DETERMINISTIC CHAOS

1refs

CHAOS, TURBULENCE
and other unsolved mysteries

"Turbulence is the most important unsolved problem of classical physics"

- Richard Feynman (1918 - 1988)

"I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment:

One is quantum electrodynamics,
and the other is the turbulent motion of fluids.

And about the former I am rather optimistic."

- Horace Lamb (1849 - 1934)

Turbulent

Laminar

Deterministic Chaos

Table 12-1. Summary of the Hierarchy of Dynamic Systems.

Type	Constraints	Description
Zero	Absolute	Constant state
I	Analytic integrals	Solvable dynamic system
II	Approximate analytic integrals	Amenable to perturbation theory
III	Quasi-deterministic; smooth but erratic trajectory	Chaotic dynamic system
IV	Rigorously defined only by averages over time or state space	Turbulent/stochastic

Table 12-2. A few examples of the types of dynamic systems.

Type	Examples
Zero	Images, gravity models, structures
I	Gear trains, 2-body problem, physical pendulum
II	Satellite orbits, lunar and planetary theories
III	Climatology, Lorenz equations, discrete logistic equation
IV	Quantum mechanics, turbulent flow, statistical mechanics

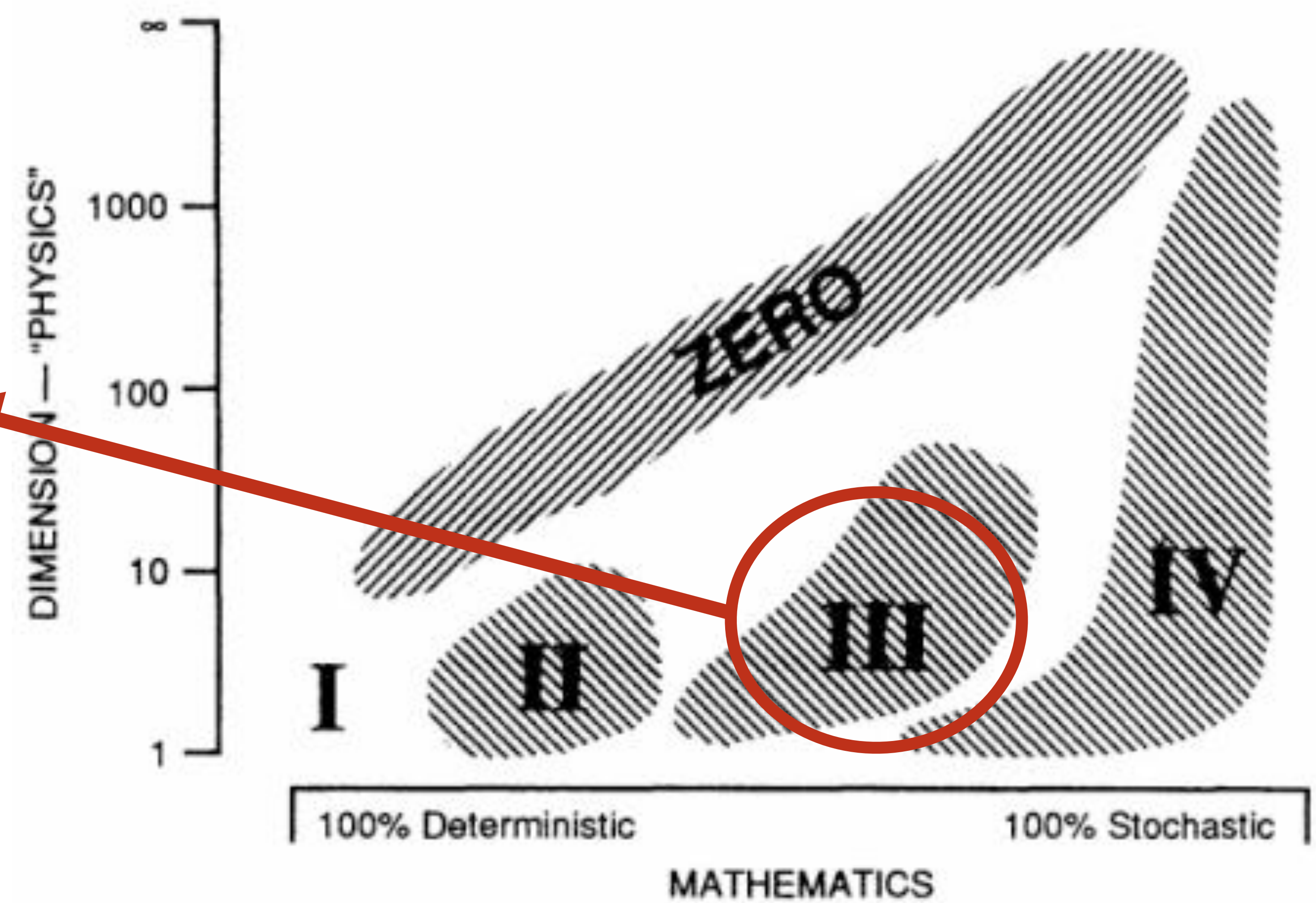


Figure 12-1. Schematic representation of the Hierarchy of Dynamic Systems.

Deterministic Chaos

There is no real definition of chaos, but there are at least four ingredients:

*The dynamics is **a-periodic** and **bounded**, and the system is **deterministic** and **sensitively depends on initial conditions**.*

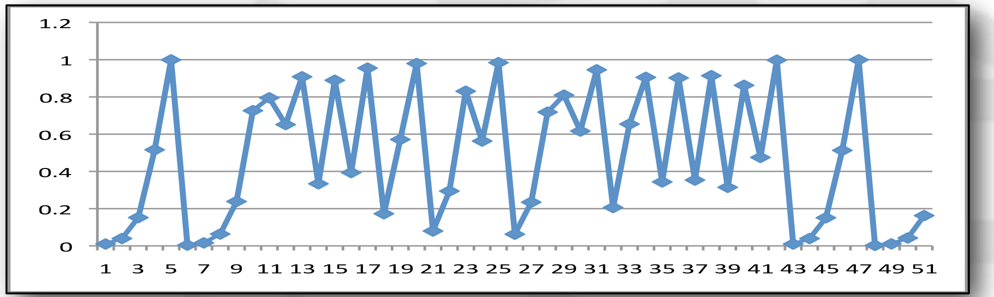
[1]refs

CHAOS, TURBULENCE
and other unsolved mysteries

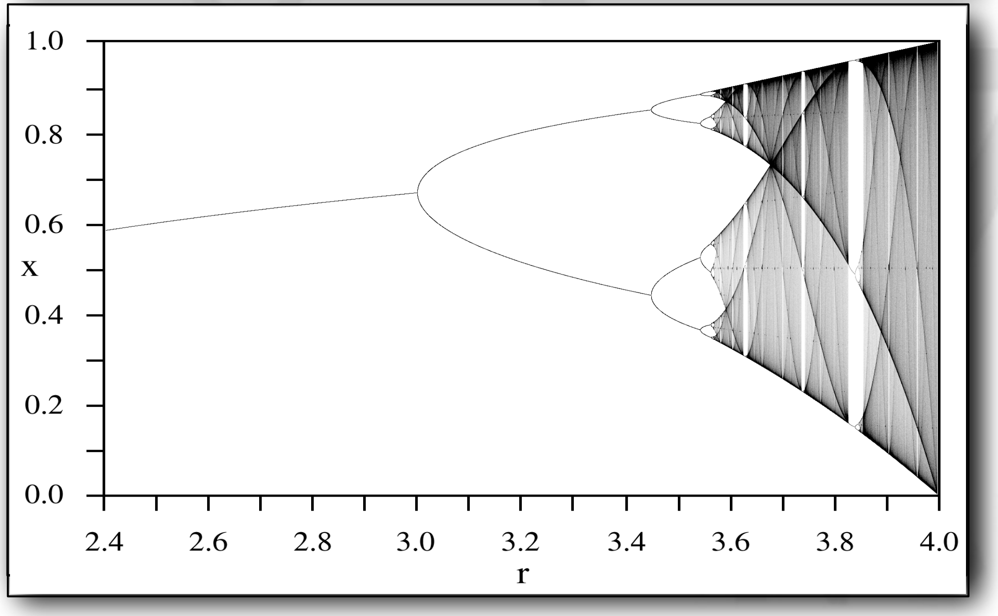
What *can* we say about chaos?

