```prolog
list([]).
list([X|Xs]) :- list(Xs).

lista([a]).
lista([a|Xs]):-lista(Xs).

member(X,[X|Xs]).
member(X,[Y|Ys]) :- member(X,Ys).

/*
From this Prolog program,
state then prove the following properties:

- there exists x s.t. list(x)
- there exists x s.t. lista(x)
- we don't have lista([a,b])
- if lista(xs) then ground(xs)
- for all xs, if lista(xs) then list(xs)
- for all xs, if lista(xs) then list(xs) terminates
- for all xs, if list(xs) then lista(xs) terminates
- for all xs, if lista(xs) then lista([a|xs])
- for all x xs, if lista(xs) then member(x,xs) terminates
- for all x xs, if lista(xs) and member(x,xs) then x = a

*/
```

File: `ws6.pr`

Lemma 1 [*list:ex1*] $\exists x\ \mathbf{S}\,\mathtt{list}(x)$.
Proof. $\exists x\ \mathbf{S}\,\mathtt{list}(x)$ by **GAP**. □

Lemma 2 [*lista:ex2*] $\exists x\ \mathbf{S}\,\mathtt{lista}([x,x])$.
Proof. $\exists x\ \mathbf{S}\,\mathtt{lista}([x,x])$ by **GAP**. □

Lemma 3 [*lista:ff*] $\mathbf{S}\,\mathtt{lista}([\mathtt{b}]) \to \bot$.
Proof. $\mathbf{S}\,\mathtt{lista}([\mathtt{b}]) \to \bot$ by **GAP**. □

Lemma 4 [*lista:f1*] $\mathbf{F}\,\mathtt{lista}([\mathtt{a},\mathtt{b}])$.
Proof. $\mathbf{F}\,\mathtt{lista}([\mathtt{a},\mathtt{b}])$ by **GAP**. □

Lemma 5 [*lista:gr*] $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to gr(xs))$.
Proof. $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to gr(xs))$ by **GAP**. □

Lemma 6 [*lista:list*] $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{S}\,\mathtt{list}(xs))$.
Proof. $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{S}\,\mathtt{list}(xs))$ by **GAP**. □

Lemma 7 [*lista:term*] $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{T}\,\mathtt{lista}(xs))$.
Proof. $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{T}\,\mathtt{lista}(xs))$ by **GAP**. □

Lemma 8 [*lista:list:term*] $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{T}\,\mathtt{list}(xs))$.
Proof. $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{T}\,\mathtt{list}(xs))$ by **GAP**. □

Lemma 9 [*list:lista:term*] $\forall xs\,(\mathbf{S}\,\mathtt{list}(xs) \to \mathbf{T}\,\mathtt{lista}(xs))$.
Proof. $\forall xs\,(\mathbf{S}\,\mathtt{list}(xs) \to \mathbf{T}\,\mathtt{lista}(xs))$ by **GAP**. □

Lemma 10 [*lista:lista*] $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{S}\,\mathtt{lista}([\mathtt{a}|xs]))$.
Proof. $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{S}\,\mathtt{lista}([\mathtt{a}|xs]))$ by **GAP**. □

Lemma 11 [*lista:member:term*] $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{T}\,\mathtt{member}(\mathtt{a},xs))$.
Proof. $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{T}\,\mathtt{member}(\mathtt{a},xs))$ by **GAP**. □

Lemma 12 [*lista:member:succ*] $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{S}\,\mathtt{member}(\mathtt{a},xs))$.
Proof. $\forall xs\,(\mathbf{S}\,\mathtt{lista}(xs) \to \mathbf{S}\,\mathtt{member}(\mathtt{a},xs))$ by **GAP**. □

Lemma 13 [*lista:app*] $\forall x,xs\,(\mathbf{S}\,\mathtt{lista}(xs) \wedge \mathbf{S}\,\mathtt{member}(x,xs) \to x = \mathtt{a})$.
Proof. $\forall x,xs\,(\mathbf{S}\,\mathtt{lista}(xs) \wedge \mathbf{S}\,\mathtt{member}(x,xs) \to x = \mathtt{a})$ by **GAP**. □