Dynamics of Complex Systems

part 1

Ergodic systems vs. Adaptive systems
Basic Timeseries Analysis
Temporal Correlations in time series

part 2

Quantifying temporal patterns:
Relative Roughness
Sample Entropy

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Story so far …

Deterministic mathematical models of change processes:

- Simulate the temporal-evolution of one or more system observables under different values of a control parameter and/or resource levels and/or coupling
- Can reveal temporal patterns that resemble a random process
- Show interaction dynamics in an N-dimensional state space when N processes are dynamically coupled or mutually dependent

- *Background:* Complexity Science / Science of the Individual / Systems Biology

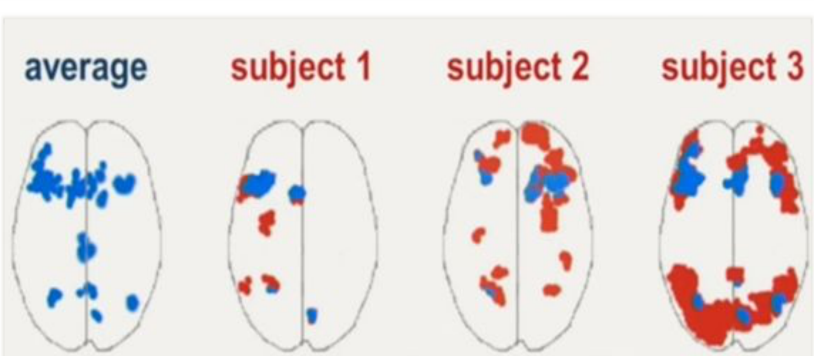
Today:

- Random events can reveal temporal patterns that resemble a deterministic process
- Analysing temporal correlations (correlation functions ~~and AR-MA models~~)
- Basic quantification of temporal patterns (ACF/PACF/CCF, Relative Roughness,
- Entropy)

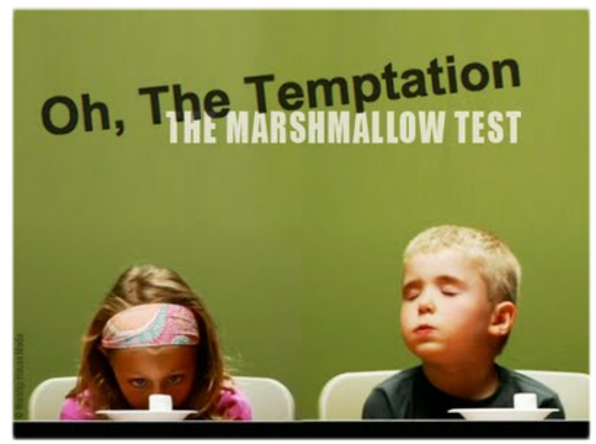
Story so far …

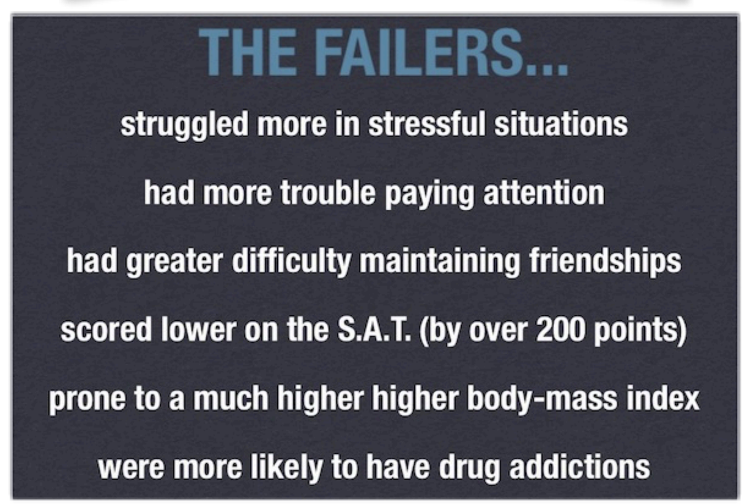
- *Background:* Complexity Science / **Science of the Individual** / Systems Biology
- **Idiographic** versus **Nomothetic** goal of scientific explanation

Principle of jaggedness
no individual corresponds to average

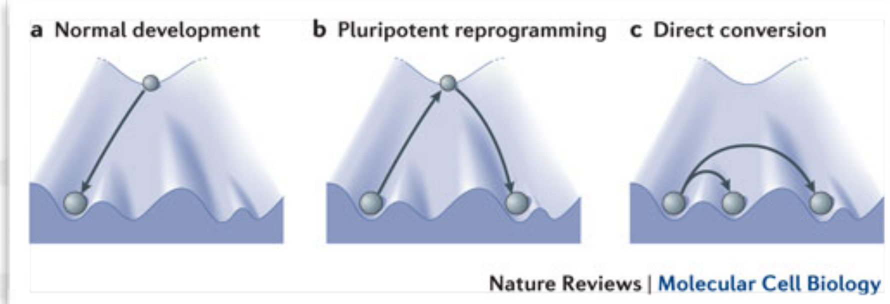


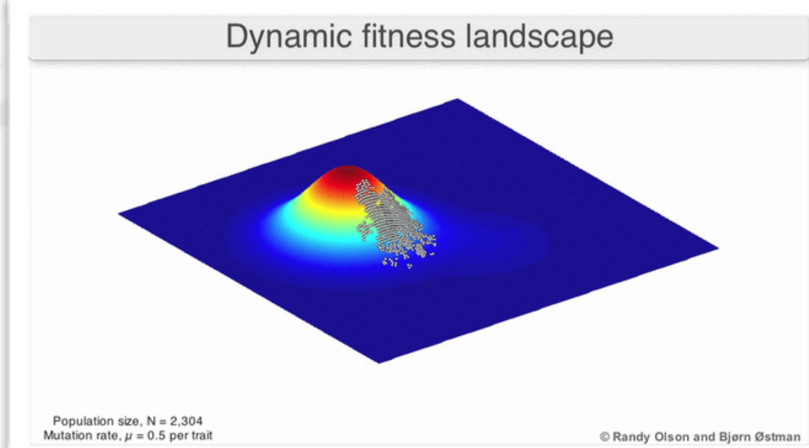
Principle of context
no behaviour is context independent



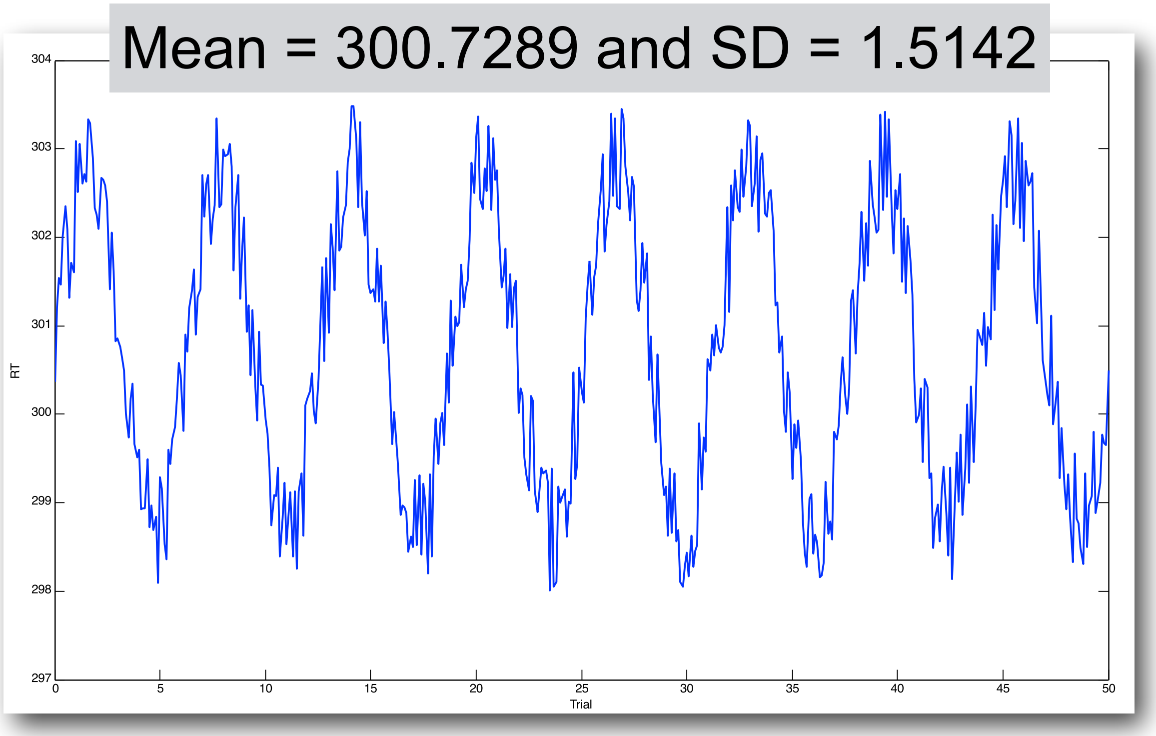


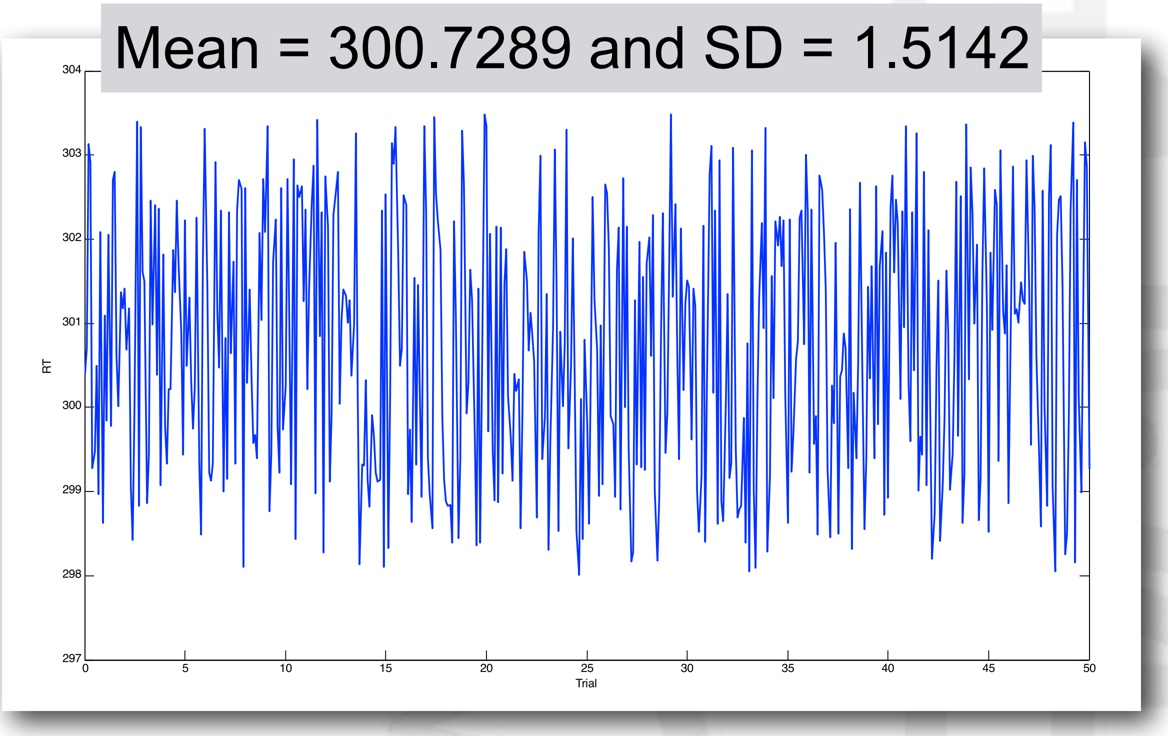
Principle of pathways
multiple trajectories to 'success'





Rose / Molenaar: "Analyse then Aggregate!"



