Dynamics of Complex Systems

Fractal Geometry in Time Series:

Scaling
Fluctuation Analyses
DFA
SDA
PSD slope

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Story so far …

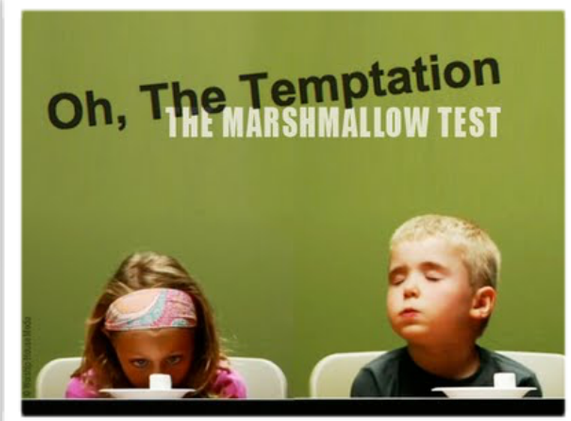
- *Background:* Complexity Science / **Science of the Individual** / Systems Biology
- **Idiographic** versus **Nomothetic** goal of scientific explanation

Principle of jaggedness

no individual corresponds to average

Principle of context

no behaviour is context independent



THE FAILERS...

struggled more in stressful situations

had more trouble paying attention

had greater difficulty maintaining friendships

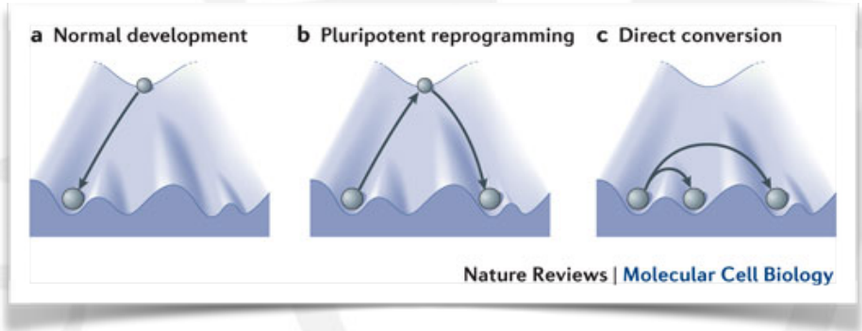
scored lower on the S.A.T. (by over 200 points)

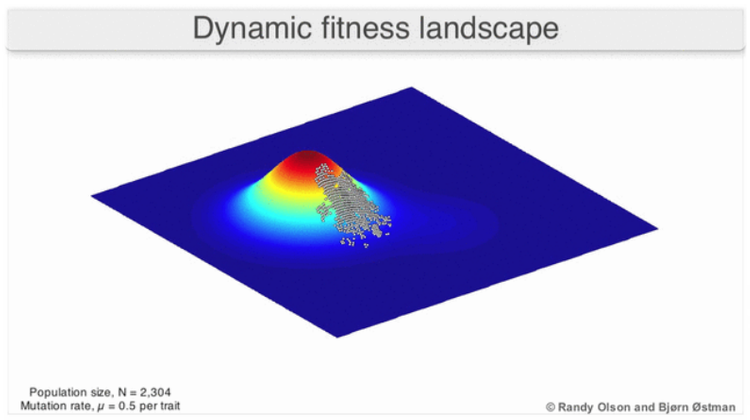
prone to a much higher higher body-mass index

were more likely to have drug addictions

Principle of pathways

multiple trajectories to 'success'





Thread 1:

At age 27-32, those who had waited longest during the Marshmallow Test in preschool had a lower BMI, a better sense of self-worth, pursued their goals more effectively, and coped better with stress and frustration.

(pg. 5)

long range correlations
attractor / repellor
resistance to change
context sensitivity

Thread 2:

Resisting temptation is difficult because the brain's "hot system" is heavily biased towards the present: it takes full account of immediate rewards, but discounts rewards that are delayed.

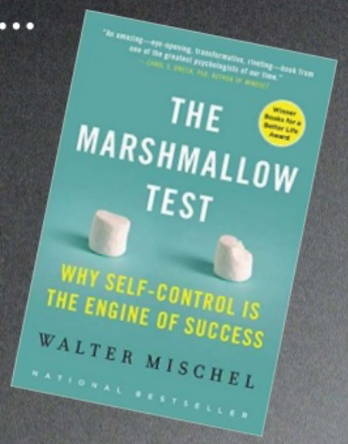
(pg. 255)

Thread 6:

The more we learn about nature and nurture, the more it is clear that they inseparably shape each other. "A predisposition does not a pre- determination make."

(pg. 91-93)

Teasing 10 threads from Walter Mischel's…



Thread 4:

Prolonged stress impairs the prefrontal cortex, essential for things like surviving high school, holding down a job, avoiding depression and refraining from decisions that seem intuitively right but turn out to be really stupid.

(pg. 49)

Thread 7:

Each child who waited successfully had a distinctive methodology for self-control. First, they had to remember and actively keep in mind their chosen goal.

(pg. 107)

complex dynamics
different paths
(multi-realisability)

metaphors from thermodynamics

Thread 10:

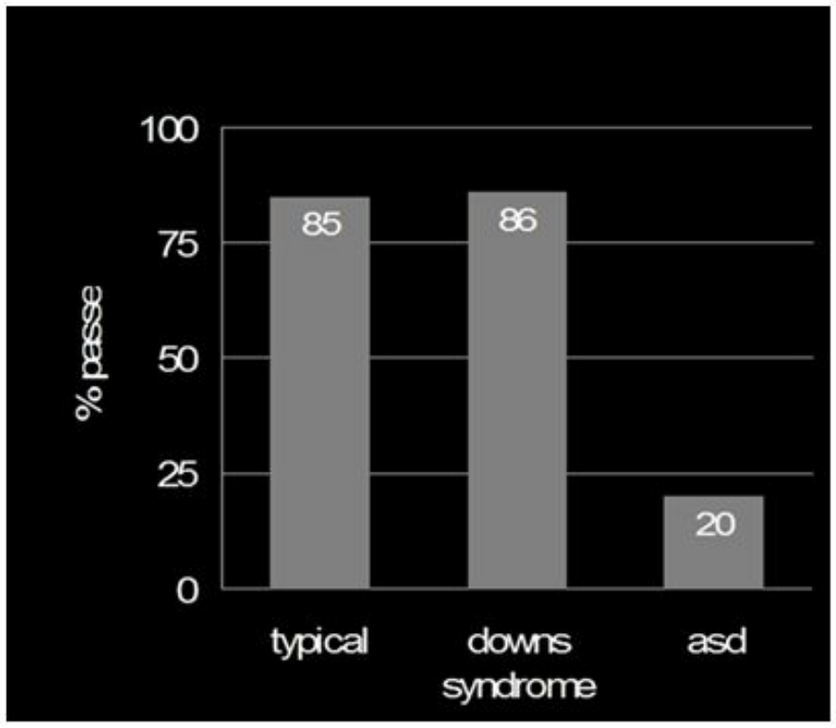
The Fundamental Marshmallow Principle: "Cool the now; heat the later!"

(pg. 256)

theory of mind?

only four of the 20 autistic children (20%) answered correctly

Sally-Anne problem



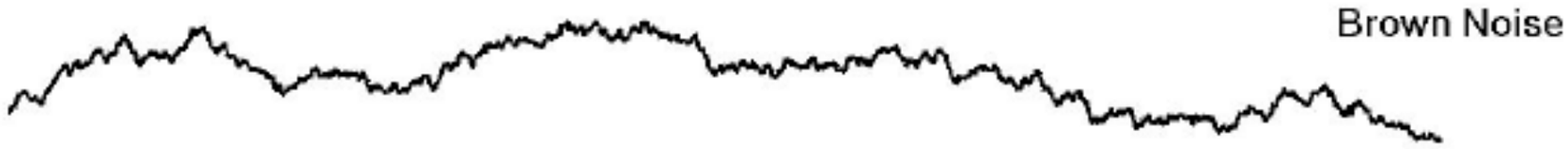
Baron-Cohen, Leslie, and Frith (1985)
Social and emotional problems secondary to cognitive problem

Instruct to keep the chocolate…. no problem!

Last week's assignments:
"Statistics": Correlations, *inter*dependece of repeated measurements

Brownian motion, or random walk processes

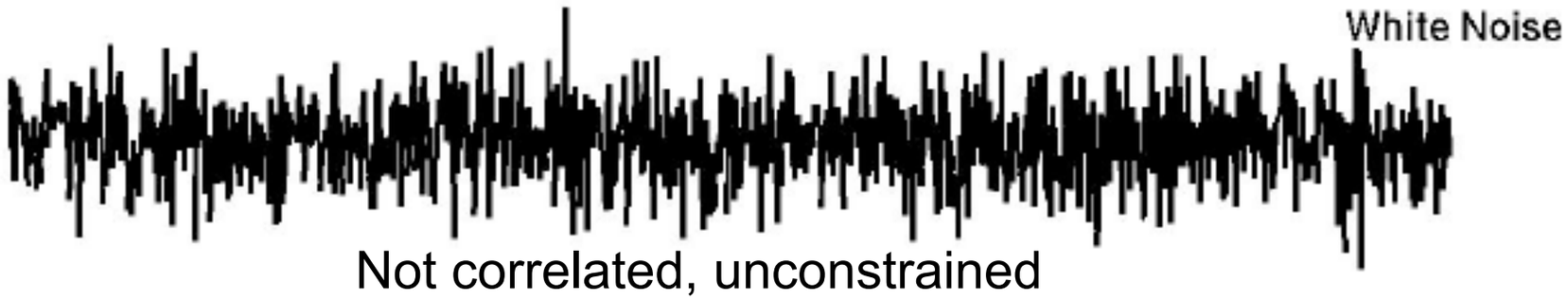
Highly correlated, constrained



Between order and random… Chaos?

Associated with Self Organizing Complex Systems

Gaussian, random processes



Not correlated, unconstrained

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A stochastic linear process model AR(fI)MA

- Notation: **ARIMA(p,d,q)**, these parameters usually take values of 0-2 indicating none, 1 or 2 components.
- **ARIMA(1,2,1)** = means One AutoRegressive parameter, a (filtered-out!) quadratic trend, and 1 Moving Average parameter.
- **AR-part:** The model tries to predict each data point **X** at time **t** based on a constant or intercept (**ξ**) + a linear combination of previous observations ($\phi_{1...p}$) + random error called random shock (**ε**):

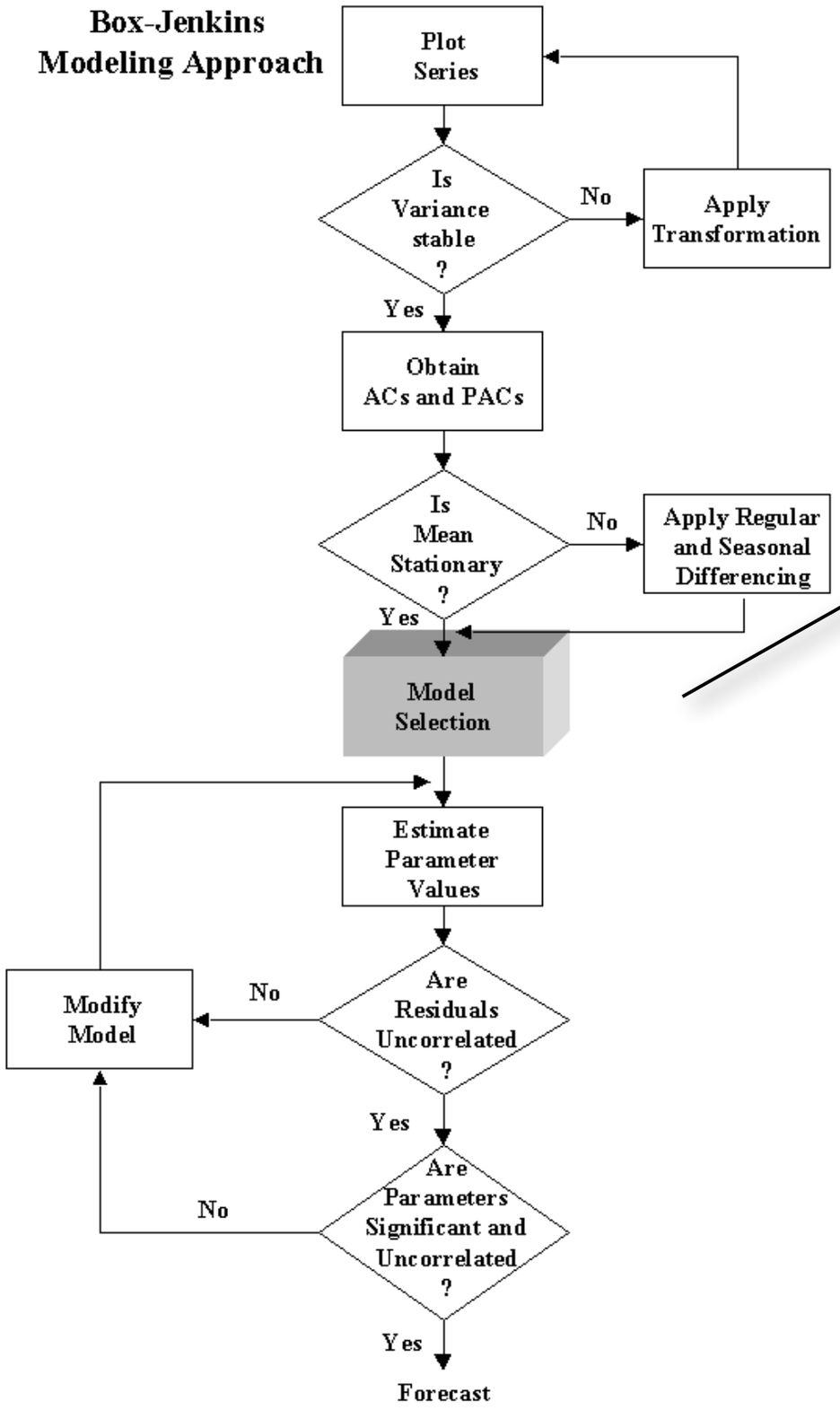
$$X_t = \xi + \phi_1 X_{(t-1)} + \phi_2 X_{(t-2)} + \phi_3 X_{(t-3)} + \ldots + \phi_p X_{(t-p)} + \varepsilon$$

- **MA-part:** The model tries to predict each data point **X** at time **t** based on the average of the time series (**μ**) + the current random error (ε_t) - a linear combination of previously observed random error ($\theta_{1...q}$):

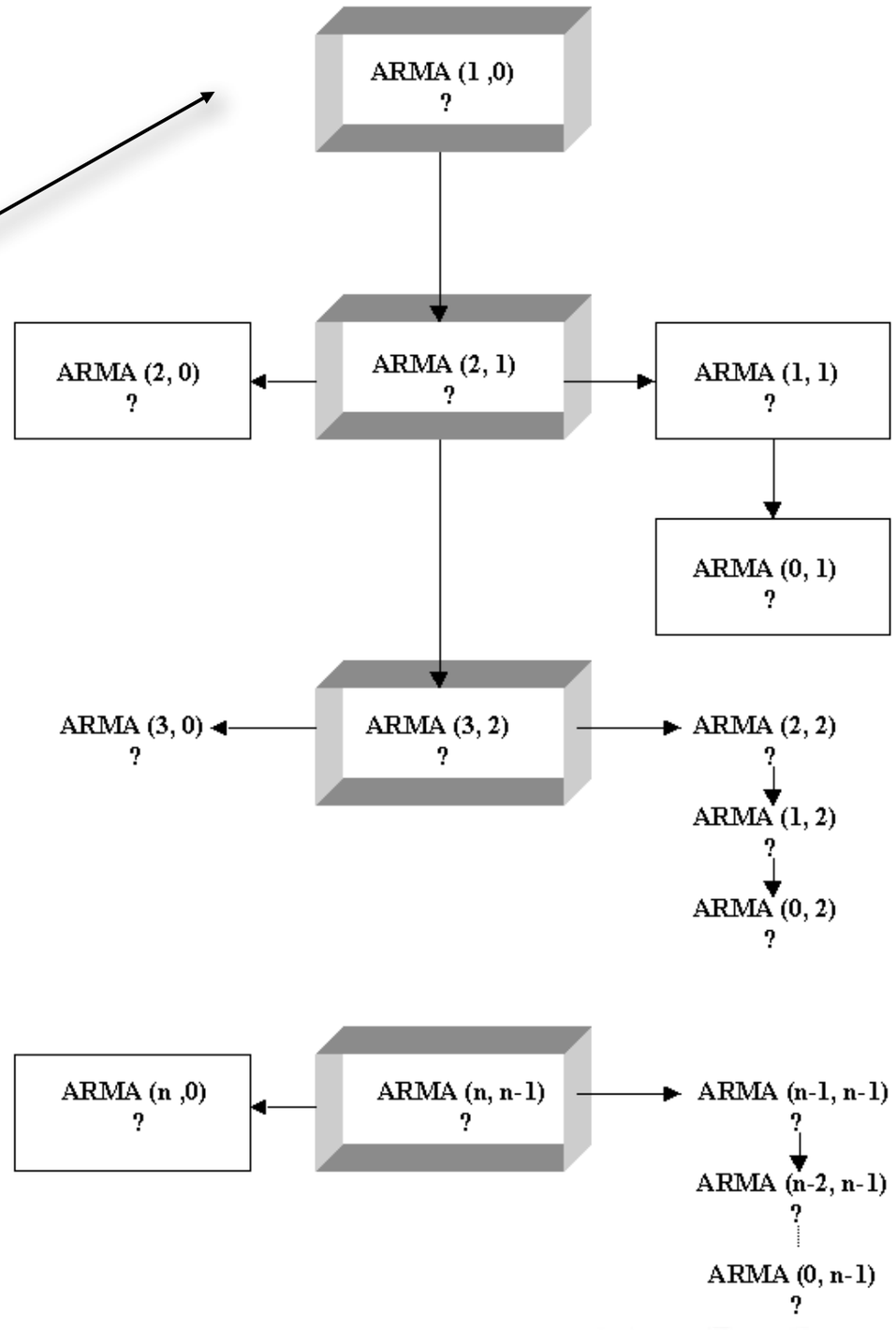
$$X_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{(t-1)} - \theta_2 \varepsilon_{(t-2)} - \theta_3 \varepsilon_{(t-3)} - \ldots - \theta_p \varepsilon_{(t-q)}$$

- **I(ntegration)-part:** The data must be stationary (and more!): 1) constant mean, 2) constant variance, 3) constant autocorrelation *through* time. This part removes correlations between data points that are a particular distance (lag) apart by differencing the time series… in other words: Trends in the data are filtered out.
- How many parameters do you need? … ACF and PACF

