A starting point for an estimate of $u_j$ would be to take the mean of $y_{ij} - \hat{\beta}_0$ for group $j$. This is sometimes called the **mean raw residual**:

$$\bar{r}_j = \bar{y}_j - \hat{\beta}_0$$

where $\bar{y}_j$ is the mean of $y_{ij}$ in group $j$.