Mathematical Models

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Contents

Holte Model

$$\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= a_S - d_S S - \beta S V \\
\frac{\mathrm{d}I}{\mathrm{d}t} &= \beta S V - \kappa I^{n+1} \\
\frac{\mathrm{d}V}{\mathrm{d}t} &= \pi I - g V - \beta S V
\end{aligned}$$

This model has been previously fit in RSIF timing paper.

Effector Model with Unproductive Cells (Reeves)

$$\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= a_S - d_S S - \beta S V \\
\frac{\mathrm{d}I_{UP}}{\mathrm{d}t} &= (1-\tau)(1-\lambda)\beta S V - d_I I_{UP} - kEI_{UP} \\
\frac{\mathrm{d}I_P}{\mathrm{d}t} &= \tau(1-\lambda)\beta S V - d_I I_P - kEI_P \\
\frac{\mathrm{d}L}{\mathrm{d}t} &= \lambda\beta S V - th_L L \\
\frac{\mathrm{d}E}{\mathrm{d}t} &= a_E + wI\frac{E}{E+E_{50}} - d_E E \\
\frac{\mathrm{d}V}{\mathrm{d}t} &= \pi I_P - g V - \beta S V \\
I &= I_P + I_{UP}
\end{aligned}$$

Table 1: EFfector model parameters

parameter	value
a_S	fitted
d_S	fitted
β	fitted (log-scale)
τ	0.05
λ	$1x10^{-4}$
d_I	fitted or fixed to 0.8
k	1
th_L	$5.2x10^{-4}$
a_E	$1x10^{-4}$
d_E	fitted
w	fitted
$E50$	fitted
π	fitted (log-scale)
g	23
S_0	a_S/d_S
E_0	a_E/d_E
V_0	$0.01/1e3$
$I_{P,0}$	gV_0/p
$\sum I_0$	$\frac{I_{P,0}}{\tau(1-\lambda)}$
$I_{UP,0}$	$(1-\tau)(1-\lambda)\sum I_0$
L_0	$\lambda \sum I_0$

Pre-cursor Immune Model (Borducchi, Prague aka Hill)

This model does not include effector (CD8) function. https://www.biorxiv.org/content/10.1101/700401v2.full.pdf https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5145754/

Unclear if we'll fit a version of this model, it is specialized and parameterized for SIV analysis, specifically the lack of CD8 effect.

$$
\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= a_S - d_S S - \beta S V \\
\frac{\mathrm{d}I}{\mathrm{d}t} &= \frac{\beta S V}{1 + \frac{E}{N_E}} - d_I I \\
\frac{\mathrm{d}P}{\mathrm{d}t} &= a_P + w(1-f)\frac{V}{V + N_P} P - d_P P \\
\frac{\mathrm{d}E}{\mathrm{d}t} &= w f \frac{V}{V + N_P} P - d_E E \\
\frac{\mathrm{d}V}{\mathrm{d}t} &= \pi I - g V
\end{aligned}
$$

Pre-cursor Effector Model

This model is the Hill model with effector function and growth depending on infected cell levels.

$$
\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= a_S - d_S S - \beta S V \\
\frac{\mathrm{d}I}{\mathrm{d}t} &= \beta S V - d_I I - k E I \\
\frac{\mathrm{d}P}{\mathrm{d}t} &= a_P + w(1-f)\frac{I}{I+N_P}P - d_P P \\
\frac{\mathrm{d}E}{\mathrm{d}t} &= wf\frac{I}{I+N_P}P - d_E E \\
\frac{\mathrm{d}V}{\mathrm{d}t} &= \pi I - gV - \beta S V
\end{aligned}
$$

Table 2: EFfector model parameters

parameter	value
a_S	fitted
d_S	fitted
β	fitted (log-scale)
d_I	fitted or fixed to 0.8
k	1
a_P	$1x10^{-5}$
d_P	fitted
d_E	fitted
f	0.9 (from Prague)
w	fitted
N_P	fitted
π	fitted (log-scale)
g	23
S_0	a_S/d_S
I_0	gV_0/p
P_0	a_P/d_P
E_0	0
V_0	$0.01/1e3$

Pre-cursor Effector Model with unproductive cells

This combines Hill and Reeves.

$$
\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= a_S - d_S S - \beta S V \\
\frac{\mathrm{d}I_{UP}}{\mathrm{d}t} &= (1-\tau)(1-\lambda)\beta S V - d_I I_{UP} - kEI_{UP} \\
\frac{\mathrm{d}I_P}{\mathrm{d}t} &= \tau(1-\lambda)\beta S V - d_I I_P - kEI_P \\
\frac{\mathrm{d}L}{\mathrm{d}t} &= \lambda \beta S V - th_L L \\
\frac{\mathrm{d}P}{\mathrm{d}t} &= a_P + w(1-f)\frac{I}{I+N_P}P - d_P P \\
\frac{\mathrm{d}E}{\mathrm{d}t} &= wf\frac{I}{I+N_P}P - d_E E \\
\frac{\mathrm{d}V}{\mathrm{d}t} &= \pi I - gV - \beta S V \\
I &= I_P + I_{UP}
\end{aligned}
$$

Notes

A key component of the immune models is the logistic growth term. There are several versions of immune growth rates represented above. Would be interested to know how these fundamentally differ by mechanistic interpretation.

$$
I\frac{E}{N_E + E}
$$

$$
P\frac{V}{N_P + V}
$$

$$
P\frac{I}{N_P + I}
$$