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Lemma 1 [p7:example4] $p \wedge q \wedge r \rightarrow q \wedge r$.

Proof.

Assumption₀: $p \wedge q \wedge r$.

Thus₀: $p \wedge q \wedge r \rightarrow q \wedge r$. \square

Lemma 2 [p8:exercise2:1] $p \wedge q \wedge r \wedge s \wedge t \rightarrow q \wedge s$.

Proof. $p \wedge q \wedge r \wedge s \wedge t \rightarrow q \wedge s$ by **GAP**. \square

Lemma 3 [p8:exercise2:2] $p \wedge q \rightarrow q \wedge p$.

Proof. $p \wedge q \rightarrow q \wedge p$ by **GAP**. \square

Lemma 4 [p8:exercise2:3] $p \wedge q \wedge r \rightarrow p \wedge q \wedge r$.

Proof. $p \wedge q \wedge r \rightarrow p \wedge q \wedge r$ by **GAP**. \square

Lemma 5 [p9:0] $p \rightarrow \neg\neg p$.

Proof.

Assumption₀: p .

Contra₁: $\neg p$. \perp .

Thus₁: $\neg\neg p$.

Thus₀: $p \rightarrow \neg\neg p$. \square

Lemma 6 [p9:1] $\neg\neg p \rightarrow p$.

Proof.

Assumption₀: $\neg\neg p$.

Indirect₁: $\neg p$. \perp .

Thus₁: p .

Thus₀: $\neg\neg p \rightarrow p$. \square

Lemma 7 [p9:example5:a] $\neg\neg(q \wedge r) \rightarrow q \wedge r$.

Proof. $\neg\neg(q \wedge r) \rightarrow q \wedge r$ by **GAP**. \square

Lemma 8 [p9:example5:b] $p \wedge \neg\neg(q \wedge r) \rightarrow \neg\neg p \wedge r$.

Proof.

Assumption₀: $p \wedge \neg\neg(q \wedge r)$. $\neg\neg p$ by Lemma 5 [p9:0]. $q \wedge r$ by Lemma 7 [p9:example5:a]. r .

Thus₀: $p \wedge \neg\neg(q \wedge r) \rightarrow \neg\neg p \wedge r$. \square

Lemma 9 [p10:0] $p \wedge (p \rightarrow q) \rightarrow q$.

Proof. $p \wedge (p \rightarrow q) \rightarrow q$ by **GAP**. \square

Lemma 10 [p10:1] $(\neg p \vee q) \wedge (\neg p \vee q \rightarrow r \vee \neg p) \rightarrow r \vee \neg p$.

Proof. $(\neg p \vee q) \wedge (\neg p \vee q \rightarrow r \vee \neg p) \rightarrow r \vee \neg p$ by **GAP**. \square

Lemma 11 [p11:0] $p \wedge (p \rightarrow q) \wedge (p \rightarrow q \rightarrow r) \rightarrow r$.

Proof. $p \wedge (p \rightarrow q) \wedge (p \rightarrow q \rightarrow r) \rightarrow r$ by **GAP**. \square

Lemma 12 [p11:example7:a] $(q \rightarrow r) \wedge \neg r \rightarrow \neg q$.

Proof. $(q \rightarrow r) \wedge \neg r \rightarrow \neg q$ by **GAP**. \square

Lemma 13 [p11:example7:b] $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$.

Proof. $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$ by **GAP**. \square

Lemma 14 [p11:example7:c] $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$.

Proof. $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$ by **GAP**. \square

Lemma 15 [p11:example7:d] $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$.

Proof. $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$ by **GAP**. \square

Lemma 16 [p12:example8:a] $(\neg p \rightarrow q) \wedge \neg q \rightarrow p$.

Proof. $(\neg p \rightarrow q) \wedge \neg q \rightarrow p$ by **GAP**. \square

Lemma 17 [p12:example8:b] $(p \rightarrow \neg q) \wedge q \rightarrow \neg p$.

