About the Course…

- **Two 'parts':** *Simulation/modeling* and *Complexity measures/analyses*

- **Exam:** Take-home assignment. Finished 2 weeks after the last session.

- **Essential part of the course:** Read Chapters / Papers / watch videos, and ask questions on BB.Your response is not graded, but you need to post at least 1 question, or answer a posted question per session in order to get the final grade. Chapters are from "The End of Average" Articles / Videos will often be applications of analysis techniques. The articles / videos you have to comment on are given in the appropriate discussion board thread.

- **Literature:**
  - Lecture slides
  - Assignments
  - In addition, at the secretariat of PWO (5th floor, Spinoza building A.05.19) selected chapters from the book "Dynamical Psychology" by Jay Friedenberg are available. It is not necessary to own the book to complete this course, but if you can find a copy, it may help to structure all the information we provide.

  - **note:** You'll be prepared for the final assignment if you make sure you study the materials for the discussion assignments and do the analysis assignments each week.

About the course…. (continued)

•“book”: https://darwin.pwo.ru.nl/skunkworks/courseware/1718_DCS/

•assignments: https://darwin.pwo.ru.nl/skunkworks/courseware/1718_DCS/assignments/

• Most assignments about analyses were designed for **R**. You do **not** need to learn to script/code, you mainly need to be able to load data and run scripts. There will be instructions in lecture notes and assignment solutions that should

• **Goals:**
  • Read and understand papers that use a complex systems approach.
  • Simulate the basic models
  • Perform the basic analyses

• Contact us if you want to use a complex systems approach in your research!

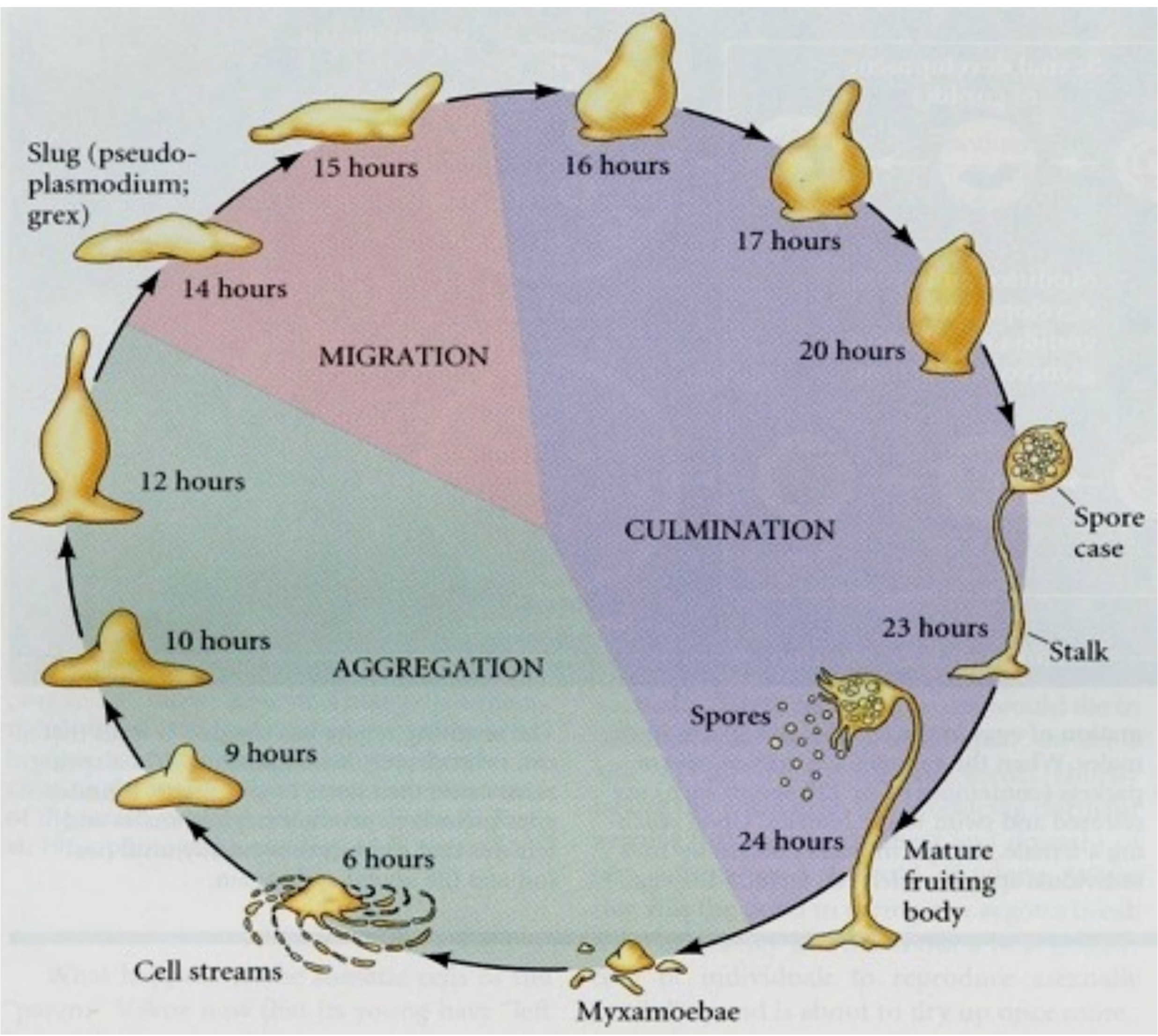
Dynamics of Complex Systems

Mathematics of Change

Simple (1D) Dynamical models

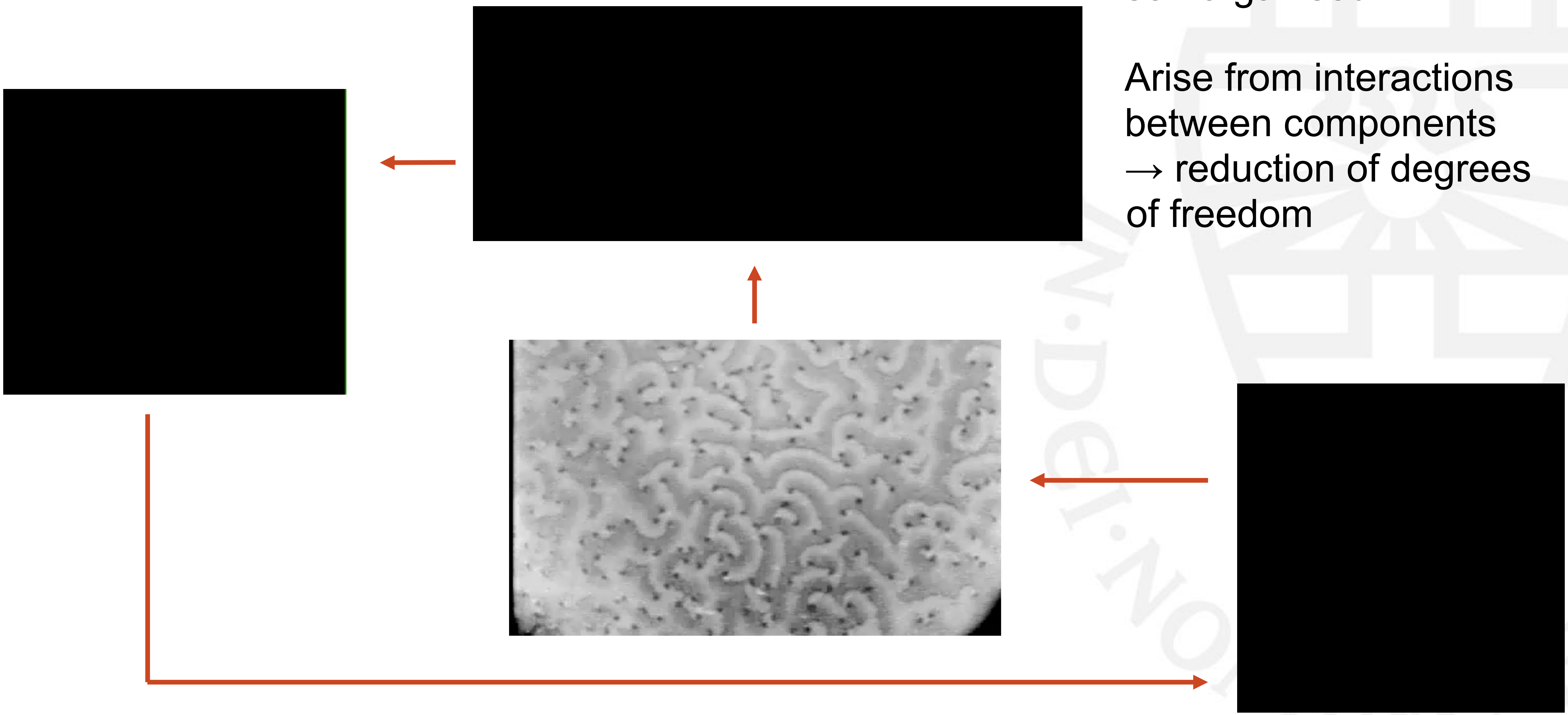
Notes on Deterministic Chaos

Emergence and Self-Organization: The life-cycle of *Dictyostelium*



1. Free living myxamoebae feed on bacteria and divide by fission.
2. When food is exhausted they aggregate to form a mound, then a multicellular slug.
3. Slug migrates towards heat and light.
4. Differentiation then ensues forming a fruiting body, containing spores.
5. It all takes just 24 hrs.
6. Released spores form new amoebae.

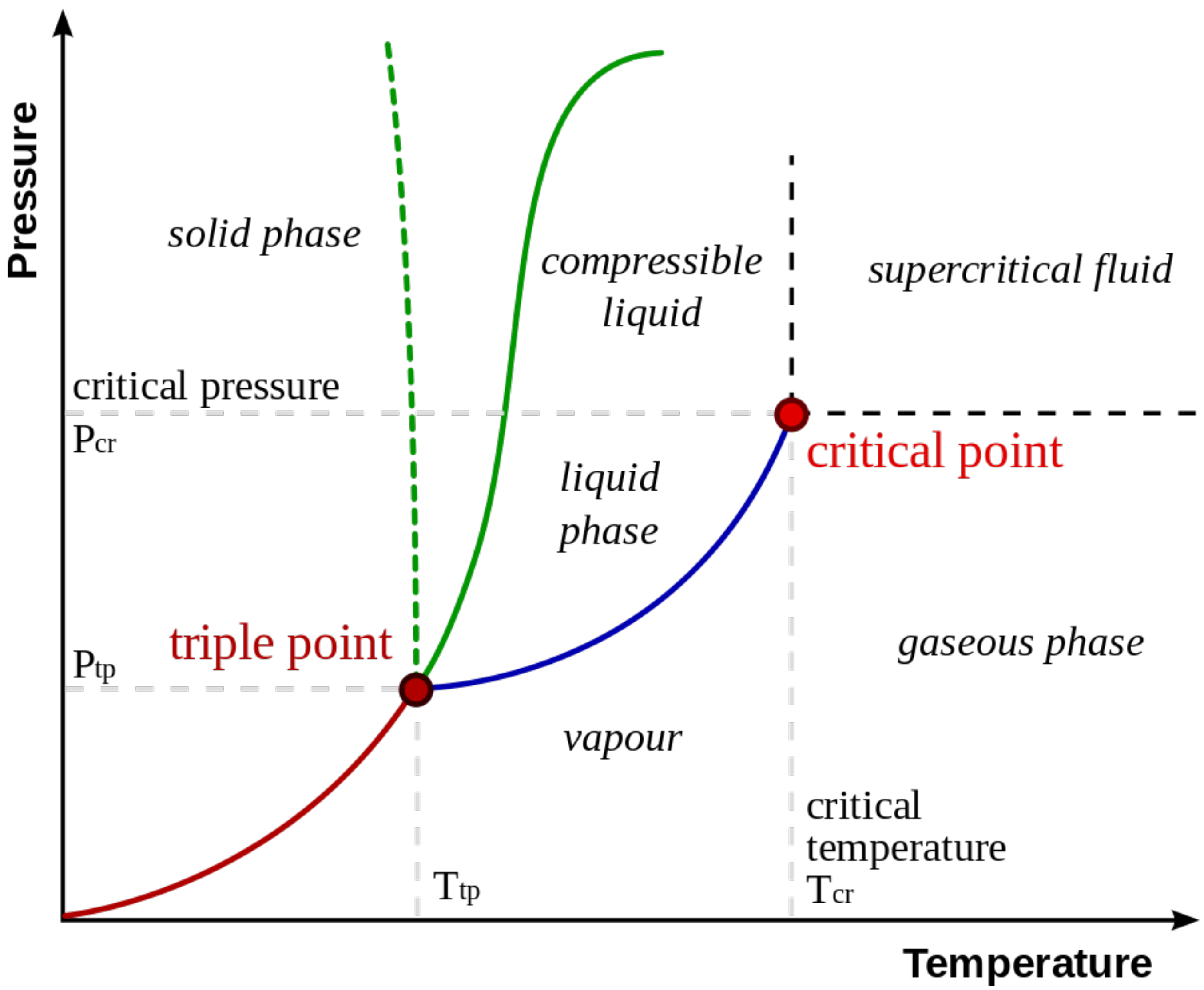
Order parameter: Labelling states of a complex system

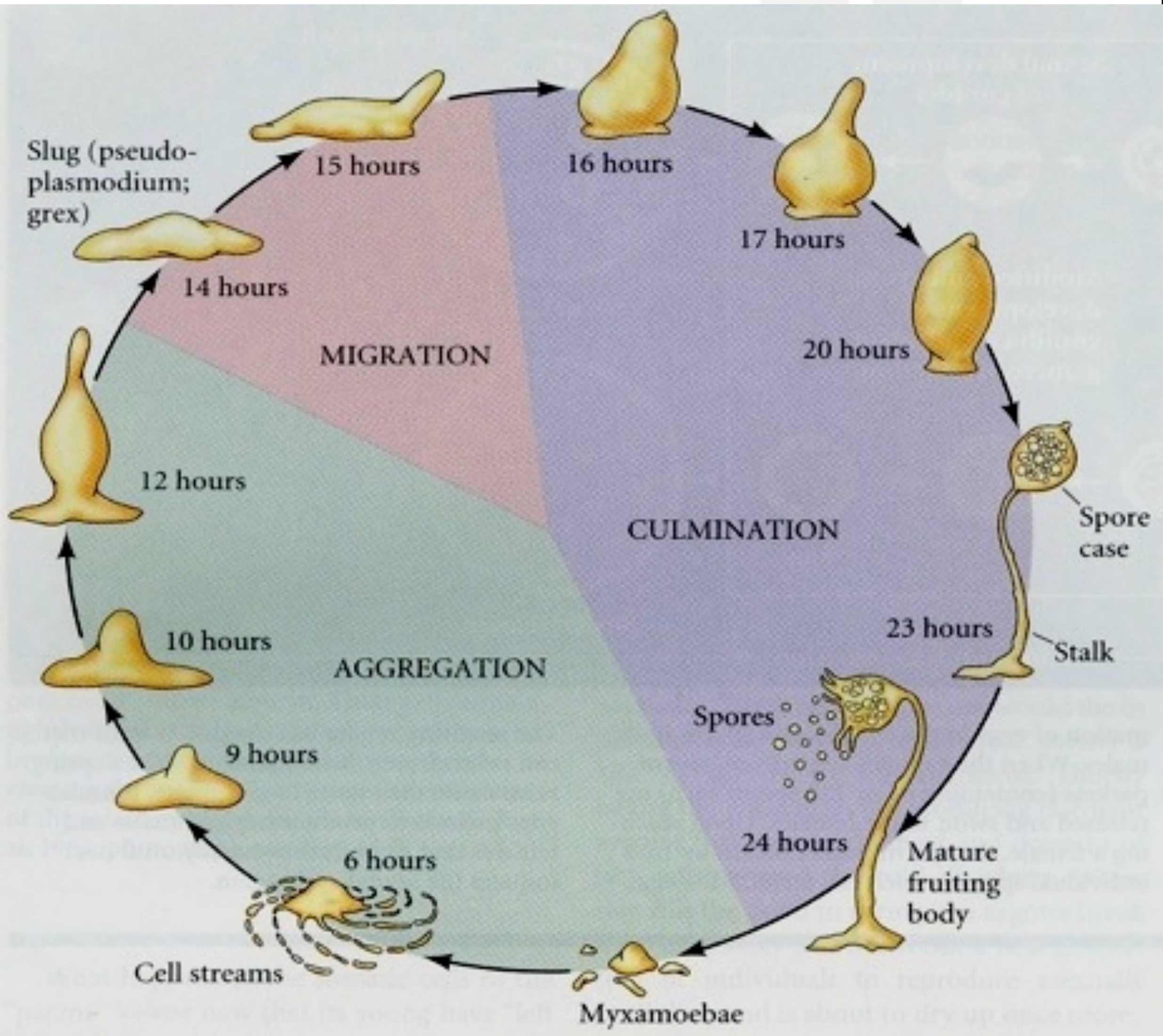


Forms are emergent, self-organised:

Arise from interactions between components → reduction of degrees of freedom

Phase Diagram & Order parameter





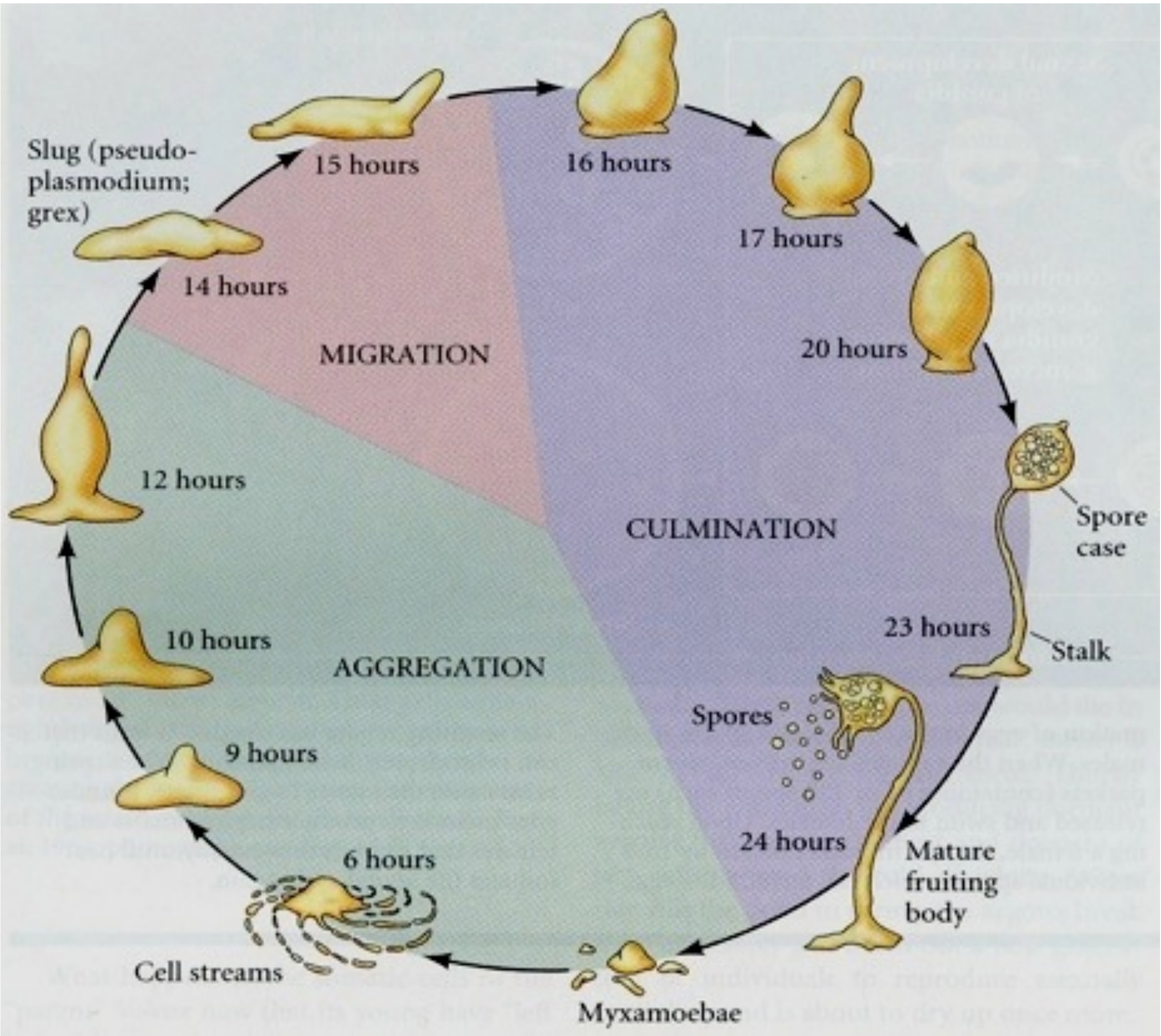
The order parameter is often a qualitative description of a macro state / global organisation of the system, conditional on the control parameters:

