Project One Template

MAT350: Applied Linear Algebra

Fred Wahab

5/19/2021

Problem 1

Develop a system of linear equations for the network by writing an equation for each router (A, B, C, D, and E). Make sure to write your final answer as A**x**=**b** where A is the 5x5 coefficient matrix, **x** is the 5x1 vector of unknowns, and **b** is a 5x1 vector of constants.

Solution:

To solve the unknown variables. Linear equations have been written for each router. They will have the sum input on the left side of the equation and the sum output on the right and then reformatted to have the uknown variables on the left side and constants on the right.

Sum of input = Sum of output Rewritten

Router A: $100 = 2x_1 + x_2$ $2x_1 + x_2 = 100$

Router B: $x_1 + x_2 = x_3 + x_5$ $x_1 + x_2 - x_3 - x_5 = 0$

Router C: $50 + x_1 = x_3 + x_5$ $-x_1 + x_3 + x_5 = 50$ (signs reversed)

Router D: $x_4 + x_5 = 120 + x_2$ $-x_2 + x_4 + x_5 = 120$

Router E: $x_2 + x_3 + x_5 = x_4$ $x_2 + x_3 - x_4 + x_5 = 0$

The equations can be used to create the 5x5 coefficient matrix A, the unknowns will be the 1x5 vector x, and the solutions will be the 1x5 vector of constants b.

Ax=b

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & -1 \\ -1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 50 \\ 120 \\ 0 \end{bmatrix}$$

Problem 2

Use MATLAB to construct the augmented matrix [A **b**] and then perform row reduction using the rref() function. Write out your **reduced matrix and identify the free and basic variables of the system**.

Solution:

```
%Create coefficent matrix A and constants vector b.
A = [2 1 0 0 0; 1 1 -1 0 -1; -1 0 1 0 1; 0 -1 0 1 1 ; 0 1 1 -1 1]
```

```
A = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
    -1     0     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
b = [100; 0; 50; 120; 0]
```

```
b = 5×1
   100
     0
    50
   120
     0
```

```
%Create augmented matrix Ab.
Ab = [A b]
```

```
Ab = 5×6
     2     1     0     0     0   100
     1     1    -1     0    -1     0
    -1     0     1     0     1    50
     0    -1     0     1     1   120
     0     1     1    -1     1     0
```

```
%Perform row reduction on Ab into echelon form.
[rowreducedAb, pivotvarsAb] = rref(Ab)
```

```
rowreducedAb = 5×6
     1     0     0     0     0    25
     0     1     0     0     0    50
     0     0     1     0     0    30
     0     0     0     1     0   125
     0     0     0     0     1    45
pivotvarsAb = 1×5
     1     2     3     4     5
```

```
%Use the size command to find the number of variables in the system of
%linear equations. Store this number in numvars.
[numeqns, numvars] = size(A)
```

```
numeqns = 5
numvars = 5
```

```
%Use the size commmand to find the number of pivot variables. Store this
%number in numpivotvars.
[numrows, numpivotvars] = size(pivotvarsAb)
```

```
numrows = 1
numpivotvars = 5
```

```
%use subtraction to find number of free variables in the solution to the
%system of linear equations. Store this number in numfreevars.
```

```
numfreevars = numvars - numpivotvars
```

```
numfreevars = 0
```

```
%There are 5 basic variables and 0 free variables.
```

Problem 3

Use MATLAB to **compute the LU decomposition of A**, i.e., find A = LU. For this decomposition, find the transformed set of equations L**y** = **b**, where **y** = U**x**. Solve the system of equations L**y** = **b** for the unknown vector **y**.

Solution:

```
%Use the lu() command to find the LU decomposition of A.
[ L, U] = lu(A)
```

```
L = 5×5
    1.0000         0         0         0         0
    0.5000   -0.5000   -1.0000    1.0000         0
   -0.5000   -0.5000    1.0000         0         0
         0    1.0000         0         0         0
         0   -1.0000    1.0000   -0.5000    1.0000
U = 5×5
    2.0000    1.0000         0         0         0
         0   -1.0000         0    1.0000    1.0000
         0         0    1.0000    0.5000    1.5000
         0         0         0    1.0000    1.0000
         0         0         0         0    1.0000
```

```
%Solve the system of linear equations Ax=b using the LU decomposition.
y = L\b
```

```
y = 5×1
   100
   120
   160
   170
    45
```

```
x = U\y
```

```
x = 5×1
    25
    50
    30
   125
    45
```

Problem 4

Use MATLAB to **compute the inverse** of U using the inv() function.