Proof. $(p \rightarrow \neg q) \wedge q \rightarrow \neg p$ by **GAP**. \square

Lemma 18 [p12:exercise3:2:a] $(p \rightarrow p \rightarrow q) \wedge p \rightarrow q$.

Proof. $(p \rightarrow p \rightarrow q) \wedge p \rightarrow q$ by **GAP**. \square

Lemma 19 [p12:exercise3:2:b] $(q \rightarrow p \rightarrow r) \wedge \neg r \wedge q \rightarrow \neg p$.

Proof. $(q \rightarrow p \rightarrow r) \wedge \neg r \wedge q \rightarrow \neg p$ by **GAP**. \square

Lemma 20 [p13:0] $(p \rightarrow q) \rightarrow \neg q \rightarrow \neg p$.

Proof. $(p \rightarrow q) \rightarrow \neg q \rightarrow \neg p$ by **GAP**. \square

Lemma 21 [p14:0] $(\neg q \rightarrow \neg p) \rightarrow p \rightarrow \neg \neg q$.

Proof. $(\neg q \rightarrow \neg p) \rightarrow p \rightarrow \neg \neg q$ by **GAP**. \square

Lemma 22 [p15:0] $p \rightarrow p$.

Proof. $p \rightarrow p$ by **GAP**. \square

Lemma 23 [p15:example11] $(q \rightarrow r) \rightarrow (\neg q \rightarrow \neg p) \rightarrow p \rightarrow r$.

Proof. $(q \rightarrow r) \rightarrow (\neg q \rightarrow \neg p) \rightarrow p \rightarrow r$ by **GAP**. \square

Lemma 24 [p17:example13] $(p \wedge q \rightarrow r) \rightarrow p \rightarrow q \rightarrow r$.

Proof. $(p \wedge q \rightarrow r) \rightarrow p \rightarrow q \rightarrow r$ by **GAP**. \square

Lemma 25 [p18:example14] $(p \rightarrow q \rightarrow r) \rightarrow p \wedge q \rightarrow r$.

Proof. $(p \rightarrow q \rightarrow r) \rightarrow p \wedge q \rightarrow r$ by **GAP**. \square

Lemma 26 [p18:example15] $(p \rightarrow q) \rightarrow p \wedge r \rightarrow q \wedge r$.

Proof. $(p \rightarrow q) \rightarrow p \wedge r \rightarrow q \wedge r$ by **GAP**. \square

Lemma 27 [p19:0] $p \vee q \rightarrow q \vee p$.

Proof.

Assumption₀: $p \vee q$.

Case₁: p . $q \vee p$.

Case₁: q . $q \vee p$.

Hence₁, in all cases: $q \vee p$.

Thus₀: $p \vee q \rightarrow q \vee p$. \square

Lemma 28 [p19:0:bis] $p \vee q \rightarrow q \vee p$.

Proof.

Assumption₀: $p \vee q$.

Thus₀: $p \vee q \rightarrow q \vee p$. \square

Lemma 29 [p20:example16] $(q \rightarrow r) \rightarrow p \vee q \rightarrow p \vee r$.

Proof. $(q \rightarrow r) \rightarrow p \vee q \rightarrow p \vee r$ by **GAP**. \square

Lemma 30 [p21:example17:0] $p \vee q \vee r \rightarrow p \vee q \vee r$.

Proof. $p \vee q \vee r \rightarrow p \vee q \vee r$ by **GAP**. \square

Lemma 31 [p21:example17] $p \vee q \vee r \rightarrow p \vee q \vee r$.

Proof.

Assumption₀: $p \vee q \vee r$.

Case₁: p . $p \vee q \vee r$.

Case₁: q . $q \vee r$. $p \vee q \vee r$.

Case₁: r . $q \vee r$. $p \vee q \vee r$.