H_2O: Ice (Solid), Water (Liquid), Steam (Vapour)

Disctyostelium: Aggregation (Mound), Migration (Slug), Culmination (Fruiting Body)

Dynamic Metaphor vs. Dynamic Measure

Metaphor: Sate Space / Order Parameter
Measures: Attractor strength / Stability



Order parameter: the qualitatively different states

Control parameter: available food (actually concentration of a chemical that is released if they are starving)

Experiments:
Find out if the process is reversible... add food

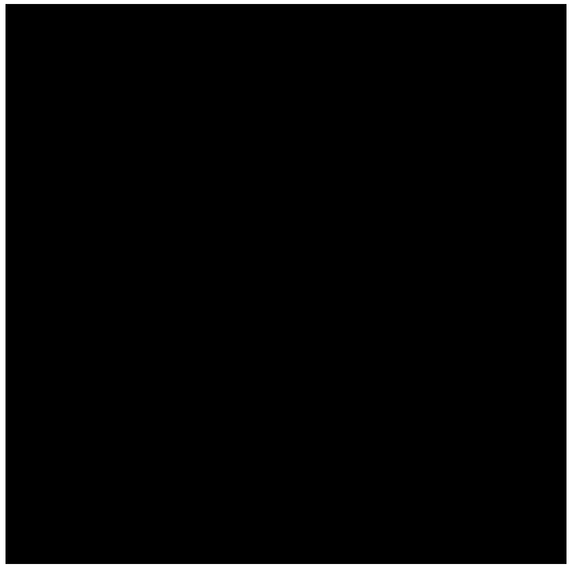
perturb the system during the various phases...

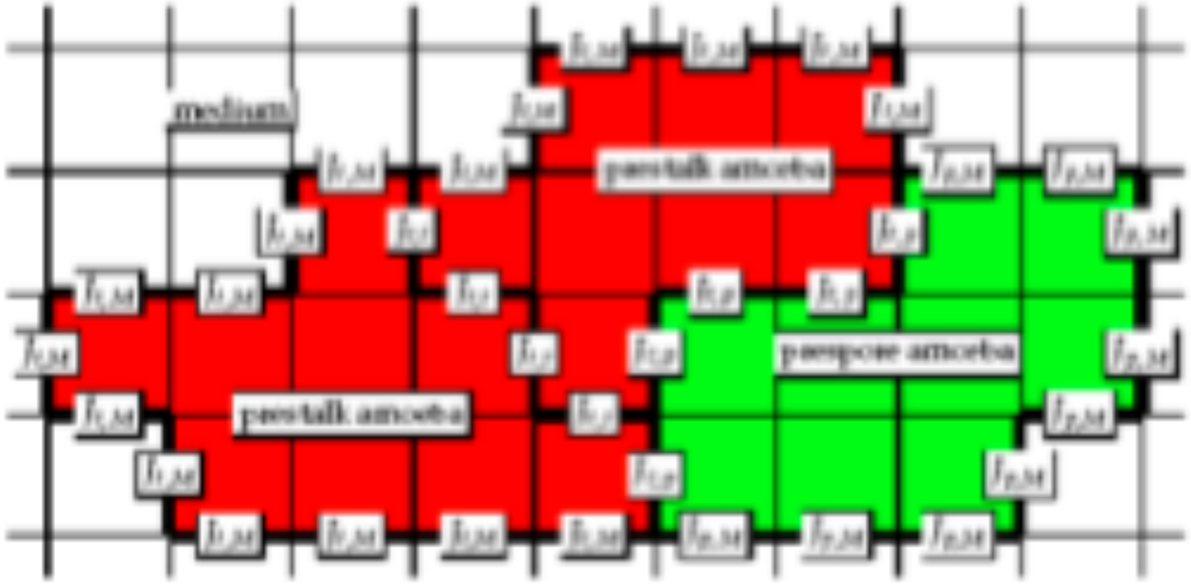
the degrees of freedom of the individual components are increasingly constrained by the interaction:

free living amoebae... slug... immovable sporing pod

nb State space and Phase Space (or: Diagram) are different concepts, but often used interchangeably to describe a State Space… see slide 18

From Pattern Formation to Morphogenesis
Multicellular Coordination in *Dictyostelium Discoideum*
A.F.M. Marée (2000). Proefschrift, UU.





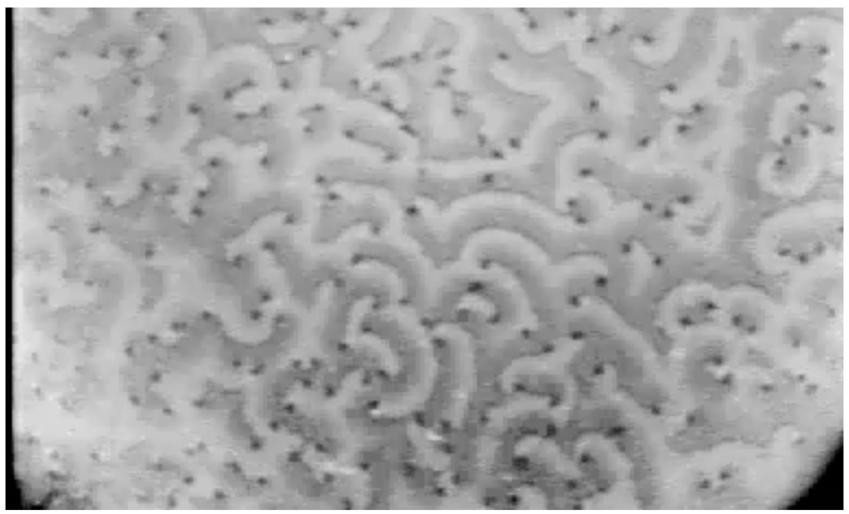
Two-Scale Cellular Automata with Differential Adhesion

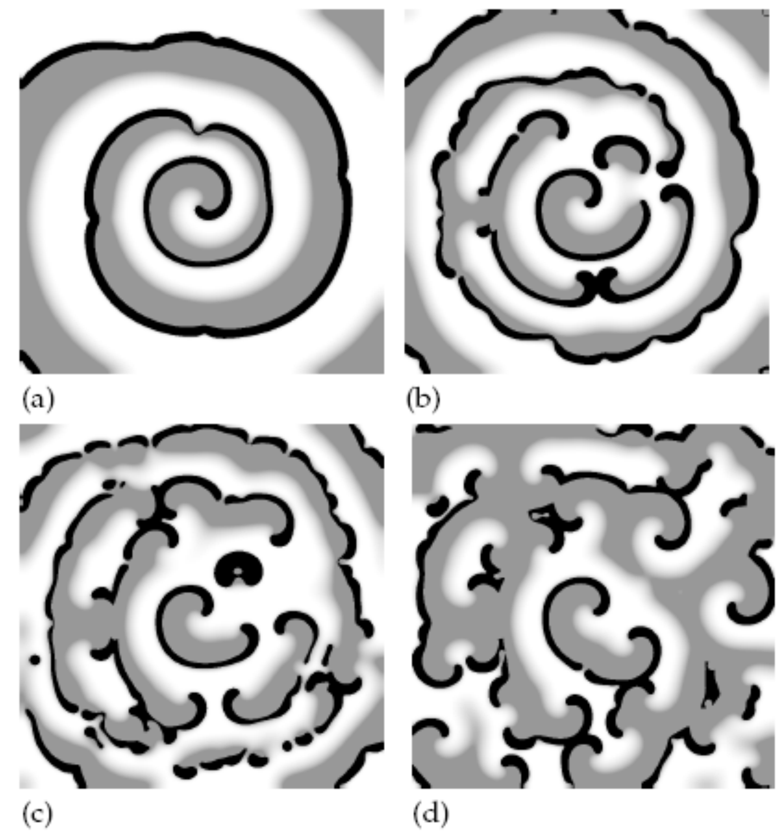
$$H_\sigma = \sum_{\text{all } \sigma,\sigma' \text{ neighbours}} \frac{J_{\tau_\sigma,\tau_{\sigma'}}}{2} + \sum_{\text{all } \sigma, \text{medium neighbours}} J_{\tau_\sigma,\tau_{\text{medium}}} + \lambda(v_\sigma - V)^2, \quad (1.1)$$

Mathematical model of Dictyostelium

Spiral Breakup in Excitable Tissue due to Lateral Instability

Marée, A. F. M., & Panlov, A.V. (1997). *Physical Review Letters, 78*,1819-1822.





$$
\begin{aligned}
\frac{\partial e}{\partial t} &= \Delta e - f(e) - g, \\
\frac{\partial g}{\partial t} &= D_g \Delta g + \varepsilon(e,g)(ke - g),
\end{aligned}
$$

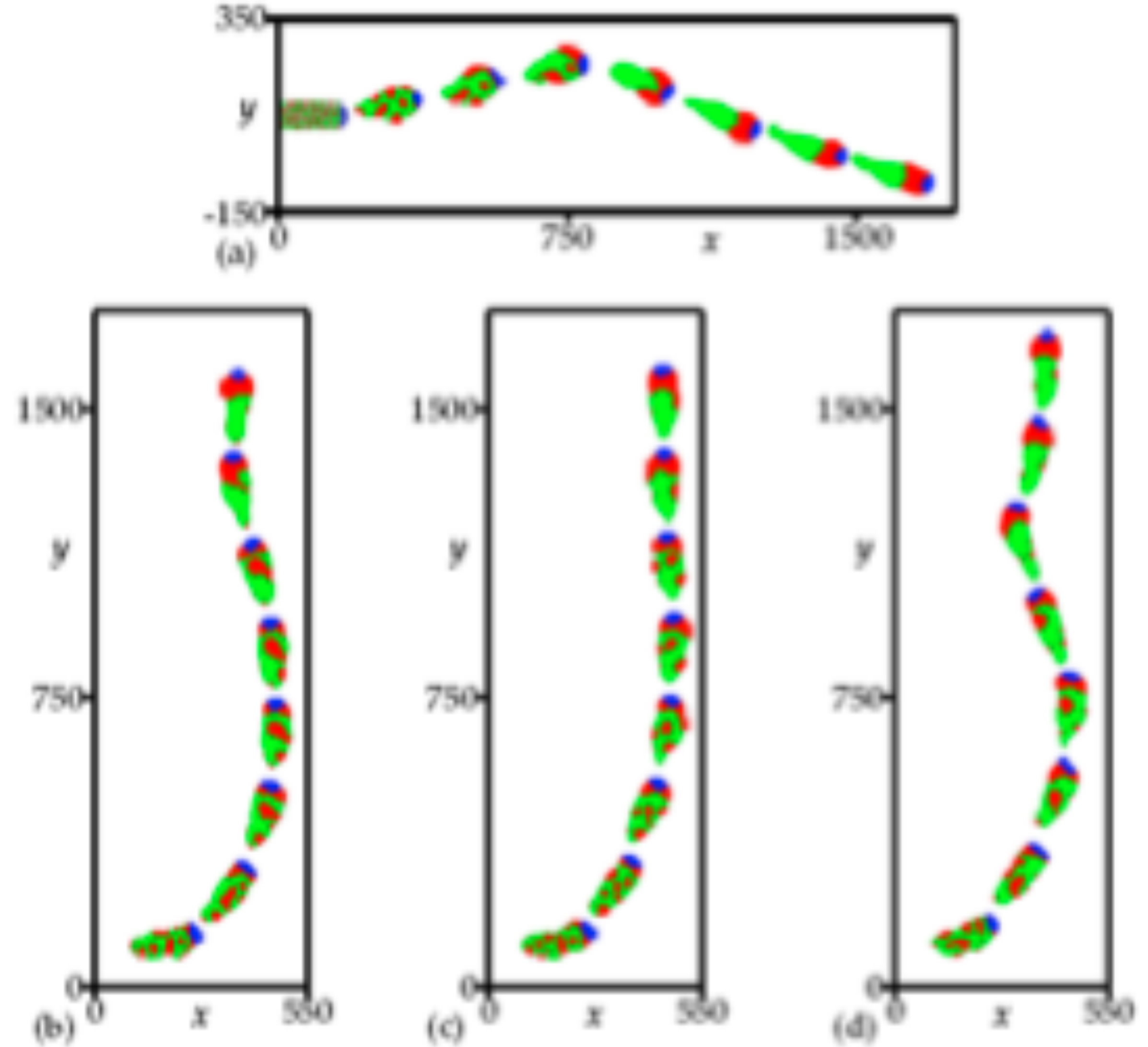
Mathematical model of Dictyostelium

$$H_\sigma = \sum \frac{J_{\text{cell,cell}}}{2} + \sum J_{\text{cell,medium}} + \lambda(v - V)^2,$$

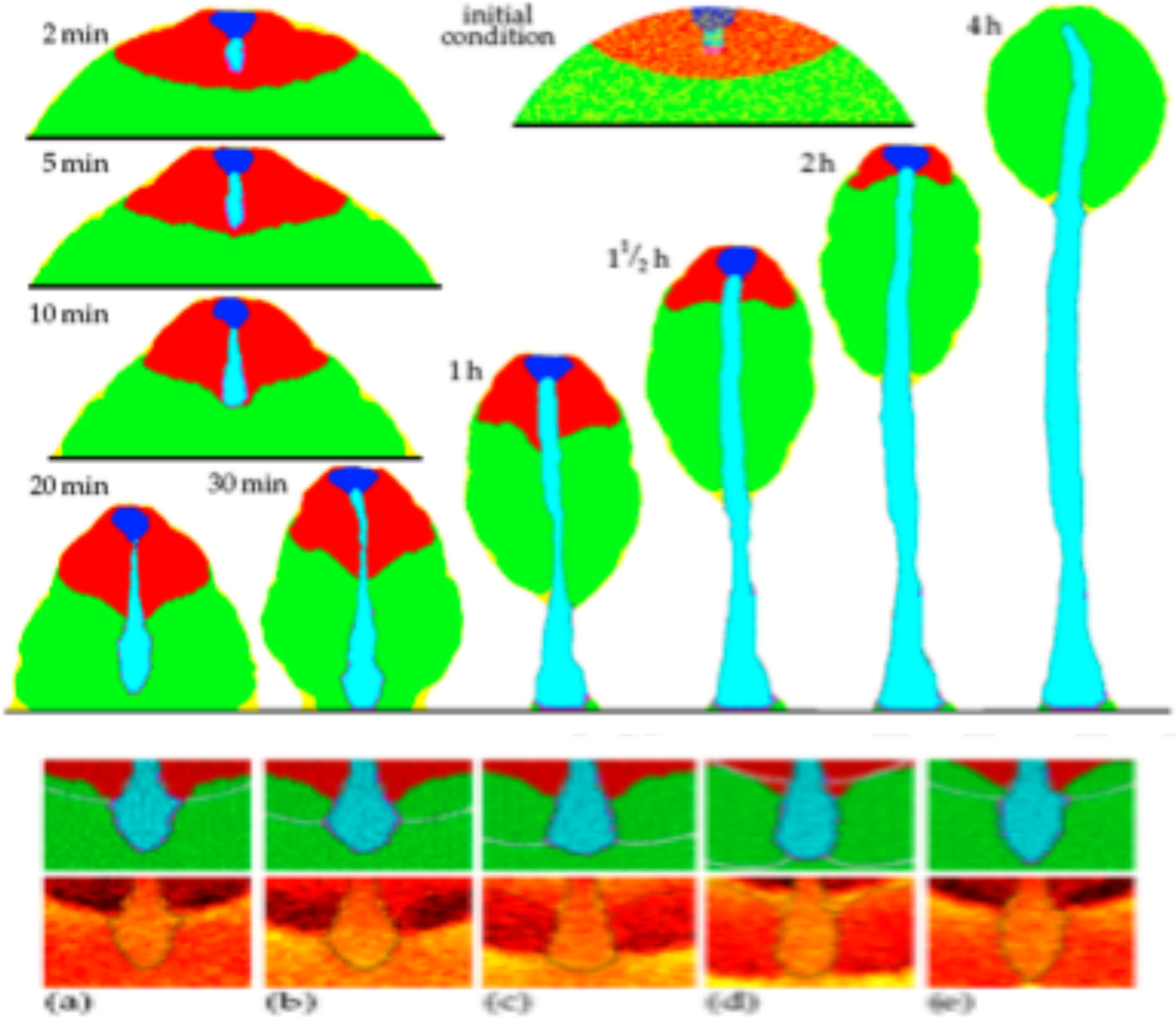
$$\left.\begin{aligned} \frac{\partial c}{\partial t} &= D_c \Delta c - f(c) - r, \\ \frac{\partial r}{\partial t} &= \varepsilon(c)(kc - r), \end{aligned}\right\} \text{ inside the amoebae}$$

$$\left.\frac{\partial c}{\partial t} = D_c \Delta c - d_c(c - c_0),\right\} \text{ outside the amoebae}$$

