Project Two Template

MAT-350: Applied Linear Algebra

Student Name

Date

Problem 1

Use the svd() function in MATLAB to compute A_1, the **rank-1 approximation of** A**.** Clearly state what A_1 is, rounded to 4 decimal places. Also, **compute** the root-mean square error (RMSE) between A and A_1.

Solution:

```
% Consider the matrix A.
A = [1 2 2; 3 4 5; 6 7 8]
```

```
A = 3×3
     1     2     2
     3     4     5
     6     7     8
```

```
% Returns 3 matricies, the mxm orthogonal matrix U,
% the mxn diagonal matrix S, and the nxn orthogonal matrix V.
[U, S, V] = svd(A)
```

```
U = 3×3
   -0.2055   -0.6658   -0.7172
   -0.4900   -0.5644    0.6643
   -0.8471    0.4880   -0.2103
S = 3×3
   14.4042         0         0
         0    0.6450         0
         0         0    0.3229
V = 3×3
   -0.4692    0.8820    0.0433
   -0.5763   -0.2687   -0.7718
   -0.6691   -0.3871    0.6344
```

```
% Compute A1 using 1st column of U, 1st column and row of S, and 1st column
% of V.
A1 = U(:,1:1)*S(1:1,1:1)*V(:,1:1)'
```

```
A1 = 3×3
    1.3889    1.7059    1.9807
    3.3118    4.0678    4.7230
    5.7253    7.0322    8.1649
```

```
% Root mean square error (RMSE) between The matrix A and The matrix A1.
RMSE = sqrt(mean((A(:)-A1(:)).^2))
```

```
RMSE = 0.2404
```

Problem 2

Use the svd() function in MATLAB to compute A_2, the **rank-2 approximation of** A. Clearly state what A_2 is, rounded to 4 decimal places. Also, **compute** the root-mean square error (RMSE) between A and A_2. Which approximation is better, A_1 or A_2? Explain.

Solution:

```
% Compute A2 use 1st 2 columns of U, 1st 2 columns and rows of S, and 1st 2
% columns of V.
A2 = U(:,1:2)*S(1:2,1:2)*V(:,1:2)'
```

```
A2 = 3×3
    1.0100    1.8213    2.1469
    2.9907    4.1656    4.8639
    6.0029    6.9476    8.0431
```

```
% Root mean square error (RMSE) between The matrix A and The matrix A2.
RMSE = sqrt(mean((A(:)-A2(:)).^2))
```

```
RMSE = 0.1076
```

Explain: RMSE (root mean square error) measures the differences between values. The lower the RSME, the more closely related the variables are. Sinc A2 is lower, is is a more accurate approximation.

Problem 3

For the 3×3 matrix A, the singular value decomposition is $A = USV'$ where $U = [\mathbf{u}_1\ \mathbf{u}_2\ \mathbf{u}_3]$. Use MATLAB to **compute** the dot product $d_1 = dot(\mathbf{u}_1, \mathbf{u}_2)$.

Also, use MATLAB to **compute** the cross product $\mathbf{c} = cross(\mathbf{u}_1, \mathbf{u}_2)$ and dot product $d_2 = dot(\mathbf{c}, \mathbf{u}_3)$. Clearly state the values for each of these computations. Do these values make sense? **Explain**.

Solution:

```
%code
% d1 = dot product of u1 and u2.
d1 = dot(U(:,1),U(:,2))
```

```
d1 = -8.3267e-17
```

```
% c = cross product of u1 and u2.
c = cross(U(:,1),U(:,2))
```

```
c = 3×1
   -0.7172
    0.6643
   -0.2103
```

```
% u3 for comparison.
u3 = U(:,3)
```

```
u3 = 3×1
```

```
   -0.7172
    0.6643
   -0.2103
```

```
% d2 = dot product of c and u3.
d2 = dot(c,U(:,3))
```

```
d2 = 1
```

Explain: u1 and u2 are perpindicular so that is why their cross product is equal to u3 and the dot product of that cross product and u3 is 1.

Problem 4

Using the matrix $U = [\mathbf{u}_1\ \mathbf{u}_2\ \mathbf{u}_3]$, d**etermine whether or not the columns of U span $\mathbb{R}^3$. Explain your approach.**

Solution:

```
%code
reducedU = rref(U)
```

```
reducedU = 3×3
     1     0     0
     0     1     0
     0     0     1
```

Explain: When reduced, it becomes an identity matrix which proves that the columns of U span R^3.

Problem 5

Use the MATLAB imshow() function to load and display the image A stored in the image.mat file, available in the Project Two Supported Materials area in Brightspace. For the loaded image, **derive the value of k** that will result in a compression ratio of $CR \approx 2$. For this value of k, **construct the rank-*k* approximation of the image**.