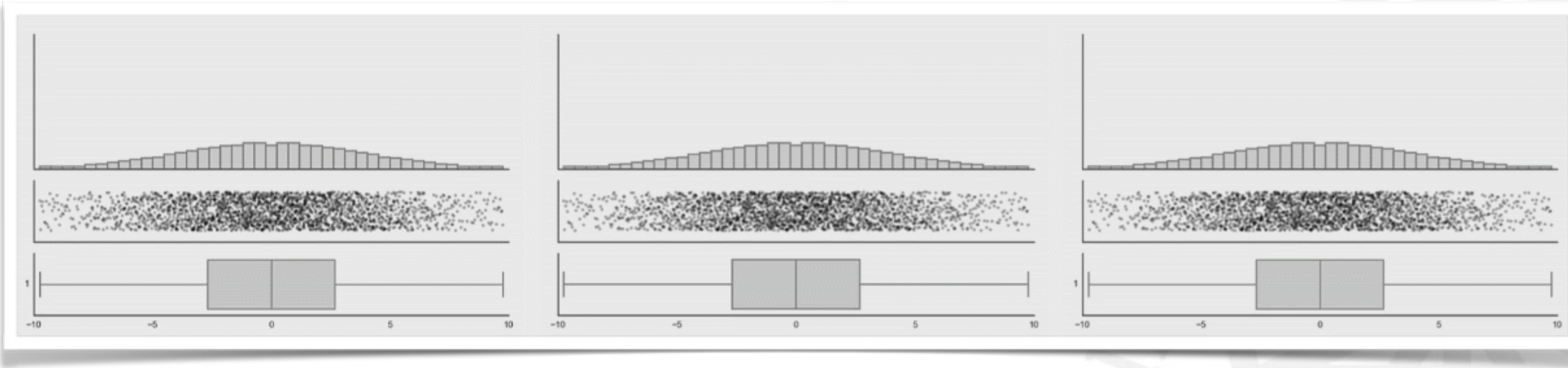


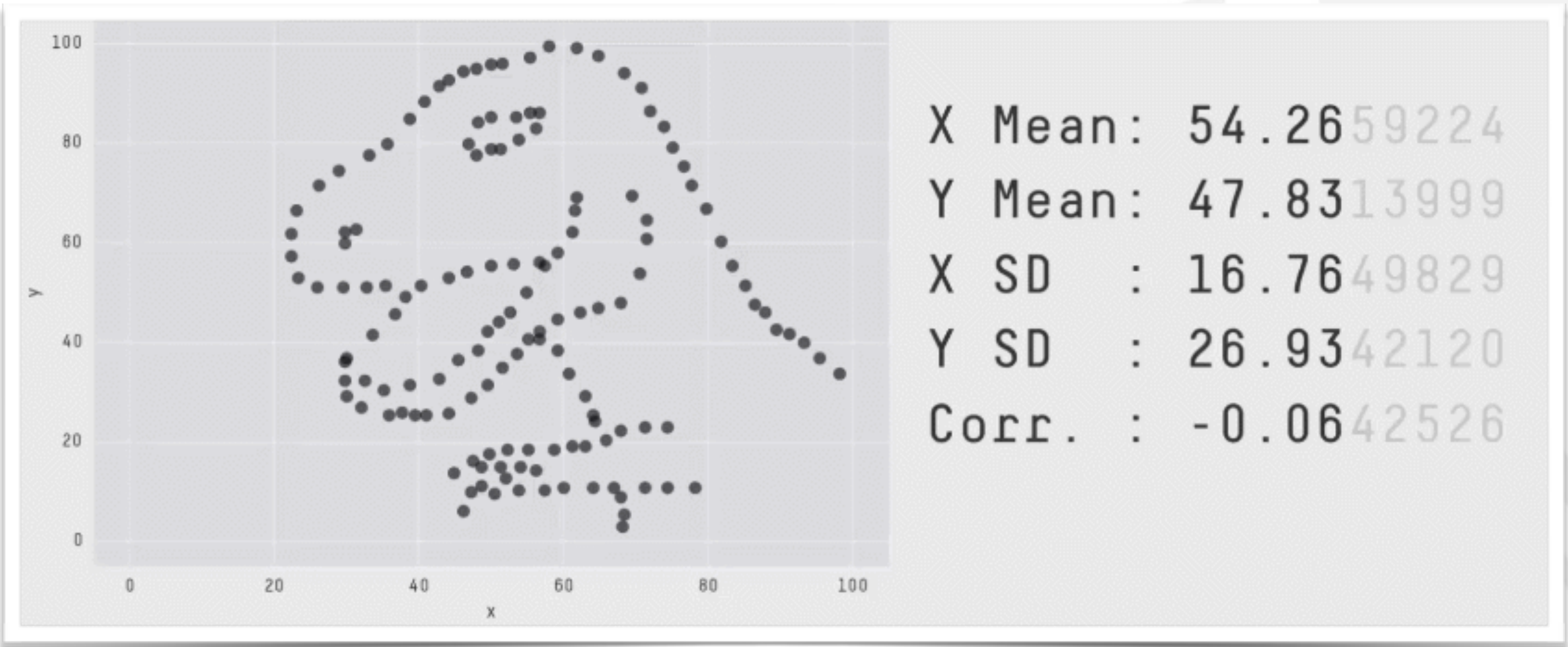
- The first process is very different from the second yet the central tendency measures are the same (MEAN, SD, etc. are equal).

- How can we characterise this difference?
    1. Quantify patterns of dependencies in the data as: deterministic/periodic/ stochastic, stable/unstable fluctuations …
    2. For each individual, each measure, in each context of interest
    3. Aggregate (if necessary)

- Basics steps to get step 1. - Correlation functions

"Analyse then Aggregate!"
same stats - different patterns





Matejka, J., & Fitzmaurice, G. (2017, May). Same Stats, Different Graphs: Generating Datasets with Varied Appearance and Identical Statistics through Simulated Annealing. In *Proceedings of the 2017 CHI Conference on Human Factors in Computing Sys*

https://www.autodeskresearch.com/publications/samestat

The Ergodic Bait and Switch

Random events / processes
Independent / Memory-less
Linear / Polynomial
Efficient causes

Random events / processes
Deterministic / Temporal correlations
Linear / Nonlinear
Efficient causes / Circular causality

component dominant dynamics

The Law of Large Numbers (Bernouiili, 1713) +
The Central Limit Theorem (de Moivre, 1733) +
The Gauss-Markov Theorem (Gauss, 1809) +
Statistics by Intercomparison *(Galton, 1875)* =
Social Physics *(Quetelet, 1840)*

Collectively known as:
The Classical Ergodic Theorems

Molenaar, P.C.M. (2008). On the implications of the classical ergodic theorems: Analysis of developmental processes has to focus on intra individual variation. *Developmental Psychobiology, 50*, 60-69

interaction dominant dynamics

Deterministic chaos (Lorenz, 1972)
(complexity, nonlinear dynamics, predictability)

Takens' Theorem (1981)
(phase space reconstruction)

Systems far from thermodynamic equilibrium
(Prigogine, & Stengers, 1984)

SOC / $\frac{1}{f^{\alpha}}$ noise (Bak, 1987)
(self-organized criticality, interdependent measurements)

Fractal geometry (Mandelbrot, 1988)
(self-similarity, scale free behaviour, infinite variance)

Aczel's Anti-Foundation Axiom (1988)
(hyperset theory, circular causality, complexity analysis)

Anomalous diffusion

"The conditions assumed by Einstein in his derivation of the diffusion equation are (i) the independence of individual particles, (ii) the existence of a sufficiently small time scale beyond which individual displacements are statistically independent, and (iii) the property that the particle displacements during this time scale correspond to a typical mean free path distributed symmetrically in positive or negative directions."

So, we need:

- ***Independence* of indivdual measurement** objects in a sample
- ***Memorylessness*** regarding any interactions between individuals and their environment. The after-effects of interactions should be short-lived and not affect behaviour in the long run (no long-range correlations).
- ***Randomly distributed deviations*** from central tendency (Gaussian *error* distribution).

Anomalous diffusion

What happens when we violate these assumptions?

In anomalous diffusion processes, at least one of these fundamental assumptions is violated, and the **strong convergence to the Gaussian according to the central limit theorem broken.** In particular, by departing from one or more of the assumptions (i)–(iii), we find that there exist many different generalisations of the Einstein–Smoluchowski diffusion picture.

Ok, but what does that mean?

The fact that we consider this large range of anomalous diffusion processes is the **non-universal nature of anomalous diffusion** itself. Once we leave the realm of Brownian motion, we lose the confines of the central limit theorem forcing the processes to converge to the Gaussian behaviour predicted by Einstein.
[…]
Quite commonly such analyses of time series from experiment or simulations are performed in terms of time averaged observables, in particular, the time averaged MSD [*Mean squared displecement = Mean squared 'error'*]. We point out that the physical interpretation of the obtained behaviour of such time averages in terms of the **typically available ensemble approaches may be treacherous** : many of the anomalous diffusion processes discussed herein lead to a **disparity between the ensemble and the time averaged observable**, for instance, between the ensemble and time averaged MSDs […] even in the limit of long measurement times. Moreover, it turns out that individual results for time averages […] appear to be **irreproducible**, despite long measurement times.

Ergodicity

- A random process $X(t)$ is ergodic if all of its statistics can be determined from a sample function of the process
- That is, the ensemble averages equal the corresponding time averages with probability one.

5

Thus, you obtain two different results: one statistical analysis over the entire ensemble of people at a certain moment in time, and one statistical analysis for one person over a certain period of time. The first one may not be representative for a longer period of time, while the second one may not be representative for all the people.

The idea is that an ensemble is ergodic if the two types of statistics give the same result. $\boxed{\text{Many ensembles, like the human populations, are not ergodic.}}$

https://larspsyll.wordpress.com/2016/11/23/what-is-ergodicity-2/

“Many ensembles like the human populations are not ergodic” what?… but… then… what? … no! … if… really? …what? no!

Relax
(a bit)

You can use the analyses we teach in this course to study ensembles of individuals :)

The Ergodic Bait and Switch

Assumptions of statistical models (GLM)

Compound Symmetry

The compound symmetry assumption requires that the variances (pooled within-group) and covariances (across subjects) of the different repeated measures are homogeneous (identical). This is a sufficient condition for the univariate F test for repeated measures to be valid (i.e., for the reported F values to actually follow the F distribution). However, it is not a necessary condition.

Sphericity

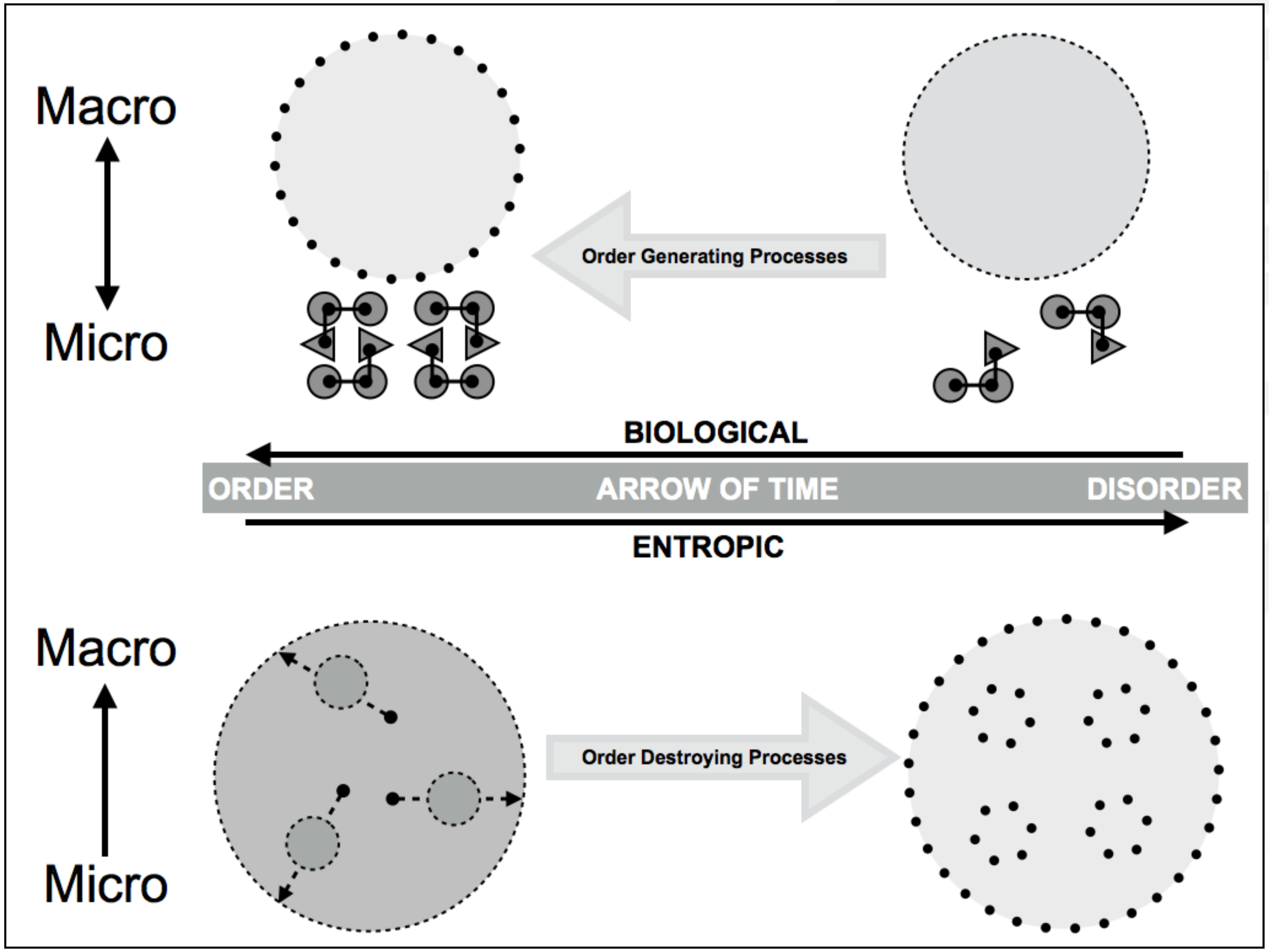
The sphericity assumption is a necessary and sufficient condition for the F test to be valid; it states that the within-subject "model" consists of independent (orthogonal) components.

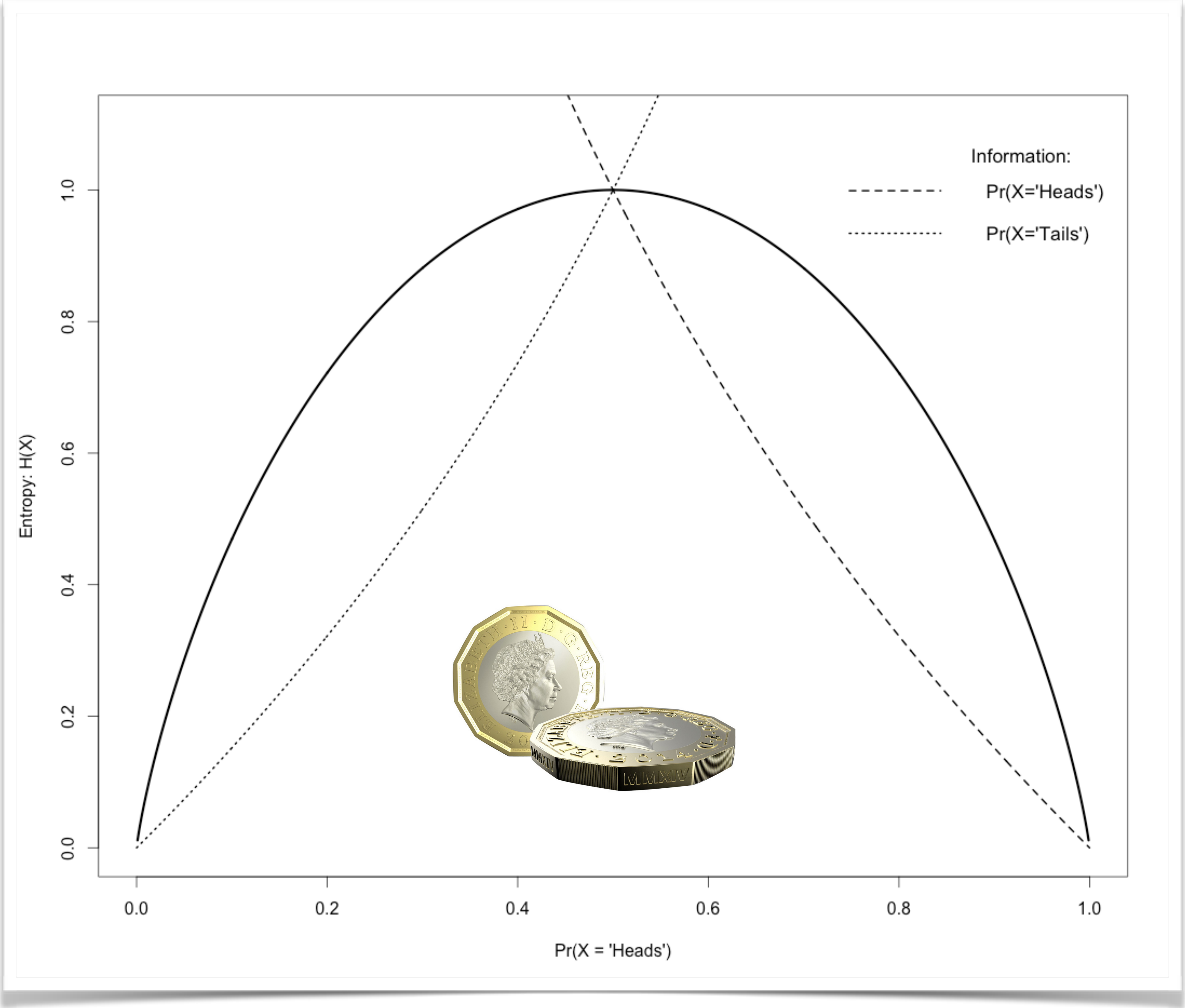
The nature of these assumptions, and the effects of violations are usually not well-described in ANOVA textbooks