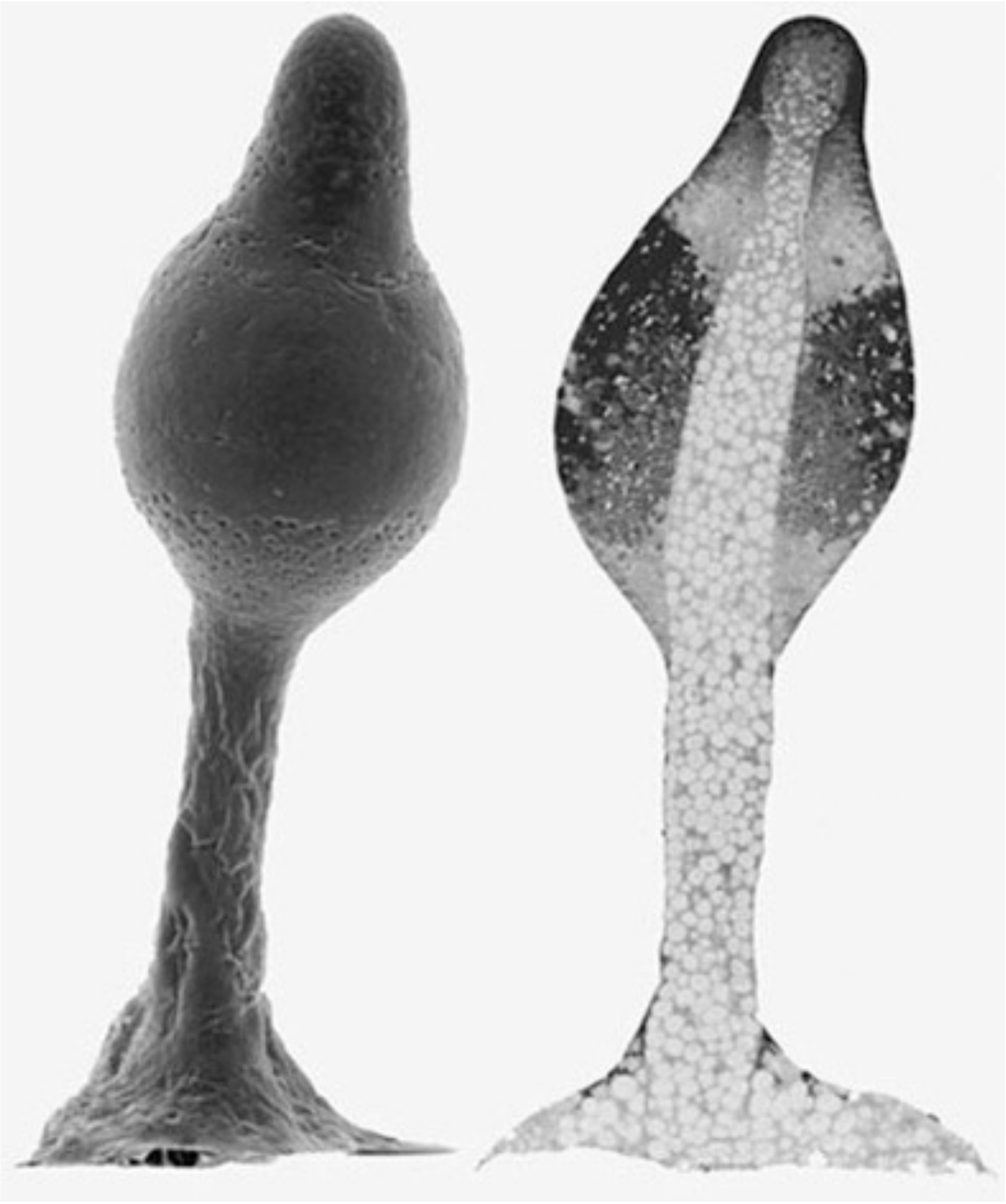
$$\Delta H' = \Delta H - \mu(c_{\text{automaton}} - c_{\text{neighbour}}),$$

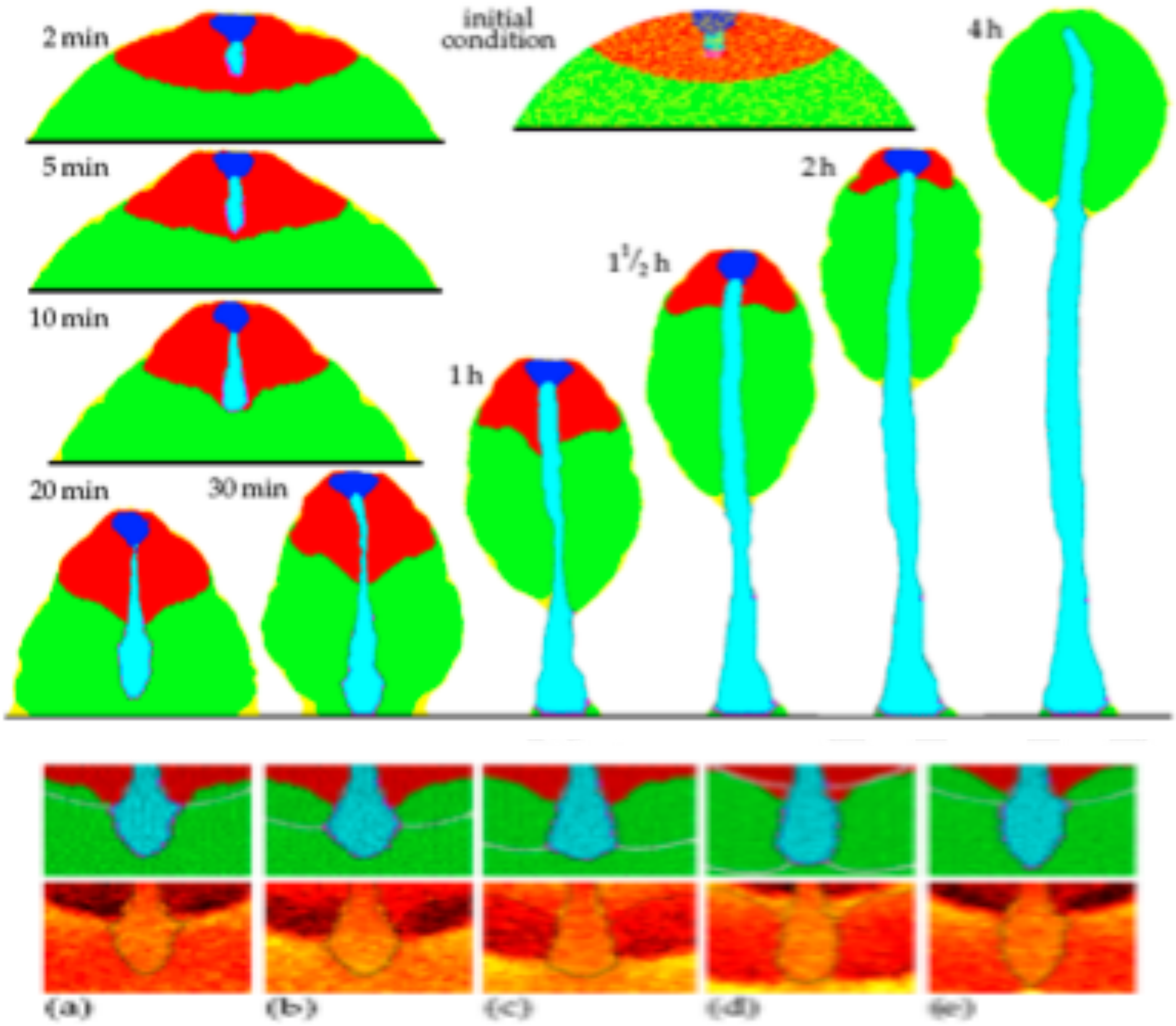


Mathematical model of Dictyostelium



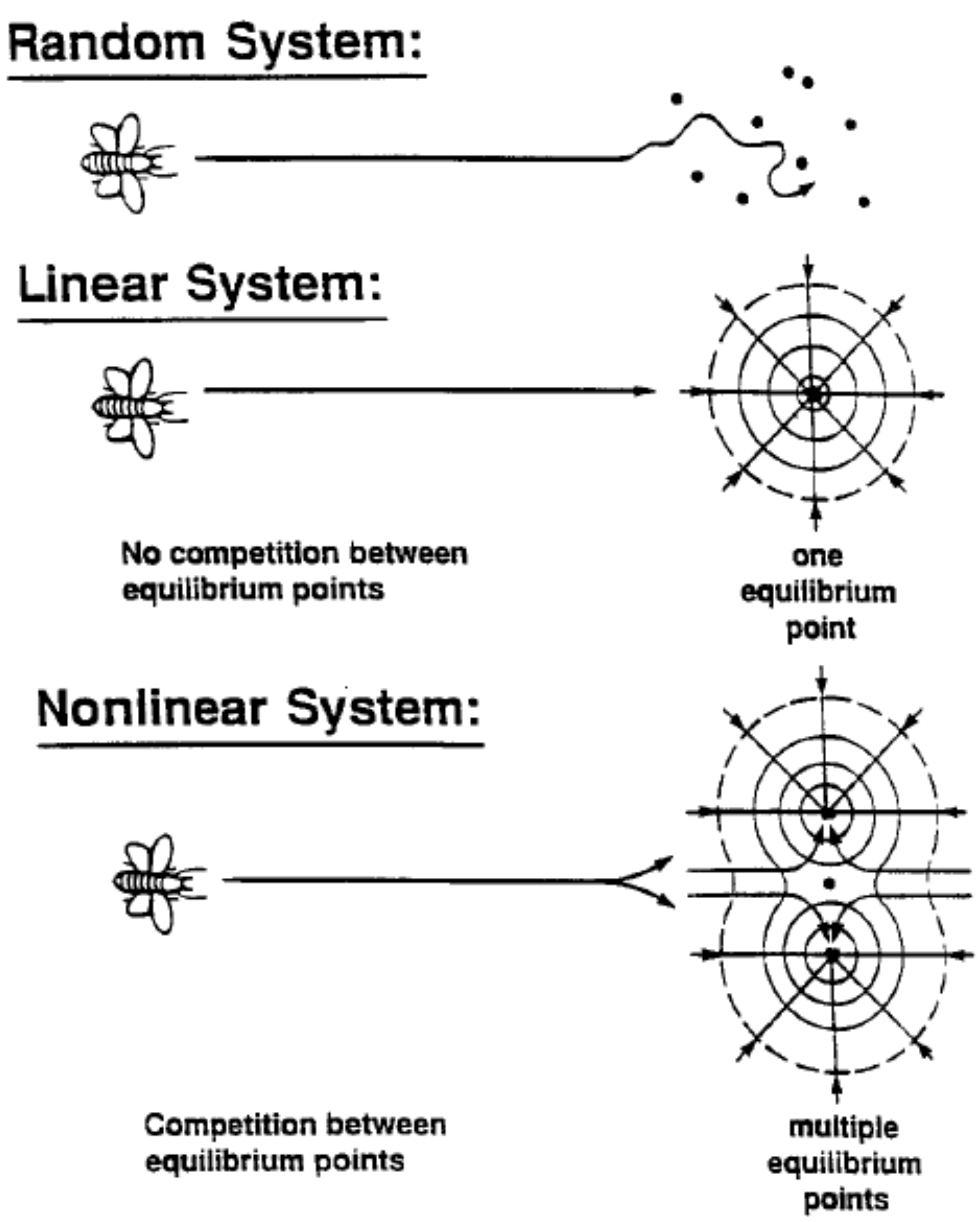
Mathematical model of Dictyostelium



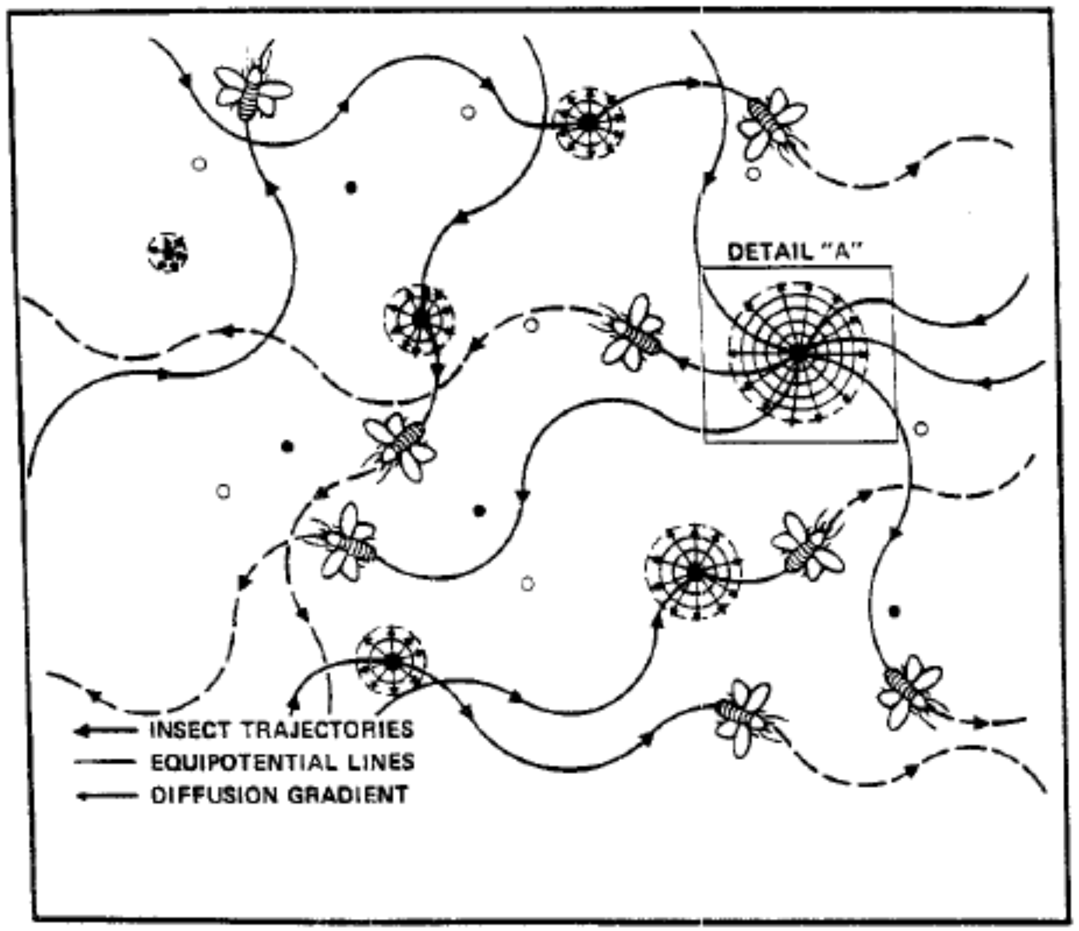


Mathematical model of Dictyostelium

Termite cathedrals: Complex structures from simple rules



Termite cathedrals: Complex structures from simple rules



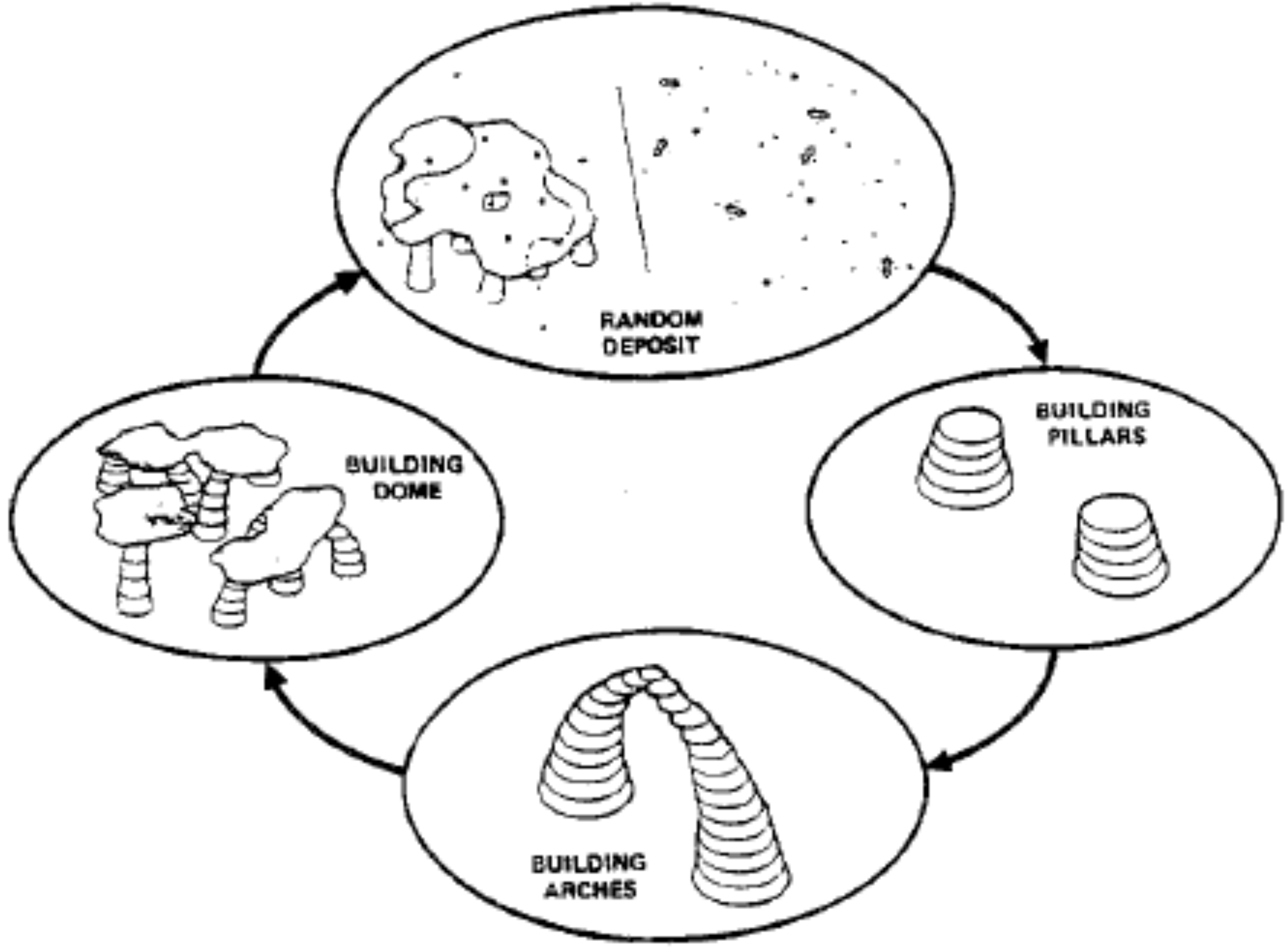


Fig. 14. Circular ring of building phases: each phase is dominated by

Termite cathedrals: Complex structures from simple rules

Can be “explained” by (local) laws of thermodynamics... termite is a particle in a gradient field...

Dissipative systems: Systems that extract energy from the environment to maintain their internal structure, their internal complexity

Usually: many simple units interact in simple ways to create complex patterns at the global, macro level...

But termites are more complex than classical particles!

Two Metaphors to explain Human Behaviour

Machine Metaphor

- Parts exist for each other, but not by means of each other
- Parts act together to meet the things purpose, but their actions have nothing to do with the thing's construction
- **Open to efficient cause (predicative logic)**
- Human behaviour: **Computation; Information processing**

Organism Metaphor

- Parts are both causes and effects of the thing, both means and end
- Parts act together but also construct and maintain themselves as a whole
- **Closed to efficient cause (impredicative logic)**
- Human Behaviour: **Concinnity; Embodied and Embedded**

Concinnity: Harmony in the arrangement or interarrangement of parts with respect to a whole.