Solution:

```
% Load and display image.
figure
image = load("image.mat")
```

```
image = struct with fields:
    A: [3072×4608 uint8]
```

```
A = image.A
```

```
A = 3072×4608 uint8 matrix
   189   192   188   189   191   193   197   195   196   193   188   193   192 ···
   188   191   190   192   193   193   197   195   198   195   189   192   190
   188   192   191   192   192   192   197   196   198   197   192   192   189
   191   194   191   191   190   190   197   198   196   198   194   194   191
   192   194   191   191   190   189   195   197   195   198   195   197   193
   190   193   191   194   193   190   194   194   195   198   196   198   194
   190   191   190   194   195   192   195   194   198   198   196   199   194
```

```
   191   191   188   193   194   193   197   197   199   199   195   199   194
   194   194   192   190   191   195   198   197   200   199   197   199   193
   193   194   194   194   195   197   199   200   199   198   194   196   190
     ⋮
```

```
imshow(A)
```

```
% Calculate SVD on A.
[U S V] = svd(double(A))
```

```
U = 3072×3072
   -0.0220    0.0337   -0.0276    0.0071   -0.0003    0.0114   -0.0108    0.0043 ···
   -0.0220    0.0335   -0.0273    0.0066   -0.0002    0.0106   -0.0112    0.0037
   -0.0220    0.0335   -0.0271    0.0062   -0.0003    0.0100   -0.0113    0.0029
   -0.0220    0.0333   -0.0271    0.0057   -0.0003    0.0094   -0.0110    0.0023
   -0.0219    0.0331   -0.0273    0.0053   -0.0003    0.0083   -0.0109    0.0020
   -0.0219    0.0329   -0.0274    0.0049   -0.0007    0.0066   -0.0107    0.0017
   -0.0219    0.0325   -0.0274    0.0041   -0.0012    0.0048   -0.0106    0.0012
   -0.0218    0.0322   -0.0277    0.0037   -0.0012    0.0027   -0.0104    0.0008
   -0.0218    0.0321   -0.0281    0.0028   -0.0020    0.0008   -0.0097    0.0003
   -0.0218    0.0319   -0.0281    0.0020   -0.0025   -0.0009   -0.0093   -0.0001
     ⋮
S = 3072×4608
10^5 ×
    5.7986         0         0         0         0         0         0         0 ···
         0    0.6755         0         0         0         0         0         0
```

```
         0         0    0.3657         0         0         0         0         0
         0         0         0    0.3129         0         0         0         0
         0         0         0         0    0.2842         0         0         0
         0         0         0         0         0    0.2423         0         0
         0         0         0         0         0         0    0.2325         0
         0         0         0         0         0         0         0    0.2217
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
       ⋮
```

```
V = 4608×4608
   -0.0159   -0.0085   -0.0079   -0.0083    0.0064   -0.0076    0.0061    0.0098 ···
   -0.0159   -0.0087   -0.0081   -0.0084    0.0064   -0.0082    0.0064    0.0100
   -0.0159   -0.0088   -0.0079   -0.0085    0.0063   -0.0078    0.0067    0.0104
   -0.0160   -0.0090   -0.0081   -0.0087    0.0063   -0.0077    0.0069    0.0106
   -0.0160   -0.0091   -0.0079   -0.0092    0.0058   -0.0077    0.0071    0.0104
   -0.0160   -0.0092   -0.0082   -0.0091    0.0055   -0.0076    0.0076    0.0104
   -0.0160   -0.0093   -0.0081   -0.0094    0.0054   -0.0078    0.0076    0.0106
   -0.0160   -0.0094   -0.0083   -0.0094    0.0053   -0.0076    0.0080    0.0107
   -0.0160   -0.0095   -0.0084   -0.0098    0.0053   -0.0075    0.0080    0.0108
   -0.0160   -0.0097   -0.0085   -0.0099    0.0052   -0.0074    0.0081    0.0112
     ⋮
```

```
% For CR to = 2, k = 922.
CR = (3072*4608)/(922*(3072 + 4608 + 1))
```

```
CR = 1.9989
```

```
% Calculate 922 approximation of A.
A922 = U(:,1:922) * S(1:922, 1:922) * V(:,1:922)'
```

```
A922 = 3072×4608
  188.9882  191.8914  188.9691  187.8169  190.8839  193.6597  196.8791  193.1967 ···
  188.8761  192.1536  189.6657  190.0302  191.8746  193.0029  196.4551  194.3982
  189.5207  192.6222  189.9735  190.5806  191.4181  192.0160  197.1477  196.4821
  191.2833  192.9773  190.2435  191.5475  190.4264  189.8804  196.5851  198.0480
  191.7268  193.2964  190.4523  193.6784  191.5790  189.5906  195.2746  197.2235
  190.4955  192.6234  190.1119  194.4576  193.5961  190.9232  195.0511  194.8653
  188.1594  191.7671  191.5152  193.0712  193.5285  192.0258  195.9567  195.3186
  188.4331  192.2572  192.4687  192.7673  192.9458  193.7013  196.6905  196.9677
  191.2003  192.7487  192.5094  192.8476  193.5783  194.3414  198.4755  198.9639
  192.5875  194.3928  194.5535  193.9942  194.0894  194.8430  197.6064  198.5257
     ⋮
```

```
% Convert back from double.
A922 = uint8(round(A922))
```

```
A922 = 3072×4608 uint8 matrix
   189   192   189   188   191   194   197   193   196   195   190   193   192 ···
   189   192   190   190   192   193   196   194   198   197   191   193   190
   190   193   190   191   191   192   197   196   198   198   192   192   188
   191   193   190   192   190   190   197   198   197   197   193   193   190
   192   193   190   194   192   190   195   197   195   196   193   196   193
   190   193   190   194   194   191   195   195   195   198   196   198   193
   188   192   192   193   194   192   196   195   197   198   196   199   194
   188   192   192   193   193   194   197   197   199   199   195   199   195
   191   193   193   193   194   194   198   199   199   199   197   200   193
   193   194   195   194   194   195   198   199   199   198   196   198   192
     ⋮
```

