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Lemma 1 [*man:odilon*] $\textsf{\textbf{S}}\,\texttt{man}(\texttt{odilon})$.

Proof. $\textsf{\textbf{S}}\,\texttt{man}(\texttt{odilon})$ by completion. □

Lemma 2 [*woman:genevieve*] $\textsf{\textbf{S}}\,\texttt{woman}(\texttt{genevieve})$.

Proof. $\textsf{\textbf{S}}\,\texttt{woman}(\texttt{genevieve})$ by completion. □

Lemma 3 [*man:exists*] $\exists x\ \textsf{\textbf{S}}\,\texttt{man}(x)$.

Proof. $\textsf{\textbf{S}}\,\texttt{man}(\texttt{odilon})$ by Lemma 1 [*man:odilon*]. □

Lemma 4 [*woman:existsnot*] $\exists x\ \textsf{\textbf{F}}\,\texttt{woman}(x)$.

Proof. $\textsf{\textbf{F}}\,\texttt{woman}(\texttt{toto})$ by completion. □

Lemma 5 [*man:term*] $\forall x\ \textsf{\textbf{T}}\,\texttt{man}(x)$.

Proof. $\textsf{\textbf{T}}\,\texttt{man}(x)$ by completion. □

Lemma 6 [*woman:term*] $\forall x\ \textsf{\textbf{T}}\,\texttt{woman}(x)$.

Proof. $\textsf{\textbf{T}}\,\texttt{woman}(x)$ by completion. □

Lemma 7 [*man:ground*] $\forall x\,(\textsf{\textbf{S}}\,\texttt{man}(x) \to gr(x))$.

Proof.
Assumption$_0$: $\textsf{\textbf{S}}\,\texttt{man}(x)$. $\textsf{\textbf{D}}\,\textsf{\textbf{S}}\,\texttt{man}(x)$ by completion.
  Case$_1$: $x = \texttt{odilon}$. $gr(\texttt{odilon})$.
  Case$_1$: $x = \texttt{remy}$. $gr(\texttt{remy})$.
  Case$_1$: $x = \texttt{edouard}$. $gr(\texttt{edouard})$.
  Hence$_1$, in all cases: $gr(x)$.
Thus$_0$: $\textsf{\textbf{S}}\,\texttt{man}(x) \to gr(x)$. □

Lemma 8 [*woman:ground*] $\forall x\,(\textsf{\textbf{S}}\,\texttt{woman}(x) \to gr(x))$.

Proof.
Assumption$_0$: $\textsf{\textbf{S}}\,\texttt{woman}(x)$. $\textsf{\textbf{D}}\,\textsf{\textbf{S}}\,\texttt{woman}(x)$ by completion.
  Case$_1$: $x = \texttt{genevieve}$. $gr(\texttt{genevieve})$.
  Case$_1$: $x = \texttt{pauline}$. $gr(\texttt{pauline})$.
  Case$_1$: $x = \texttt{melaine}$. $gr(\texttt{melaine})$.
  Hence$_1$, in all cases: $gr(x)$.
Thus$_0$: $\textsf{\textbf{S}}\,\texttt{woman}(x) \to gr(x)$. □

Lemma 9 [*man:notwoman*] $\forall x\, (\mathbf{S}\,\mathtt{man}(x) \rightarrow \mathbf{F}\,\mathtt{woman}(x))$.

Proof.

Assumption$_0$: $\mathbf{S}\,\mathtt{man}(x)$.

Contra$_1$: $x = \mathtt{genevieve}$. $\mathbf{D}\,\mathbf{S}\,\mathtt{man}(x)$ by completion.

Case$_2$: $x = \mathtt{odilon}$.

Case$_2$: $x = \mathtt{remy}$.

Case$_2$: $x = \mathtt{edouard}$.

Hence$_2$, in all cases: $\bot$.

Thus$_1$: $\neg x = \mathtt{genevieve}$.

Contra$_1$: $x = \mathtt{pauline}$. $\mathbf{D}\,\mathbf{S}\,\mathtt{man}(x)$ by completion.

Case$_2$: $x = \mathtt{odilon}$.

Case$_2$: $x = \mathtt{remy}$.

Case$_2$: $x = \mathtt{edouard}$.

Hence$_2$, in all cases: $\bot$.

Thus$_1$: $\neg x = \mathtt{pauline}$.

Contra$_1$: $x = \mathtt{melaine}$. $\mathbf{D}\,\mathbf{S}\,\mathtt{man}(x)$ by completion.

Case$_2$: $x = \mathtt{odilon}$.

Case$_2$: $x = \mathtt{remy}$.

Case$_2$: $x = \mathtt{edouard}$.

Hence$_2$, in all cases: $\bot$.