Box-Jenkins Modeling Approach

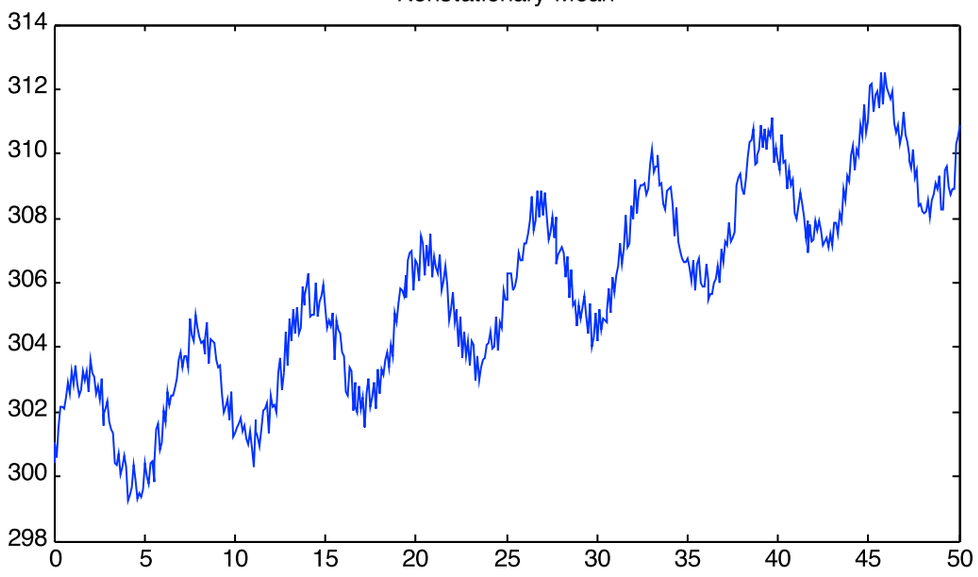


Model Selection Process in Box-Jenkins Modeling Approach

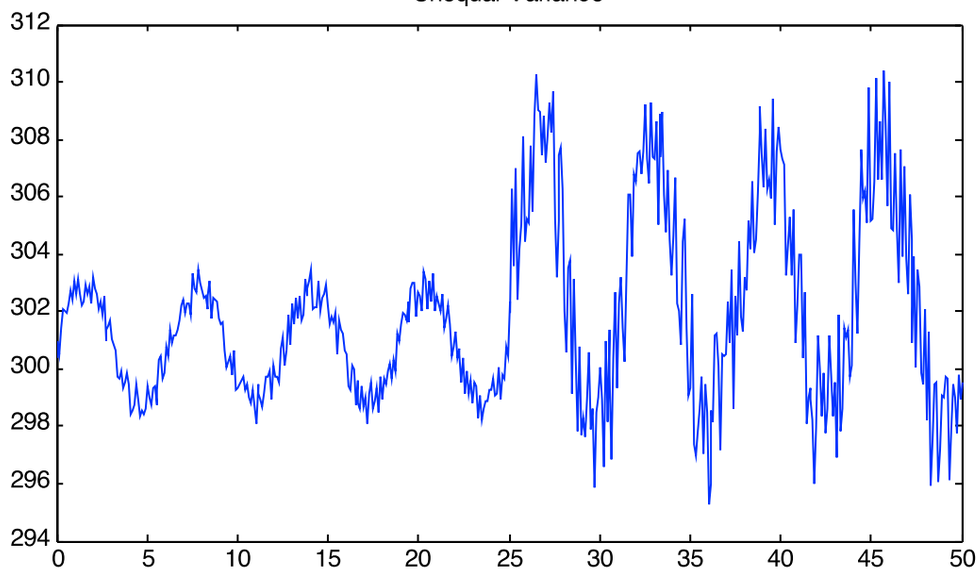


Problems with ARfIMA (data assumptions)

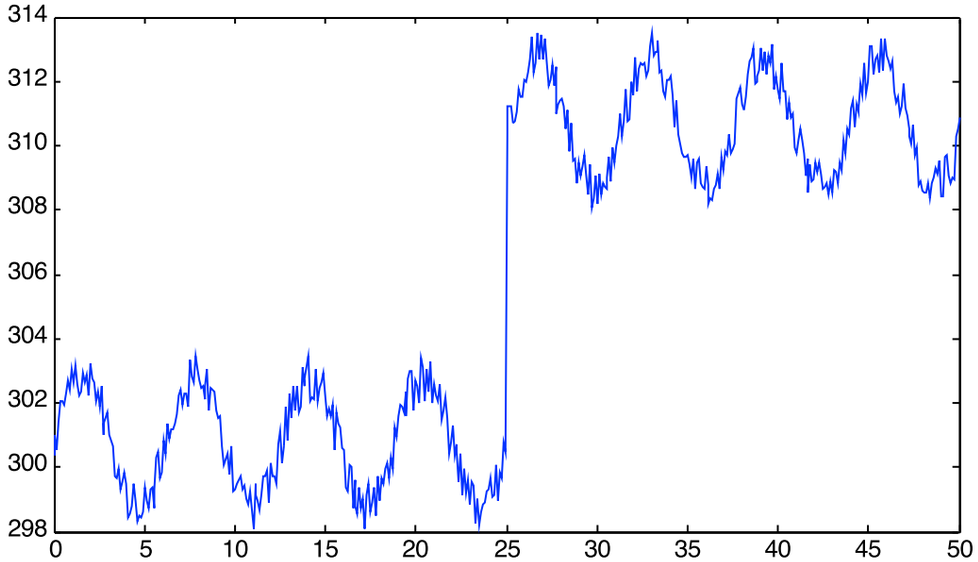
Nonstationary Mean



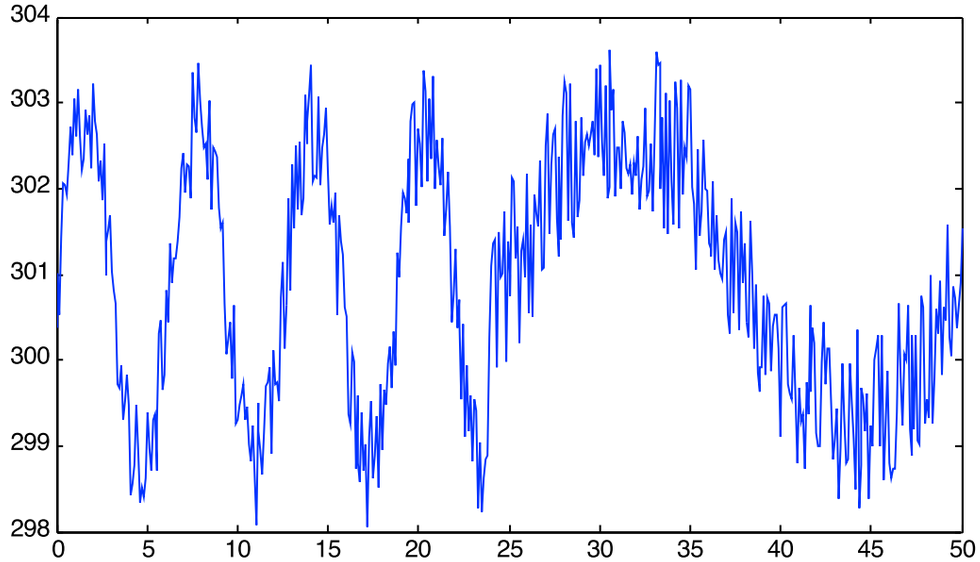
Unequal Variance



Jump of Mean

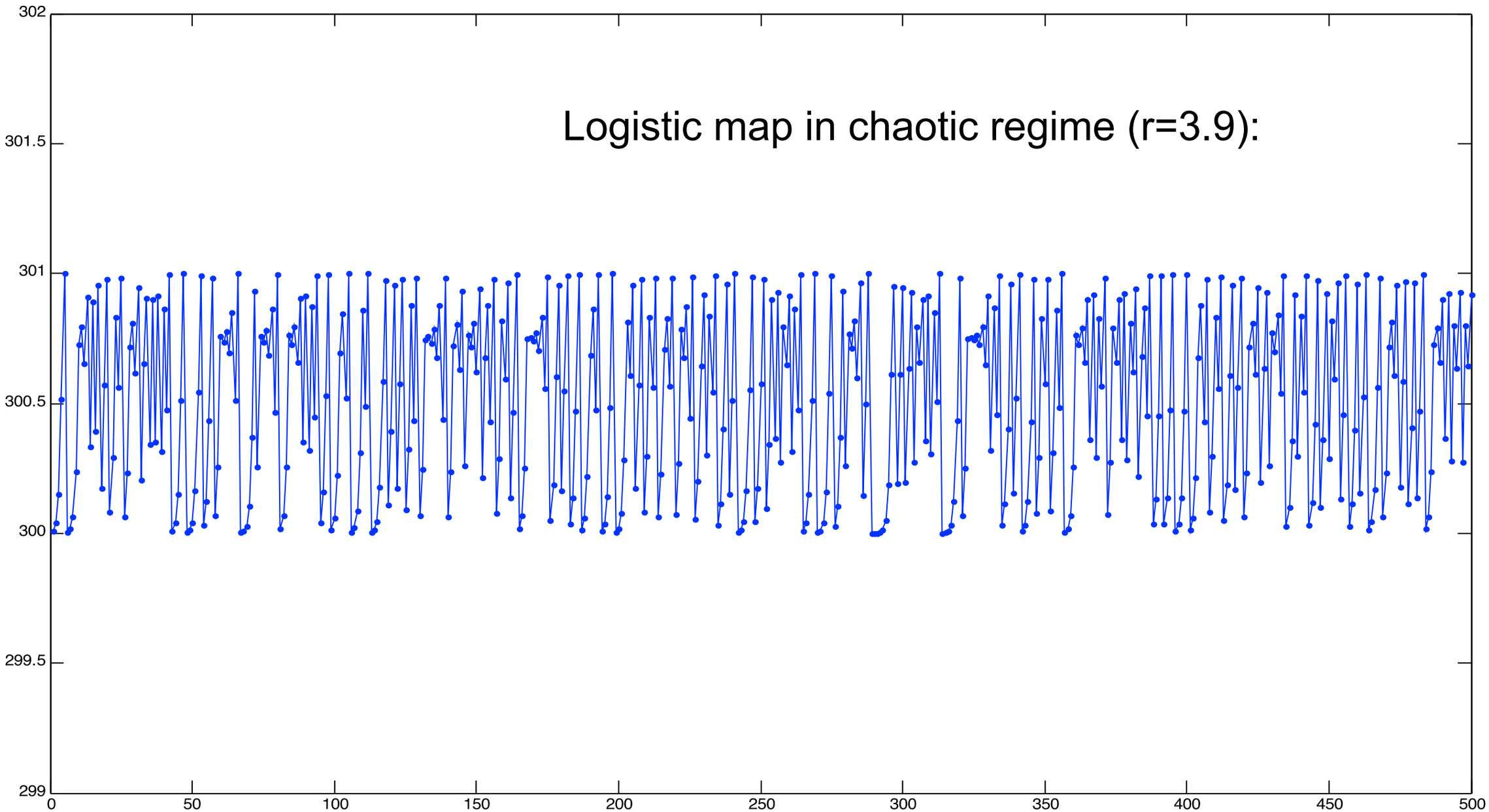


Unequal Periodicity



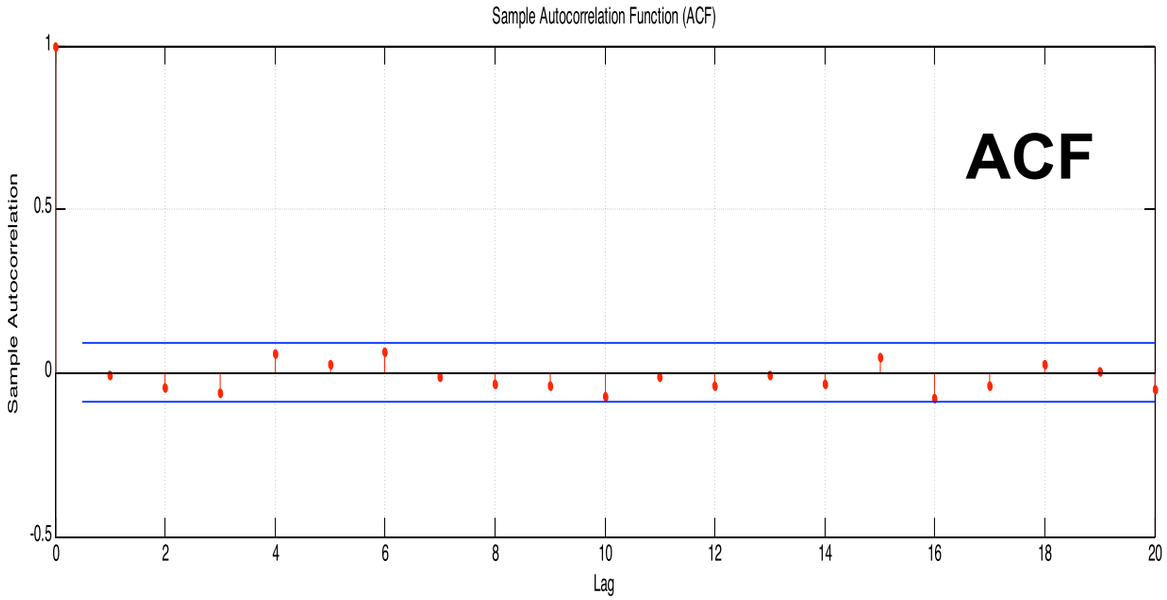
Problems with ARfIMA (data assumptions)

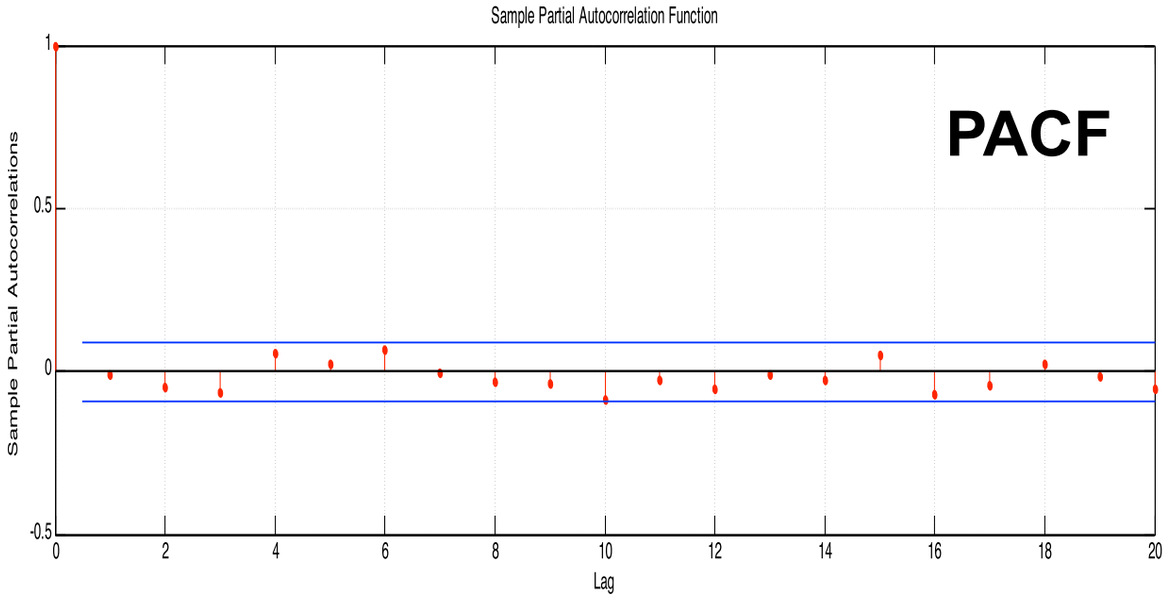
And what about deterministic CHAOS?



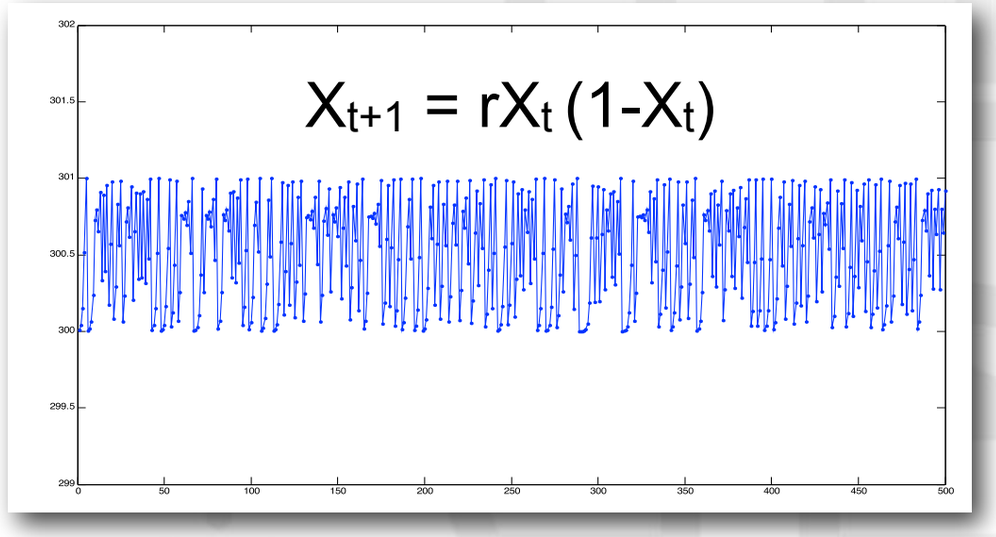
Problems with AR[fIMA] models (data assumptions)

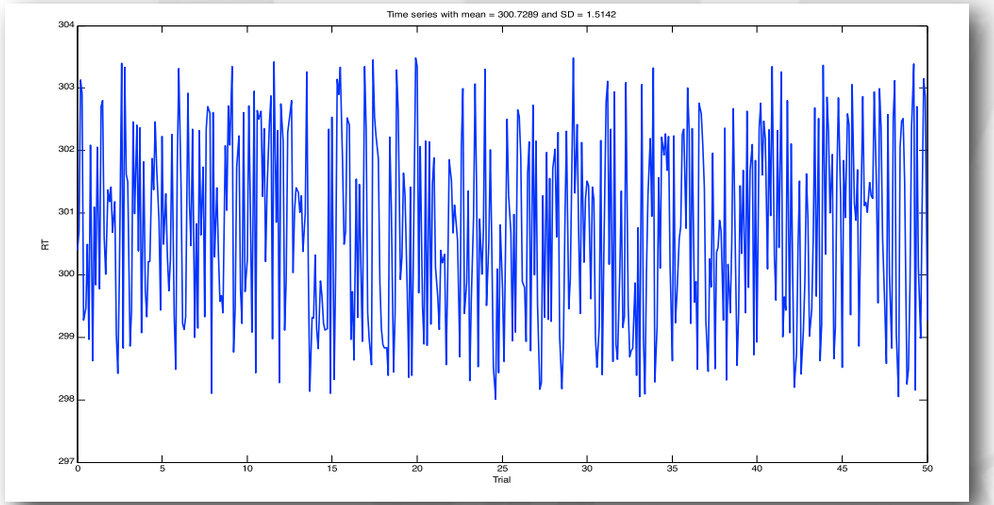
ARIMA(0,0,0) ??? - A Random Process ??? - But we know the equation !!!!