Explain: k is derived from the equation CR = MN/ 6 K(M + N + 1). Got SVD from image.

Problem 6

Display the image and compute the root mean square error (RMSE) between the approximation and the original image. Make sure to include a copy of the approximate image in your report.

Solution:

```
% Display image.
imshow(A922)
```

```
% Calculate RSME for A and A922.
RSME922 = norm(double(A)-double(A922),'fro')/(3072 * 4608)
```

```
RSME922 = 4.0620e-04
```

Problem 7

Repeat Problems 5 and 6 for $CR \approx 10$, $CR \approx 25$, and $CR \approx 75$. **Explain** what trends you observe in the image approximation as CR increases and provide your recommendation for the best CR based on your observations. Make sure to include a copy of the approximate images in your report.

Solution:

```
% Test k for CR = 10.
CR = (3072*4608)/(184*(3072 + 4608 + 1))
```

```
CR = 10.0161
```

```
% Calculate Rank 184 approximation of A.
A184 = U(:,1:184) * S(1:184, 1:184) * V(:,1:184)'
```

```
A184 = 3072×4608
  189.7432  191.5483  189.1171  188.5879  189.2042  190.4093  192.2473  192.2744 ···
  189.9357  191.6766  189.2152  189.0111  189.4455  190.7028  192.9499  193.0818
  190.3505  191.8218  189.2282  189.5003  189.7682  190.8722  193.6113  194.0608
  190.4580  191.6506  189.2845  190.0465  190.6301  191.5918  194.6700  195.6617
  189.7958  190.8026  188.7556  190.2446  190.8688  191.7150  195.1483  196.7306
  189.5912  190.5549  188.8867  190.6705  191.0324  191.8086  195.1119  196.9319
  189.8759  190.7995  189.5386  191.3636  191.7975  192.5087  195.5045  197.6150
  190.6872  191.5916  190.2235  192.3257  192.6240  193.2553  195.9026  198.1159
  192.4771  193.3784  192.0164  194.2350  194.6985  195.1316  197.0148  198.9931
  194.2754  195.2644  193.8108  195.8900  195.8527  196.3854  197.4867  199.1781
     ⋮
```

```
% Convert back.
A184 = uint8(round(A184))
```

```
A184 = 3072×4608 uint8 matrix
   190   192   189   189   189   190   192   192   195   195   193   195   192 ···
   190   192   189   189   189   191   193   193   195   195   193   196   193
   190   192   189   190   190   191   194   194   196   196   193   196   193
   190   192   189   190   191   192   195   196   197   197   195   197   194
   190   191   189   190   191   192   195   197   198   198   196   198   195
   190   191   189   191   191   192   195   197   198   198   196   198   194
   190   191   190   191   192   193   196   198   199   199   196   199   194
   191   192   190   192   193   193   196   198   199   199   197   199   194
   192   193   192   194   195   195   197   199   200   200   198   200   195
   194   195   194   196   196   196   197   199   200   200   198   200   194
     ⋮
```

```
% Display image.
imshow(A184)
```

```
% Calculate RSME for A and A184.
RSME184 = norm(double(A)-double(A184),'fro')/(3072 * 4608)
```

```
RSME184 = 9.1735e-04
```

```
%------------------------------------------------------------
% Test k for CR = 25.
CR = (3072*4608)/(74*(3072 + 4608 + 1))
```

```
CR = 24.9049
```

```
% Calculate Rank 74 approximation of A.
A74 = U(:,1:74) * S(1:74, 1:74) * V(:,1:74)'
```

```
A74 = 3072×4608
  195.2146  194.2607  194.6176  195.0690  195.0659  195.4609  196.9847  196.5227 ···
  195.0544  194.0670  194.3006  194.8159  194.8360  195.1776  196.6900  196.1473
  194.9326  193.9499  194.1028  194.6309  194.7149  194.9625  196.5167  195.8726
  194.6416  193.6869  193.8314  194.3918  194.5278  194.7520  196.3028  195.6323
  194.2353  193.2686  193.3772  193.9606  194.2055  194.3836  195.9494  195.2664
  193.4457  192.4704  192.5364  193.0812  193.4071  193.5124  195.1114  194.4368
  192.9854  191.9917  192.0466  192.5400  192.8802  192.9862  194.5540  193.8953
