

Assignment 1

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1 Ex 1.1

$$\begin{array}{c}
 \begin{array}{ccc|ccc}
 1 & & & 1 & 2 & \\
 2 & & & 3 & & \\
 3 & & & 4 & & \\
 4 & & & 1 & & \\
 \hline
 A & B & C & A & B & C
 \end{array} \\
 \begin{array}{ccc|ccc}
 3 & & & 3 & 1 & \\
 4 & 2 & 1 & 4 & 2 & \\
 A & B & C & A & B & C
 \end{array} \\
 \begin{array}{ccc|ccc}
 1 & & & 1 & 2 & \\
 4 & 2 & 3 & 4 & 3 & \\
 A & B & C & A & B & C
 \end{array} \\
 \begin{array}{ccc|ccc}
 1 & 2 & & 1 & 2 & \\
 & 4 & 3 & & 4 & \\
 A & B & C & A & B & C
 \end{array} \\
 \begin{array}{ccc|ccc}
 1 & 3 & & 1 & 3 & \\
 2 & 4 & 1 & 2 & 4 & \\
 A & B & C & A & B & C
 \end{array} \\
 \begin{array}{ccc|ccc}
 2 & & & 2 & & \\
 3 & & & 3 & & \\
 4 & 1 & & 4 & & \\
 A & B & C & A & B & C
 \end{array}
 \end{array}$$

Figure 1: Exe 1.1 Solution.

2 Ex 1.2

2.a

Beginners will find the smallest ring first and move it to either A or C. And then find the second small one and figure out which pole should move to.

2.b

Suppose I have a $n-1$ DTH solver called *minorDTHSolver*, I will test if pole B contains the ring n first. If it is true, move $n-1$ rings to pole B and problem solved. If ring n is on pole A, I will move all $n-1$ rings to pole C with the solver and then move the ring n from pole A to B, then move all $n-1$ rings to B with the solver. If ring n is on pole C, it's the same.

2.c

```

procedure DTH( $n$ , A, B, C);
  if  $n$  equals 1 then
    if IsItThere(1, A) equals True then
      move ring 1 from the top of A to the top of B
    elseif IsItThere(1, C) equals True then
      move ring 1 from the top of C to the top of B
    endif
  else
    if IsItThere( $n$ , A) equals True then
      DTH( $n-1$ , A, C, B)
      move ring  $n$  from the top of A to the top of B
      DTH( $n-1$ , A, B, C)
    elseif IsItThere( $n$ , C) equals True then
      DTH( $n-1$ , B, A, C)
      move ring  $n$  from the top of C to the top of B
      DTH( $n-1$ , A, B, C)
    elseif IsItThere( $n$ , B) equals True then
      DTH( $n-1$ , A, B, C)
    endif
  endif
end_DTH;

```

2.d

```

procedure DTH( $n$ , A, B, C);
  if  $n$  equals 1 then
    if IsItThere(1, A) equals True then

```

```

    move ring 1 from the top of A to the top of B
elseif IsItThere(1, C) equals True then
    move ring 1 from the top of C to the top of B
endif
else
    if IsItThere(n, A) equals True then
        DTH(n-1, A, C, B)
        move ring n from the top of A to the top of B
        TowersOfHanoi(n-1, C, B, A)
    elseif IsItThere(n, C) equals True then
        DTH(n-1, B, A, C)
        move ring n from the top of C to the top of B
        TowersOfHanoi(n-1, A, B, C)
    elseif IsItThere(n, B) equals True then
        DTH(n-1, A, B, C)
    endif
endif
end_DTH;

```

3 Ex 1.3

3.a

Ring 1 (the smallest)

3.b

How can I use the pole D to make the TowersOfHanoi procedure more efficient in *EACH* recursive round. Make the size of subproblem as small as possible.