$$X_{i+1} = r X_i (1 - X_i)$$

1. Deterministic, not stochastic



2. A-periodic, or Quasi-periodic



3. Bounded

4. Sensitive to initial conditions

CHAOS, TURBULENCE
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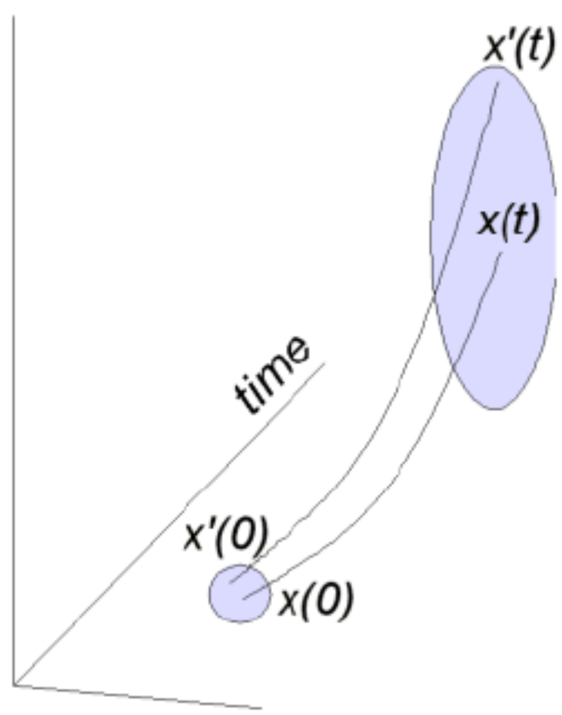
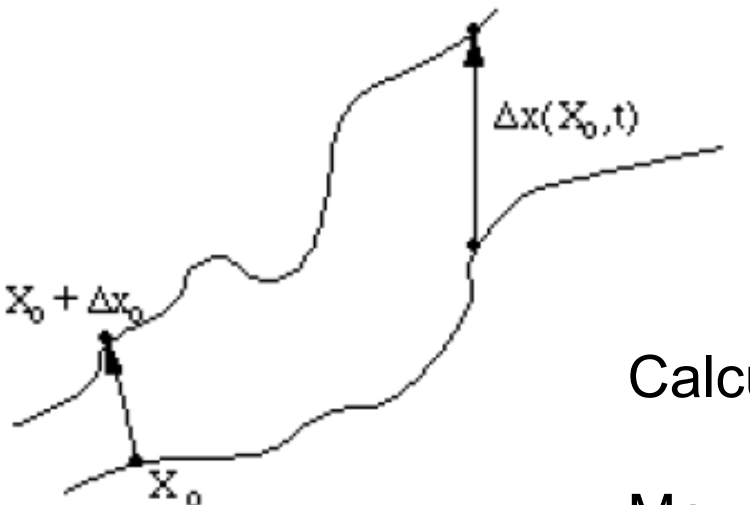
What *can* we say about chaos?

4. Sensitive dependence on initial conditions

The *Lyapounov Exponent* characterises (quantifies) the rate of separation of two infinitesimally close trajectories in state space.





Calculate if you have a model

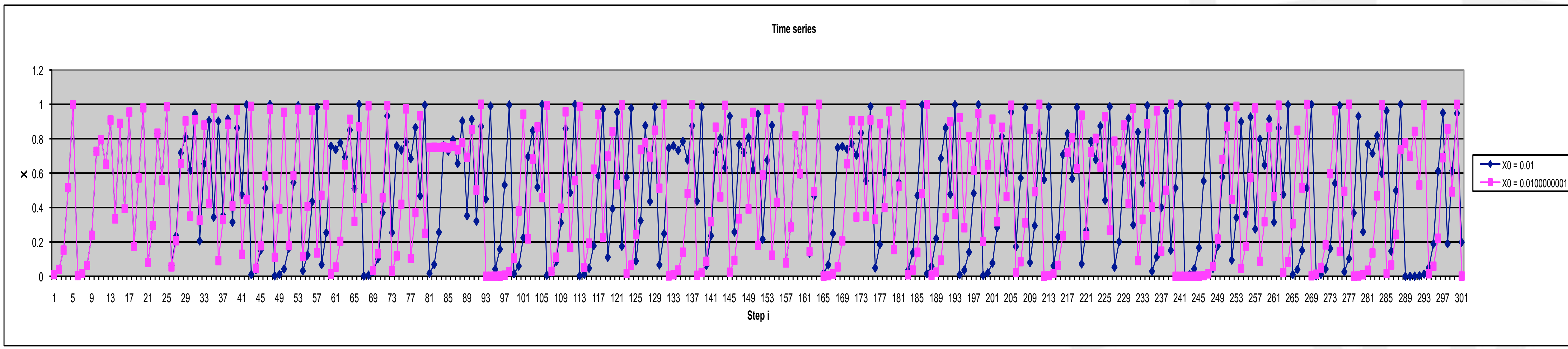
May be experimentally accessible

Analytic techniques (in R) are available

Sensitive Dependence on Initial Conditions

What *can* we say about deterministic chaos and complexity?

$X_0 = 0.01$ **$X_0 = 0.0100000001$**



Tiny differences in initial conditions can yield diverging time-evolutions of system states

Lorenz observed this in his models of the upper atmosphere:
The divergence was so extreme it resembled a butterfly flapping its wings -or not-
could be the difference between weather developing as a hurricane or a summer breeze