Adapted from Turvey (2007)

Two types of mathematical formalism:

Random events / processes
Linear
Efficient causes

Random events / processes
Deterministic events / processes
Linear / Nonlinear
Efficient causes / Circular causality

component dominant dynamics

The Law of Large Numbers (Bernouiili, 1713) +
The Central Limit Theorem (de Moivre, 1733) +
The Gauss-Markov Theorem (Gauss, 1809) +
Statistics by Intercomparison *(Galton, 1875)* =
Social Physics *(Quetelet, 1840)*

Collectively known as:
The Classical Ergodic Theorems

Molenaar, P.C.M. (2008). On the implications of the classical ergodic theorems: Analysis of developmental processes has to focus on intra individual variation. *Developmental Psychobiology, 50*, 60-69

interaction dominant dynamics

Deterministic chaos (Lorenz, 1972)
(complexity, nonlinear dynamics, predictability)

Takens' Theorem (1981)
(phase space reconstruction)

Systems far from thermodynamic equilibrium
(Prigogine, & Stengers, 1984)

SOC / $\frac{1}{f^{\alpha}}$ **noise** (Bak, 1987)
(self-organized criticality, interdependent measurements)

Fractal geometry (Mandelbrot, 1988)
(self-similarity, scale free behaviour, infinite variance)

Aczel's Anti-Foundation Axiom (1988)
(hyperset theory, circular causality, complexity analysis)

Two types of mathematical formalism for two types of systems

component dominant dynamics

Jakob Bernouiili (1654-1704): [The application of the Law of large numbers in chance theory] *to predict the weather next month or year, predicting the winner of a game which depends partly on psychological and or physical factors or to the investigation of matters which depend on hidden causes, which can interact in a multitude of ways is* ***completely futile!*** *"* Vervaet (2004)

A system is ***ergodic*** *iff:*

The average dynamical behaviour of an ensemble of components is reducible to the dynamical behaviour of the components in the ensemble

(dynamical behaviour: change of behaviour over time)

f.i. The developmental trajectory of a cognitive variable of one individual measured from age 1-80 should be the same as measured in 80 different individuals, aged 1-80.

interaction dominant dynamics

Deterministic chaos (Lorenz, 1972)
(complexity, nonlinear dynamics, predictability)

Takens' Theorem (1981)
(phase space reconstruction)

Systems far from thermodynamic equilibrium
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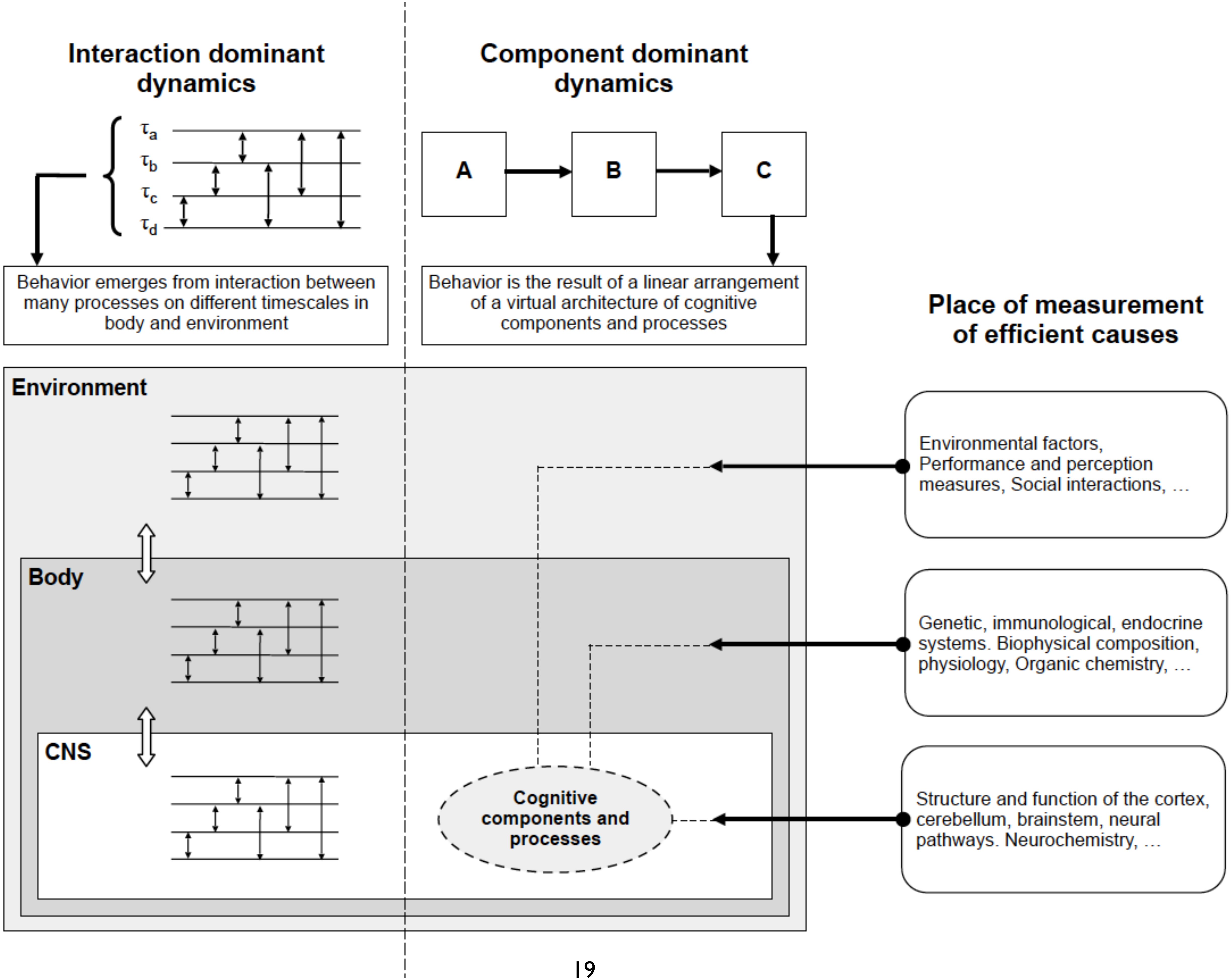

Interaction dominant dynamics
τ_a
τ_b
τ_c
τ_d
Behavior emerges from interaction between many processes on different timescales in body and environment
Component dominant dynamics
A
B
C
Behavior is the result of a linear arrangement of a virtual architecture of cognitive components and processes
Place of measurement of efficient causes
Environment
Body
CNS
Cognitive components and processes
Environmental factors, Performance and perception measures, Social interactions, …
Genetic, immunological, endocrine systems. Biophysical composition, physiology, Organic chemistry, …
Structure and function of the cortex, cerebellum, brainstem, neural pathways. Neurochemistry, …

Systems-thinking vs. Component thinking

A system is an entity that can be described as a composition of components, according to one or more organising principles.

The organising principles can take many different forms, but essentially they decide the three important features of systems that have to do with the relationship between parts and wholes:

* *What are the relevant scales of observation of the system*

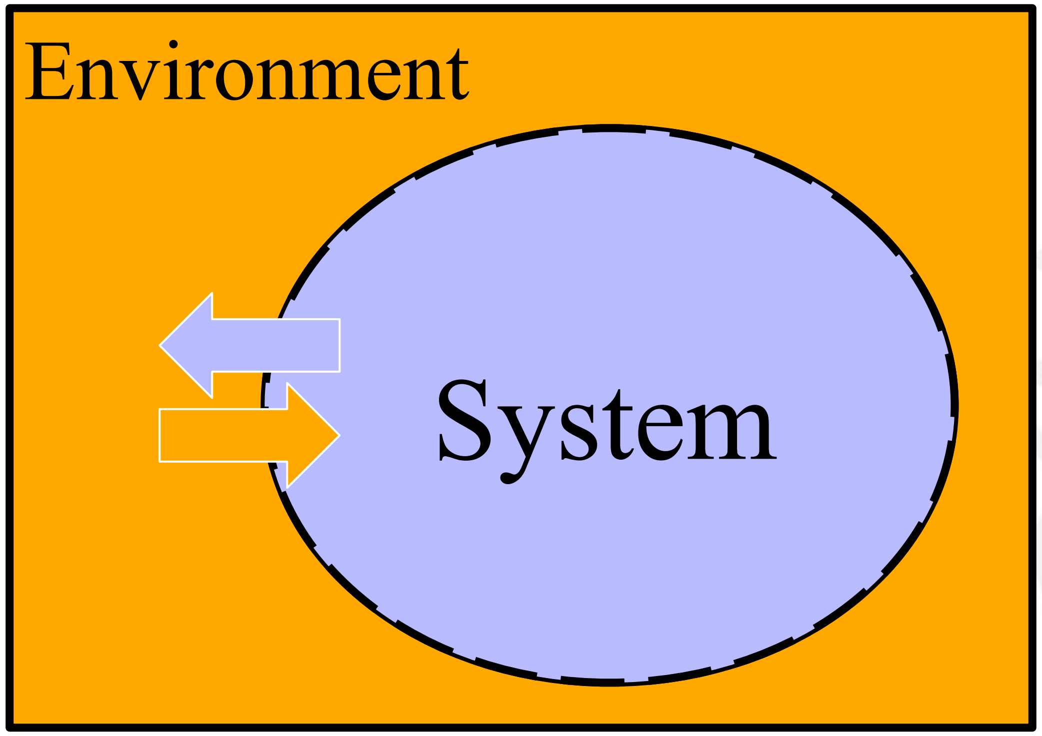
* *What are the relevant phenomena that may be observed at the different scales*

* *Can interactions with the internal and external environment occur, and if so, do interactions have any effects on the structure and/or behaviour of the system*

DYNAMICAL RULES / PROCESSES

What is a system?

Closed and Open Systems



Continuous exchange of matter, energy, and information with the environment.

What is a system?

Open vs. Closed

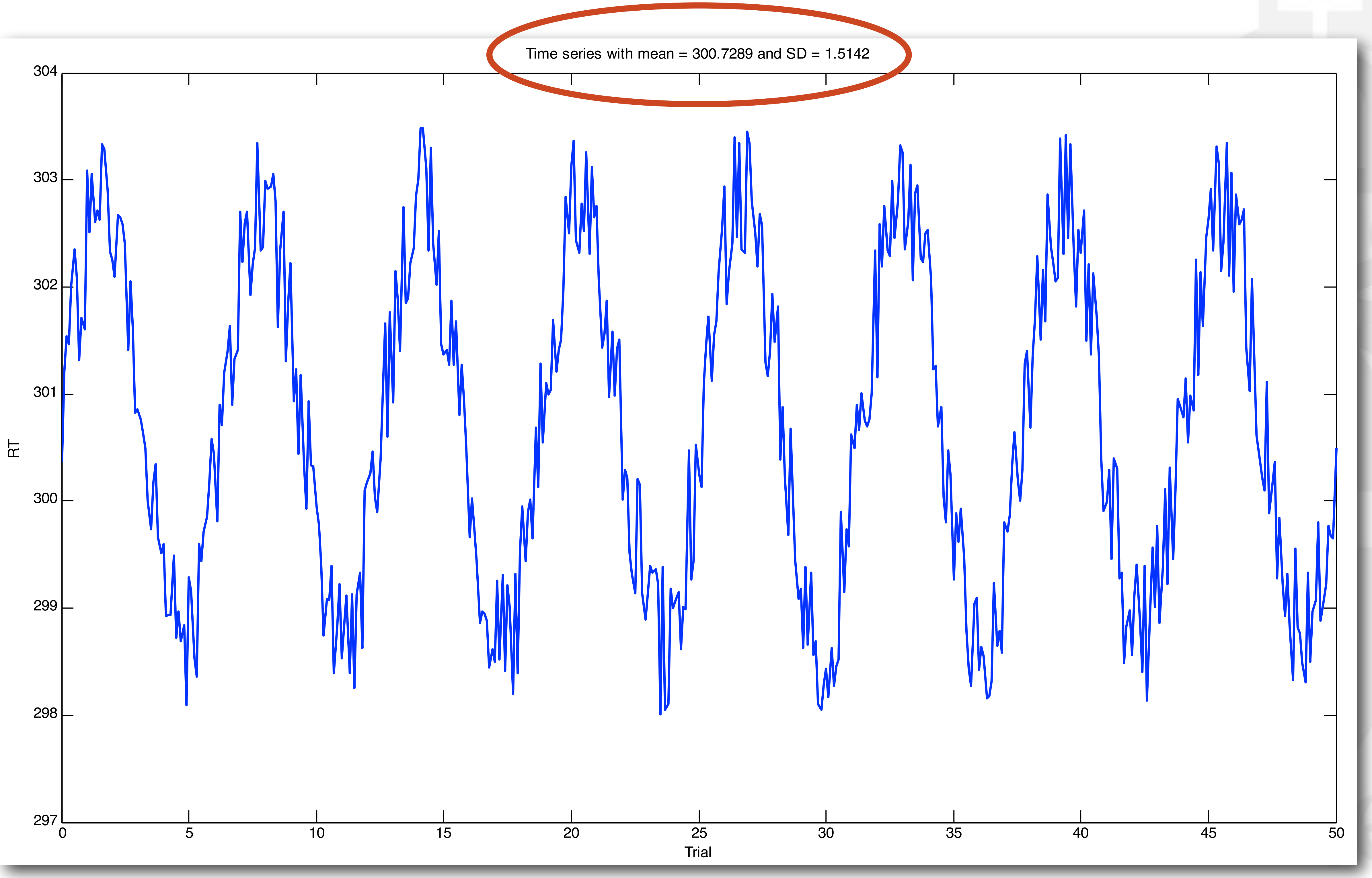
Simple vs. Complex

Ergodic vs. Non-ergodic

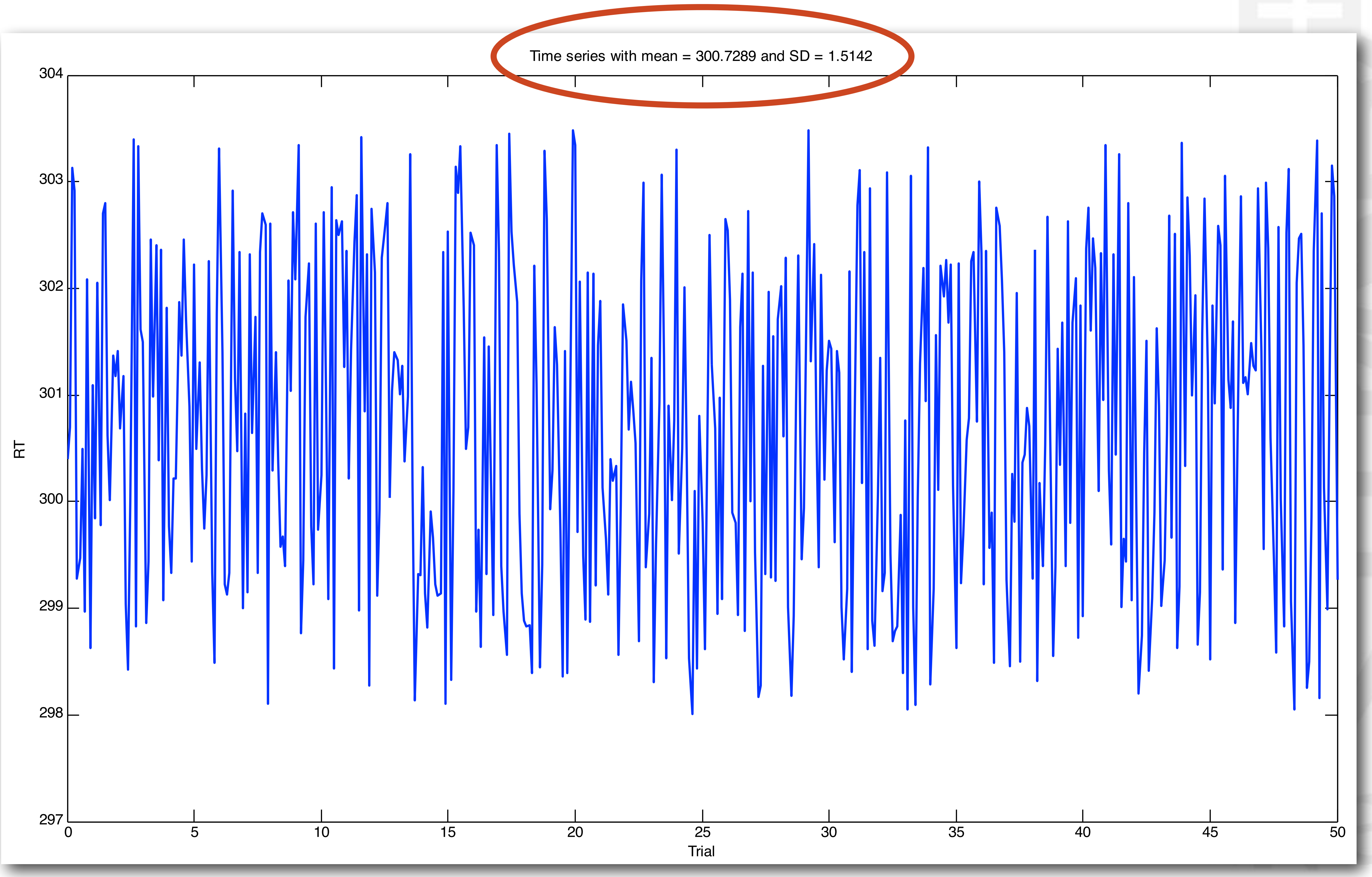
Equilibrium vs. Far-from-equilibrium

Order vs. Disorder

Importance of TIME-evolutionary processes



Importance of TIME-evolutionary processes



Order vs. Disorder

"order is essentially the arrival of redundancy in a system, a reduction of possibilities"

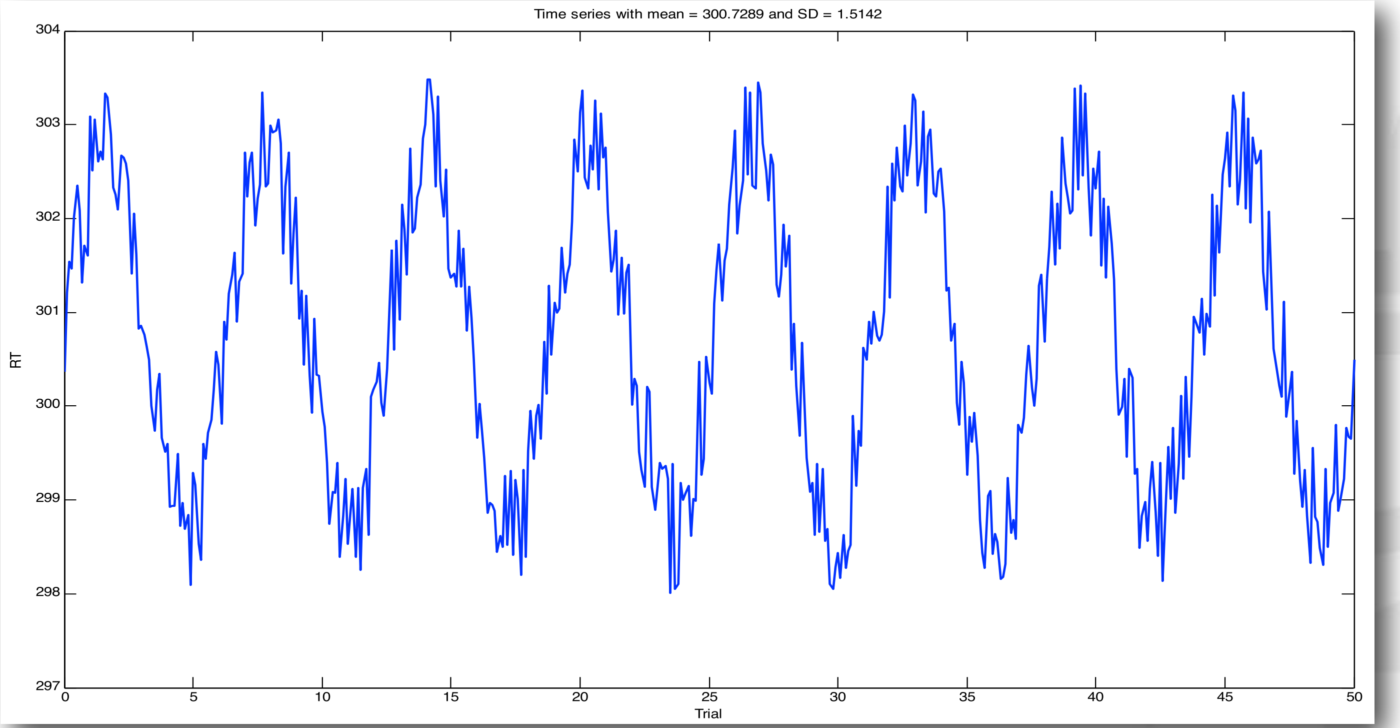
Von Förster (2003)