Hence₁, in all cases: $p \vee q \vee r$.

Thus₀: $p \vee q \vee r \rightarrow p \vee q \vee r$. \square

Lemma 32 [p21:example18] $p \wedge (q \vee r) \rightarrow p \wedge q \vee p \wedge r$.

Proof. $p \wedge (q \vee r) \rightarrow p \wedge q \vee p \wedge r$ by **GAP**. \square

Lemma 33 [p22:exercise4:1:a] $p \wedge q \rightarrow p$.

Proof. $p \wedge q \rightarrow p$ by **GAP**. \square

Lemma 34 [p22:exercise4:1:b] $p \rightarrow q \rightarrow p \wedge q$.

Proof. $p \rightarrow q \rightarrow p \wedge q$ by **GAP**. \square

Lemma 35 [p22:exercise4:1:c] $p \rightarrow (p \rightarrow q) \rightarrow q$.

Proof. $p \rightarrow (p \rightarrow q) \rightarrow q$ by **GAP**. \square

Lemma 36 [p23:0] $p \wedge \neg p \rightarrow q$.

Proof. $p \wedge \neg p \rightarrow q$ by **GAP**. \square

Lemma 37 [p25:example20] $\neg p \vee q \rightarrow p \rightarrow q$.

Proof. $\neg p \vee q \rightarrow p \rightarrow q$ by **GAP**. \square

Lemma 38 [p25:example21] $(p \rightarrow q) \wedge (p \rightarrow \neg q) \rightarrow \neg p$.

Proof. $(p \rightarrow q) \wedge (p \rightarrow \neg q) \rightarrow \neg p$ by **GAP**. \square

Lemma 39 [p26:example21] $(p \rightarrow \neg p) \rightarrow \neg p$.

Proof. $(p \rightarrow \neg p) \rightarrow \neg p$ by **GAP**. \square

Lemma 40 [p26:example22] $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$.

Proof. $(p \rightarrow q \rightarrow r) \wedge p \wedge \neg r \rightarrow \neg q$ by **GAP**. \square

Lemma 41 [p26:example23] $(p \wedge \neg q \rightarrow r) \wedge \neg r \wedge p \rightarrow q$.

Proof. $(p \wedge \neg q \rightarrow r) \wedge \neg r \wedge p \rightarrow q$ by **GAP**. \square

Lemma 42 [p26:example23:bis] $(p \wedge \neg q \rightarrow r) \wedge \neg r \wedge p \rightarrow q$.

Proof. $(p \wedge \neg q \rightarrow r) \wedge \neg r \wedge p \rightarrow q$ by **GAP**. \square

Lemma 43 [p30:example24] $(p \rightarrow q) \rightarrow \neg p \vee q$.

Proof.

Assumption₀: $p \rightarrow q$. $\neg p \vee p$.

Case₁: $\neg p$. $\neg p \vee q$.

Case₂: p . q . $\neg p \vee q$.

Hence₁, in all cases: $\neg p \vee q$.

Thus₀: $(p \rightarrow q) \rightarrow \neg p \vee q$. \square

Lemma 44 [p34:exequiv1:gd] $\neg(p \wedge q) \rightarrow \neg p \vee \neg q$.

Proof. $\neg(p \wedge q) \rightarrow \neg p \vee \neg q$ by **GAP**. \square

Lemma 45 [p34:exequiv1:dg] $\neg p \vee \neg q \rightarrow \neg(p \wedge q)$.

Proof. $\neg p \vee \neg q \rightarrow \neg(p \wedge q)$ by **GAP**. \square

Lemma 46 [p34:exequiv1] $\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$.

Proof. $\neg(p \wedge q) \rightarrow \neg p \vee \neg q$ by Lemma 44 [p34:exequiv1:gd]. $\neg p \vee \neg q \rightarrow \neg(p \wedge q)$ by Lemma 45 [p34:exequiv1:dg]. \square