CSCI 2824, Fall 2015

Challenge Problem 2, due Thursday October 8 (hard copy, in class)

(a) This is a "pile-splitting problem". Here is the situation: you start with n objects in a pile, and split it into two smaller piles. Then, for each new pile, you continue the splitting process until (finally) there are n piles of size one. At each splitting operation, you compute the product of the size of the two smaller piles. Once there are n piles, sum up all the products computed. The result will be a function of n, and will not (as it turns out) depend on how each pile is split along the way. Conjecture what this function is, and prove that your conjecture is correct. (You will need to use "strong induction", as discussed in class when we proved the Fundamental Theorem of Arithmetic.)

Here's an example, for $n = 5$.

Split the pile into 3 and 2. (The product of this split is 6.)

Split the 3 pile into 2 and 1. (The product is 2.)

Split the first 2-pile into 1 and 1. (The product is 1.)

Split the second 2-pile into 1 and 1. (The product is 1.)

Now, the total is 6 + 2 + 1 + 1 = 10.

(b) A second pile-splitting problem. You start with n objects in a pile, and split it into two smaller piles (call them size j and k). Now, for each splitting operation, you sum the reciprocals–that is, compute the sum $(1/j) + (1/k)$. Once there are n piles of size 1, multiply all the sums computed. Again, the result will depend only on n. Conjecture the function of n associated with this process and prove that your conjecture is correct.

Here's and example, for $n = 5$.

Split the pile into 4 and 1. (The sum of reciprocals is 5/4.)

Split the 4 pile into 2 and 2. (The sum of reciprocals is 1.)

Split each of the two 2-piles into 1 and 1. (The sum of reciprocals in each case is 2.)

Multiply all the sums together: 2 * 2 * 1 * 5/4 = 5.