**CSCI 2824: Discrete Structures Problem Set 3**
**Due Tuesday 11/10 in class (*hard copy*)**

**Problem 1. (60 points)**

(a) (12 points) Recall that Mastermind codes consist of an ordered row of four colors. (For instance, RED BLUE BLUE ORANGE is one code; BLUE RED BLUE ORANGE is a different code.) There are six different colors to choose from. How many potential Mastermind codes have 2 or 3 pegs the same color (i.e., one pair or one trio, but not two pairs)?

(b) (12 points) How many ways are there of choosing a Mastermind code that uses the color orange exactly once?

(c) (12 points) How many distinct arrangements can be made from the letters in the word ANTEATER? (For example, ANTTEAER is one such arrangement.)

(d) (12 points) Suppose you have 10 disks all of the same size (indistinguishable). How many ways are there of placing all these disks on the three Tower of Hanoi pegs? Note that the order in which disks are placed on a peg doesn't matter, since all the disks are indistinguishable; the only thing that matters is the total number of disks placed on each of the three pegs. Thus, (7 2 1) is one way of placing the pegs; (1 2 7) is another.

(e) (12 points) How many distinct ways are there of ordering a deck of 52 cards such that all hearts occur before any other card? (That is, the first 13 cards are all hearts.)

**Problem 2. (20 points)**
Suppose you are given a set of 10 cities (including Boston, New York, and Chicago), and asked to plan an itinerary that visits each of the cities exactly once.

(a) (6 points) How many distinct itineraries begin with one of Boston, New York, or Chicago (but no other possible first cities)?
(b) (7 points) How many distinct itineraries have the property that you visit Boston either immediately before or immediately after New York?
(c) (7 points) How many distinct itineraries have the property that you visit Boston before New York (though not necessarily immediately before), and visit New York before Chicago (though not necessarily immediately before)?

**Problem 3. (20 points)** Suppose you have a standard (8-by-8) chessboard, and you place a marker on the bottom left square, which we'll call (1, 1). Your marker can now take single moves: either the marker can move one square to the right, or one square upward at any step. Thus, it will take 14 moves to get the marker to the upper right corner, which we'll call (8, 8).

(a) (6 points) How many distinct 14-move paths are there by which the marker can go from the bottom left corner to the upper right corner?
(b) (7 points) Suppose we want to avoid landing on the square (4, 4): that is, the square in the fourth row from the bottom and the fourth column from the left. How many distinct 14-move paths can get the marker from bottom left to upper right, subject to this constraint?
(c) (7 points) Suppose we disallow paths that include six or seven successive upward moves (for example, we do not allow the path that begins by moving the marker up seven rows, and then moves it rightward after that; and note that we don't care about rightward moves, just upward moves). Now how many legal paths are there?