

File: ws5.pr

Lemma 1 [nat:ground] $\forall x (\mathbf{S}\text{nat}(x) \rightarrow gr(x))$.

Proof. \perp by **GAP**. \square

Lemma 2 [add:ground:1] $\forall x, y, z (\mathbf{S}\text{add}(x, y, z) \rightarrow gr(x))$.

Proof. \perp by **GAP**. \square

Lemma 3 [add:ground:2] $\forall x, y, z (\mathbf{S}\text{add}(x, y, z) \wedge gr(y) \rightarrow gr(z))$.

Proof. \perp by **GAP**. \square

Lemma 4 [add:ground:3] $\forall x, y, z (\mathbf{S}\text{add}(x, y, z) \wedge gr(z) \rightarrow gr(y))$.

Proof. \perp by **GAP**. \square

Lemma 5 [add:types:2] $\forall x, y, z (\mathbf{S}\text{add}(x, y, z) \wedge \mathbf{S}\text{nat}(y) \rightarrow \mathbf{S}\text{nat}(z))$.

Proof. \perp by **GAP**. \square

Lemma 6 [add:types:3] $\forall x, y, z (\mathbf{S}\text{add}(x, y, z) \wedge \mathbf{S}\text{nat}(z) \rightarrow \mathbf{S}\text{nat}(y))$.

Proof. \perp by **GAP**. \square

Lemma 7 [nat:termination] $\forall x (\mathbf{S}\text{nat}(x) \rightarrow \mathbf{T}\text{nat}(x))$.

Proof. \perp by **GAP**. \square

Lemma 8 [add:termination:1] $\forall x, y, z (\mathbf{S}\text{nat}(x) \rightarrow \mathbf{T}\text{add}(x, y, z))$.

Proof. \perp by **GAP**. \square

Lemma 9 [add:termination:2] $\forall x, y, z (\mathbf{S}\text{nat}(z) \rightarrow \mathbf{T}\text{add}(x, y, z))$.

Proof. \perp by **GAP**. \square

Lemma 10 [add:termination:3] $\forall x, y, z (\mathbf{S}\text{add}(x, y, z) \rightarrow \mathbf{T}\text{add}(x, y, z))$.

Proof. \perp by **GAP**. \square