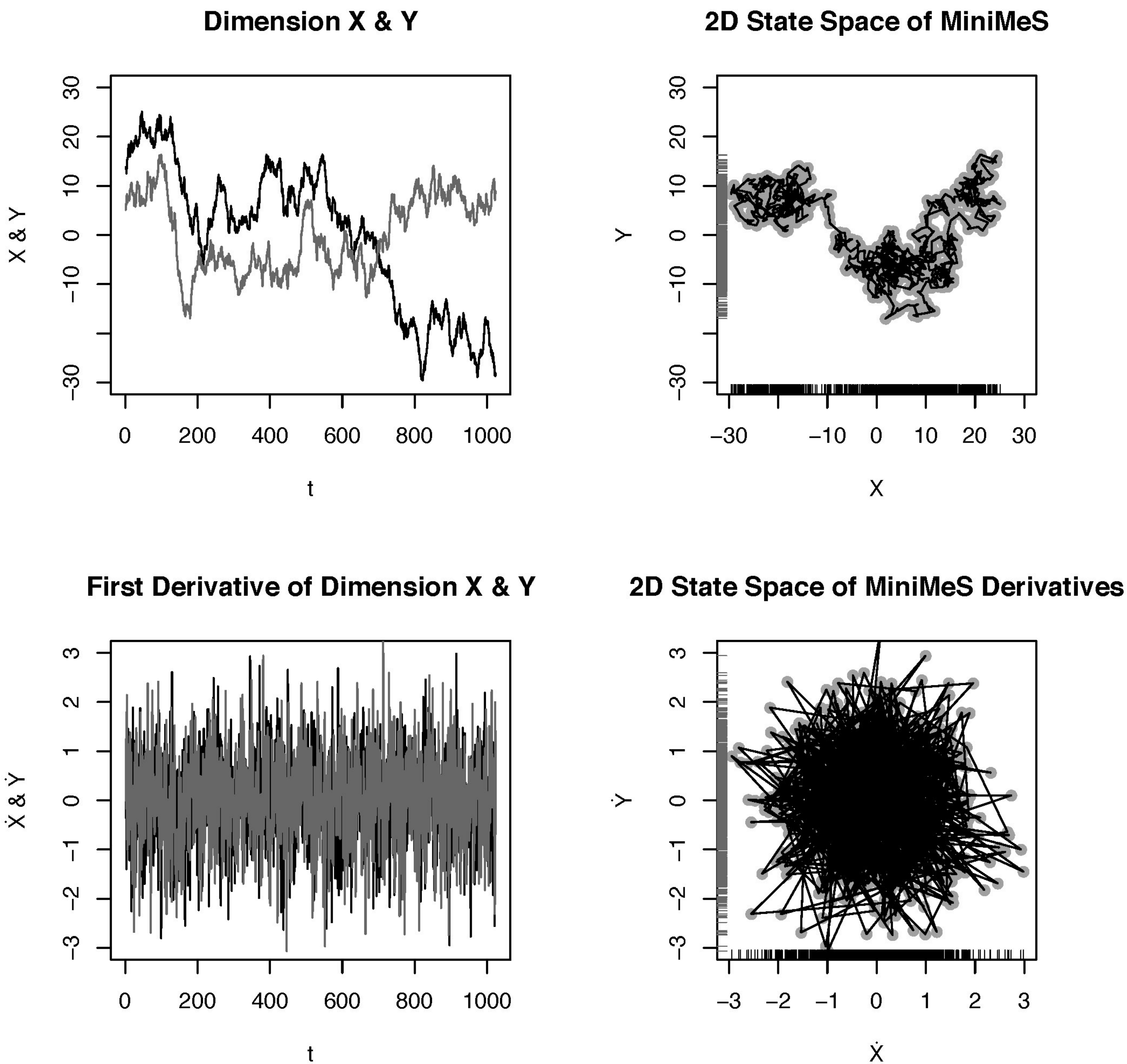


- State = X,Y coordinate
- Minimal Memory System can move around within the boundary.
- When would you infer randomness, when a deterministic rule?
- What kind of succession of states?
- What kind of trajectory through space?

MINIME SYSTEM



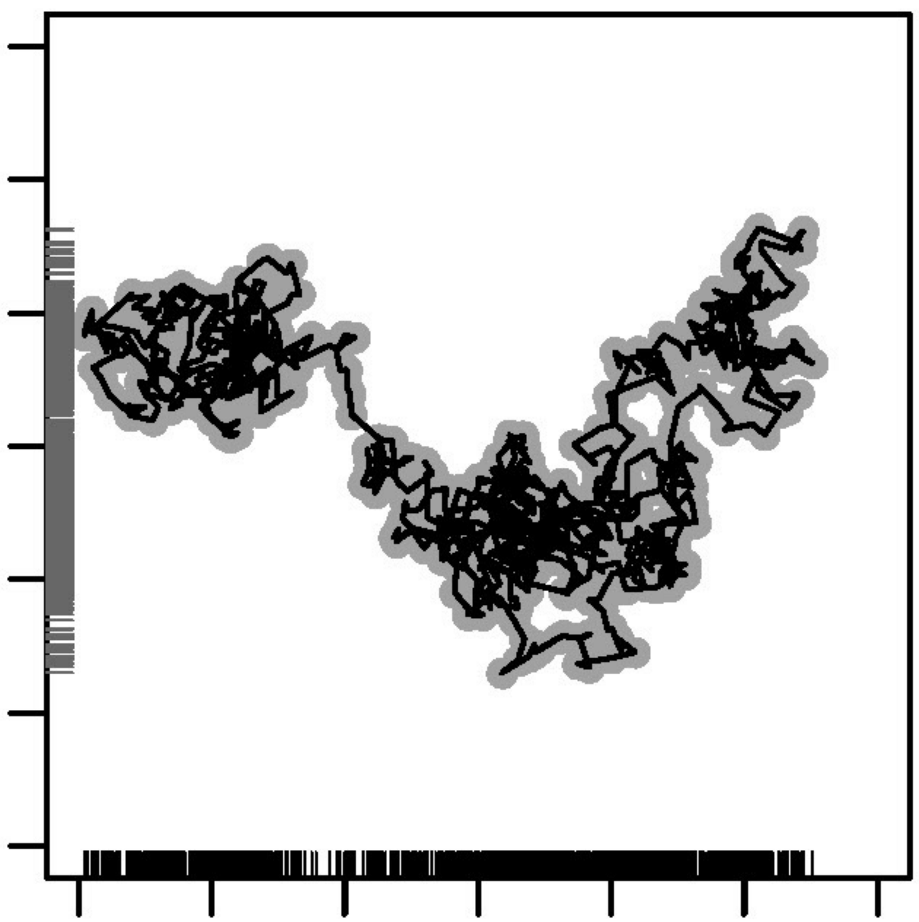
MINIME SYSTEM

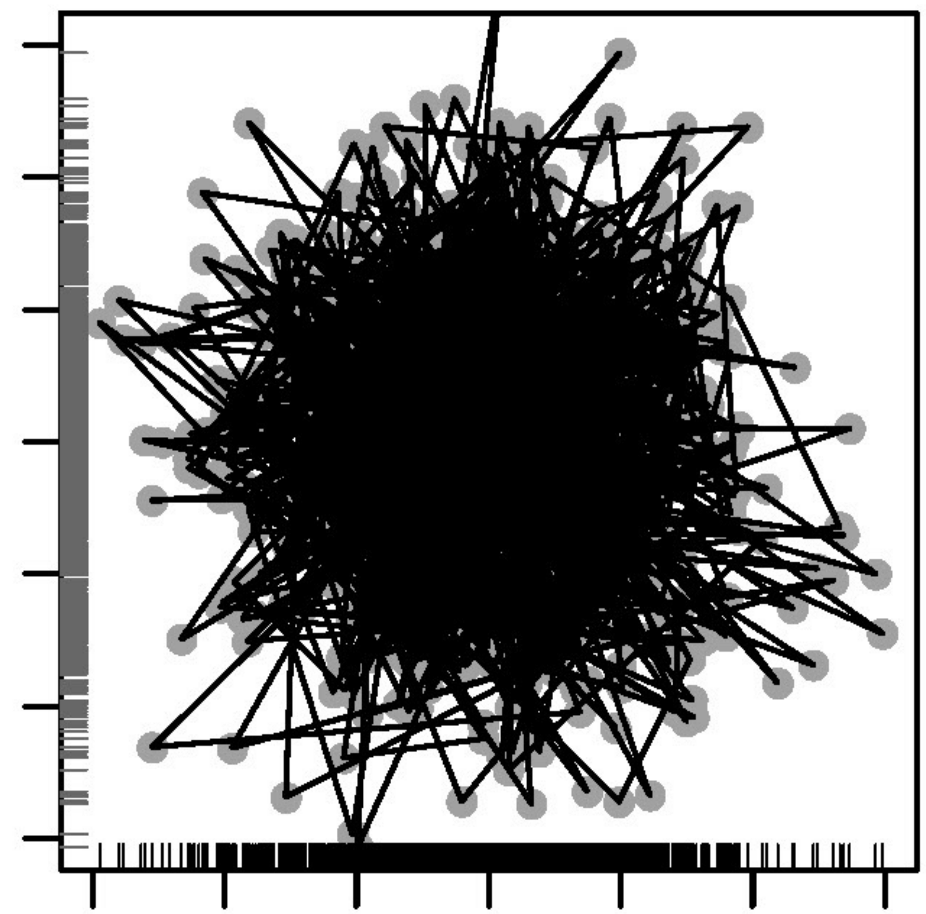


- State Space (X & Y): The degrees of freedom MiniMe has to generate its behaviour (move)
- This is a random walk, Brownian motion: Add a random number drawn from normal distribution to current number.
- Where does the apparent order come from? It's a random process!!!!

‘Simple’ rule reduces degrees of freedom to move around:

Matter has to occupy finite space & movement takes time (no teleportation yet)





Minimal form of ‘physical memory’ through ‘natural computation’: summation / counting

Emergence of structure / temporal correlations / redundancies / dependencies

Brownian motion / Levy flights are very common in nature (diffusion, percolation, foraging)

How to characterise the nature of the dependencies?

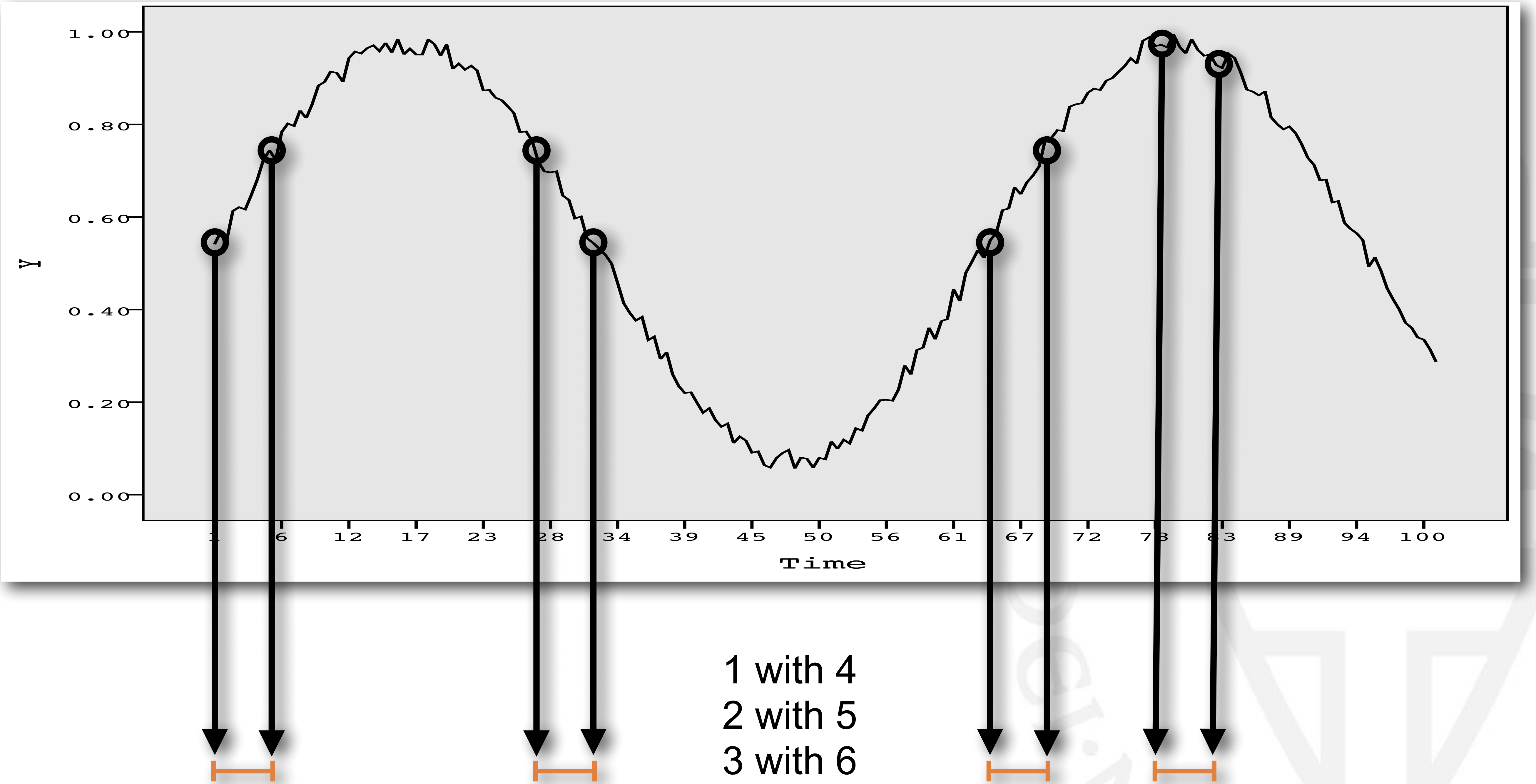
(Partial) Autocorrelation Function - (P)ACF

$$R(k) = \frac{E[(X_i - \mu)(X_{i+k} - \mu)]}{\sigma^2}$$

The average correlation **r** between data points that are a distance (lag) **k** apart in time

This holds only for *stationary*, *random processes.* So X measured here is a *random variable*.

ACF and the Partial ACF are used to decide which AR(fI)MA model you need (how many AR and/or MA parameters you need).



Lag = 3

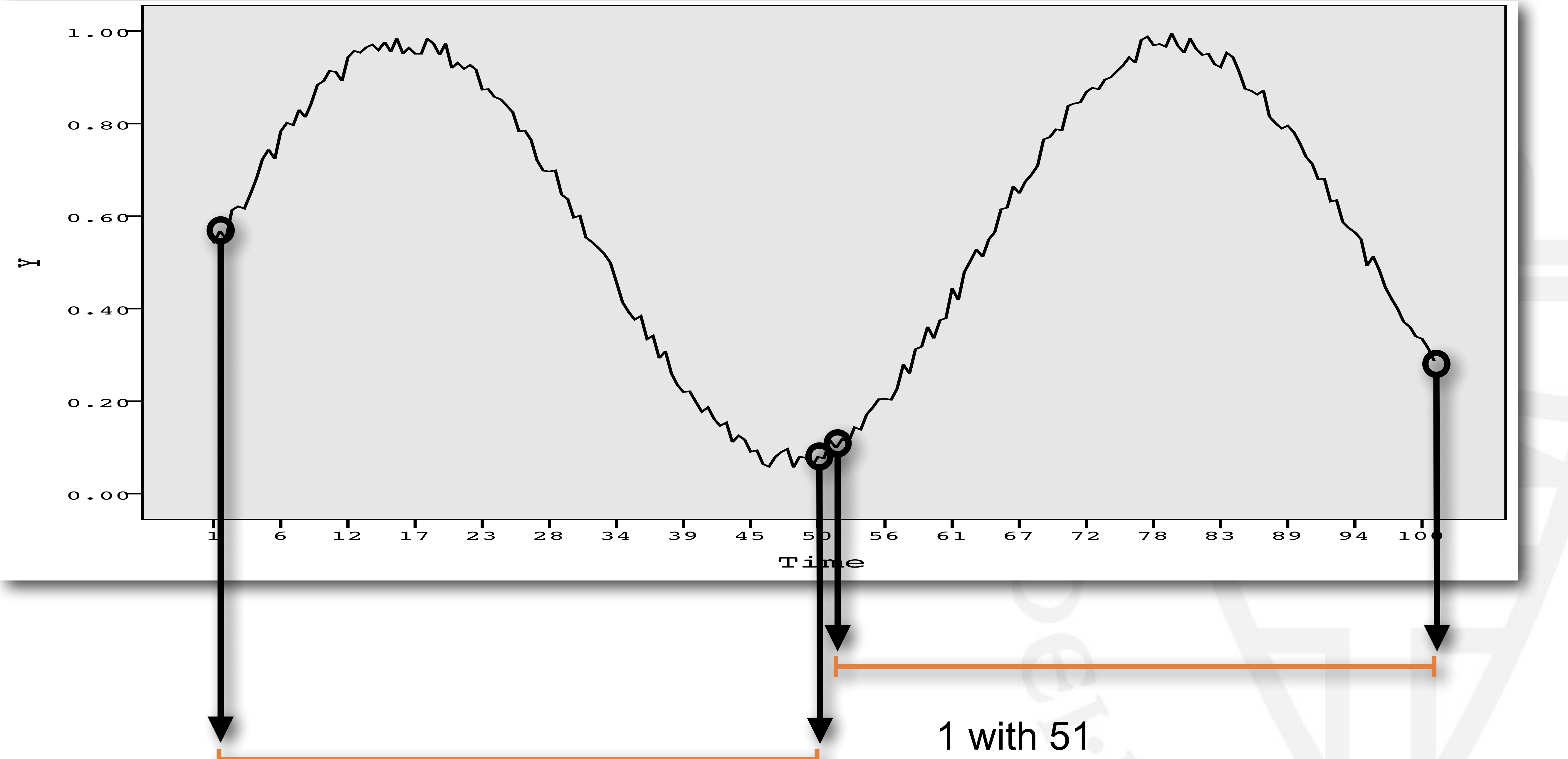
How many correlations of lag 3?

TS length = 100 data points

1 with 4
2 with 5
3 with 6
…
96 with 99
97 with 100

Low or High at lag 3?

r_3= 0.895 (*SD* = *0.095*)



Lag = 50

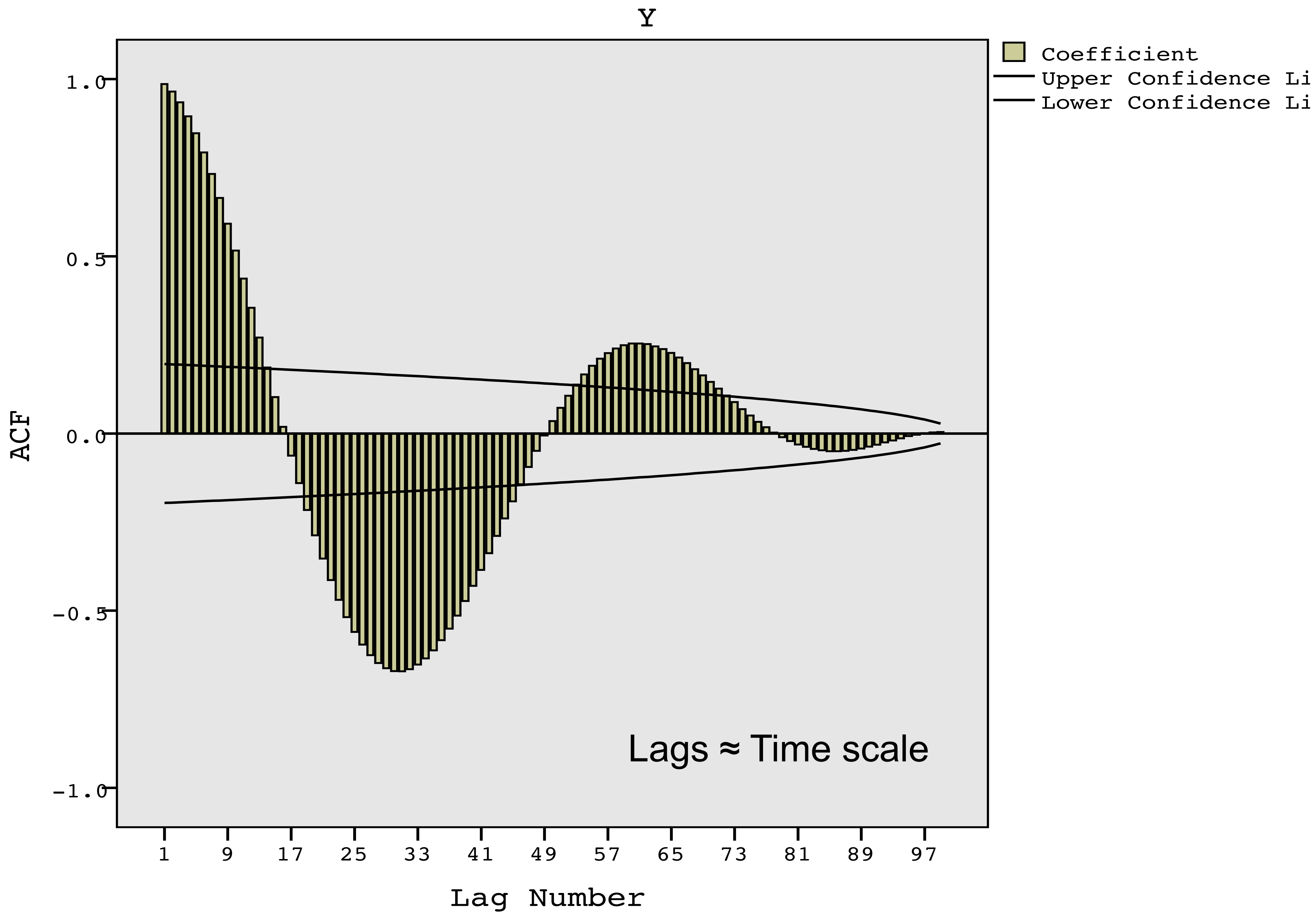
1 with 51
2 with 52
3 with 53
…
50 with 100

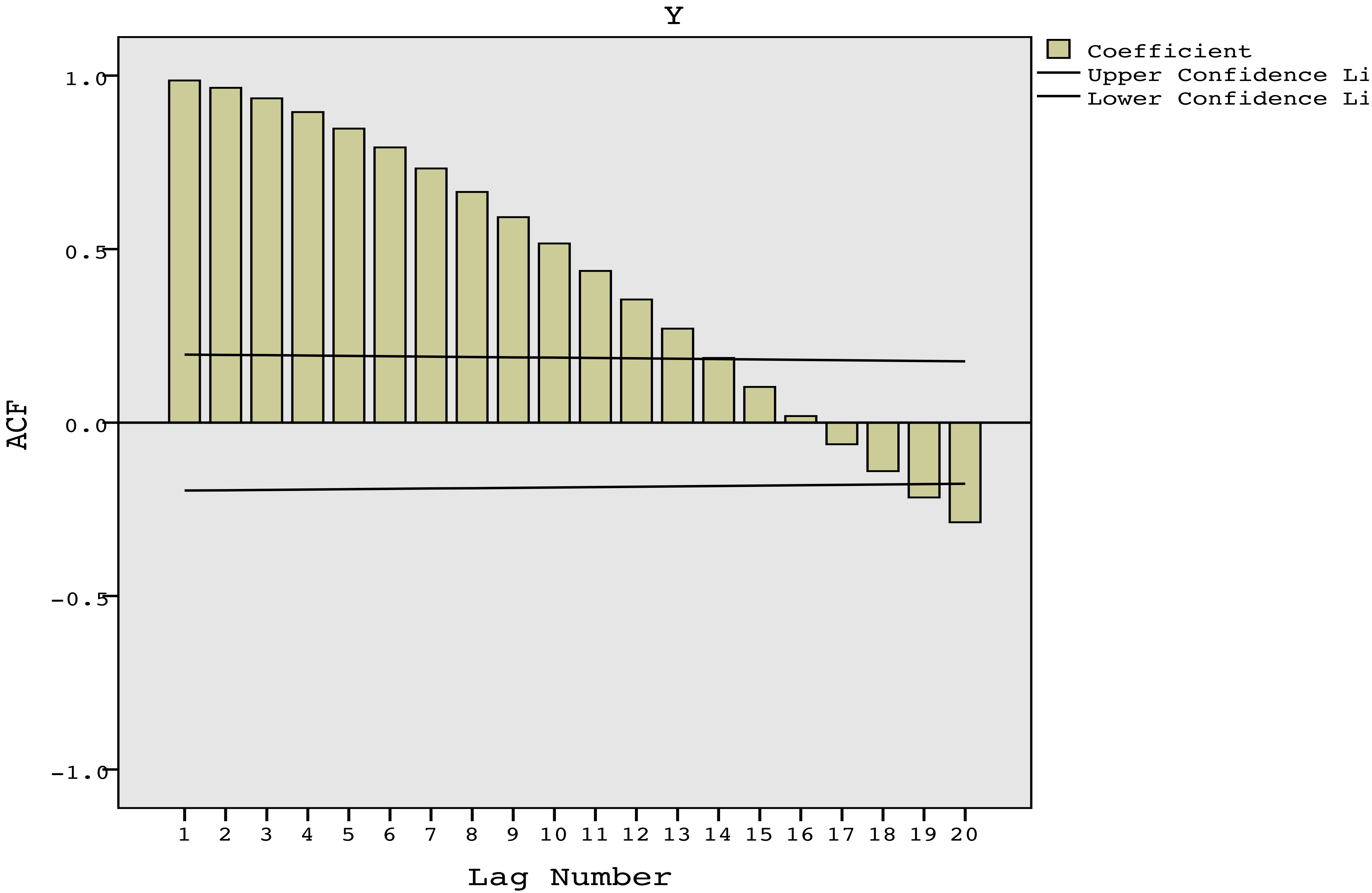
How many correlations of lag 50?

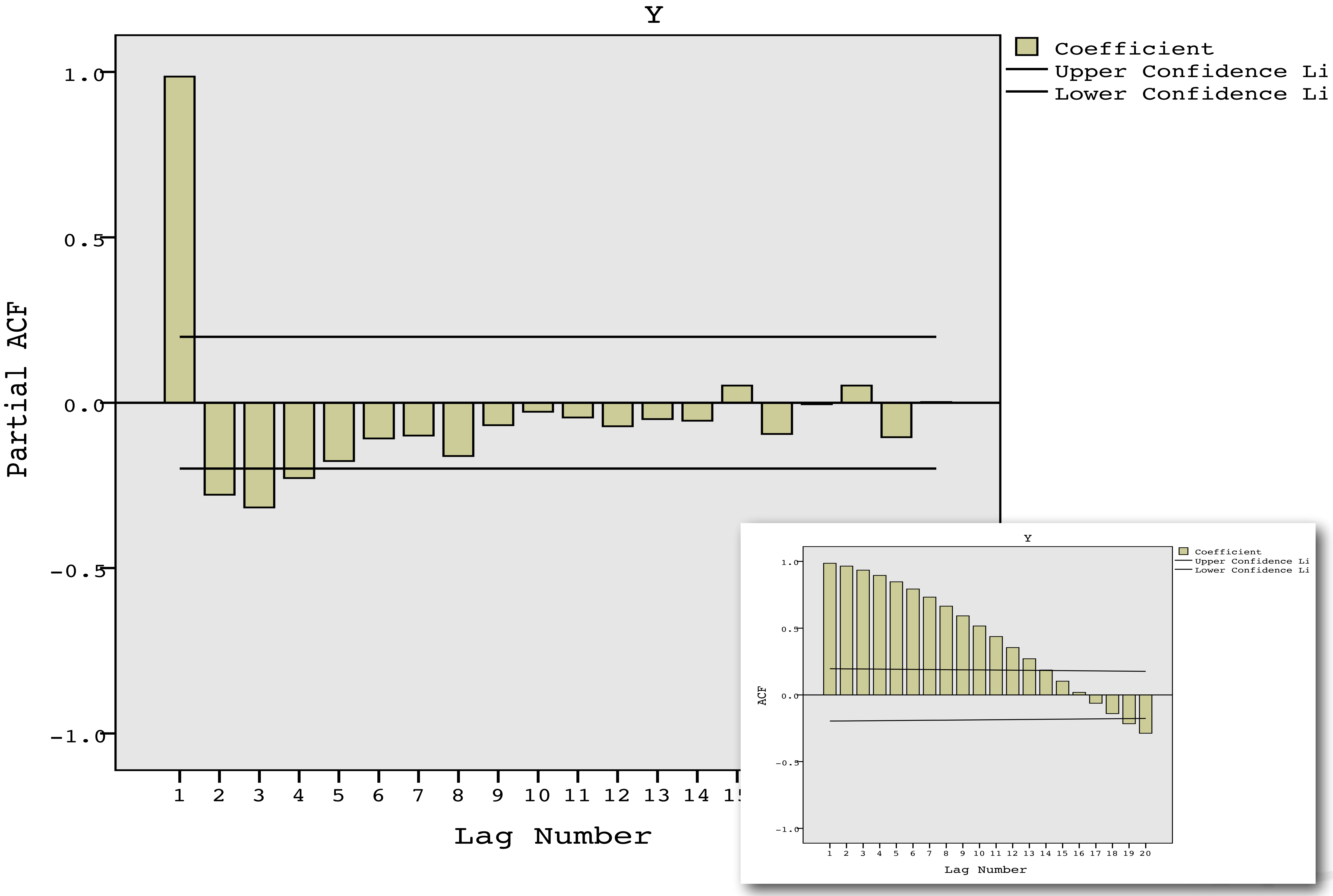
Low or High at lag 50?

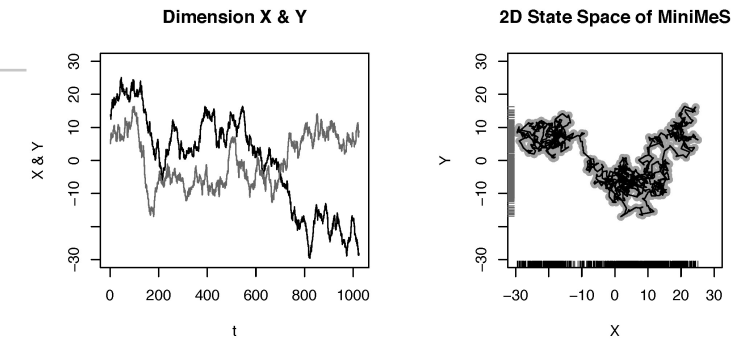
r_{50} = 0.035 (*SD = 0.070*)

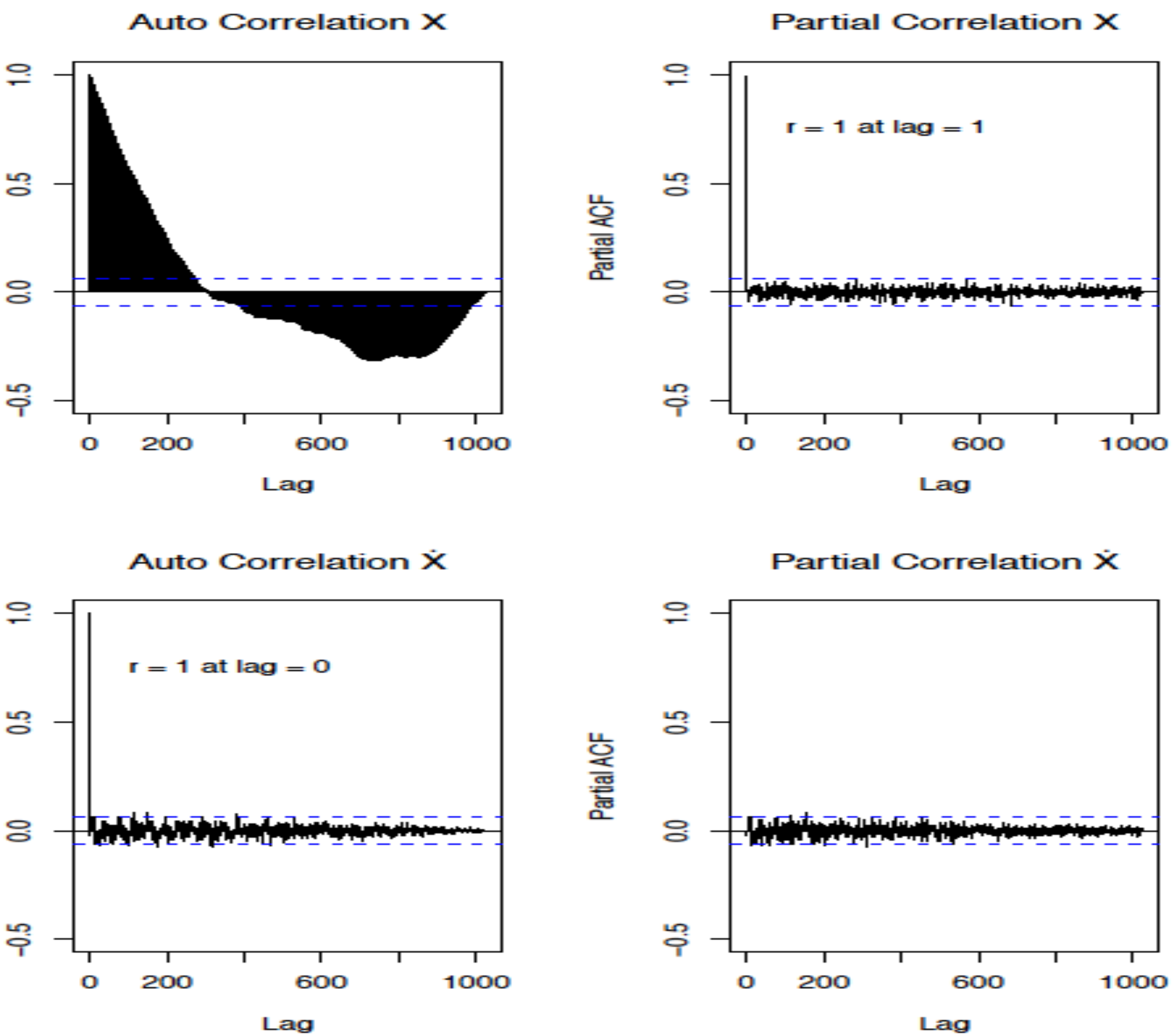
TS length = 100 data points


Y
Coefficient
Upper Confidence Li
Lower Confidence Li
1.0
0.5
0.0
-0.5
-1.0
ACF
Lags ≈ Time scale
1
9
17
25
33
41
49
57
65
73
81
89
97
Lag Number


Y
Coefficient
Upper Confidence Li
Lower Confidence Li
1.0
0.5
0.0
-0.5
-1.0
ACF
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Lag Number


Y
Coefficient
Upper Confidence Li
Lower Confidence Li
Partial ACF
1.0
0.5
0.0
-0.5
-1.0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Lag Number
Y
Coefficient
Upper Confidence Li
Lower Confidence Li
ACF
1.0
0.5
0.0
-0.5
-1.0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Lag Number





- ACF and PACF: representation of redundancies in the time series
- ACF: Correlations across multiple time scales?
- PACF: Nope, correlation at lag 1

A stochastic linear process model AR(fI)MA

- Notation: **ARIMA(p,d,q)**, these parameters usually take values of 0-2 indicating none, 1 or 2 components.

- **ARIMA(1,2,1)** = means One AutoRegressive parameter, a (filtered-out!) quadratic trend, and 1 Moving Average parameter.

- **AR-part:** The model tries to predict each data point **X** at time **t** based on a constant or intercept (ξ) + a linear combination of previous observations ($\phi_{1\ldots p}$) + random error called random shock (ε):

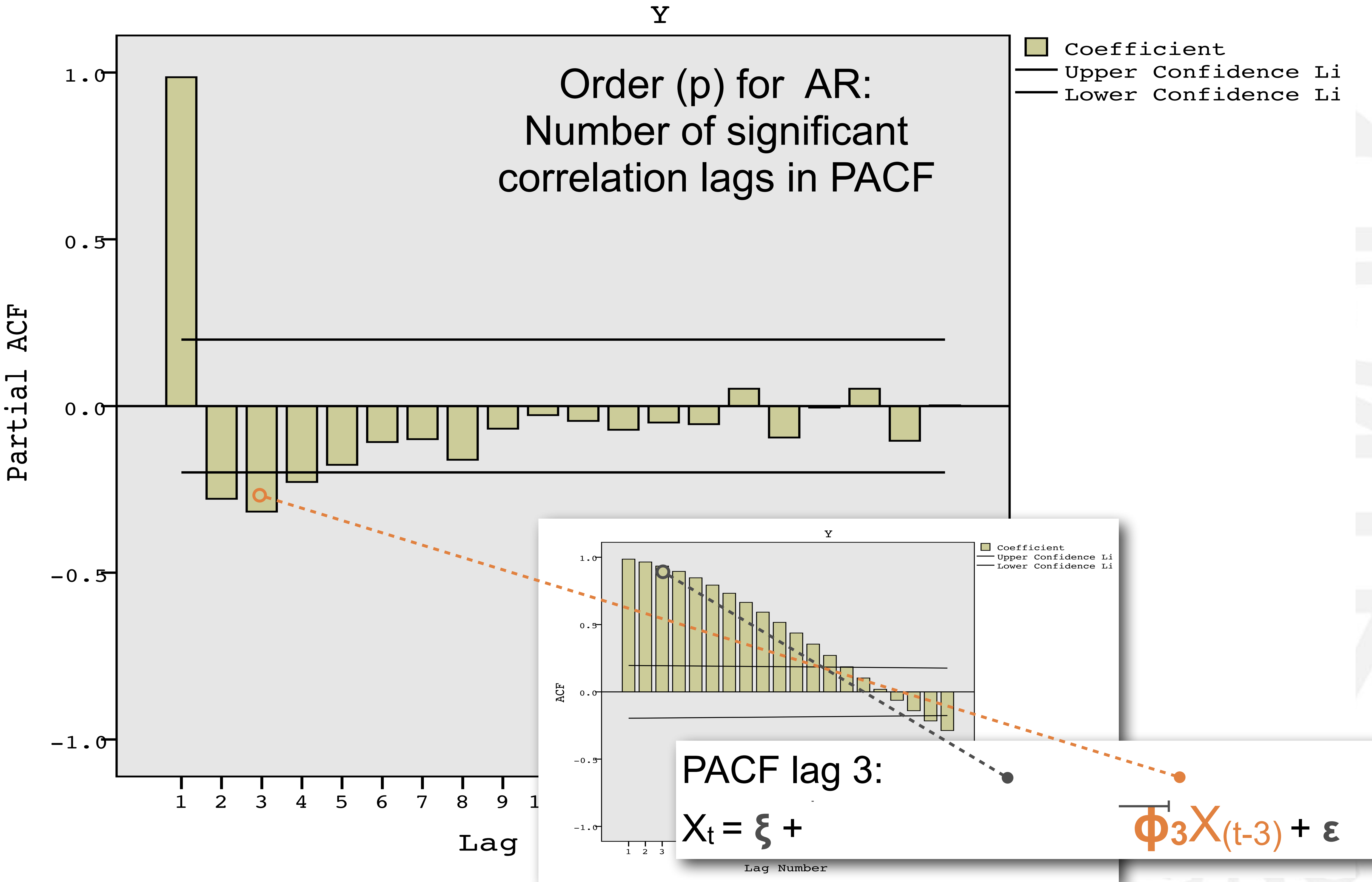
$$X_t = \xi + \phi_1 X_{(t-1)} + \phi_2 X_{(t-2)} + \phi_3 X_{(t-3)} + \ldots + \phi_p X_{(t-p)} + \varepsilon$$

- **MA-part:** The model tries to predict each data point **X** at time **t** based on the average of the time series (μ) + the current random error (ε_t) - a linear combination of previously observed random error ($\theta_{1\ldots q}$):

$$X_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{(t-1)} - \theta_2 \varepsilon_{(t-2)} - \theta_3 \varepsilon_{(t-3)} - \ldots - \theta_p \varepsilon_{(t-q)}$$

- **I(ntegration)-part:** The data must be stationary (and more!): 1) constant mean, 2) constant variance, 3) constant autocorrelation *through* time. This part removes correlations between data points that are a particular distance (lag) apart by differencing the time series… in other words: Trends in the data are filtered out.