3.c

```

procedure DPoleTH(n, A, B, C, D);
    if n equals 1 then
        move ring 1 from the top of A to the top of B
    elseif n equals 2 then
        move ring 1 from the top of A to the top of D
        move ring 2 from the top of A to the top of B
        move ring 1 from the top of D to the top of B
    else
        DPoleTH(n-2, A, C, B, D);
        move ring n-1 from A to D;
        move ring n from A to B;
        move ring n-1 from D to B;
        DPoleTH(n-1, C, B, A, D)
    end

```

```

endif
end_DPoleTH;

```

4 EX 1.4

4.a

Ring n 's top side is white, the other $n-1$ rings' top sides color are red cause they are flipped twice.

4.b

```

procedure RTH(n, A, B, C);
  if n equals 1 then
    move ring 1 from the top of A to the top of C
    move ring 1 from the top of C to the top of B
  else
    RTH(n-1, A, C, B);
    move ring n from the top of A to the top of B;
    RTH(n-1, C, B, A)
  endif
end_RTH;

```

4.c

```

procedure WTH(n, A, B, C);
  if n equals 1 then
    move ring 1 from the top of A to the top of B
  else
    WTH(n-1, A, C, B);
    move ring n from the top of A to the top of B;
    WTH(n-1, C, A, B)
    WTH(n-1, A, B, C)
  endif
end_WTH;

```

5 EX 1.5

```

procedure CTHOneJump(n, A, B, C);
  if n equals 1 then
    move ring 1 from the top of A to the top of B
  else
    CTHOneJump(n-1, A, B, C);
    CTHOneJump(n-1, B, C, A);
    move ring n from the top of A to the top of B;

```

```

    CTHOneJump(n-1, C, A, B);
    CTHOneJump(n-1, A, B, C);
  endif
end_CTHOneJump;

```

6 EX 1.6

6.a

```

procedure GColor(n, Adj[1..n], ColorOf[1..n] );
  input: the vertices 1,2,..., n, where for each  $j$ ,  $Adj[j]$  is a list of  $j$ 's neighbouring vertices;
  output:  $Color[j]$  is the color assigned to vertex  $j$ 
1  Create empty sets of Colorable
2  Create an array MostRecentNeighborColor[1..n] with initial value is NIL Copy the n vertices into
   Colorable
3  while Colorable not empty do
4    Select a new color for this new round and call it  $c$ ;
5    foreach vertex  $v$  in the set Colorable do
6      if  $MostRecentNeighborColor[v]$  not equals  $c$  then
7         $ColorOf[v] \leftarrow c$ ;
8        Remove  $v$  from Colorable;
9        foreach vertex  $w$  in the list  $Adj[v]$  do
10          $MostRecentNeighborColor[w] \leftarrow c$ 
11       endfor
12     endif
13   endfor
14 endwhile
end_GColor;

```

6.b

```

procedure GColor(n, Adj[1..n], ColorOf[1..n] );
  input: the vertices 1,2,..., n, where for each  $j$ ,  $Adj[j]$  is a list of  $j$ 's neighbouring vertices;
  output:  $Color[j]$  is the color assigned to vertex  $j$ 
1  Create an array  $Color[1..m]$  with initial value is 0
2  while Colorable not empty do
3    Select a new color for this new round and call it  $c$ ;
4    foreach vertex  $i$  do
5      foreach vertex  $j$  in  $Adj[i]$  do
6         $Color[ColorOf[j]] \leftarrow 1$ 
7      endfor
8      foreach  $c$  in  $Color[m]$  do
9        if  $c$  equals 0 then
10          $ColorOf[i] \leftarrow c$ 
11       endif
12     foreach vertex  $j$  in  $Adj[i]$  do

```

```

13         Color[ColorOf[j]] ← 0
14     endfor
15
16 endfor
17 endwhile
18 end_GColor;

```

6.c

As we can see there are 3 *Foreach* loop with each vertex, if they traverse all neighbor of vertex v then there are maximum $3*13$ array access. At last each vertex should be colored which needs two array access. Hence, the pseudocode solve all subgoals in no more than $3*13+2$ array access.