Non-ergodic systems / Anomalous diffusion: Far-from-thermodynamic-equilibrium systems Dissipative systems Complex Adaptive Systems

Simple

Complex





Entropy

Information

Uncertainty

Redundancy

Probability

Symmetry breaking

Correlation Functions

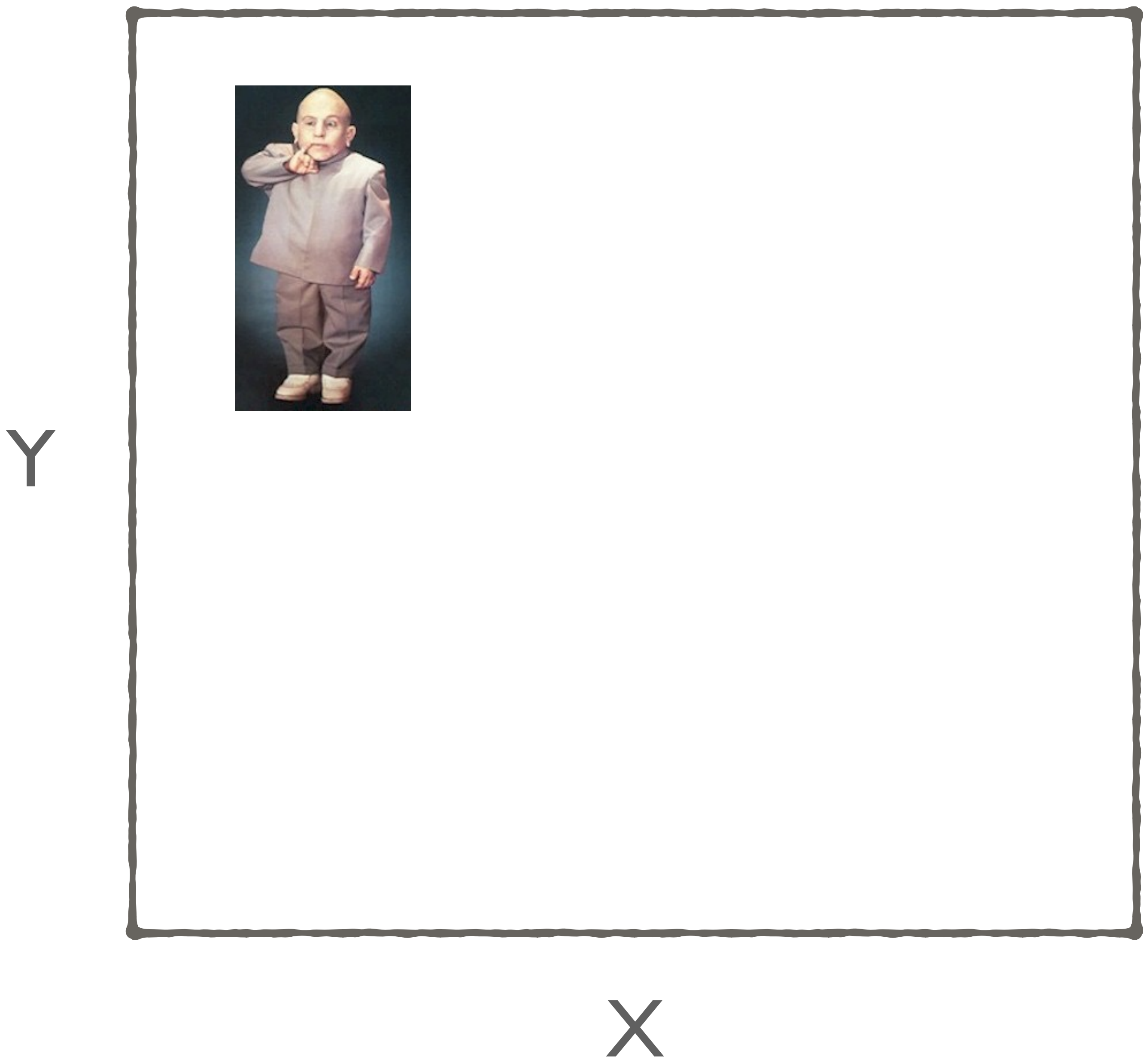
State space

Time scales

Linear v. nonlinear

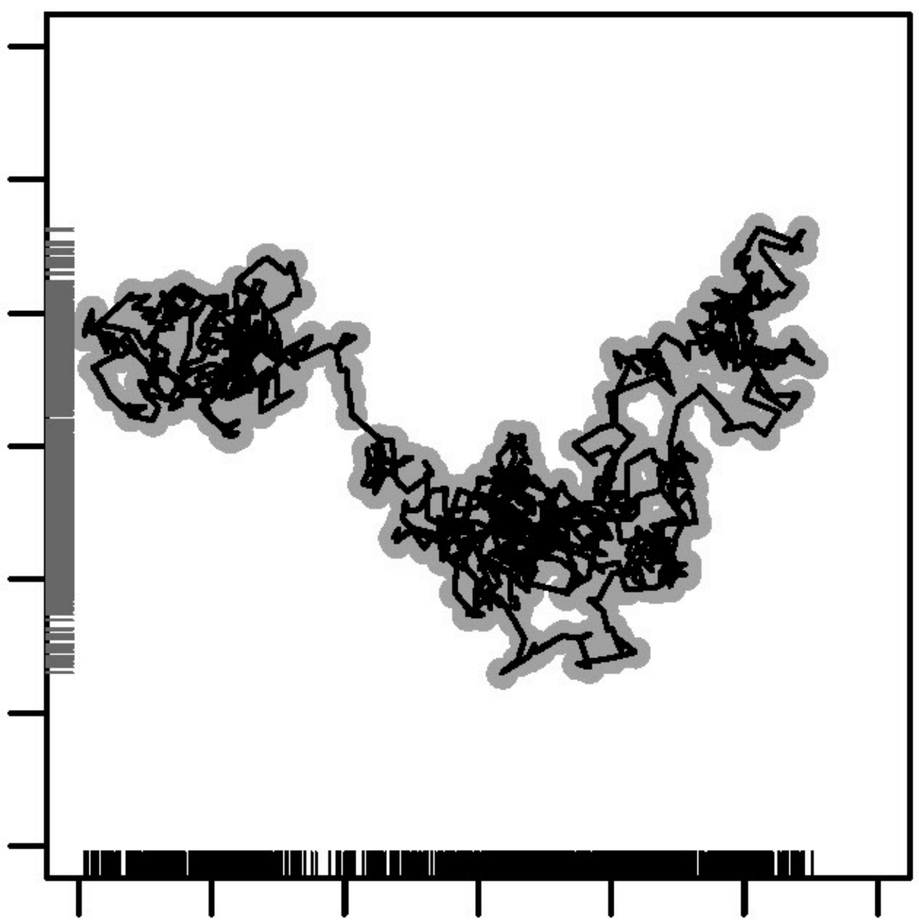
Homogeneous v. non-homogeneous

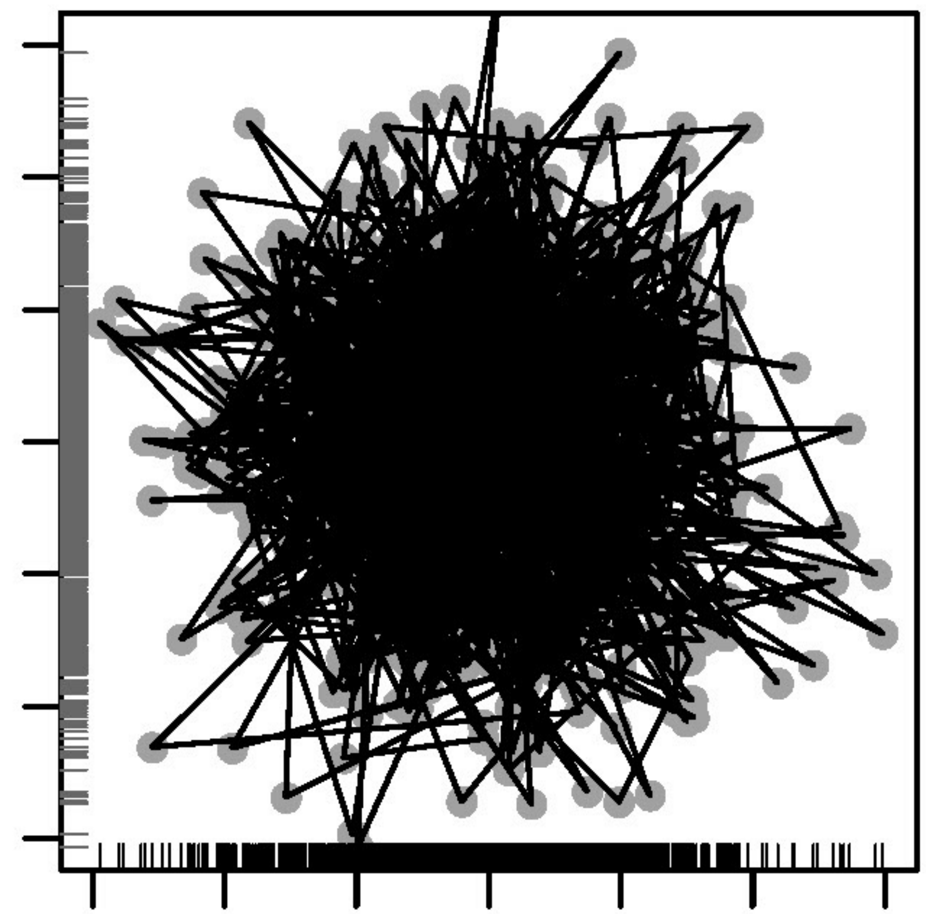
MINIME SYSTEM



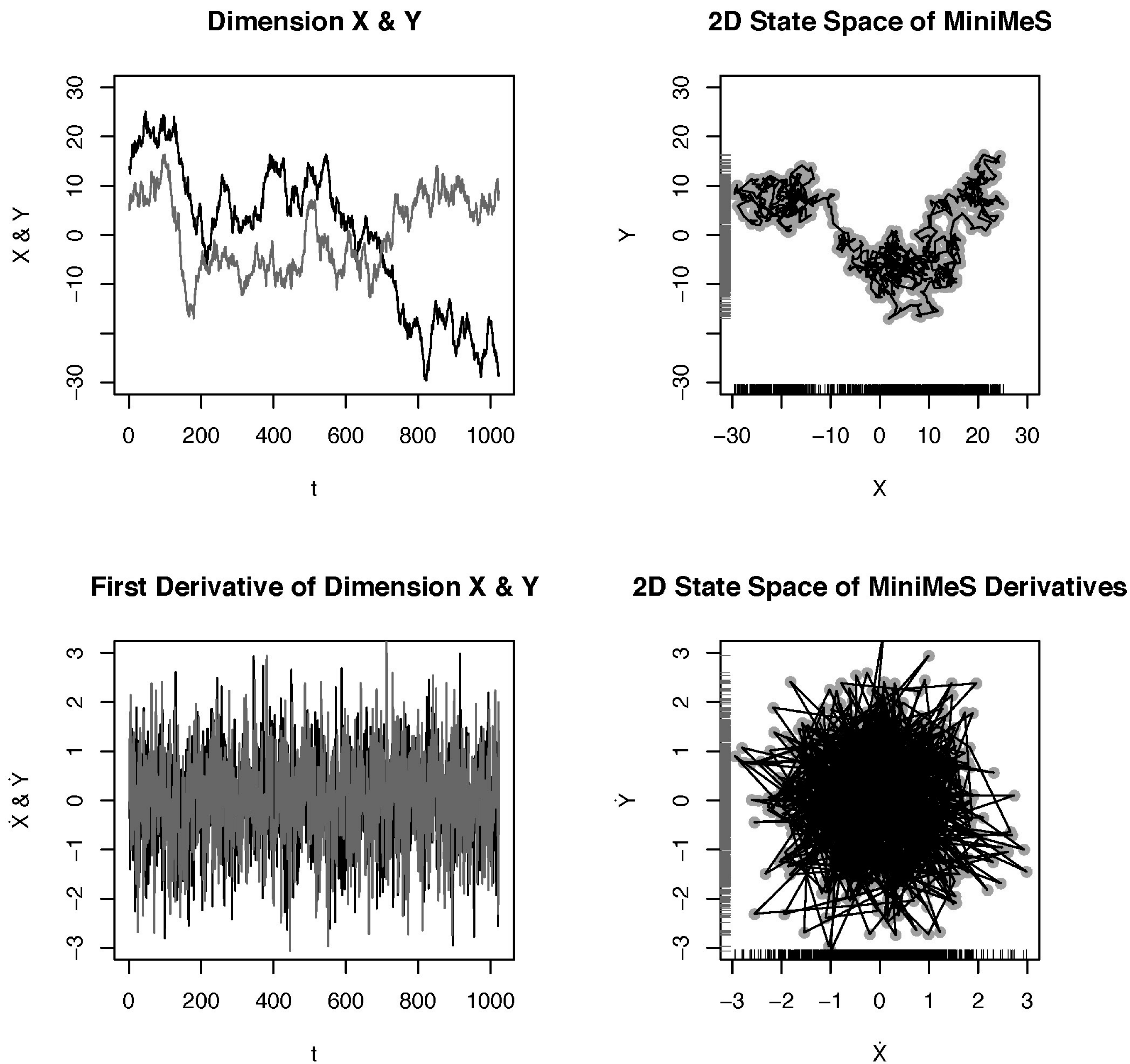
- State = X,Y coordinate
- Minimal Memory System can move around within the boundary.
- When would you infer randomness, when a deterministic rule?
- What kind of succession of states?
- What kind of trajectory through space?