- How many parameters do you need? … ACF and PACF

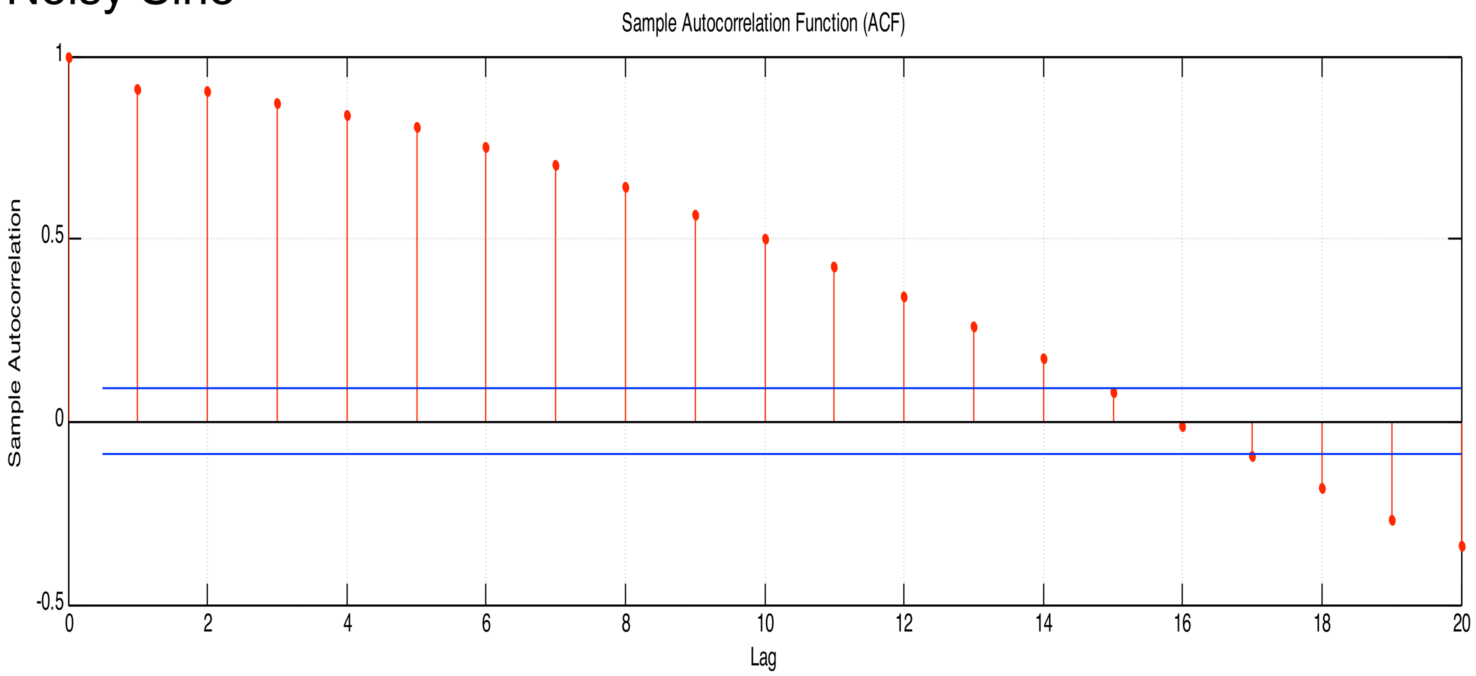

Y
Coefficient
Upper Confidence Li
Lower Confidence Li
Order (p) for AR:
Number of significant
correlation lags in PACF
1.0
0.5
0.0
-0.5
-1.0
Partial ACF
1 2 3 4 5 6 7 8 9 1
Lag
ACF
Lag Number
PACF lag 3:
$X_t = \xi + \phi_3 X_{(t-3)} + \varepsilon$

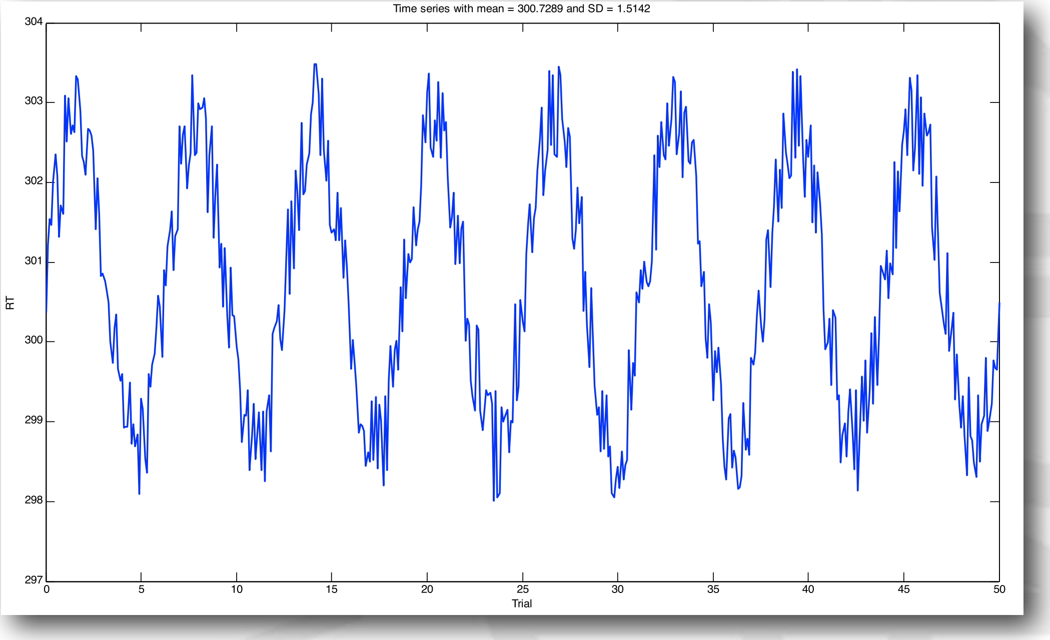
Rules of thumb
(http://www.itl.nist.gov/div898/handbook/pmc/section4/pmc446.htm)

SHAPE of ACF	INDICATED MODEL
Exponential, decaying to zero	Autoregressive model. Use the partial autocorrelation plot to identify the order of the autoregressive model.
Alternating positive and negative, decaying to zero	Autoregressive model. Use the partial autocorrelation plot to help identify the order.
One or more spikes, rest are essentially zero	Moving average model, order identified by where plot becomes zero.
Decay, starting after a few lags	Mixed autoregressive and moving average model.
All zero or close to zero	Data is essentially random.
High values at fixed intervals	Include seasonal autoregressive term.
No decay to zero	Series is not stationary.

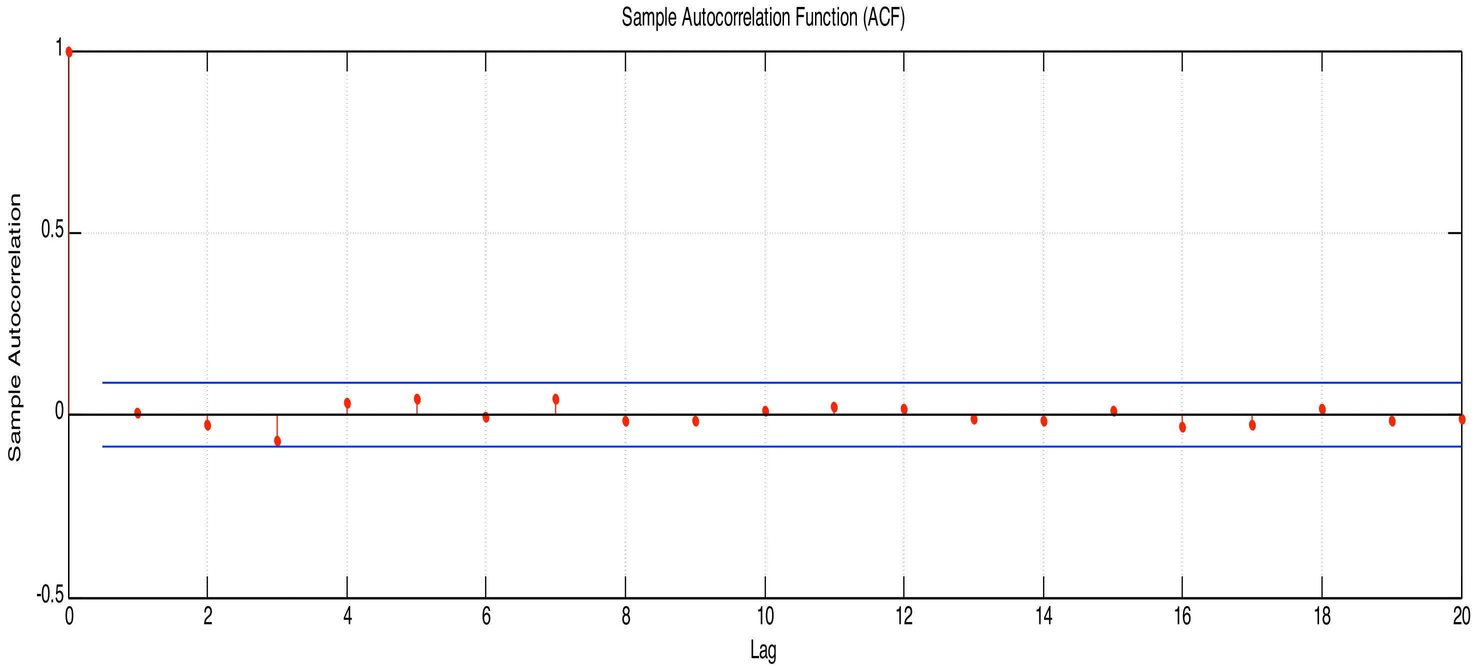
Randomising temporal order = Destroying correlations in the data

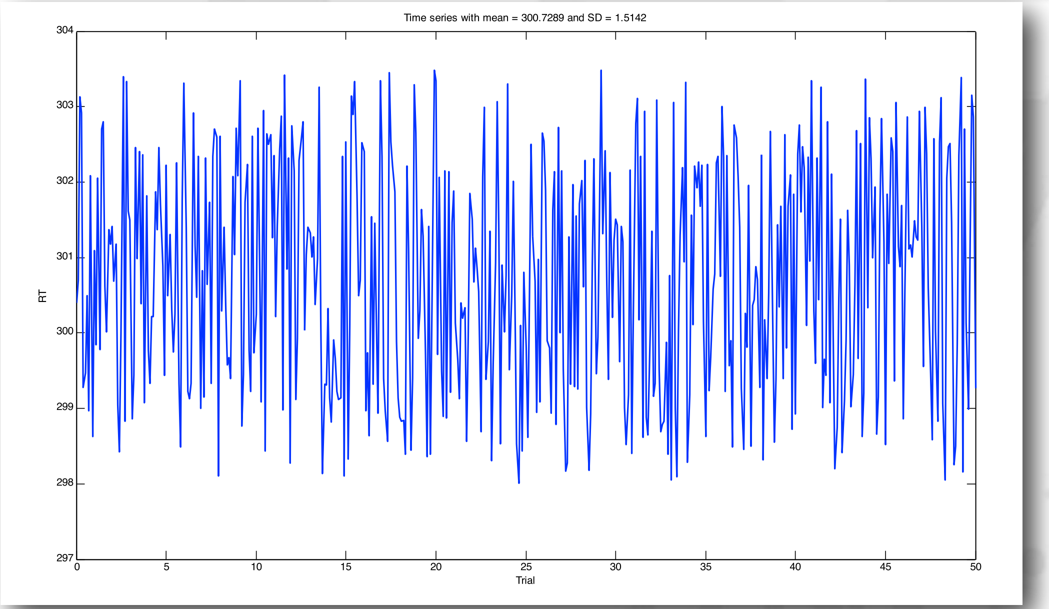
Noisy Sine





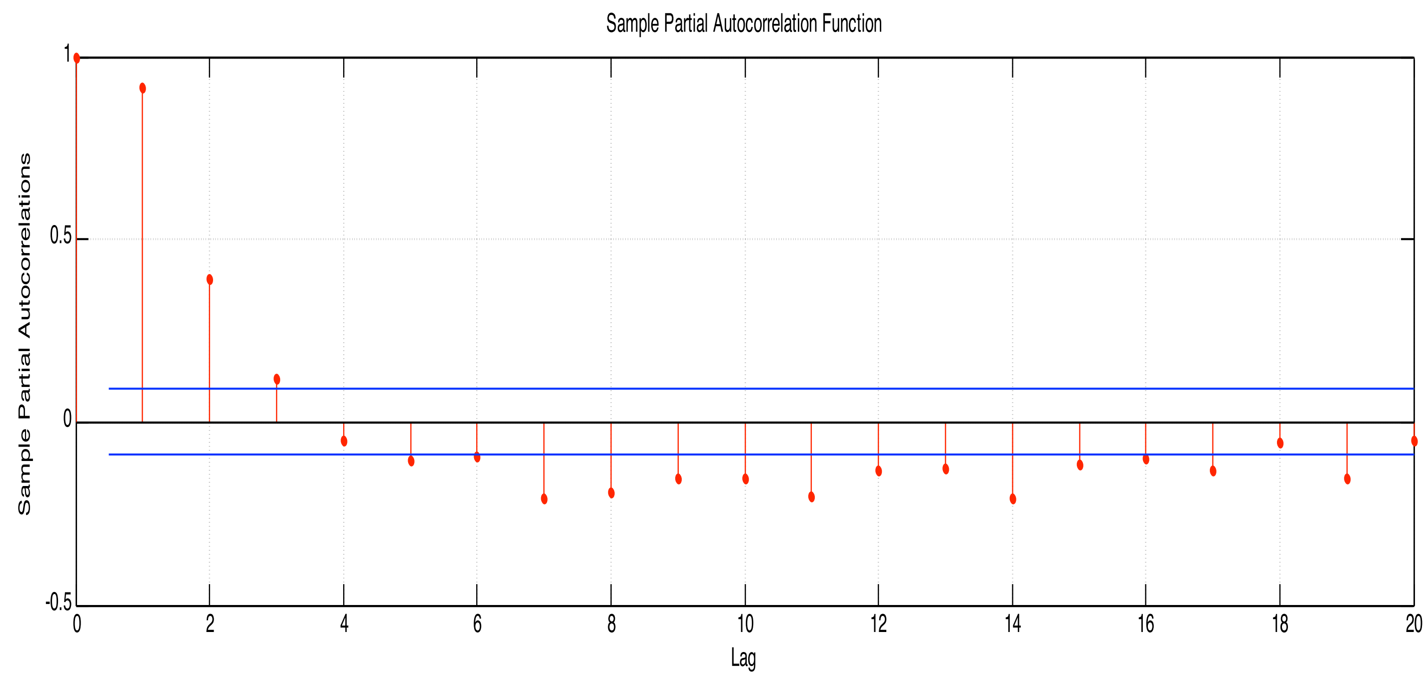
Randomised Noisy Sine

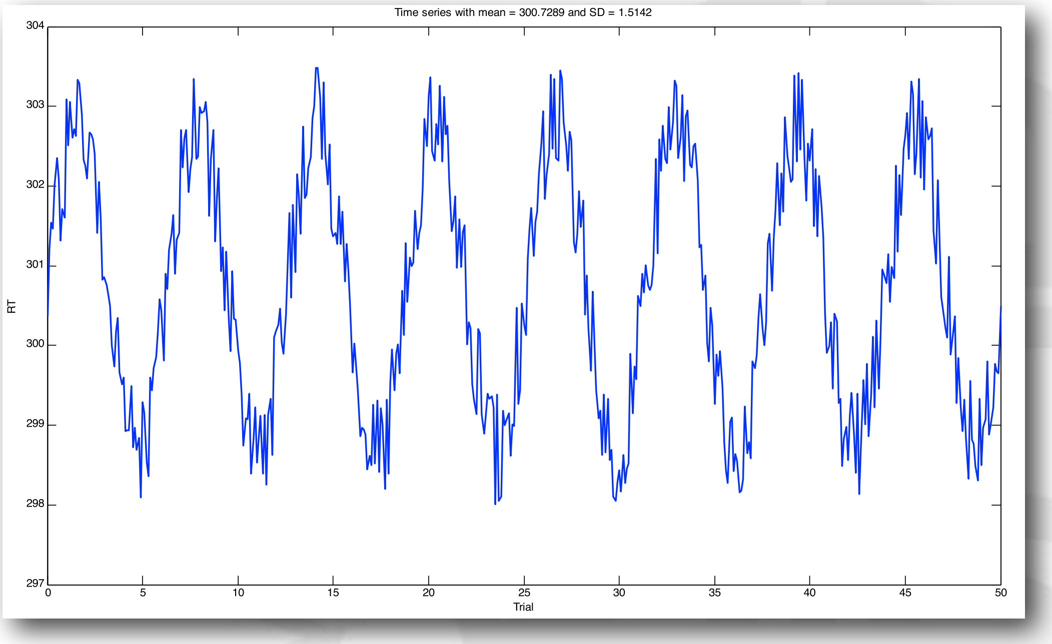




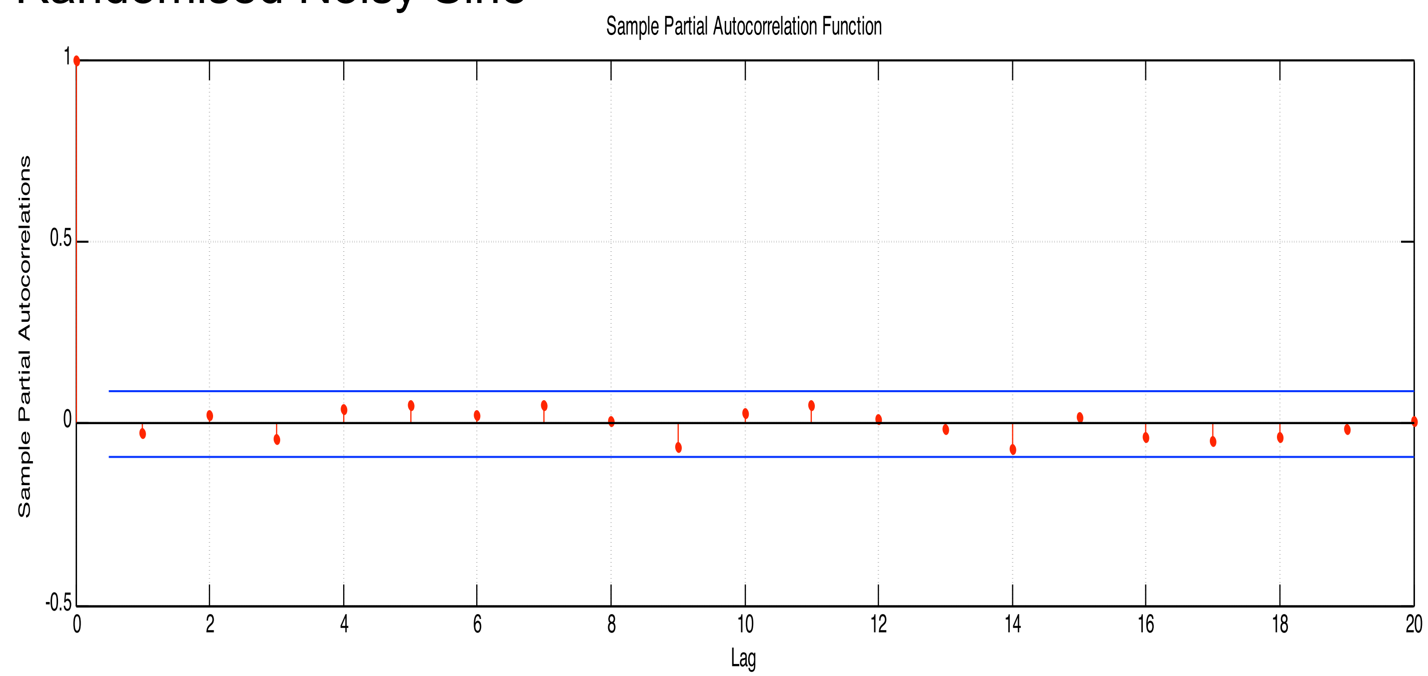
Randomising temporal order = Destroying correlations in the data

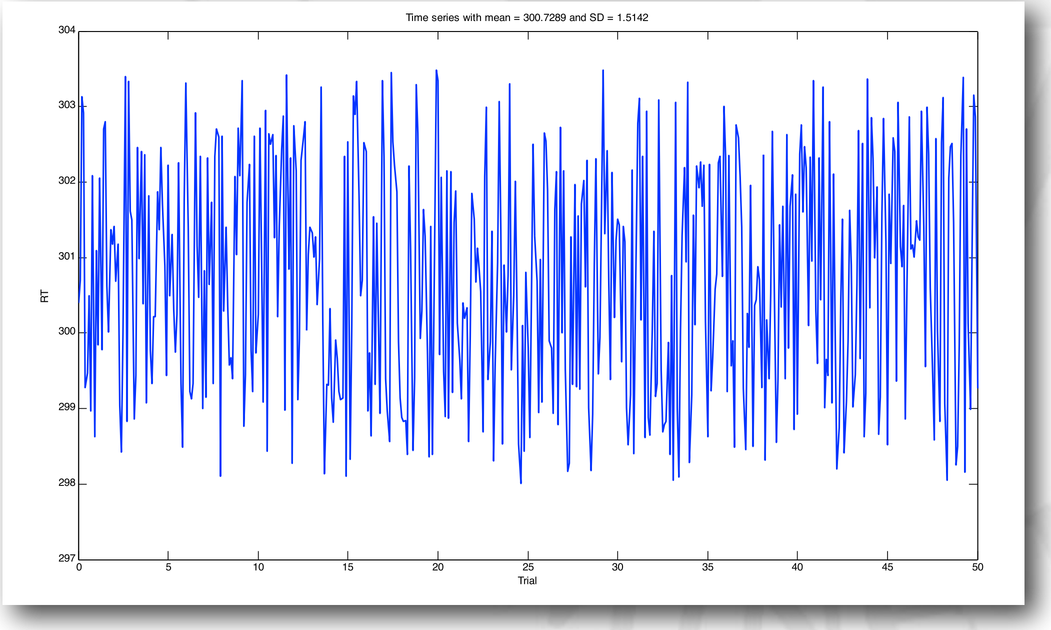
Noisy Sine

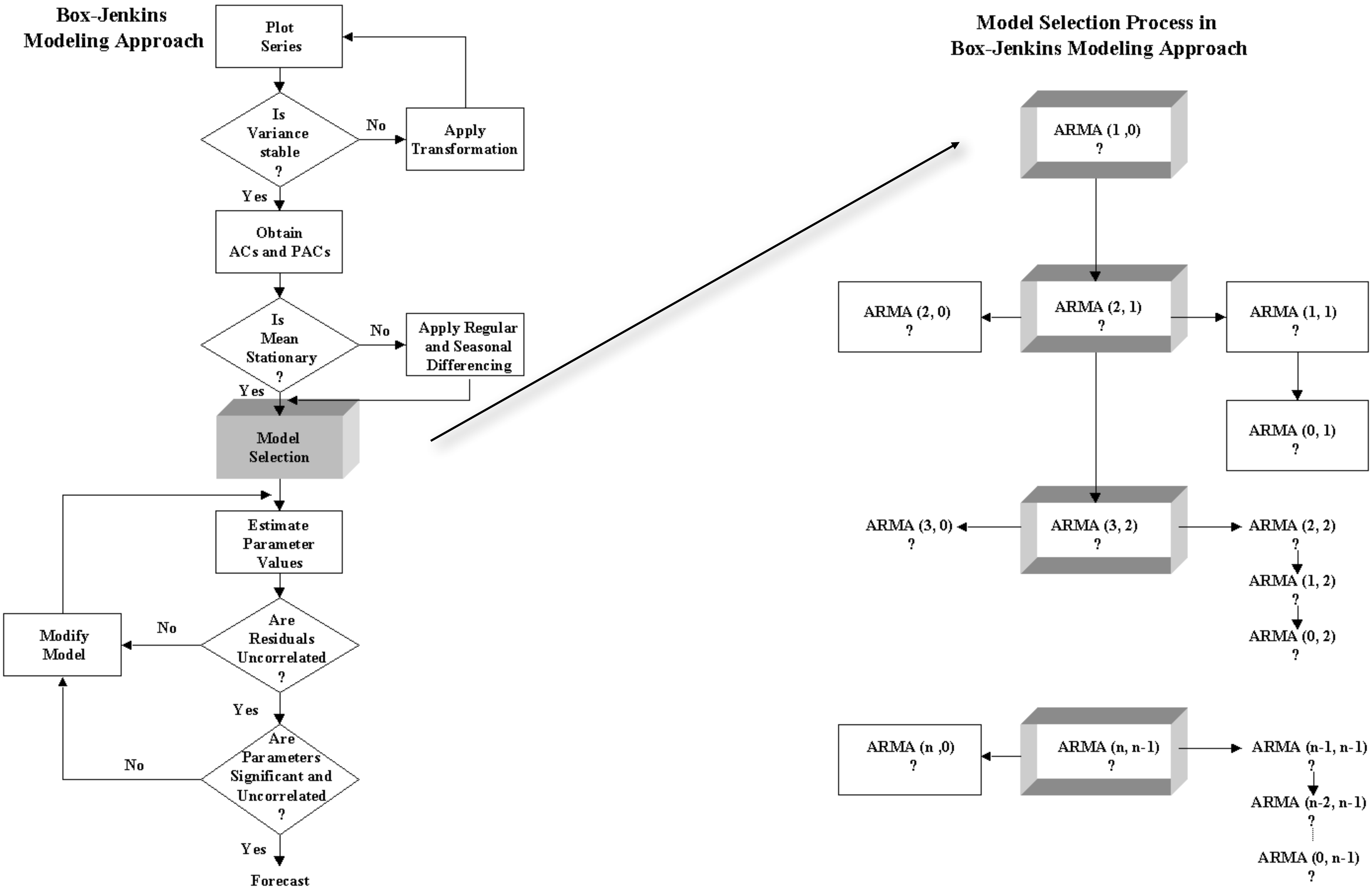




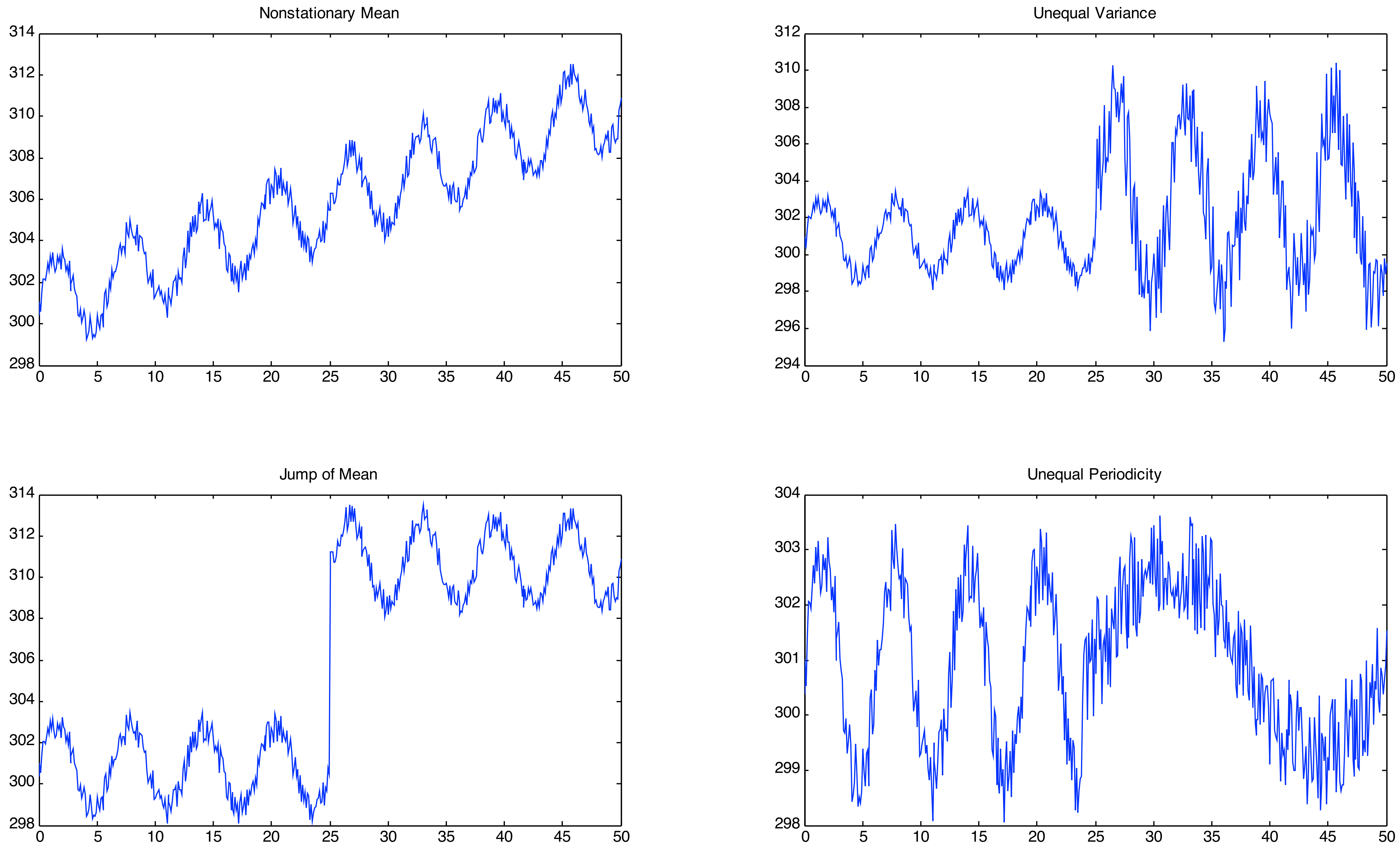
Randomised Noisy Sine





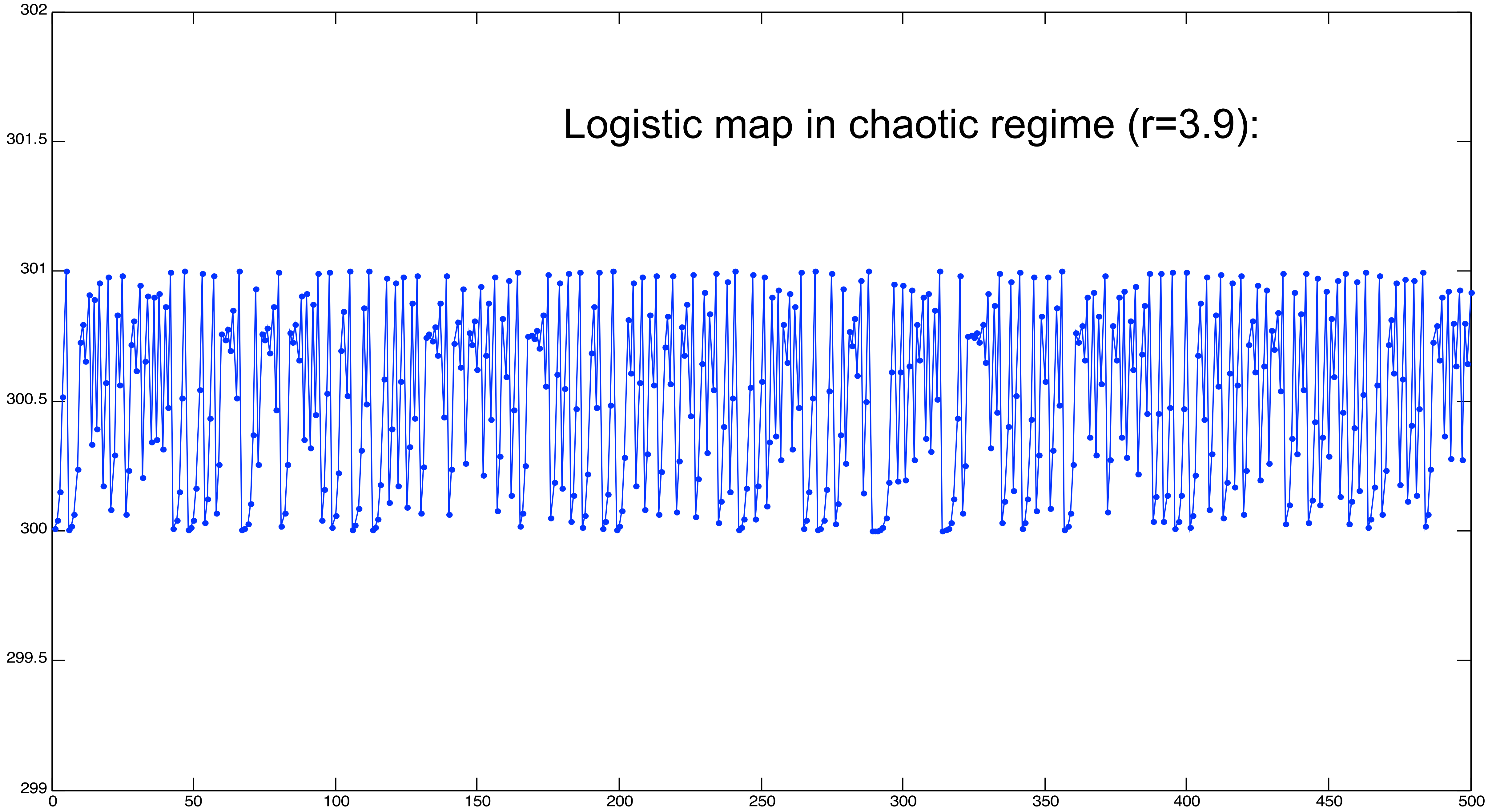

Box-Jenkins
Modeling Approach
Plot
Series
Is
Variance
stable
?
No
Apply
Transformation
Yes
Obtain
ACs and PACs
Is
Mean
Stationary
?
No
Apply Regular
and Seasonal
Differencing
Yes
Model
Selection
Estimate
Parameter
Values
Are
Residuals
Uncorrelated
?
No
Modify
Model
Yes
Are
Parameters
Significant and
Uncorrelated
?
No
Yes
Forecast
Model Selection Process in
Box-Jenkins Modeling Approach
ARMA (1 ,0)
?
ARMA (2, 0)
?
ARMA (2, 1)
?
ARMA (1, 1)
?
ARMA (0, 1)
?
ARMA (3, 0)
?
ARMA (3, 2)
?
ARMA (2, 2)
?
ARMA (1, 2)
?
ARMA (0, 2)
?
ARMA (n ,0)
?
ARMA (n, n-1)
?
ARMA (n-1, n-1)
?
ARMA (n-2, n-1)
?
ARMA (0, n-1)
?

Problems with ARfIMA (data assumptions)



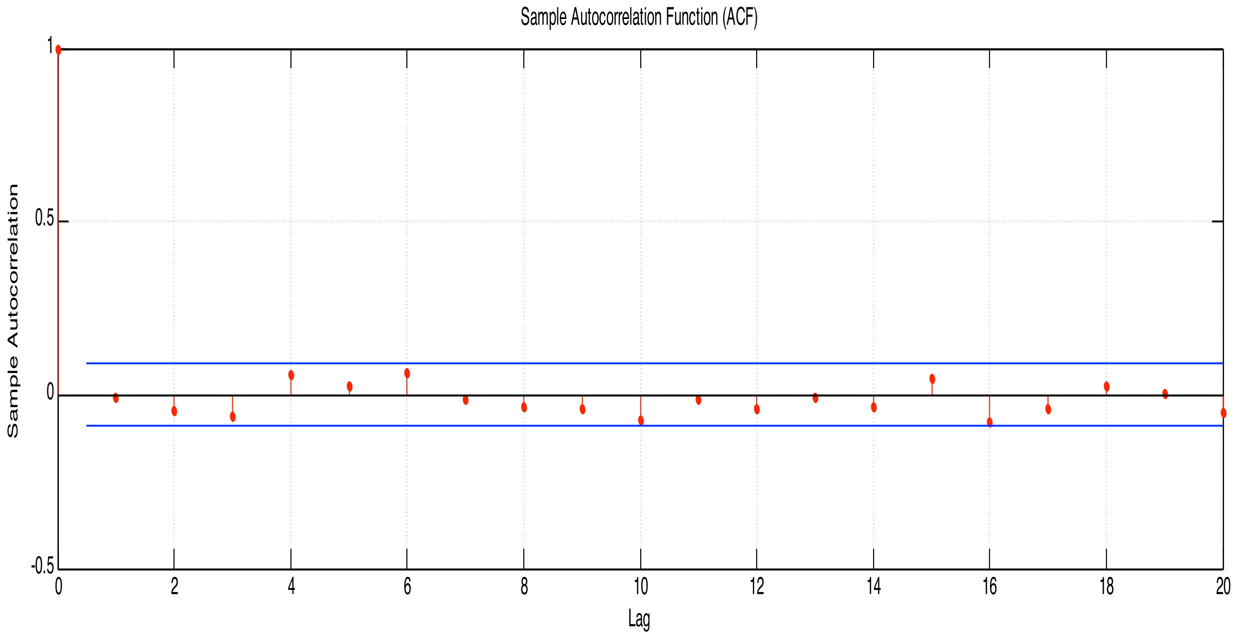
Problems with ARfIMA (data assumptions)

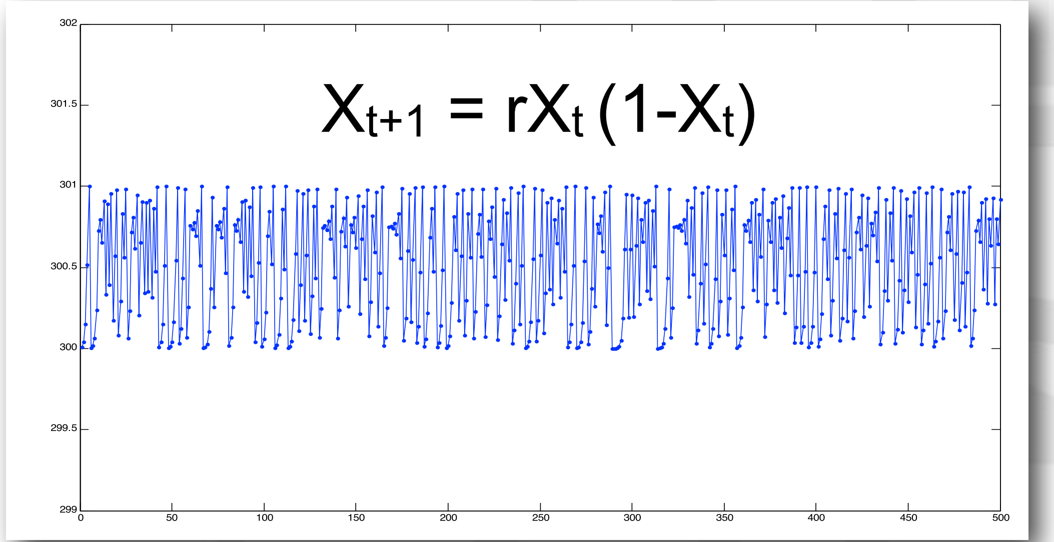
And what about deterministic CHAOS?



