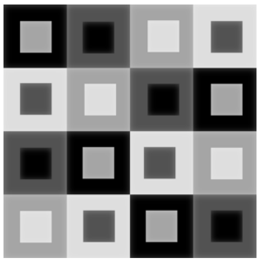
Frederik Lohner
CSCI 2824

Discrete Structures Problem Set 1

1a.



I started by filling out 2,1 Outer as blue since 4,4 made 1,4 Outer Blue impossible. Therefore 1,4 Outer must be Gold. This means that 2,4 Outer is Black. Which means that 4,1 Outer must be Red and 2,3 Outer must be Blue. We then continue this Sudoku like logic until we fill in all of the outer spaces. From then on it is a simple task to fill out the inner squares, beginning with 4,3 as Red since 1,1 Inner stops 4,1 Inner Red as a possibility, which in turn means that 4,1 Inner must be Gold, etc.
Apologies for the lack of color, printer was not cooperating.

1b. Assuming we don't have to declare our original starting position then we have 22 propositions. If we do have to declare our starting positions, then 32 propositions.

1c. $\text{SQ11CRed} \land \neg (\text{SQ11CGold} \lor \text{SQ11CBlue} \lor \text{SQ11CBlack})$

1d.
$$\begin{aligned}
&(\text{SQ14BGold} \land \neg (\text{SQ14BBlack} \lor \text{SQ14BBlue} \lor \text{SQ14BRed})) \land \\
&(\text{SQ24BBlack} \land \neg (\text{SQ24BGold} \lor \text{SQ24BBlue} \lor \text{SQ24BRed})) \land \\
&(\text{SQ34BRed} \land \neg (\text{SQ34BBBlack} \lor \text{SQ34BBlue} \lor \text{SQ34BGold})) \land \\
&(\text{SQ44BBlue} \land \neg (\text{SQ44BBlack} \lor \text{SQ44BGold} \lor \text{SQ44BRed}))
\end{aligned}$$

2a. All students live in the dormitories.
$\forall s \in S\ s \text{ live in dormitories}$
Not all students live in the dormitories. Some students do not live in the dormitories.
$\exists s \in S\ s\ \neg(\text{live in the dormitories})$

2b. All mathematics majors are males.
$\forall x \in X\ x \text{ are male}$
Not all mathematics majors are males. Some mathematics majors are not male.
$\exists x \in X\ x \text{ are } \neg(\text{male})$

2c. Some students are 25 years or older.
$\exists s \in S\ s \text{ are 25 years} \lor \text{older}$
All students are not 25 years or older.
$\forall s \in S\ s \text{ is } \neg (25 \lor \text{older})$

3a. $\neg (p \land q)$

3b. $\neg (p \land q \land r)$

3c. $(\neg p \land (q \land r)) \lor (\neg q \land (p \land r)) \lor (\neg r \land (p \land q))$

3d. $(p \land \neg (q \land r)) \lor (q \land \neg (p \land r)) \lor (r \land \neg (p \land q))$

3.2.

A	B	Stranger 1	Stranger 2
T	T	F	T
T	F	T	T
F	T	T	F
F	F	F	F

Both people are lying

4. $F(1)^2 + F(2)^2 + F(3)^2 + \ldots F(N)^2 = F(N) * F(N+1)$

For n = 1

$F(1)^2 = F(1) * F(2)$

$1 = 1 * (F(2-1) + F(2-2))$

$1 = 1 * (1+0)$

$1 = 1$

Is true for n = 1

For n=k

$F(1)^2 + F(2)^2 + F(3)^2 + \ldots F(k)^2 = F(k) * F(k+1)$

For n=k+1

$F(1)^2 + F(2)^2 + F(3)^2 + \ldots F(k)^2 + F(k+1)^2 = F(k+1) * F(k+1+1)$

$[F(1)^2 + F(2)^2 + F(3)^2 + \ldots F(k)^2] + F(k+1)^2 = F(k+1) * F(k+1+1)$

$[F(k) * F(k+1)] + F(K+1)^2 = F(k+1) * F(k+1+1)$

$[F(k) * F(k+1)] + [F(k+1) * F(k+1)] = F(k+1) * F(k+1+1)$

Divide both sides by F(k+1):