Solution:

```
%compute the inverse.
inv(U)
```

```
ans = 5×5
    0.5000    0.5000         0   -0.5000         0
         0   -1.0000         0    1.0000         0
         0         0    1.0000   -0.5000   -1.0000
         0         0         0    1.0000   -1.0000
         0         0         0         0    1.0000
```

Problem 5

Compute the solution to the original system of equations by transforming **y** into **x**, i.e., compute **x** = inv(U)**y**.

Solution:

```
x = inv(U)*y
```

```
x = 5×1
    25
    50
    30
   125
    45
```

Problem 6

Check your answer for x_1 using Cramer's Rule. Use MATLAB to compute the required determinants using the det() function.

Solution:

```
%Inititalize the matrices A1-A5 as matrix A.
A1 = A
```

```
A1 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
    -1     0     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A2 = A
```

```
A2 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
    -1     0     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A3 = A
```

```
A3 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
    -1     0     1     0     1
     0    -1     0     1     1
```

```
     0     1     1    -1     1
```

```
A4 = A
```

```
A4 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
    -1     0     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A5 = A
```

```
A5 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
    -1     0     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
%Replace columns with column vector of constants b.
A1(:,1) = b
```

```
A1 = 5×5
   100     1     0     0     0
     0     1    -1     0    -1
    50     0     1     0     1
   120    -1     0     1     1
     0     1     1    -1     1
```

```
A2(:,2) = b
```

```
A2 = 5×5
     2   100     0     0     0
     1     0    -1     0    -1
    -1    50     1     0     1
     0   120     0     1     1
     0     0     1    -1     1
```

```
A3(:,3) = b
```

```
A3 = 5×5
     2     1   100     0     0
     1     1     0     0    -1
    -1     0    50     0     1
     0    -1   120     1     1
     0     1     0    -1     1
```

```
A4(:,4) = b
```

```
A4 = 5×5
     2     1     0   100     0
     1     1    -1     0    -1
    -1     0     1    50     1
     0    -1     0   120     1
     0     1     1     0     1
```

```
A5(:,5) = b
```

```
A5 = 5×5
     2     1     0     0   100
```

```
     1     1    -1     0     0
    -1     0     1     0    50
     0    -1     0     1   120
     0     1     1    -1     0
```

```
%Find the solution for x1-x5 using ratios of determinants.
x1 = det(A1)/det(A)
```

```
x1 = 25.0000
```

```
x2 = det(A2)/det(A)
```

```
x2 = 50
```

```
x3 = det(A3)/det(A)
```

```
x3 = 30.0000
```

```
x4 = det(A4)/det(A)
```

```
x4 = 125.0000
```

```
x5 = det(A5)/det(A)
```

```
x5 = 45
```

Problem 7

The Project One Table Template, provided in the Project One Supporting Materials section in Brightspace, shows the recommended throughput capacity of each link in the network. Put your solution for the system of equations in the third column so it can be easily compared to the maximum capacity in the second column. In the fourth column of the table, provide recommendations for how the network should be modified based on your network throughput analysis findings. The modification options can be No Change, Remove Link, or Upgrade Link. In the final column, explain how you arrived at your recommendation.

Solution:

Southern New Hampshire University

MAT 350 Project One Table Template

Complete this template by replacing the bracketed text with the relevant information.

Network Link	Recommended Capacity (Mbps)	Solution	Recommendation	Explanation
x_1	60	25	Upgrade Link	The data rate on this link is less than the recommended capacity so it needs to be upgraded to meet the recommended capacity.
x_2	50	50	No Change	The data rate is equal, so no change is necessary.
x_3	100	30	Upgrade Link	The data rate on this link is less than the recommended capacity so it needs to be upgraded to meet the recommended capacity.
x_4	100	125	Remove Link	The data rate on this link exceeds the recommended capacity which could cause network instability. It needs to be lowered to meet the recommended capacity.
x_5	50	45	Upgrade Link	The data rate on this link is less than the recommended capacity so it needs to be upgraded to meet the recommended capacity.