MINIME SYSTEM





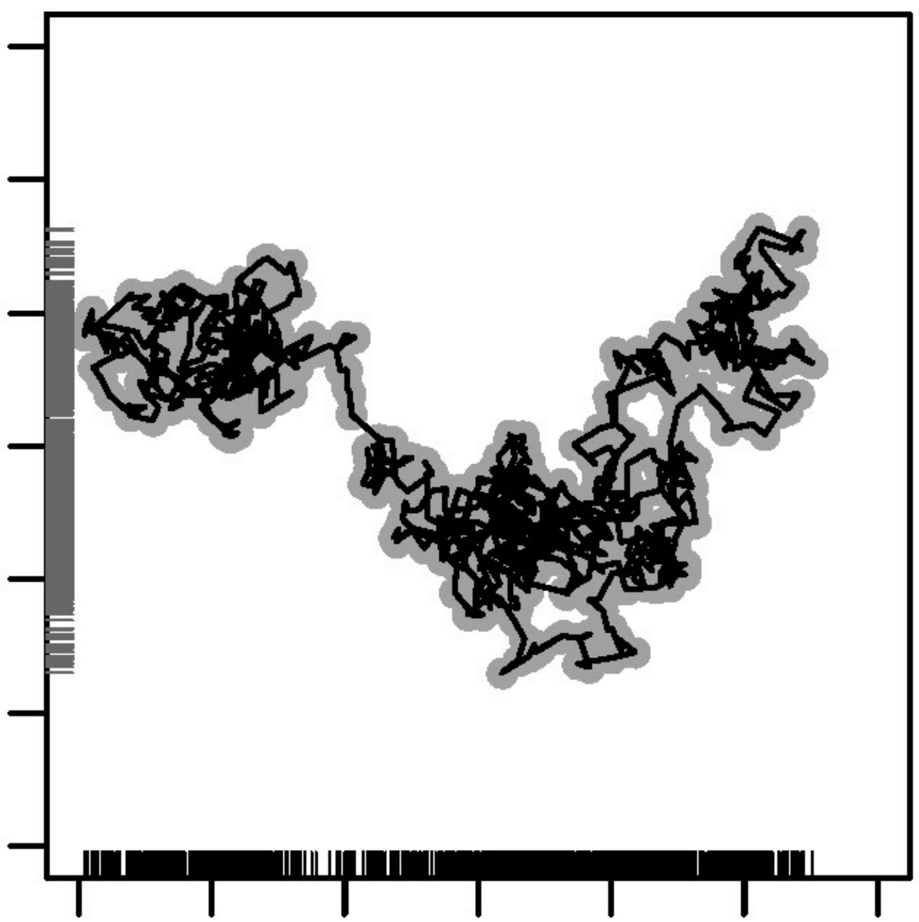
MINIME SYSTEM

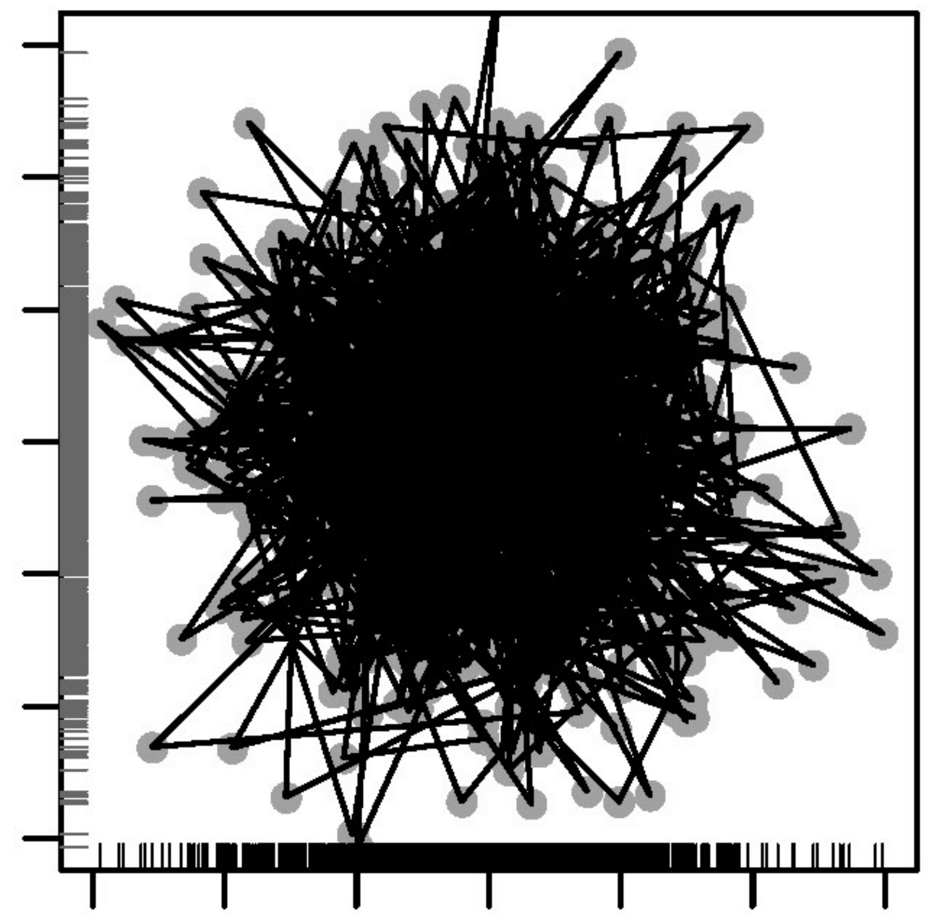


- State Space (X & Y): The degrees of freedom MiniMe has to generate its behaviour (move)
- This is a random walk, Brownian motion: Add a random number drawn from normal distribution to current number.
- Where does the apparent order come from? It’s a random process!!!!

‘Simple’ rule reduces degrees of freedom to move around:

Matter has to occupy finite space & movement takes time (no teleportation yet)





Minimal form of ‘physical memory’ through ‘natural computation’: summation / counting

Emergence of structure / temporal correlations / redundancies / dependencies

Brownian motion / Levy flights are very common in nature (diffusion, percolation, foraging)

How to characterise the nature of the dependencies?

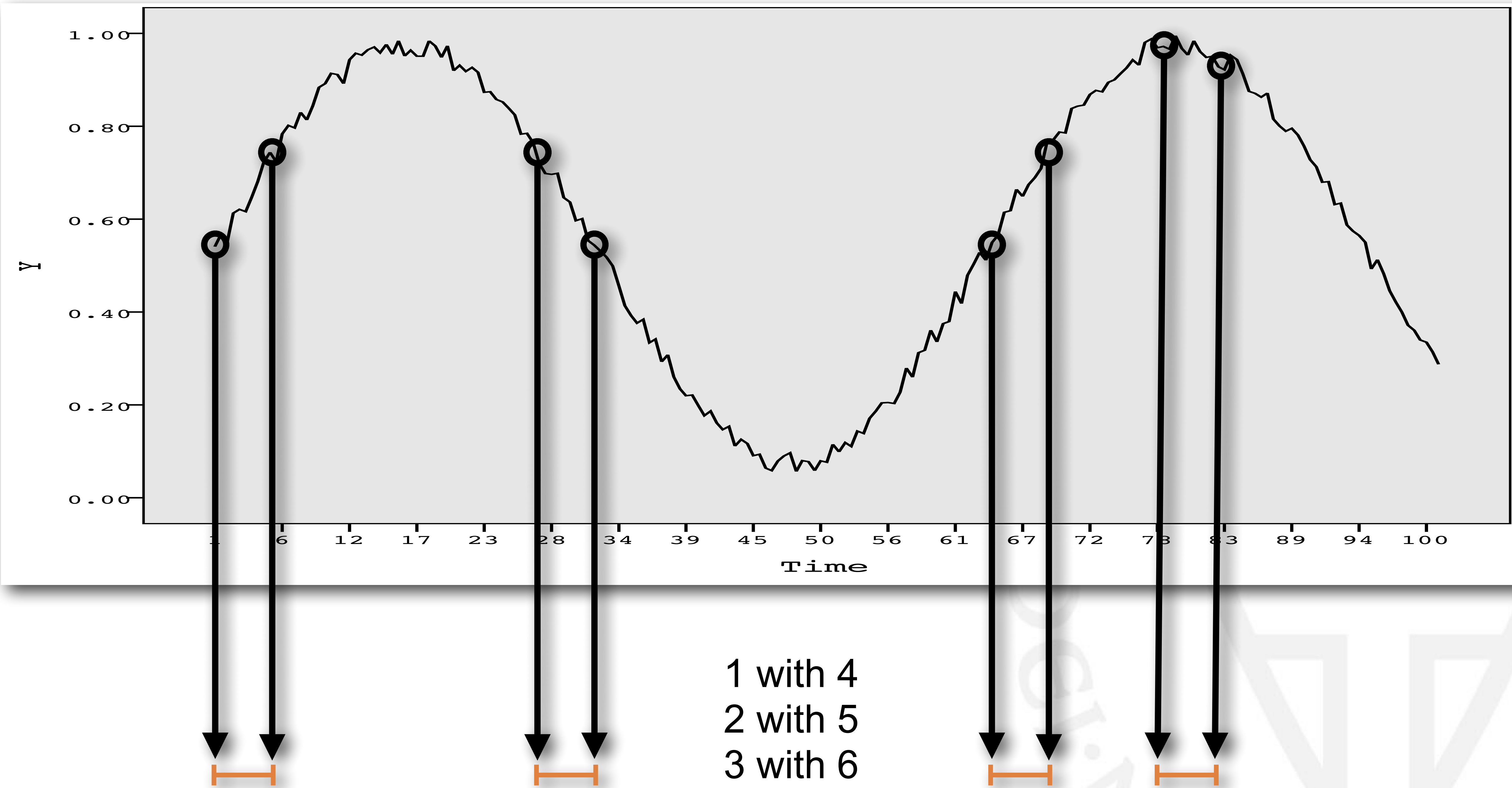
(Partial) Autocorrelation Function - (P)ACF

$$R(k) = \frac{E[(X_i - \mu)(X_{i+k} - \mu)]}{\sigma^2}$$

The average correlation **r** between data points that are a distance (lag) **k** apart in time

This holds only for *stationary*, *random processes.* So X measured here is a *random variable*.

ACF and the Partial ACF are used to decide which AR(fI)MA model you need (how many AR and/or MA parameters you need).



1 with 4
2 with 5
3 with 6
…
96 with 99
97 with 100

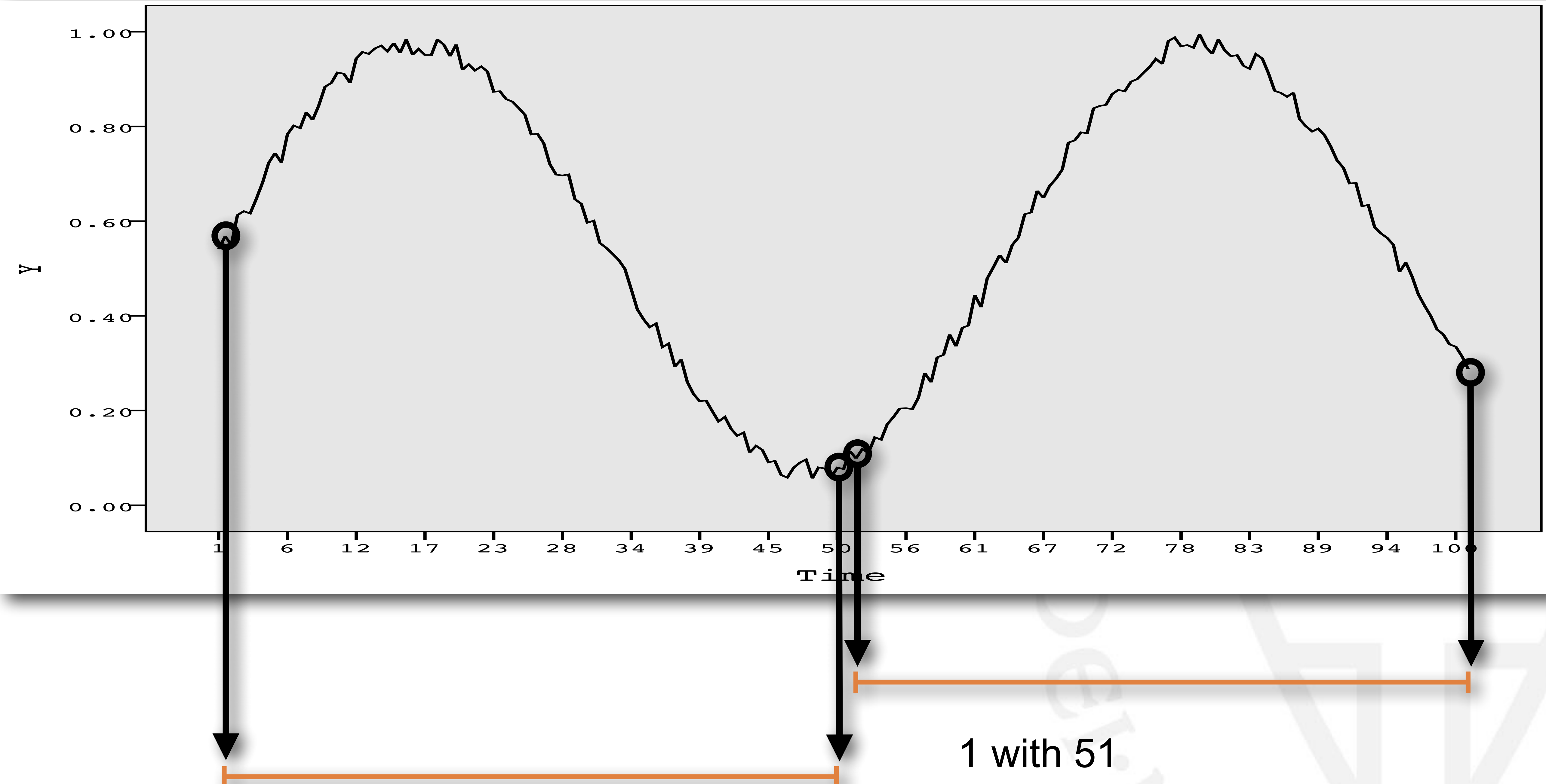
Lag = 3

How many correlations of lag 3?