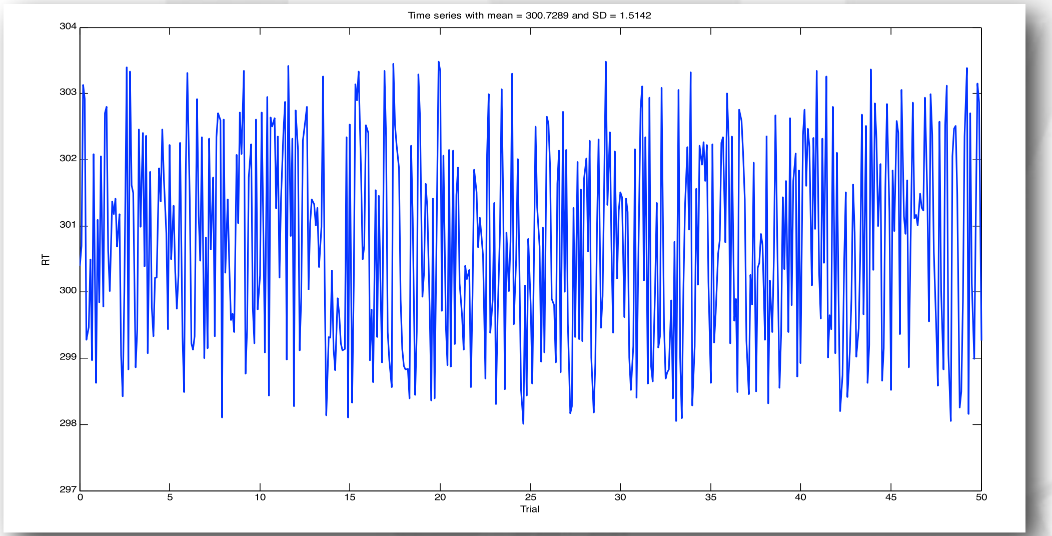
Problems with ARfIMA (data assumptions)

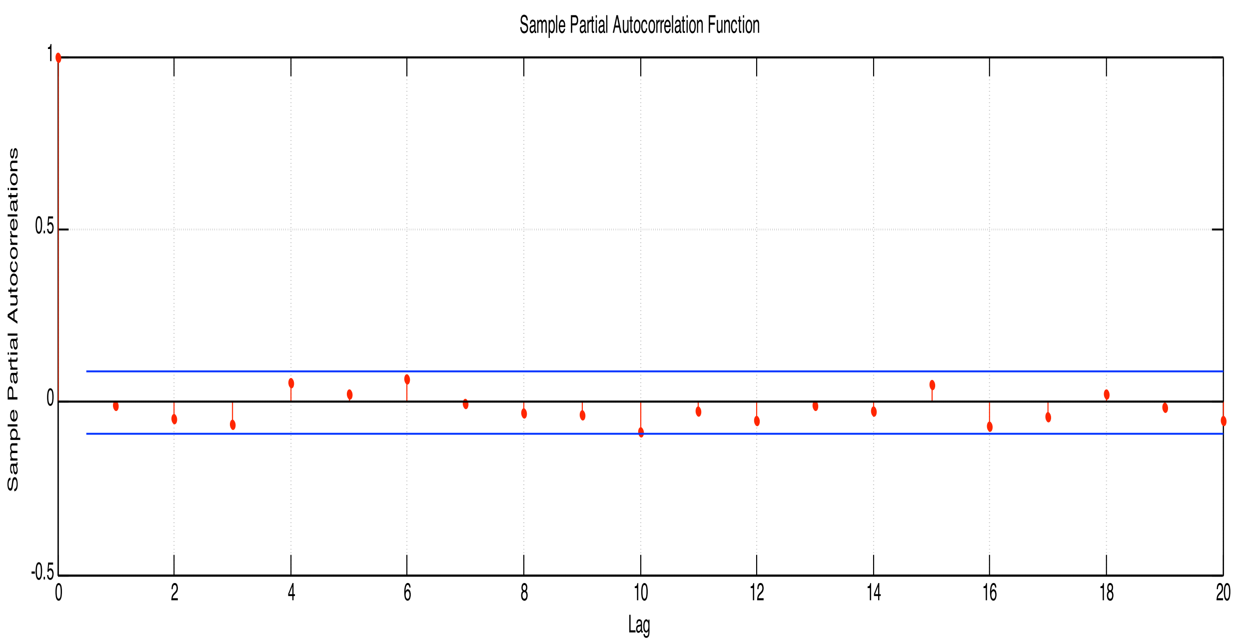
ARIMA(0,0,0) ??? - A Random Process ??? - But we know the equation !!!!





?





“Things that look random, but are not” (Lorenz, 1972)

Other problems with ARfIMA
(many have to do with modelling scaling)

frontiers in
PHYSIOLOGY

GENERAL COMMENTARY
published: 05 February 2014
doi: 10.3389/fphys.2014.00028

A comment on "Measuring fractality" by Stadnitski (2012)

Maarten L. Wijnants *

Behavioural Science Institute, Radboud University Nijmegen, Nijmegen, Netherlands
Correspondence: m.wijnants@pwo.ru.nl

Edited by:
Radhakrishnan Nagarajan, University of Kentucky, USA

Reviewed by:
Rathinaswamy B. Govindan, Children's National Medical Center, USA

Keywords: 1/f noise, fractal, ARFIMA, response time, long memory, cognitive science

frontiers in
PHYSIOLOGY

OPINION ARTICLE
published: 08 April 2013
doi: 10.3389/fphys.2013.00075

When the blind curve is finite: dimension estimation and model inference based on empirical waveforms

Fred Hasselman*

Learning and Plasticity, Behavioural Science Institute, Radboud University Nijmegen, Nijmegen, Netherlands
Correspondence: f.hasselman@bsi.ru.nl

Edited by:
Michael A. Riley, University of Cincinnati, USA

Reviewed by:
Michael A. Riley, University of Cincinnati, USA
John G. Holden, University of Cincinnati, USA
Sebastian Wallot, Aarhus University, Denmark

Scaling phenomena

How long is the coast of Great-Britain?

Scaling phenomena

scale	scale	scale
11.5 x 200 = 2300 km	28 x 100 = 2800 km	70 x 50 = 3500 km

Length systematically depends on the size of the measurement stick you use!

https://en.wikipedia.org/wiki/Fractal_dimension

Scaling phenomena

"scaling of bulk with size"

(Theiler, 1990)

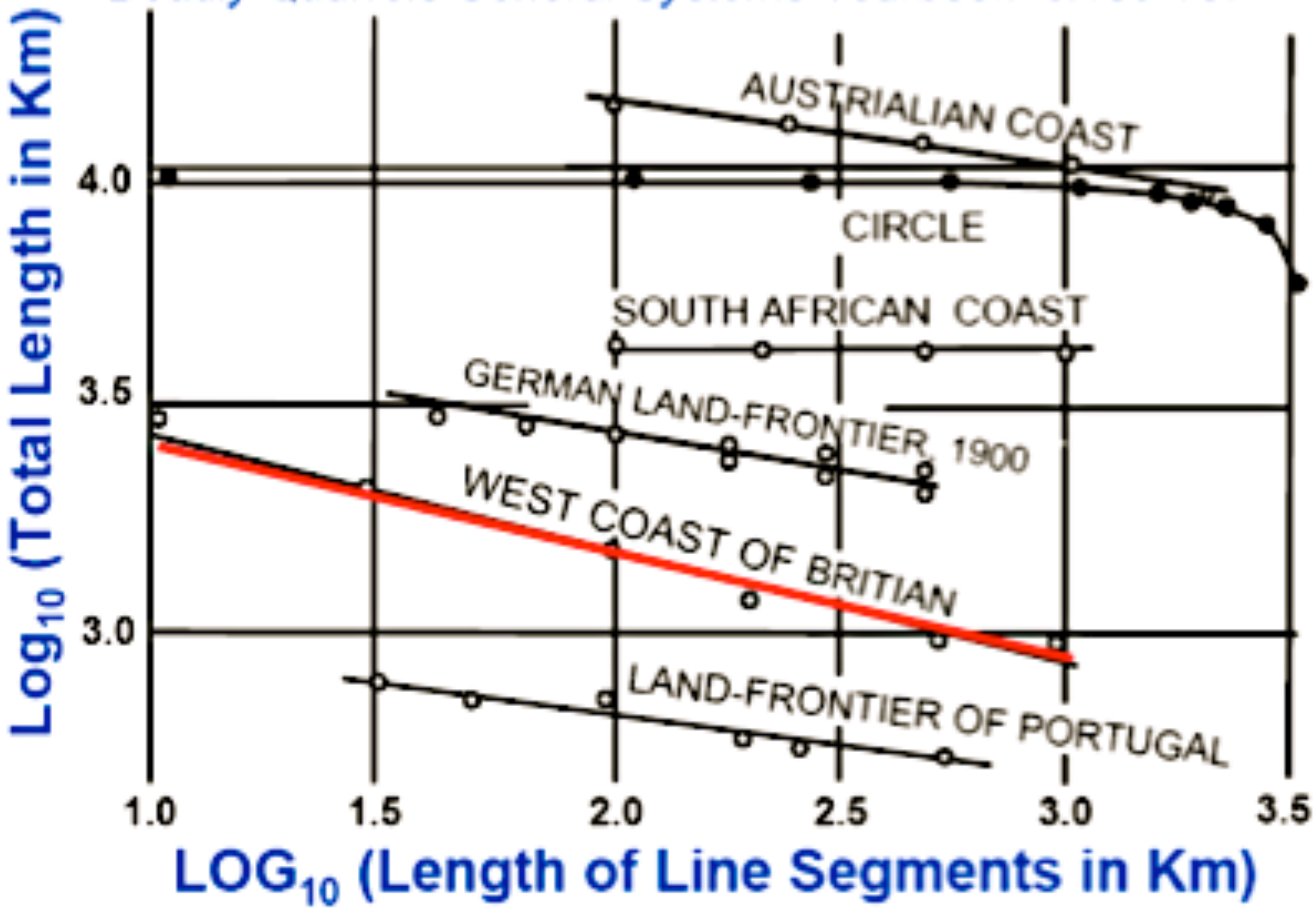
The formal answer to the question is:

"There is no characteristic scale at which the length of the coast of GB can be expressed"

Mandelbrot, B. B. (1967). How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science, 156*(3775), 636–8.

How Long is the Coastline of Britain?

Richardson 1961 The problem of contiguity: An Appendix to Statistics of Deadly Quarrels General Systems Yearbook 6:139-187



Scale invariance…

no meaningful central moments can be defined

Mean and SD characterise the data only relative to the scale of observation (e.g. sample size)

A *power law* scaling relation (**LOG scale**):
There is no characteristic length, just an indication of complexity

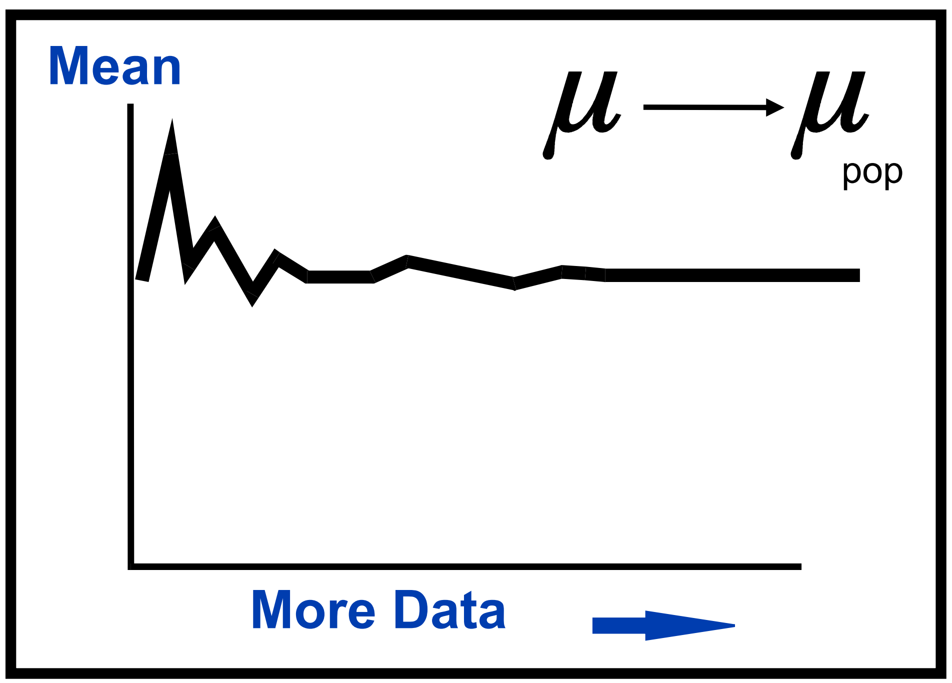
Scaling phenomena

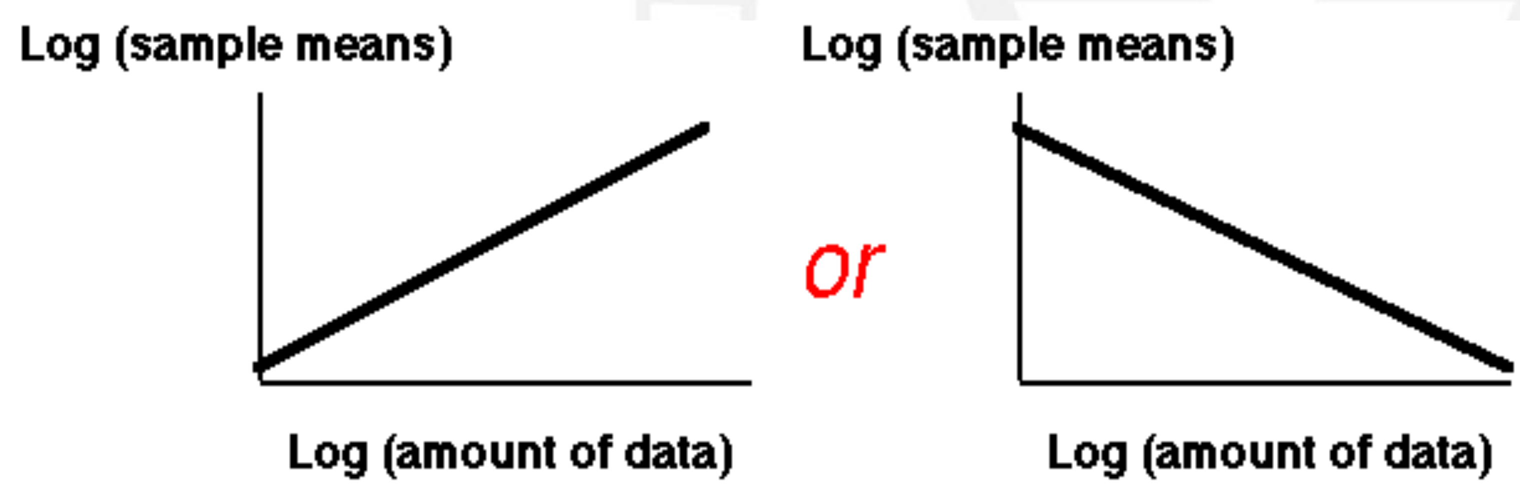
Independent observations of random variables

μ±σ are sufficient to characterise absence of dependencies in the data:
e.g. Expected value of **μ** for **N** = 100, given **σ**

Interdependent observations across different scales

μ±σ are insufficient to characterise dependencies in the data:
e.g. Sample estimates of **μ** change with **N**





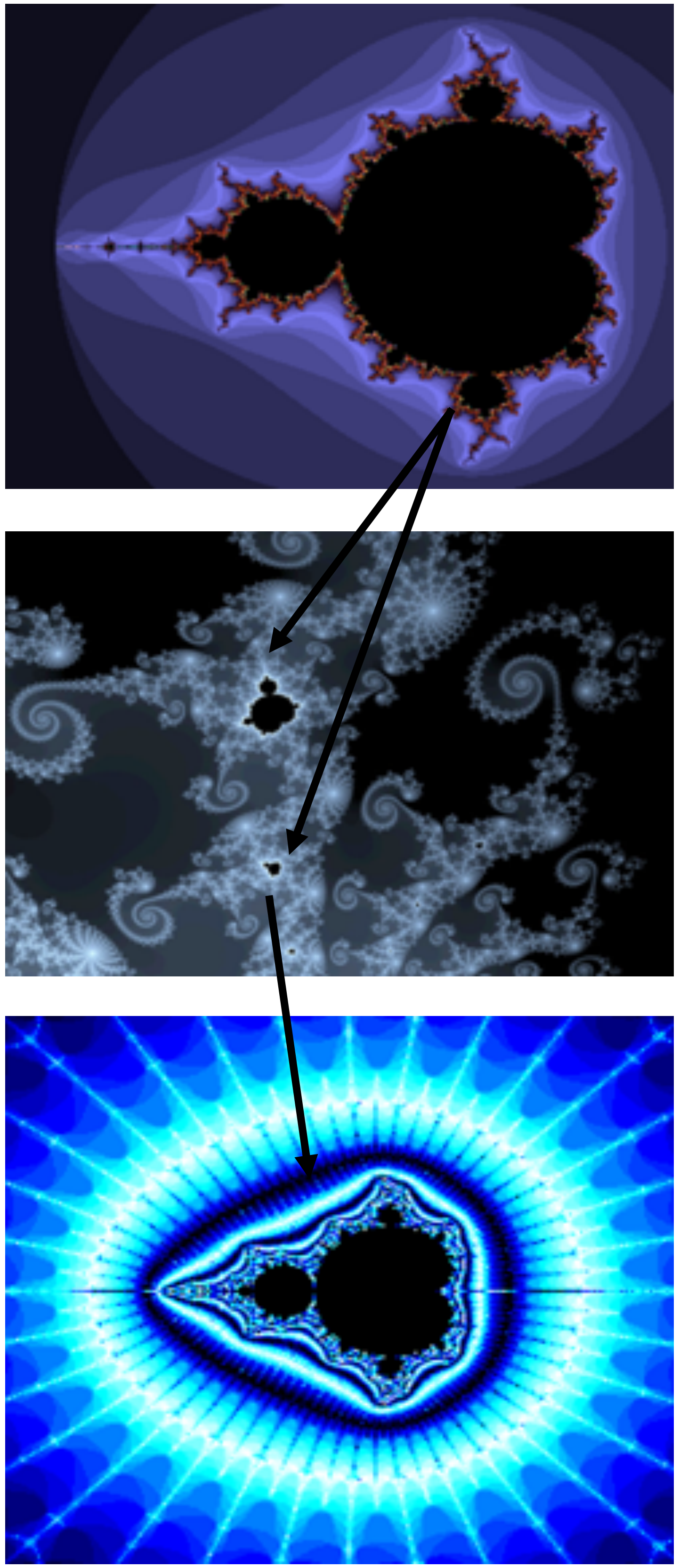
Help! How can I do science without **μ** or **σ**?

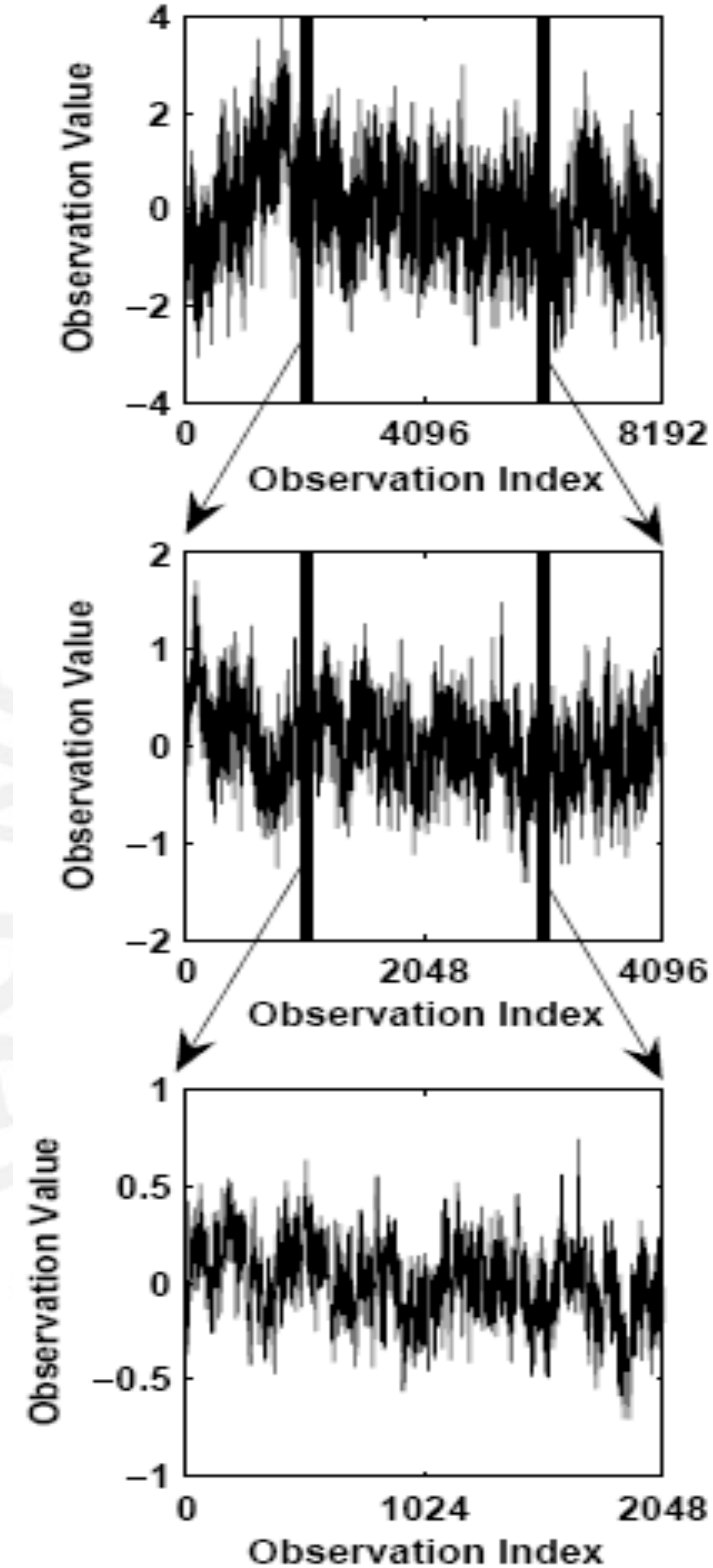
What is scaling?
Self-similarity and Self-affinity

Object looks roughly the same on all scales = (Statistical) **self-similarity** ("zoom similarity")

(Statistical) self-similarity is observed after affine transformation = **self-affinity** ("warp similarity")

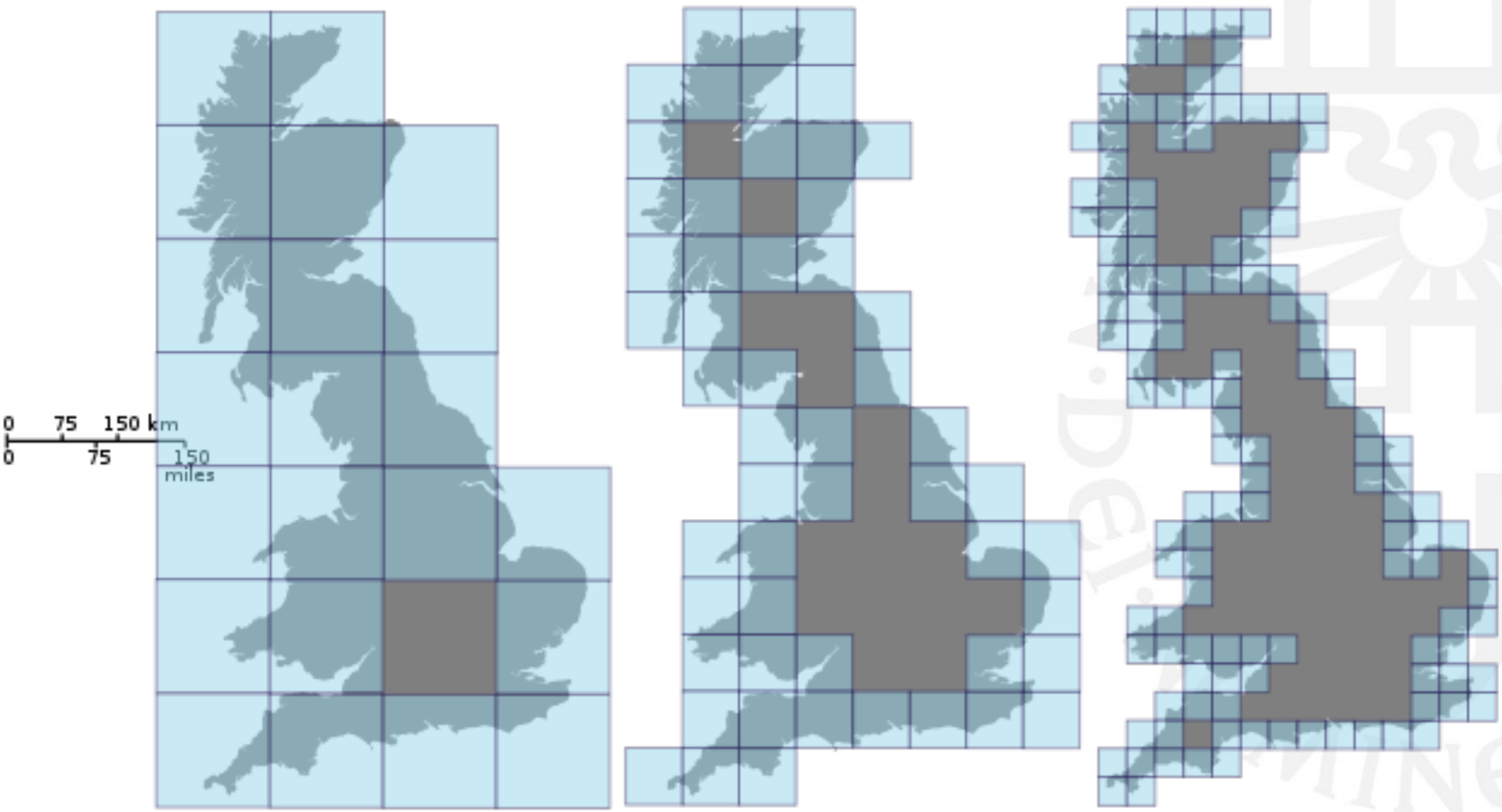
Degree of invariance across scales = Dependencies/regularities/correlations across scales



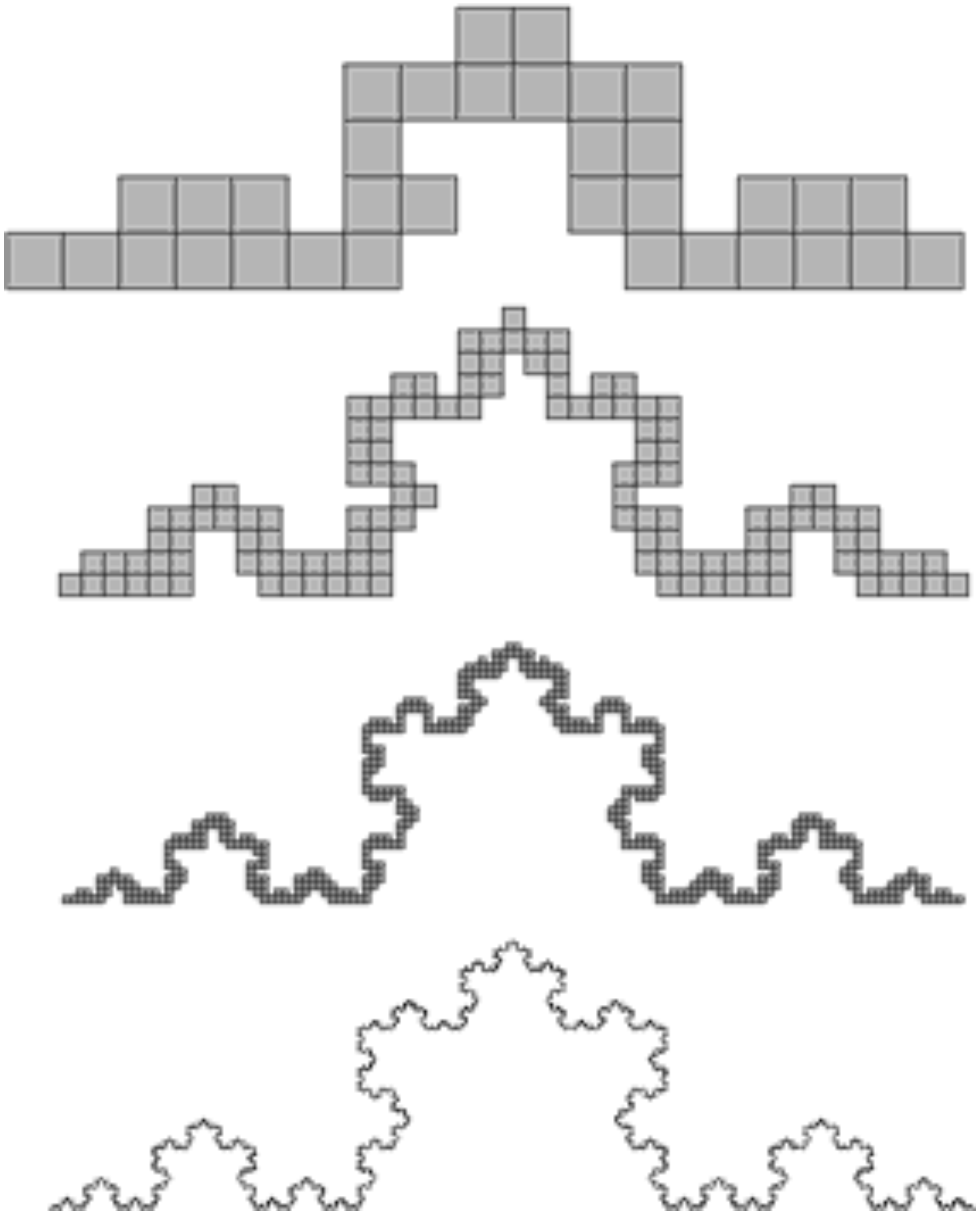


aka: “Fractal scaling”

How to describe scaling relations:
Calculate a “fractional” dimension, e.g. box-counting dimension



Solution:
Calculate a “fractional” dimension, e.g. box-counting dimension



Hausdorff-Besicovitch dimension

(box-counting dimension, covering dimension, packing dimension, mass-radius, circle-counting, etc, etc)

$$D = \frac{\log \mathbf{N_h}}{\log 1/\mathbf{h}}$$

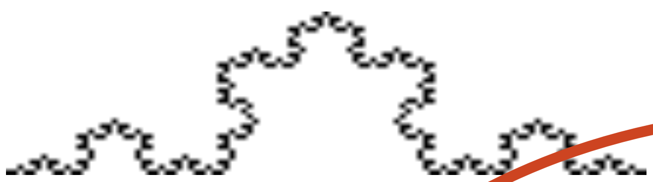
N= number of blocks of size **h** needed to cover the object

Relation between *measure stick* and *measurement outcome*, or: *“scaling of bulk with size”*

Scaling phenomena

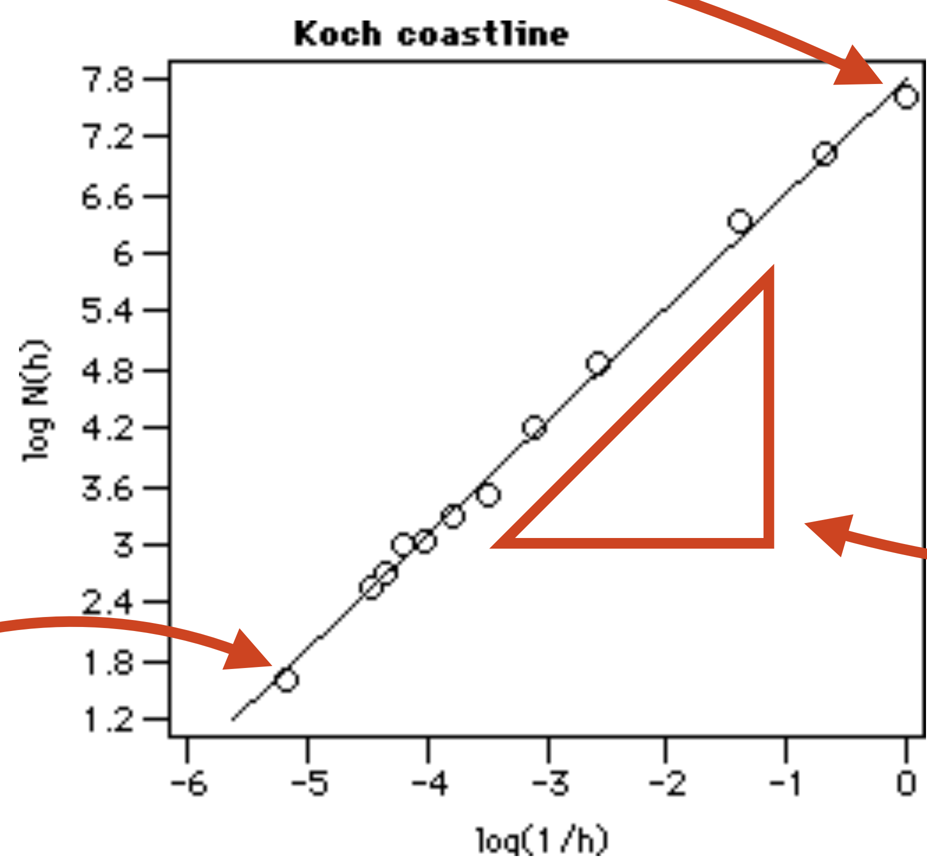
Measuring dimension

Koch Coastline



log (1/h)	log N(h)
0.00	7.61
-0.69	7.04
-1.39	6.33
-2.56	4.86
-3.09	4.22
-3.50	3.53
-3.78	3.30
-4.01	3.04
-4.19	3.00
-4.34	2.71
-4.48	2.56
-5.18	1.61

dimension (experimental) = **1.18**
dimension (analytical) = **1.26**
deviation = 6%

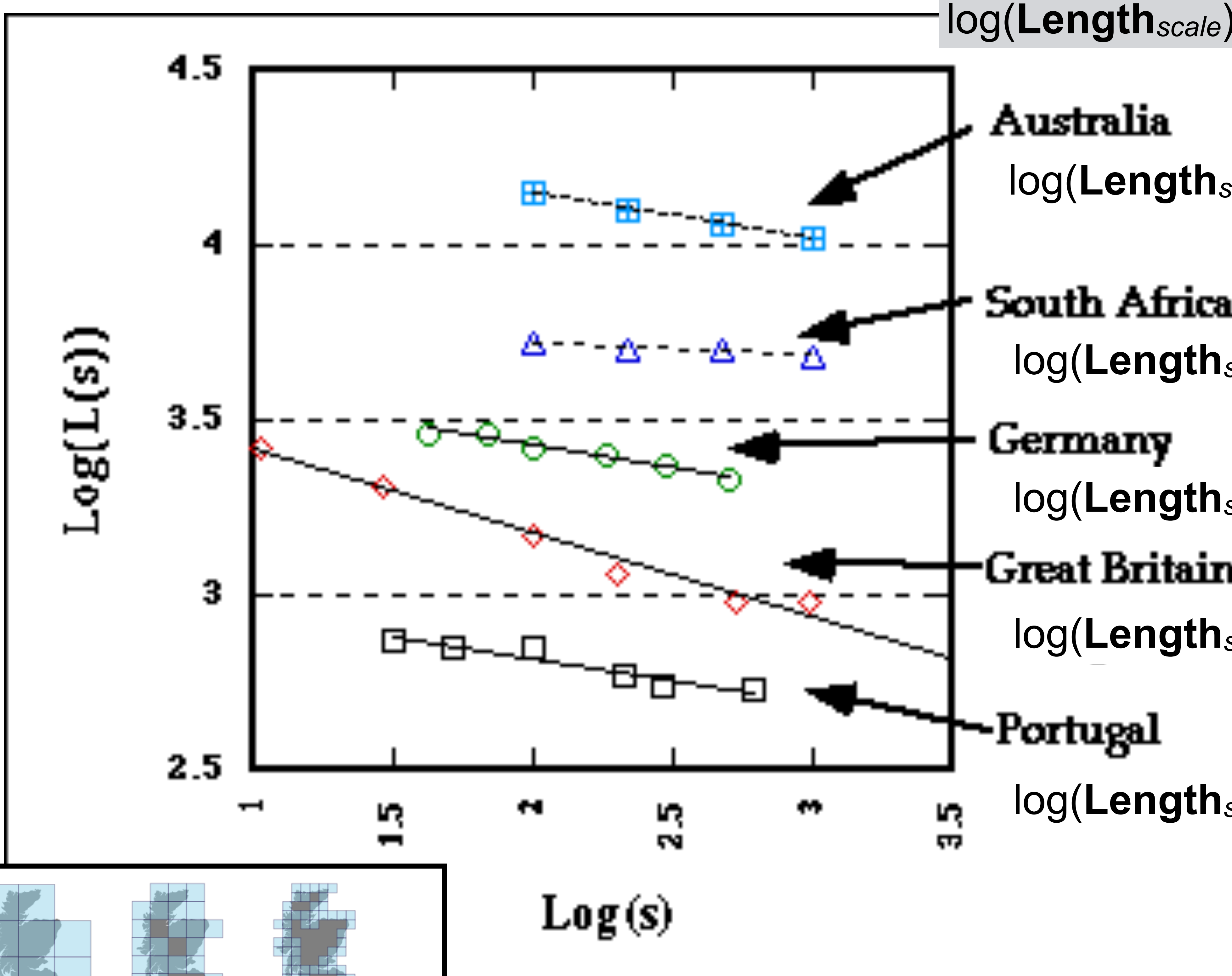


Fractal dimension
it's a fraction!

```
Call:
lm(formula = L ~ invS, data = df)

Residuals:
     Min       1Q   Median       3Q      Max
-0.18777 -0.06292  0.02390  0.06059  0.16703

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  7.79777    0.07318  106.55  < 2e-16 ***
invS         1.17611    0.02109   55.75 8.35e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.'
0.1 ' ' 1

Residual standard error: 0.1137 on 10 degrees of
freedom
Multiple R-squared:  0.9968,    Adjusted R-squared:
0.9965
F-statistic:  3109 on 1 and 10 DF,  p-value: 8.355e-14
```

Scaling phenomena

log(**Length**$_{scale}$) = b*log(*scale*)+a



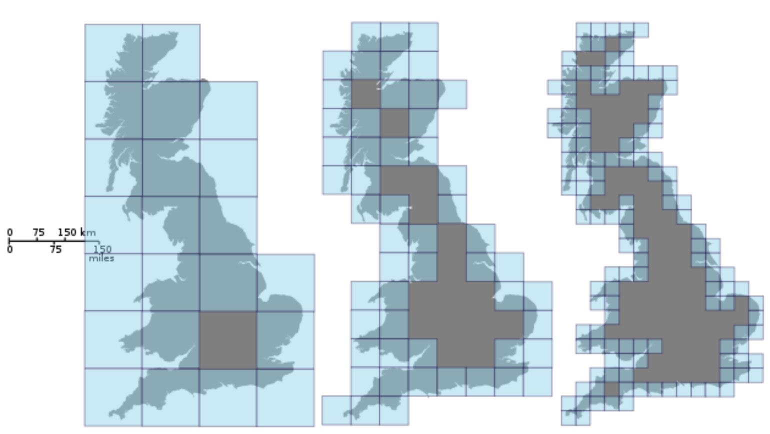
Australia: log(**Length**$_s$) = -.13*log(*s*)+4.4

