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Lemma 1 [*p7:example4*] $p \wedge q \wedge r \rightarrow q \wedge r$.

Proof.

Assumption₀: $p \wedge q \wedge r$.

Thus₀: $p \wedge q \wedge r \rightarrow q \wedge r$. \square

Lemma 2 [*p8:exercise2:1*] $p \wedge q \wedge r \wedge s \wedge t \rightarrow q \wedge s$.

Proof. $p \wedge q \wedge r \wedge s \wedge t \rightarrow q \wedge s$ by **GAP**. \square

Lemma 3 [*p8:exercise2:2*] $p \wedge q \rightarrow q \wedge p$.

Proof. $p \wedge q \rightarrow q \wedge p$ by **GAP**. \square

Lemma 4 [*p8:exercise2:3*] $p \wedge q \wedge r \rightarrow p \wedge q \wedge r$.

Proof. $p \wedge q \wedge r \rightarrow p \wedge q \wedge r$ by **GAP**. \square

Lemma 5 [*p9:0*] $p \rightarrow \neg\neg p$.

Proof.

Assumption₀: p .

Contra₁: $\neg p$. \perp .

Thus₁: $\neg\neg p$.

Thus₀: $p \rightarrow \neg\neg p$. \square

Lemma 6 [*p9:1*] $\neg\neg p \rightarrow p$.

Proof.

Assumption₀: $\neg\neg p$.

Indirect₁: $\neg p$. \perp .

Thus₁: p .

Thus₀: $\neg\neg p \rightarrow p$. \square

Lemma 7 [*p9:example5:a*] $\neg\neg(q \wedge r) \rightarrow q \wedge r$.

Proof. $\neg\neg(q \wedge r) \rightarrow q \wedge r$ by **GAP**. \square

Lemma 8 [*p9:example5:b*] $p \wedge \neg\neg(q \wedge r) \rightarrow \neg\neg p \wedge r$.

Proof.

Assumption₀: $p \wedge \neg\neg(q \wedge r)$. $\neg\neg p$ by Lemma 5 [*p9:0*]. $q \wedge r$ by Lemma 7 [*p9:example5:a*]. r .

Thus₀: $p \wedge \neg\neg(q \wedge r) \rightarrow \neg\neg p \wedge r$. \square

Lemma 9 [*p10:0*] $p \wedge (p \rightarrow q) \rightarrow q$.

Proof. $p \wedge (p \rightarrow q) \rightarrow q$ by **GAP**. \square

Lemma 10 [*p10:1*] $(\neg p \vee q) \wedge (\neg p \vee q \rightarrow r \vee \neg p) \rightarrow r \vee \neg p$.

Proof. $(\neg p \vee q) \wedge (\neg p \vee q \rightarrow r \vee \neg p) \rightarrow r \vee \neg p$ by **GAP**. \square

Lemma 11 [*p11:0*] $p \wedge (p \rightarrow q) \wedge (p \rightarrow q \rightarrow r) \rightarrow r$.

Proof. $p \wedge (p \rightarrow q) \wedge (p \rightarrow q \rightarrow r) \rightarrow r$ by **GAP**. \square

Lemma 12 [*p11:example7:a*] $(q \rightarrow r) \wedge \neg r \rightarrow \neg q$.

Proof. $(q \rightarrow r) \wedge \neg r \rightarrow \neg q$ by **GAP**. \square

Lemma 13 [*p11:example7:b*] $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$.

Proof. $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$ by **GAP**. \square

Lemma 14 [*p11:example7:c*] $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$.

Proof. $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$ by **GAP**. \square

Lemma 15 [*p11:example7:d*] $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$.

Proof. $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$ by **GAP**. \square

Lemma 16 [*p12:example8:a*] $(\neg p \rightarrow q) \wedge \neg q \rightarrow p$.

Proof. $(\neg p \rightarrow q) \wedge \neg q \rightarrow p$ by **GAP**. \square

Lemma 17 [*p12:example8:b*] $(p \rightarrow \neg q) \wedge q \rightarrow \neg p$.