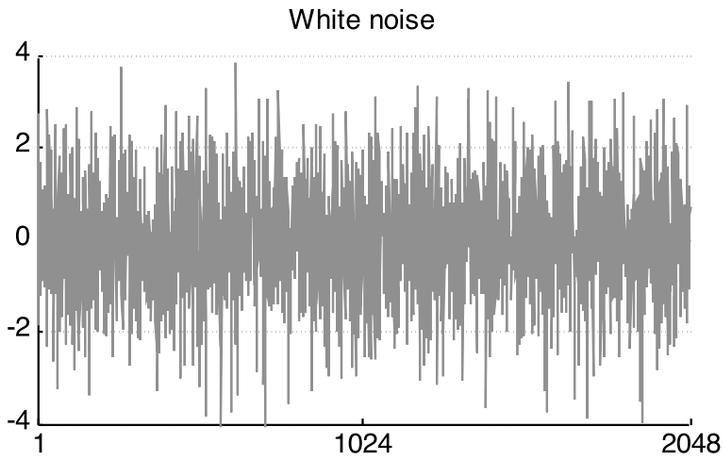


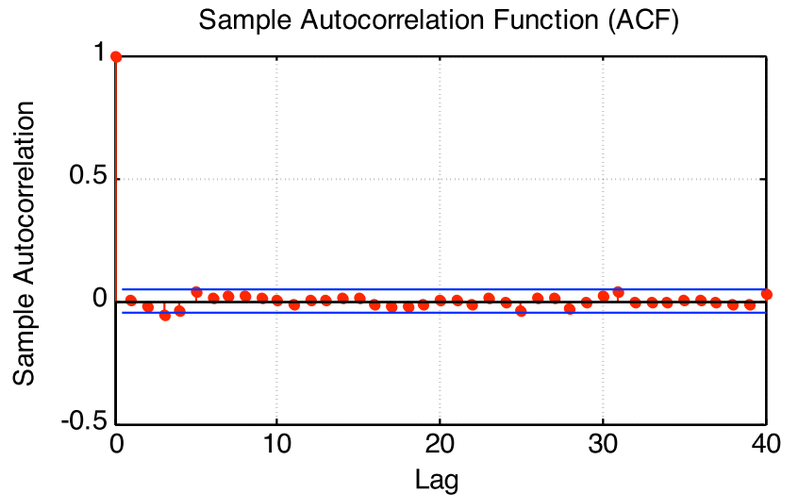
?

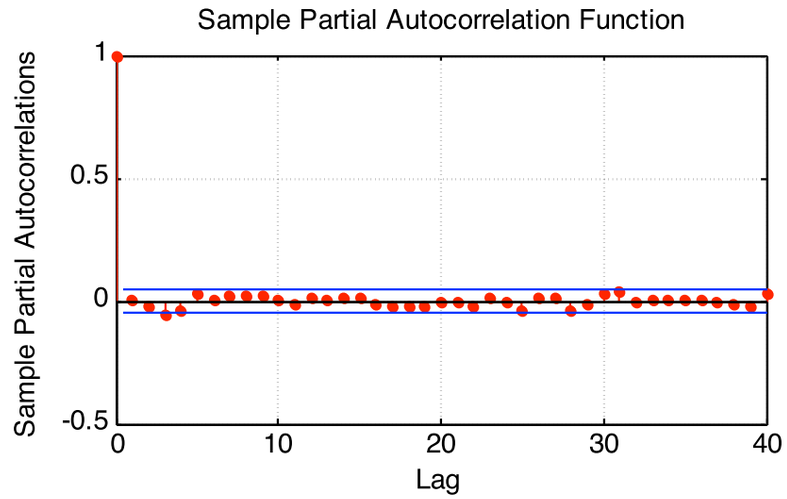


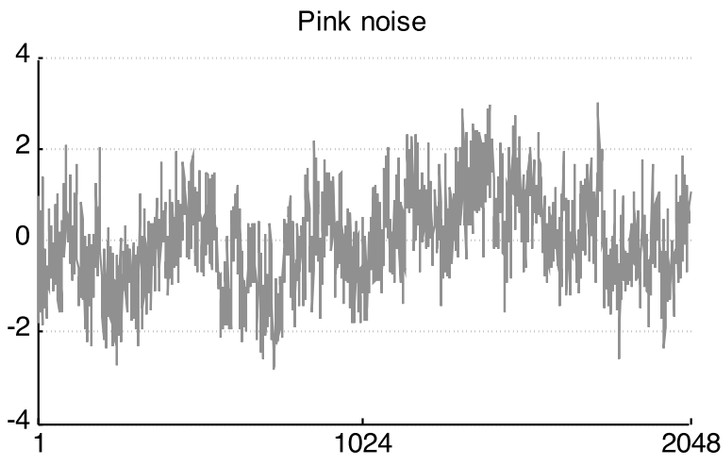


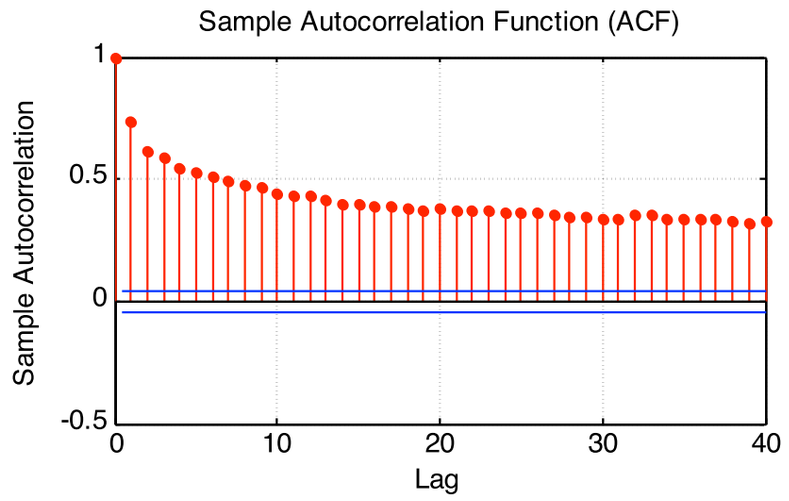
"Things that look random, but are not" (Lorenz, 1972)

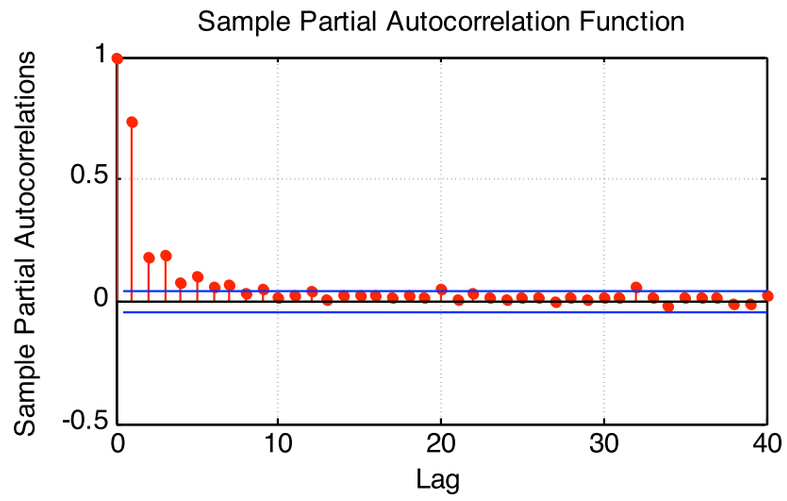

White noise
4
2
0
-2
-4
1
1024
2048

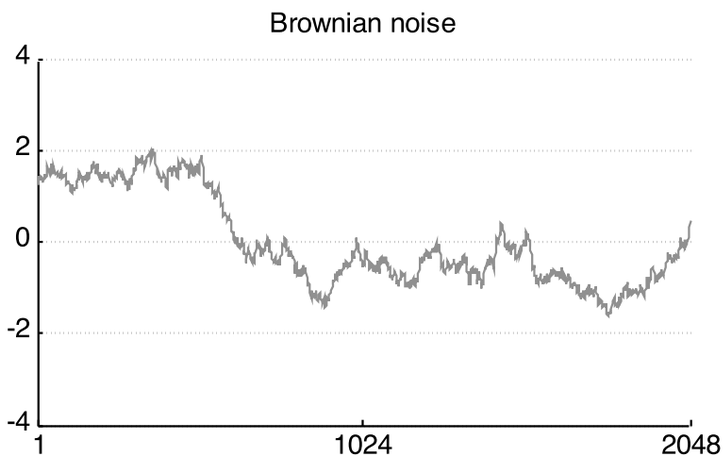

Sample Autocorrelation Function (ACF)
Sample Autocorrelation
1
0.5
0
-0.5
0
10
20
30
40
Lag

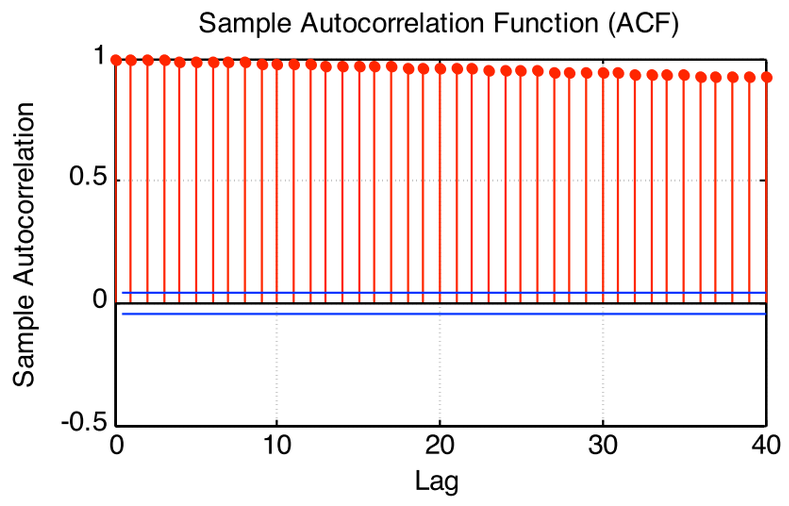

Sample Partial Autocorrelation Function
Sample Partial Autocorrelations
1
0.5
0
-0.5
0
10
20
30
40
Lag

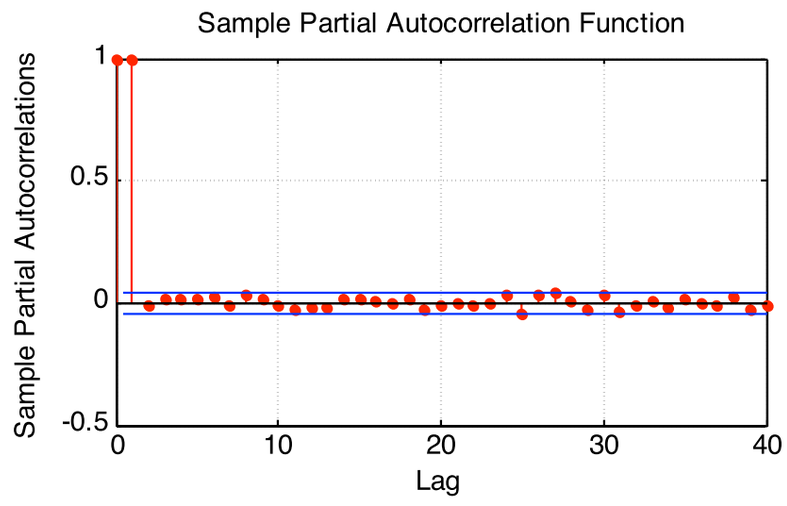

Pink noise
4
2
0
-2
-4
1
1024
2048


Sample Autocorrelation Function (ACF)
Sample Autocorrelation
1
0.5
0
-0.5
0
10
20
30
40
Lag


Sample Partial Autocorrelation Function
Sample Partial Autocorrelations
1
0.5
0
-0.5
0
10
20
30
40
Lag

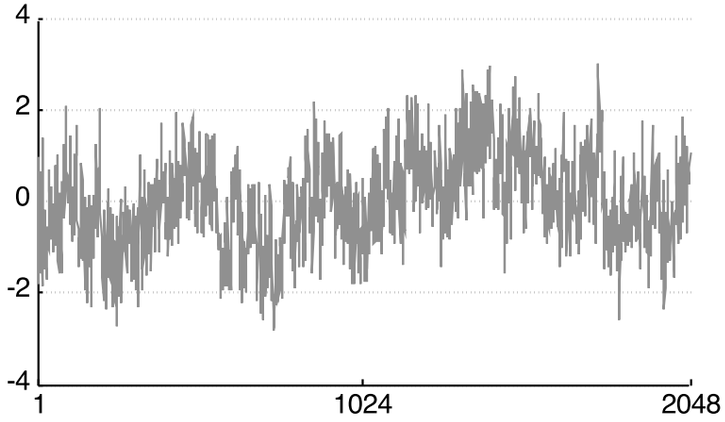

Brownian noise
4
2
0
-2
-4
1
1024
2048

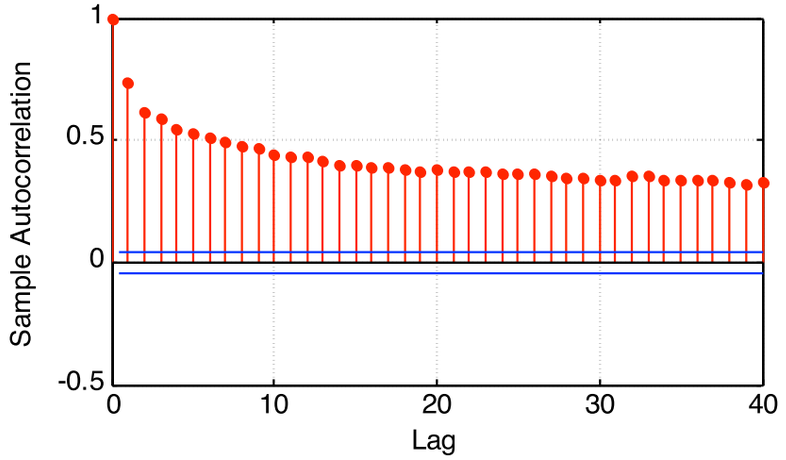

Sample Autocorrelation Function (ACF)
Sample Autocorrelation
1
0.5
0
-0.5
0
10
20
30
40
Lag

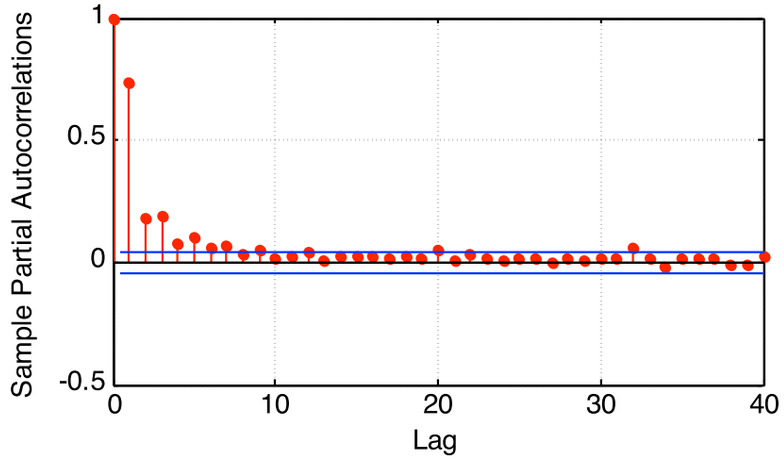

Sample Partial Autocorrelation Function
Sample Partial Autocorrelations
1
0.5
0
-0.5
0
10
20
30
40
Lag

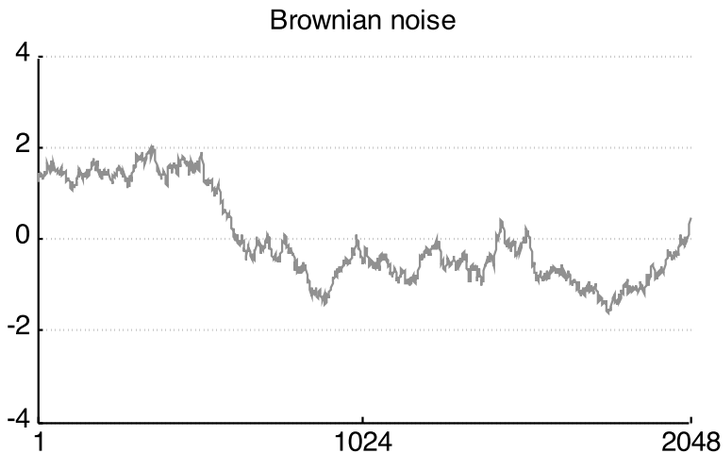
Correlations at all lags = Correlations at all time scales
There is not 1 characteristic time scale

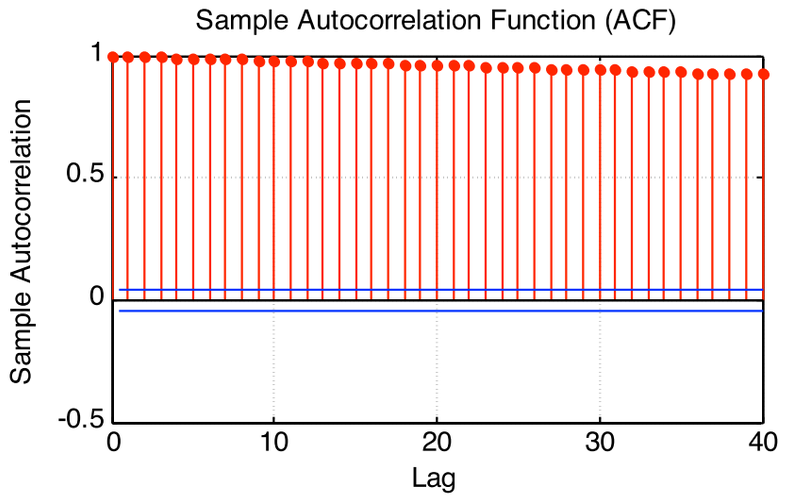
Each data point participates in the larger pattern,
but the pattern does not reduce to the properties of individual measurements.