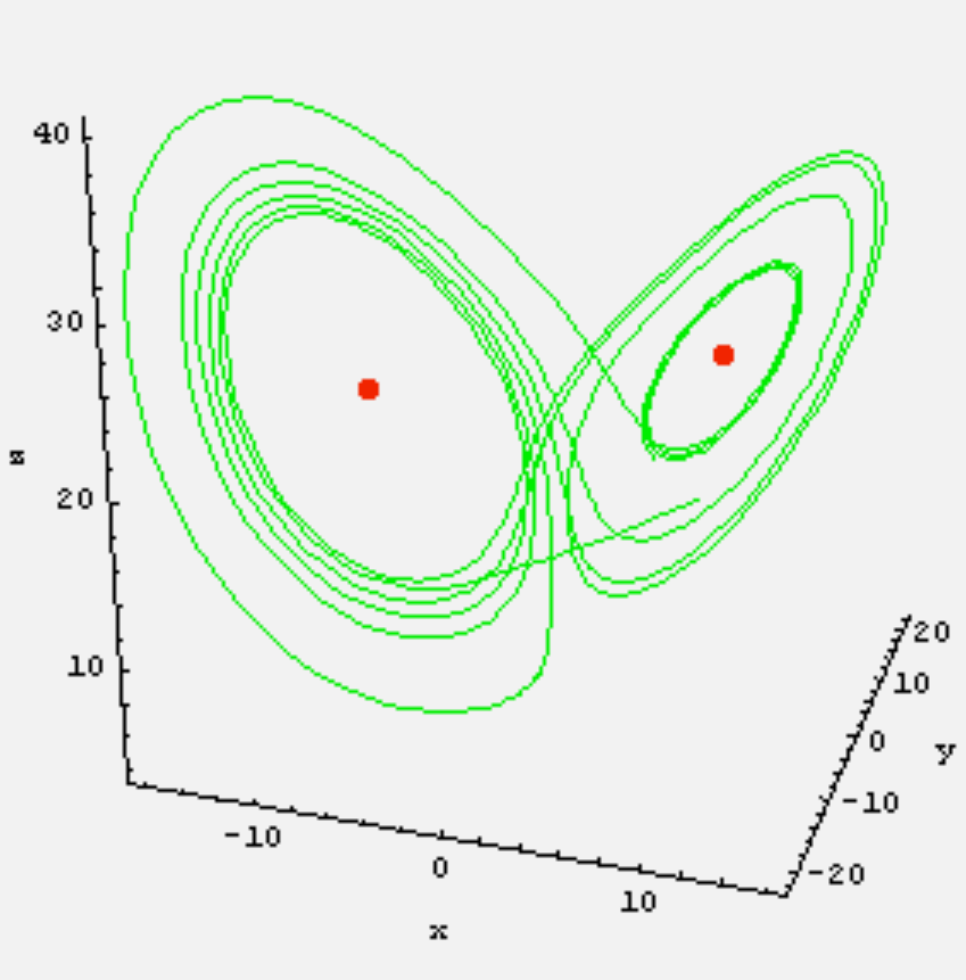
Lorenz Attractor

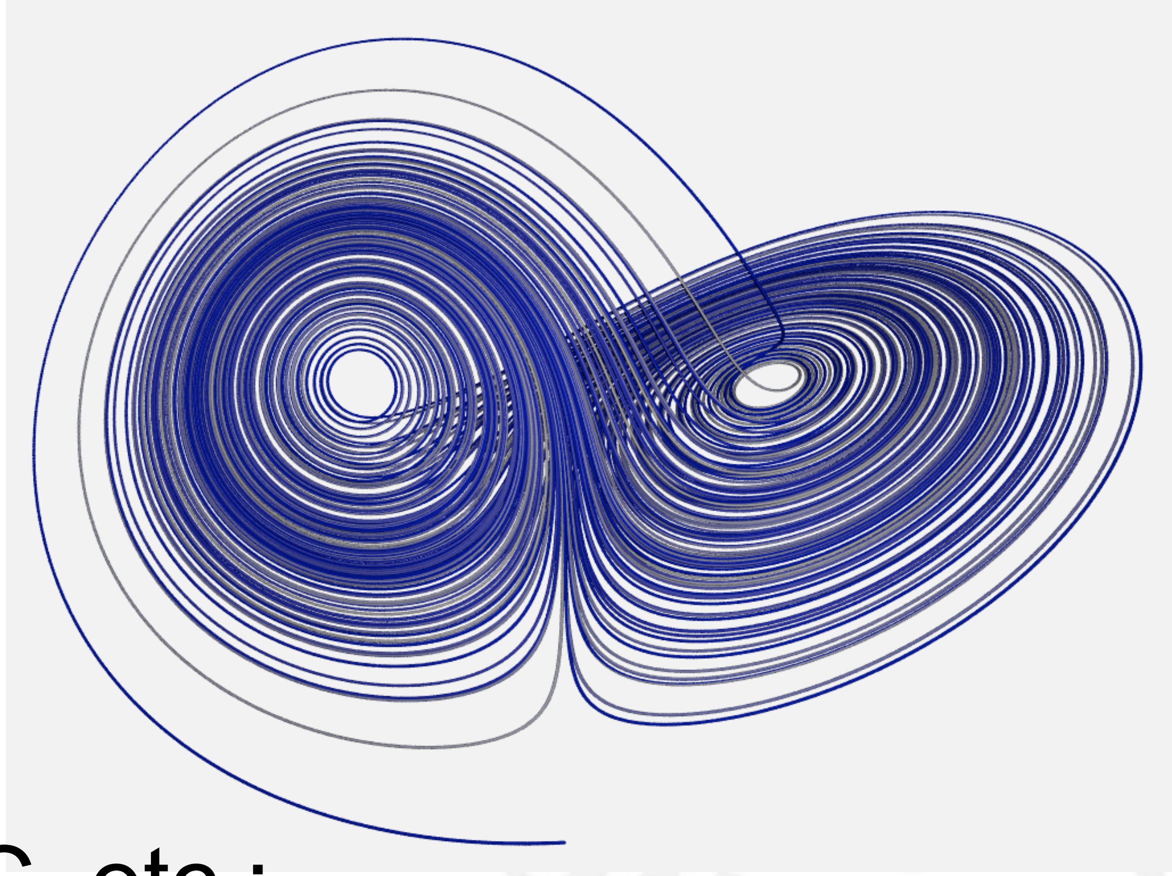
$$\frac{dx}{dt} = a(y - x),$$
$$\frac{dy}{dt} = x(b - z) - y,$$
$$\frac{dz}{dt} = xy - cz.$$

Deterministic Chaos

Maps: linear map, 1D state space

Flows: Need 3 coupled ODEs (ordinary differential equations) Minimum is 3D state space





Lorenz about chaos, fractals, SOC, etc.:
"*Study of things that look random -but are not*"

Advanced Data Analysis:

Dynamics of Complex Systems

Potential Theory

f.hasselman@pwo.ru.nl m.wijnants@pwo.ru.nl

Behavioural Science Institute
Radboud University Nijmegen

Nonlinear Phenomena
Example: Relapse in Addictive Alcohol Use

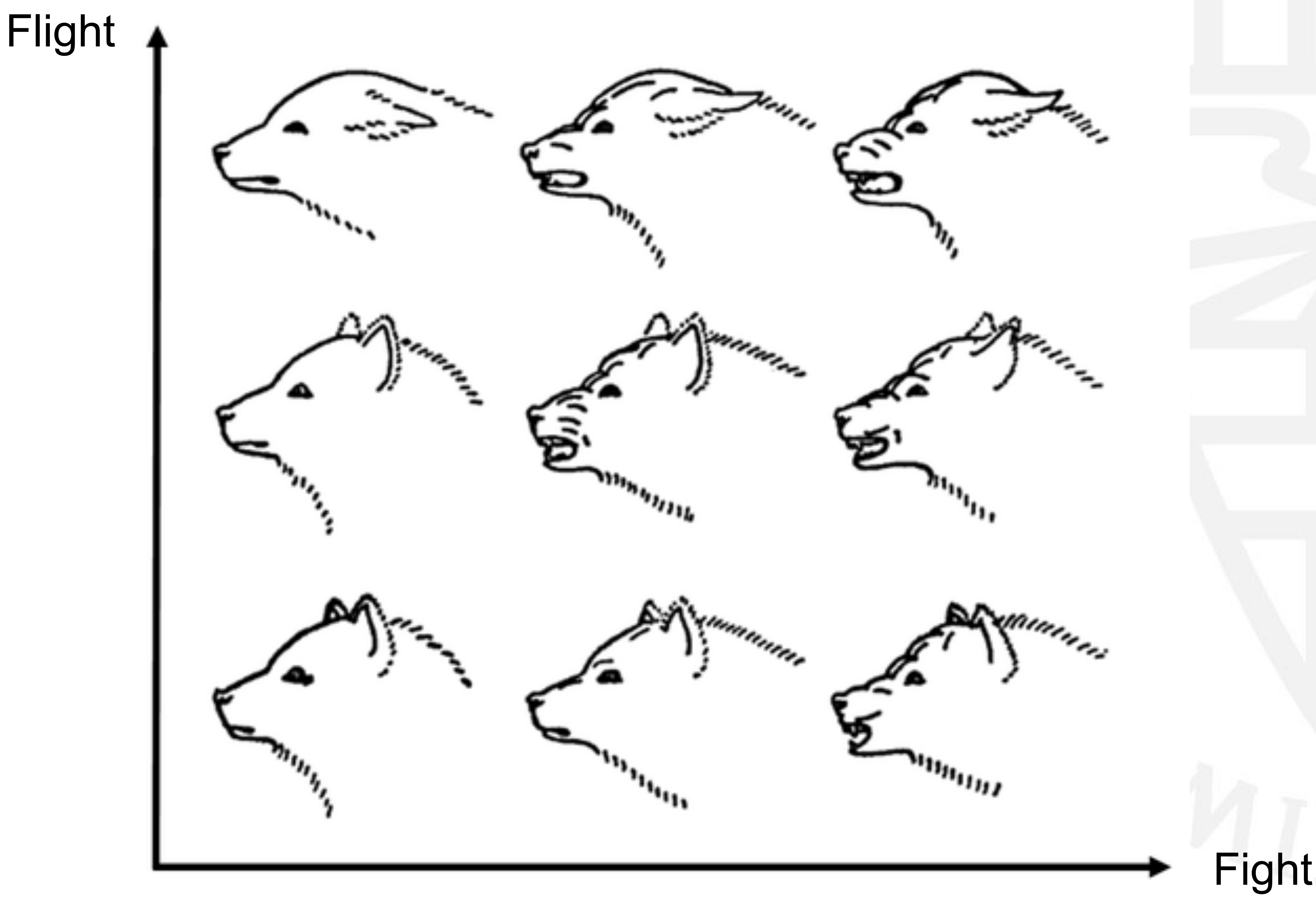
What we know…

- Relatively small changes in the risk factors can strongly influence the possibility of relapse (**sudden jumps due to gradual change in control parameter**)

- Risk factors for relapse are not the same as the success factors for the switch back to abstinence (**hysteresis, enhanced contrast: nonlinear interactions**)

- Seemingly unrelated events and decisions can (over time) lead to relapse (**nonlinear interactions across timescales, divergence**)

Konrad Lorenz: Aggression in dogs



Potential Functions (1)

$$\boxed{\frac{dx}{dt} = f(x)} \qquad \boxed{f(x) = -\frac{dV(x)}{dx}}$$

- x is the (psychological) variable that we are interested in.
- V is a ***potential function*** that describes / determines the possible states that x can attain.

→ Attractor states

Potential Functions (2)

$$\boxed{\frac{dx}{dt} = -\frac{dV(x;\alpha)}{dx}}$$

For different values of the parameter(s) α there are qualitatively different behavioral regimes ('possibilities') of the dynamical system.

→ ***Bifurcations***

Haken, Kelso & Bunz (1985)

Model for the relative phase dynamics, using the following potential function V(Φ):

$$\boxed{V(\phi) = -a\cos\phi - b\cos 2\phi.}$$

- *a* and *b* are ***coupling constants***
- The ratio *b/a* is inversely related to frequency

Potential models - Nonlinear Dynamics

Potential Function of the HKB model

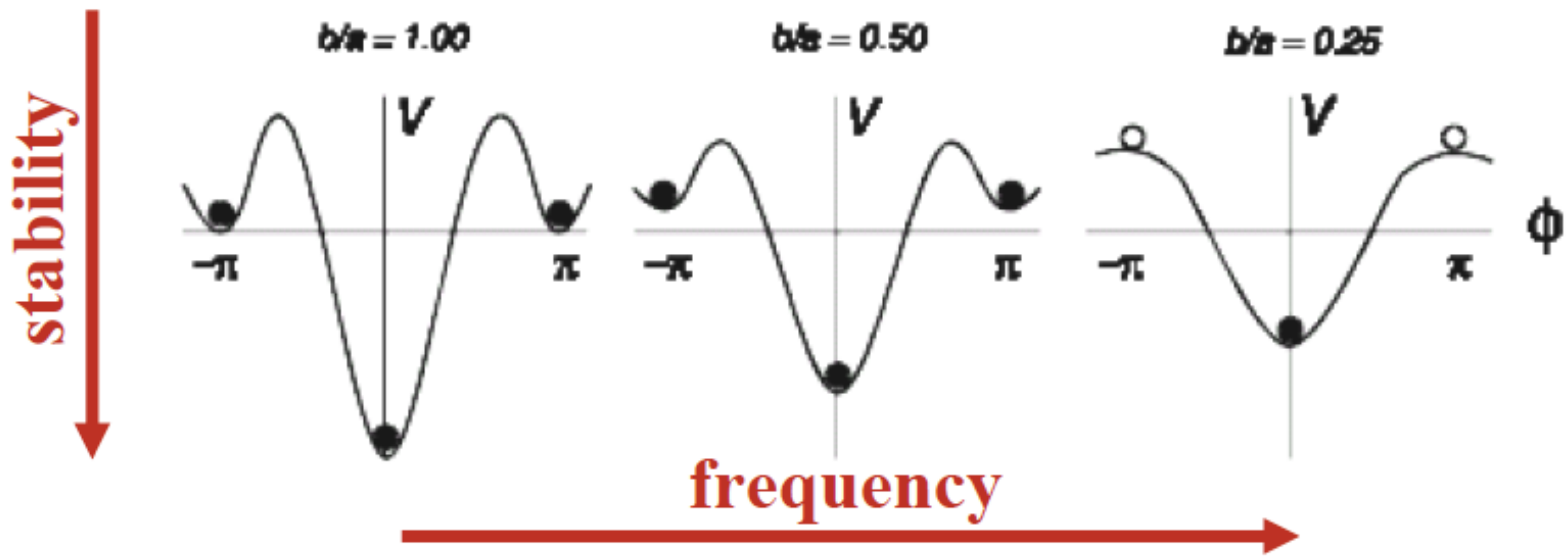


FIGURE 3.1. In the potential $V(\phi) = -a \cos\phi - b \cos 2\phi$, the dynamical "landscape" or "attractor lay-out" of ϕ changes as a function of the coupling ratio b/a. Black balls represent stable points, and white balls represent unstable points in the potential (adapted from Haken, Kelso, & Bunz, 1985).