Thus$_1$: $\neg x = \mathtt{melaine}$. $\mathbf{F}\,\mathtt{woman}(x)$ by completion.

Thus$_0$: $\mathbf{S}\,\mathtt{man}(x) \rightarrow \mathbf{F}\,\mathtt{woman}(x)$. $\square$

Lemma 10 [*child:groundground*] $\forall x, y\, (\mathbf{S}\,\mathtt{child}(x, y) \rightarrow gr(x) \wedge gr(y))$.

Proof.

Assumption$_0$: $\mathbf{S}\,\mathtt{child}(x, y)$. $\mathbf{D}\,\mathbf{S}\,\mathtt{child}(x, y)$ by completion.

Case$_1$: $x = \mathtt{melaine} \wedge y = \mathtt{odilon}$. $gr(\mathtt{melaine})$.

Case$_1$: $x = \mathtt{melaine} \wedge y = \mathtt{genevieve}$. $gr(\mathtt{melaine})$.

Case$_1$: $x = \mathtt{pauline} \wedge y = \mathtt{odilon}$. $gr(\mathtt{pauline})$.

Case$_1$: $x = \mathtt{pauline} \wedge y = \mathtt{genevieve}$. $gr(\mathtt{pauline})$.

Case$_1$: $x = \mathtt{edouard} \wedge y = \mathtt{odilon}$. $gr(\mathtt{edouard})$.

Case$_1$: $x = \mathtt{edouard} \wedge y = \mathtt{genevieve}$. $gr(\mathtt{edouard})$.

Hence$_1$, in all cases: $gr(x)$. $\mathbf{D}\,\mathbf{S}\,\mathtt{child}(x, y)$ by completion.

Case$_1$: $x = \mathtt{melaine} \wedge y = \mathtt{odilon}$. $gr(\mathtt{odilon})$.

Case$_1$: $x = \mathtt{melaine} \wedge y = \mathtt{genevieve}$. $gr(\mathtt{genevieve})$.

Case$_1$: $x = \mathtt{pauline} \wedge y = \mathtt{odilon}$. $gr(\mathtt{odilon})$.

Case$_1$: $x = \mathtt{pauline} \wedge y = \mathtt{genevieve}$. $gr(\mathtt{genevieve})$.

Case$_1$: $x = \mathtt{edouard} \wedge y = \mathtt{odilon}$. $gr(\mathtt{odilon})$.

Case$_1$: $x = \mathtt{edouard} \wedge y = \mathtt{genevieve}$. $gr(\mathtt{genevieve})$.

Hence$_1$, in all cases: $gr(y)$.

Thus$_0$: $\mathbf{S}\,\mathtt{child}(x, y) \rightarrow gr(x) \wedge gr(y)$. $\square$

Lemma 11 [*father:groundground*] $\forall x, y\, (\mathbf{S}\,\mathtt{father}(x, y) \rightarrow gr(x) \wedge gr(y))$.

Proof.

Assumption$_0$: $\mathbf{S}\,\mathtt{father}(x, y)$. $\mathbf{D}\,\mathbf{S}\,\mathtt{father}(x, y)$ by completion. $gr(x)$ by Lemma 7 [*man:ground*].

$\mathbf{D}\,\mathbf{S}\,\mathtt{father}(x, y)$ by completion. $gr(y)$ by Lemma 10 [*child:groundground*].

Thus$_0$: $\mathbf{S}\,\mathtt{father}(x, y) \rightarrow gr(x) \wedge gr(y)$. $\square$

Lemma 12 [*father:man*] $\forall x, y\,(\mathsf{S}\,\mathtt{father}(x,y) \to \mathsf{S}\,\mathtt{man}(x))$.

Proof.
Assumption$_0$: $\mathsf{S}\,\mathtt{father}(x,y)$. $\mathsf{D}\,\mathsf{S}\,\mathtt{father}(x,y)$ by completion.
Thus$_0$: $\mathsf{S}\,\mathtt{father}(x,y) \to \mathsf{S}\,\mathtt{man}(x)$. □

Lemma 13 [*father:child*] $\forall x, y\,(\mathsf{S}\,\mathtt{father}(x,y) \to \mathsf{S}\,\mathtt{child}(y,x))$.

Proof.
Assumption$_0$: $\mathsf{S}\,\mathtt{father}(x,y)$. $\mathsf{D}\,\mathsf{S}\,\mathtt{father}(x,y)$ by completion. $\mathsf{S}\,\mathtt{child}(y,x)$.
Thus$_0$: $\mathsf{S}\,\mathtt{father}(x,y) \to \mathsf{S}\,\mathtt{child}(y,x)$. □

Lemma 14 [*brother:brother_or_sister*] $\forall x, y\,(\mathsf{S}\,\mathtt{brother}(x,y) \to \mathsf{S}\,\mathtt{brother\_or\_sister}(x,y))$.