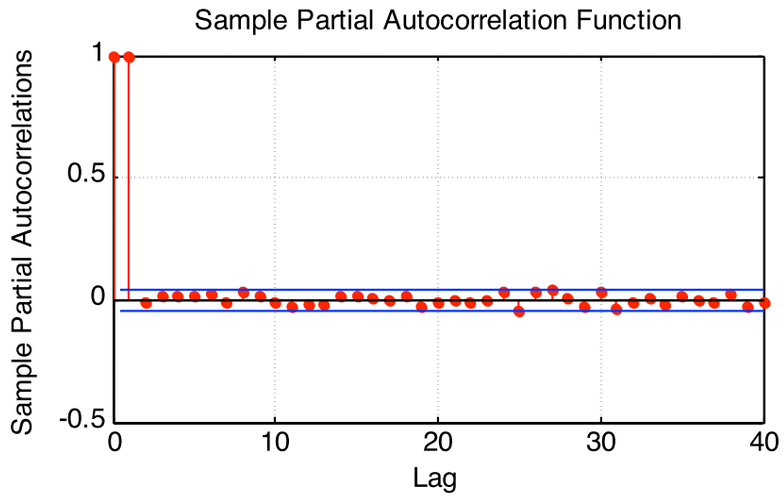


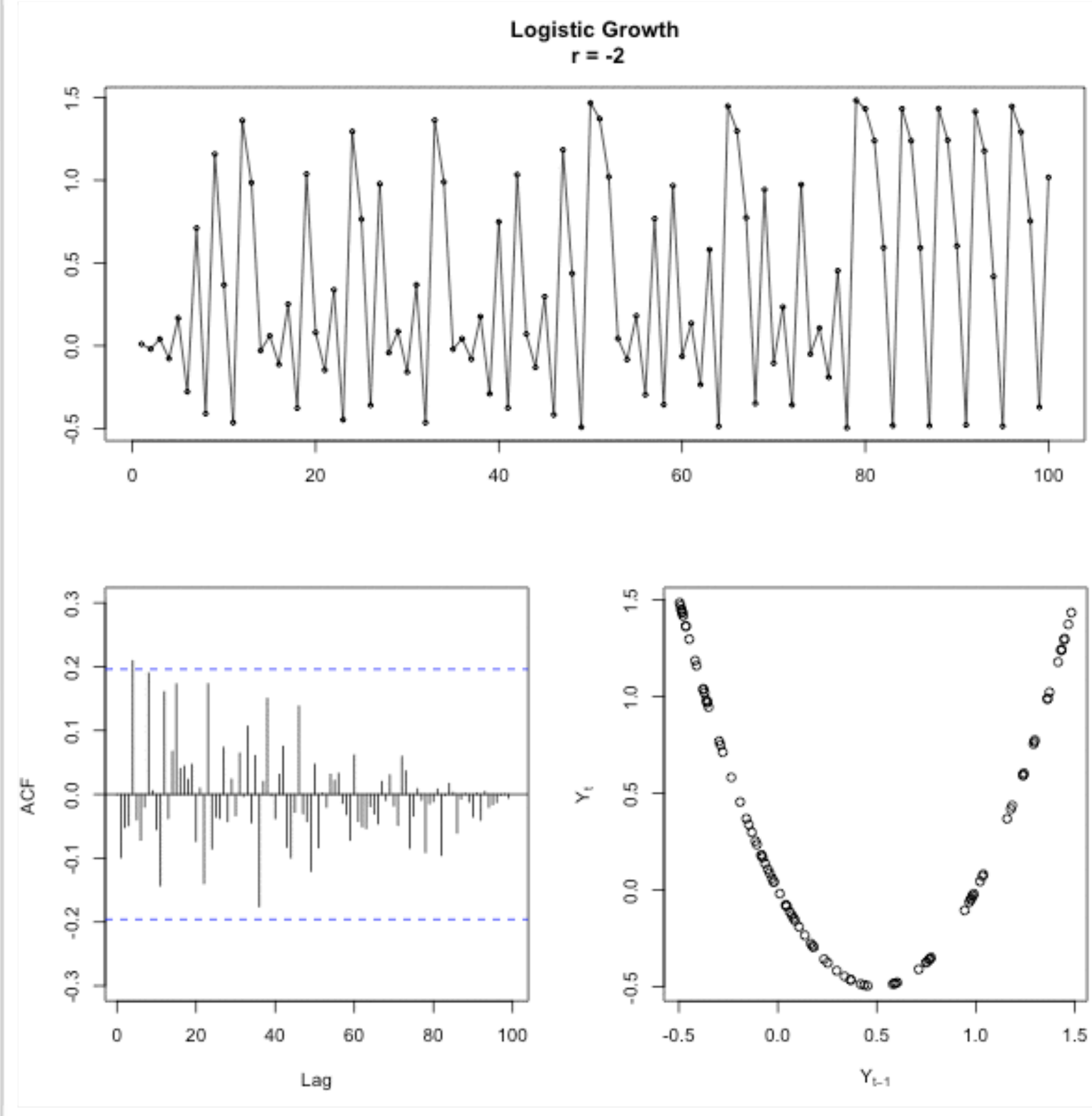











Logistic Growth
r = -2
1.5
1.0
0.5
0.0
-0.5
0
20
40
60
80
100
ACF
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
Lag
Y_t
Y_{t-1}

Testing for ergodicity

Testing for **stationarity**

Testing for **homogeneity**

http://fredhasselman.com/post/2017-05-19-testing-assumptions-of-the-data-generating-process-underlying-experience-sampling/

Scaling phenomena

How long is the coast of Great-Britain?

Scaling phenomena

scale	scale	scale
11.5 x 200 =	28 x 100 =	70 x 50 = 3500
2300 km	2800 km	km

Length <u>systematically</u> depends on the size of the measurement stick you use!

https://en.wikipedia.org/wiki/Fractal_dimension

Scaling phenomena

"scaling of bulk with size"

(Theiler, 1990)

The formal answer to the question is:

"There is no characteristic scale at which the length of the coast of GB can be expressed"

Mandelbrot, B. B. (1967). How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science, 156*(3775), 636–8.

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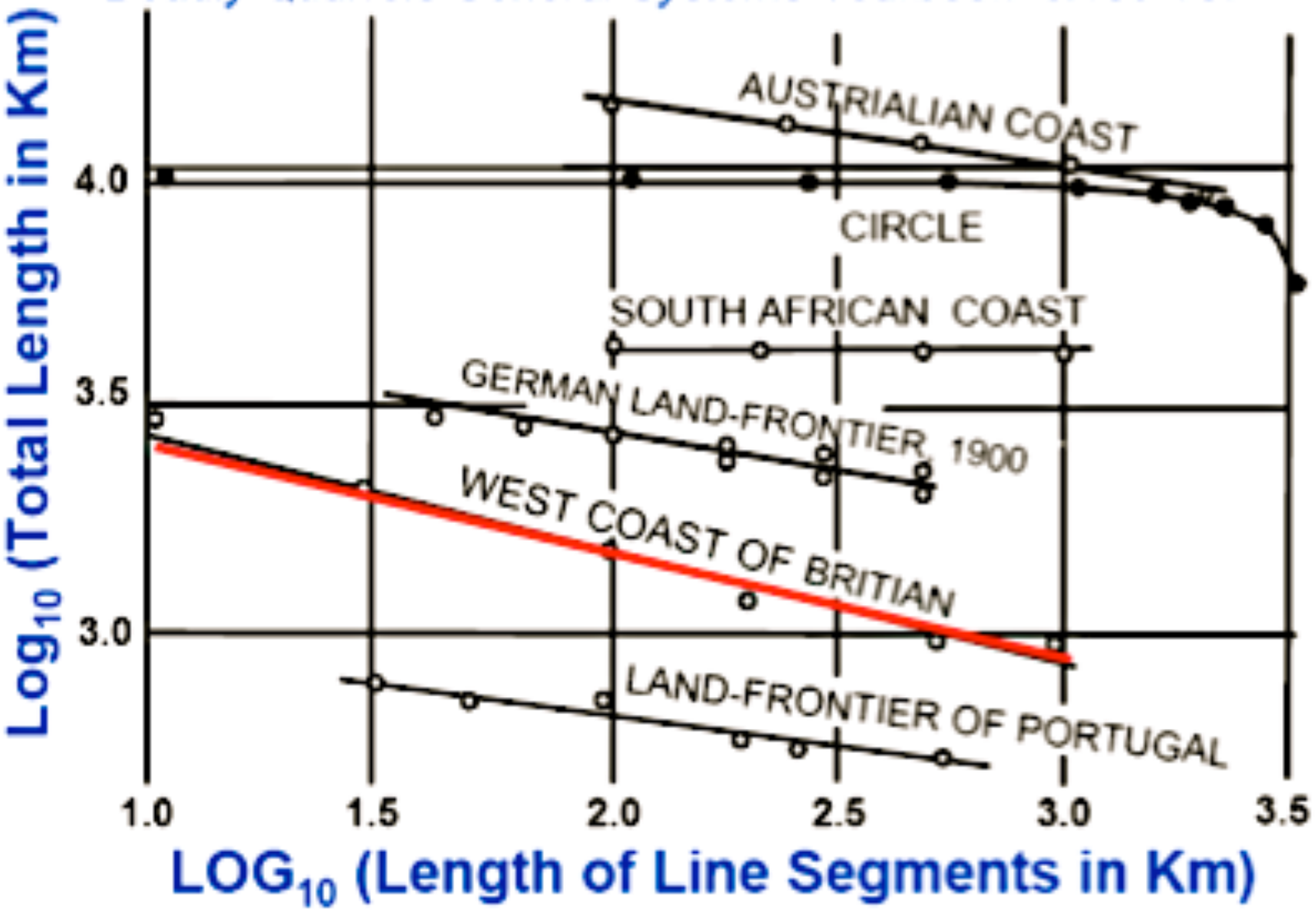
How Long is the Coastline of Britain?

Richardson 1961 The problem of contiguity: An Appendix to Statistics of Deadly Quarrels General Systems Yearbook 6:139-187



Scale invariance…

no meaningful central moments can be defined

Mean and SD characterise the data only relative to the scale of observation (e.g. sample size)

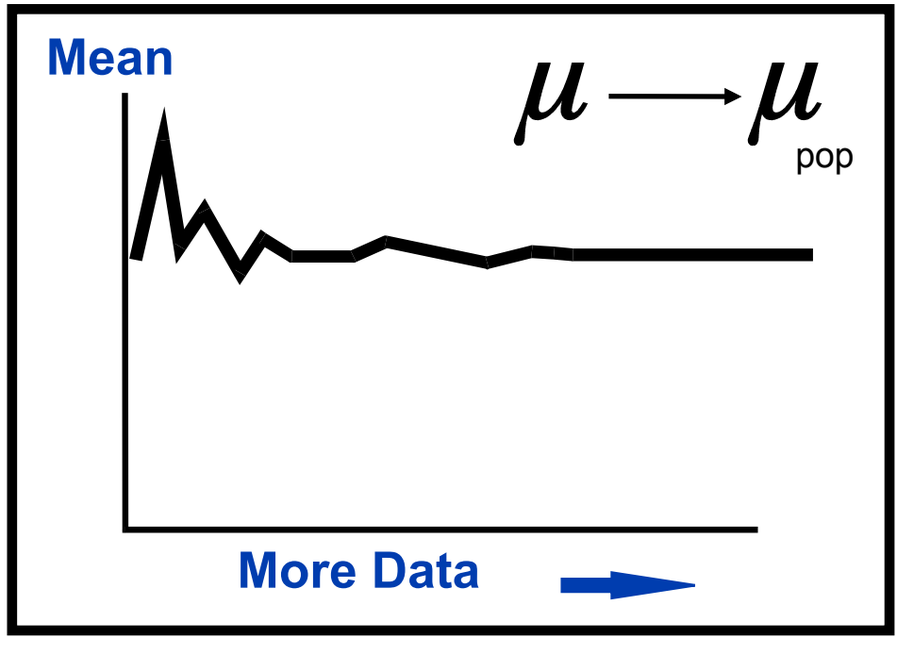
A *power law* scaling relation (**LOG scale**):
There is no characteristic length, just an indication of complexity

Scaling phenomena

Independent observations of random variables

μ±σ are sufficient to characterise absence of dependencies in the data:

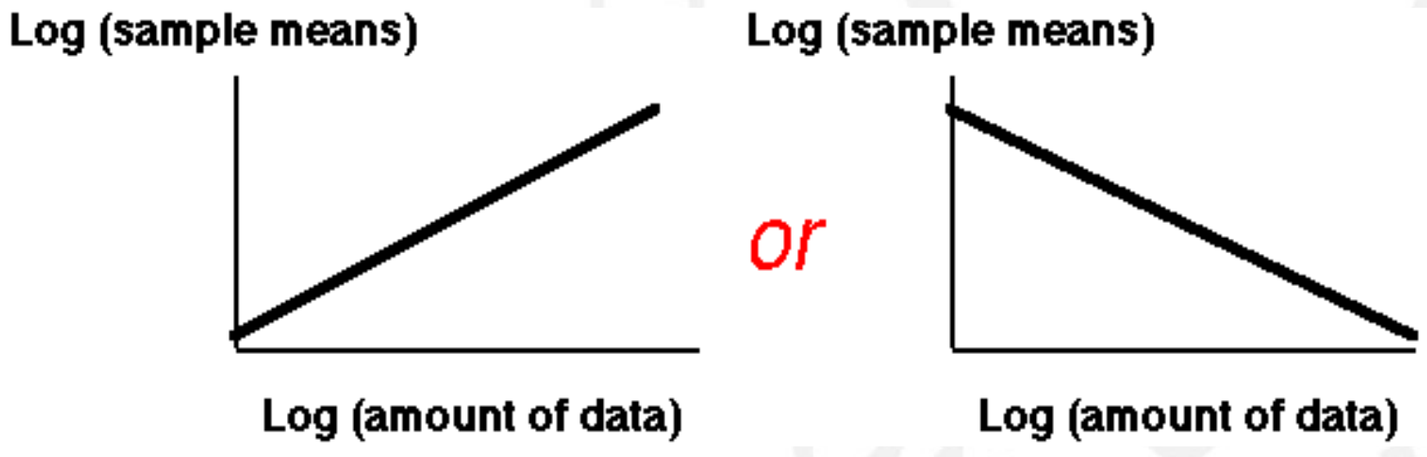
e.g. Expected value of **μ** for **N** = 100, given **σ**



Interdependent observations across different scales

μ±σ are insufficient to characterise dependencies in the data:

e.g. Sample estimates of **μ** change with **N**



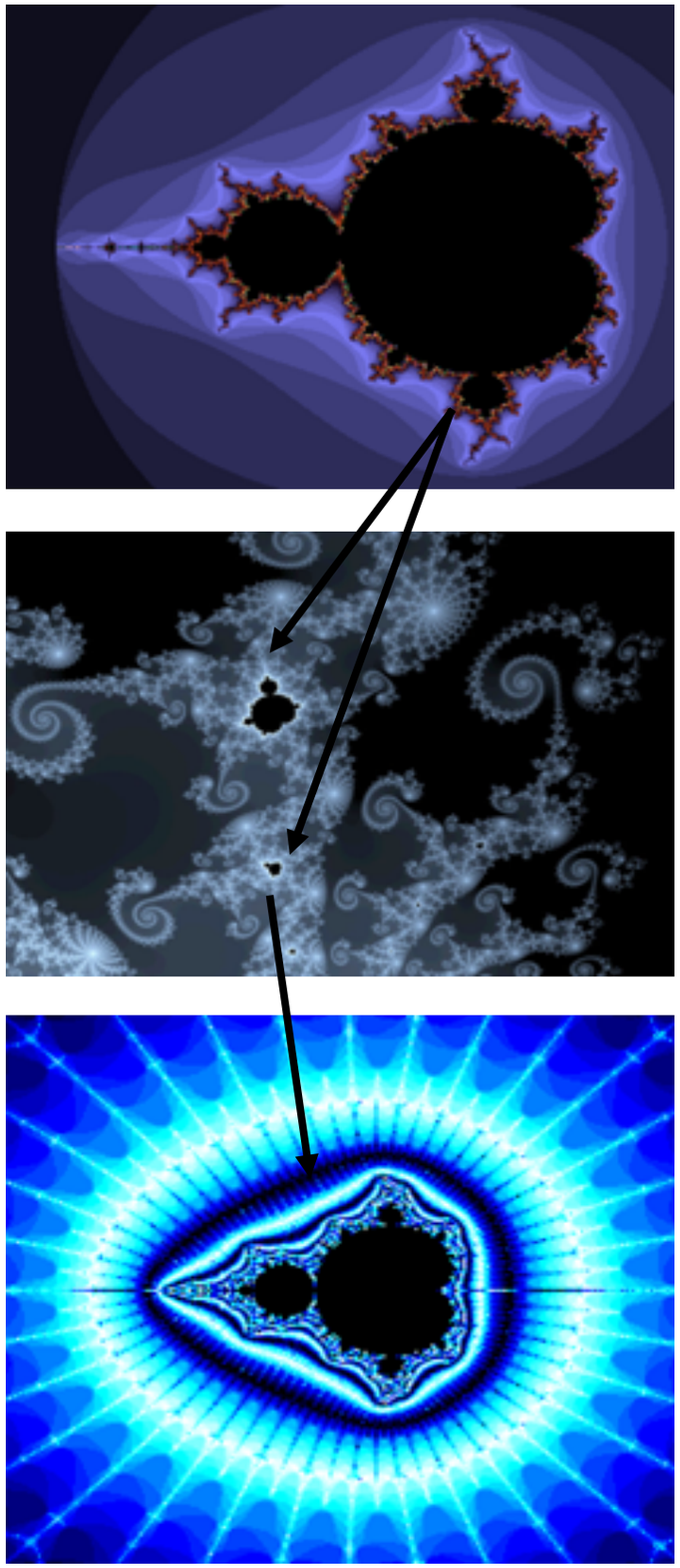
Help! How can I do science without **μ** or **σ**?

