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Lemma 1 $[nat:ground]$ $\forall x (\mathbf{S\,nat}(x) \rightarrow gr(x))$.

Proof. \perp by **GAP**. \square

Lemma 2 $[add:ground:1]$ $\forall x, y, z (\mathbf{S\,add}(x, y, z) \rightarrow gr(x))$.

Proof. \perp by **GAP**. \square

Lemma 3 $[add:ground:2]$ $\forall x, y, z (\mathbf{S\,add}(x, y, z) \wedge gr(y) \rightarrow gr(z))$.

Proof. \perp by **GAP**. \square

Lemma 4 $[add:ground:3]$ $\forall x, y, z (\mathbf{S\,add}(x, y, z) \wedge gr(z) \rightarrow gr(y))$.

Proof. \perp by **GAP**. \square

Lemma 5 $[add:types:2]$ $\forall x, y, z (\mathbf{S\,add}(x, y, z) \wedge \mathbf{S\,nat}(y) \rightarrow \mathbf{S\,nat}(z))$.

Proof. \perp by **GAP**. \square

Lemma 6 $[add:types:3]$ $\forall x, y, z (\mathbf{S\,add}(x, y, z) \wedge \mathbf{S\,nat}(z) \rightarrow \mathbf{S\,nat}(y))$.

Proof. \perp by **GAP**. \square

Lemma 7 $[nat:termination]$ $\forall x (\mathbf{S\,nat}(x) \rightarrow \mathbf{T\,nat}(x))$.

Proof. \perp by **GAP**. \square

Lemma 8 $[add:termination:1]$ $\forall x, y, z (\mathbf{S\,nat}(x) \rightarrow \mathbf{T\,add}(x, y, z))$.

Proof. \perp by **GAP**. \square

Lemma 9 $[add:termination:2]$ $\forall x, y, z (\mathbf{S\,nat}(z) \rightarrow \mathbf{T\,add}(x, y, z))$.

Proof. \perp by **GAP**. \square

Lemma 10 $[add:termination:3]$ $\forall x, y, z (\mathbf{S\,add}(x, y, z) \rightarrow \mathbf{T\,add}(x, y, z))$.

Proof. \perp by **GAP**. \square