$[F(k) * 1] + [1 * F(k+1)] = F(k+1+1)$

$F(k) + F(k+1) = F(k+1+1)$

5. For n = 1

$$F(n) = \frac{((1+\sqrt{5})^{n}-(1-\sqrt{5})^{n})}{2^{n}\sqrt{5}}$$

$$F(1) = \frac{((1+\sqrt{5})^{1}-(1-\sqrt{5})^{1})}{2^{1}\sqrt{5}}$$

$$F(1) = \frac{2\sqrt{5}}{2^{1}\sqrt{5}}$$

1 = 1

For n=2

$$F(2) = \frac{((1+\sqrt{5})^{2}-(1-\sqrt{5})^{2})}{2^{2}\sqrt{5}}$$

$$(1+\sqrt{5})^{2} = 1+\sqrt{5}+\sqrt{5}+5 = 6+2\sqrt{5}$$

$$(1-\sqrt{5})^{2} = 1-\sqrt{5}+\sqrt{5}+5 = 6-2\sqrt{5}$$

$$F(2) = \frac{6+2\sqrt{5}-6+2\sqrt{5}}{2^{2}\sqrt{5}}$$

$$F(2) = \frac{2\sqrt{5}+2\sqrt{5}}{2^{2}\sqrt{5}}$$

$$F(2) = \frac{4\sqrt{5}}{4\sqrt{5}}$$

1=1

For n=k

$$F(k) = \frac{((1+\sqrt{5})^{k}-(1-\sqrt{5})^{k})}{2^{k}\sqrt{5}}$$

For n=k+1

$$F(k+1) = F(k-1)+F(k)$$

$$F(k+1) = \frac{((1+\sqrt{5})^{k-1}-(1-\sqrt{5})^{k-1})}{2^{k-1}\sqrt{5}} + \frac{((1+\sqrt{5})^{k}-(1-\sqrt{5})^{k})}{2^{k}\sqrt{5}}$$

$$F(k+1) = \frac{4(1+\sqrt{5})^{k-1}-4(1-\sqrt{5})^{k-1}+2(1+\sqrt{5})^{k}-2(1-\sqrt{5})^{k}}{2^{k+1}\sqrt{5}}$$

$$F(k+1) = \frac{(1+\sqrt{5})^{k-1}\left(4+2(1+\sqrt{5})\right)-(1-\sqrt{5})^{k-1}(4+2(1-\sqrt{5}))}{2^{k+1}\sqrt{5}}$$

$$F(k+1) = \frac{(1+\sqrt{5})^{k-1}(6+2\sqrt{5})-(1-\sqrt{5})^{k-1}(6-2\sqrt{5})}{2^{k+1}\sqrt{5}}$$

$$F(k+1) = \frac{(1+\sqrt{5})^{k-1}(1+\sqrt{5})^{2}-(1-\sqrt{5})^{k-1}(1-\sqrt{5})^{2}}{2^{k+1}\sqrt{5}}$$

$$F(k+1) = \frac{(1+\sqrt{5})^{k+1}-(1-\sqrt{5})^{k+1}}{2^{k+1}\sqrt{5}}$$

6a.

$$L(n-1)+L(n-2)= F(n-1)+F(n+1)$$

For n=2

$$L(2-1)+L(2-2)= F(2-1)+F(2+1)$$
$$L(1)+L(0)= F(1)+F(3)$$
$$1+2= 1+2$$

For n=k

$$L(k)= F(k-1)+F(k+1)$$

For n=k+1

$$L(k+1)=L(k-1)+L(k)$$

Using $L(k)= F(k-1)+F(k+1)$

$$L(k+1)=\big(F(k-2)+F(k)\big)+(F(k-1)+F(k+1))$$
$$L(k+1)=(F(k-2)+F(k-1))+(F(k)+F(k+1))$$
$$L(k+1)=F(k)+F(k+2)$$

6b.

For n=0

$$F(0)+L(0)=2F(0+1)$$
$$0+2= 2*1$$

For n=k

$$F(k)+L(k)=2F(k+1)$$

For n=k+1

$$F(k+1)+L(k+1)=F(k)+F(k-1)+L(k)+ L(k-1)$$
$$F(k+1)+L(k+1)=2F(k+1)+F(k-1)+L(k-1)$$
$$F(k+1)+L(k+1)=2F(k+1)+2F(k)$$
$$F(k+1)+L(k+1)=2F(k+2)$$