TS length = 100 data points

Low or High at lag 3?

r_3= 0.895 (*SD = 0.095*)



Lag = 50

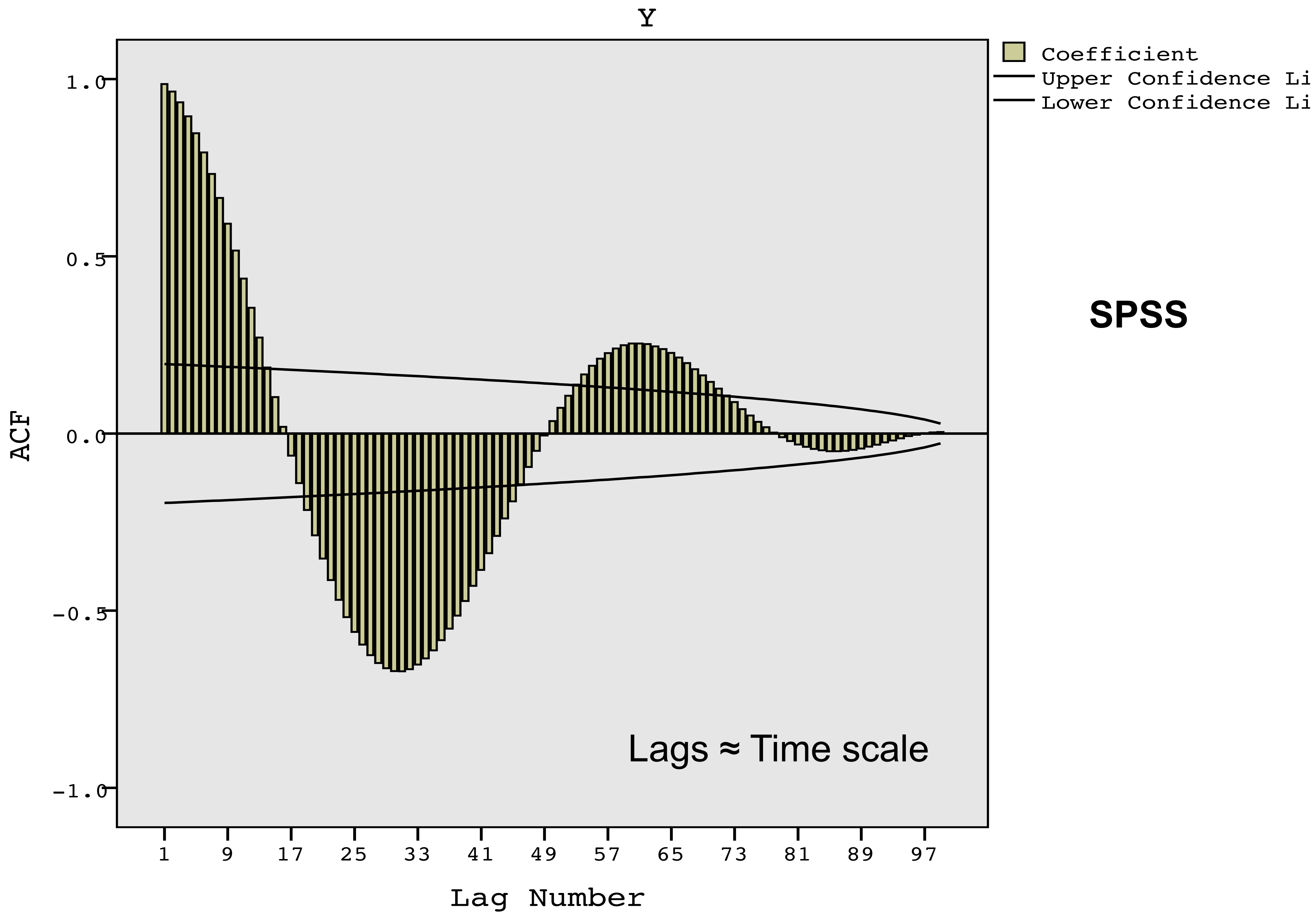
1 with 51
2 with 52
3 with 53
…
50 with 100

How many correlations of lag 50?

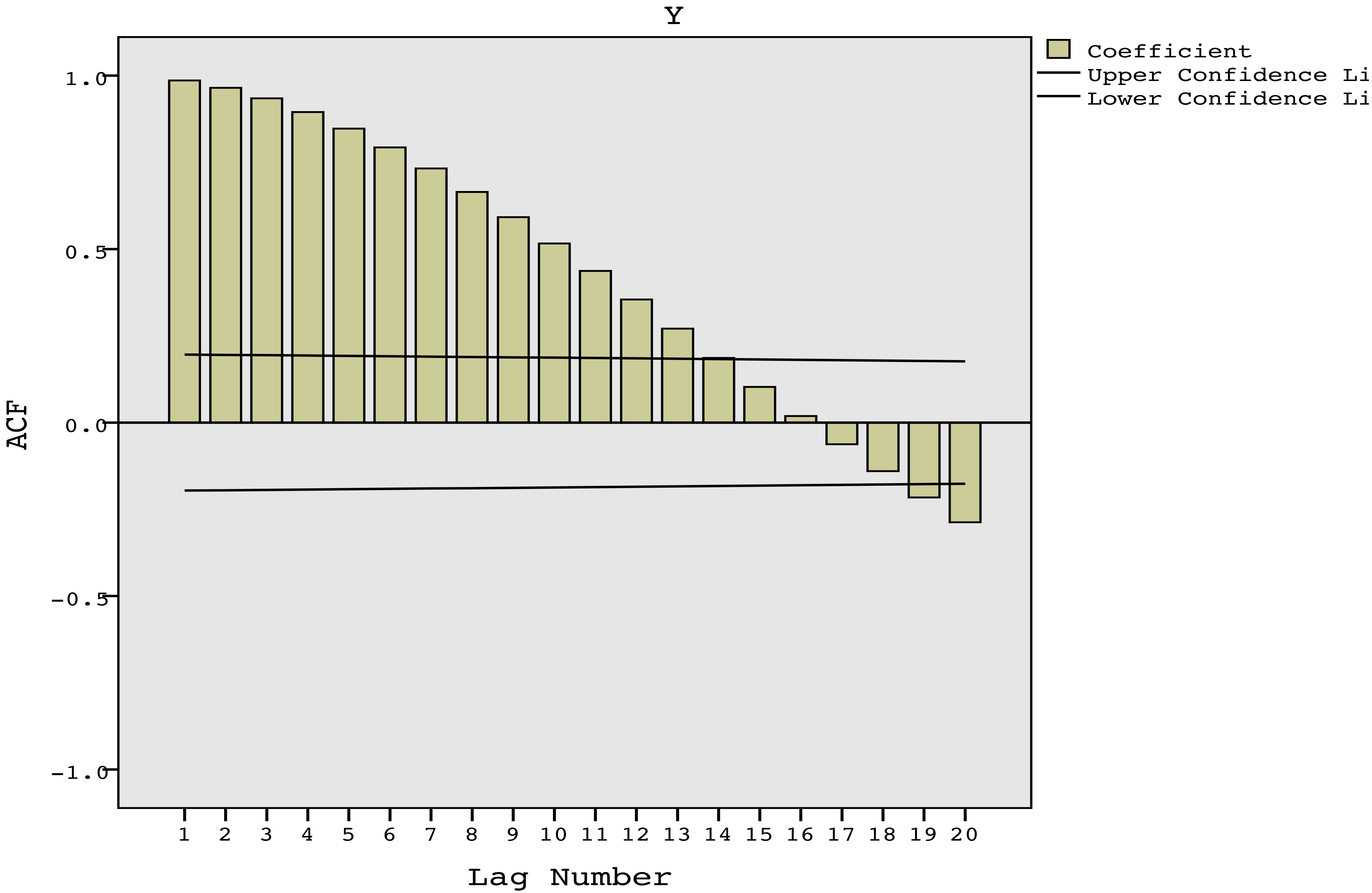
TS length = 100 data points

Low or High at lag 50?

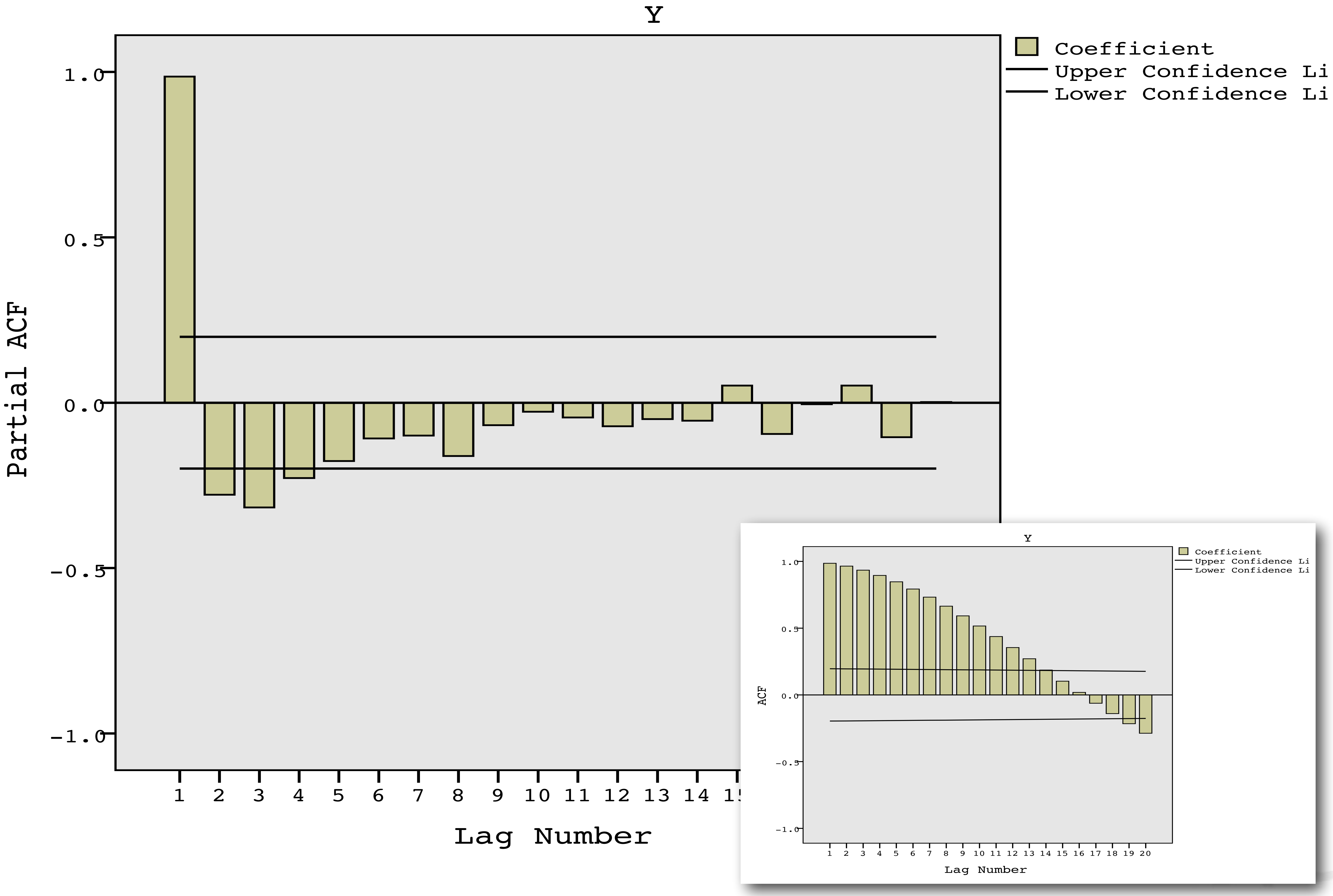
r_{50} = 0.035 (*SD* = *0.070*)

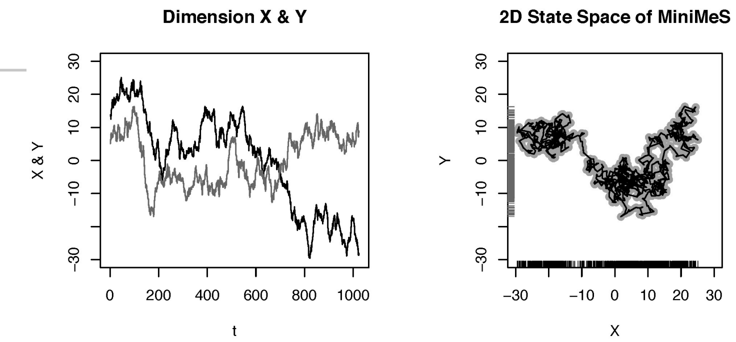

Y
Coefficient
Upper Confidence Li
Lower Confidence Li
1.0
0.5
0.0
-0.5
-1.0
ACF
1
9
17
25
33
41
49
57
65
73
81
89
97
Lag Number
Lags ≈ Time scale

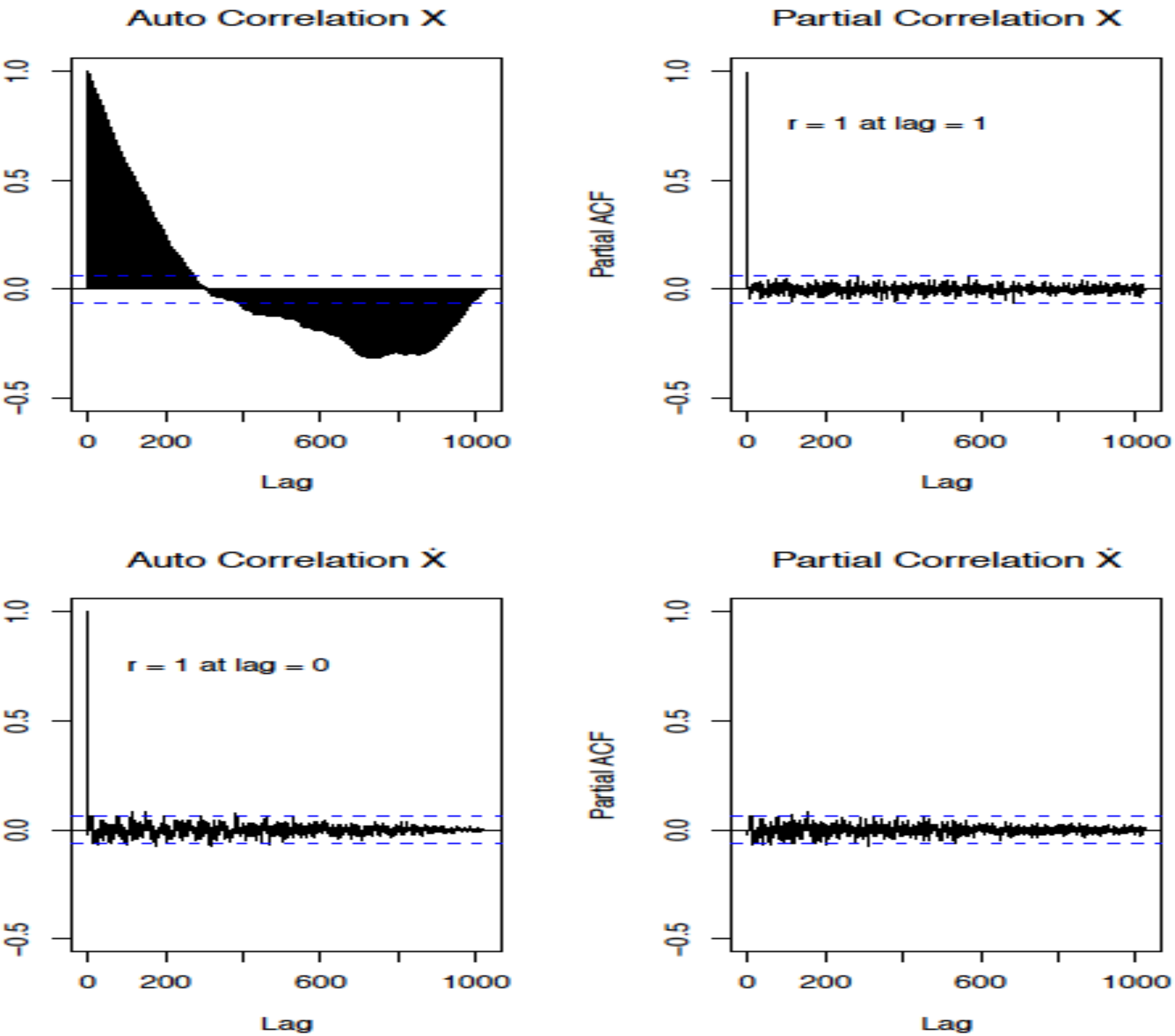
SPSS



Y
Coefficient
Upper Confidence Li
Lower Confidence Li
1.0
0.5
0.0
-0.5
-1.0
ACF
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Lag Number