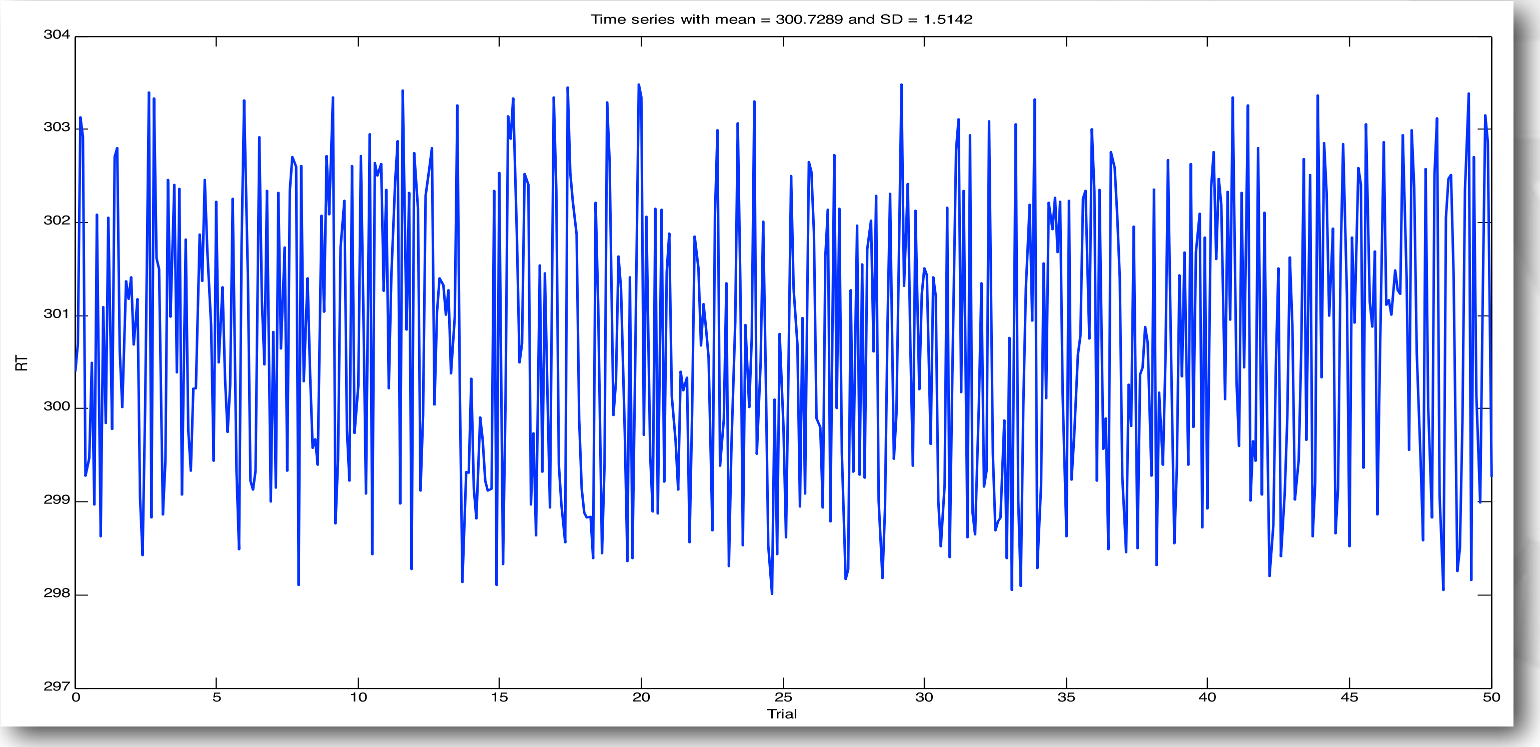
Entropy: Measure of 'disorder' in a system

However see Information Theory: "The amount of information we need to describe the different states (behaviour) of a system."

General definition: Relative absence of order/redundancy in a system. The degrees of freedom a system has available for generating its behaviour: Possibility.

Which series has a higher entropy?

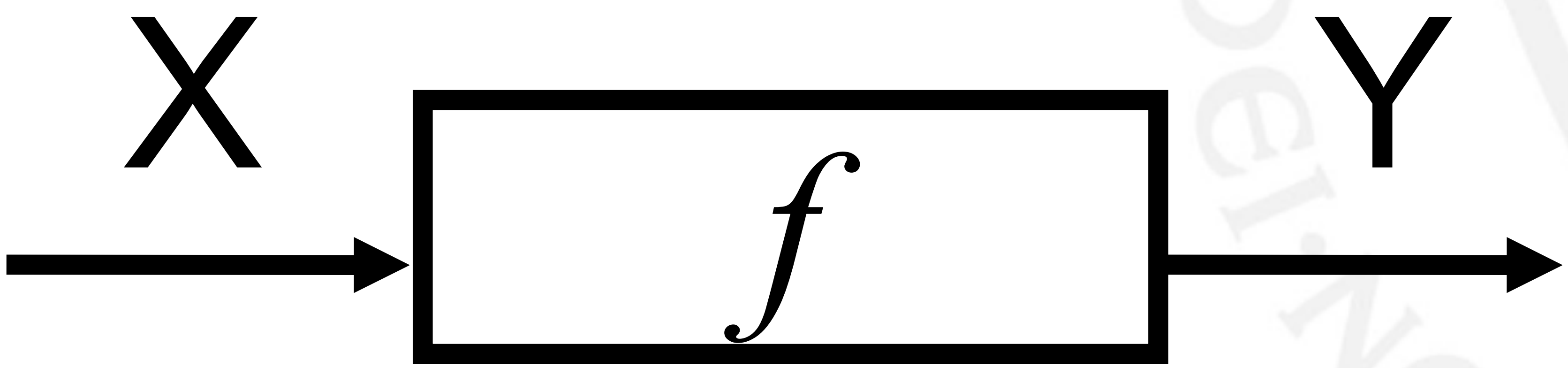




The mathematics of change

Traditional: Functional relations

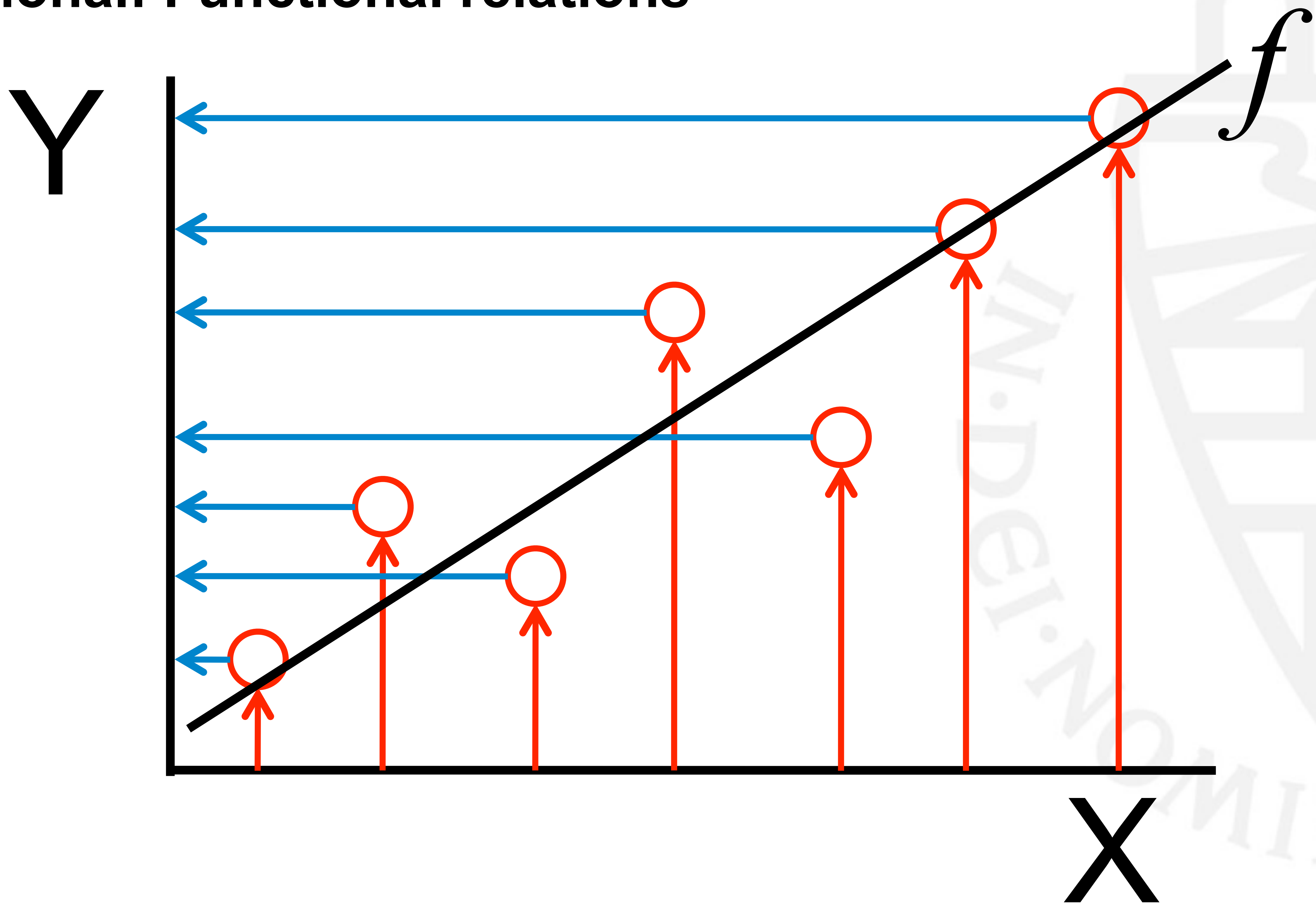
$$Y = f(X)$$



1refs

The mathematics of change

Traditional: Functional relations



1refs

The mathematics of change

Complex systems however:

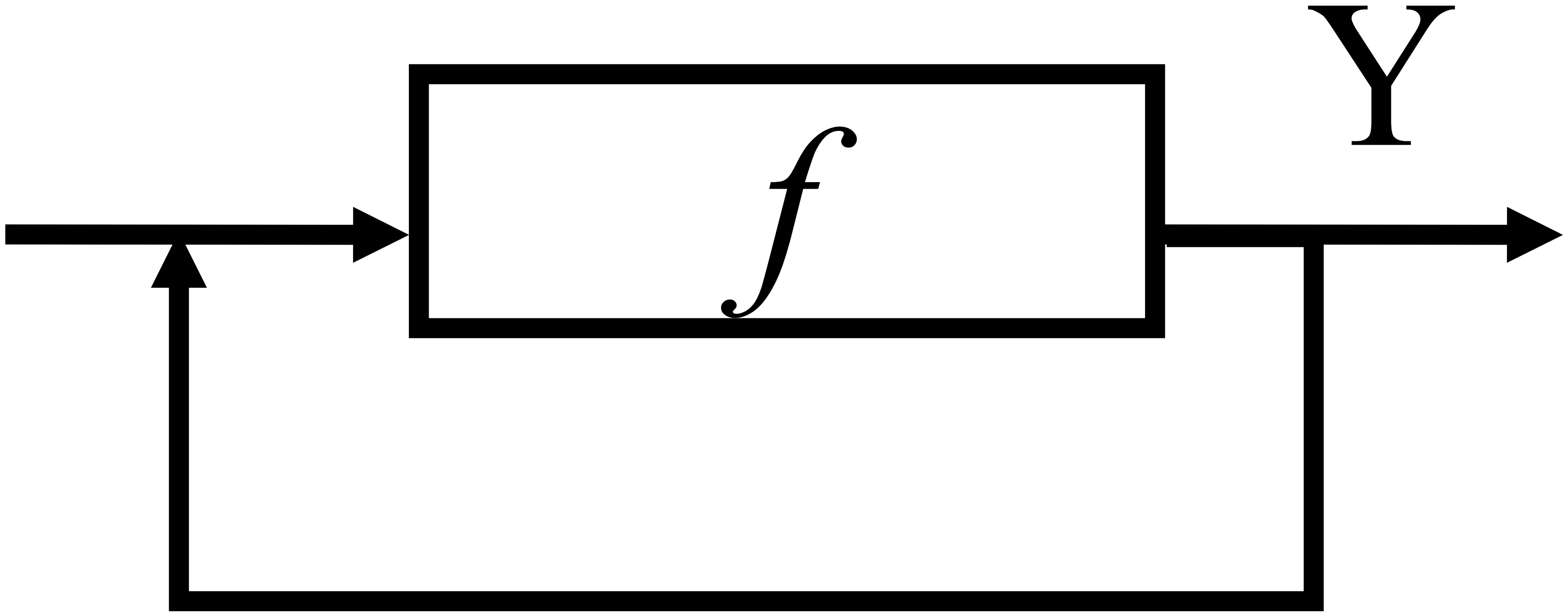
- Consist of feedback loops
- Are recurrent / recursive
- Have history
- Are characterised by multiplicative interactions between components

[1]refs

The mathematics of change

Complex systems: Recurrent processes / Feedback

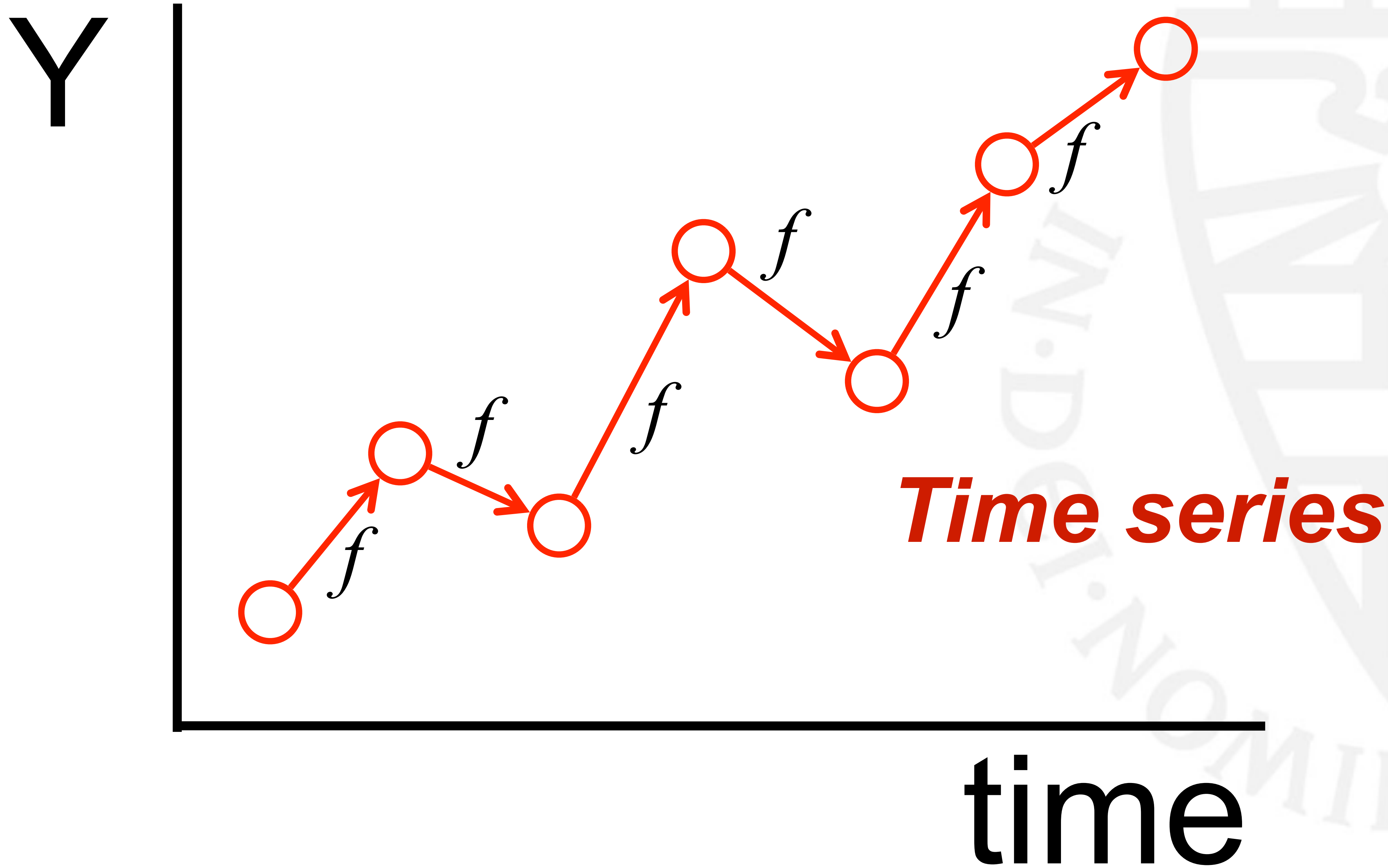
$$\hat{Y} = f(Y)$$



1refs

The mathematics of change

Complex systems: Recurrent processes / Feedback



[1]refs

Two Flavors: Flows & Maps

Dynamical models of psychological processes can be formulated in:

‘Clock’ time	**‘Metronome’ time**
Continuous System	Discrete System
~ Flow ~	… Map ...
(Differen*tial* equation)	(Differen*ce* equation)

1refs

Parameters & Bifurcations

Example 1:

The Linear Map

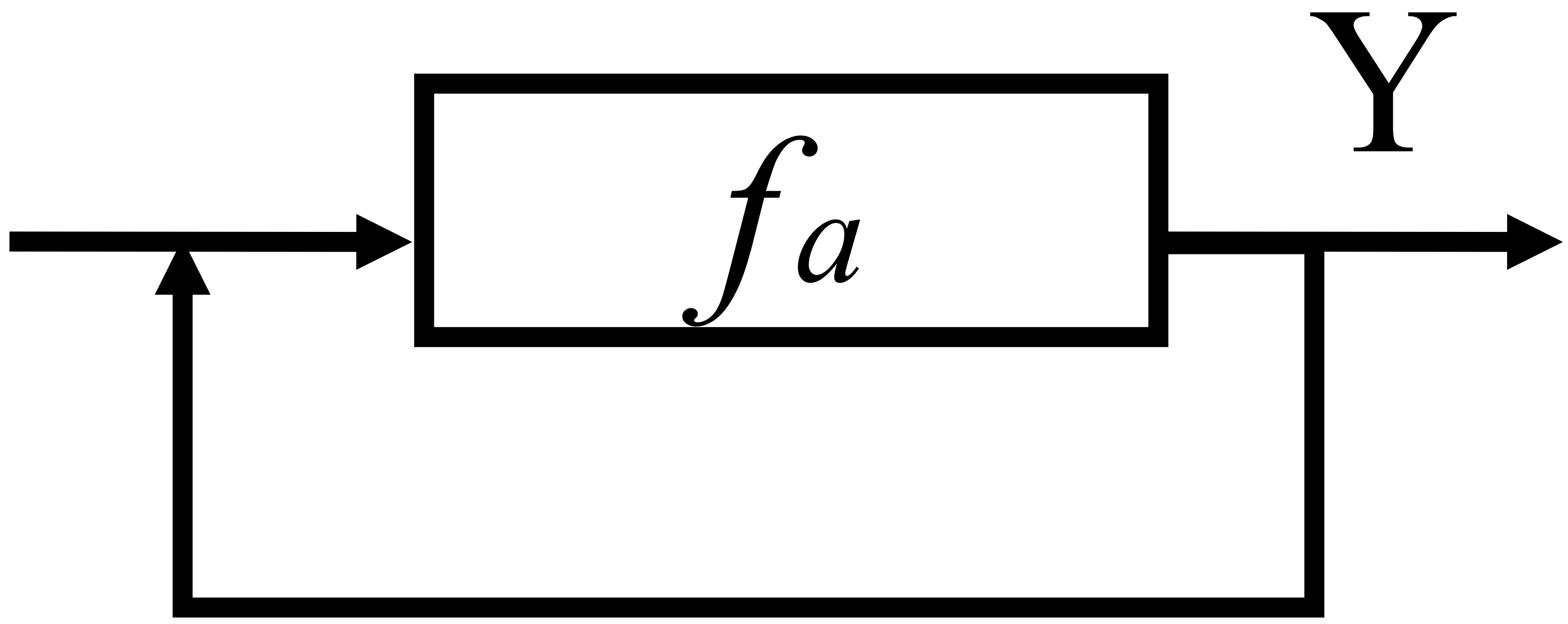
(Linear Growth)

1refs

The linear map

Dynamic Models: Parameter

$$\hat{Y} = f_a(Y)$$



The Linear Map …

The (rate of) change of the state of a system is proportional to its current state:

$$\boxed{Y_i+1 = a \cdot Y_i}$$

...Iteration...