The Ontogenesis of a Competence

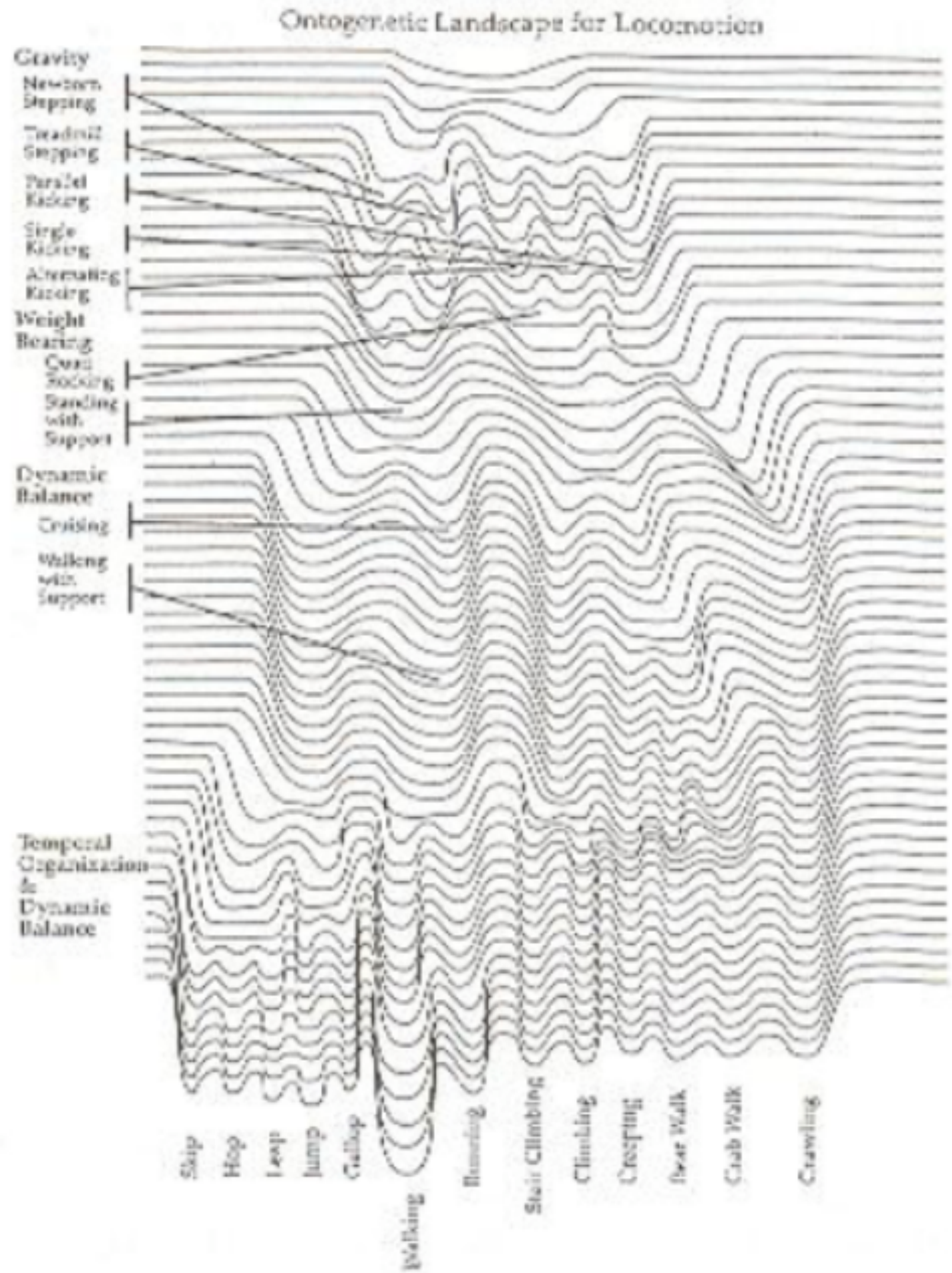
A (hypothetical) dynamical landscape for the different phases of locomotion. →

The groves mark the stable locomotor organizations that are possible at a certain age (lines).

Y-axis: Age (time)

X-axis: Forms of locomotion

Z-axis: Measure of stability



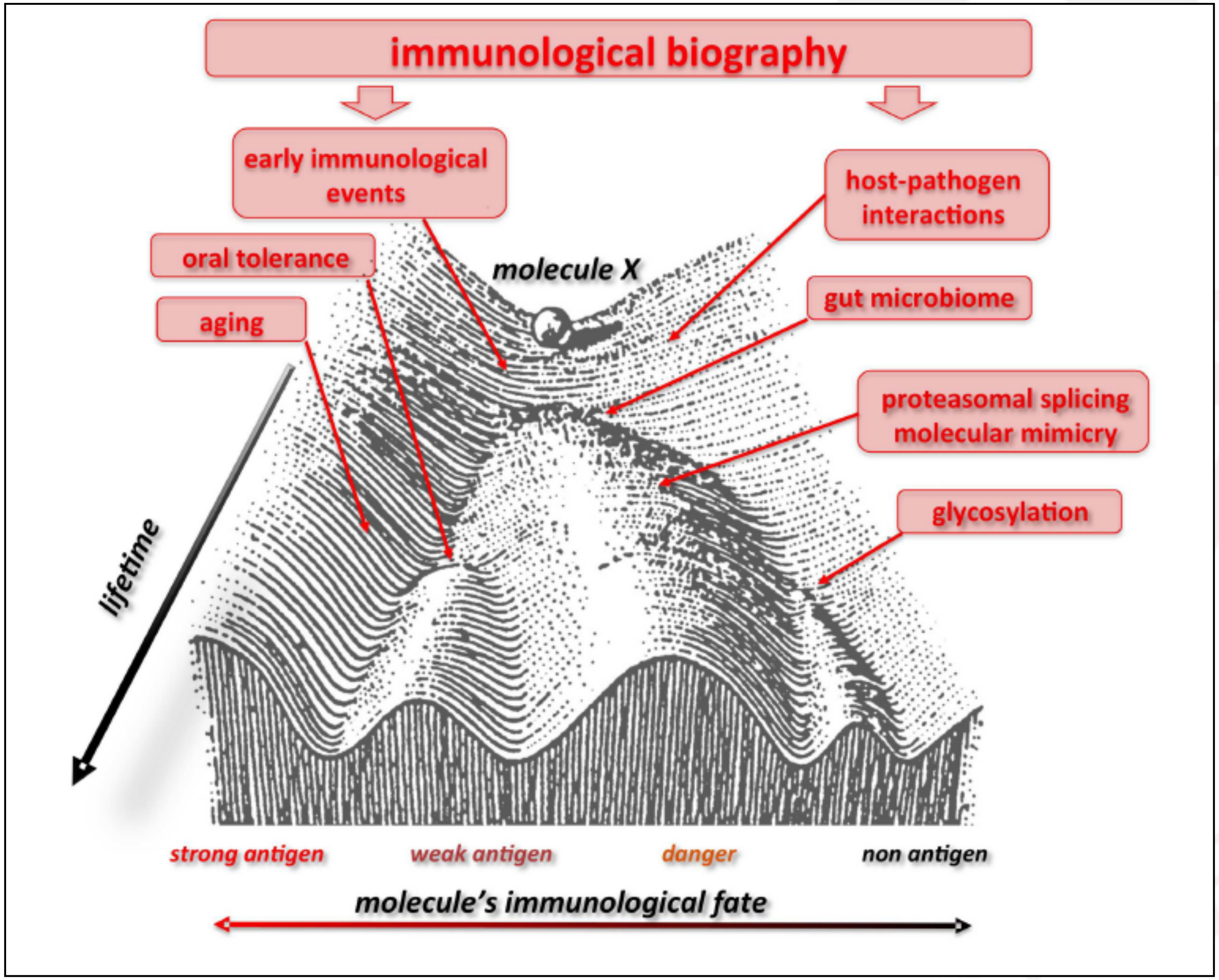
More general: Potential Landscape

Can be used to understand individual trajectories:

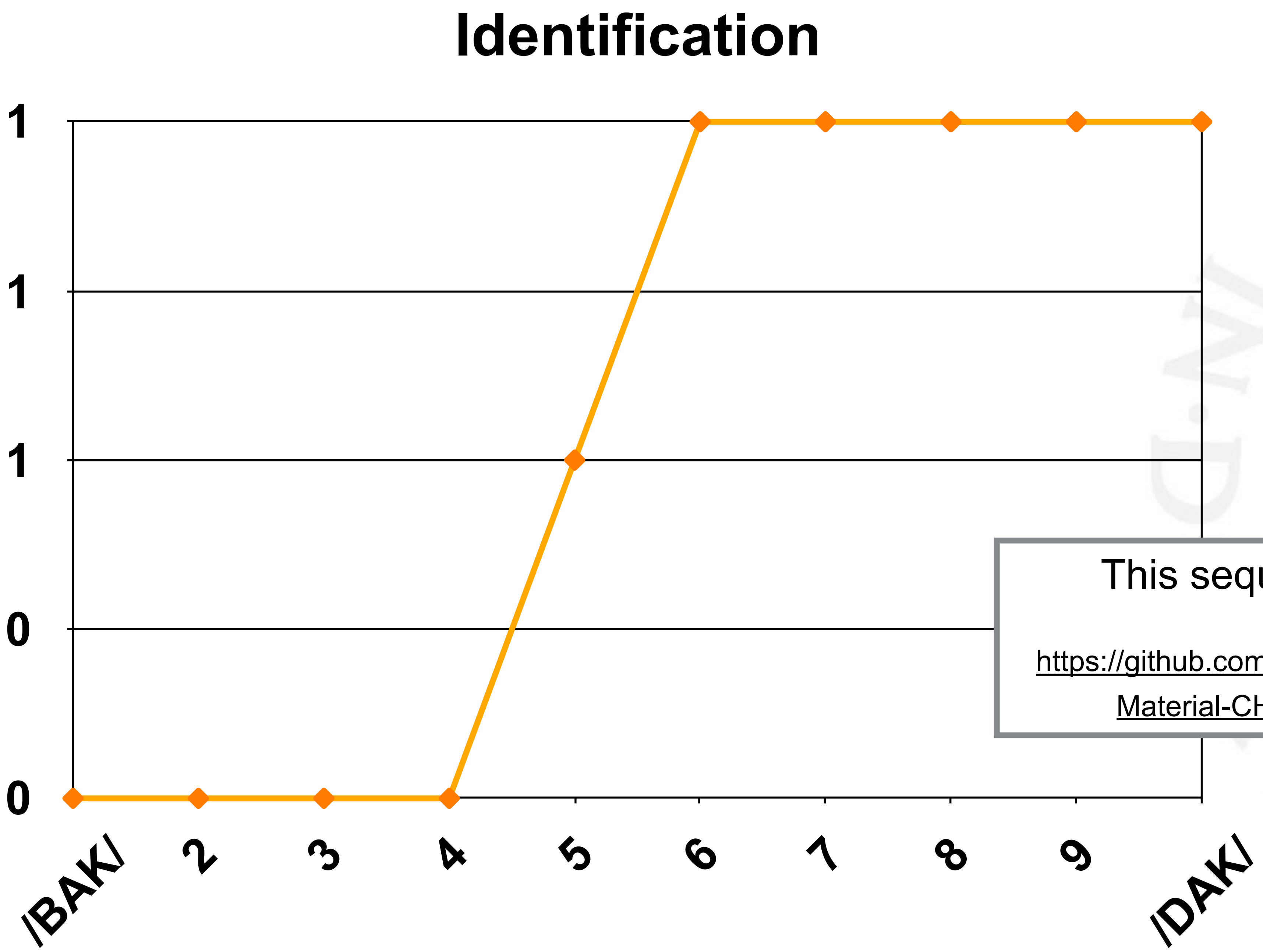
immune systems

development

learning



Potential models - Nonlinear Dynamics



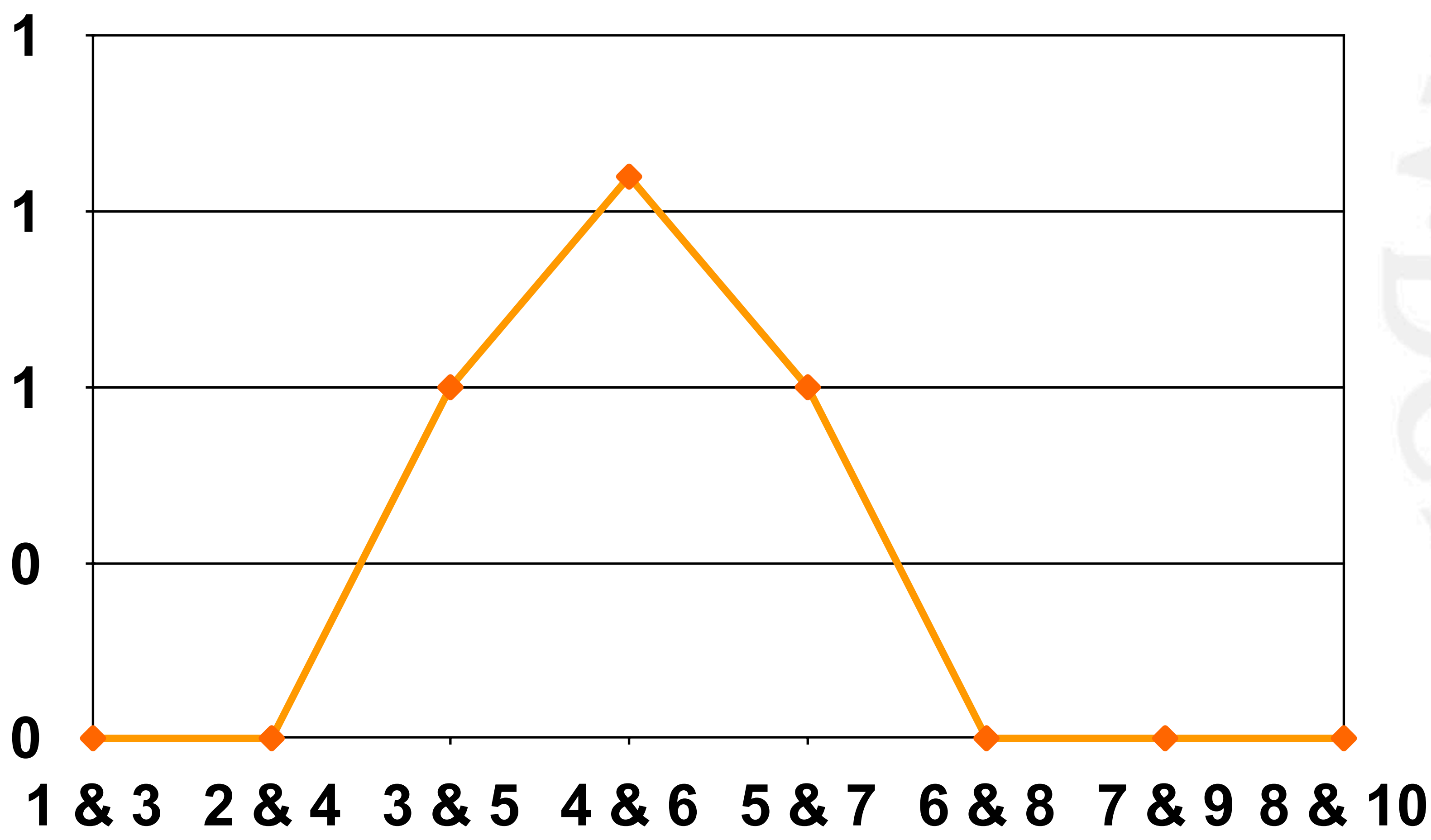
This sequence in is available at:

https://github.com/FredHasselman/BTB-Supplemental-Material-CHAPTER4/tree/master/STIMULI