South Africa: log(**Length**$_s$) = **-.03***log(s)+3.8

Germany: log(**Length**$_s$) = -.12*log(s)+3.7

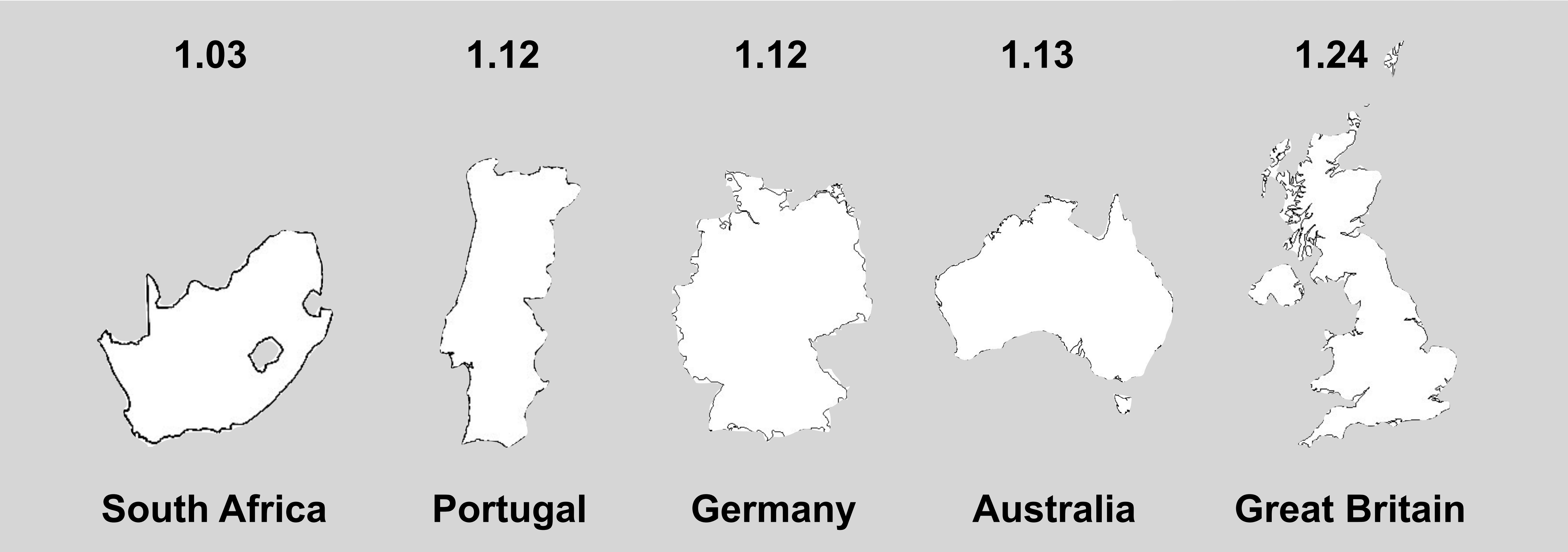
Great Britain: log(**Length**$_s$) = **-.24***log(s)+3.4

Portugal: log(**Length**$_s$) = -.12*log(s)+3.3



Scaling phenomena

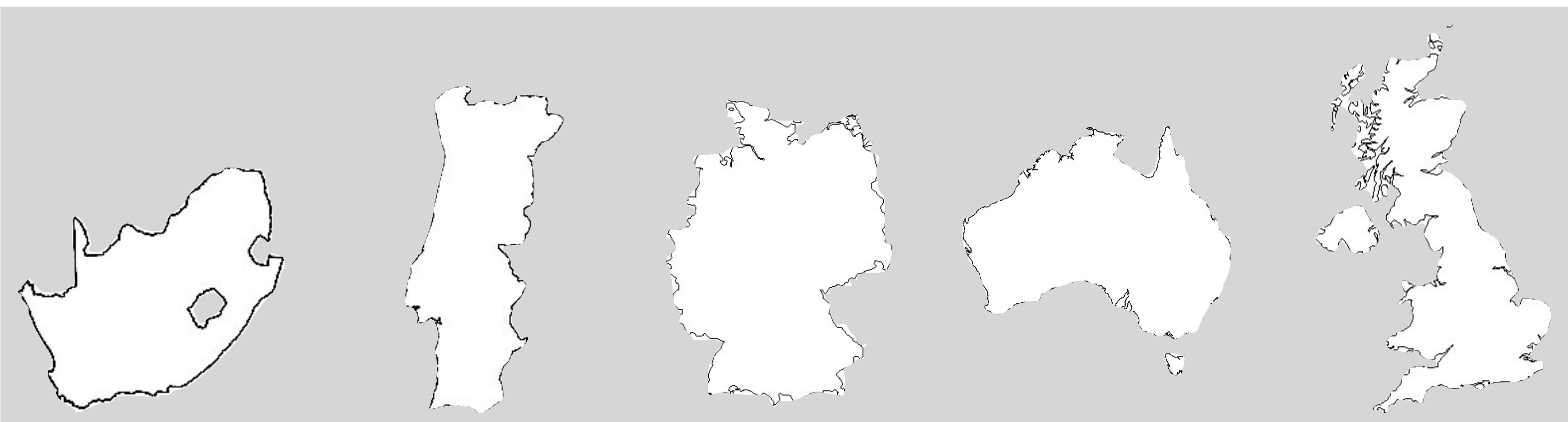
Scaling and Complexity



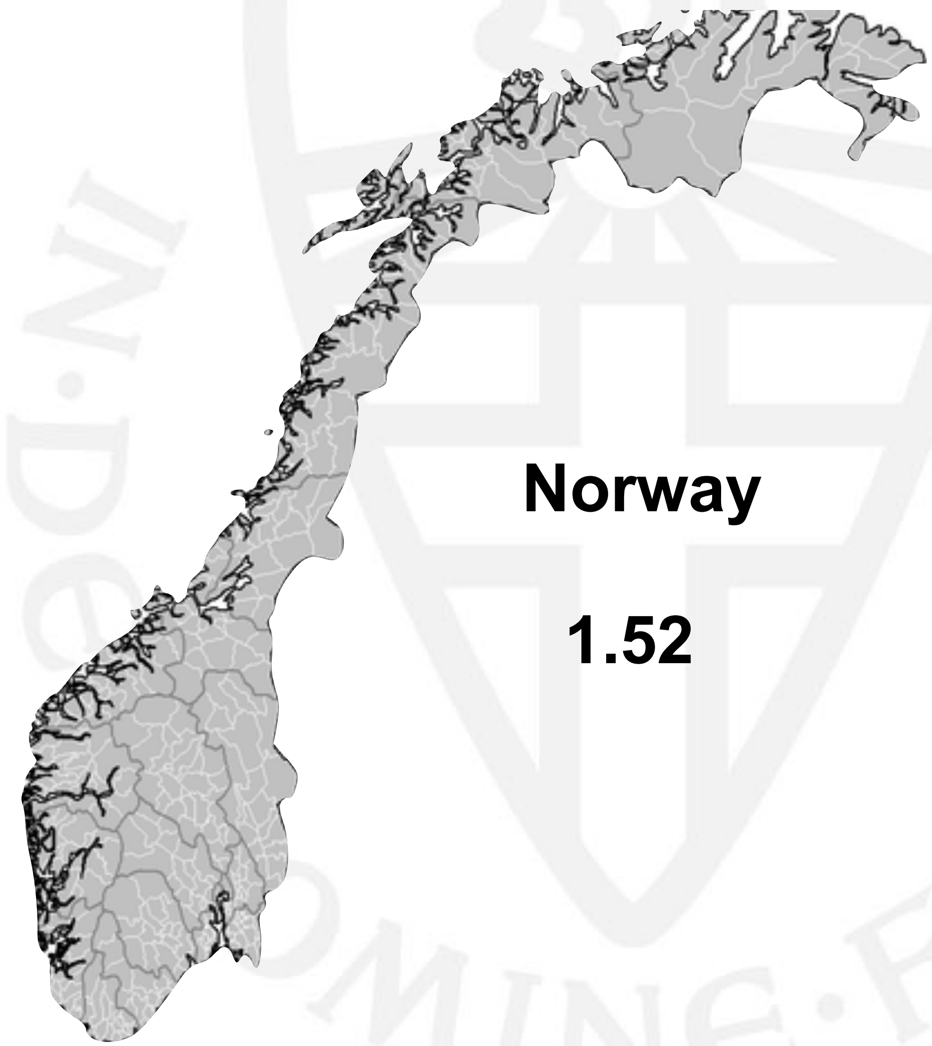
Ordered by scaling exponent, the log-log slope

Scaling phenomena

Scaling and Complexity



Ordered by scaling exponent, the log-log slope

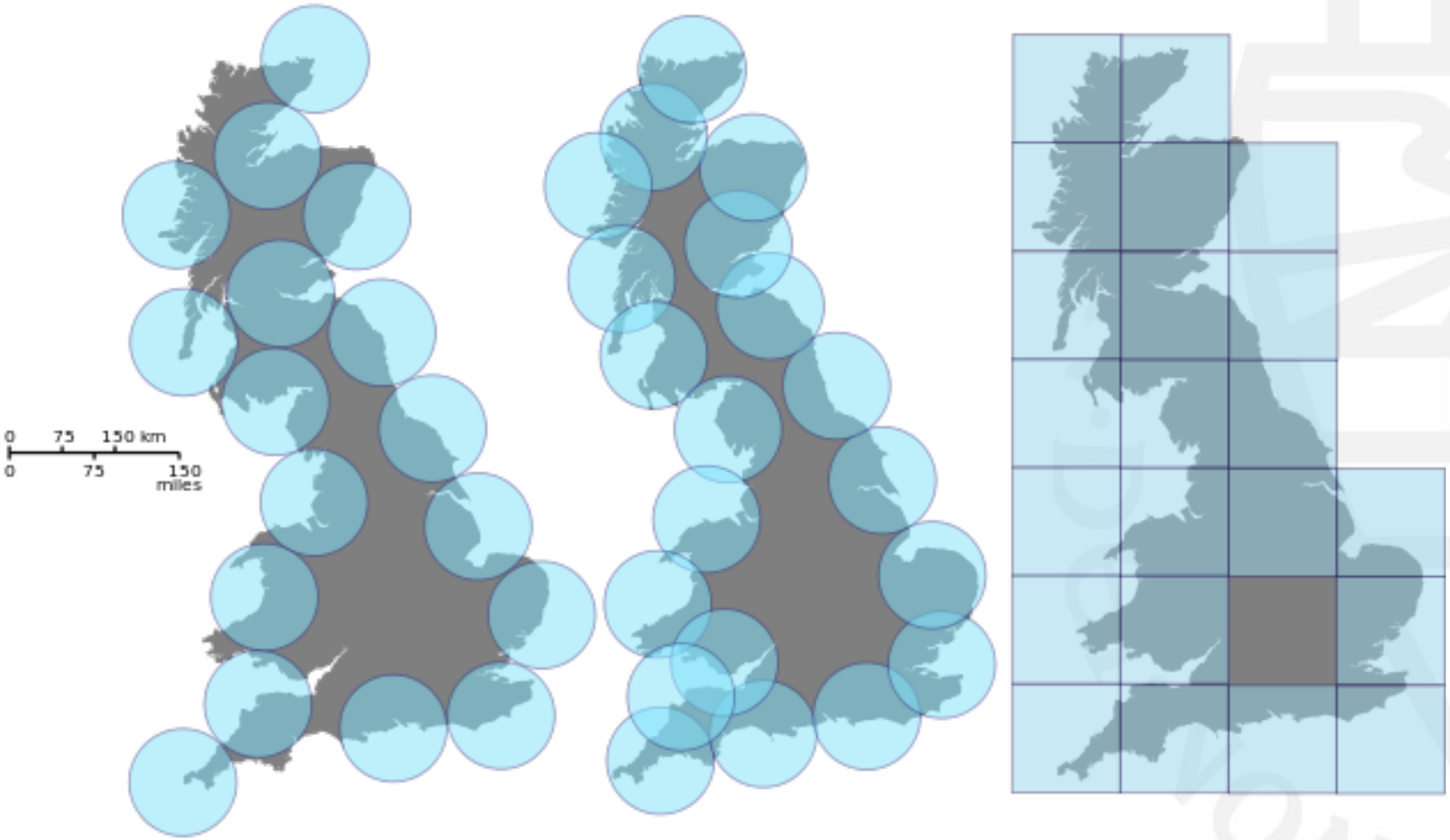


"one of the essential features of a fractal is that its Hausdorff dimension strictly exceeds its dimension"

https://en.wikipedia.org/wiki/List_of_fractals_by_Hausdorff_dimension

Scaling phenomena

Many variants of “covering” dimensions



ball packing < ball covering < box covering

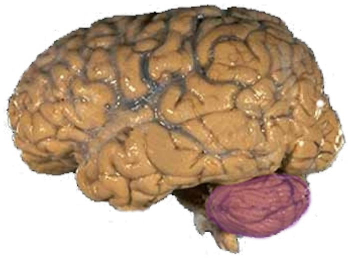
Scaling phenomena

Measuring dimension

Packing Cubes or Spheres and Wrapping Blankets:

3D spatial scaling relations in nature - Cauliflower
Fractal dimension = 2.33

Surface of human brain: 2.79



Surface of human lungs: 2.97

Scaling phenomena

Scaling relations are common in nature:

Earthquakes (Richter-Law) and the distribution of mass in the Universe

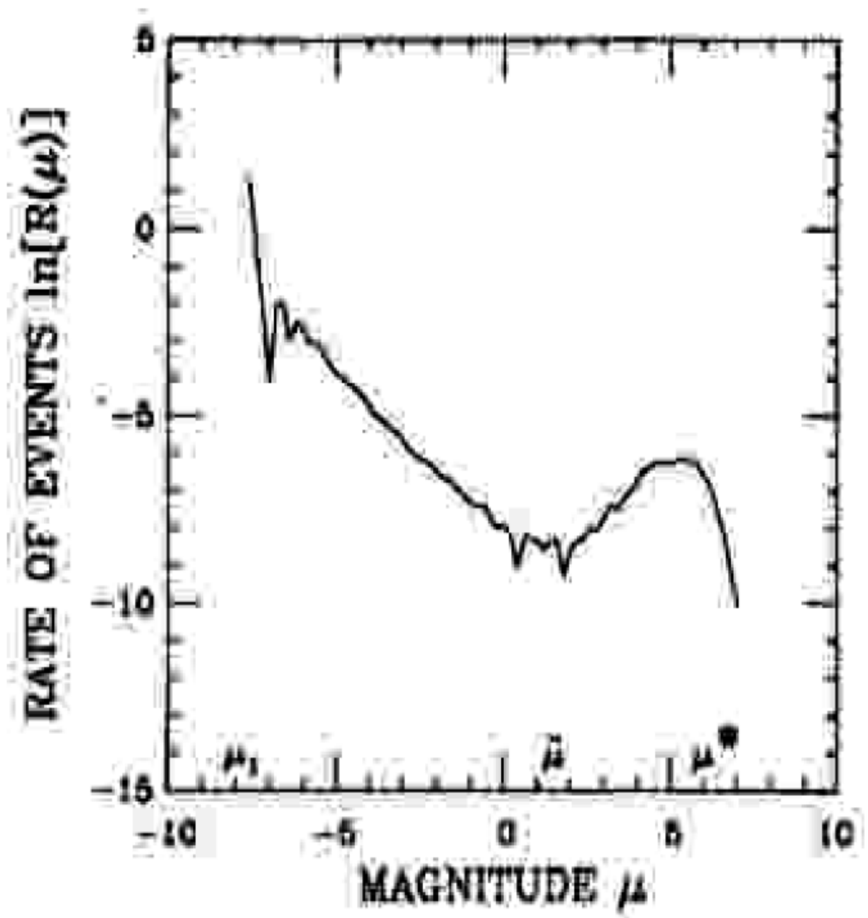
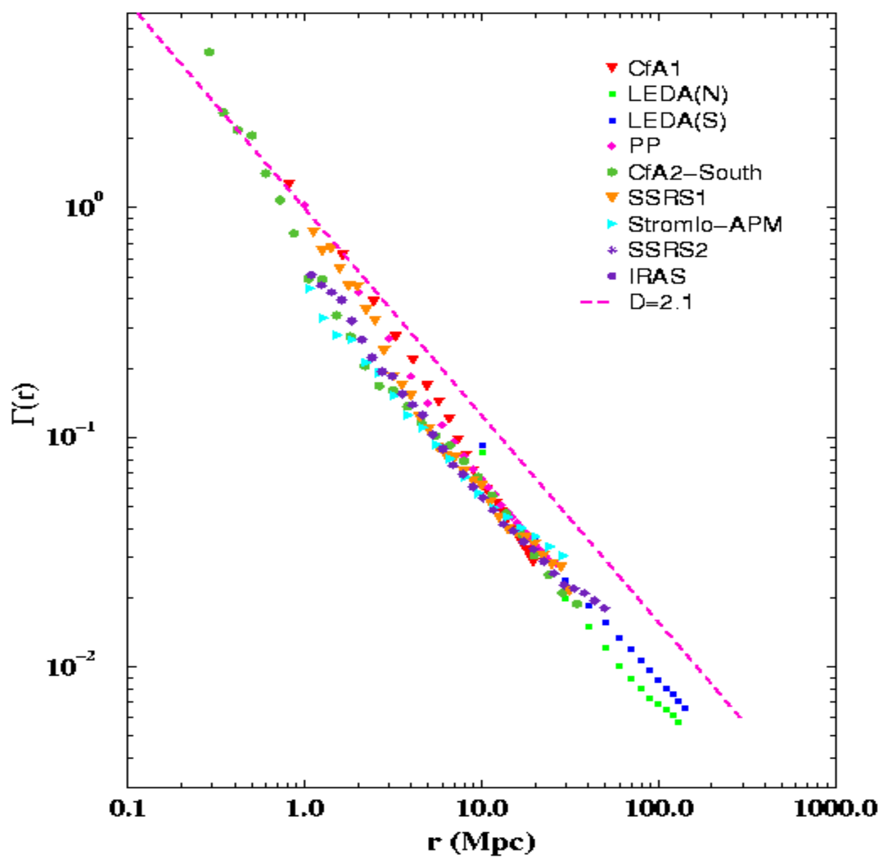


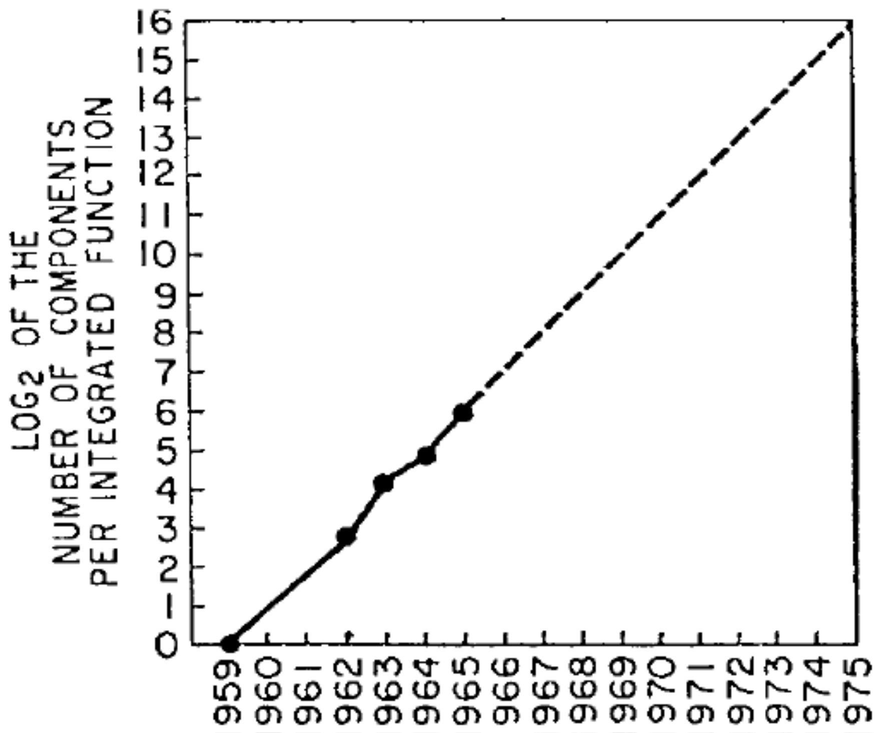
Figure 13: frequency distribution of the slip events (earthquakes) of magnitude μ taken from [53]. Notice the large bump that corresponds to an excess of events of high magnitude.



Scaling phenomena

Moore's Law:

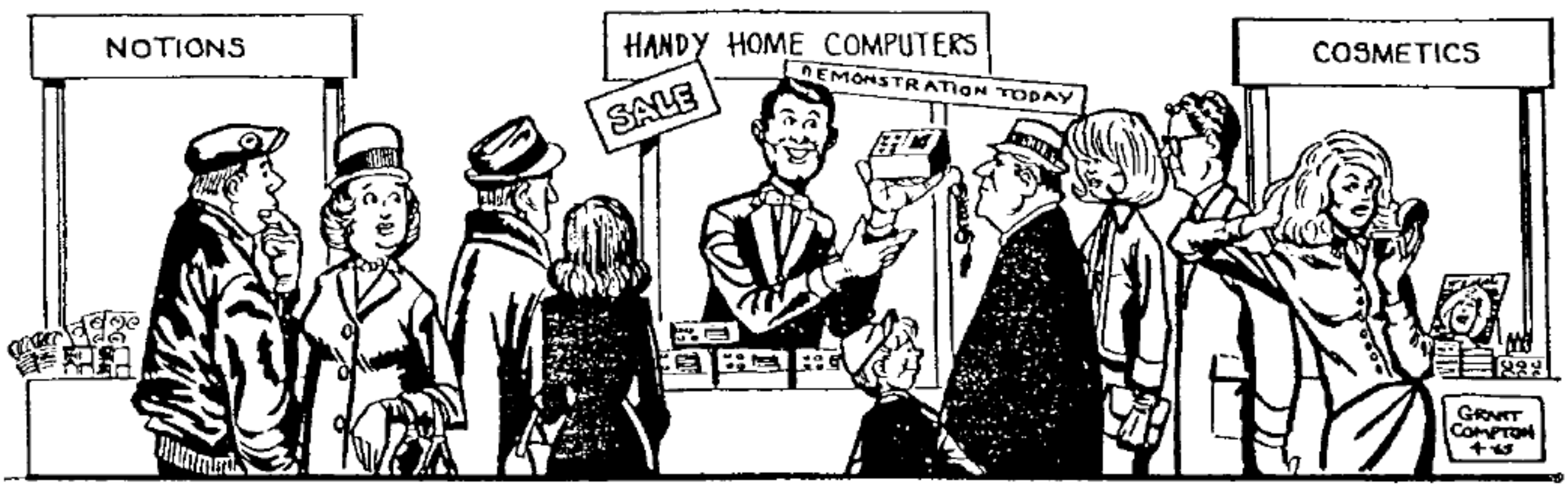
Predicted if speed of innovations in "cramming more components onto integrated circuits" kept up …



Moore's Law:

… we would soon be buying computers at the local market …

which apparently was a preposterous idea



Moore, Gordon E. (1965). "Cramming more components onto integrated circuits"


VANAF WOENSDAG 26-02
ONZE AANBIEDINGEN
Tablet PC cover
6.99
GSM AT-B26D
FM-RADIO
BLUETOOTH 2.1
OP=OP
Per stuk
49.99
10" Tablet MD98516
16 GB GEHEUGEN
Per stuk
179.00
ALDI TALK
ALDI
Aperitiefbiscuits
0.99
Basis voor soep
1.29
Munt- of honingdrop
0.99
Mini-stroopwafels
1.89
Spijsbroodjes of appelflappen
Smulgerechten
Chips patatje kapsalon of hete kip
0.99
Wireless speaker adapter
14.99
Bevestigingsset
2.99
Hortensia
4.79
6-7/16
Index
Trefwoord


Kruidvat
voordeel magazine
Goed gekleed
DAMESJASJE
M t/m XL
39.99 19.99
Stralende lippen
MAX FACTOR LIPSTICK, -GLOSS OF -LINER
ALLES VOOR € 10.00 PER STUK
Perfect haar
NIVEA HAARVERZORGING OF STYLING
1+1 GRATIS
Lekker schoon
ROBOTSTOFZUIGER
Dirt Devil
129.99 79.99
STEEDS VERRASSEND, ALTIJD VOORDELIG!
Geldig van dinsdag 25 februari t/m zondag 9 maart 2014

The Dynamics of Complex Systems

#6 Fractal Geometry in Time Series: Analysis of the Colour of Noise

Fred Hasselman

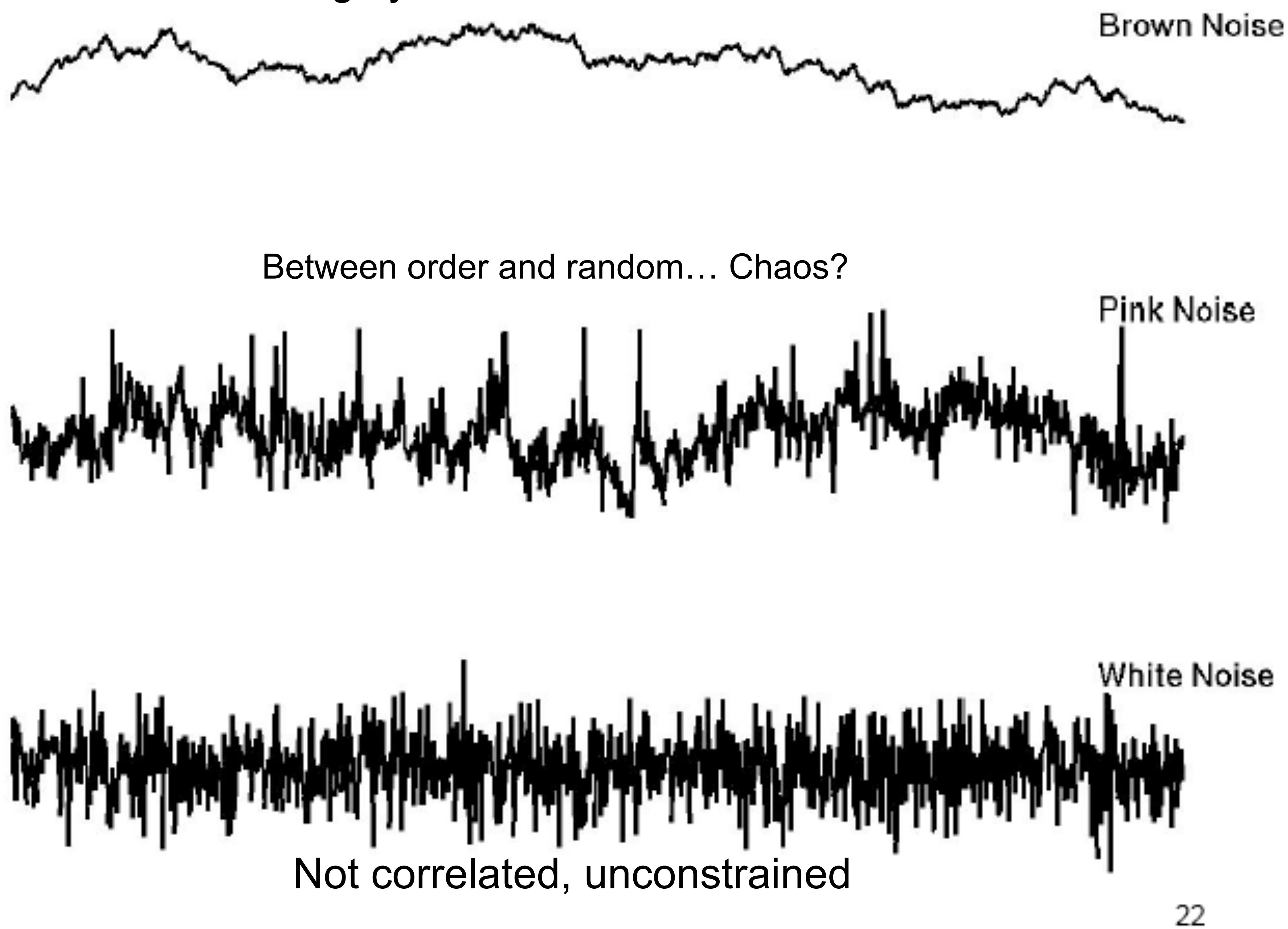
“Statistics”: Correlations, *inter*dependece of repeated measurements

Highly correlated, constrained

Brownian motion, or random walk processes

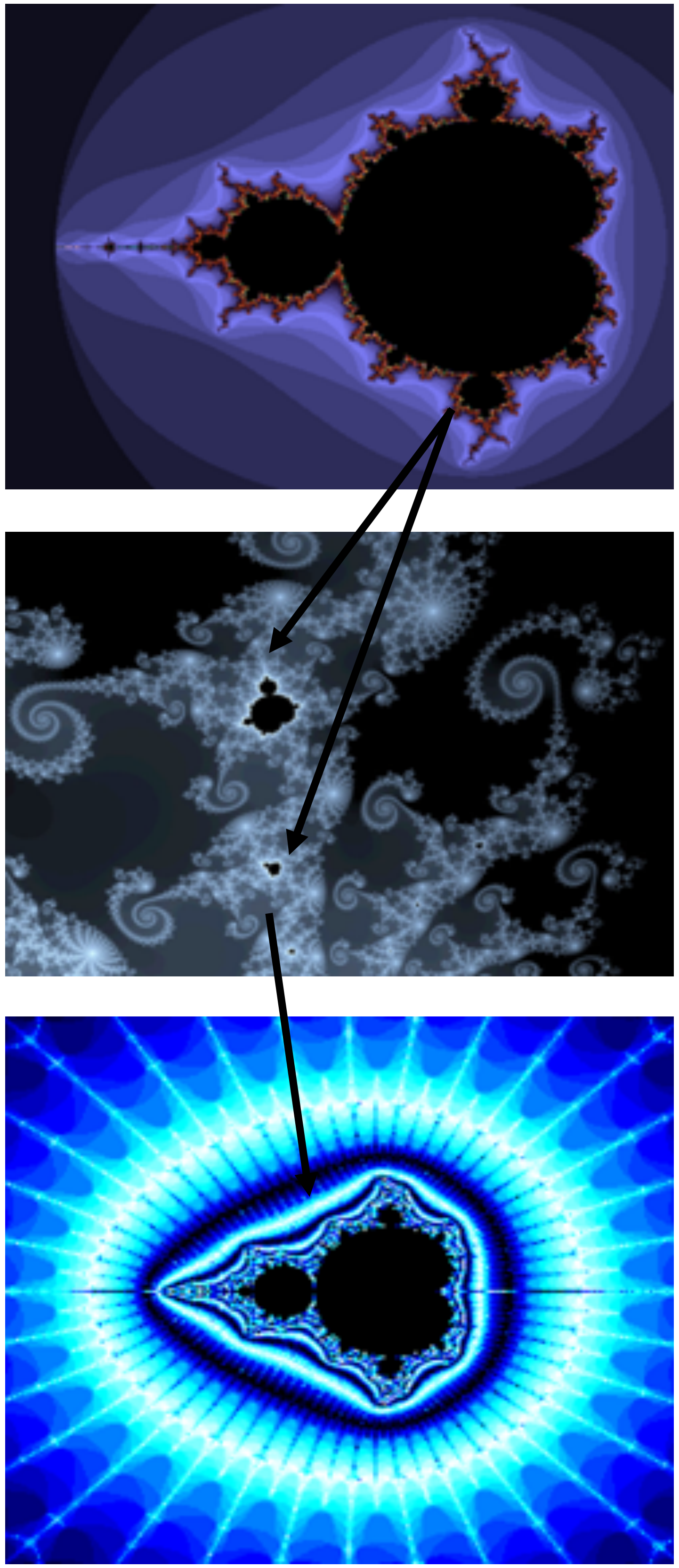
Between order and random… Chaos?

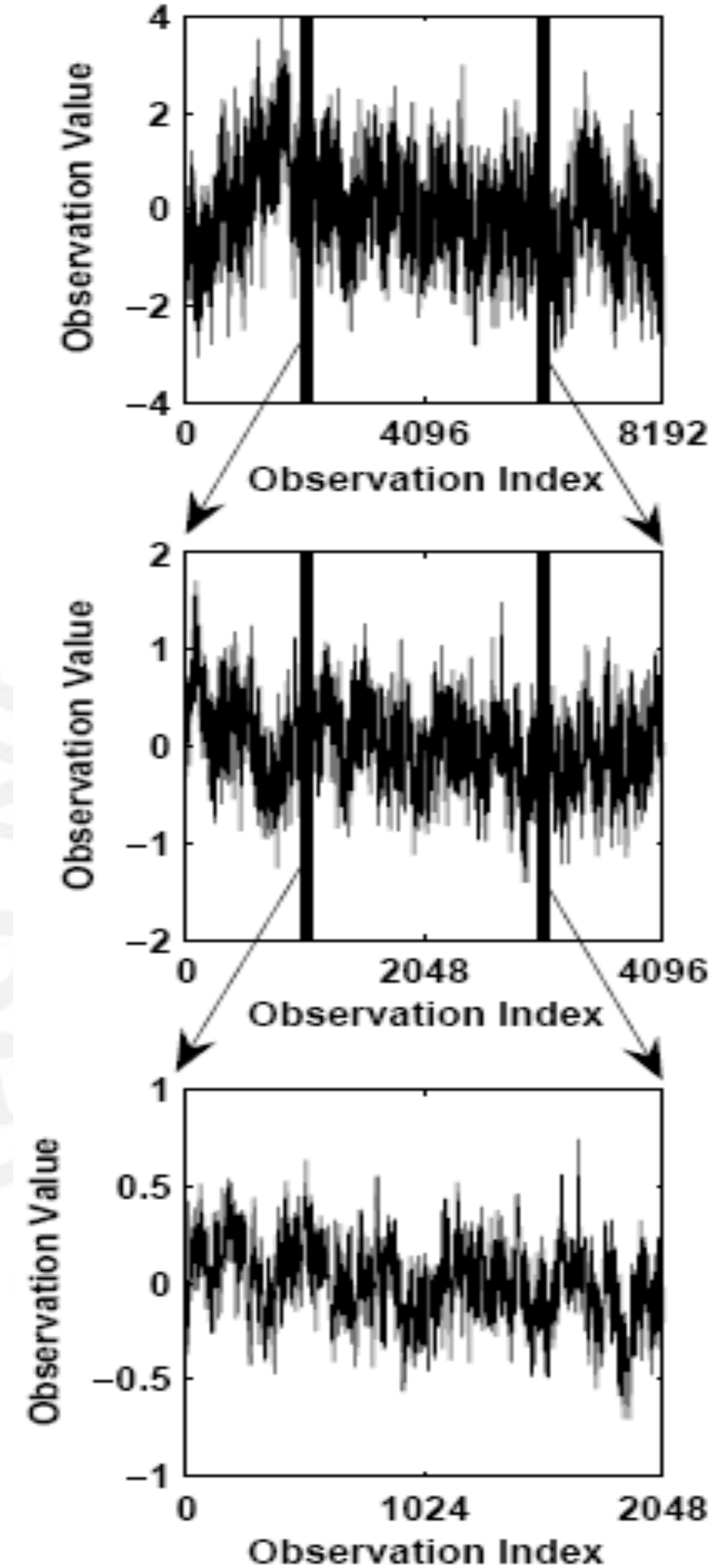
Associated with Self Organizing Complex Systems



Gaussian, random processes

Not correlated, unconstrained





Observation Value
4
2
0
−2
−4
0
4096
8192
Observation Index
Observation Value
2
1
0
−1
−2
0
2048
4096
Observation Index
Observation Value
1
0.5
0
−0.5
−1
0
1024
2048
Observation Index

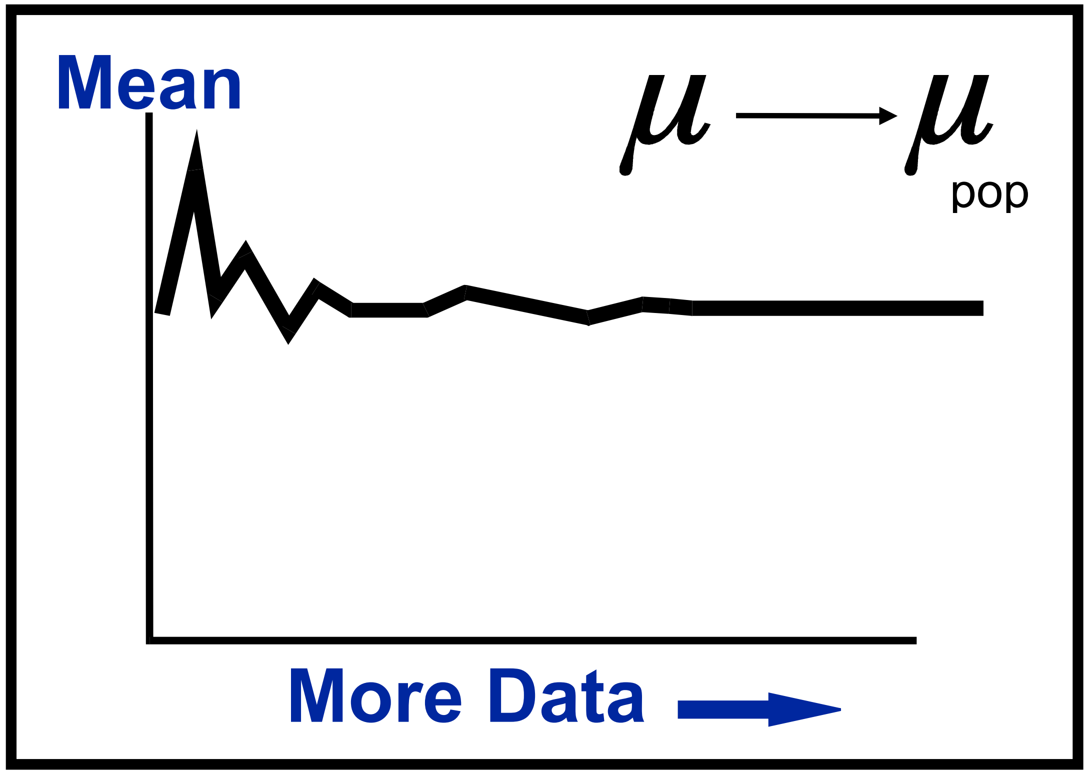
“Statistics”: Variance will not decrease with more observations!!!