  192.0525  191.1378  191.2363  191.7405  192.1727  192.2654  193.8367  193.2267
  191.1395  190.2445  190.3425  190.8276  191.2458  191.3305  192.8443  192.2891
  189.9394  189.1318  189.2626  189.7751  190.2202  190.2856  191.8472  191.2855
     ⋮
```

```
% Convert back.
A74 = uint8(round(A74))
```

```
A74 = 3072×4608 uint8 matrix
   195   194   195   195   195   195   197   197   197   198   197   199   196 ···
   195   194   194   195   195   195   197   196   197   198   197   198   196
   195   194   194   195   195   195   197   196   196   197   197   198   195
   195   194   194   194   195   195   196   196   196   197   196   198   195
   194   193   193   194   194   194   196   195   196   197   196   197   195
   193   192   193   193   193   194   195   194   195   196   195   196   193
   193   192   192   193   193   193   195   194   194   195   195   196   193
   192   191   191   192   192   192   194   193   194   195   194   195   192
   191   190   190   191   191   191   193   192   193   194   193   194   191
   190   189   189   190   190   190   192   191   192   193   192   193   190
     ⋮
```

```
% Display image.
imshow(A74)
```

```
% Calculate RSME for A and A74.
RSME74 = norm(double(A)-double(A74),'fro')/(3072 * 4608)
```

```
RSME74 = 0.0018
```

```
%------------------------------------------------------------
% Test k for CR = 75.
```

```
CR = (3072*4608)/(25*(3072 + 4608 + 1))
```

```
CR = 73.7184
```

```
% Calculate Rank 25 approximation of A.
A25 = U(:,1:25) * S(1:25, 1:25) * V(:,1:25)'
```

```
A25 = 3072×4608
  183.8670  183.5431  184.1842  184.1103  183.9570  183.8832  184.7547  184.9945 ···
  182.6307  182.3624  183.0260  182.9594  182.8929  182.8136  183.7167  183.9619
  181.1605  180.9303  181.6012  181.5267  181.5471  181.4601  182.3846  182.6361
  180.2502  180.0614  180.7289  180.6456  180.7168  180.6368  181.5718  181.8339
  179.3194  179.2104  179.8921  179.8020  179.9650  179.9045  180.8685  181.1550
  178.0185  178.0003  178.6870  178.5843  178.8720  178.8329  179.8312  180.1462
  176.7755  176.8469  177.5490  177.4406  177.8662  177.8534  178.8894  179.2291
  175.4634  175.6402  176.3655  176.2486  176.8075  176.8247  177.9027  178.2709
  174.8463  175.1319  175.8624  175.7428  176.4252  176.4820  177.6023  178.0109
  174.2013  174.5574  175.2860  175.1742  175.9547  176.0315  177.1715  177.6014
     ⋮
```

```
% Convert back.
A25 = uint8(round(A25))
```

```
A25 = 3072×4608 uint8 matrix
   184   184   184   184   184   184   185   185   185   185   186   186   185 ···
   183   182   183   183   183   183   184   184   184   184   185   185   184
   181   181   182   182   182   181   182   183   183   183   183   184   183
   180   180   181   181   181   181   182   182   182   182   183   183   182
   179   179   180   180   180   180   181   181   181   182   182   182   182
   178   178   179   179   179   179   180   180   180   181   181   181   181
   177   177   178   177   178   178   179   179   180   180   180   181   180
   175   176   176   176   177   177   178   178   179   179   179   180   179
   175   175   176   176   176   176   178   178   178   179   179   179   179
   174   175   175   175   176   176   177   178   178   178   179   179   179
     ⋮
```

```
% Display image.
imshow(A25)
```

```
% Calculate RSME for A and A25.
RSME25 = norm(double(A)-double(A25),'fro')/(3072 * 4608)
```

```
RSME25 = 0.0039
```

Explain: As compression is increased, the quality of the image is decreased. Assuming there are no limitations to the file size, a CR of 10 would be ideal for image quality purposes. Other considerations would be the image width and height neeeded and any transfer speed requirements.