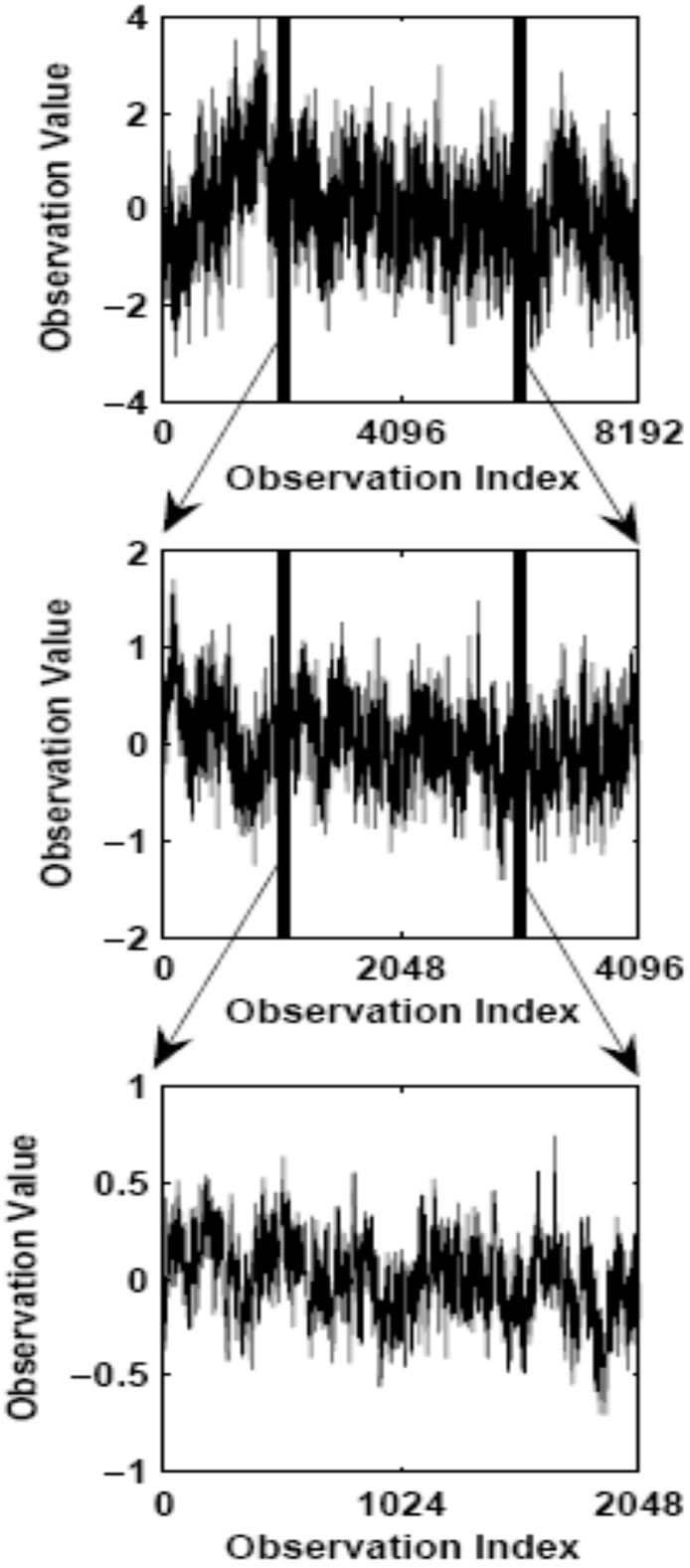
What is scaling?
Self-similarity and Self-affinity

Object looks roughly the same on all scales = (Statistical) **self-similarity** (“zoom similarity”)

(Statistical) self-similarity is observed after affine transformation = **self-affinity** (“warp similarity”)

Degree of invariance across scales = Dependencies/regularities/correlations across scales





aka: “Fractal scaling”

How to describe scaling relations:
Calculate a “fractional” dimension, e.g. box-counting dimension

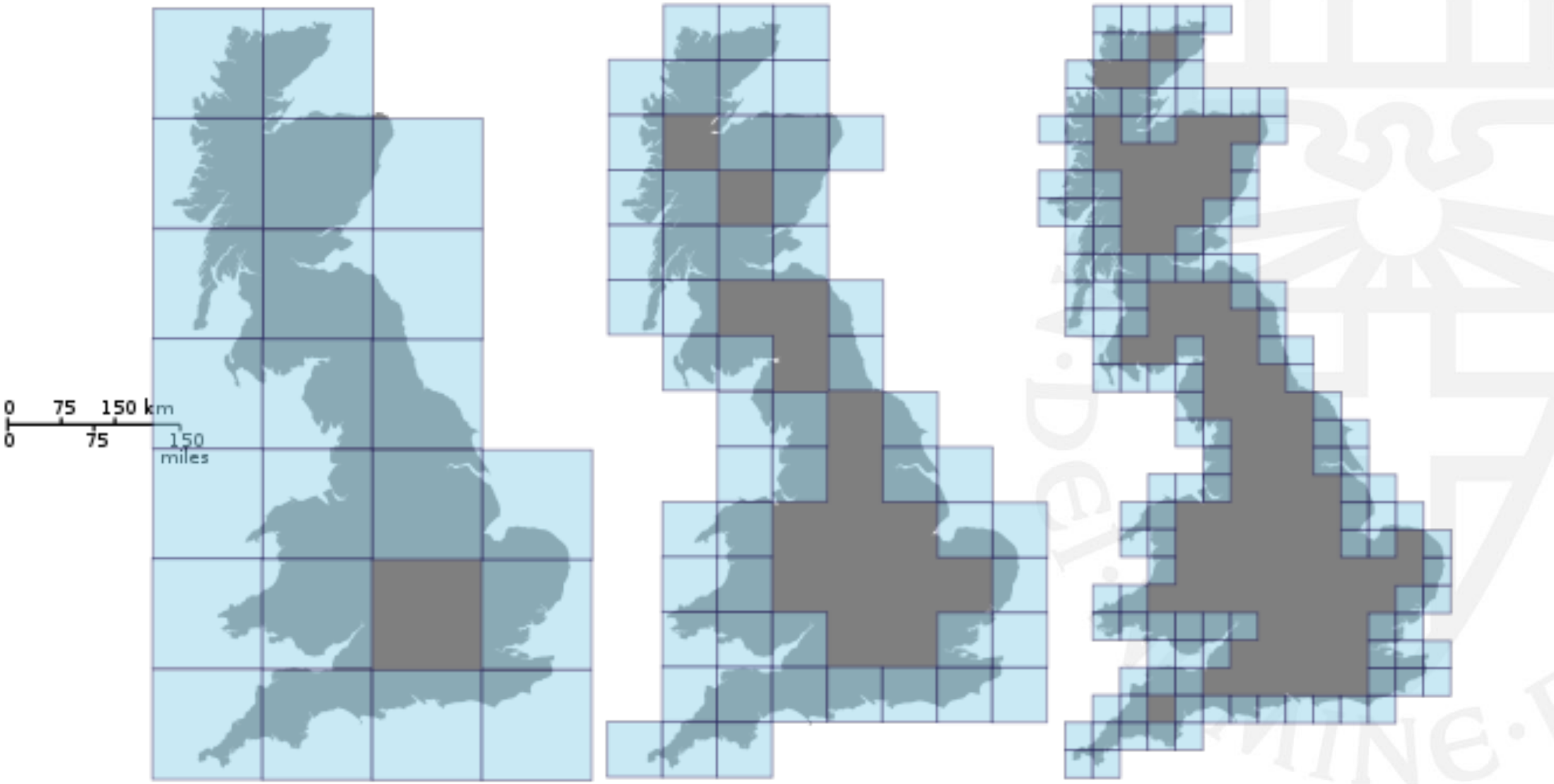


Image by Prokofiev - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=12042116

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Scaling phenomena

Measuring dimension

What is the dimension of these objects?:

Definition (Euclidian) dimension: *The number of degrees of freedom you have to move through a space.*

Definition of a space: *a collection of points*

0 1 2 3

Scaling phenomena

Measuring dimension

What is the dimension of these objects?:

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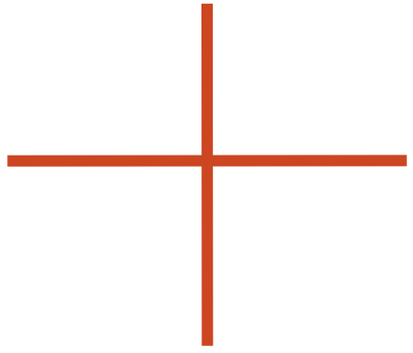


Locally 1 dimensional
On the cross-section 2 dimensional

Which one is it, the smallest?

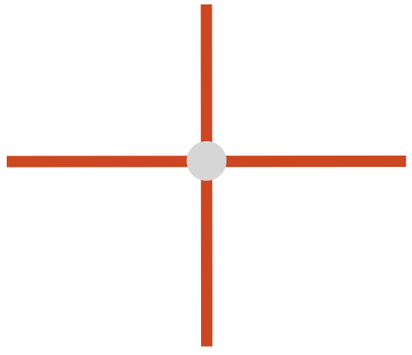
Then this space would be 0 dimensionaal, because a point = 0

Scaling phenomena

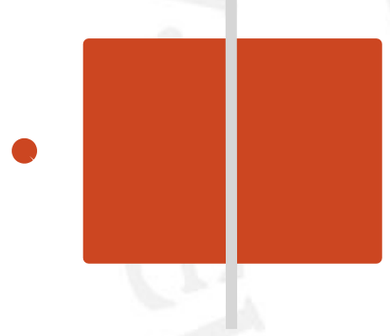
Measuring dimension

Topological Dimension

Introduces *local* and *global* dimension.
Global: Take the dimension of the object with which you can divide the space in two parts and add one.
Constraint: **The dimension must stay the same over linear transformations**



Point = 0 (+ 1)
1 dimensional



Local 0 and 2 dimensional
Take the highest
Global = 2 dimensional

Scaling phenomena

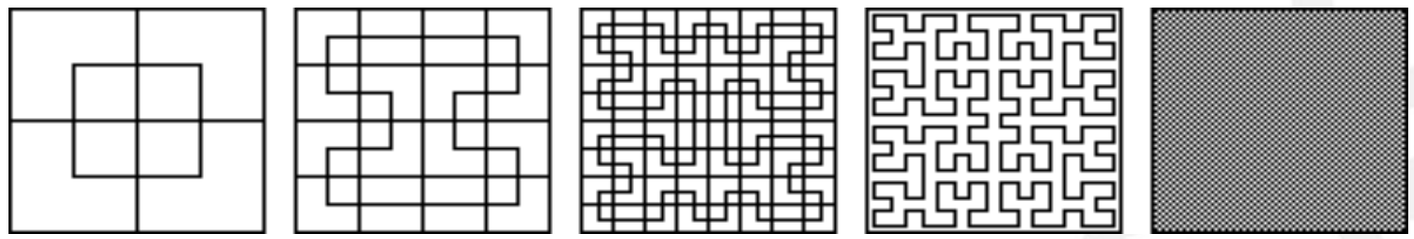
Measuring dimension

Constraint: **The dimension must stay the same over linear transformations**

Consider these linear transformations

Peano's Monster

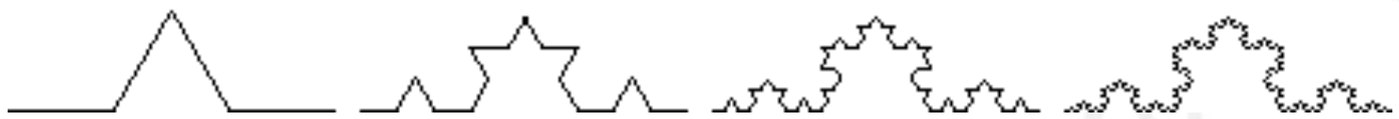
line-like



plane-like

Koch's Coastline

line-like



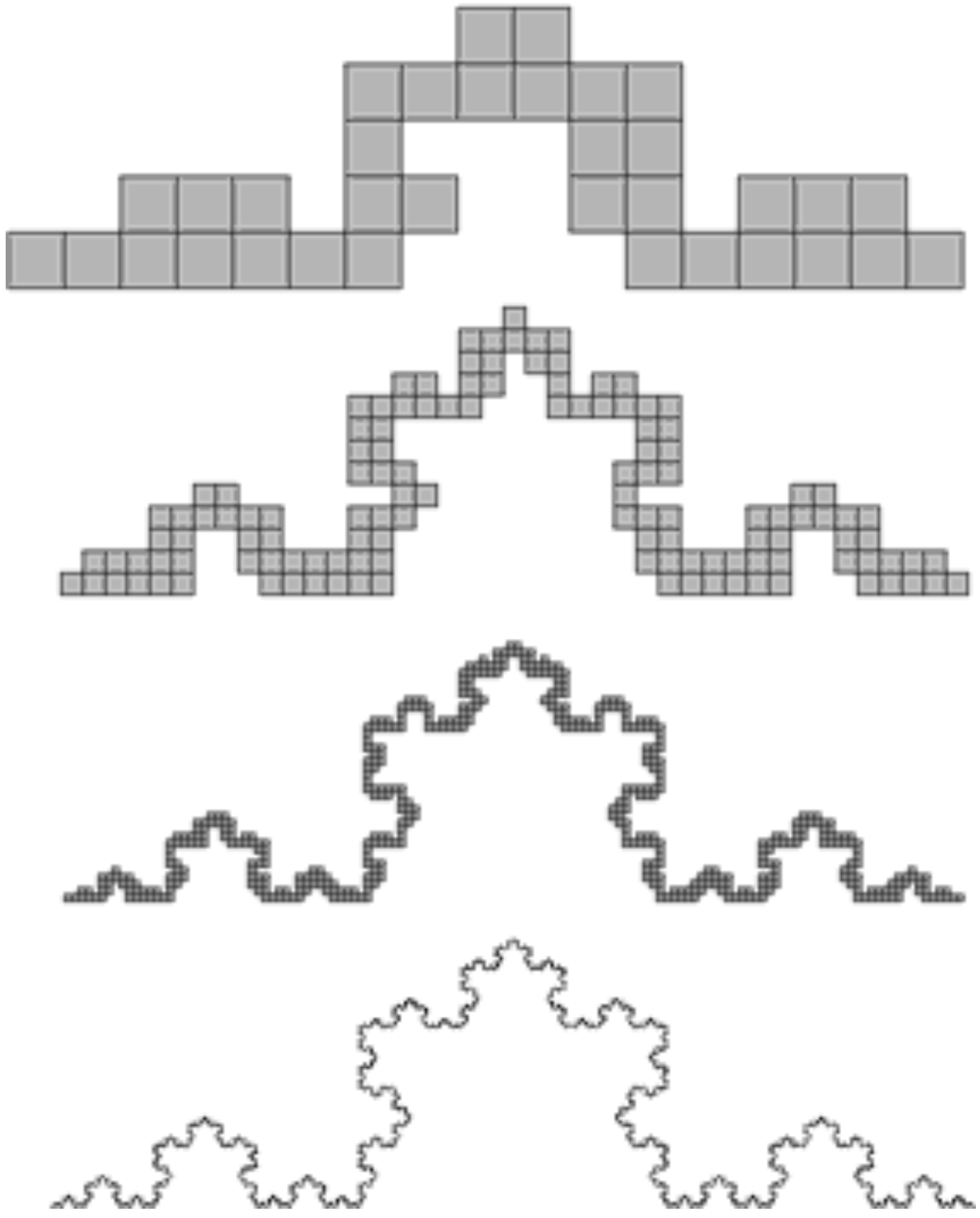
less line-like
(more complex)

Sierpinski's Gasket

plane-like

less plane-like
(more complex)

Solution:
Calculate a “fractional” dimension, e.g. box-counting dimension



Hausdorff-Besicovitch dimension

(box-counting dimension, covering dimension, packing dimension, mass-radius, circle-counting, etc, etc)

$$D = \frac{\log \mathbf{N_h}}{\log 1/\mathbf{h}}$$

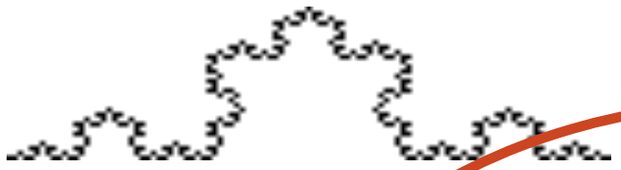
N= number of blocks of size **h** needed to cover the object

Relation between *measure stick* and *measurement outcome*, or:
“scaling of bulk with size”

Scaling phenomena

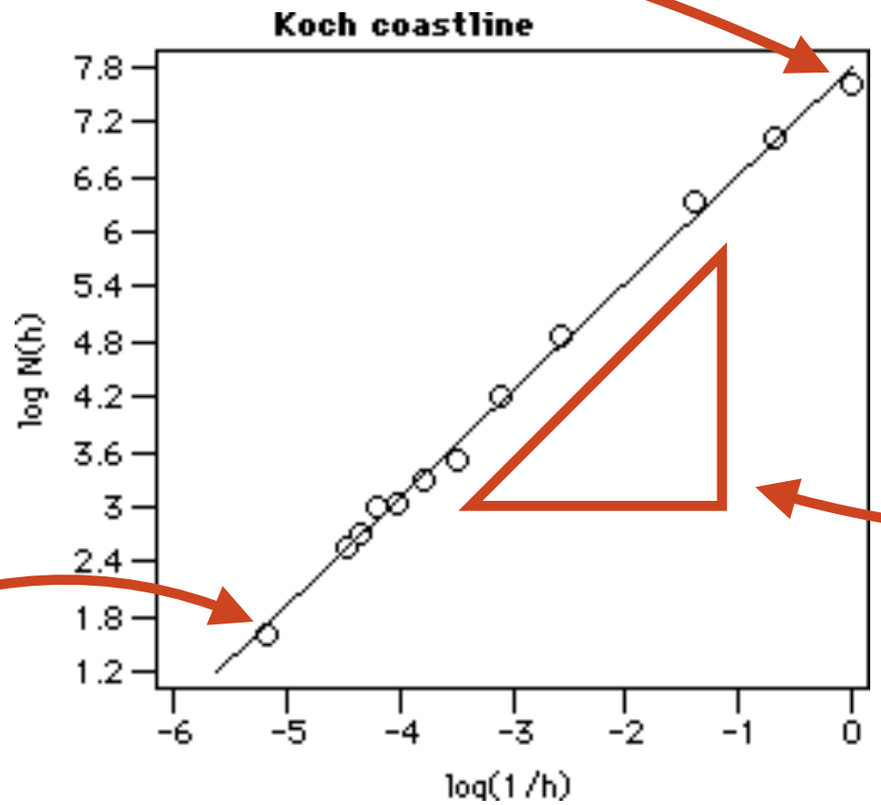
Measuring dimension

Koch Coastline



dimension (experimental) = **1.18**
dimension (analytical) = **1.26**
deviation = 6%

log (1/h)	log N(h)
0.00	7.61
-0.69	7.04
-1.39	6.33
-2.56	4.86
-3.09	4.22
-3.50	3.53
-3.78	3.30
-4.01	3.04
-4.19	3.00
-4.34	2.71
-4.48	2.56
-5.18	1.61