Speech Perception: *The Basics*

It is possible to create a continuum from BAK to DAK by slowly increasing F2 onset

Discrimination



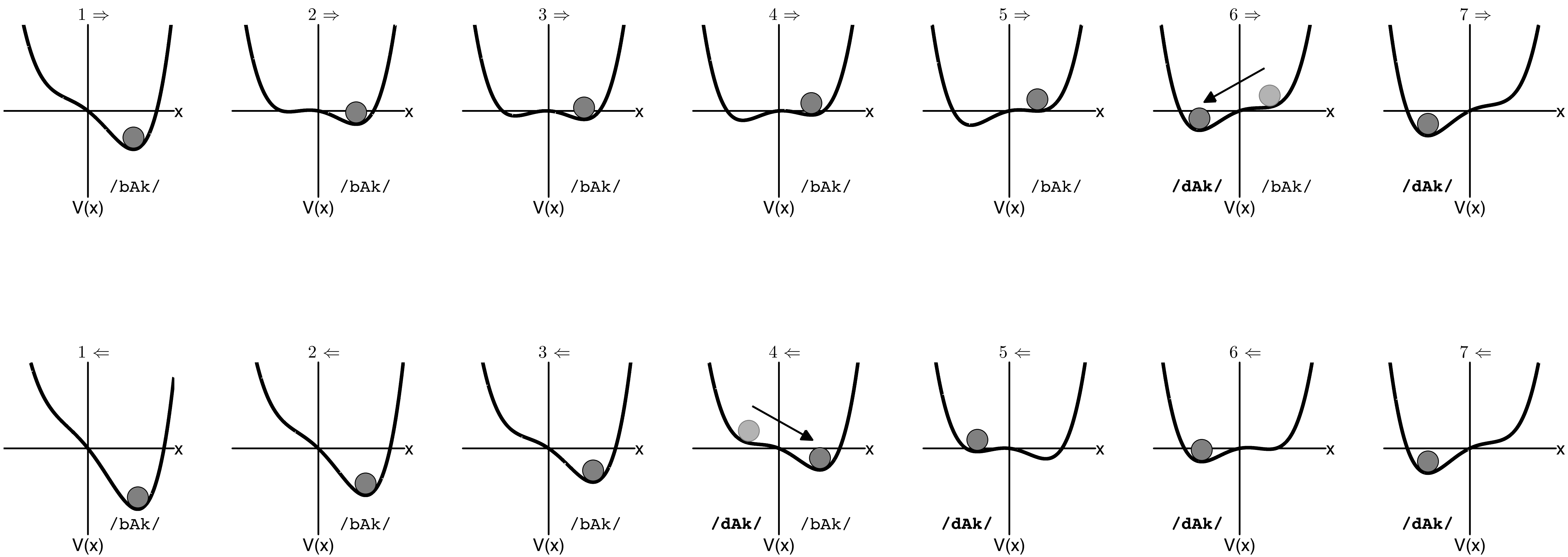
Phoneme boundary:

Innate?

Static?

>> Frequency input
>> Brain processing
>> Recognise sound by boundary frequency

Nonlinear Dynamics of Speech Categorization



BAK > DAK > BAK

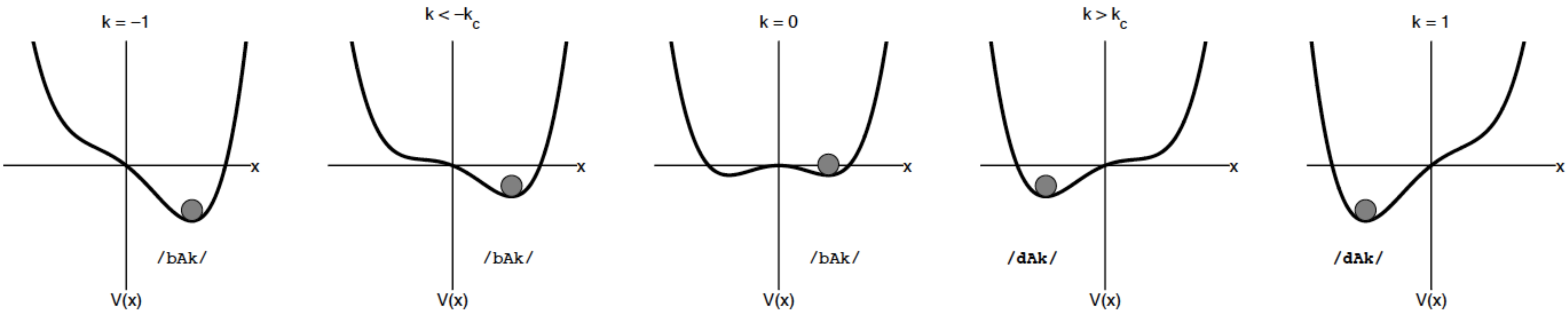
"Boundary" at 6

"Boundary" at 4

Potential models - Nonlinear Dynamics

$$\frac{dx}{dt} = -\frac{\partial V}{\partial x} \qquad V(x) = kx - \alpha\frac{x^2}{2} + \beta\frac{x^4}{4}$$

$$k(\lambda) = k_0 + \lambda + \frac{\varepsilon}{2} + \varepsilon\theta(n - n_c)(\lambda - \lambda_f)$$