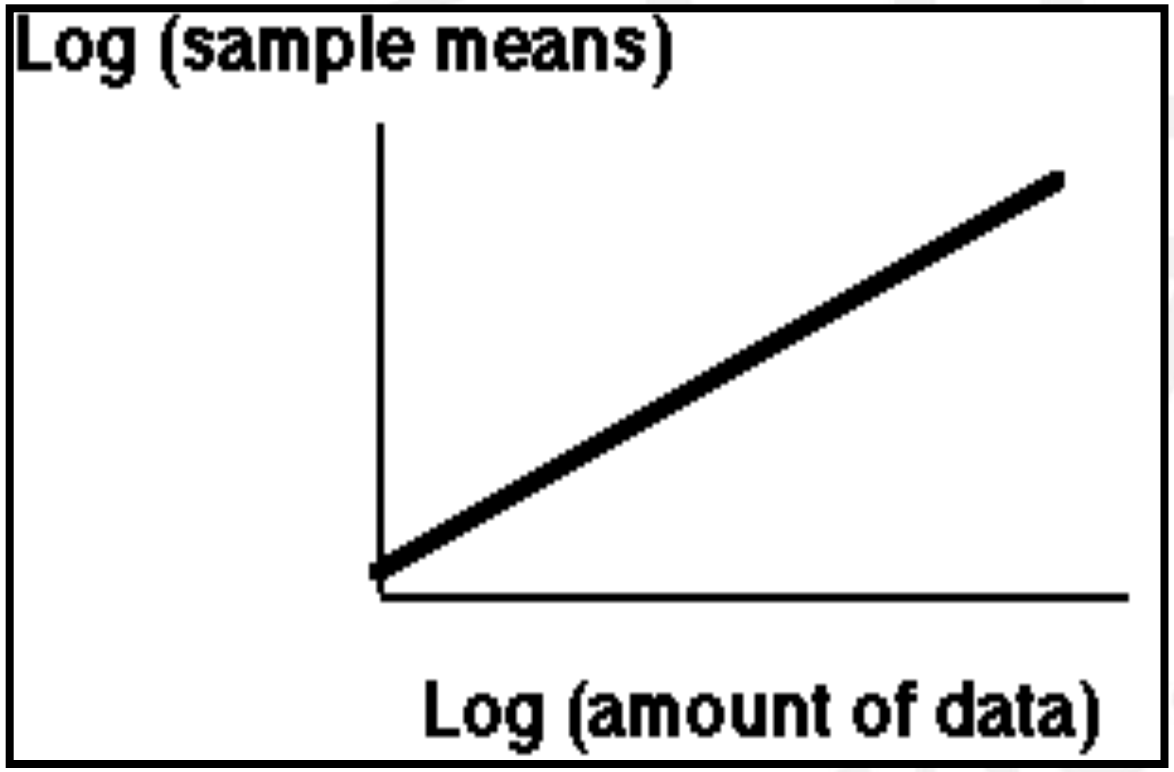
Random processes
Random variables

Independent observations
(no-similarity = random)
Characteristic scale: **T**
(the population)

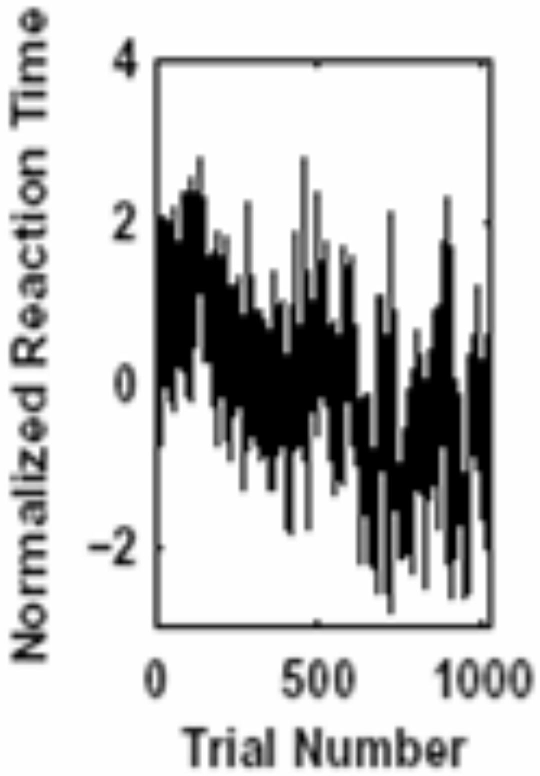
Fractal processes
Fractal variables

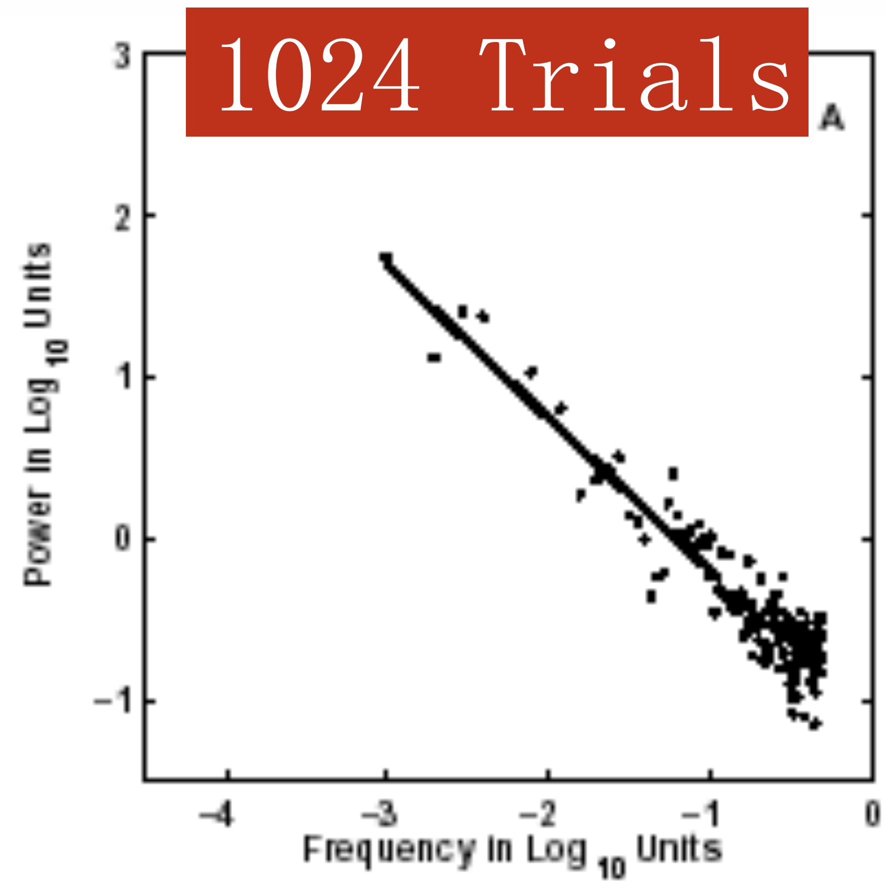
Interdependent observations over different scales!
(self-similarity = “correlated”)
No characteristic scale means: **T** does not exist!
(at least not on 1 scale)

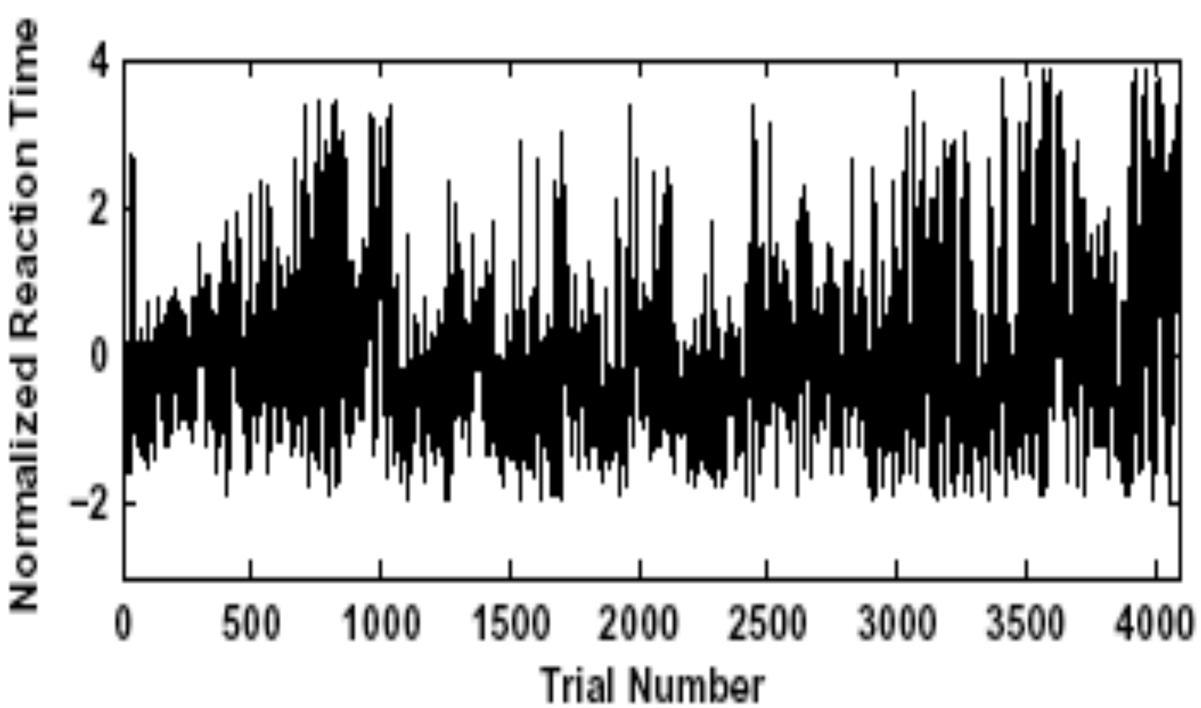


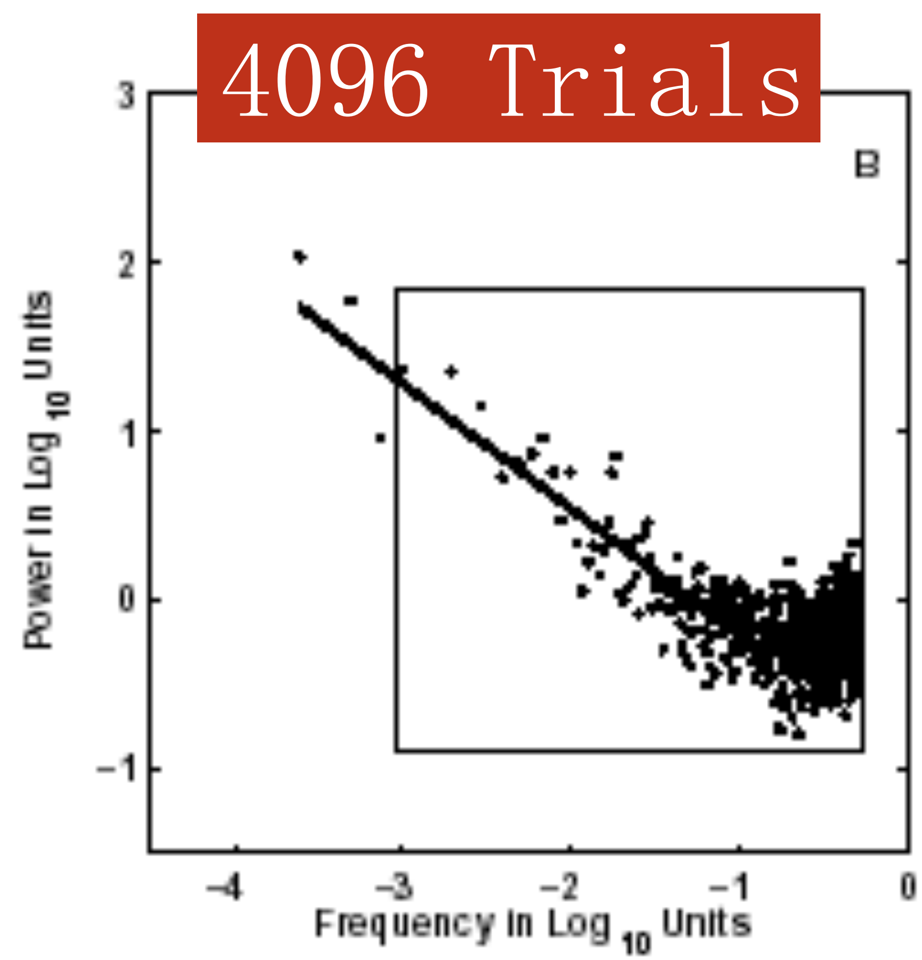


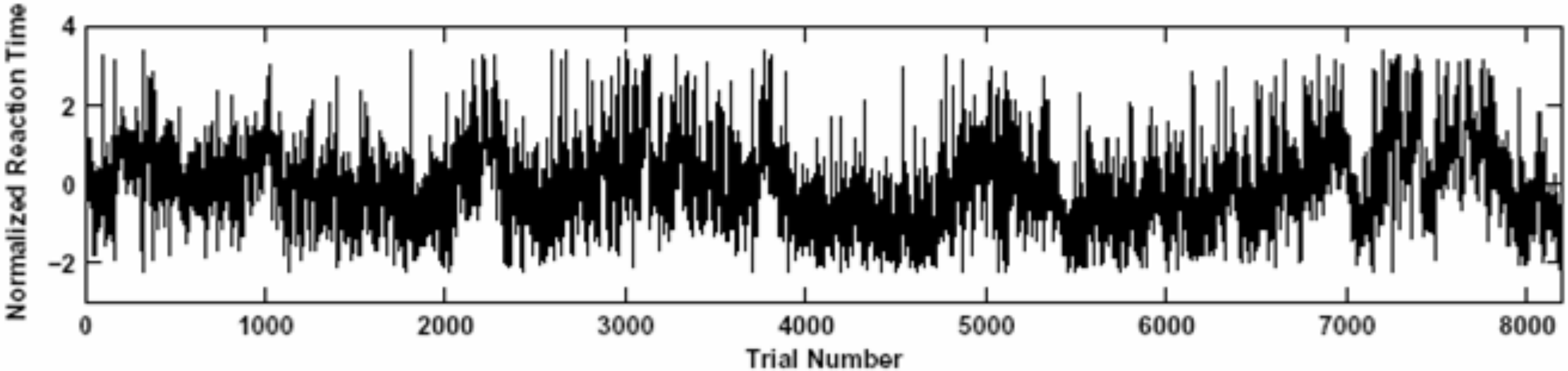
“Statistics”: More data = more variance



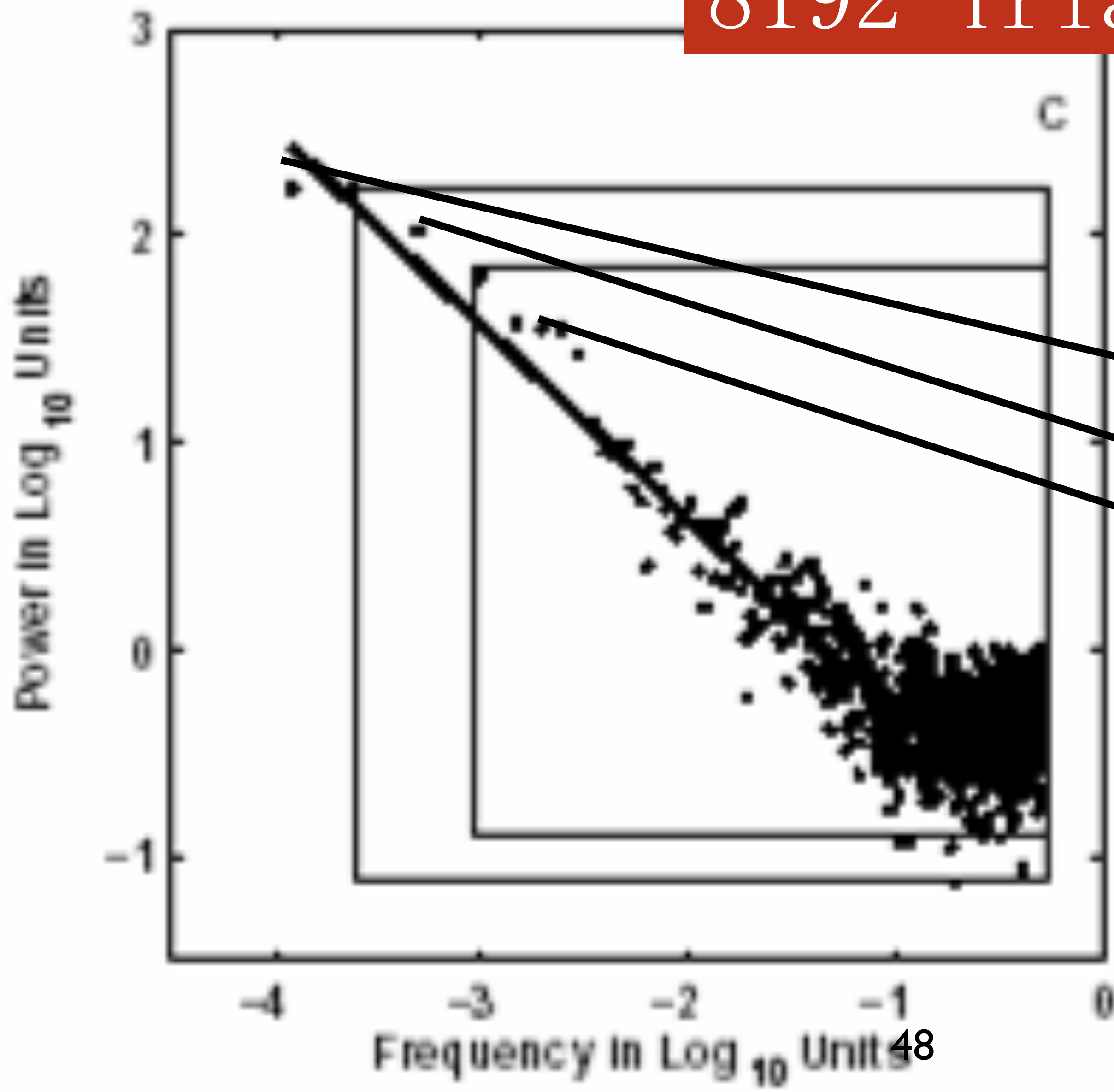






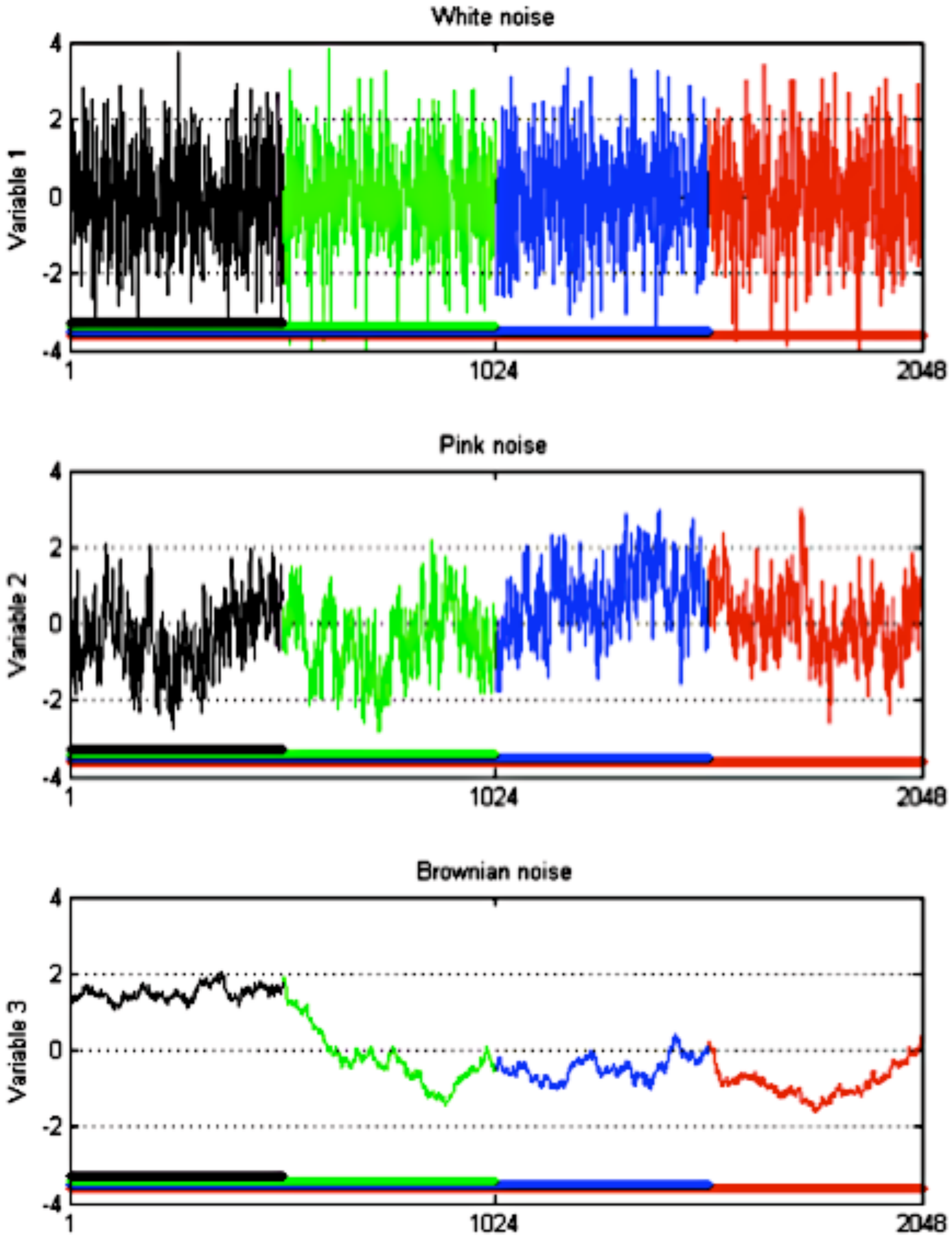


8192 Trials (3 HOURS)

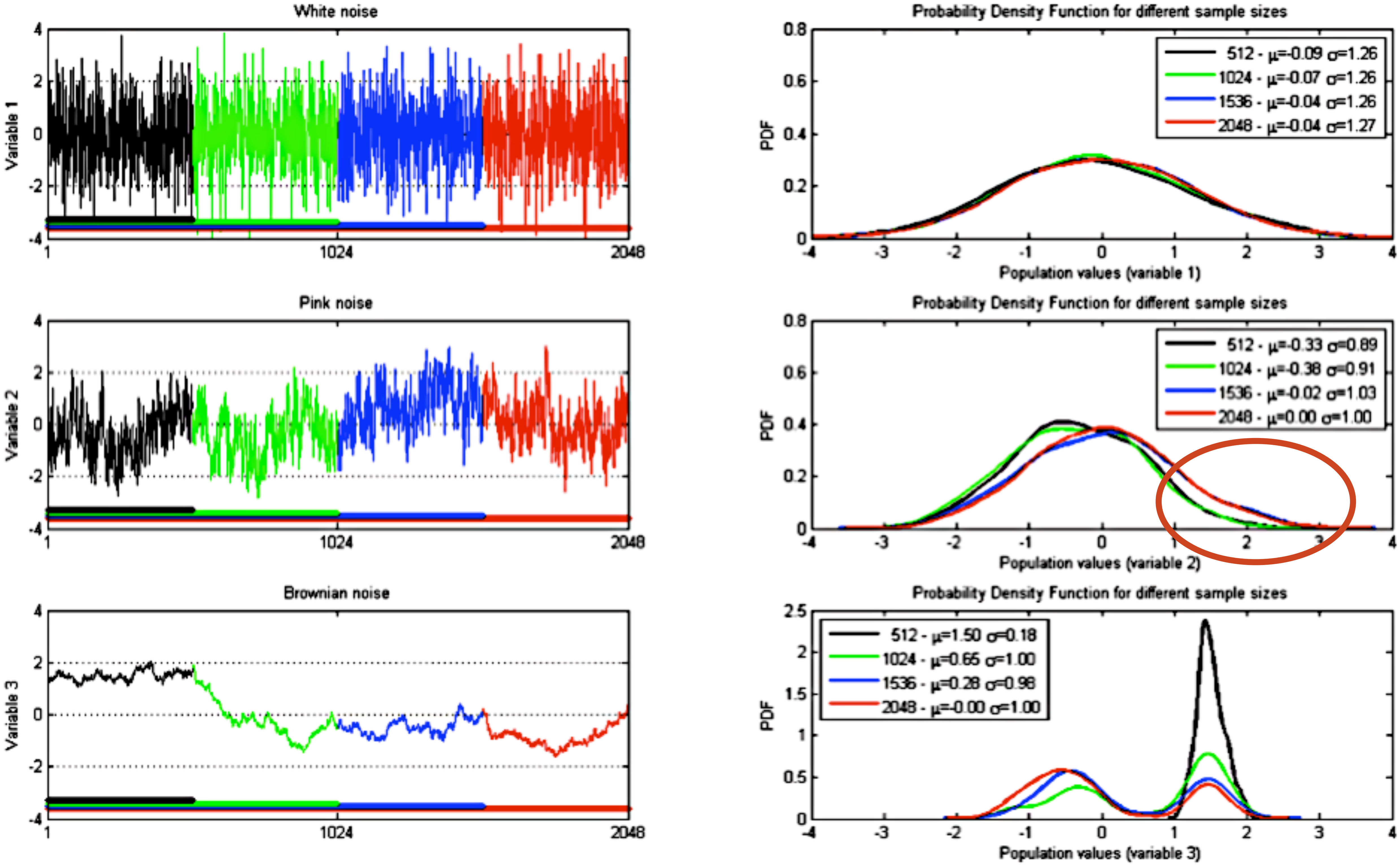


More data points = more variability!

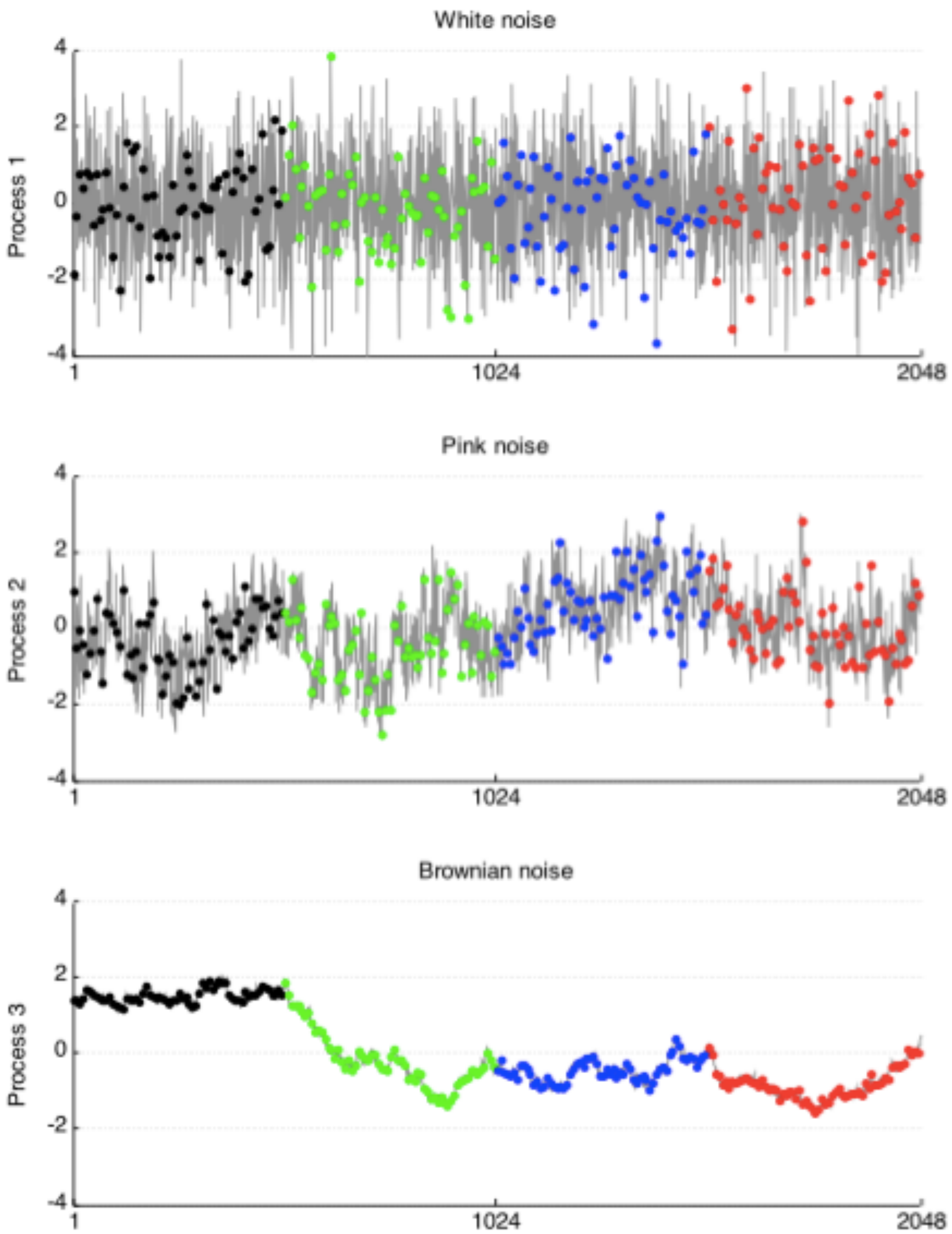
“Statistics”: Distribution of a noisy measurement series



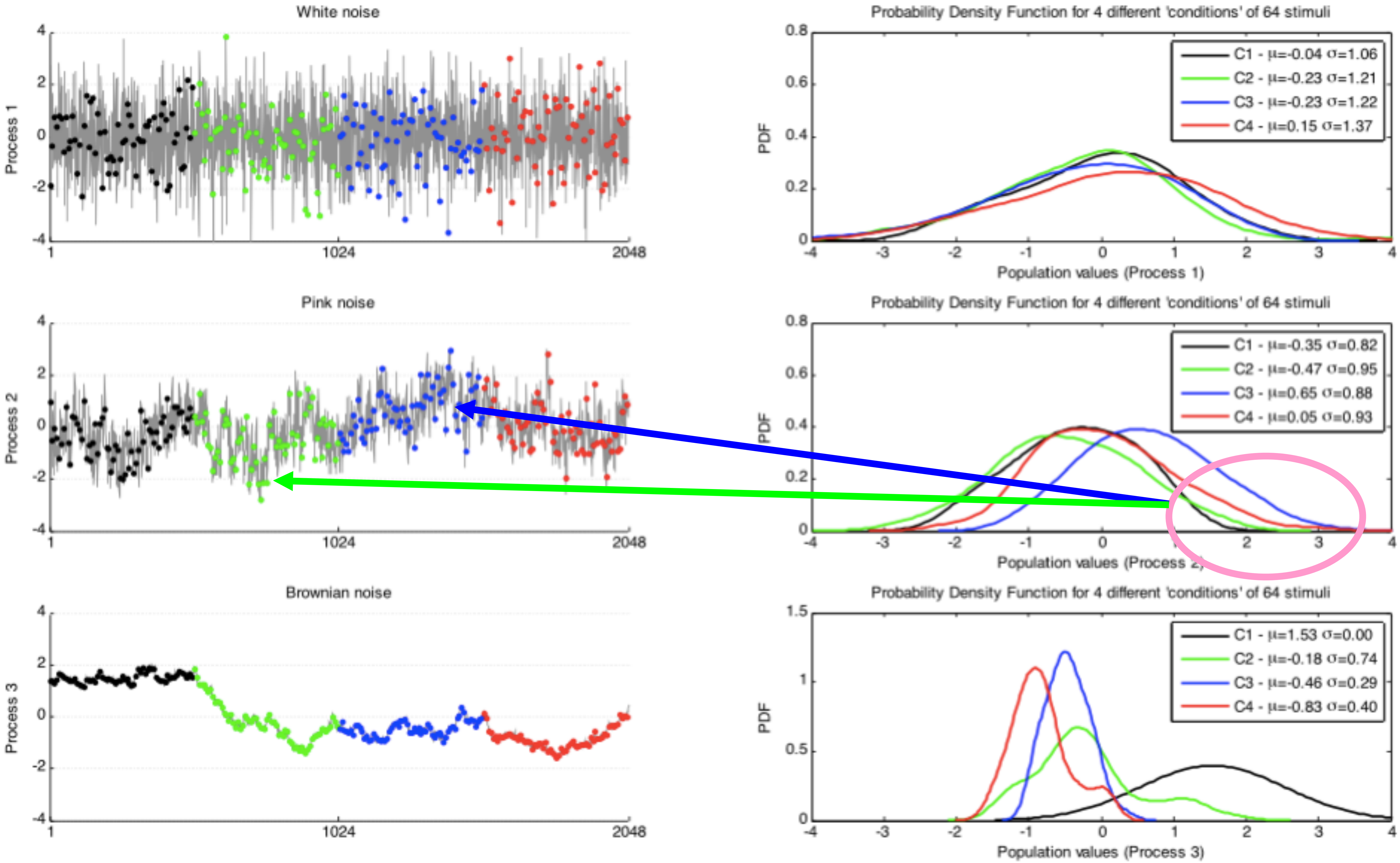
"Statistics": Distribution of a noisy measurement series

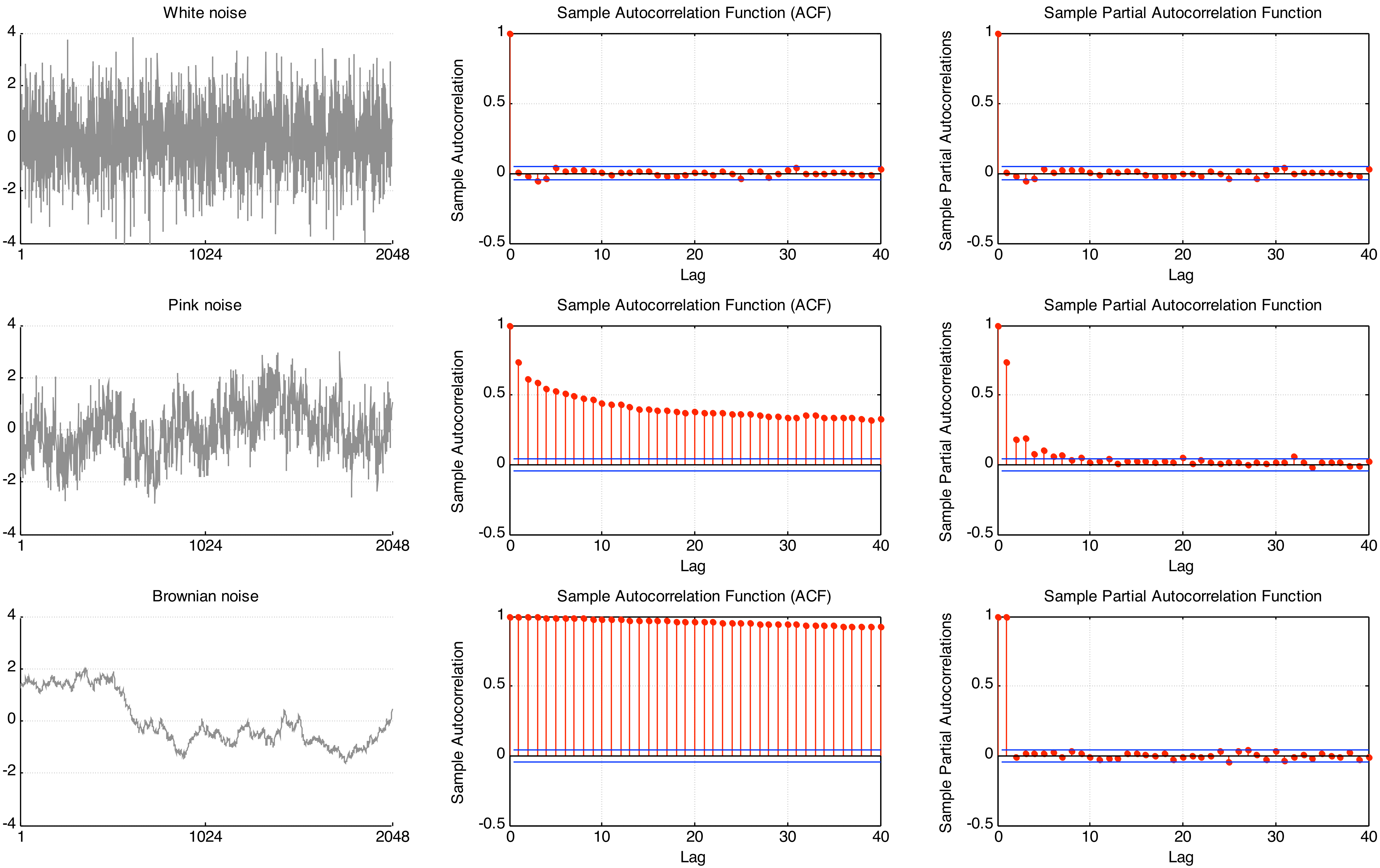


“Statistics”: Distribution of sampling from an autonomous noisy process



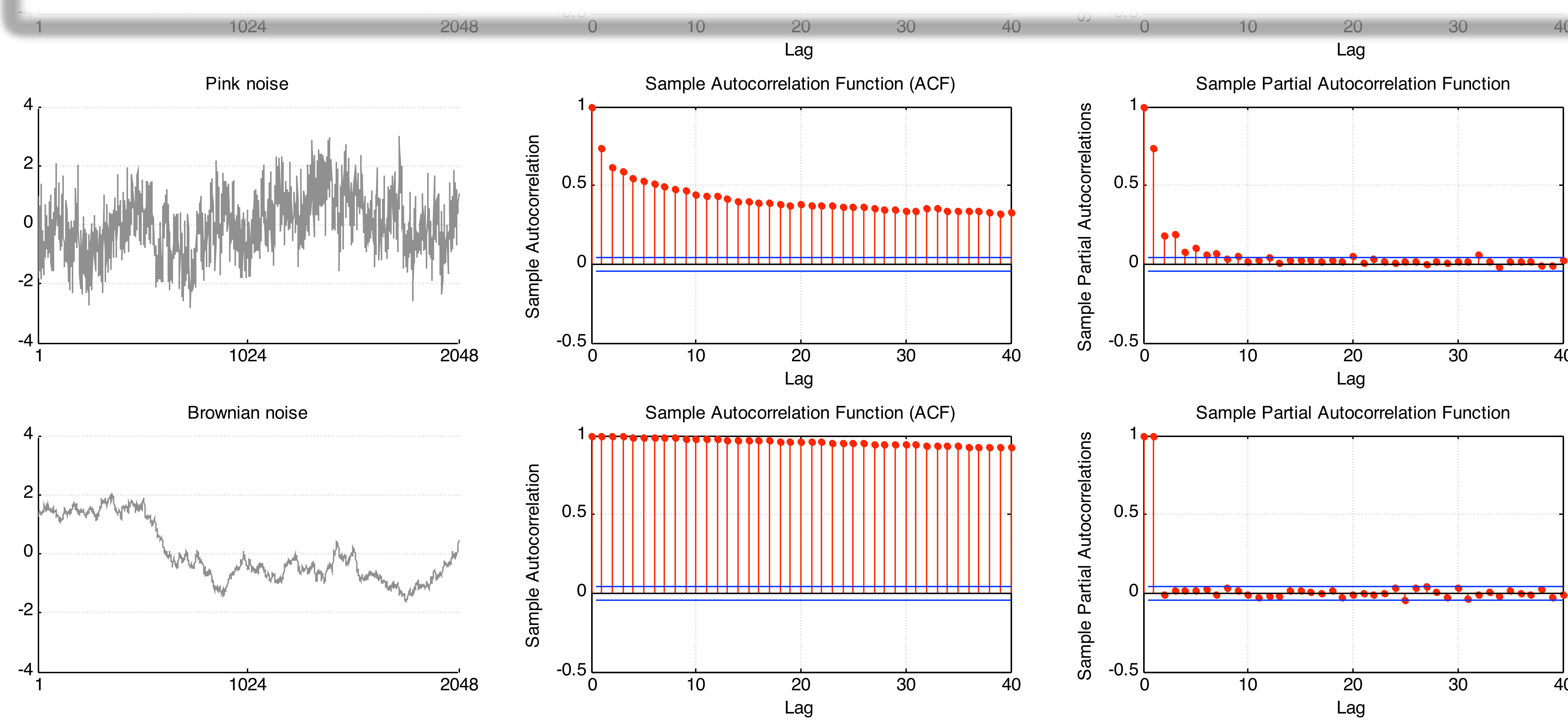
“Statistics”: Distribution of sampling from an autonomous noisy process

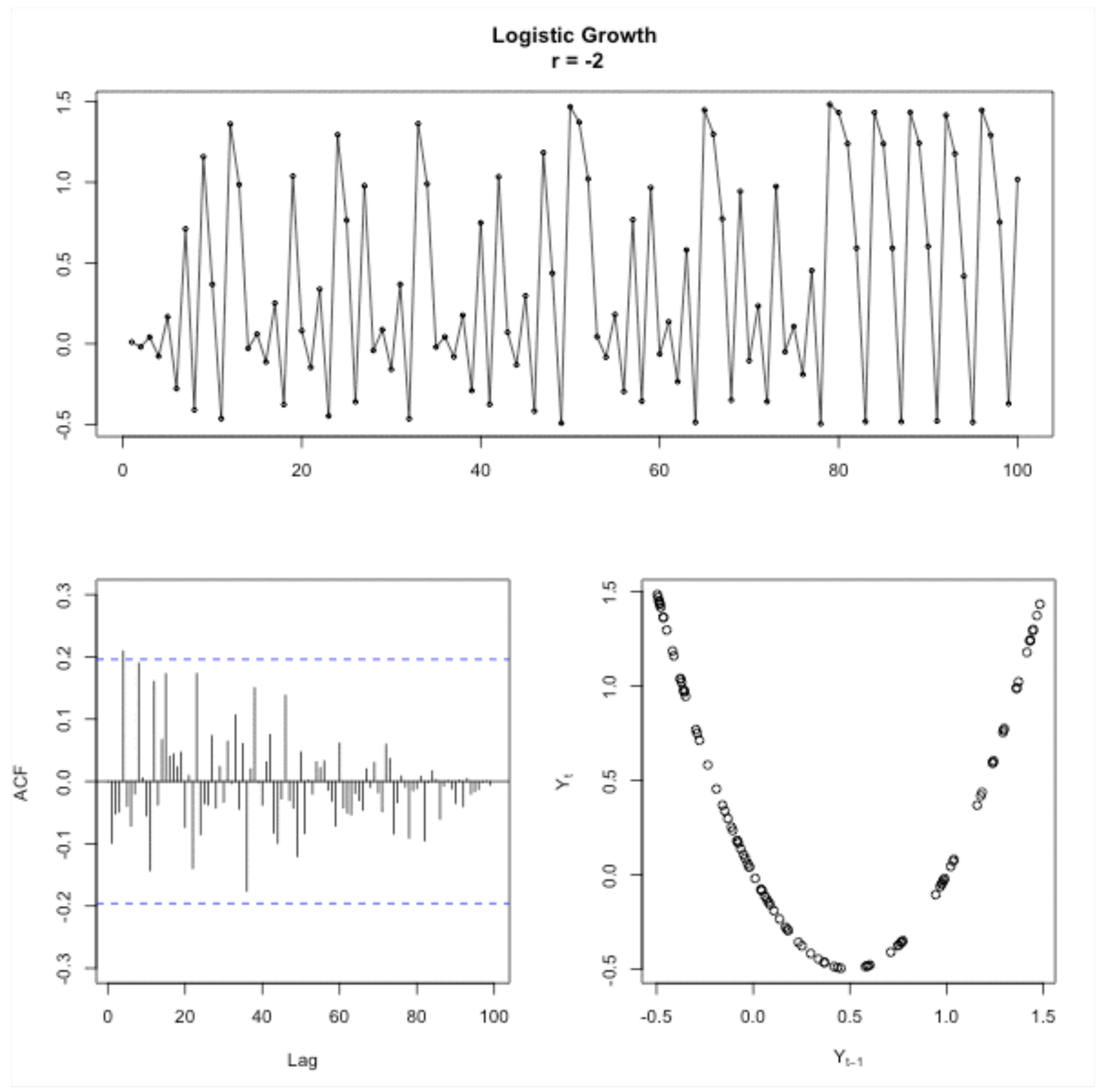


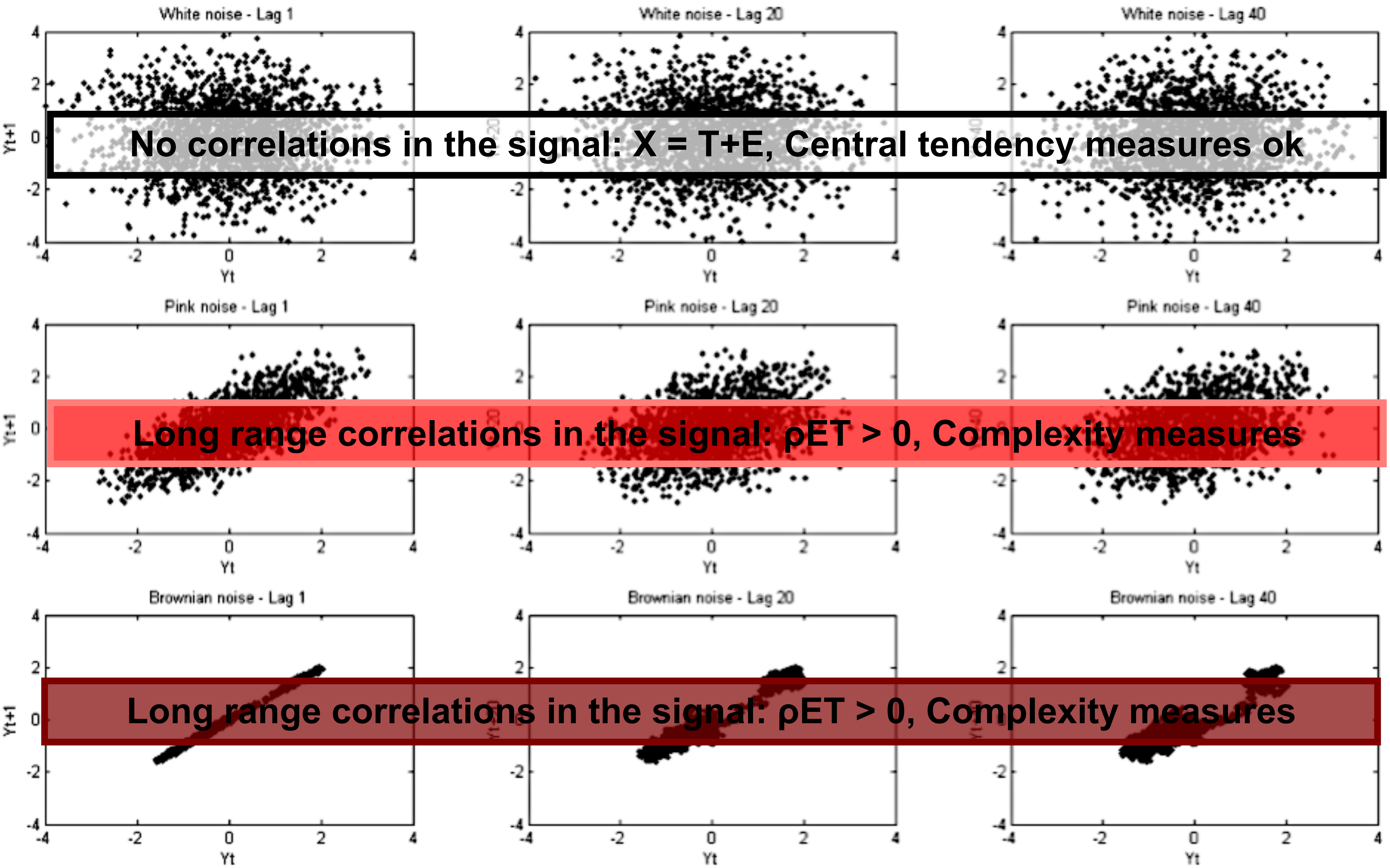

White noise
Sample Autocorrelation Function (ACF)
Sample Partial Autocorrelation Function
Pink noise
Brownian noise
Sample Autocorrelation
Sample Partial Autocorrelations
Lag
1
1024
2048
4
2
0
-2
-4
0.5
-0.5
10
20
30
40

Correlations at all lags = Correlations at all time scales
There is not 1 characteristic time scale

Each data point participates in the larger pattern,
but the pattern does not reduce to the properties of individual measurements.