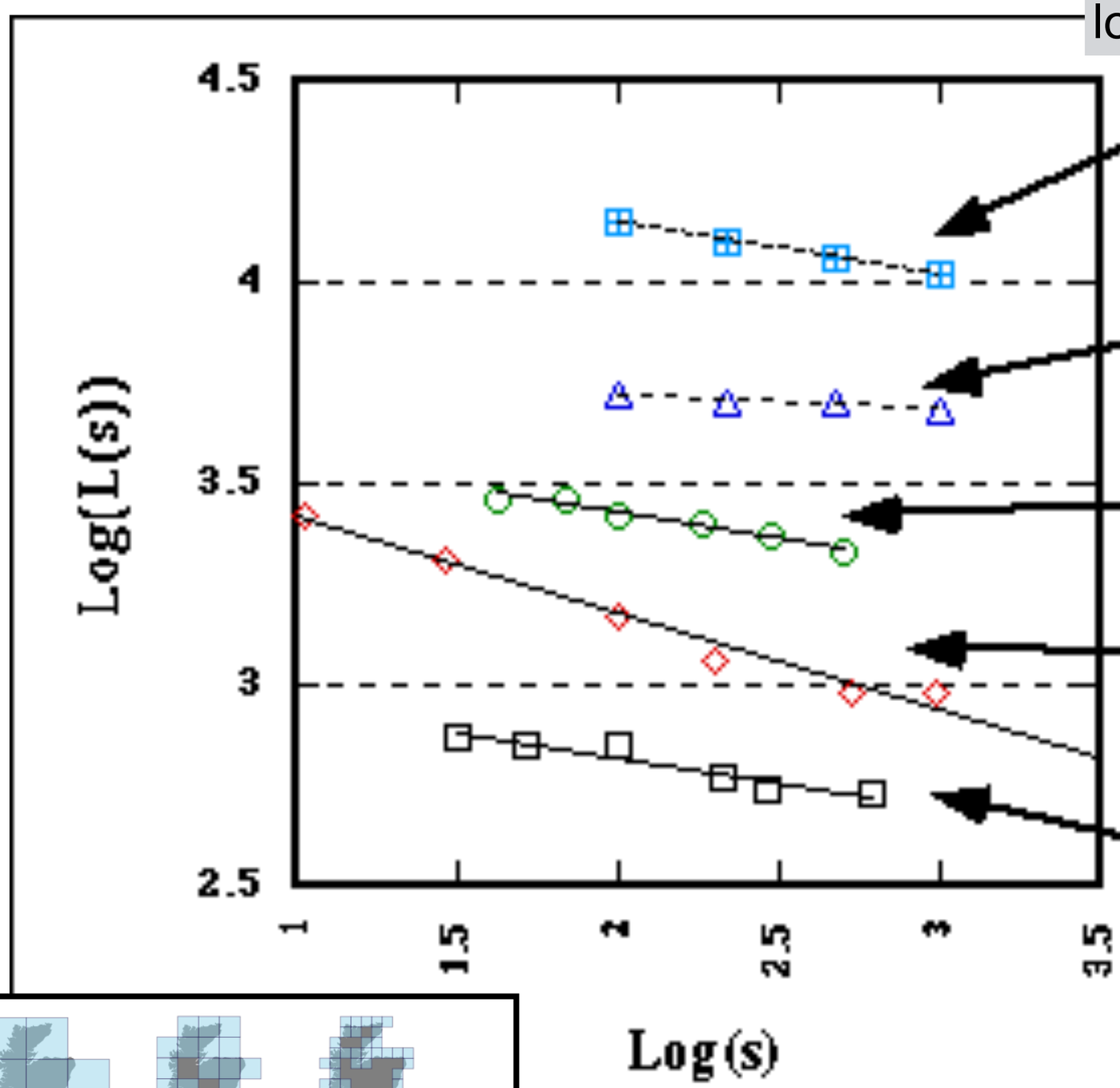



```
Call:
lm(formula = L ~ invS, data = df)

Residuals:
      Min        1Q    Median        3Q       Max
-0.18777 -0.06292  0.02390  0.06059  0.16703

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  7.79777    0.07318  106.55  < 2e-16 ***
invS         1.17611    0.02109   55.75 8.35e-14 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’
0.1 ‘ ’ 1

Residual standard error: 0.1137 on 10 degrees of
freedom
Multiple R-squared:  0.9968,    Adjusted R-squared:
0.9965
F-statistic:  3109 on 1 and 10 DF,  p-value: 8.355e-14
```

Fractal dimension
it's a fraction!

Scaling phenomena

log(**Length**$_{scale}$) = b*log(*scale*)+a



Australia

log(**Length**$_s$) = -.13*log(*s*)+4.4

South Africa

log(**Length**$_s$) = **-.03***log(s)+3.8

Germany

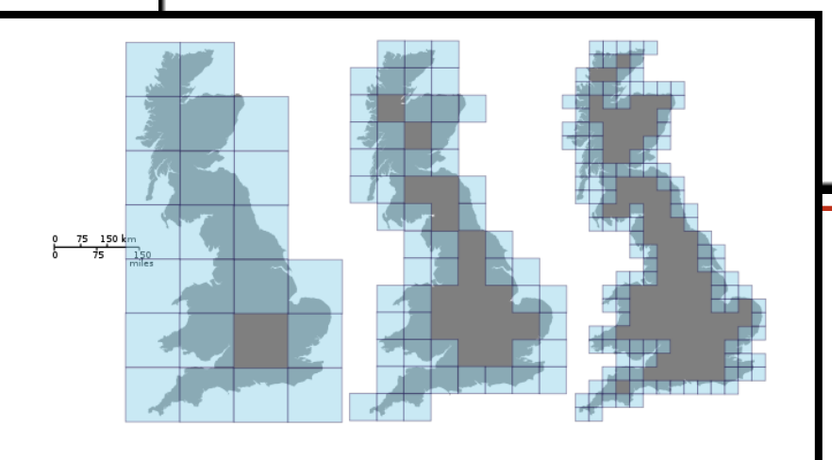
log(**Length**$_s$) = -.12*log(s)+3.7

Great Britain

log(**Length**$_s$) = **-.24***log(s)+3.4

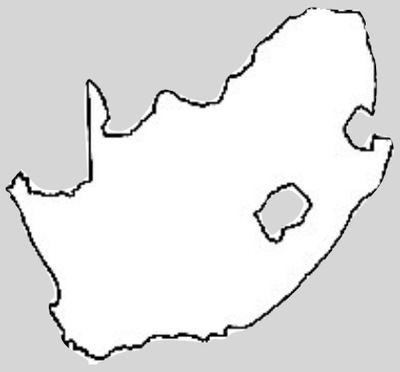
Portugal

log(**Length**$_s$) = -.12*log(s)+3.3



Scaling phenomena

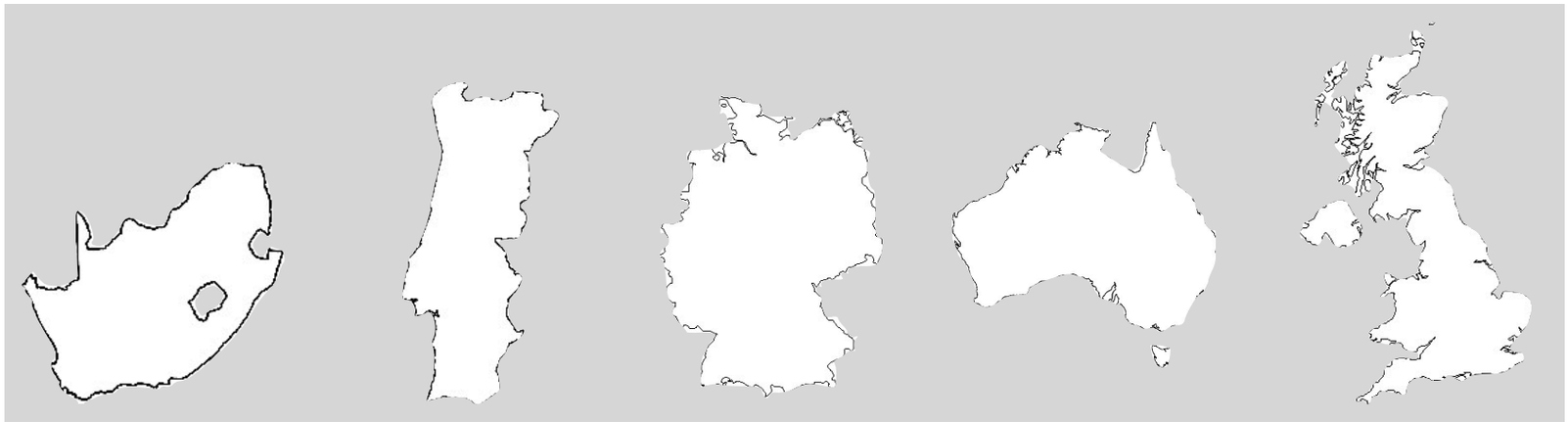
Scaling and Complexity

1.03	1.12	1.12	1.13	1.24
				
South Africa	Portugal	Germany	Australia	Great Britain

Ordered by scaling exponent, the log-log slope

Scaling phenomena

Scaling and Complexity



Ordered by scaling exponent, the log-log slope

Norway

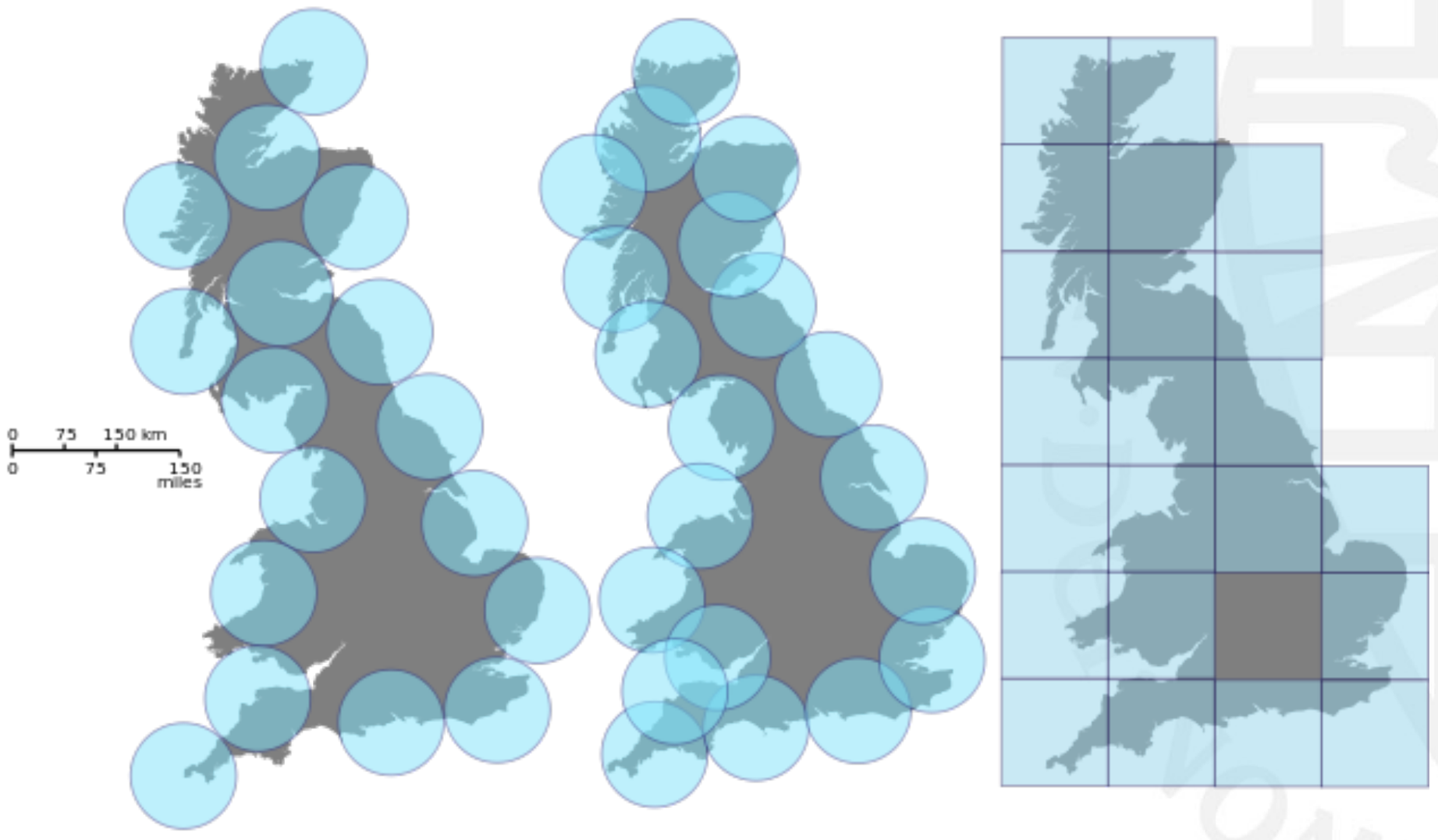
1.52

"one of the essential features of a fractal
is that its Hausdorff dimension strictly
exceeds its dimension"

https://en.wikipedia.org/wiki/List_of_fractals_by_Hausdorff_dimension

Scaling phenomena

Many variants of “covering” dimensions



ball packing < ball covering < box covering

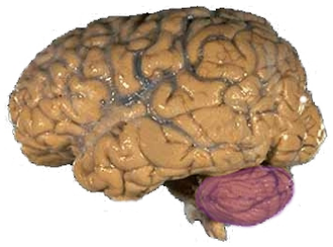
Scaling phenomena

Measuring dimension

Packing Cubes or Spheres and Wrapping Blankets:

3D spatial scaling relations in nature - Cauliflower
Fractal dimension = 2.33

Surface of human brain: 2.79

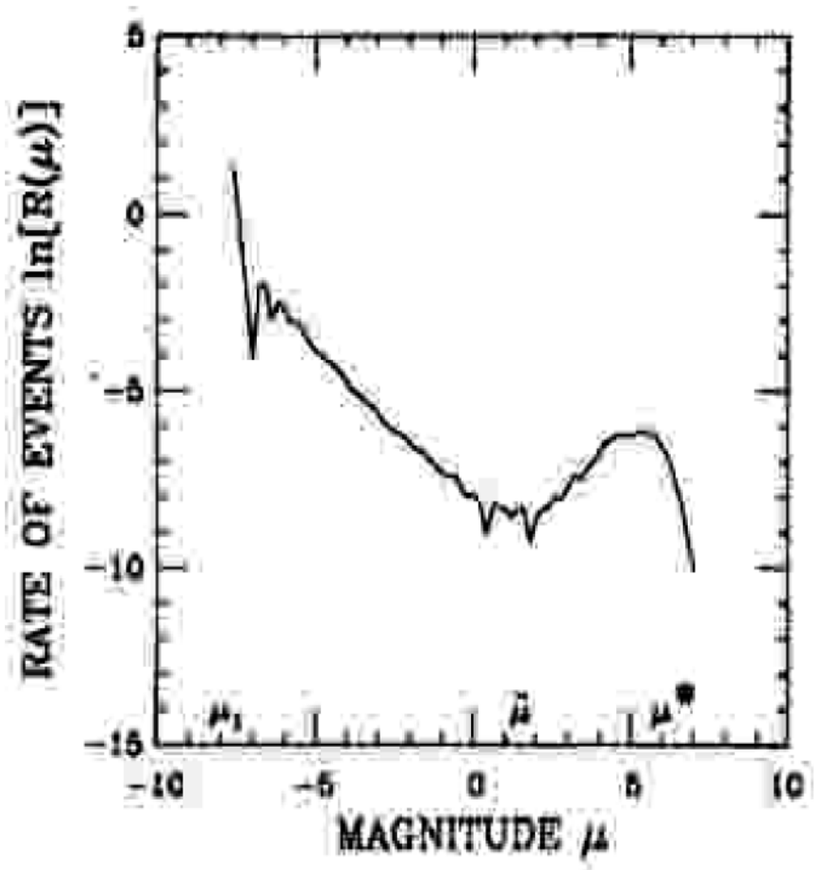


Surface of human lungs: 2.97

Scaling phenomena

Scaling relations are common in nature:

Earthquakes (Richter-Law) and the distribution of mass in the Universe



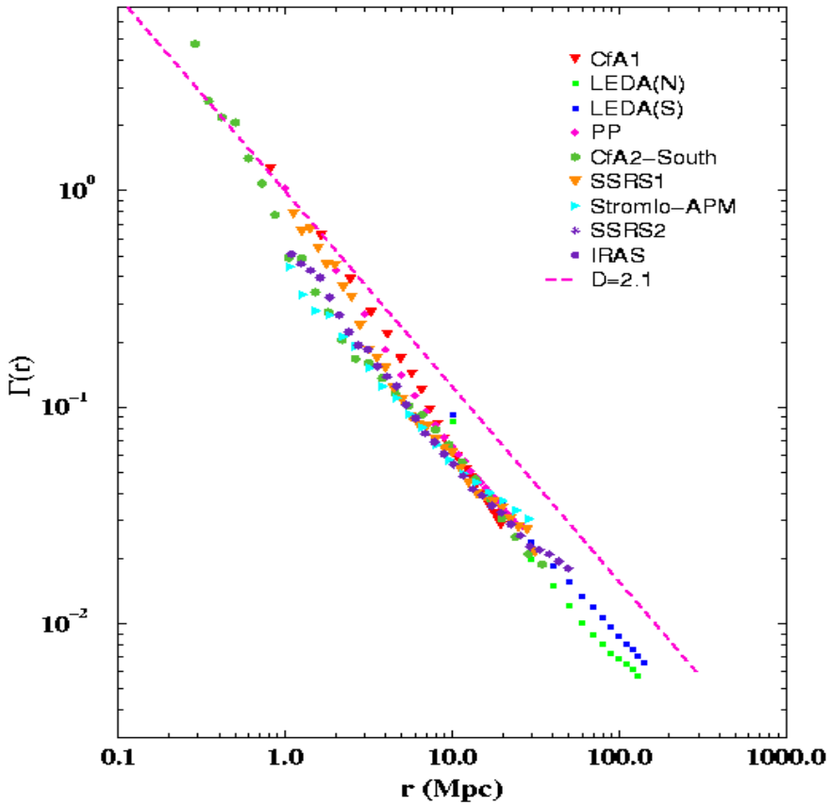
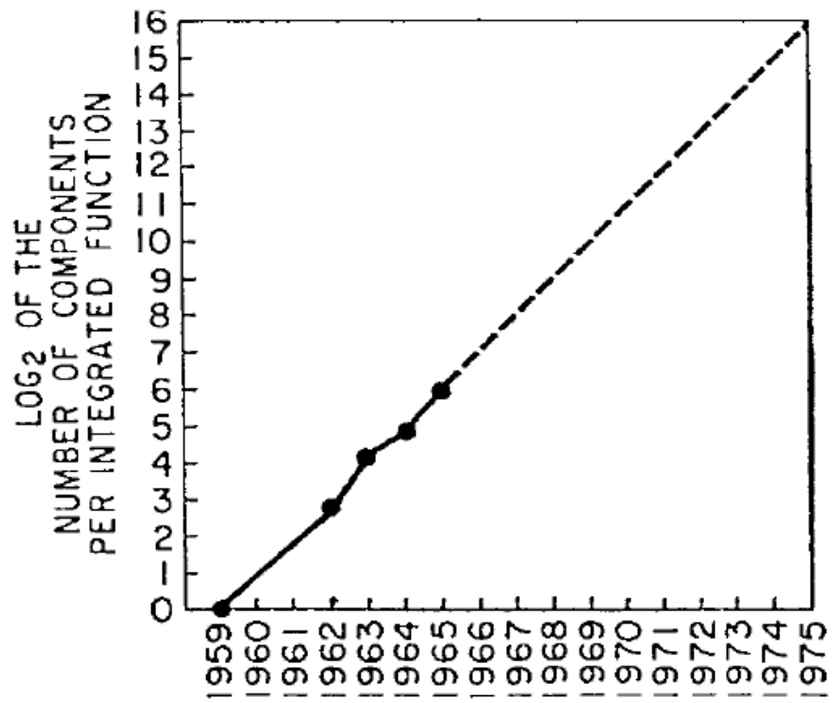


Figure 13: frequency distribution of the slip events (earthquakes) of magnitude μ taken from [53]. Notice the large bump that corresponds to an excess of events of high magnitude.