Proof. $(p \rightarrow \neg q) \wedge q \rightarrow \neg p$ by **GAP**. \square

Lemma 18 [*p12:exercise3:2:a*] $(p \rightarrow p \rightarrow q) \wedge p \rightarrow q$.

Proof. $(p \rightarrow p \rightarrow q) \wedge p \rightarrow q$ by **GAP**. \square

Lemma 19 [*p12:exercise3:2:b*] $(q \rightarrow p \rightarrow r) \wedge \neg r \wedge q \rightarrow \neg p$.

Proof. $(q \rightarrow p \rightarrow r) \wedge \neg r \wedge q \rightarrow \neg p$ by **GAP**. \square

Lemma 20 [*p13:0*] $(p \rightarrow q) \rightarrow \neg q \rightarrow \neg p$.

Proof. $(p \rightarrow q) \rightarrow \neg q \rightarrow \neg p$ by **GAP**. \square

Lemma 21 [*p14:0*] $(\neg q \rightarrow \neg p) \rightarrow p \rightarrow \neg \neg q$.

Proof. $(\neg q \rightarrow \neg p) \rightarrow p \rightarrow \neg \neg q$ by **GAP**. \square

Lemma 22 [*p15:0*] $p \rightarrow p$.

Proof. $p \rightarrow p$ by **GAP**. \square

Lemma 23 [*p15:example11*] $(q \rightarrow r) \rightarrow (\neg q \rightarrow \neg p) \rightarrow p \rightarrow r$.

Proof. $(q \rightarrow r) \rightarrow (\neg q \rightarrow \neg p) \rightarrow p \rightarrow r$ by **GAP**. \square

Lemma 24 [*p17:example13*] $(p \wedge q \rightarrow r) \rightarrow p \rightarrow q \rightarrow r$.

Proof. $(p \wedge q \rightarrow r) \rightarrow p \rightarrow q \rightarrow r$ by **GAP**. \square

Lemma 25 [*p18:example14*] $(p \rightarrow q \rightarrow r) \rightarrow p \wedge q \rightarrow r$.

Proof. $(p \rightarrow q \rightarrow r) \rightarrow p \wedge q \rightarrow r$ by **GAP**. \square

Lemma 26 [*p18:example15*] $(p \rightarrow q) \rightarrow p \wedge r \rightarrow q \wedge r$.

Proof. $(p \rightarrow q) \rightarrow p \wedge r \rightarrow q \wedge r$ by **GAP**. \square

Lemma 27 $[p19:0]$ $p \vee q \rightarrow q \vee p$.

Proof.

Assumption₀: $p \vee q$.

Case₁: p . $q \vee p$.

Case₁: q . $q \vee p$.

Hence₁, in all cases: $q \vee p$.

Thus₀: $p \vee q \rightarrow q \vee p$. \square

Lemma 28 $[p19:0:bis]$ $p \vee q \rightarrow q \vee p$.

Proof.

Assumption₀: $p \vee q$.

Thus₀: $p \vee q \rightarrow q \vee p$. \square

Lemma 29 $[p20:example16]$ $(q \rightarrow r) \rightarrow p \vee q \rightarrow p \vee r$.

Proof. $(q \rightarrow r) \rightarrow p \vee q \rightarrow p \vee r$ by **GAP**. \square

Lemma 30 $[p21:example17:0]$ $p \vee q \vee r \rightarrow p \vee q \vee r$.

Proof. $p \vee q \vee r \rightarrow p \vee q \vee r$ by **GAP**. \square

Lemma 31 $[p21:example17]$ $p \vee q \vee r \rightarrow p \vee q \vee r$.

Proof.

Assumption₀: $p \vee q \vee r$.

Case₁: p . $p \vee q \vee r$.

Case₁: q . $q \vee r$. $p \vee q \vee r$.