Y
Coefficient
Upper Confidence Li
Lower Confidence Li
1.0
0.5
0.0
-0.5
-1.0
Partial ACF
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Lag Number
Y
Coefficient
Upper Confidence Li
Lower Confidence Li
1.0
0.5
0.0
-0.5
-1.0
ACF
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Lag Number

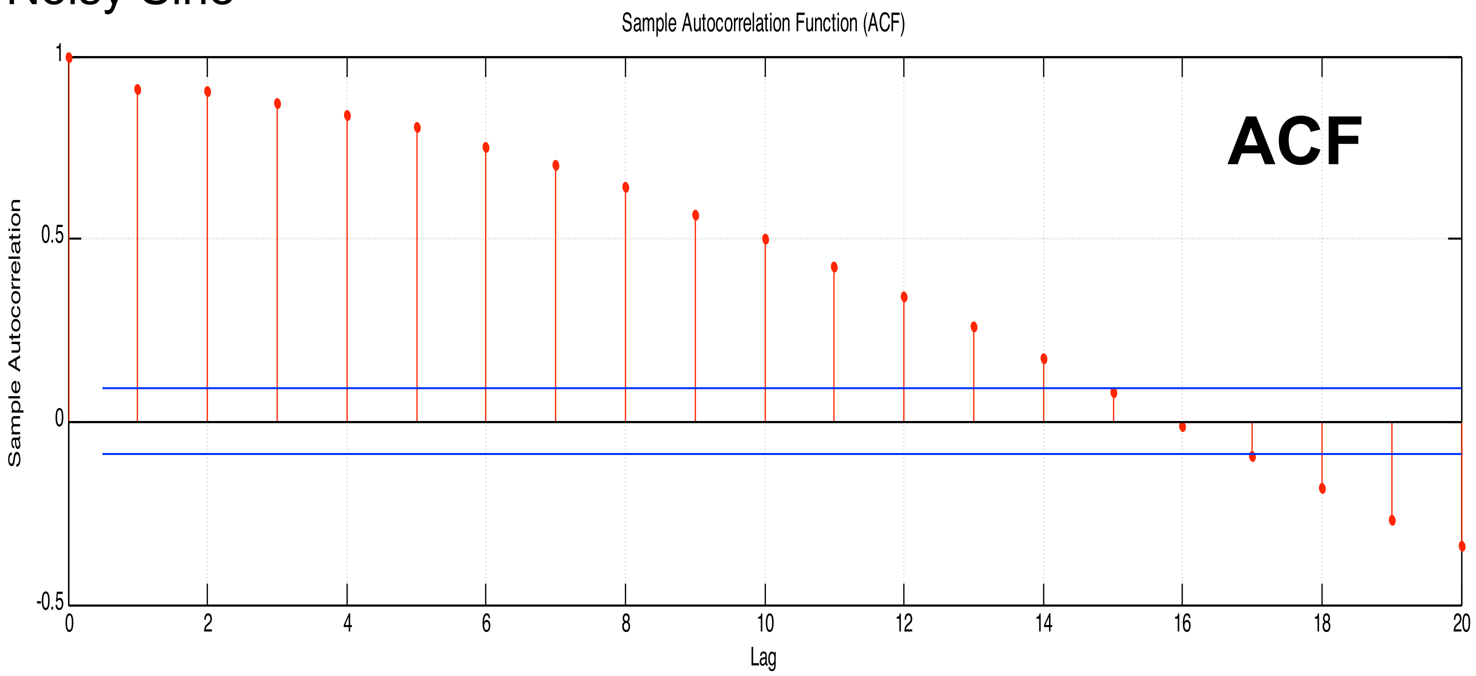


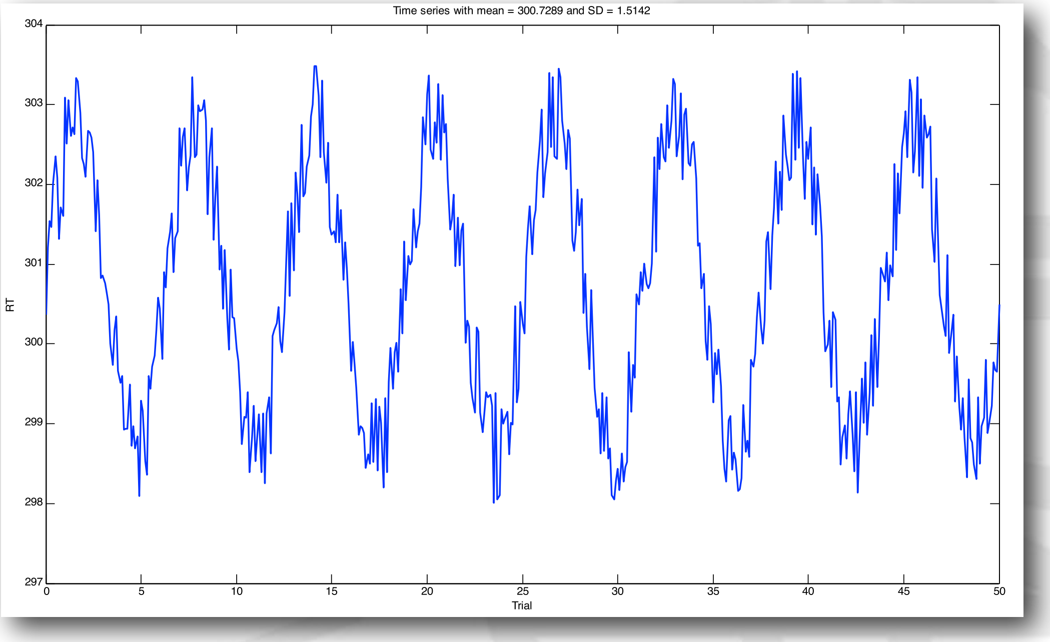


- ACF and PACF: representation of redundancies in the time series
- ACF: Correlations across multiple time scales?
- PACF: Nope, correlation at lag 1

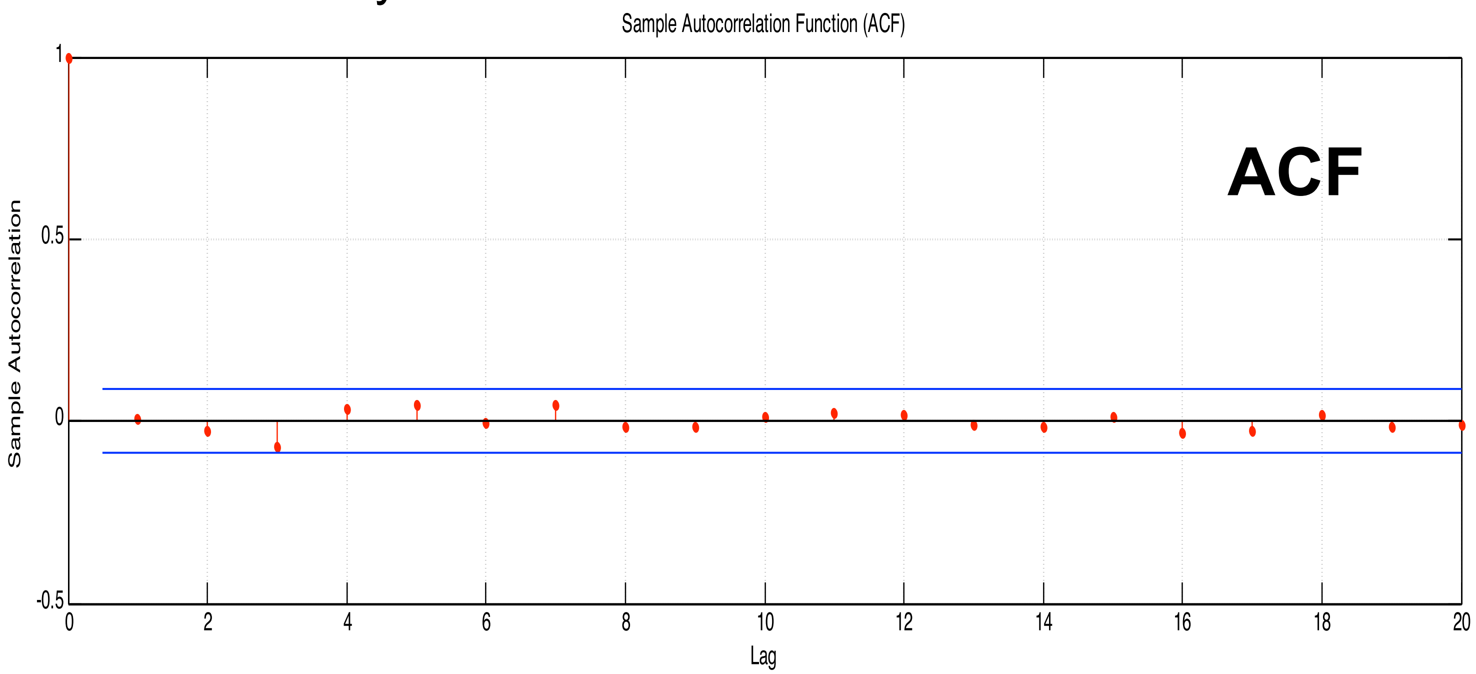
Randomising temporal order = Destroying correlations in the data