Scaling phenomena

Moore's Law:

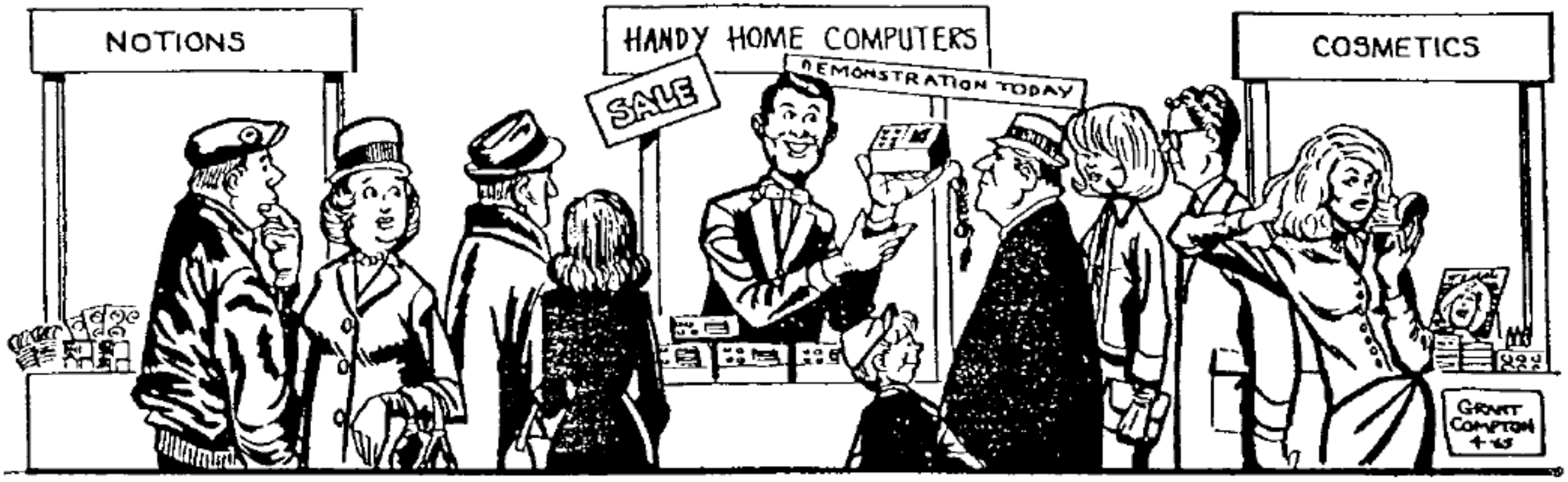
Predicted if speed of innovations in "cramming more components onto integrated circuits" kept up …



Moore's Law:

… we would soon be buying computers at the local market …

which apparently was a preposterous idea



Moore, Gordon E. (1965). "Cramming more components onto integrated circuits"

VANAF WOENSDAG **26-02** **ONZE AANBIEDINGEN**

Tablet PC cover

Universele tabletcover voor 10" tablets. Met ingebouwde standaard en microvezel binnenzijde. Blauw of zwart.

Per stuk **6.99***

GSM AT-B26D

- simlockvrije telefoon
- GSM 900/1800 MHz
- Dual-Sim
- micro-SD-lezer
- afmetingen: hoogte: 12.4 cm, breedte: 6.4 cm, dikte: 1.1 cm
- gewicht: 140 g (incl. accu)
- accu: 2100 mAh
- kleurenscherm
- 0.3MP-camera
- zaklampfunctie
- micro-USB

FM-RADIO

BLUETOOTH 2.1

OP=OP

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10" Tablet MD98516

- 10.1" HD-scherm (1280x800)
- Android 4.2
- 1.6 GHz Quad-core processor
- WiFi b/g/n
- 1 GB werkgeheugen
- mini HDMI
- micro-USB met host
- micro-SD-lezer tot 32 GB
- Bluetooth 2.1
- 2 camera's
- accuduur: tot 6 uur
- gewicht: 575 g
- afmetingen: 26x17x1 cm

16 GB GEHEUGEN

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DATAMAANDBUNDEL 750 €19.99 750 MB

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Wireless speaker adapter

Geef uw bestaande apparatuur streaming mogelijkheid. Sluit de adapter aan op uw stereo-installatie of de 30-pinversie op uw iPhone-dock en speel muziek draadloos af via Bluetooth.

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Ophangset voor lijsten, met o.a. diverse muurhaken, schroeven en spijkers.

Per set **2.99***

Hortensia

Veel bloemknoppen en kleurtonende of open bloemen. Blauw, roze/rood of wit.

Per stuk **4.79***

* Ondanks zorgvuldige planning kan het voorkomen dat actieartikelen door de grote vraag snel zijn uitverkocht – wij vr

ALDI

Aperitiefbiscuits

Ham/kaassoesjes, kaaswafelbolletjes of Gouda kaasbiscuits. 70-125 g

70-125 g **0.99*** 7.92-14.14/kg

Munt- of honingdrop

250 g

250 g **0.99*** 3.96/kg

Mini-stroopwafels

Bereid met echte roomboter. 300 g

300 g **1.89*** 6.30/kg

Chips patatje kapsalon of hete kip

250 g

250 g **0.99*** 3.96/kg

Basis voor soep

Tomaat, kip of runderlevers met groenten. 0.485 l

0.485 l **1.29*** 2.66/l

Spijsbroodjes of appelflappen

Bereid met echte roomboter. Banketspijs- of appelvulling. 220 g

220 g **1.19***

Smulgerechten

2-in-1-mix. Kebab met yoghurt-knoflooksaus gyros met paprikasaus of kipnuggets met BBQ-saus. Per pak

Kruidvat voordeel magazine

Goed gekleed

DAMESJASJE M t/m XL

true SPIRIT 39.99 **19.99**

Perfect haar

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STEEDS VERRASSEND, ALTIJD VOORDELIG!

Geldig van dinsdag 25 februari t/m zondag 9 maart 2014

Fluctuation Analyses

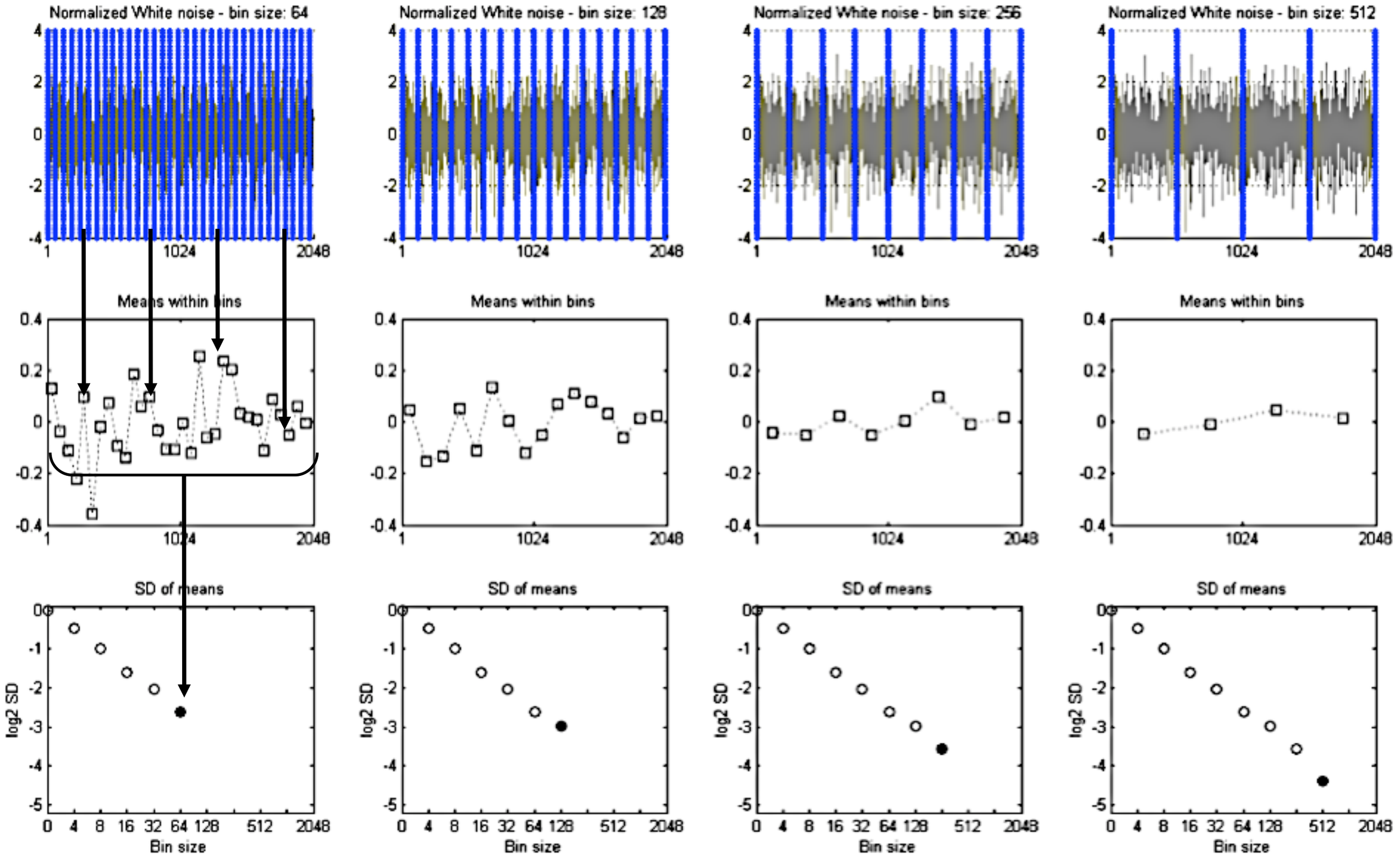
Standardised Dispersion Analysis

Detrended Fluctuation Analysis

Slope in log-log Power Spectral Density

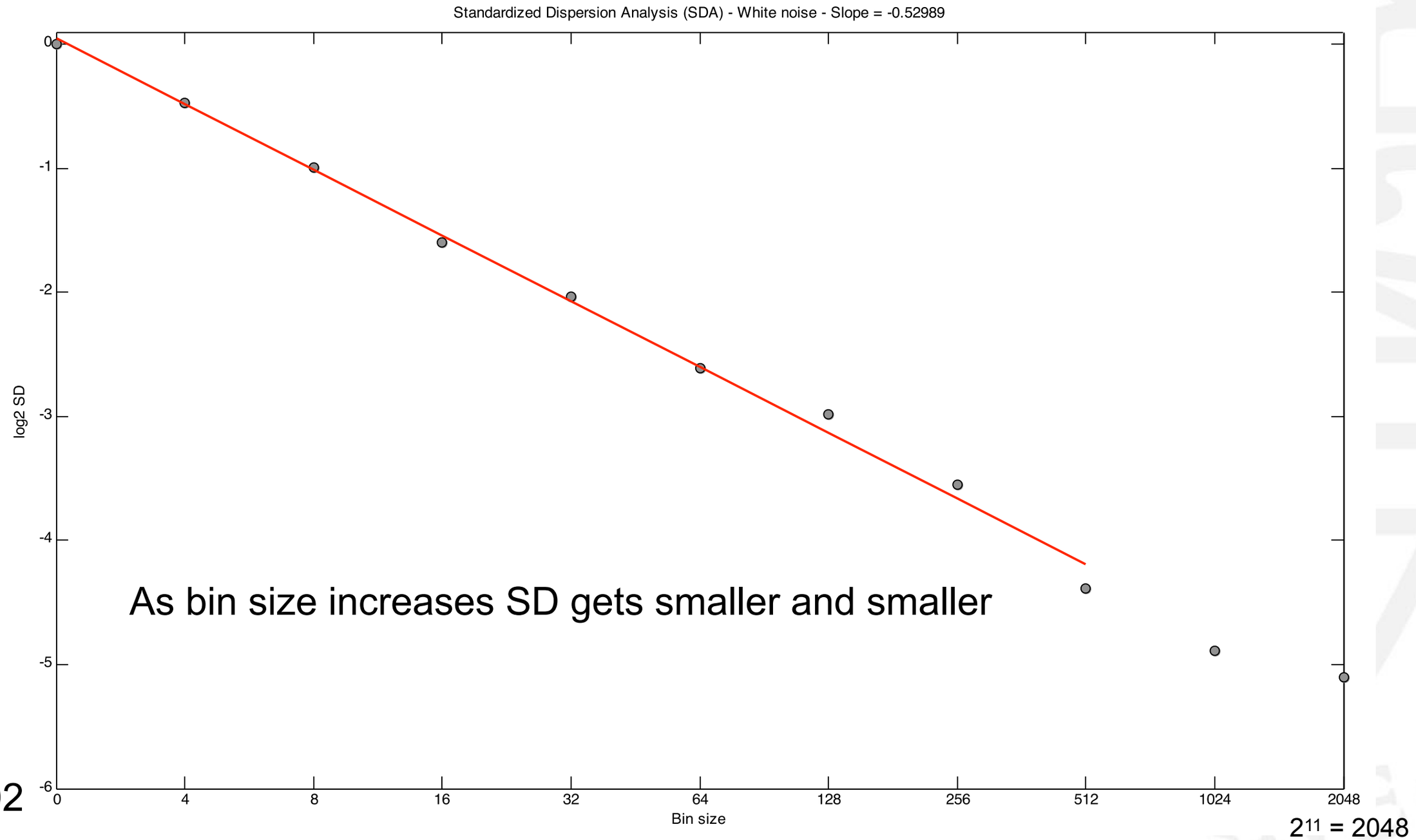
Standardised dispersion analysis
Scaling of “bulk” with “size”

- Divide TS in bins of size N
- Get some indication of variation
- repeat


Normalized White noise - bin size: 64
Normalized White noise - bin size: 128
Normalized White noise - bin size: 256
Normalized White noise - bin size: 512
Means within bins
SD of means
log2 SD
Bin size

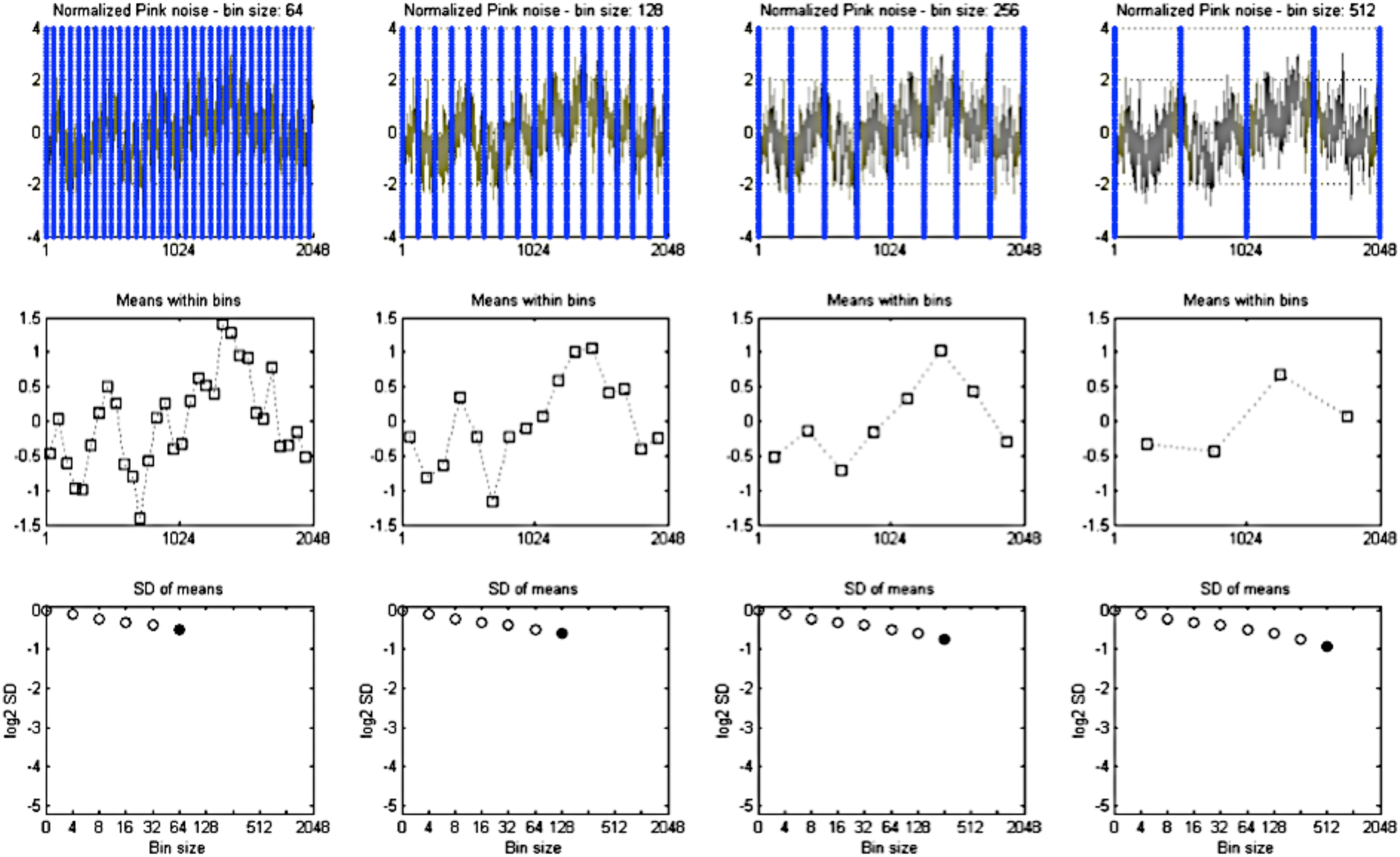
Fractal dimension = 1 – (slope of line)
FD = 1 - (-0.53) = 1.53