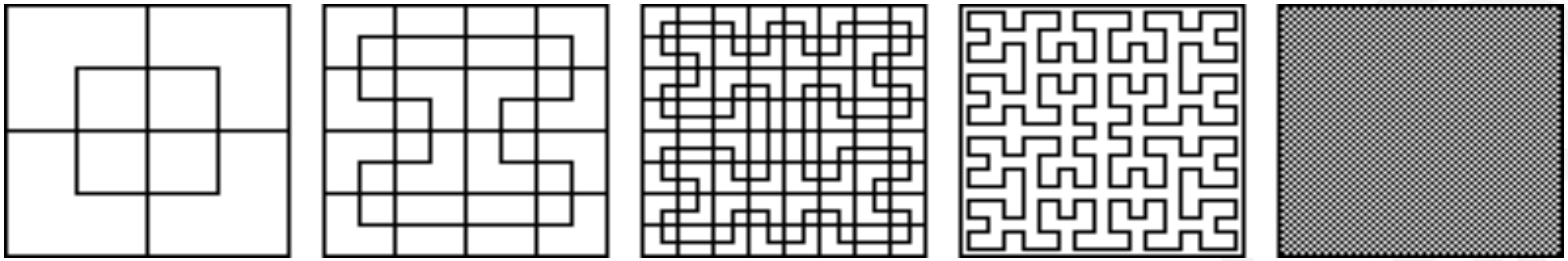



Logistic Growth
r = -2
ACF
Lag
Y_t
Y_{t-1}


White noise - Lag 1
White noise - Lag 20
White noise - Lag 40
Pink noise - Lag 1
Pink noise - Lag 20
Pink noise - Lag 40
Brownian noise - Lag 1
Brownian noise - Lag 20
Brownian noise - Lag 40
Yt
Yt+1
No correlations in the signal: X = T+E, Central tendency measures ok
Long range correlations in the signal: ρET > 0, Complexity measures
Long range correlations in the signal: ρET > 0, Complexity measures

Analyze scaling relations: Fractal dimension

Peano's Monster



Dimension is an intrinsic property of a mathematical object that indicates to what extent it occupies the topological space in which it is embedded.

Assessing self-similarity in time using scaling exponents

- Spectral Slope: Frequency vs. Power, small waves are nested within larger waves (of greater power).

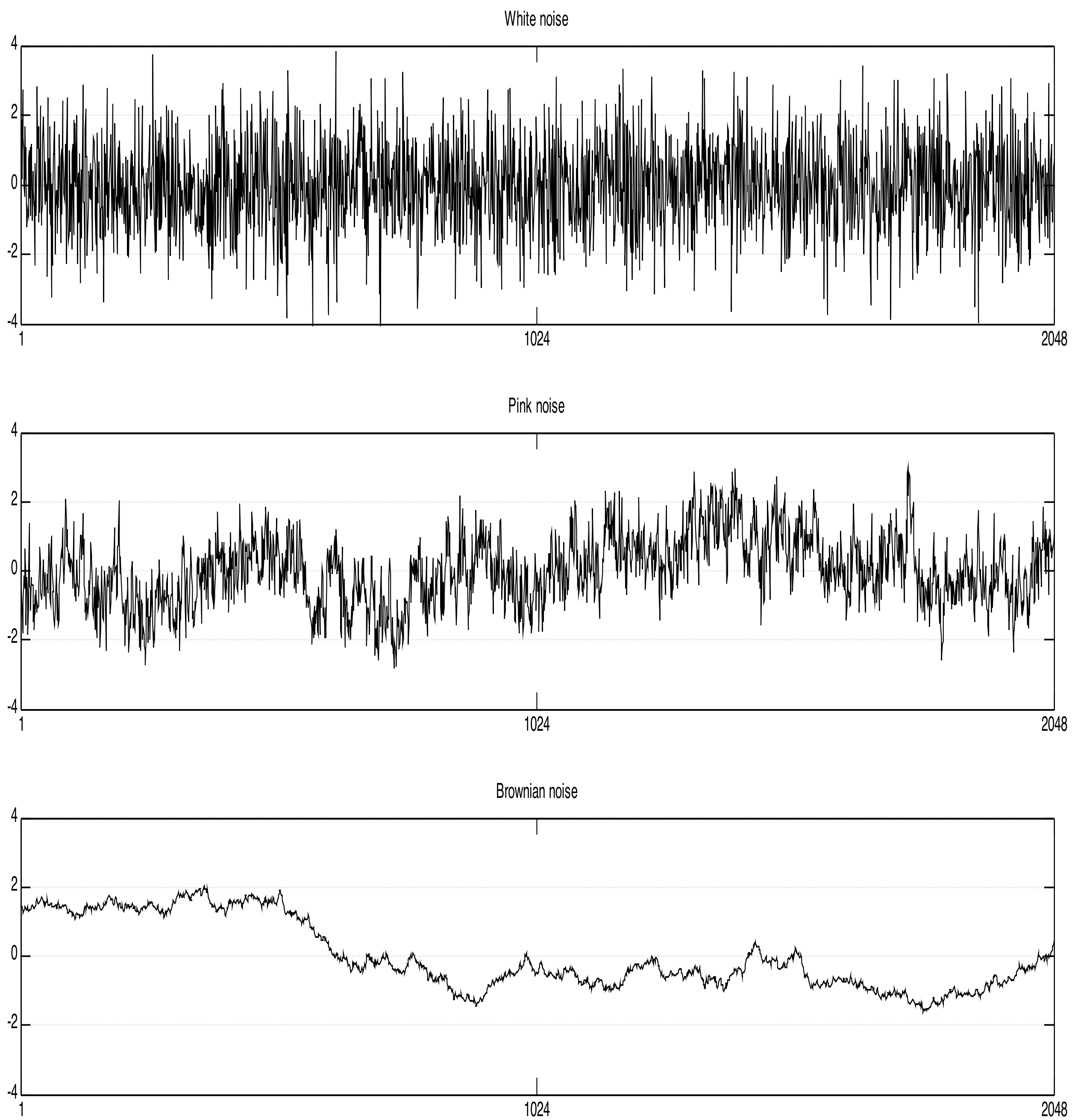
- All other methods are based on the following:

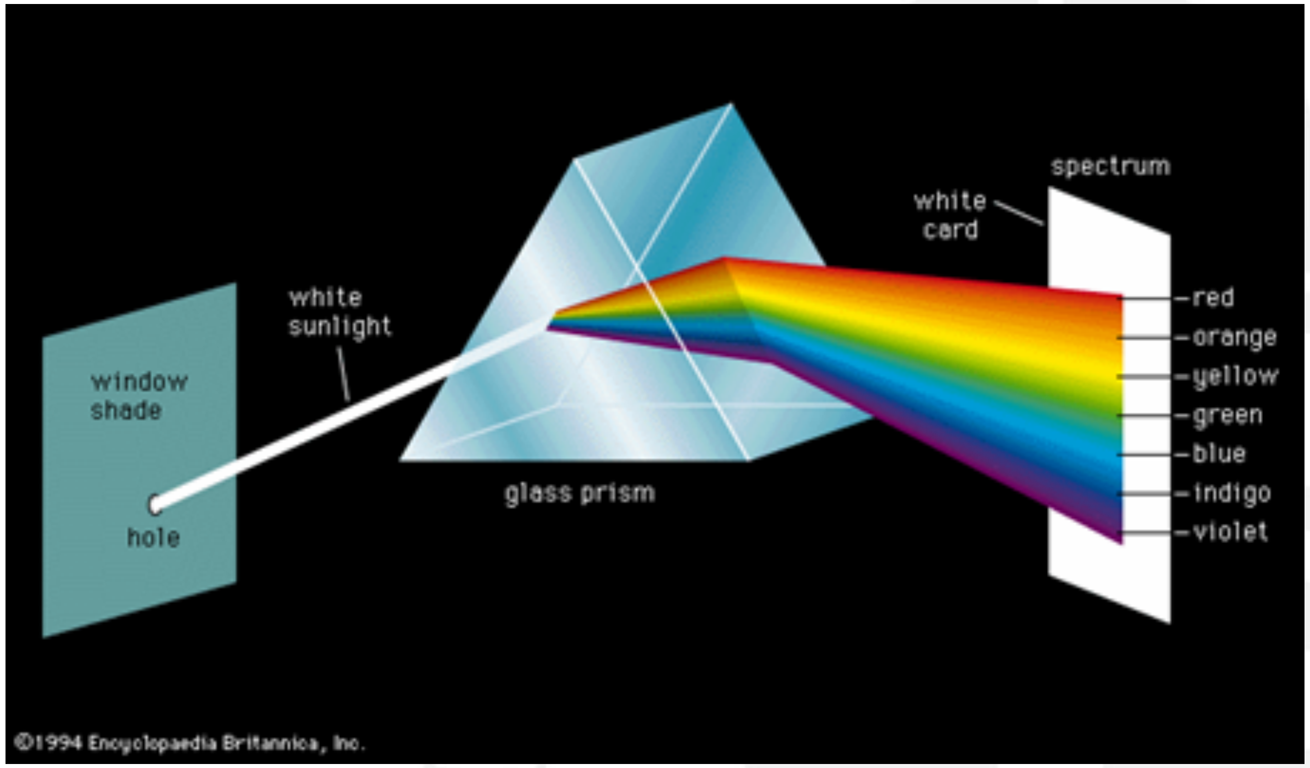
  1. Divide the time series in equal ‘bins’
  2. Examine the variability in the bins
  3. Repeat for other bin sizes
  4. Plot variability versus bin size

- This should remind you of the box-counting.

- NOTE: technically, self-similarity refers to the fact that there is one "generator" object from which larger structures arise by copying the generator. What we will be discussing is self-affinity, which refers to the fact that one axis (in 2D) needs to be scaled by a different factor (H) than the other axis in order to observe self-similarity... In other words, "*statistical self-similarity*"

Spectral analysis: Frequencies in the signal



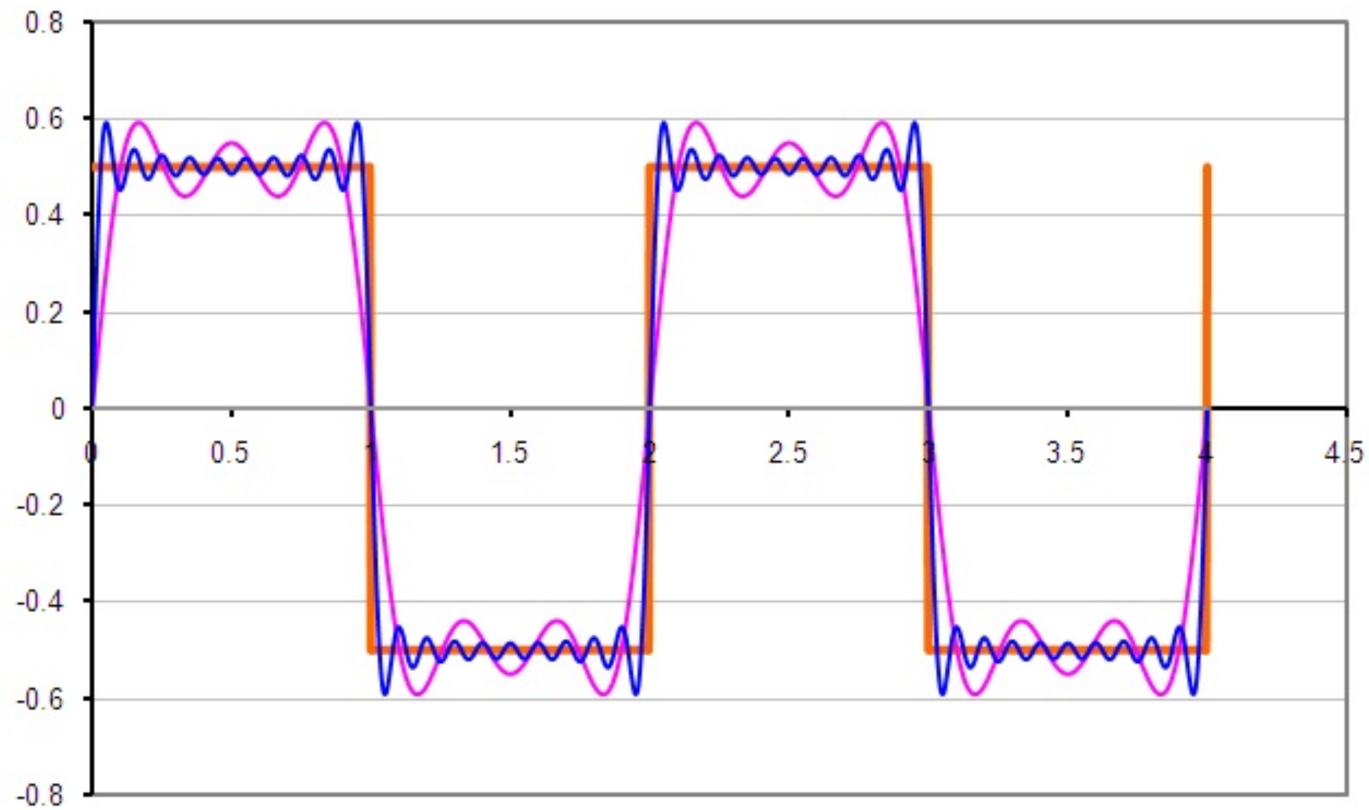


Characterize a signal by its dominant frequencies: Spectral analysis-Fourier transform

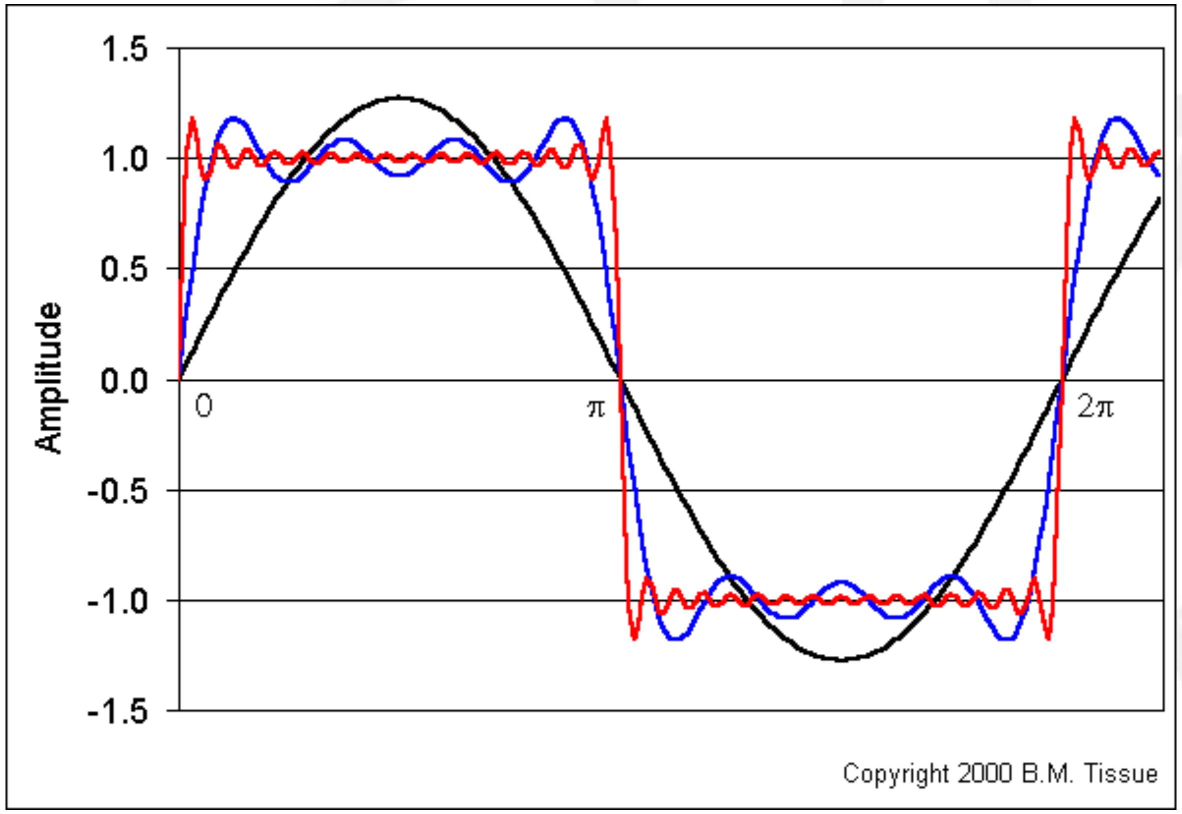
$$g(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$= \sum_{m=0}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

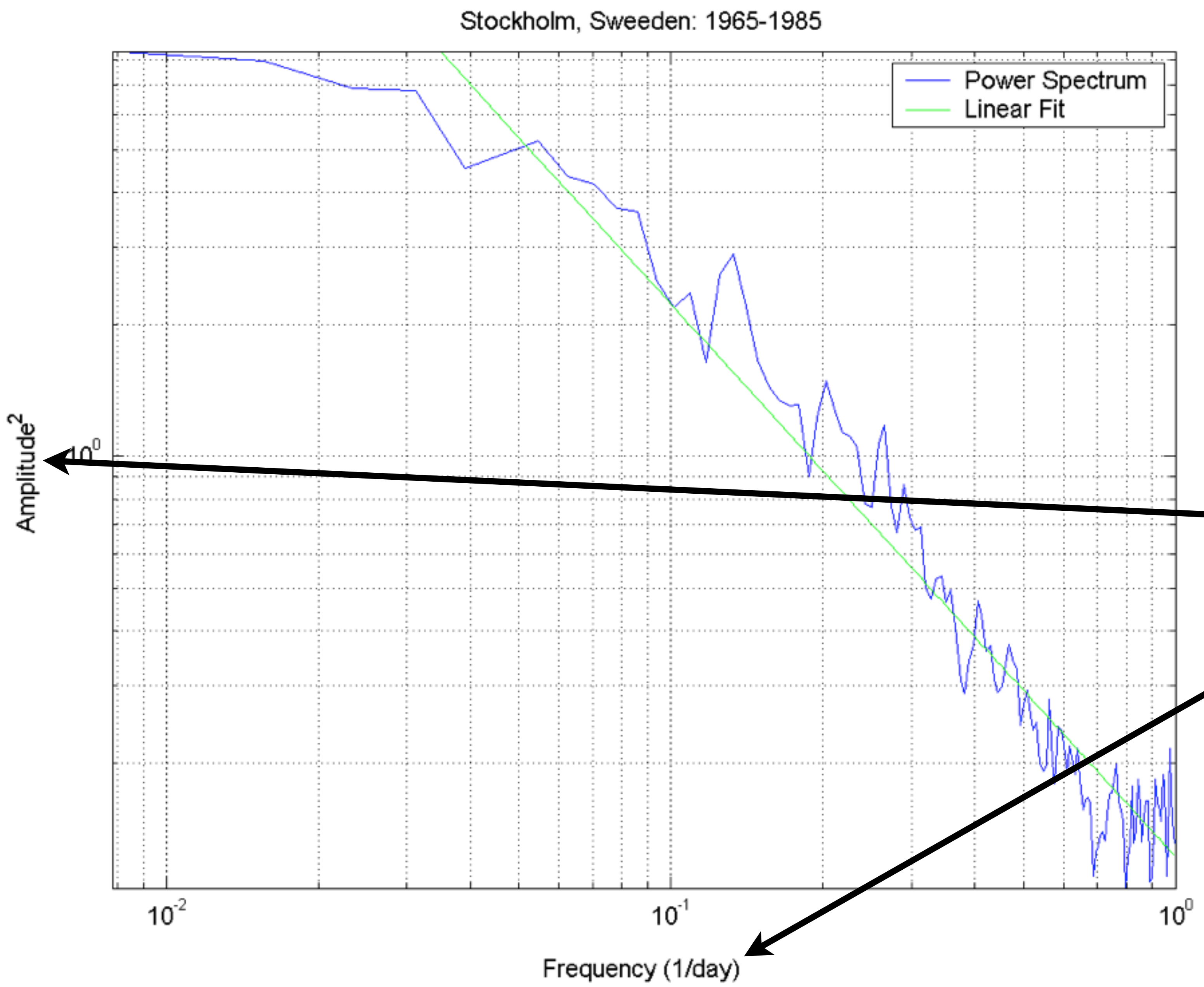
Spectral analysis: Fourier transform



Reconstruct a waveform by adding many sine and cosine waves of different frequencies and amplitudes

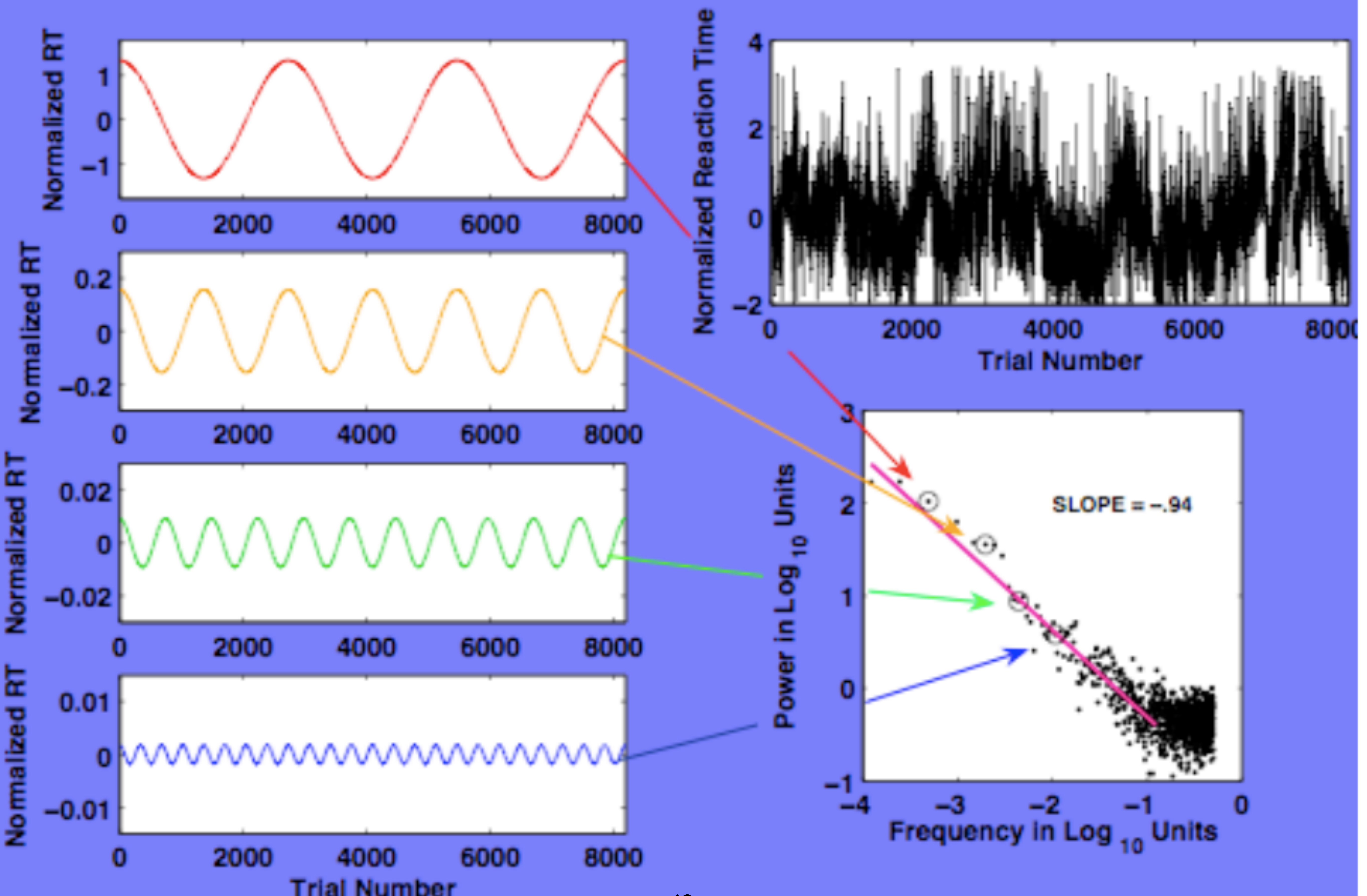


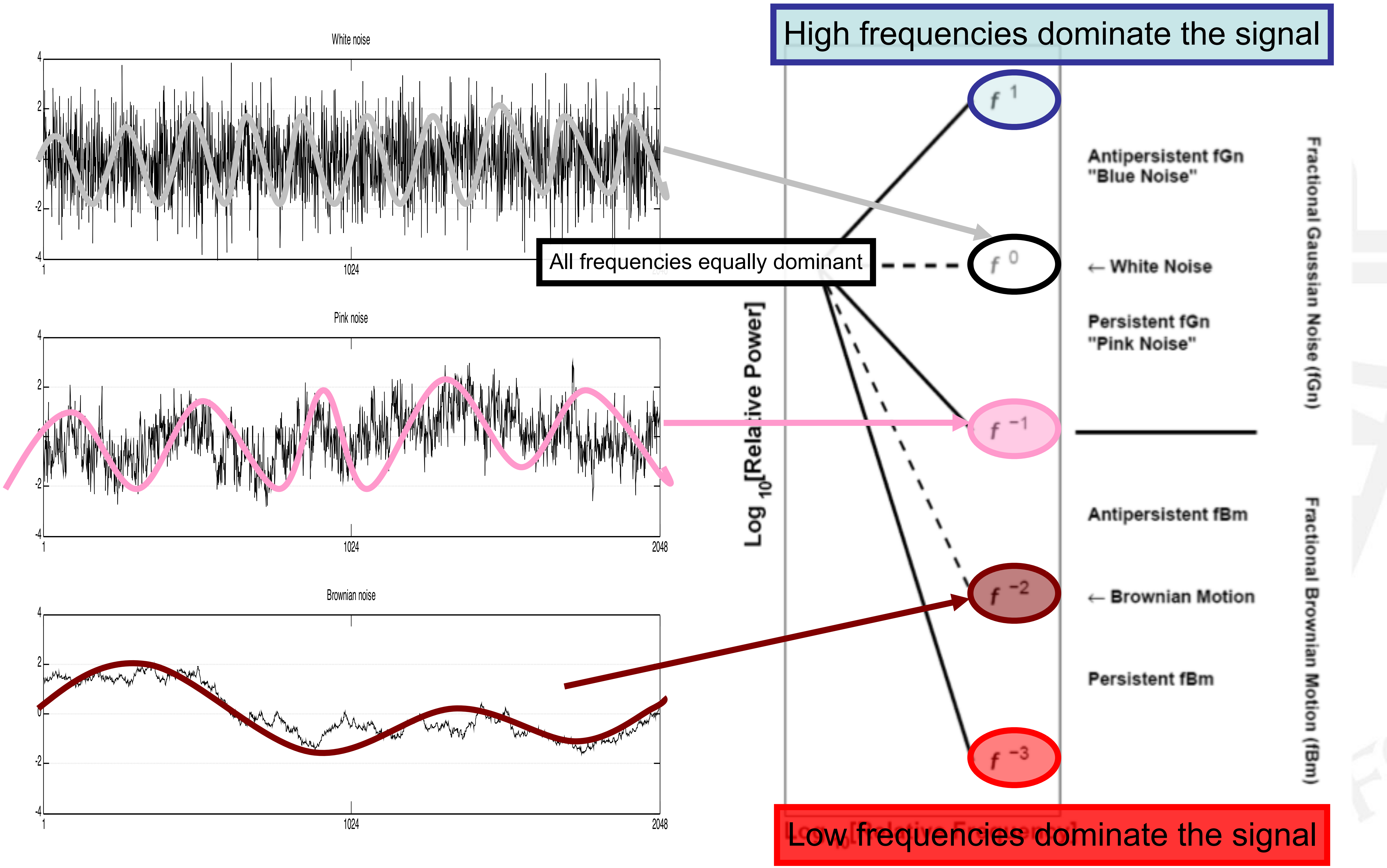
Spectral analysis: Fourier transform -> Frequency domain



Reconstruct a waveform by adding many sine and cosine waves of different **frequencies** and **amplitudes**

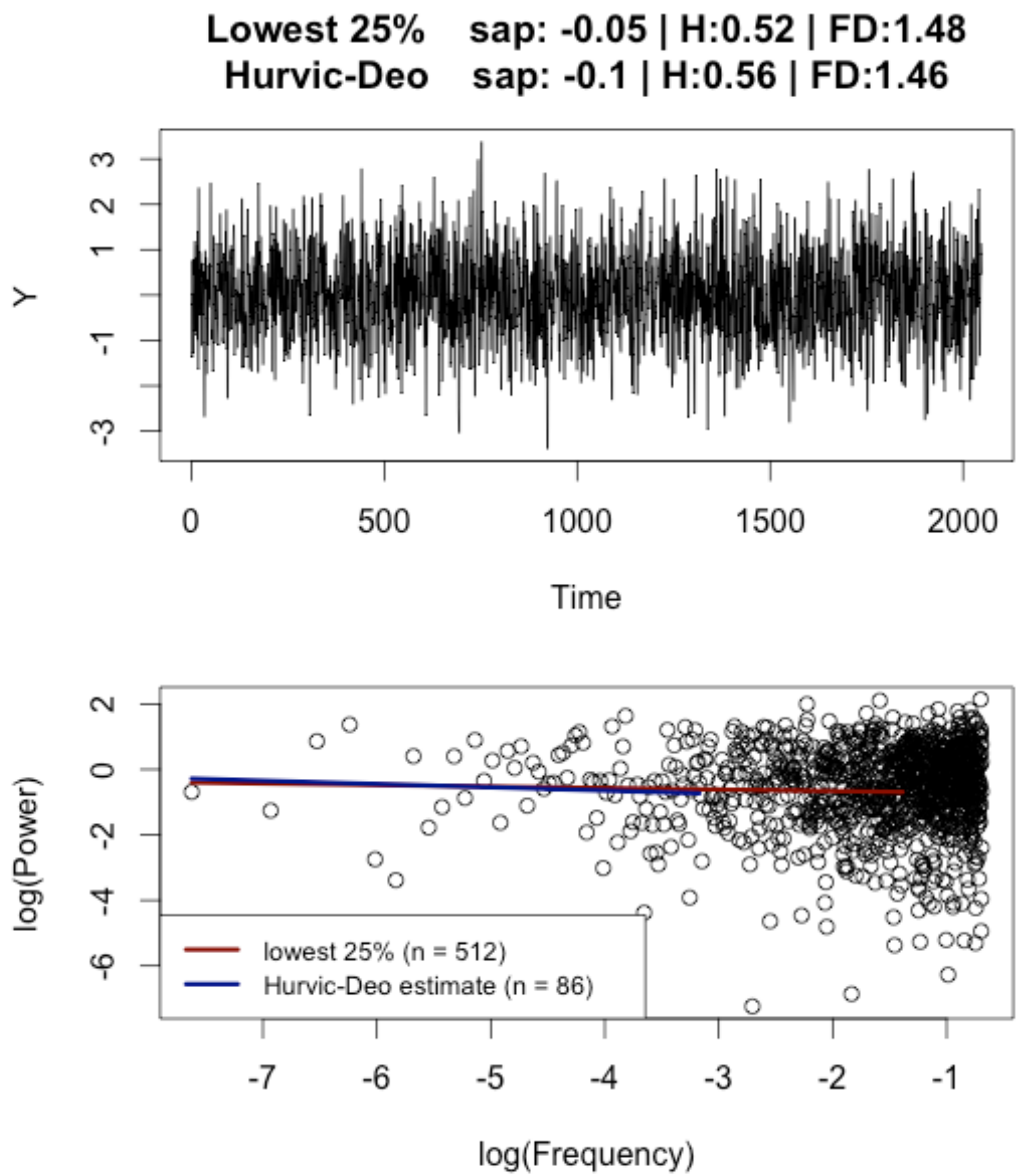
Fractal Time: Scale Free Variation in Repeated Measures