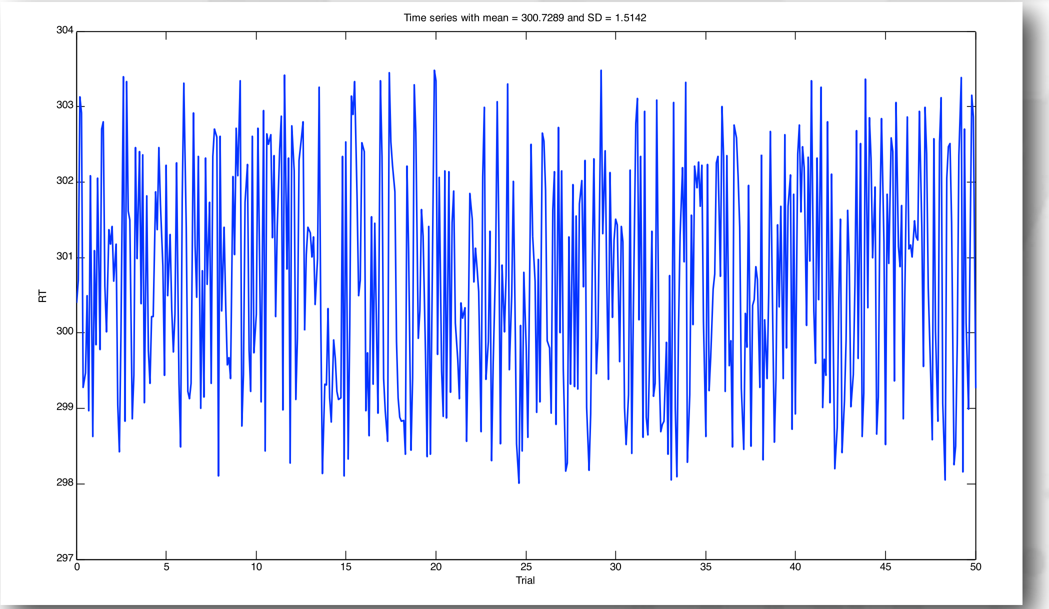
Noisy Sine





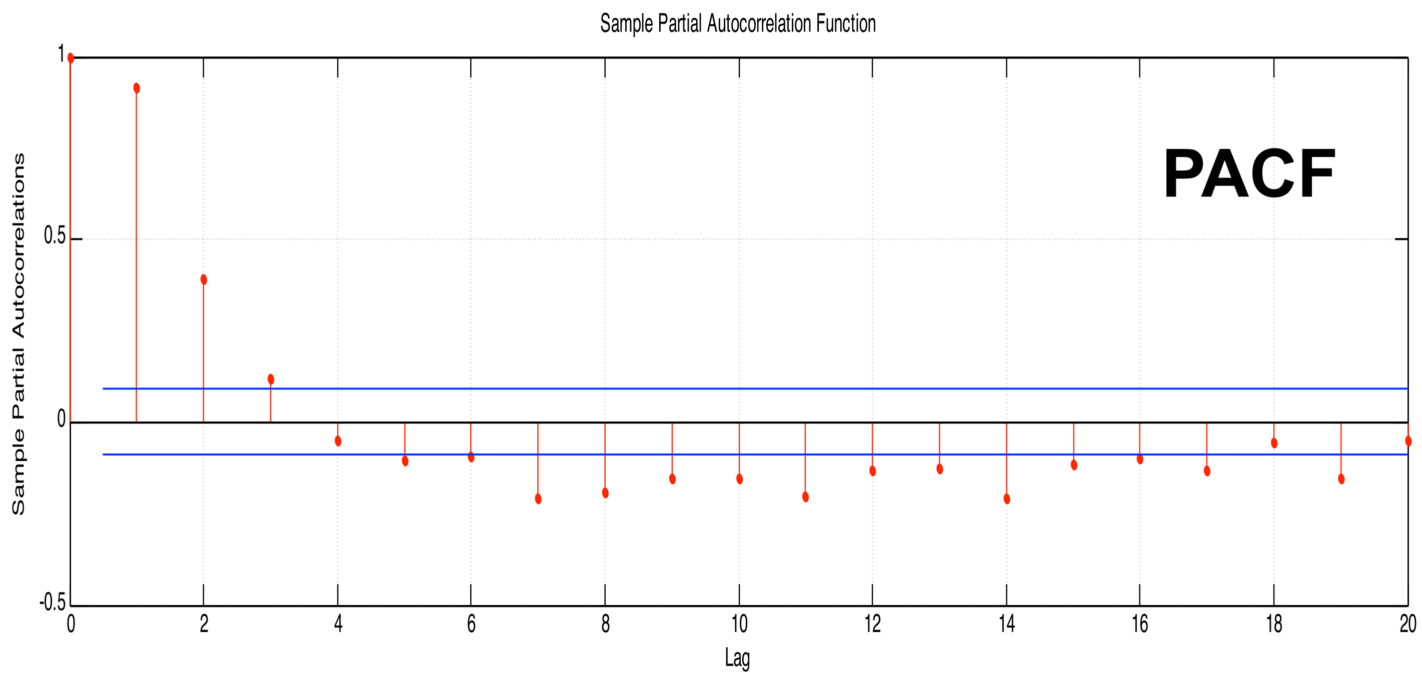
Randomised Noisy Sine

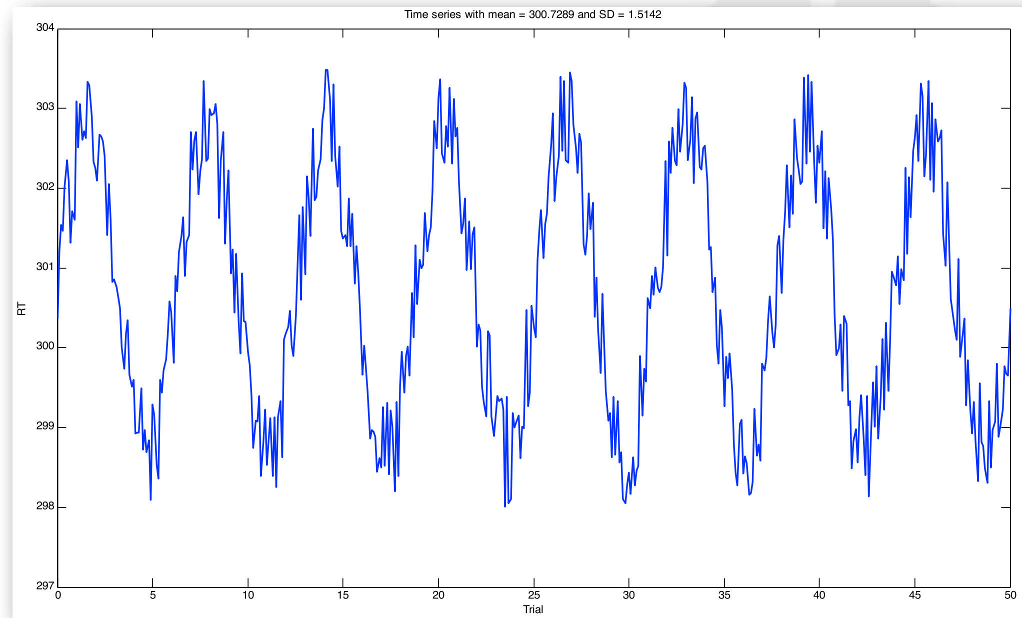




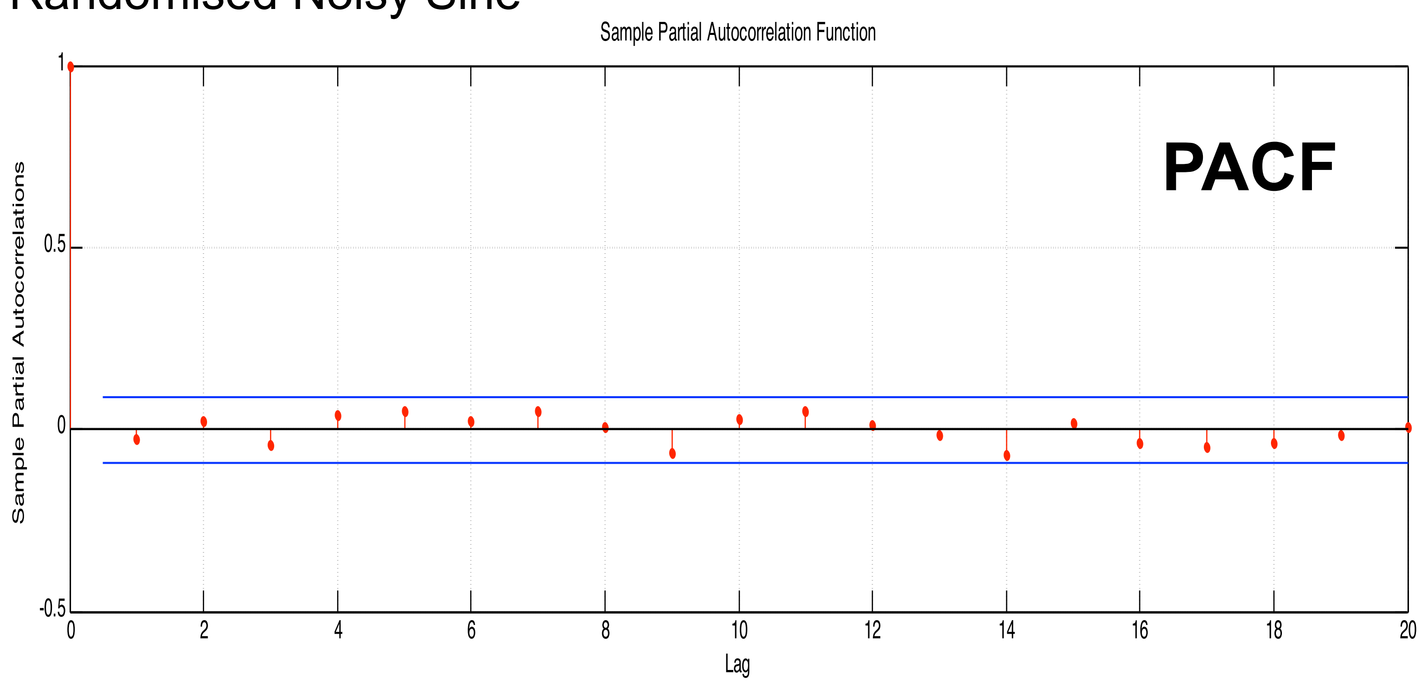
Randomising temporal order = Destroying correlations in the data