Case₁: r . $q \vee r$. $p \vee q \vee r$.

Hence₁, in all cases: $p \vee q \vee r$.

Thus₀: $p \vee q \vee r \rightarrow p \vee q \vee r$. \square

Lemma 32 $[p21:example18]$ $p \wedge (q \vee r) \rightarrow p \wedge q \vee p \wedge r$.

Proof. $p \wedge (q \vee r) \rightarrow p \wedge q \vee p \wedge r$ by **GAP**. \square

Lemma 33 $[p22:exercise4:1:a]$ $p \wedge q \rightarrow p$.

Proof. $p \wedge q \rightarrow p$ by **GAP**. \square

Lemma 34 $[p22:exercise4:1:b]$ $p \rightarrow q \rightarrow p \wedge q$.

Proof. $p \rightarrow q \rightarrow p \wedge q$ by **GAP**. \square

Lemma 35 $[p22:exercise4:1:c]$ $p \rightarrow (p \rightarrow q) \rightarrow q$.

Proof. $p \rightarrow (p \rightarrow q) \rightarrow q$ by **GAP**. \square

Lemma 36 $[p23:0]$ $p \wedge \neg p \rightarrow q$.

Proof. $p \wedge \neg p \rightarrow q$ by **GAP**. \square

Lemma 37 $[p25:example20]$ $\neg p \vee q \rightarrow p \rightarrow q$.

Proof. $\neg p \vee q \rightarrow p \rightarrow q$ by **GAP**. \square

Lemma 38 [*p25:example21*] $(p \rightarrow q) \wedge (p \rightarrow \neg q) \rightarrow \neg p$.

Proof. $(p \rightarrow q) \wedge (p \rightarrow \neg q) \rightarrow \neg p$ by **GAP**. \square

Lemma 39 [*p26:example21*] $(p \rightarrow \neg p) \rightarrow \neg p$.

Proof. $(p \rightarrow \neg p) \rightarrow \neg p$ by **GAP**. \square

Lemma 40 [*p26:example22*] $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$.

Proof. $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$ by **GAP**. \square

Lemma 41 [*p26:example23*] $(p \wedge \neg q \rightarrow r) \wedge \neg r \wedge p \rightarrow q$.

Proof. $(p \wedge \neg q \rightarrow r) \wedge \neg r \wedge p \rightarrow q$ by **GAP**. \square

Lemma 42 [*p26:example23:bis*] $(p \wedge \neg q \rightarrow r) \wedge \neg r \wedge p \rightarrow q$.

Proof. $(p \wedge \neg q \rightarrow r) \wedge \neg r \wedge p \rightarrow q$ by **GAP**. \square

Lemma 43 [*p30:example24*] $(p \rightarrow q) \rightarrow \neg p \vee q$.

Proof.

Assumption₀: $p \rightarrow q$. $\neg p \vee p$.

Case₁: $\neg p$. $\neg p \vee q$.

Case₁: p . q . $\neg p \vee q$.

Hence₁, in all cases: $\neg p \vee q$.

Thus₀: $(p \rightarrow q) \rightarrow \neg p \vee q$. \square

Lemma 44 [*p34:exequiv1:gd*] $\neg(p \wedge q) \rightarrow \neg p \vee \neg q$.

Proof. $\neg(p \wedge q) \rightarrow \neg p \vee \neg q$ by **GAP**. \square

Lemma 45 [*p34:exequiv1:dg*] $\neg p \vee \neg q \rightarrow \neg(p \wedge q)$.

Proof. $\neg p \vee \neg q \rightarrow \neg(p \wedge q)$ by **GAP**. \square

Lemma 46 [*p34:exequiv1*] $\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$.

Proof. $\neg(p \wedge q) \rightarrow \neg p \vee \neg q$ by Lemma 44 [*p34:exequiv1:gd*]. $\neg p \vee \neg q \rightarrow \neg(p \wedge q)$ by Lemma 45 [*p34:exequiv1:dg*]. \square