[1]refs

The Linear Map

Initial value: Y_0

$$Y_1 = f(Y_0)$$

$$Y_2 = f(Y_1)$$

$$Y_3 = f(Y_2)$$

Iteration in general just means applying the function over and over again starting with an initial value

and subsequently to the result of the previous step

[1]refs

The Linear Map

$$\boxed{Y_{i+1} = f(Y_i)}$$

$$i = 0: \quad Y_0 \rightarrow Y_1 = f(Y_0)$$

$$i = 1: \quad Y_1 \rightarrow Y_2 = f(Y_1) = f(f(Y_0)) = f^2(Y_0)$$

$$i = 2: \quad Y_2 \rightarrow Y_3 = f(Y_2) = \ldots = f^3(Y_0)$$

$$\vdots \qquad \vdots$$

[1]refs

$$i = n: \quad Y_n \rightarrow Y_{n+1} = f(Y_n) = \ldots = f^n(Y_0)$$

Linear Map: Iteration with a parameter

$$\boxed{Y_{i+1} = a \cdot Y_i}$$

i = 0: $Y_0 \rightarrow Y_1 = a \cdot Y_0$

i = 1: $Y_1 \rightarrow Y_2 = a \cdot Y_1 = a \cdot a \cdot Y_0 = a^2 \cdot Y_0$

i = 2: $Y_2 \rightarrow Y_3 = a \cdot Y_2 = \ldots = a^3 \cdot Y_0$

$\vdots \qquad \vdots$

i = n: $Y_n \rightarrow Y_{n+1} = a \cdot Y_n = \ldots = a^{n+1} \cdot Y_0$

[1] refs

Linear Map: Iteration with a Parameter

$$Y_{i+1} = a \cdot Y_i$$

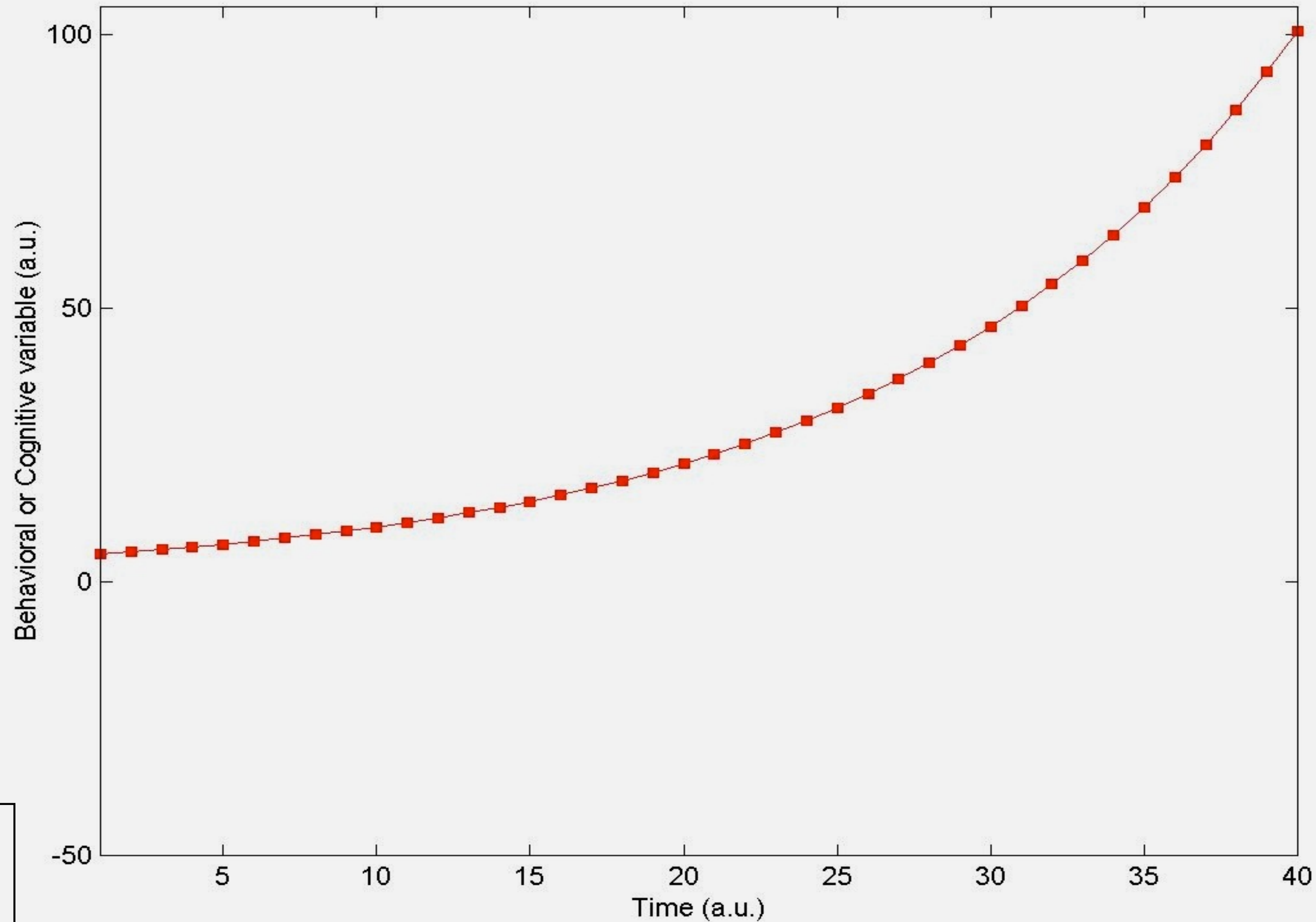
$0 < a < 1$	$-1 < a < 0$
$a > 1$	$a < -1$
$a = 1$	$a = -1$