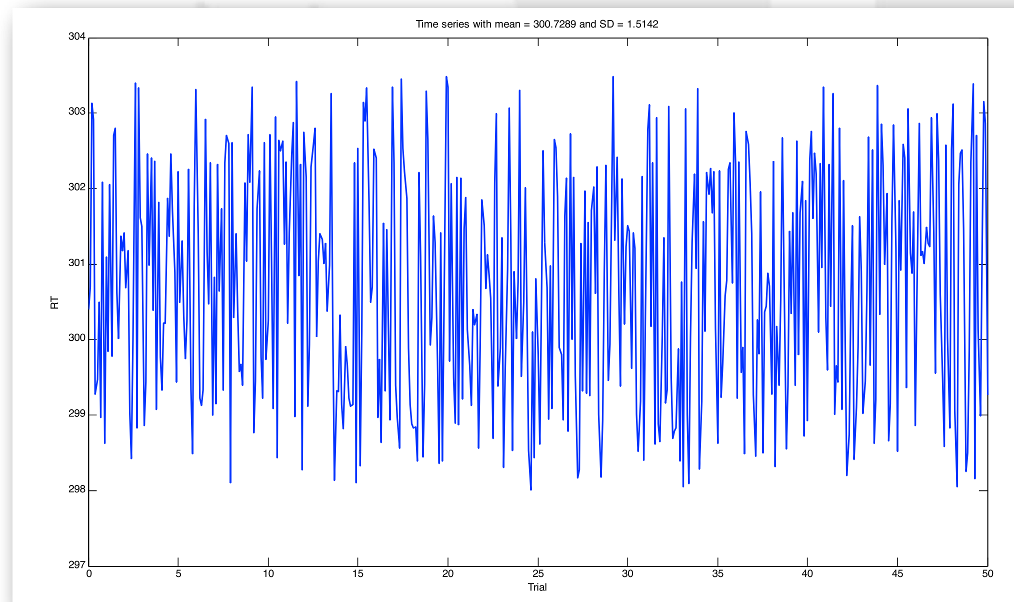
Noisy Sine

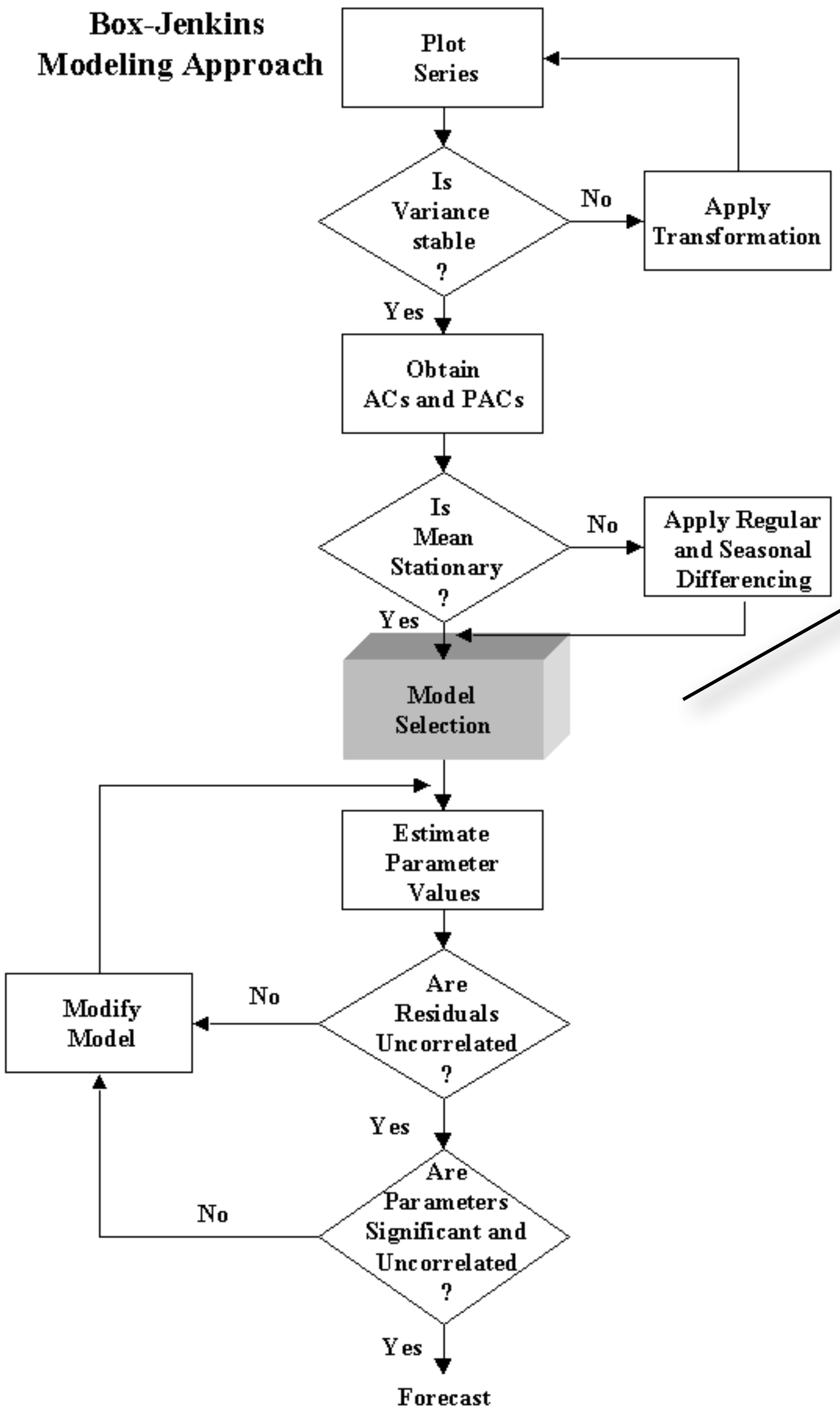




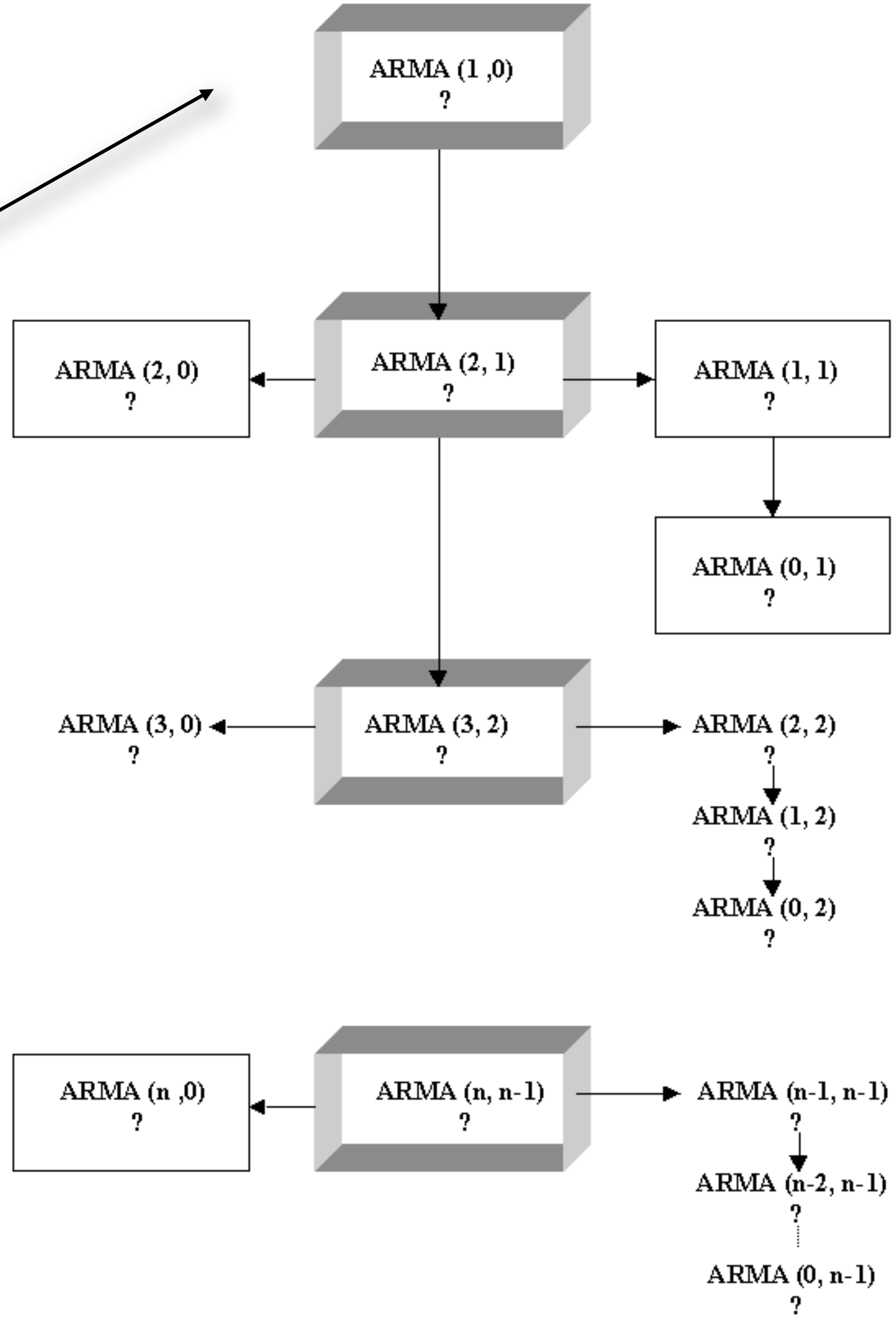
Randomised Noisy Sine





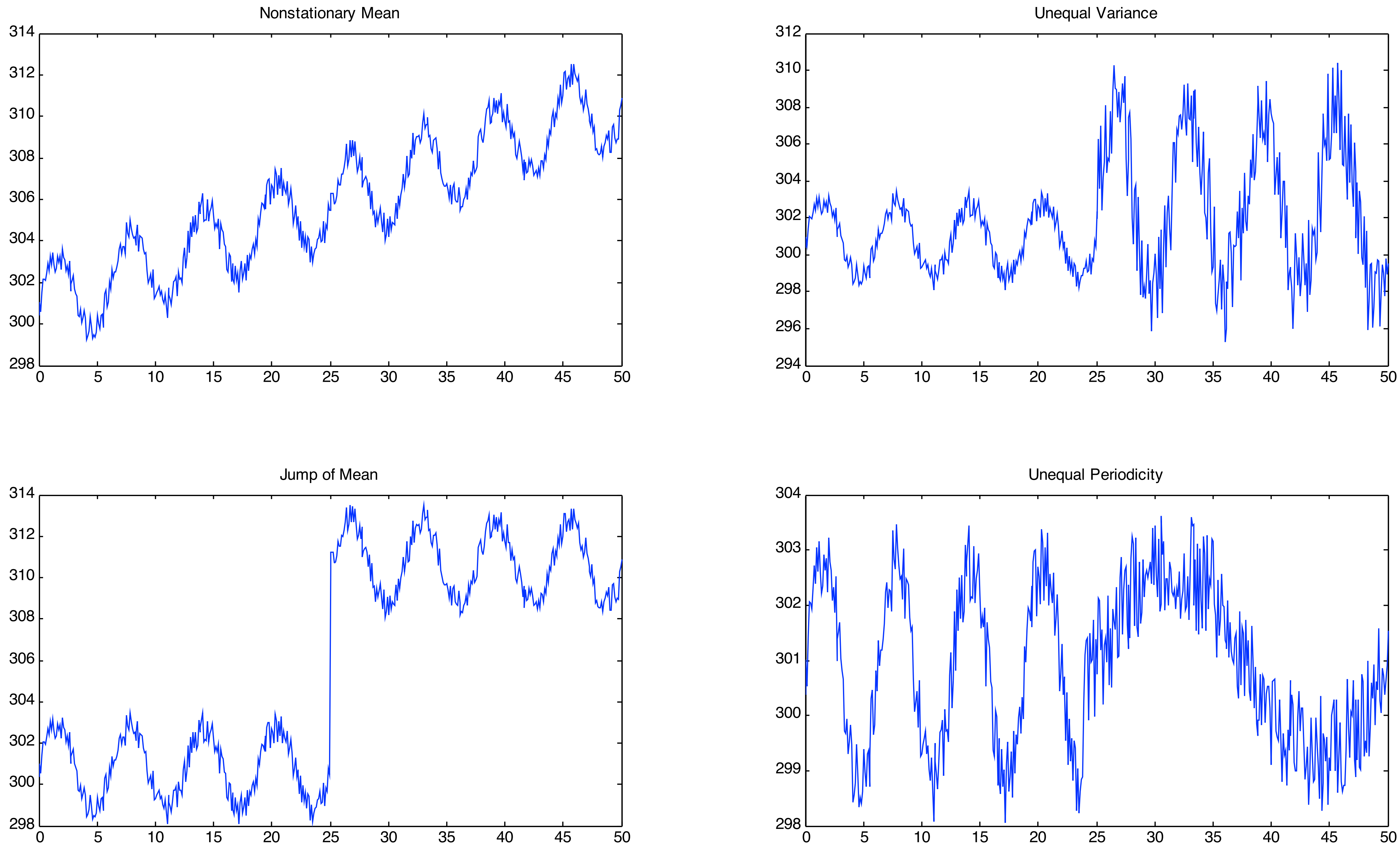


Box-Jenkins
Modeling Approach
Plot
Series
Is
Variance
stable
?
No
Apply
Transformation
Yes
Obtain
ACs and PACs
Is
Mean
Stationary
?
No
Apply Regular
and Seasonal
Differencing
Yes
Model
Selection
Estimate
Parameter
Values
Are
Residuals
Uncorrelated
?
No
Modify
Model
Yes
Are
Parameters
Significant and
Uncorrelated
?
No
Yes
Forecast



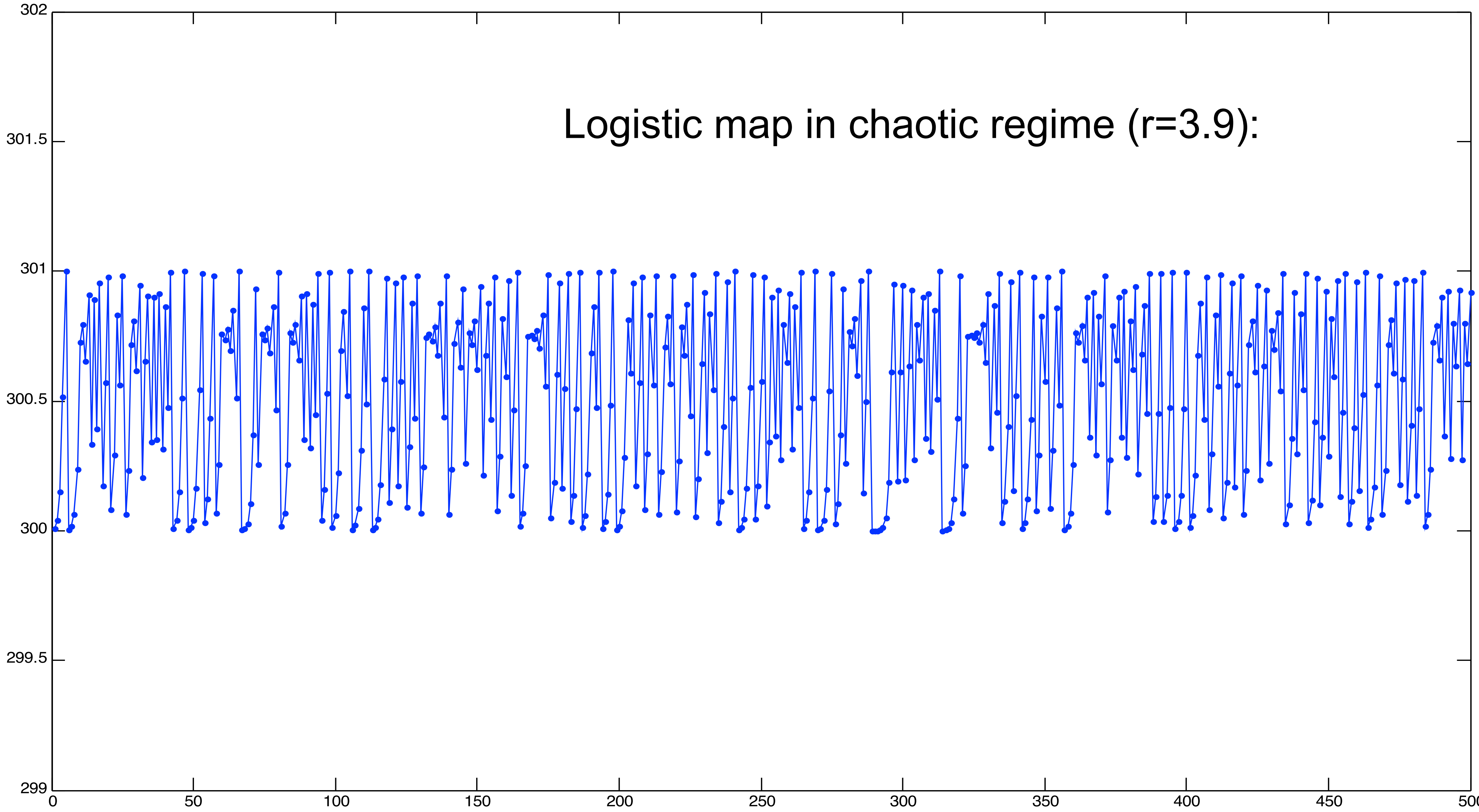
Model Selection Process in
Box-Jenkins Modeling Approach
ARMA (1 ,0)
?
ARMA (2, 0)
?
ARMA (2, 1)
?
ARMA (1, 1)
?
ARMA (0, 1)
?
ARMA (3, 0)
?
ARMA (3, 2)
?
ARMA (2, 2)
?
ARMA (1, 2)
?
ARMA (0, 2)
?
ARMA (n ,0)
?
ARMA (n, n-1)
?
ARMA (n-1, n-1)
?
ARMA (n-2, n-1)
?
ARMA (0, n-1)
?

Problems with ARfIMA (data assumptions)



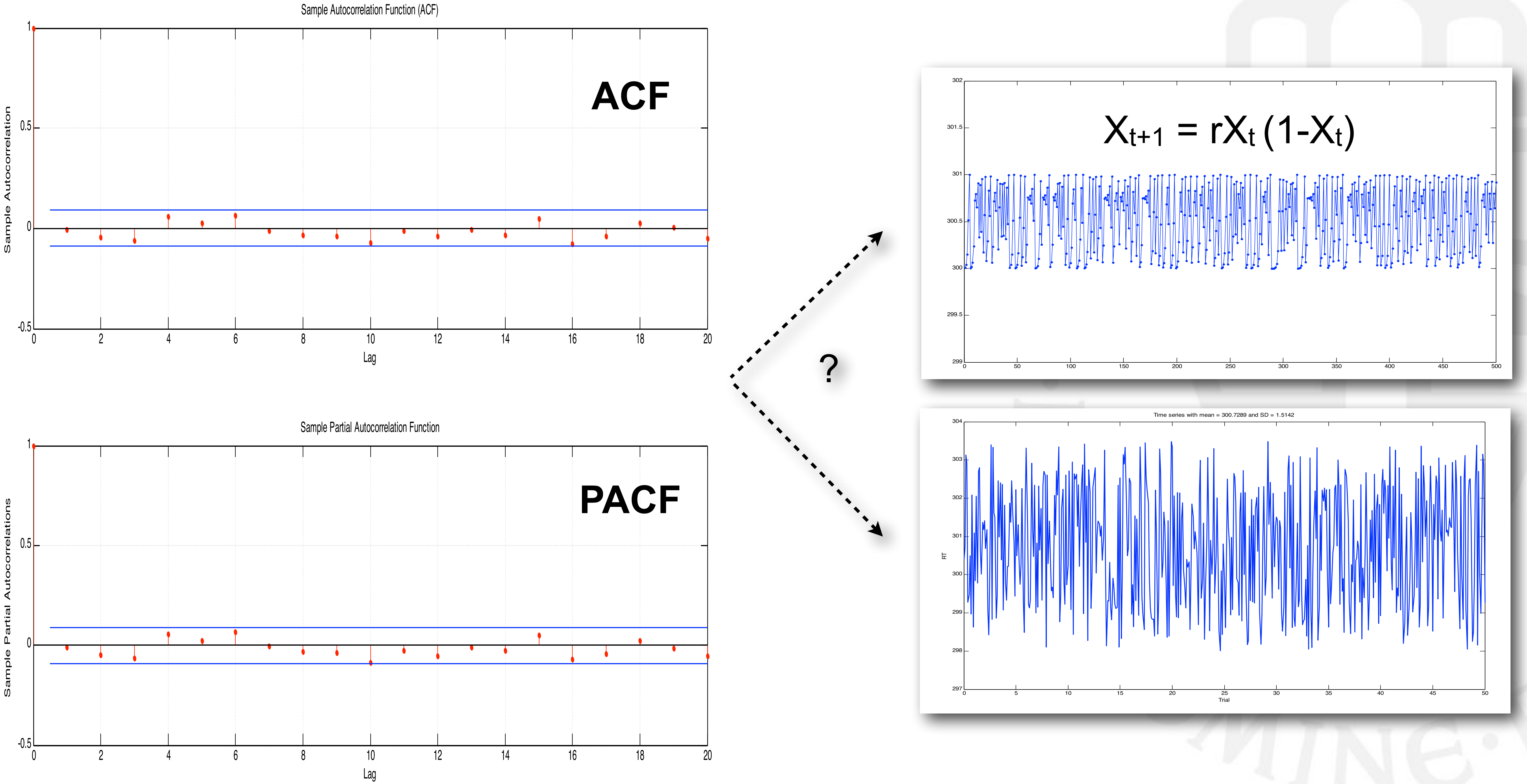
Problems with ARfIMA (data assumptions)

And what about deterministic CHAOS?



Problems with AR[fIMA] models (data assumptions)

ARIMA(0,0,0) ??? - A Random Process ??? - But we know the equation !!!!



"Things that look random, but are not" (Lorenz, 1972)

Testing for ergodicity

Testing for **stationarity**

Testing for **homogeneity**

http://fredhasselman.com/post/2017-05-19-testing-assumptions-of-the-data-generating-process-underlying-experience-sampling/