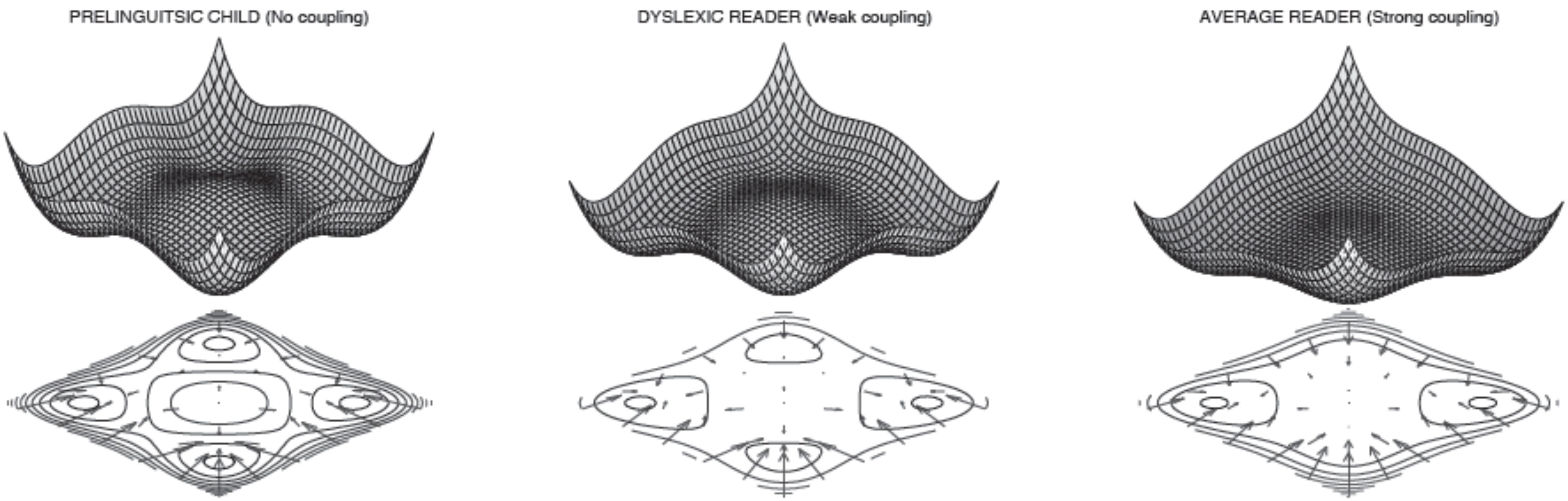


Direction of traversal is relevant:
Sequential presentation -> Hysteresis! There is no critical boundary!

Potential functions represent the attractor landscape for different values of the control parameter

Case, P., Tuller, B., Ding, M., & Kelso, J. (1995). Evaluation of a dynamical model of speech perception. *Perception & psychophysics*, *57*(7), 977-88. Retrieved from http://www.ncbi.nlm.nih.gov/pubmed/8532501

An Interaction Dominant Version of the "Coupling Hypothesis": A 2-D Potential Landscape

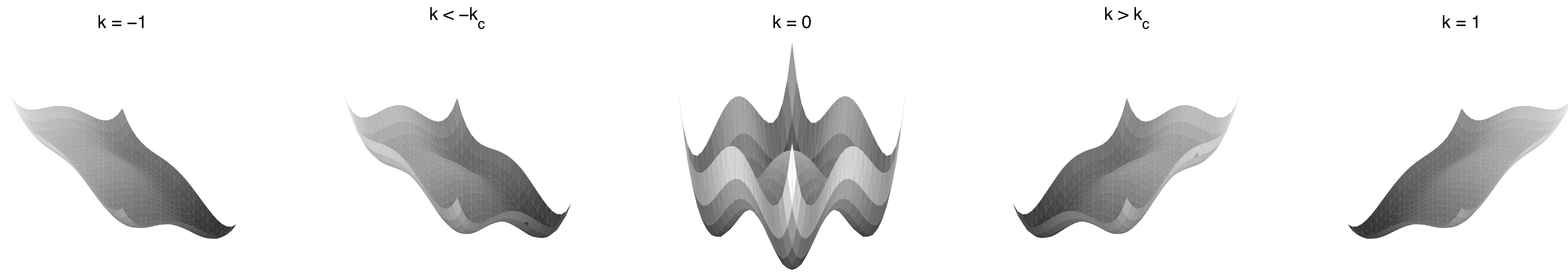


Speech perception is multimodal
(coupling between motor, auditory & visual processes)

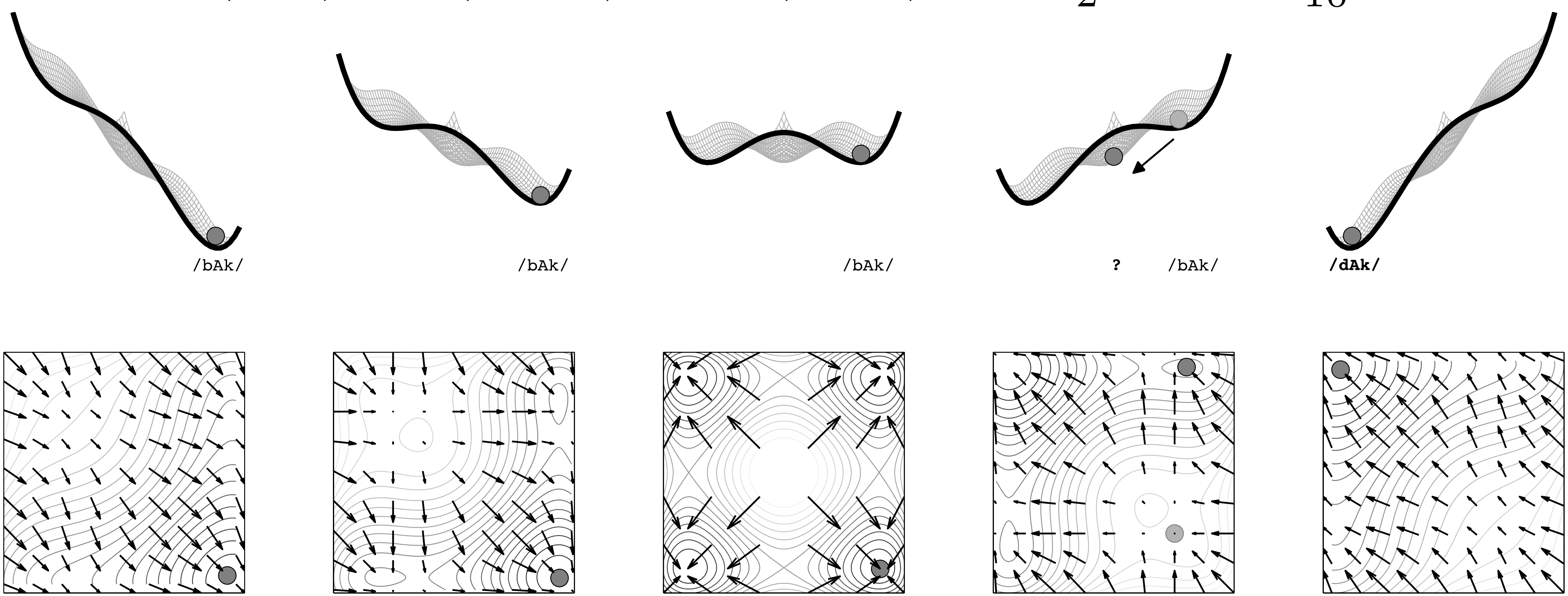
Reading is multimodal
(coupling between motor, auditory & visual processes)

Speech sounds like phonemes
are likely "ghosts of letters"
rather than something fundamentally
related to natural language...
Reading in an alphabetic language
causes perception of phoneme categories

Chapter 5: https://osf.io/ephca/



$$V(x,y) = k(x-y) + \gamma_{cs}(x*y) - \frac{(x^2+y^2)}{2} + \frac{x^4+y^4)}{10}$$



Chapter 5: https://osf.io/ephca/

This is nonlinear behaviour is very common in the cognitive and behavioural sciences....

Self-judgment of moods in bipolar patients
Postural control in baby's
Drinking behavior in adolescents
Smoking behavior in adolescents
Aggression in animals (dogs)
Development of anorexia nervosa
Development of movement coordination and control
Developmental stages
Transitions in attitudes
Fallback in addiction
Leadership in organisations

(See Optional Literature)