$2^0 = 1$

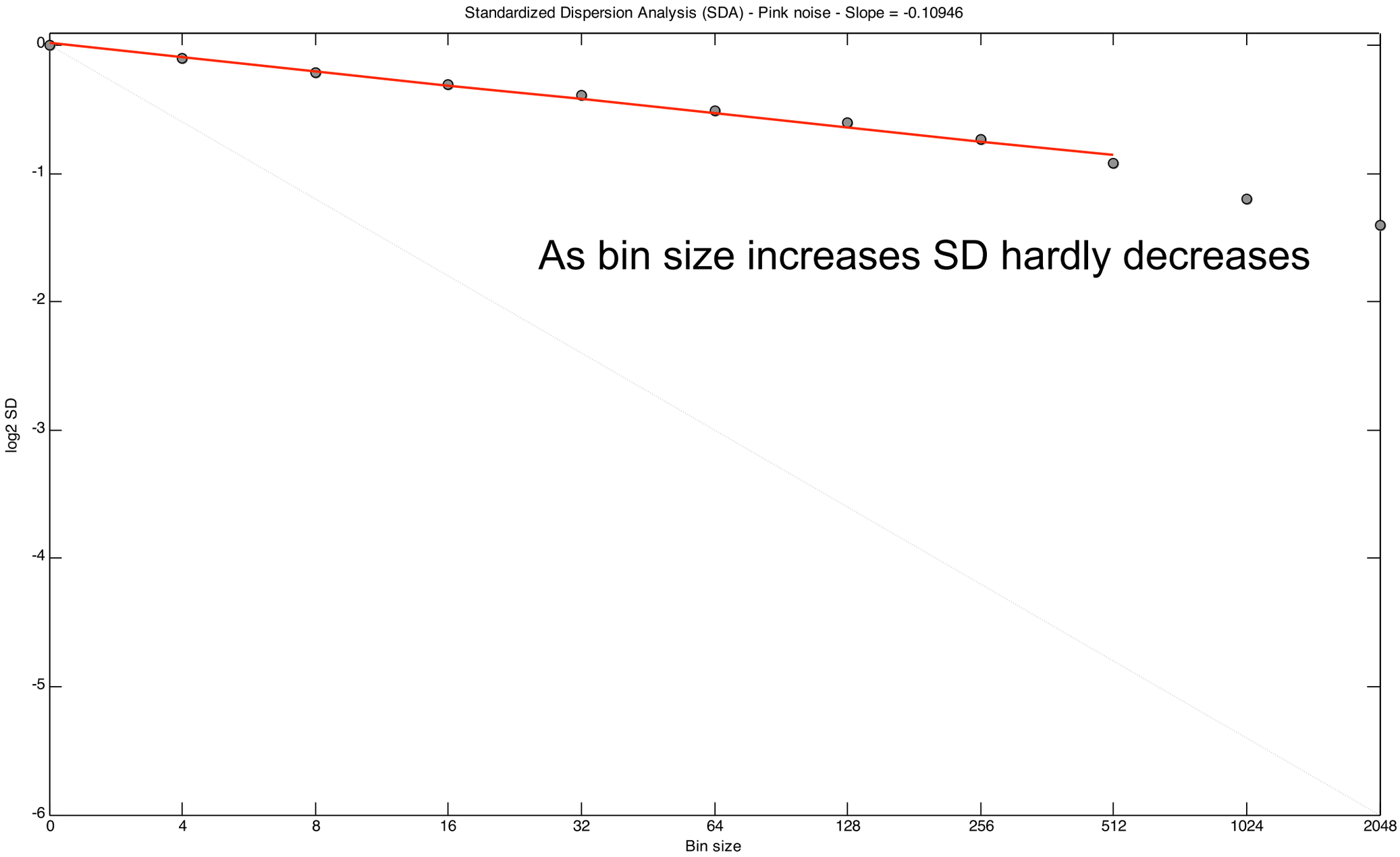


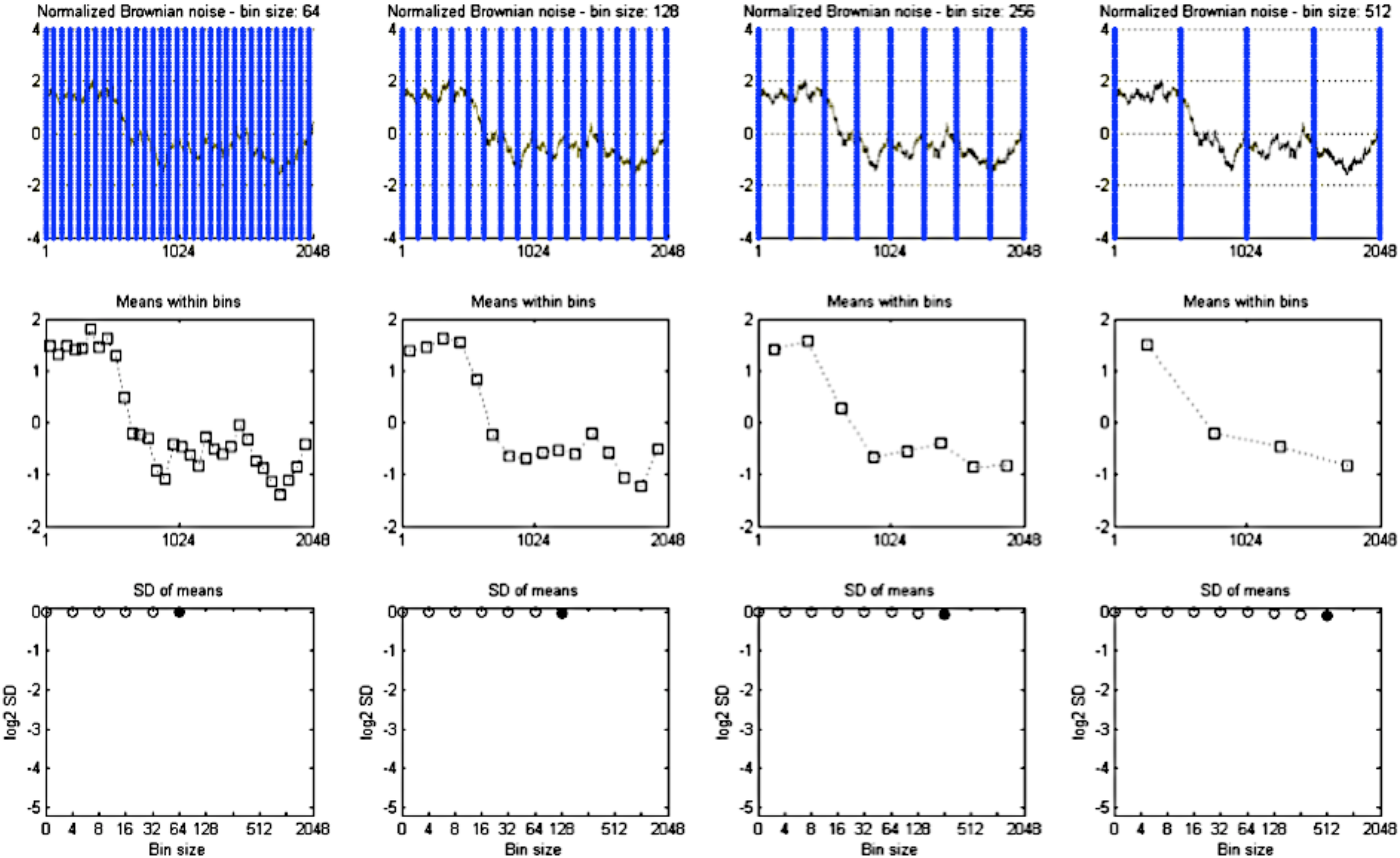
$2^{-6} = 0.02$

$2^{11} = 2048$


Normalized Pink noise - bin size: 64
Normalized Pink noise - bin size: 128
Normalized Pink noise - bin size: 256
Normalized Pink noise - bin size: 512
Means within bins
SD of means
log2 SD
Bin size

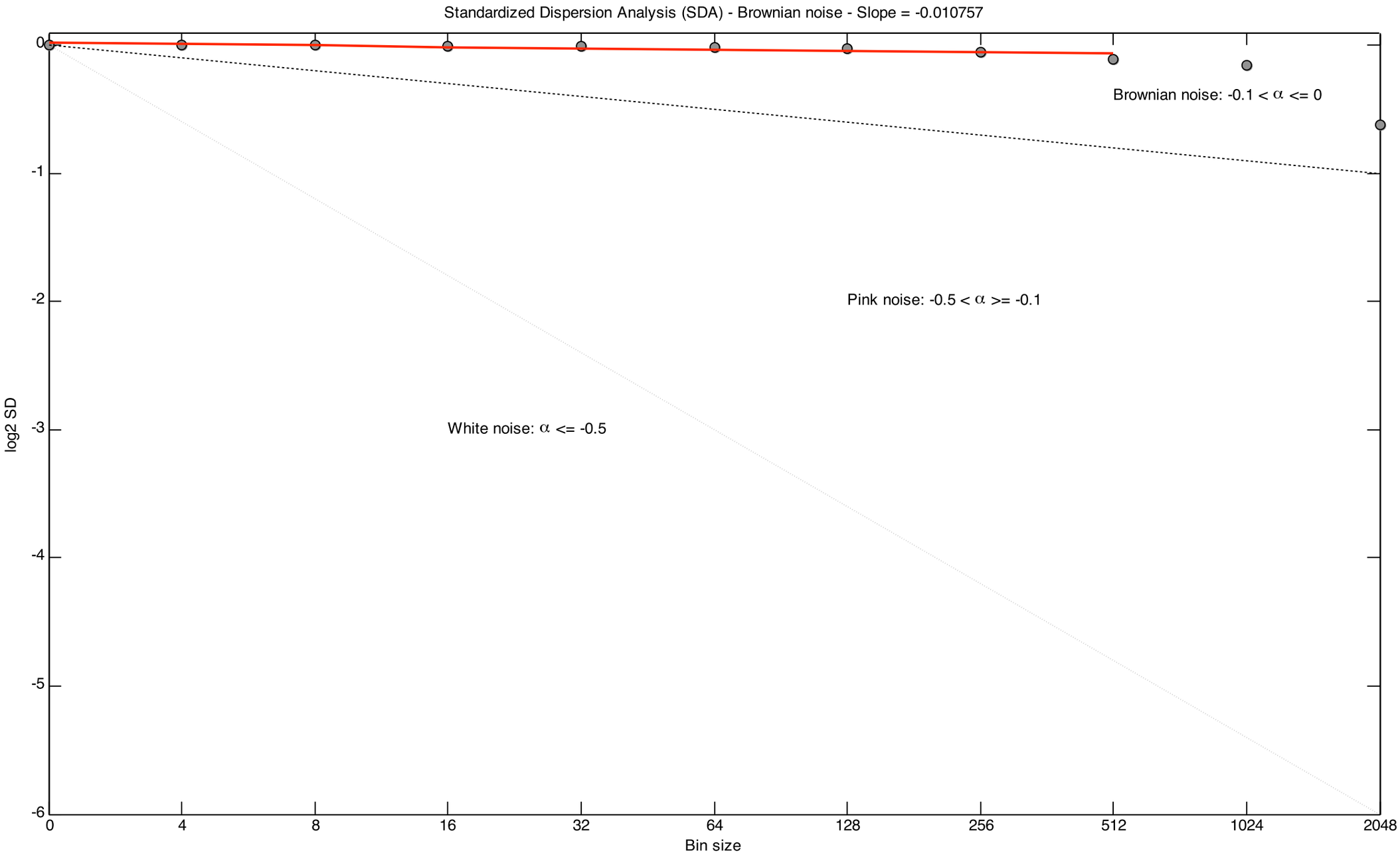
Fractal dimension = 1 – (slope of line)
FD = 1 - (-0.11) = 1.11

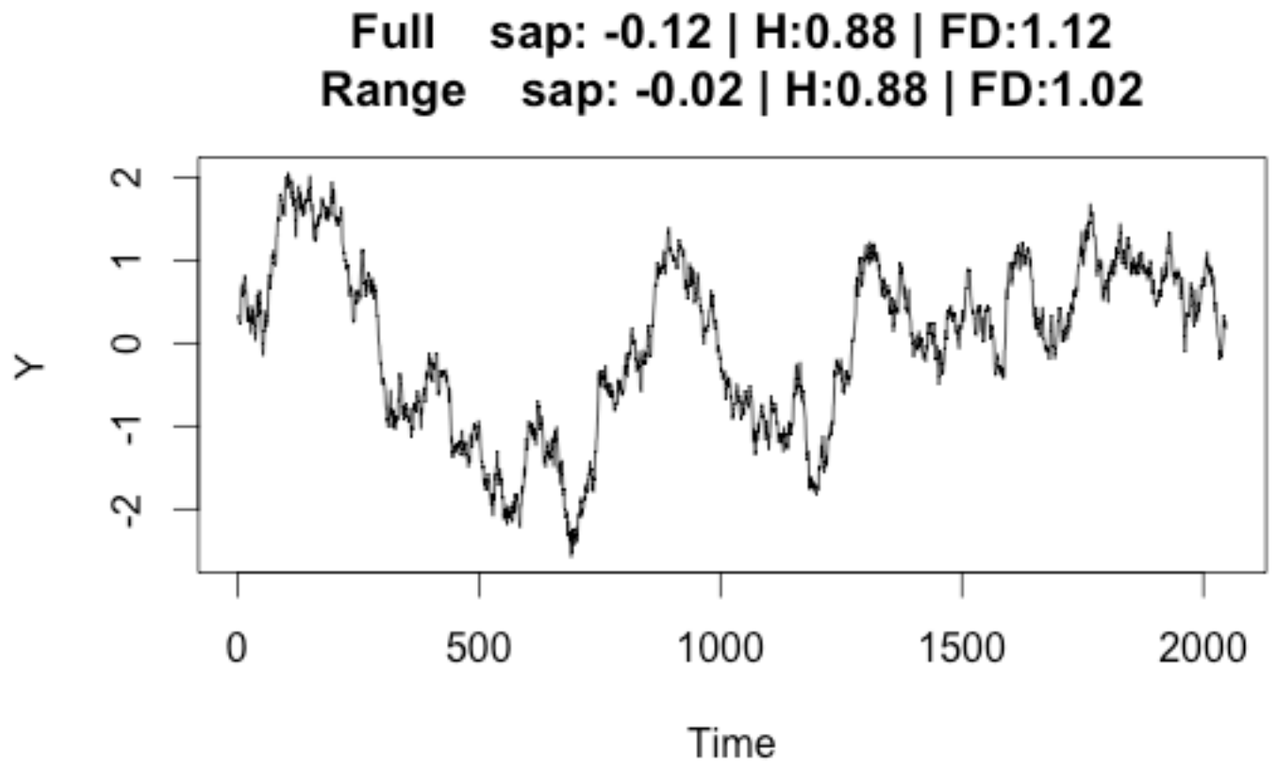


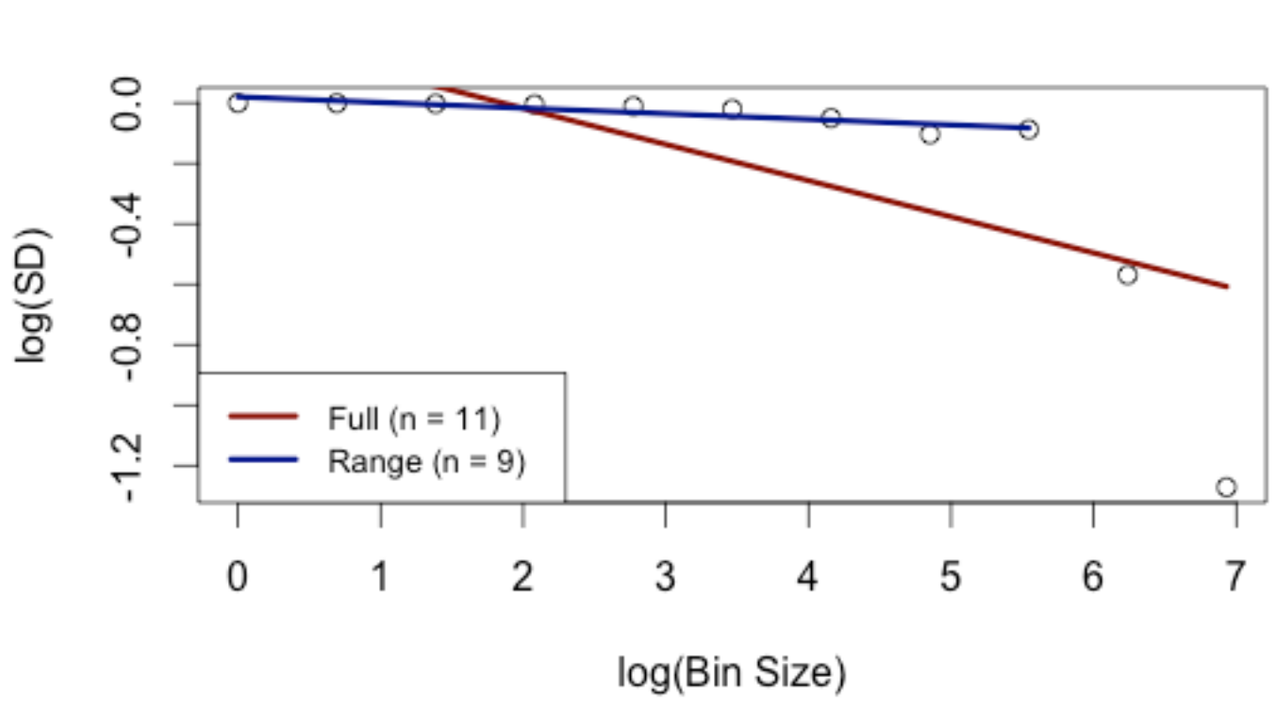


Radboud University Nijmegen

Fractal dimension = 1 – (slope of line)
FD = 1 - (-0.01) = 1.01




Full sap: -0.12 | H:0.88 | FD:1.12
Range sap: -0.02 | H:0.88 | FD:1.02
Y
2
1
0
-1
-2
0
500
1000
1500
2000
Time


log(SD)
0.0
-0.4
-0.8
-1.2
Full (n = 11)
Range (n = 9)
0
1
2
3
4
5
6
7
log(Bin Size)