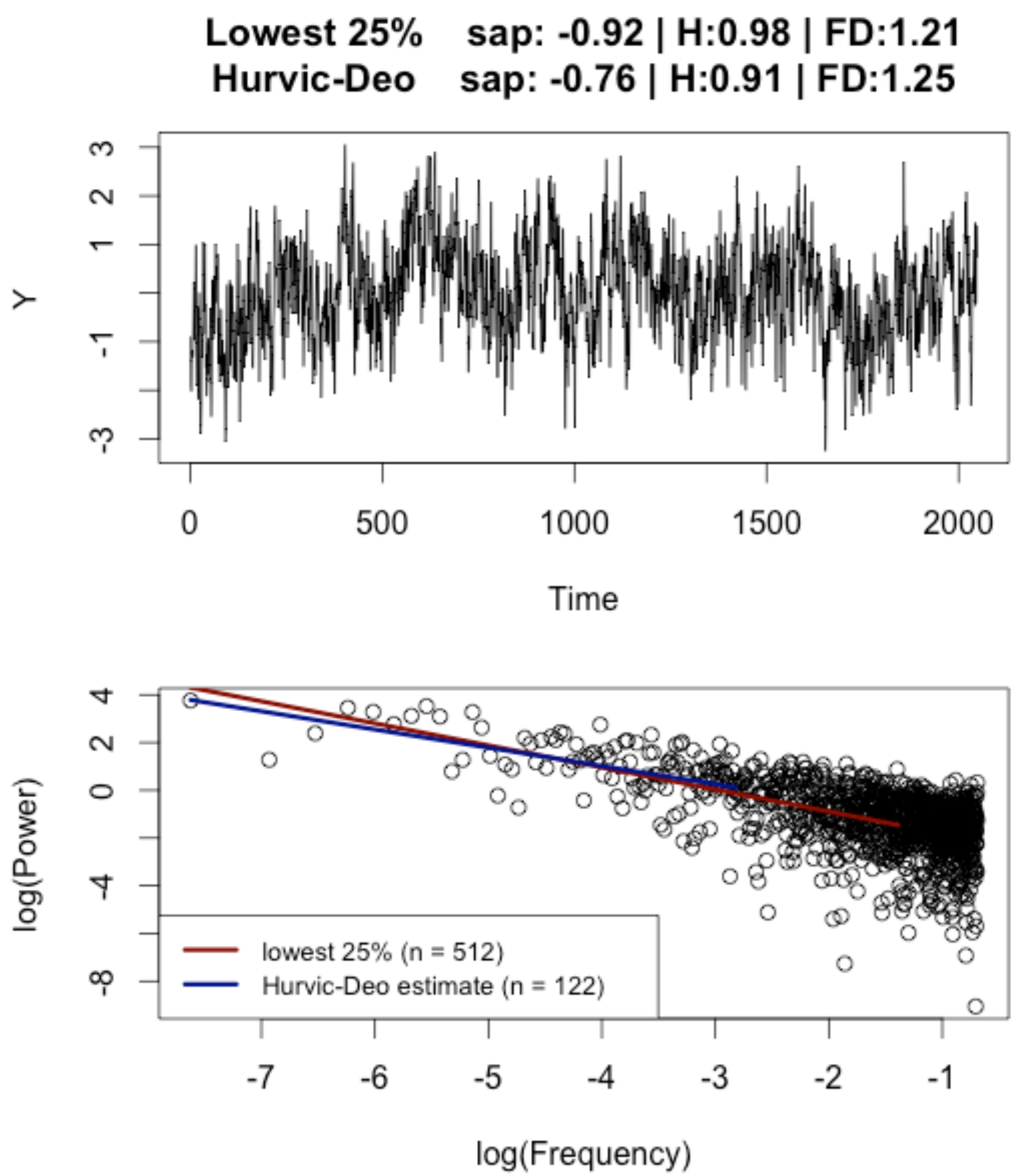

High frequencies dominate the signal
White noise
Pink noise
Brownian noise
All frequencies equally dominant
Log 10[Relative Power]
Antipersistent fGn "Blue Noise"
← White Noise
Persistent fGn "Pink Noise"
Antipersistent fBm
← Brownian Motion
Persistent fBm
Fractional Gaussian Noise (fGn)
Fractional Brownian Motion (fBm)
Low frequencies dominate the signal

sap:
self-affinity parameter
slope in some log-log space

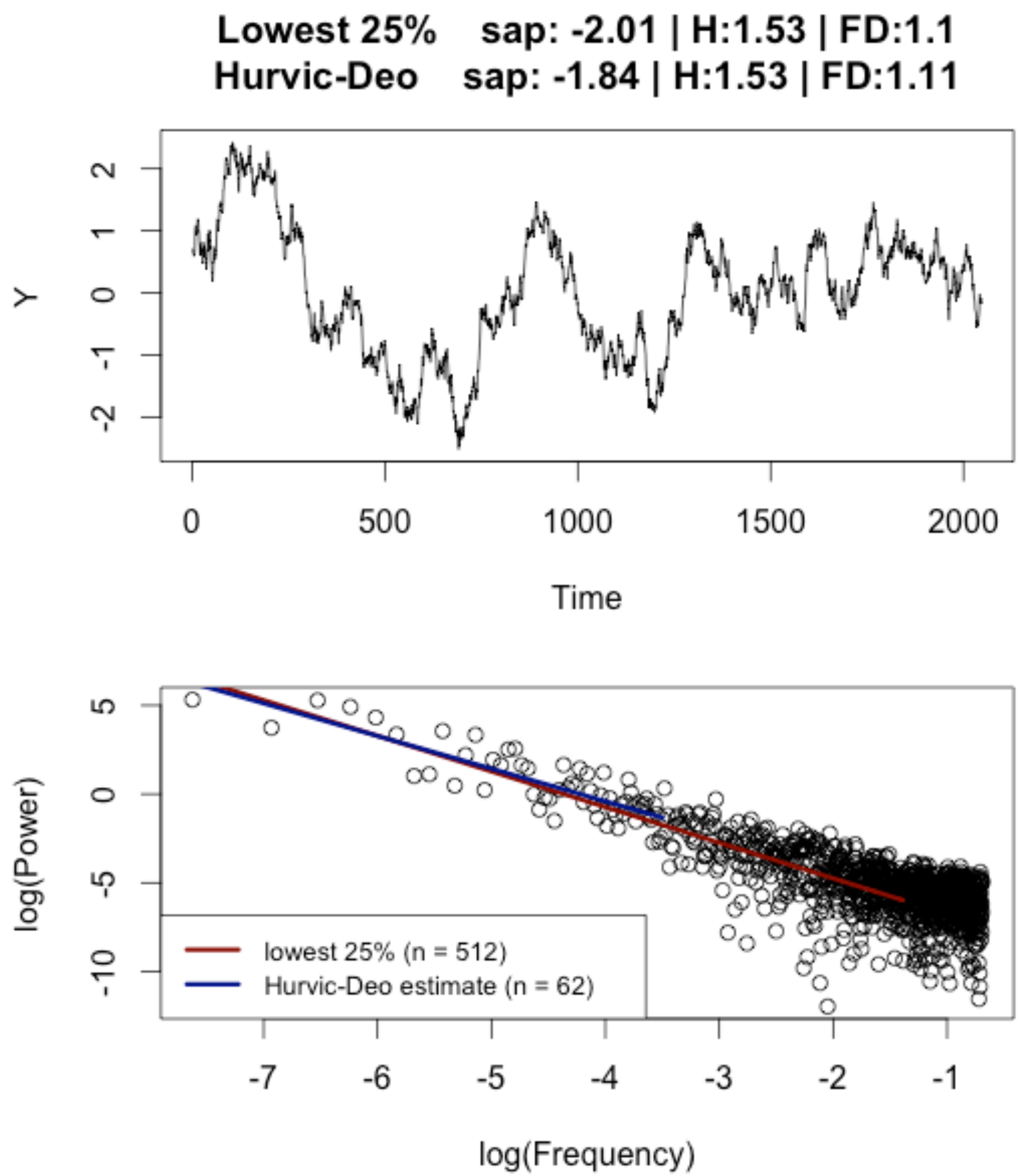
Spectral slope of white noise:



Spectral slope of pink noise:

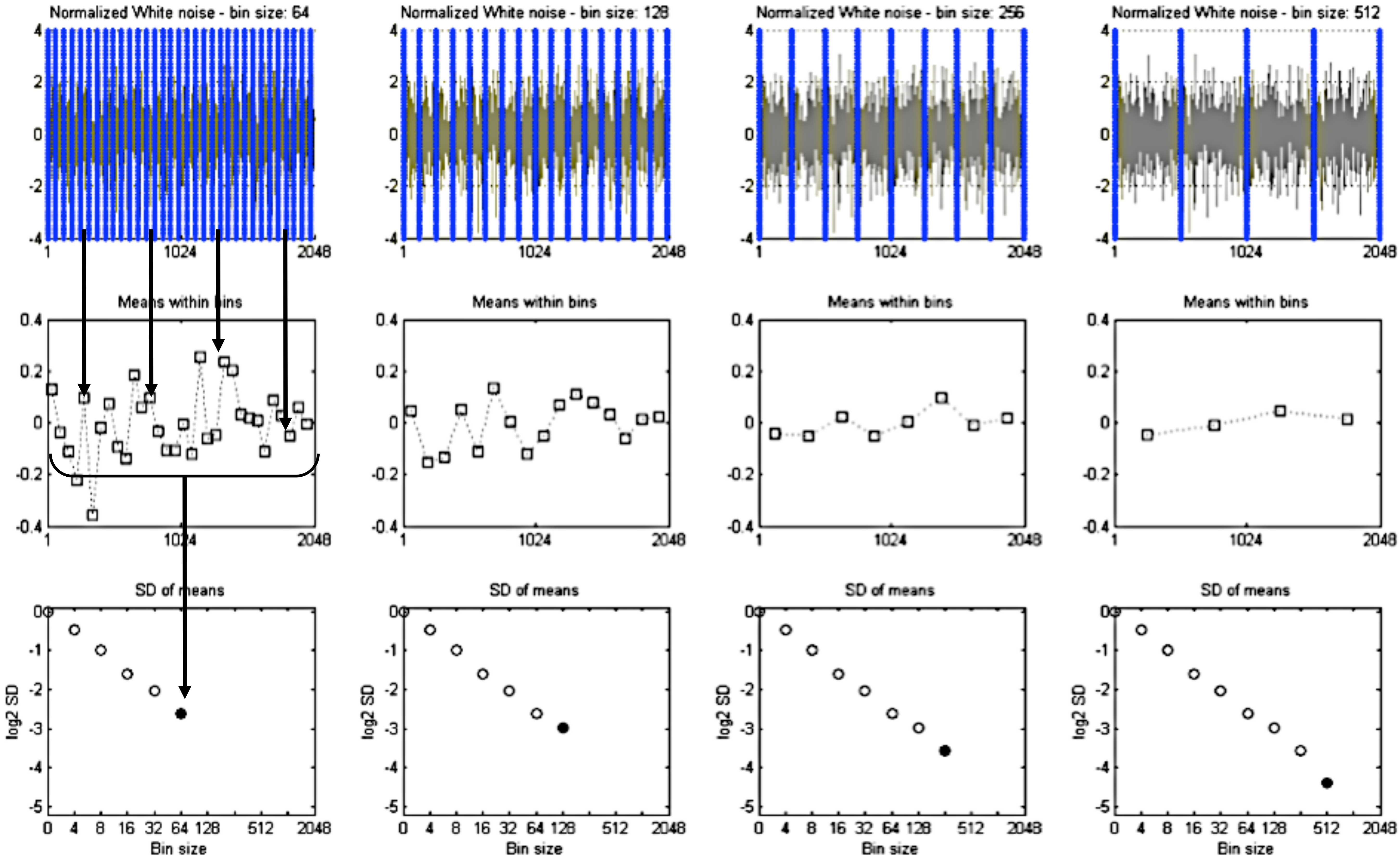


Spectral slope of Brownian motion:



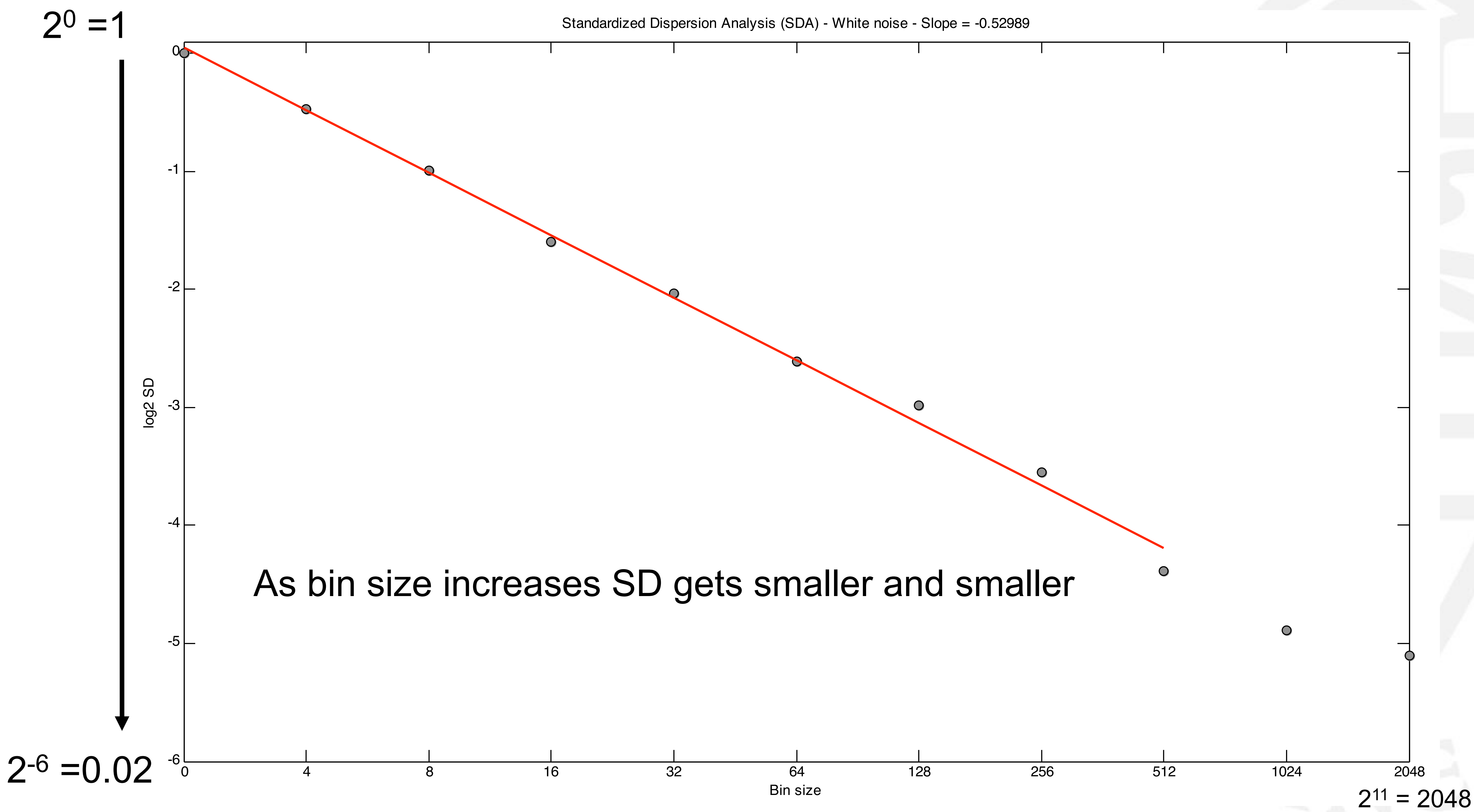
Standardised dispersion analysis

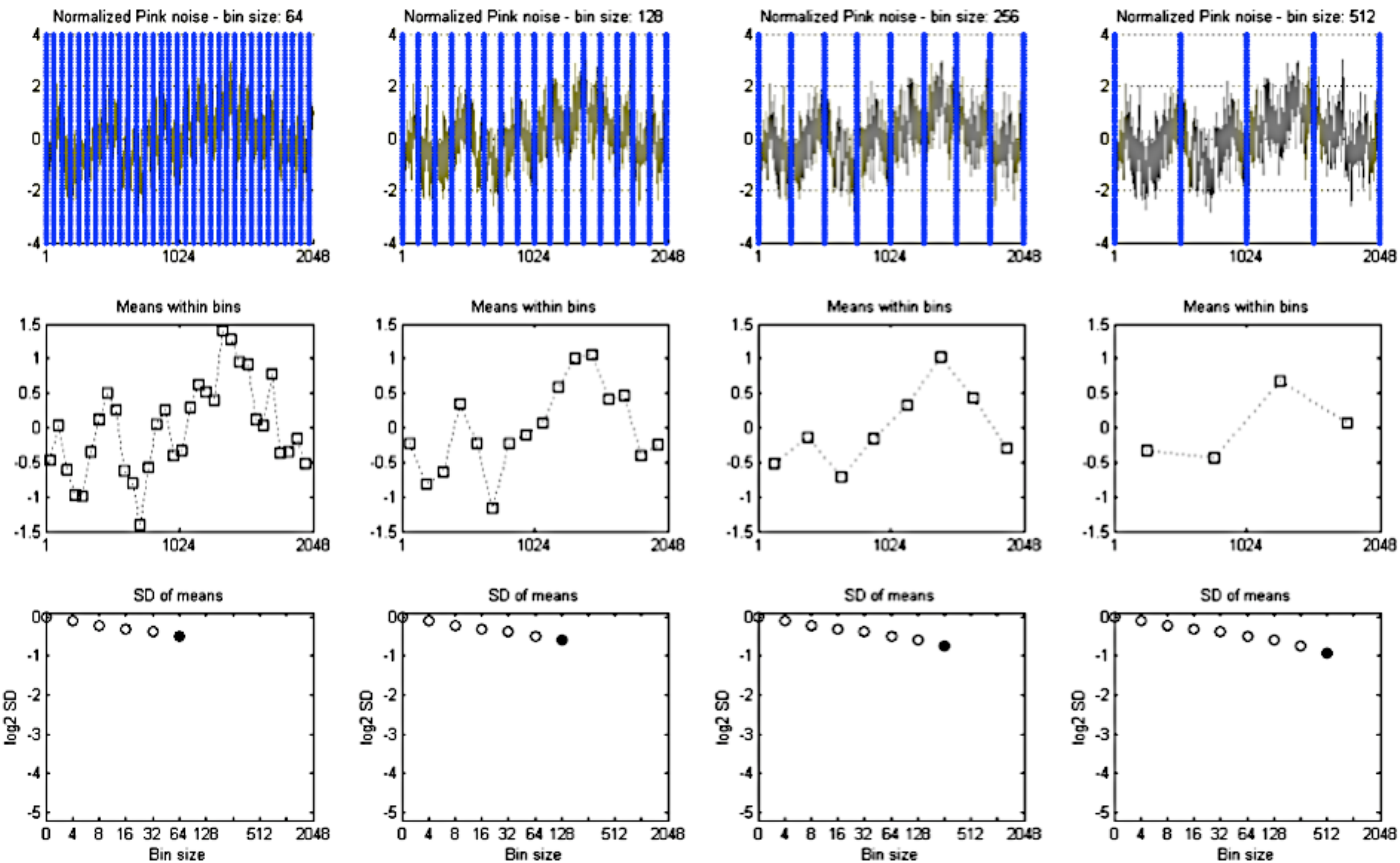
- Divide TS in bins of size N
- Get some indication of variation
- repeat



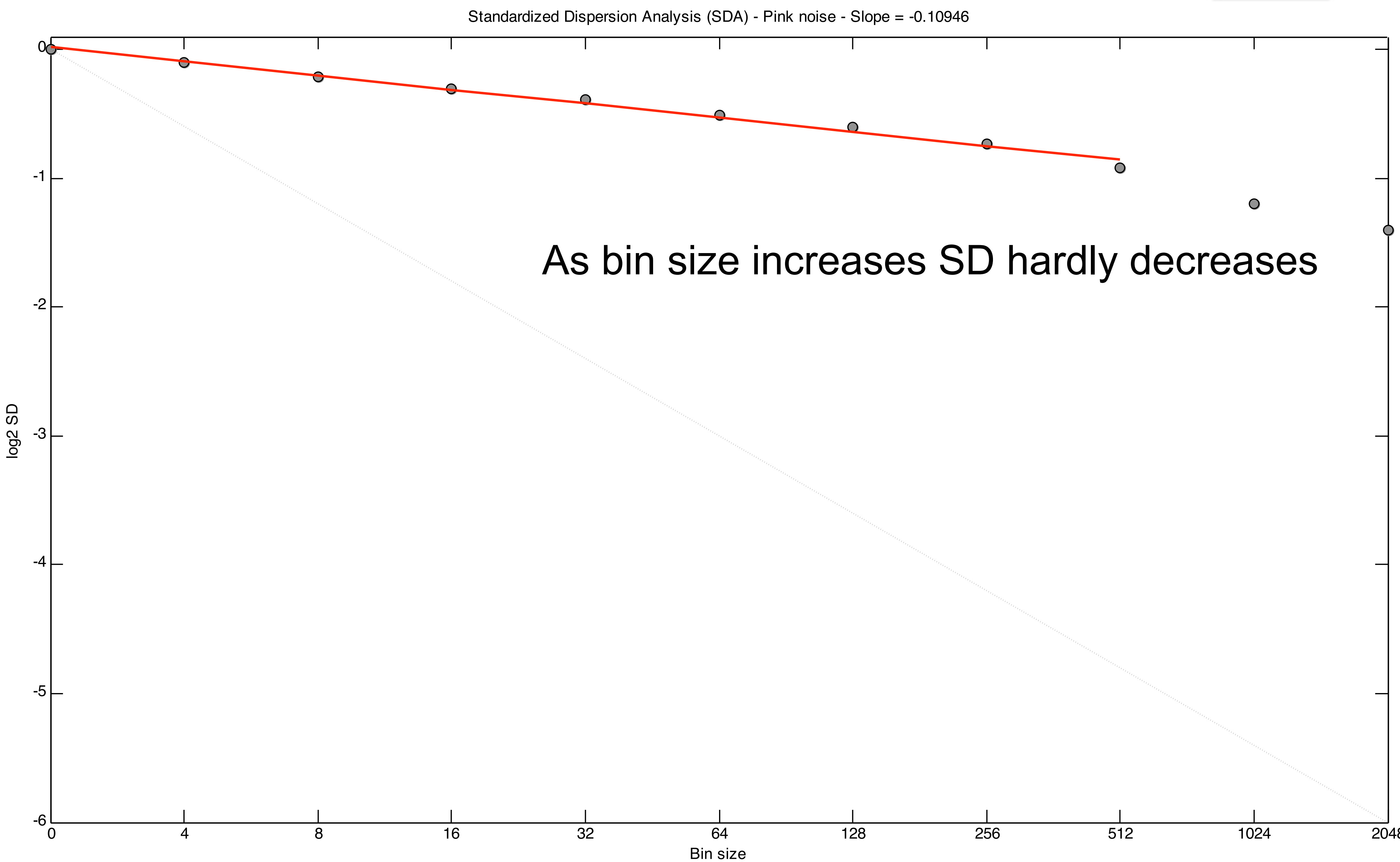
Normalized White noise - bin size: 64
Normalized White noise - bin size: 128
Normalized White noise - bin size: 256
Normalized White noise - bin size: 512
Means within bins
SD of means
log2 SD
Bin size

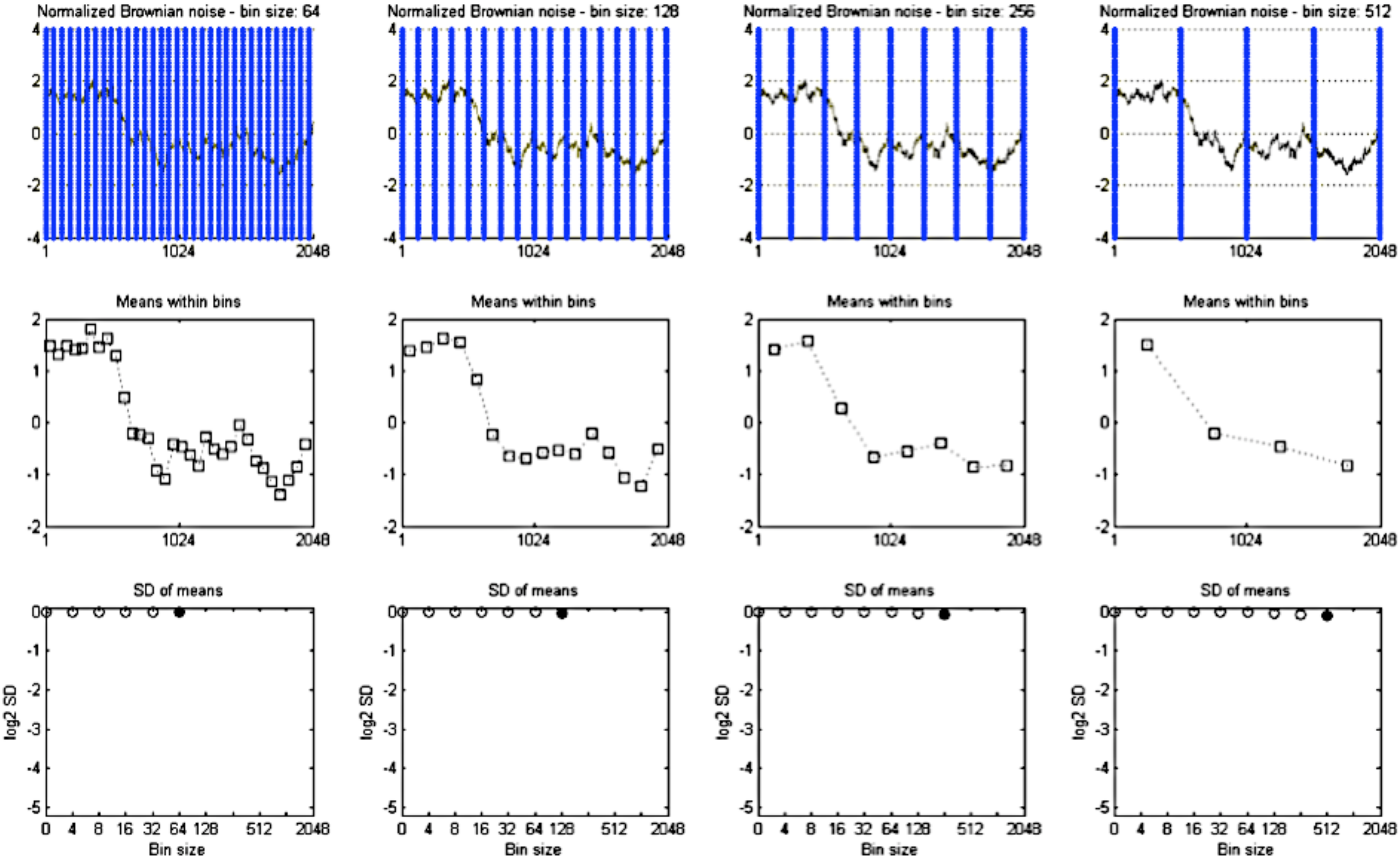
Fractal dimension = 1 – (slope of line)
FD = 1 - (-0.53) = 1.53



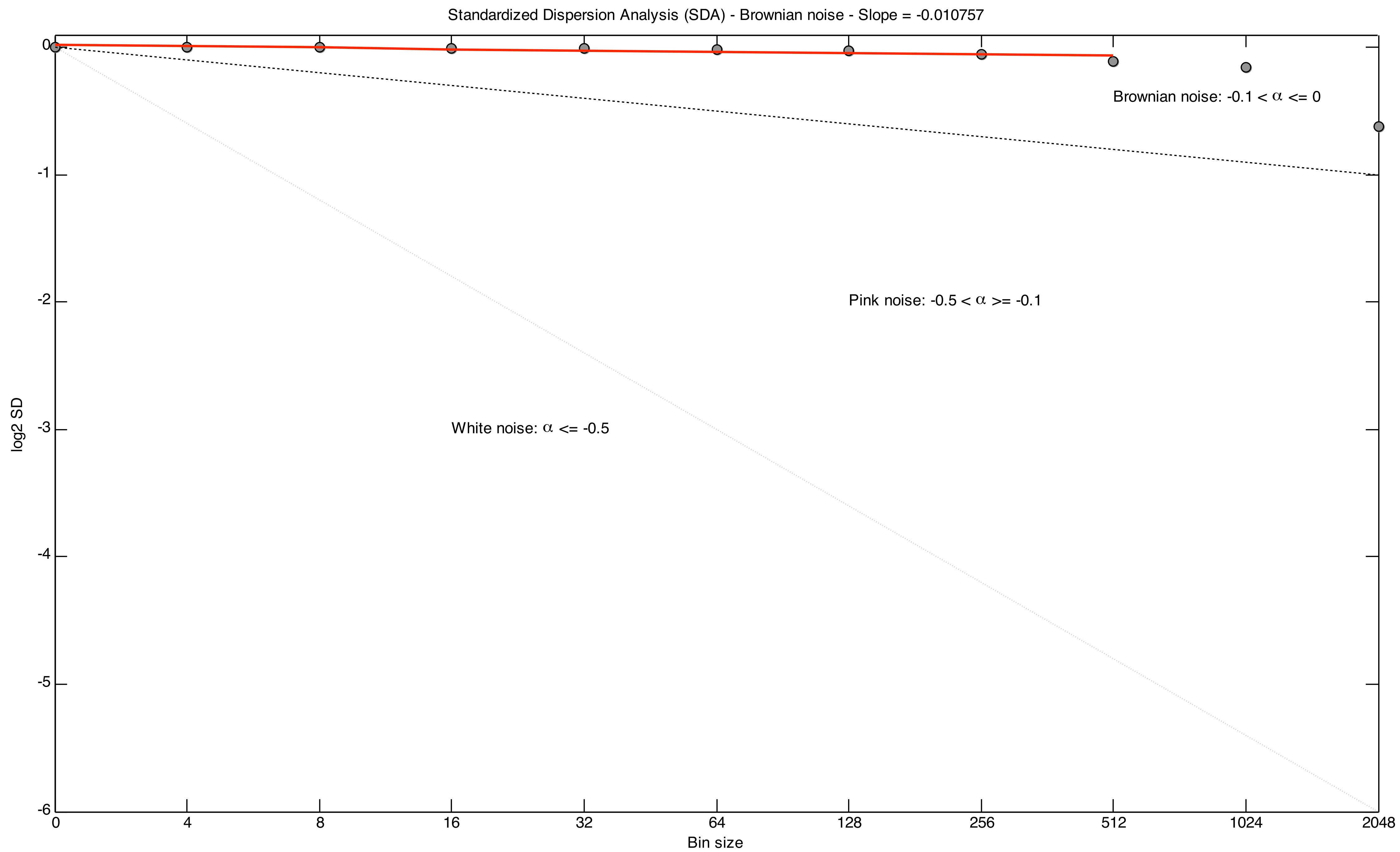

Normalized Pink noise - bin size: 64
Normalized Pink noise - bin size: 128
Normalized Pink noise - bin size: 256
Normalized Pink noise - bin size: 512
Means within bins
SD of means
log2 SD
Bin size

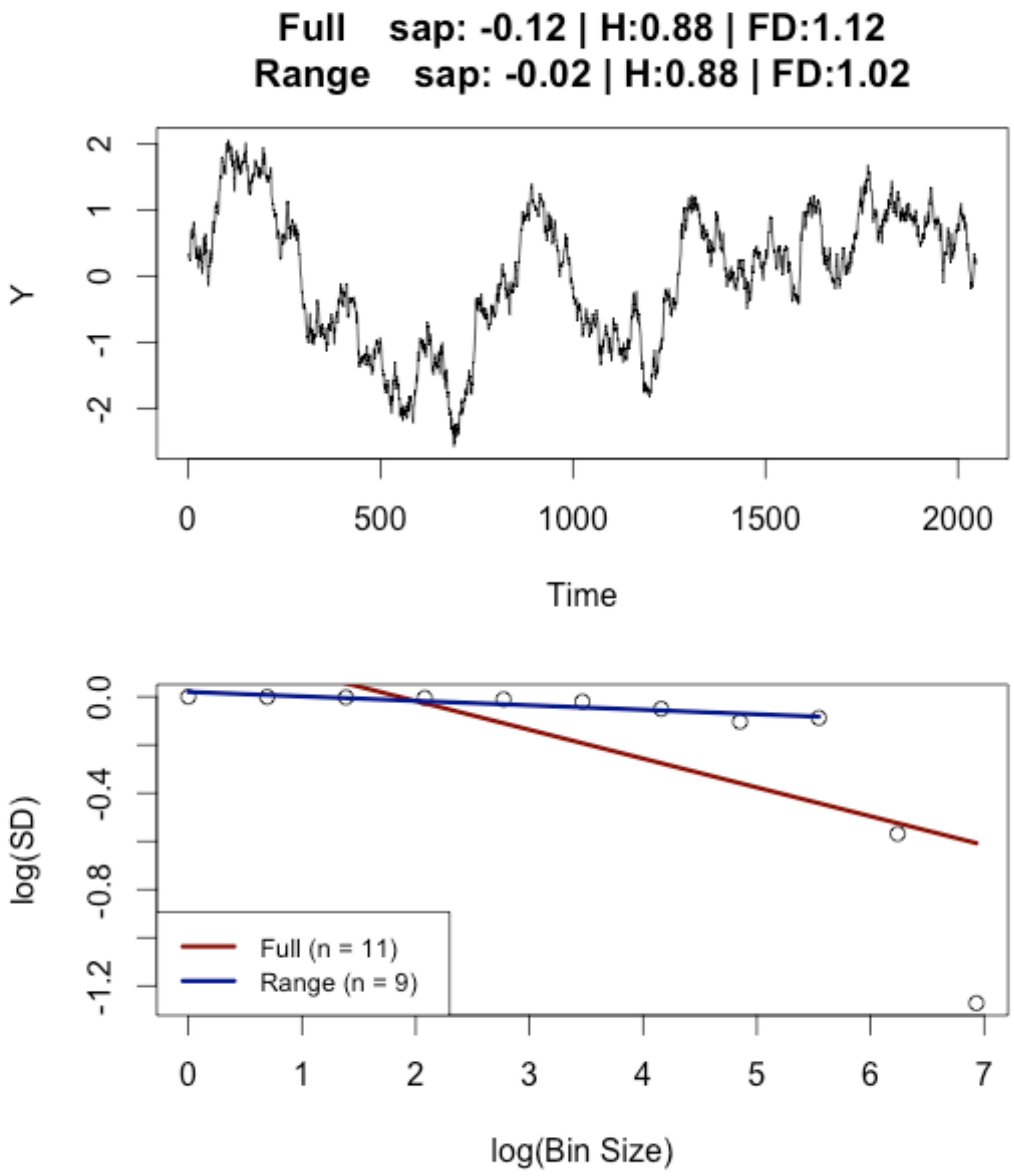
Fractal dimension = 1 – (slope of line)
FD = 1 - (-0.11) = 1.11




Normalized Brownian noise - bin size: 64
Normalized Brownian noise - bin size: 128
Normalized Brownian noise - bin size: 256
Normalized Brownian noise - bin size: 512
Means within bins
SD of means
log2 SD
Bin size
1
1024
2048
0
4
8
16
32
64
128
512

Fractal dimension = 1 – (slope of line)
FD = 1 - (-0.01) = 1.01




Full sap: -0.12 | H:0.88 | FD:1.12
Range sap: -0.02 | H:0.88 | FD:1.02
Y
Time
log(SD)
log(Bin Size)
Full (n = 11)
Range (n = 9)

Detrended Fluctuation Analysis

Same logic as SDA except:

- **A.** Signal is integrated

- **B.** Divided into bins

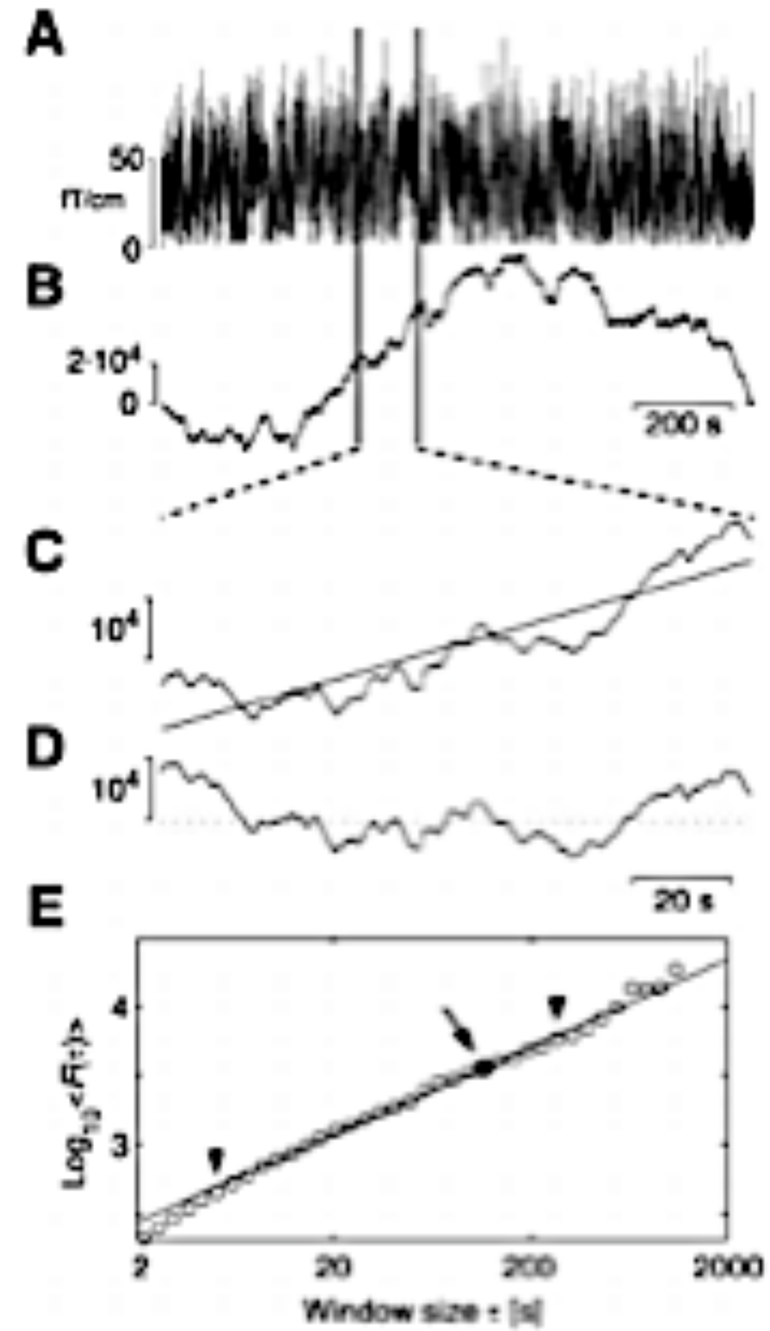
- **C.** Detrended (linear)

- **D.** Remaining SD is the dispersion measure

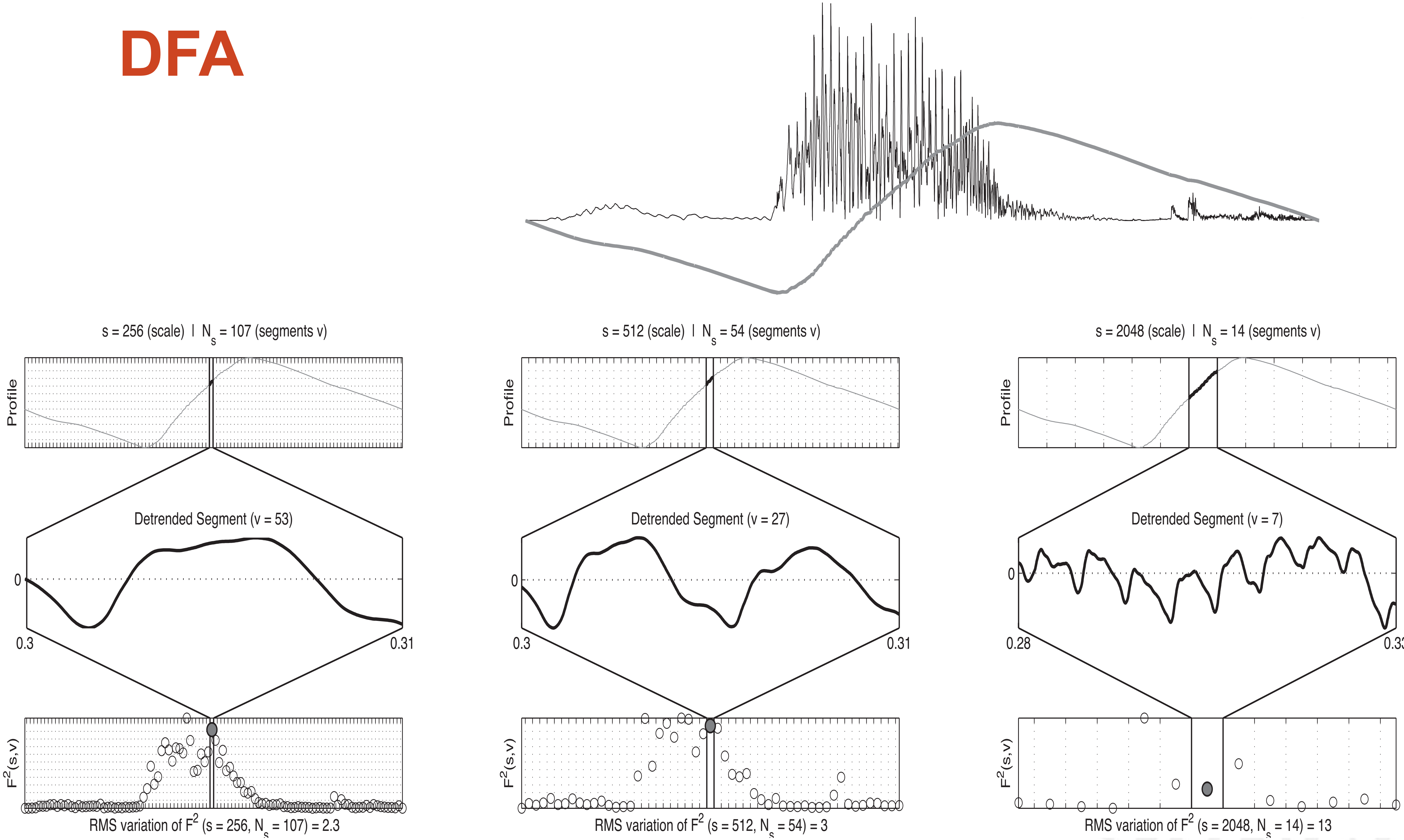
- **E.** Plot on Log_{10} scale and calculate slope (alpha)

- FD = 2 - (slope)

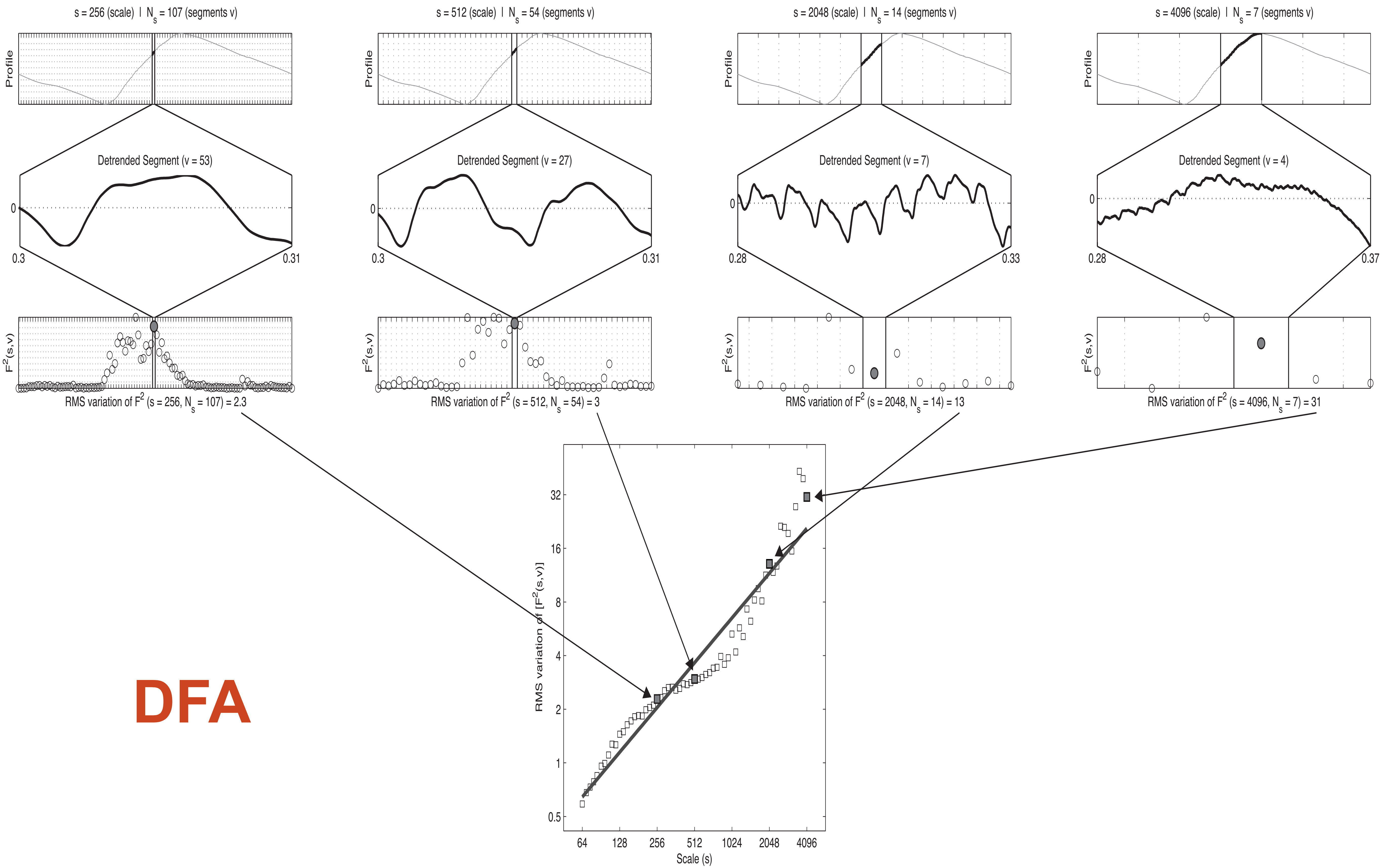
Or **C** & **D** in one step: fit a line in the bin and take SD of residuals… same result.