Y_0 nonspecific

[1]refs

Linear Map: Iteration with a Parameter

a = 1.08
Y0 = 5

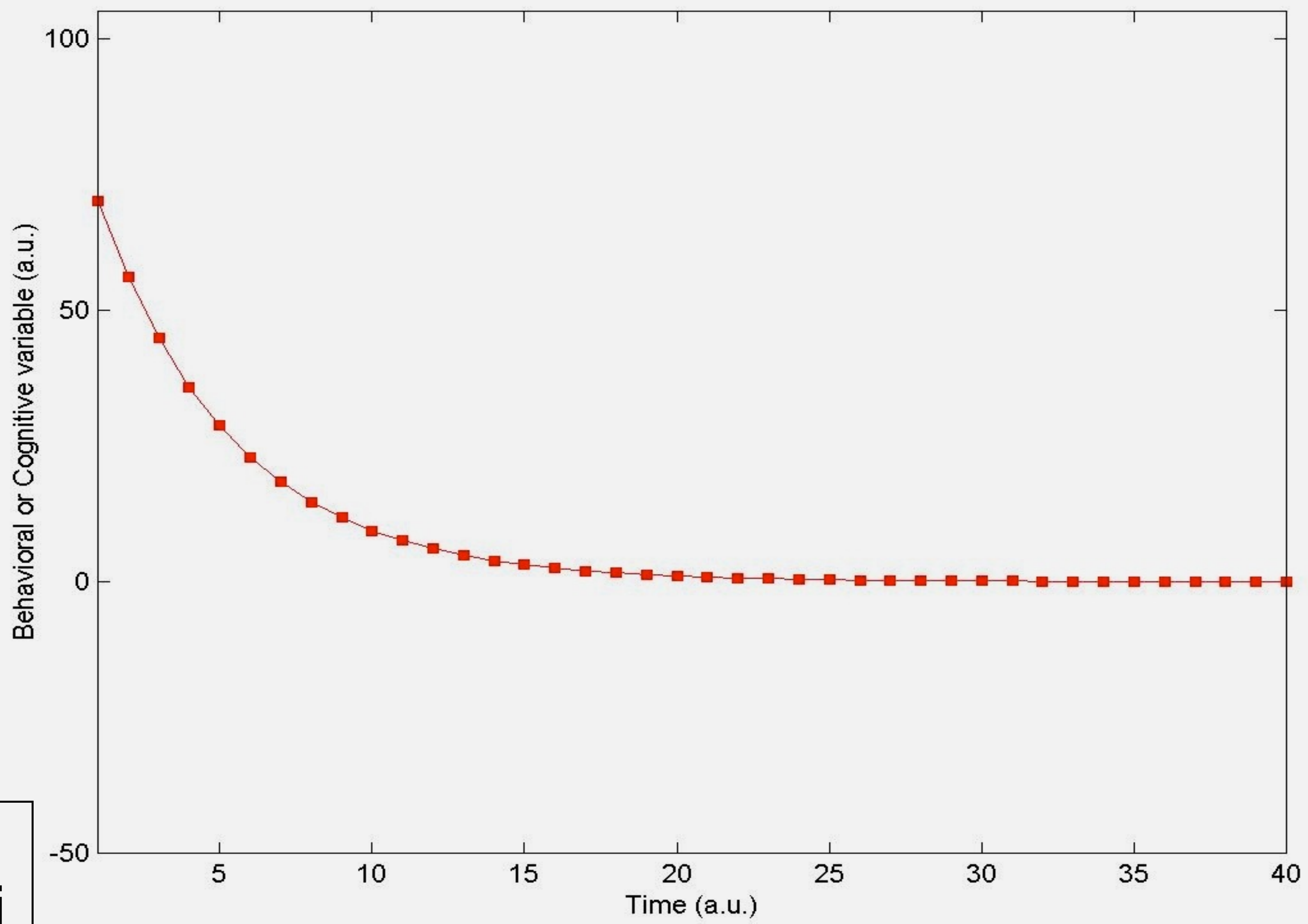


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

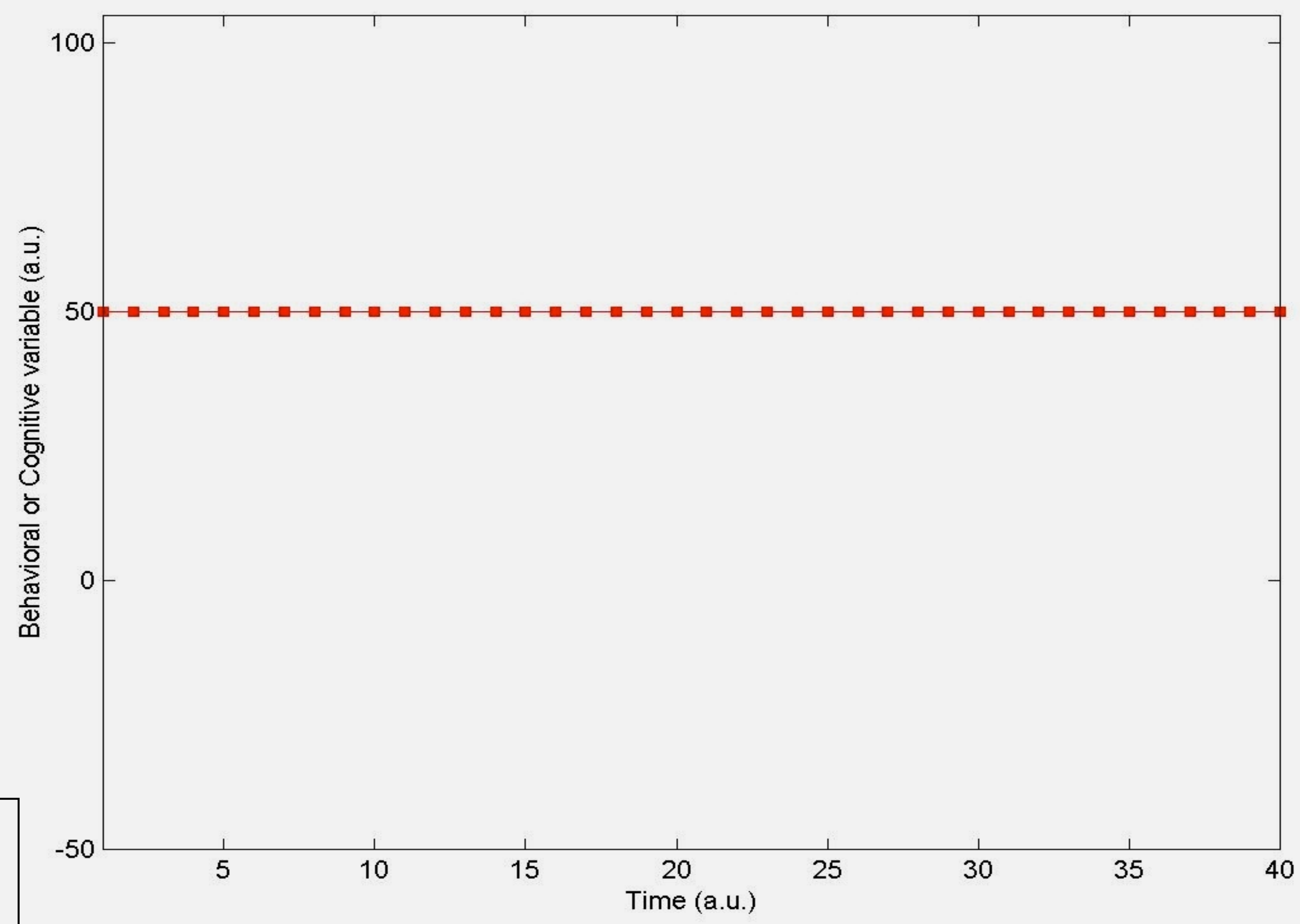
a = 0.8
Y0 = 70



$$Y_{i+1} = a \cdot Y_i$$

Linear Map: Iteration with a Parameter

a = 1.00
Y0 = 50

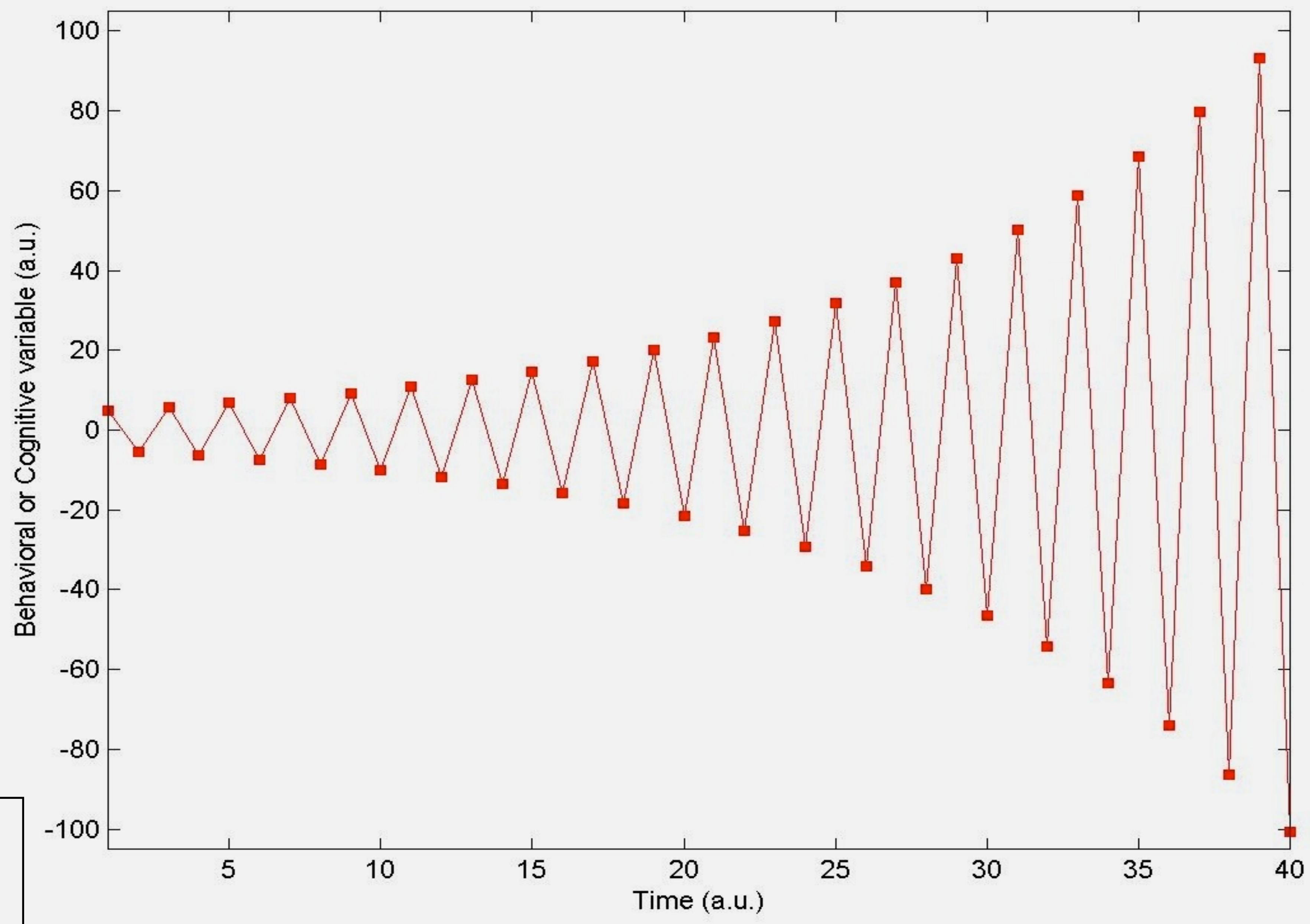


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

a = -1.08
Y0 = 5

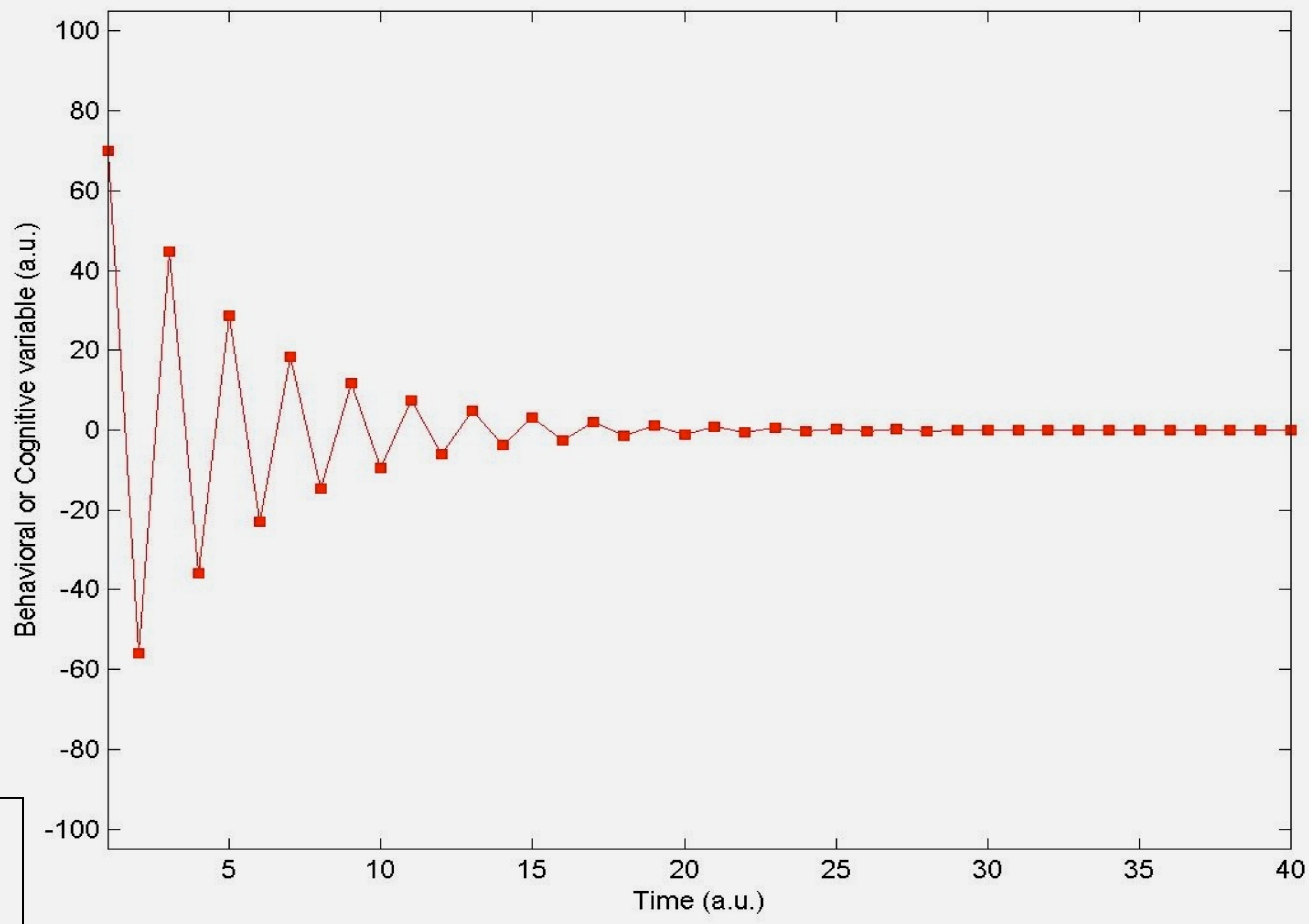


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

a = -0.8
Y0 = 70

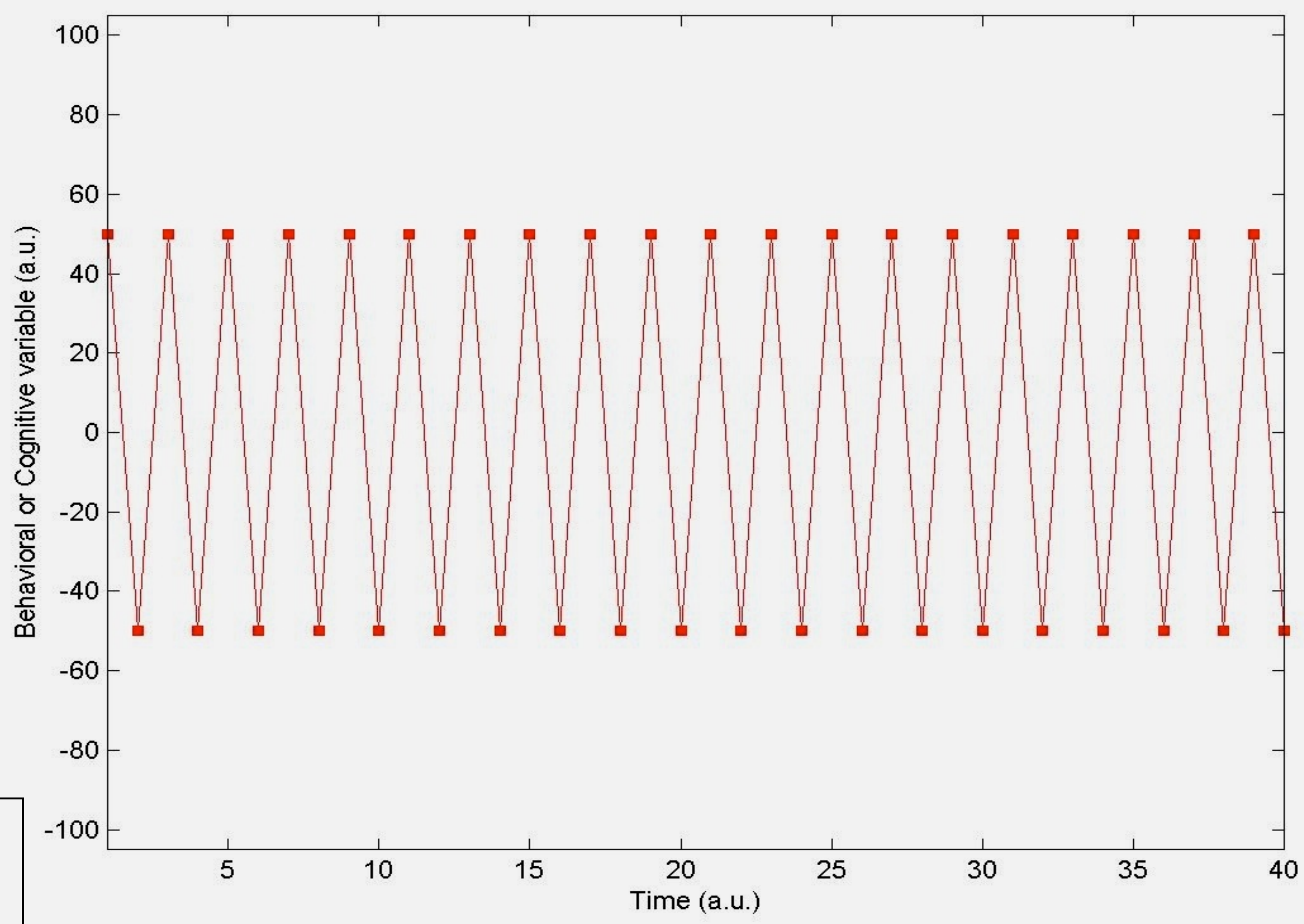


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

a = -1.00
Y0 = 50



$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

Some interesting differences compared to a linear model:

- Change of behaviour over iterations
  - *Simple model vs. "time" or "occasion" as a predictor*
- Qualitatively different behaviour
  - *One model produces at least four different types of behaviour*
  - *Not by adding predictors (components), by changing one parameter*

[1]refs

Parameters & Bifurcations

Example 2:

The Logistic Map

(restricted growth)

Logistic Map …

$$\boxed{L_{i+1} = r L_i (1 - L_i)}$$

- Simplest nontrivial model often used as an introduction to DST and Chaos theory.
- Well-known model in ecology, physics, economics and social sciences.
- ‘Styled’ version of Van Geert’s model for language growth. (*Next meeting*)

[1]refs

Logistic Map: Iteration

$$\boxed{L_{i+1} = r L_i (1 - L_i)}$$

i = 0: $L_0 \rightarrow L_1 = r L_0 (1 - L_0)$

i = 1: $L_1 \rightarrow L_2 = r L_1 (1 - L_1)$

$$= r\ rL_0(1-L_0)\ (1 - rL_0(1-L_0)\ )$$

$$= -r^3 L_0^4 + 2r^3 L_0^3 - r^2(1+r) L_0^2 + r^2 L_0$$

[1]refs

Logistic Map: Parameter

$$L_{i+1} = r L_i (1 - L_i)$$

$r = 0.90$
$r = 1.90$
$r = 2.90$

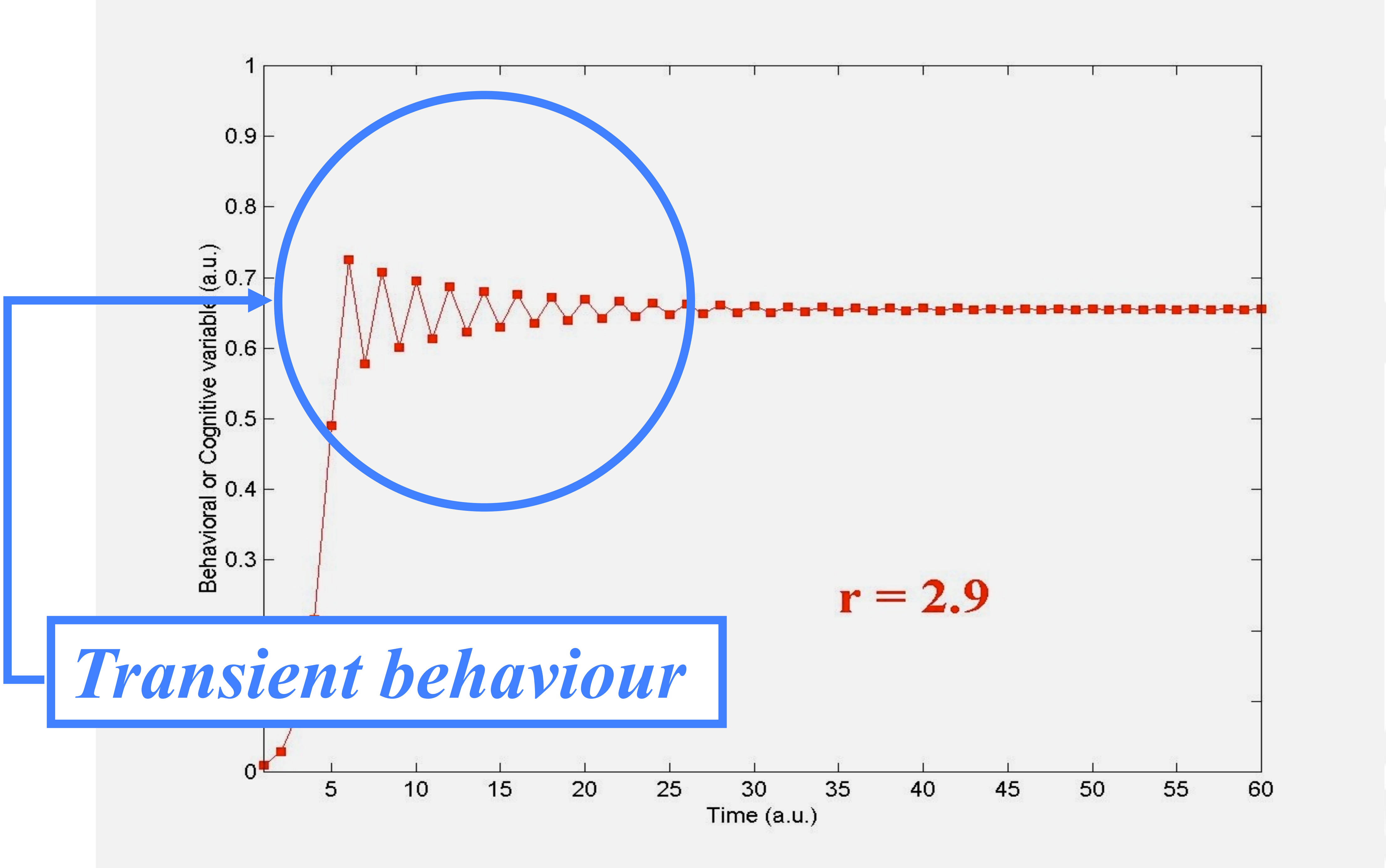
$r = 3.30$
$r = 3.52$
$r = 3.90$

L_0 small

[1]refs

Logistic Map: Graphs

$$L_{i+1} = r L_i (1 - L_i)$$



[1]refs

An ecology of growth models?
Same principle!

Basic Growth Models: Exponential + Restricted Growth

$$Population = rN \times (\frac{K-N}{K})$$

Additional Parameter: Carrying Capacity

$$CognitiveGrowth = L_i(1 + r \times \frac{K-L_i}{K})$$

$$StylizedLogistic = rY_i \times (\frac{1-Y_i}{1})$$

Bifurcation Diagram

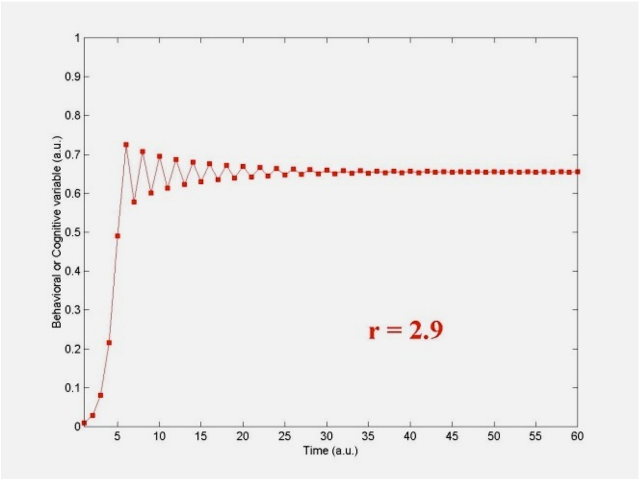
Bifurcation Diagram - Phase Diagram

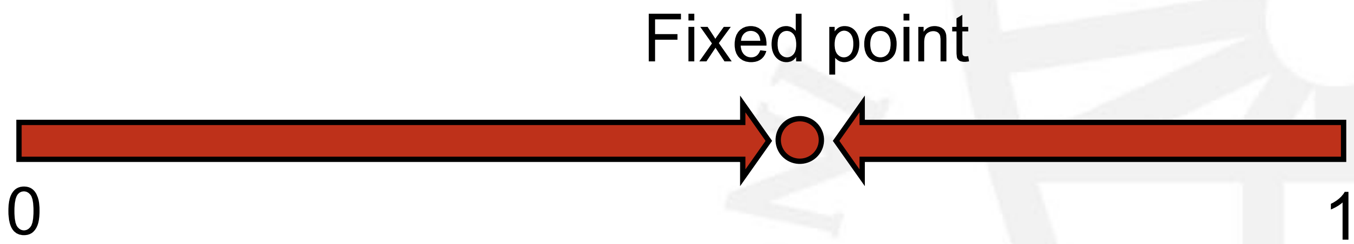
A graphical representation of the possible states a dynamical system can end up in for different values of one or more parameters.

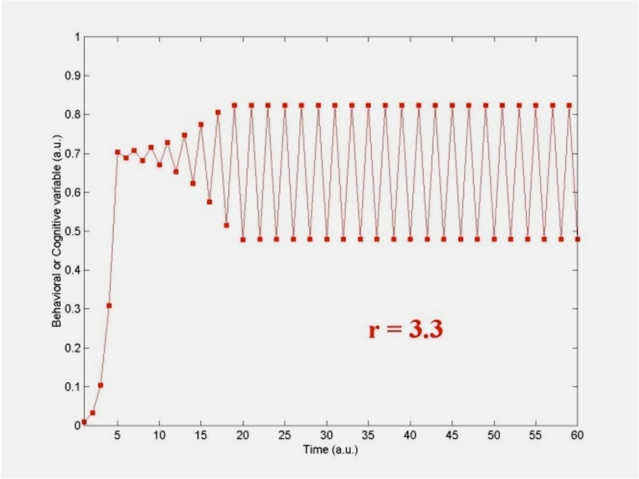
- The parameter is called the ***control parameter***.
- The end states are called ***attractors***.
- The change from one attractor (or set) to another is called a ***bifurcation***.

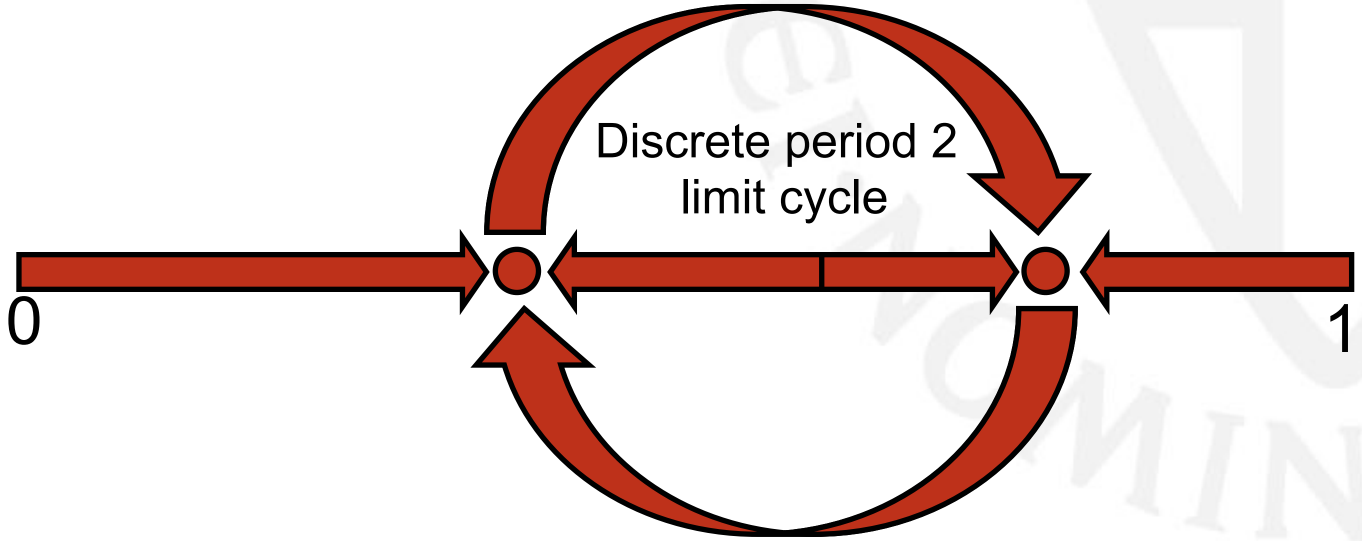
End states are attractors in state space: Attractor types

State Space is an abstract space used to represent the behaviour of a system. Its dimensions are the variables of the system. Thus a point in the phase space defines a potential state of the system. The points actually achieved by a system depend on its iterative function and initial condition (starting point).