Proof.
Assumption$_0$: $\mathsf{S}\,\mathtt{brother}(x,y)$. $\mathsf{D}\,\mathsf{S}\,\mathtt{brother}(x,y)$ by completion.
Thus$_0$: $\mathsf{S}\,\mathtt{brother}(x,y) \to \mathsf{S}\,\mathtt{brother\_or\_sister}(x,y)$. □

Lemma 15 [*sister:brother_or_sister*] $\forall x, y\,(\mathsf{S}\,\mathtt{sister}(x,y) \to \mathsf{S}\,\mathtt{brother\_or\_sister}(x,y))$.

Proof.
Assumption$_0$: $\mathsf{S}\,\mathtt{sister}(x,y)$. $\mathsf{D}\,\mathsf{S}\,\mathtt{sister}(x,y)$ by completion.
Thus$_0$: $\mathsf{S}\,\mathtt{sister}(x,y) \to \mathsf{S}\,\mathtt{brother\_or\_sister}(x,y)$. □

Lemma 16 [*brother_or_sister:brother_or_sister*] $\forall x, y\,(\mathsf{S}\,\mathtt{brother}(x,y) \lor \mathsf{S}\,\mathtt{sister}(x,y) \to \mathsf{S}\,\mathtt{brother\_or\_sister}(x,y))$.

Proof.
Assumption$_0$: $\mathsf{S}\,\mathtt{brother}(x,y) \lor \mathsf{S}\,\mathtt{sister}(x,y)$.
Case$_1$: $\mathsf{S}\,\mathtt{brother}(x,y)$. $\mathsf{D}\,\mathsf{S}\,\mathtt{brother}(x,y)$ by completion. $\mathsf{D}\,\mathsf{S}\,\mathtt{brother\_or\_sister}(x,y)$ by completion.
Case$_1$: $\mathsf{S}\,\mathtt{sister}(x,y)$. $\mathsf{D}\,\mathsf{S}\,\mathtt{sister}(x,y)$ by completion. $\mathsf{D}\,\mathsf{S}\,\mathtt{brother\_or\_sister}(x,y)$ by completion.
Hence$_1$, in all cases: $\exists z\,(\mathsf{S}\,\mathtt{child}(x,z) \land \mathsf{S}\,\mathtt{child}(y,z) \land \mathsf{S}\,\mathtt{\neq}(x,y))$. $\mathsf{S}\,\mathtt{brother\_or\_sister}(x,y)$ by completion.
Thus$_0$: $\mathsf{S}\,\mathtt{brother}(x,y) \lor \mathsf{S}\,\mathtt{sister}(x,y) \to \mathsf{S}\,\mathtt{brother\_or\_sister}(x,y)$. □

Lemma 17 [*child:man_ou_woman*] $\forall x, y\,(\mathsf{S}\,\mathtt{child}(x,y) \to \mathsf{S}\,\mathtt{man}(x) \lor \mathsf{S}\,\mathtt{woman}(x))$.

Proof.
Assumption$_0$: $\mathsf{S}\,\mathtt{child}(x,y)$. $\mathsf{D}\,\mathsf{S}\,\mathtt{child}(x,y)$ by completion.
Case$_1$: $x = \mathtt{melaine} \land y = \mathtt{odilon}$. $\mathsf{S}\,\mathtt{woman}(x)$ by completion.
Case$_1$: $x = \mathtt{melaine} \land y = \mathtt{genevieve}$. $\mathsf{S}\,\mathtt{woman}(x)$ by completion.
Case$_1$: $x = \mathtt{pauline} \land y = \mathtt{odilon}$. $\mathsf{S}\,\mathtt{woman}(x)$ by completion.
Case$_1$: $x = \mathtt{pauline} \land y = \mathtt{genevieve}$. $\mathsf{S}\,\mathtt{woman}(x)$ by completion.
Case$_1$: $x = \mathtt{edouard} \land y = \mathtt{odilon}$. $\mathsf{S}\,\mathtt{man}(x)$ by completion.
Case$_1$: $x = \mathtt{edouard} \land y = \mathtt{genevieve}$. $\mathsf{S}\,\mathtt{man}(x)$ by completion.
Hence$_1$, in all cases: $\mathsf{S}\,\mathtt{man}(x) \lor \mathsf{S}\,\mathtt{woman}(x)$.
Thus$_0$: $\mathsf{S}\,\mathtt{child}(x,y) \to \mathsf{S}\,\mathtt{man}(x) \lor \mathsf{S}\,\mathtt{woman}(x)$. □