DFA

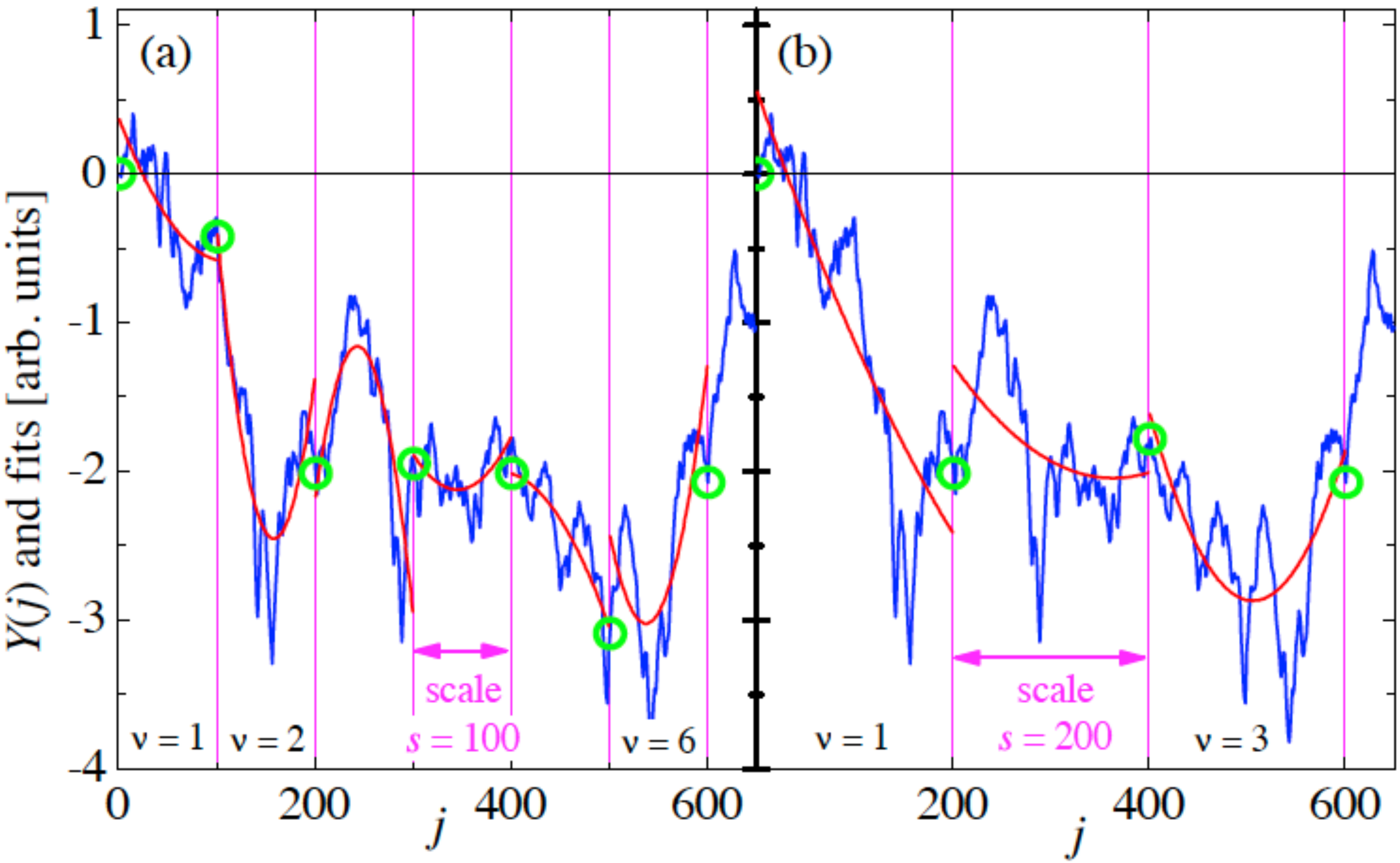


s = 256 (scale) | N_s = 107 (segments v)
Profile
Detrended Segment (v = 53)
0
0.3
0.31
F²(s,v)
RMS variation of F² (s = 256, N_s = 107) = 2.3
s = 512 (scale) | N_s = 54 (segments v)
Profile
Detrended Segment (v = 27)
0
0.3
0.31
F²(s,v)
RMS variation of F² (s = 512, N_s = 54) = 3
s = 2048 (scale) | N_s = 14 (segments v)
Profile
Detrended Segment (v = 7)
0
0.28
0.33
F²(s,v)
RMS variation of F² (s = 2048, N_s = 14) = 13

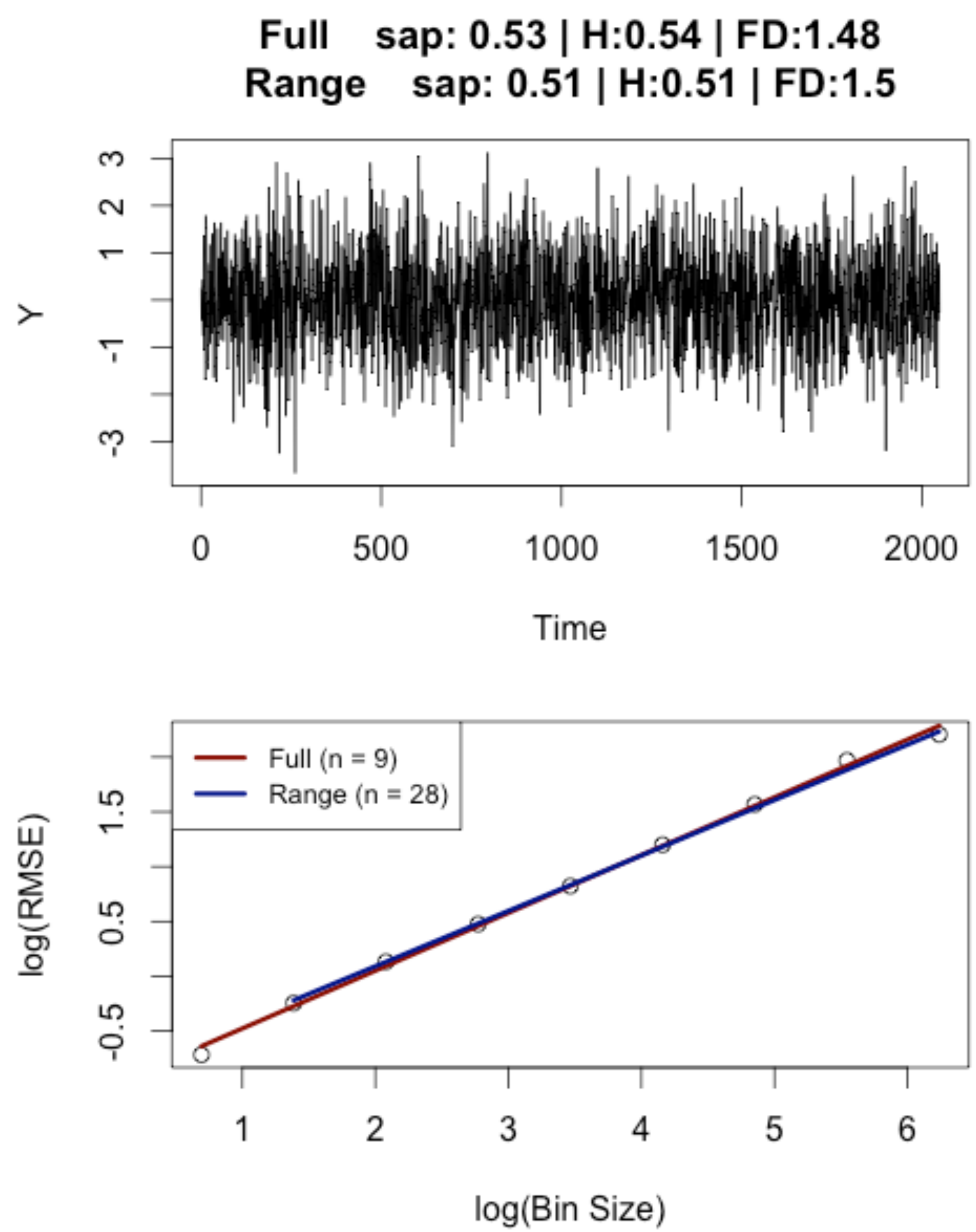

s = 256 (scale) | N_s = 107 (segments v)
s = 512 (scale) | N_s = 54 (segments v)
s = 2048 (scale) | N_s = 14 (segments v)
s = 4096 (scale) | N_s = 7 (segments v)
Profile
Detrended Segment (v = 53)
Detrended Segment (v = 27)
Detrended Segment (v = 7)
Detrended Segment (v = 4)
$F^2(s,v)$
RMS variation of F^2 (s = 256, N_s = 107) = 2.3
RMS variation of F^2 (s = 512, N_s = 54) = 3
RMS variation of F^2 (s = 2048, N_s = 14) = 13
RMS variation of F^2 (s = 4096, N_s = 7) = 31
RMS variation of $[F^2(s,v)]$
Scale (s)

DFA

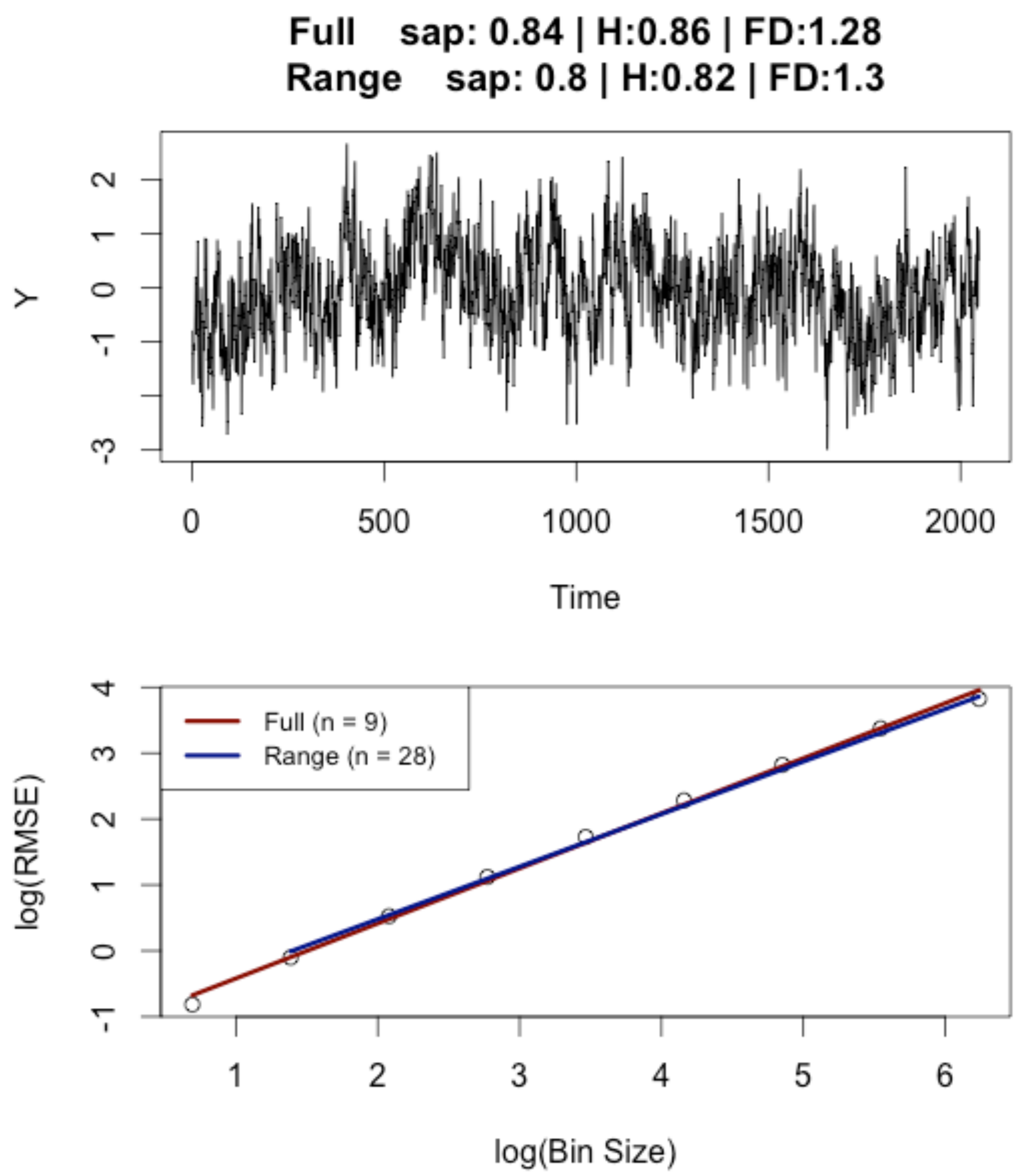
Detrended Fluctuation Analysis: Different ‘orders’ of detrending



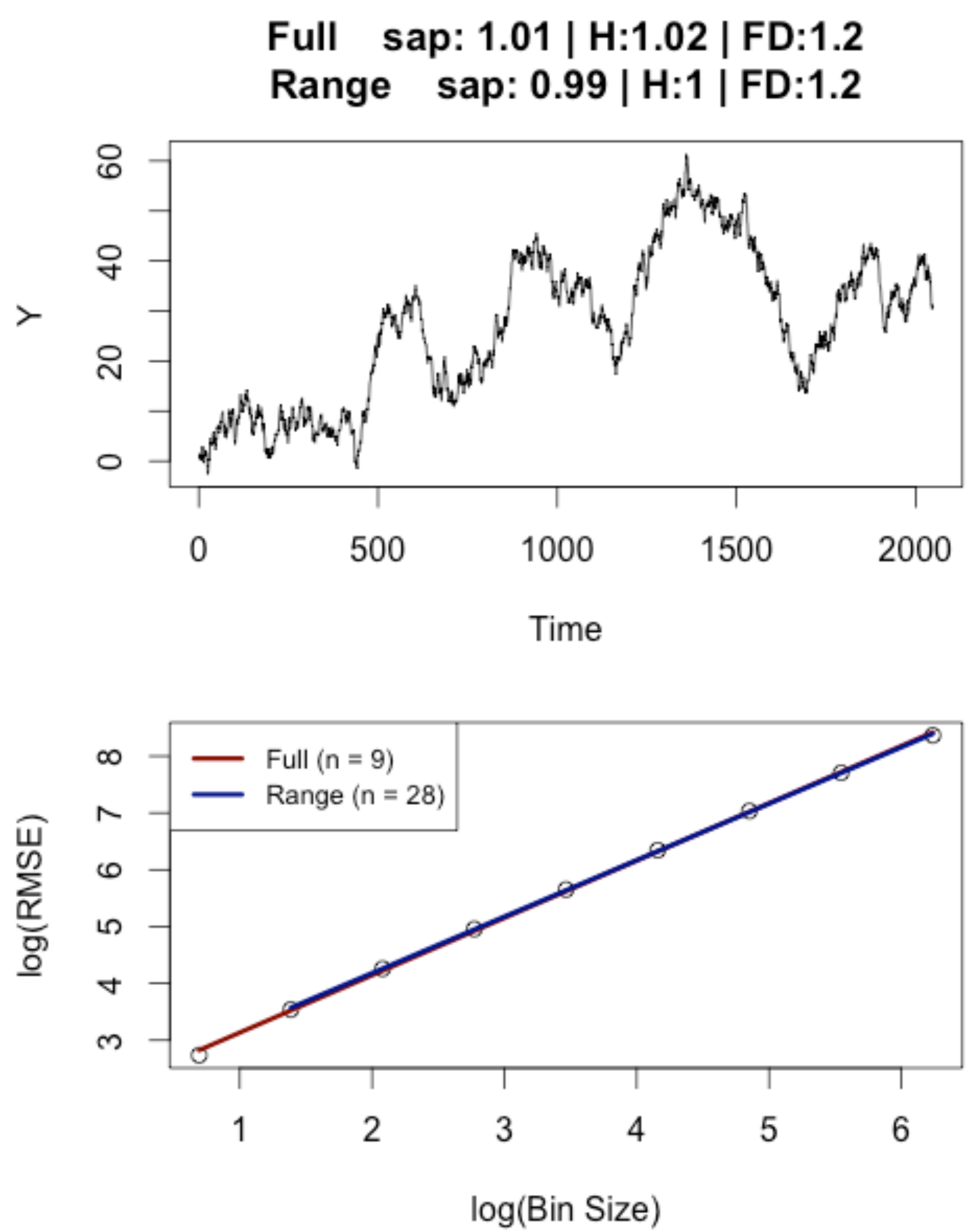
Detrended Fluctuation Analysis:



Detrended Fluctuation Analysis:



Detrended Fluctuation Analysis:



Report three values

	FD from Spectral Slope	FD from SDA	FD from DFA
White noise			
Pink noise			
Brownian noise			

Some data censoring and normalising is necessary

- **All analyses:**
  - Data points: 2^n, minimum 1024 (n=10):

  "*If you have not found the fractal pattern, you have not taken enough data*" (Machlup, 1977)

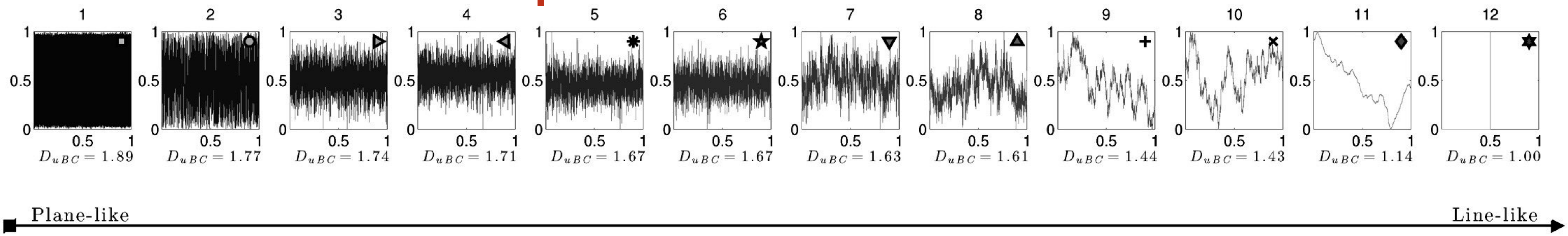
  - Remove 3 SD if necessary

- **Spectral analysis:**
  - Normalize the data (z-score transform: (**X**-mean(**X**))/SD(**X**)
  - Remove linear trend if necessary (detrend)
  - Decide number of frequencies to estimate, min. 512

- **SDA:**
  - Normalize the data (z-score transform: (**X**-mean(**X**))/SD(**X**)

- **DFA:**
  - Nothing extra, analysis integrates and detrends the signal

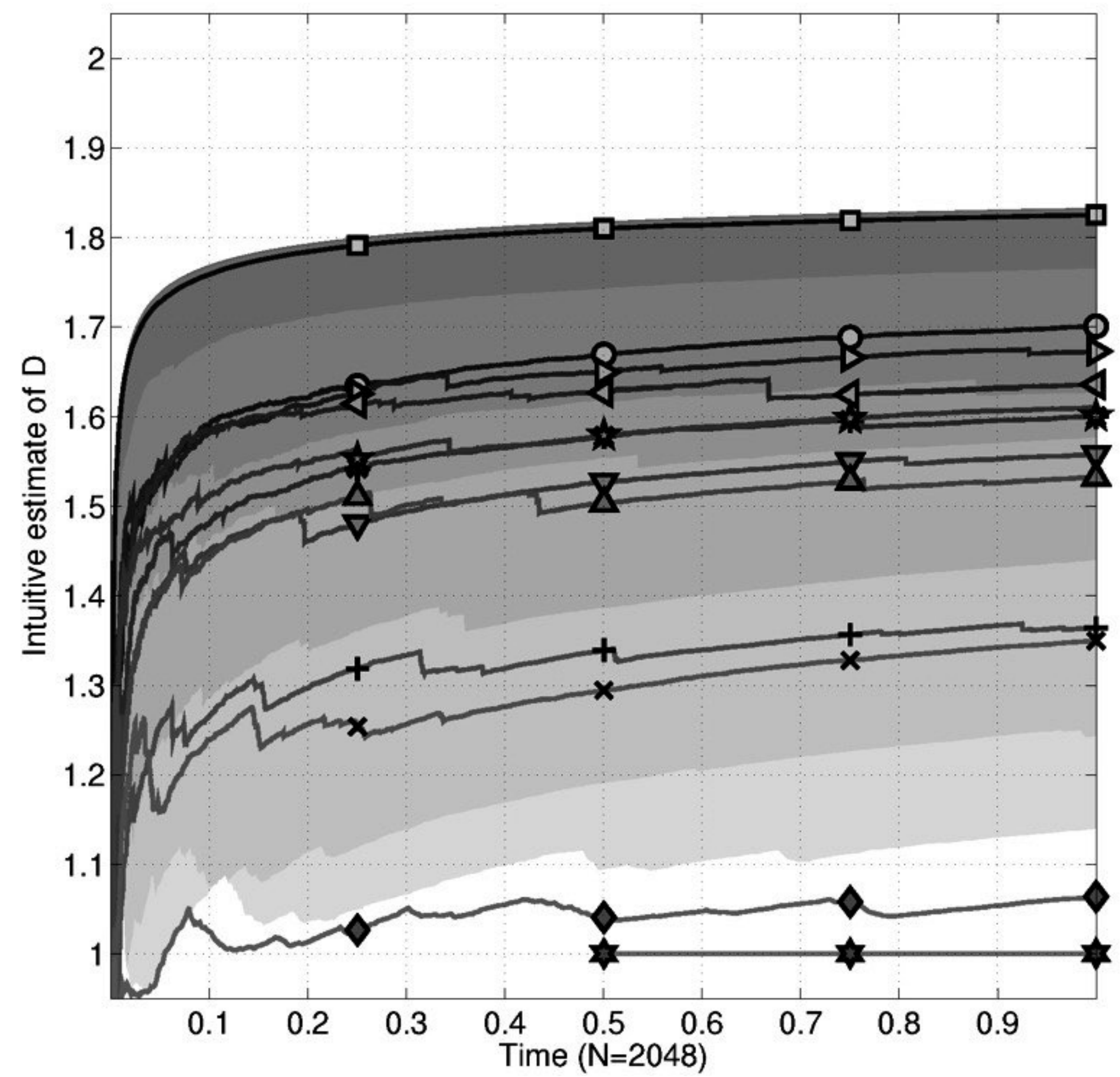
Planar Extent: How plane-like is a waveform?



Sevcik method:

$$D \approx \lim_{N' \to \infty} \left[1 + \frac{\ln(L) - \ln(2)}{\ln(2 * N')} \right]$$

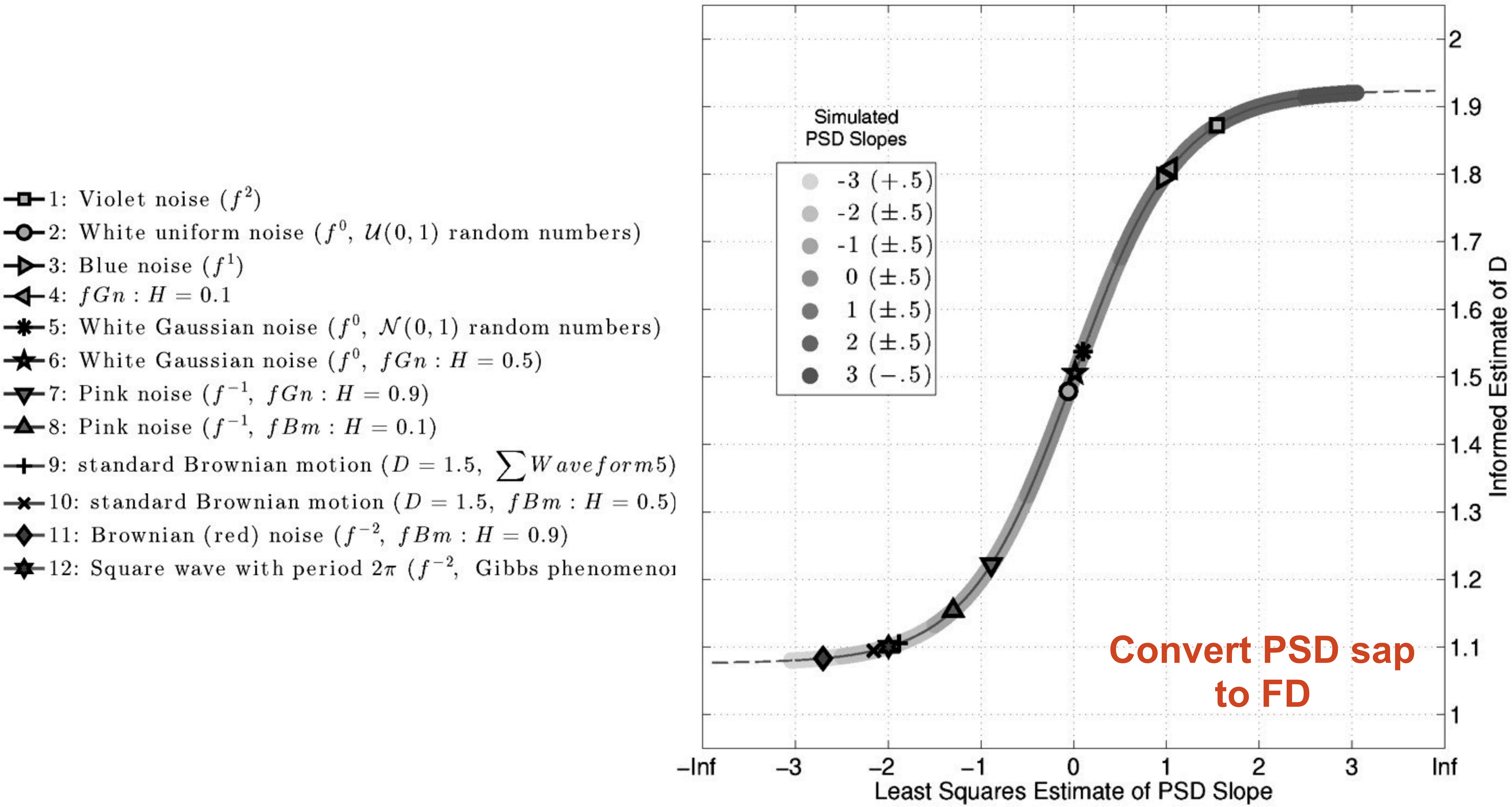
- 1: Violet noise (f^2)
- 2: White uniform noise (f^0, $\mathcal{U}(0,1)$ random numbers)
- 3: Blue noise (f^1)
- 4: $fGn : H = 0.1$
- 5: White Gaussian noise (f^0, $\mathcal{N}(0,1)$ random numbers)
- 6: White Gaussian noise (f^0, $fGn : H = 0.5$)
- 7: Pink noise (f^{-1}, $fGn : H = 0.9$)
- 8: Pink noise (f^{-1}, $fBm : H = 0.1$)
- 9: standard Brownian motion ($D = 1.5$, $\sum Waveform5$)
- 10: standard Brownian motion ($D = 1.5$, $fBm : H = 0.5$)
- 11: Brownian (red) noise (f^{-2}, $fBm : H = 0.9$)
- 12: Square wave with period 2π (f^{-2}, Gibbs phenomenon)

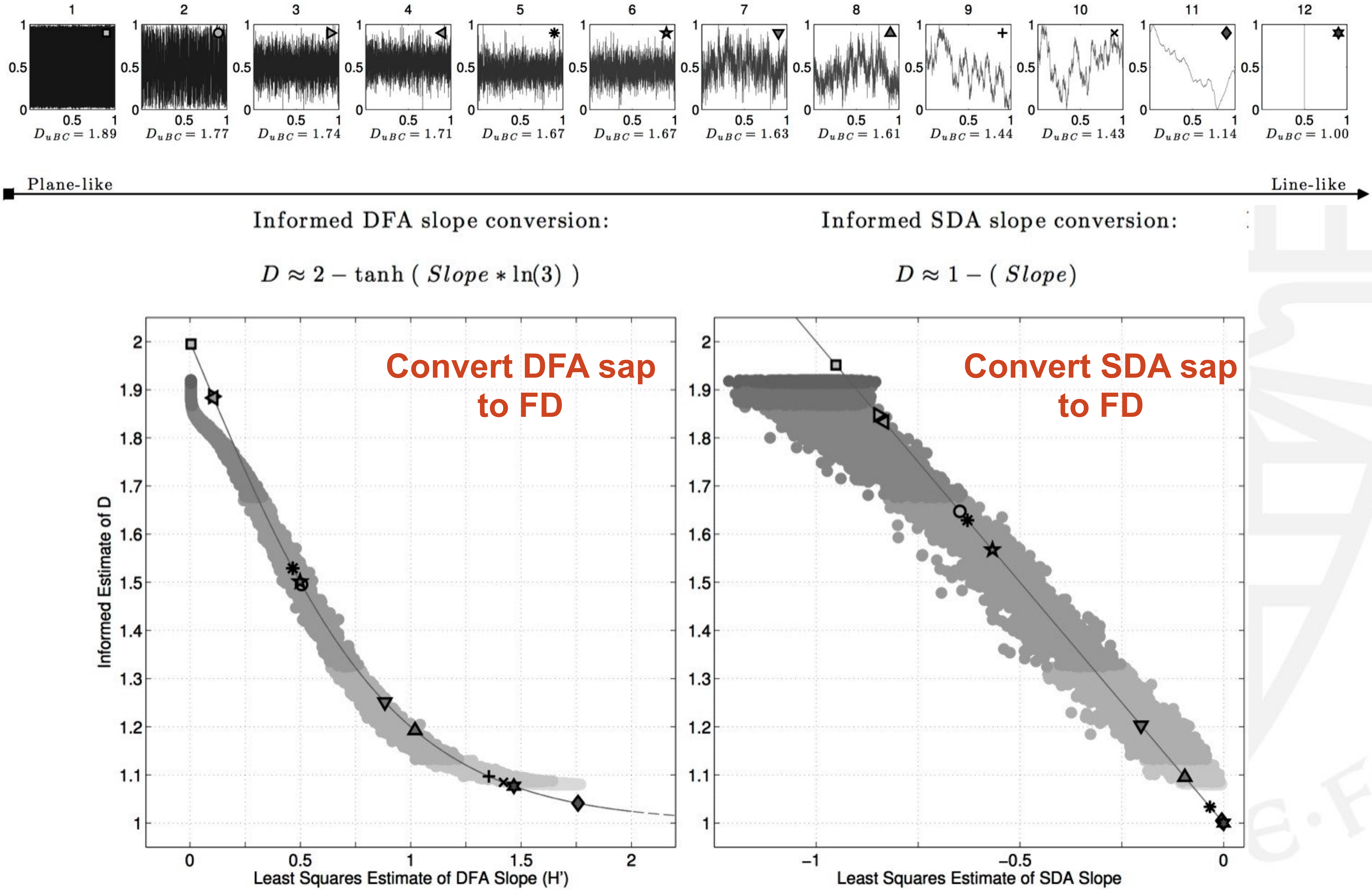


Convert sap to FD

Informed PSD slope conversion:

$$D \approx \frac{3}{2} + \frac{14}{33} * \tanh\left(Slope * \ln(1+\sqrt{2}) \right)$$





Convert DFA sap to FD

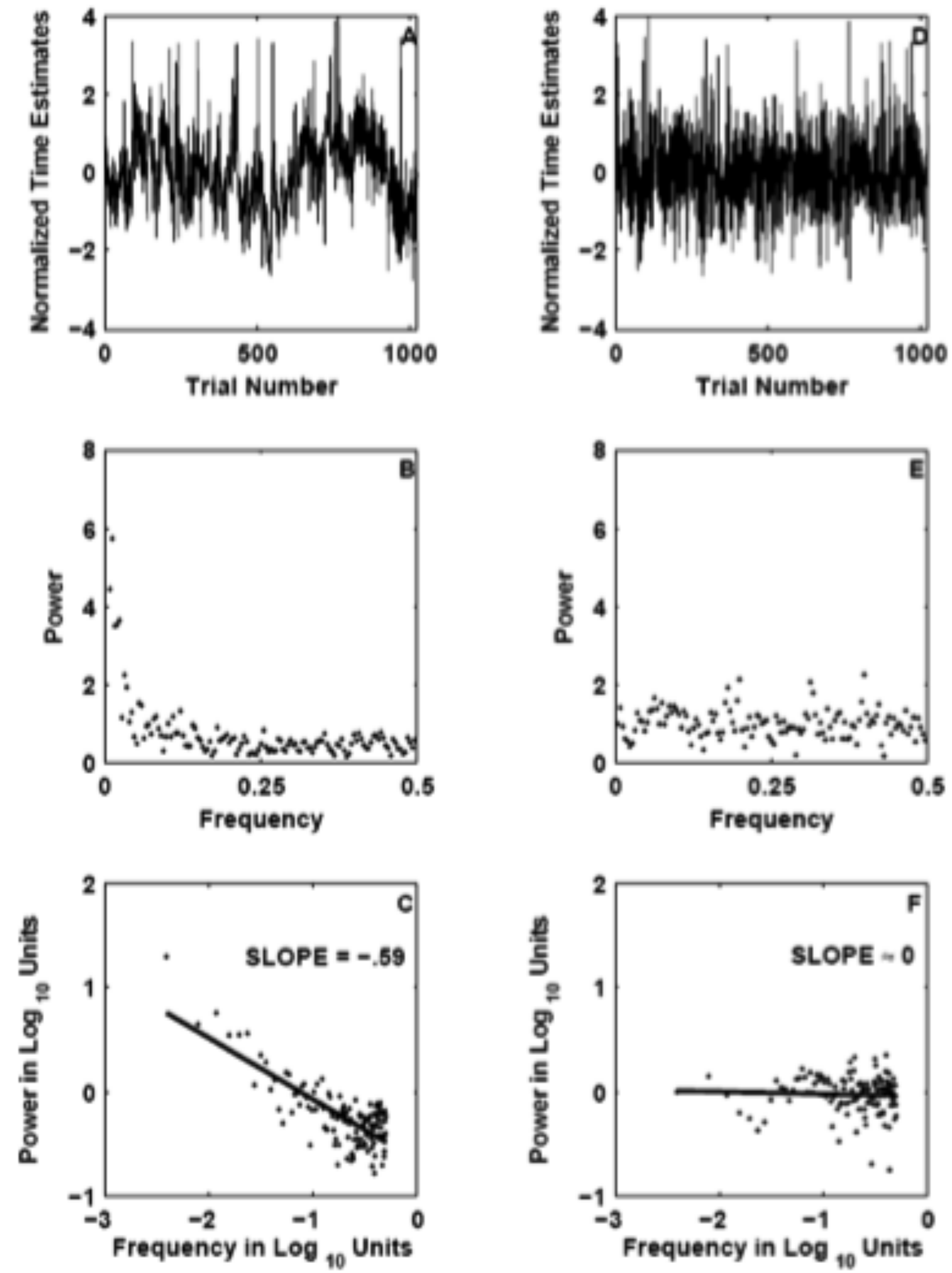
Convert SDA sap to FD

Important: The scaling relation exists only
In the time series as it was recorded.

If you randomise the timeseries after it was recorded and loose the temporal structure (dynamics) white noise appears!

So even if you randomised your stimuli, the noise is in the temporal, dynamical structure of the time series.

Test wether your fractal slope deviates from white noise by calculating a lot of random slopes.



Success!

- We have successfully characterised the data in terms of (one) complexity measure
- This is not possible with traditional measures
- Next: That's all neat stuff, but what can I do with it?