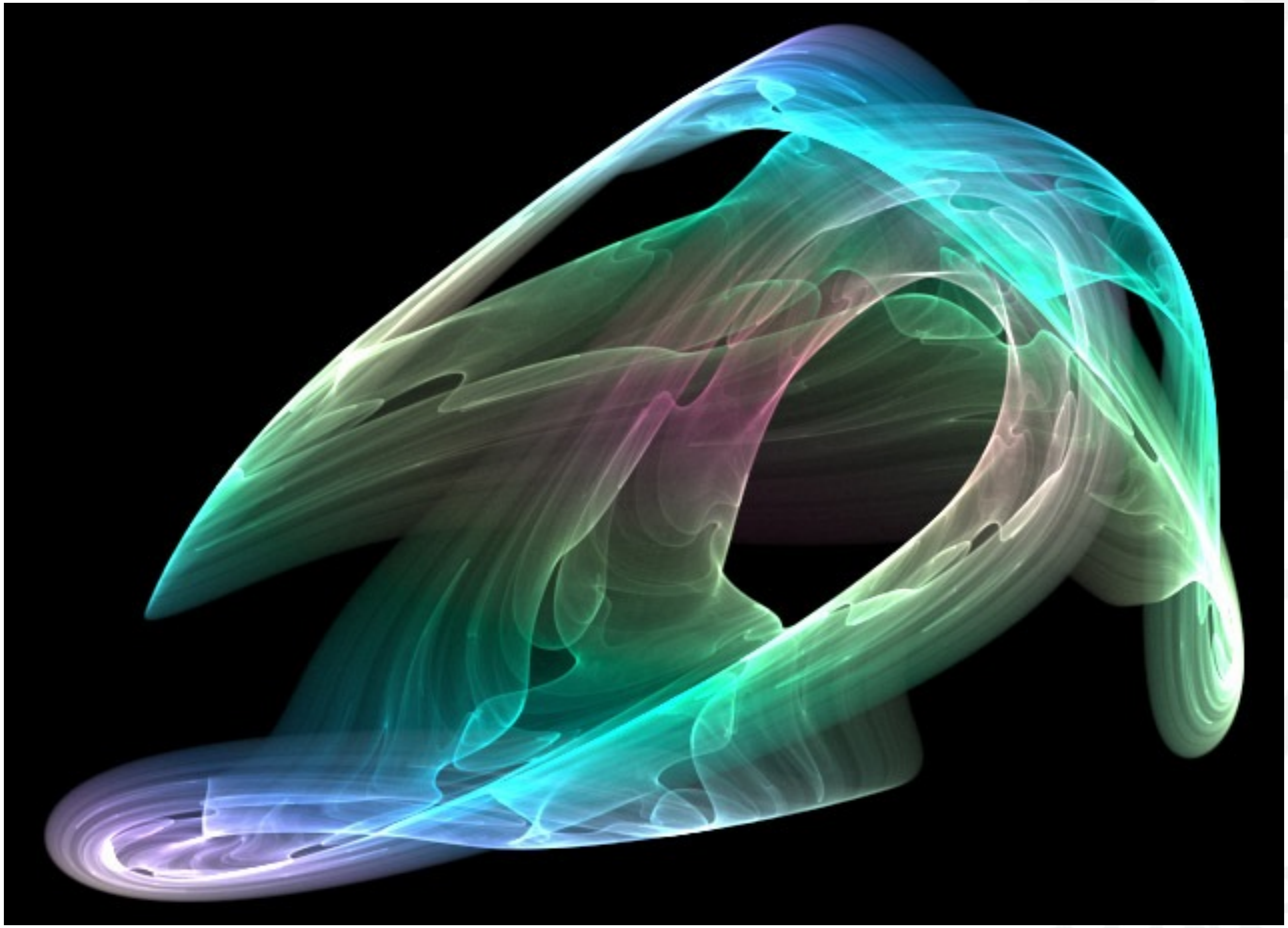
State space, Attractor types

"Saturn"
attractor

Strange attractors
are quasi periodic
and bounded

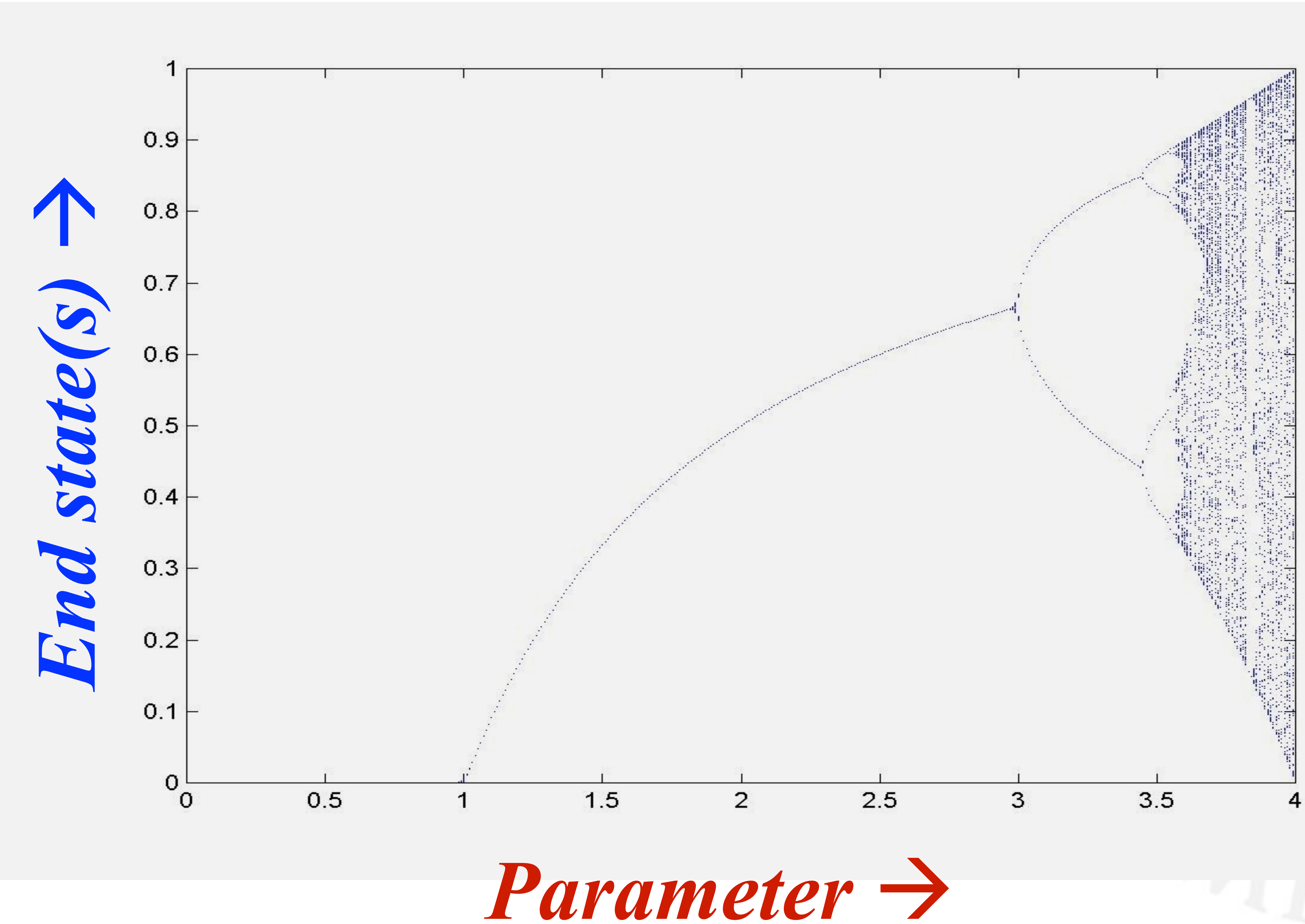
Bottom line:

An attractor means
a limited region
of state space
is visited.
Not all DF actually
available
to the system
are used.



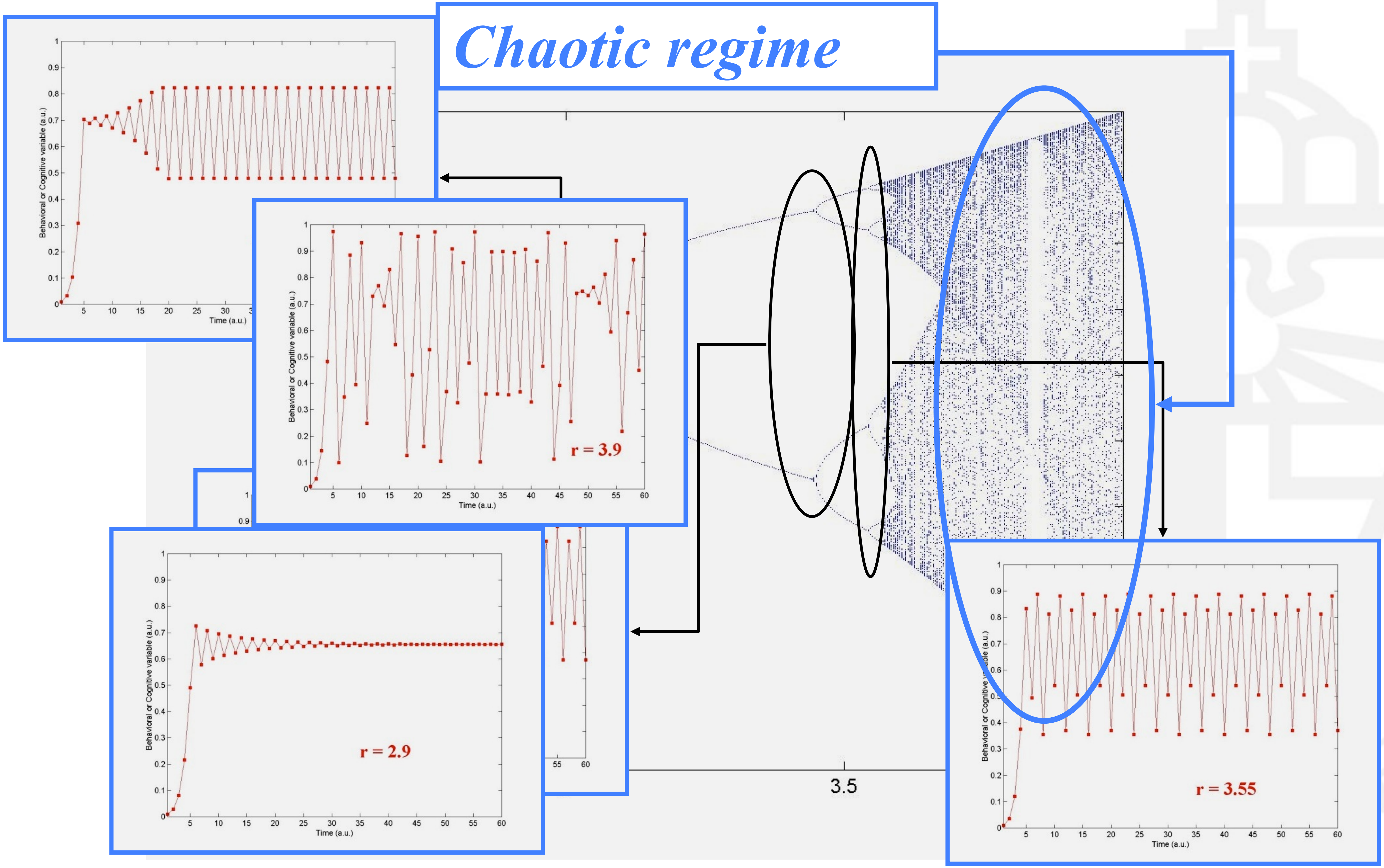
http://www.da4ga.nl/wp-content/uploads/2012/03/PastedGraphic-2-1.jpg

Logistic Map: Bifurcation Diagram

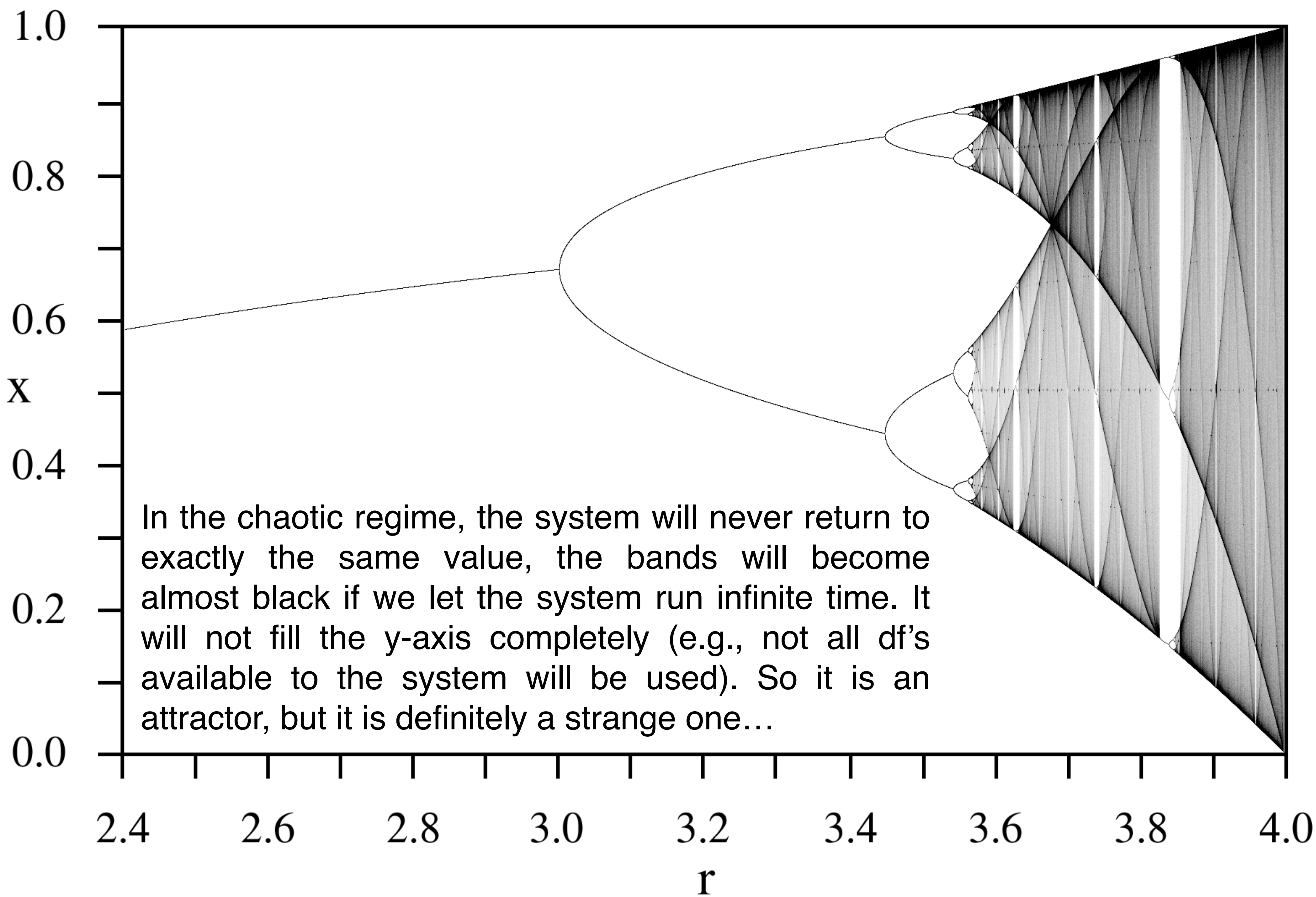


[1]refs

Chaotic regime



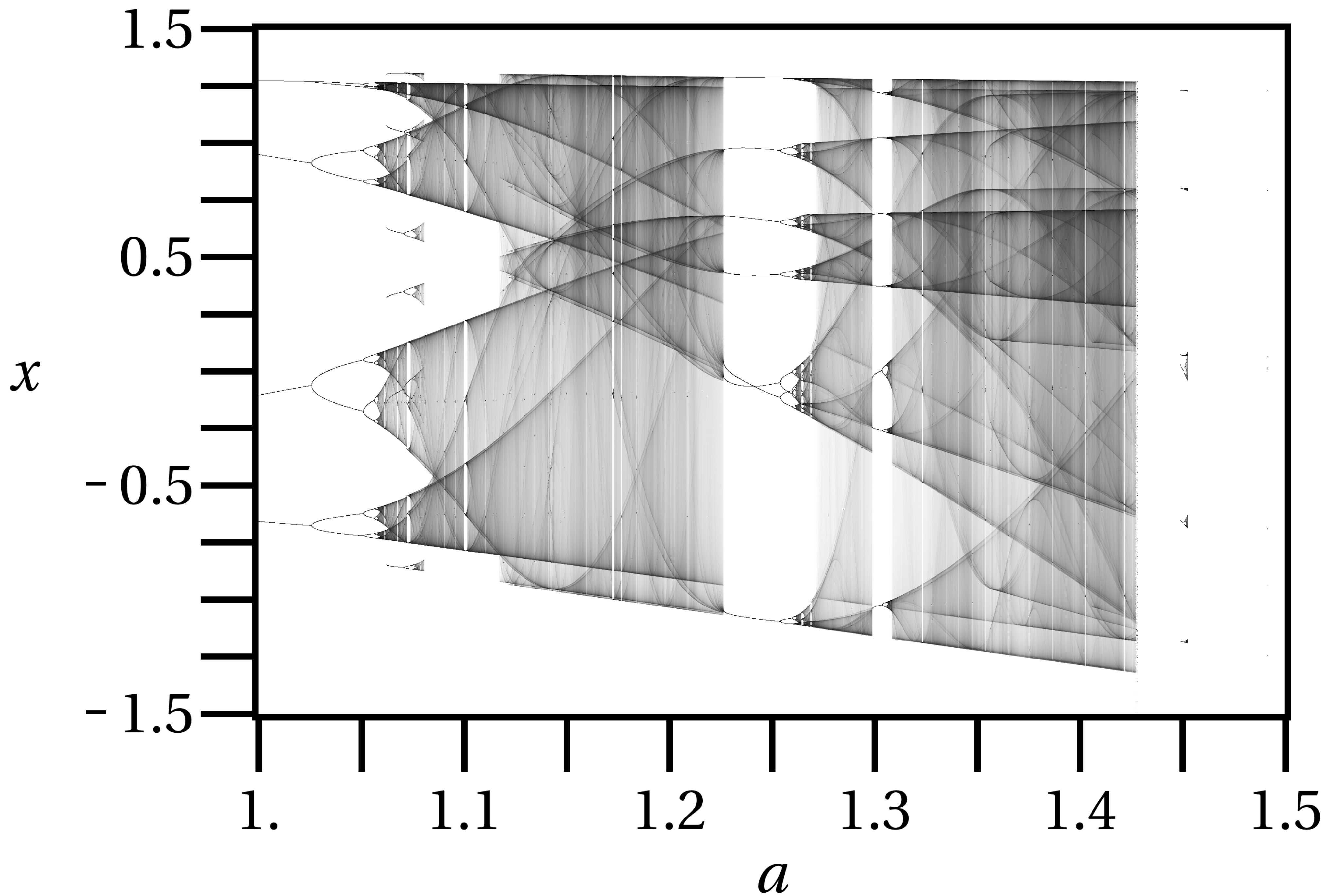
Logistic Map: Bifurcation Diagram



In the chaotic regime, the system will never return to exactly the same value, the bands will become almost black if we let the system run infinite time. It will not fill the y-axis completely (e.g., not all df's available to the system will be used). So it is an attractor, but it is definitely a strange one…

http://upload.wikimedia.org/wikipedia/commons/7/7d/LogisticMap_BifurcationDiagram.png

Henon Map: Bifurcation Diagram



http://upload.wikimedia.org/wikipedia/commons/c/cd/Henon_bifurcation_map_b%3D0.3.png