7 EX 1.7

7.a

```

1  y ← Y;
2  m ← M;
3  temp ← y.next
4  y.next ← m
5  m ← y
6  y ← temp

```

7.b

```

procedure Reverse(Y);
    p ← Y;
1   q ← Y.next
2   p.next = Nil;
3   while q not equals Nil do
4       temp ← q.next;
5       q.next ← p;
6       p ← q;
7       q ← temp;
8   endwhile
9   Y ← p;
end Reverse;

```

7.c

Reverse the list.

Use the Part 0 solution change each record's dat;

Reverse the list again;

7.d

Iterate all record and get the sum of all record's data, called SumOfAll.

Iterate all record, calculate the sum of all precede records' dat called SumOfPre, modify the each record's dat with SumOfAll - SumOfPre. Which is the sum of succeed records' dat.

7.e

```

procedure SumSucWithReverseList(Y);
1   Reverse(Y);
2   SumPre(Y);
3   Reverse(Y);
end SumSucWithReverseList;

```

7.f

```

procedure SumPreWithRecur(Y, sum);
1   p  $\leftarrow$  Y;
2   sum  $\leftarrow$  sum + p.dat;
3   p.dat  $\leftarrow$  sum;
4   if p.next not equals Nil then
5       SumPreWithRecur(p.next, sum);
6   endif
end SumPreWithRecur;

```

7.g

```

function SumSucWithRecur(Y);
1   p  $\leftarrow$  Y;
2   if p.next equals Nil then
3       return(p.dat) ;
4   else
5       p.dat  $\leftarrow$  SumSucWithRecur(p.next) + p.dat;
6   endif
end SumSucWithRecur;

```

7.h

```

procedure TestCircle(Y);
1   p  $\leftarrow$  Y.next;
2   q  $\leftarrow$  Y.next.next;
3   while q not equals p do

```



```

4      if  $q$  equals Nil then
5          return(False)
6      endif
7       $p \leftarrow p.next$ ;
8       $q \leftarrow q.next.next$ ;
9  endwhile
10     return(True);
end_TestCircle;

```

7.i

```

procedure CircleSize( $Y$ );
1  cnt1  $\leftarrow$  1;
2   $p \leftarrow Y.next$ ;
3   $q \leftarrow Y.next.next$ ;
4  while  $q$  not equals  $p$  do
5       $p \leftarrow p.next$ ;
6       $q \leftarrow q.next.next$ ;
7      cnt1  $\leftarrow$  cnt1 + 1;
8  endwhile
9  return(cnt1);
end_CircleSize;

```

7.j

```

procedure CircleLeaderSize( $Y$ );
1  cnt1  $\leftarrow$  1;
2  cnt2  $\leftarrow$  0;
3   $p \leftarrow Y.next$ ;
4   $q \leftarrow Y.next.next$ ;
5   $r \leftarrow Y$ ;
6  while  $q$  not equals  $p$  do
7       $p \leftarrow p.next$ ;
8       $q \leftarrow q.next.next$ ;
9      cnt1  $\leftarrow$  cnt1 + 1;
10 endwhile
11 while  $r$  not equals  $p$  do
12      $p \leftarrow p.next$ ;
13      $r \leftarrow r.next$ ;
14     cnt2  $\leftarrow$  cnt2 + 1;
15 endwhile
16 return(cnt1, cnt2);
end_CircleLeaderSize;

```

8 EX 1.12

8.a

```

procedure SloTH(n, A, B, C);
  if n equals 1 then
    move ring 1 from the top of A to the top of C
    move ring 1 from the top of C to the top of B
  else
    SloTH(n-1, A, B, C);
    move ring n from the top of A to the top of C;
    SloTH(n-1, B, A, C);
    move ring n from the top of C to the top of B;
    SloTH(n-1, A, B, C);
  endif
end_SloTH;

```

8.b

```

procedure SloTH(n, A, B, C);
  output: the n rings are moved from A to B in a way where all legal ring configurations are achieved
  for n rings on poles A, B and C, but just once for each configuration;
  1   if n equals 1 then
  2     move ring 1 from the top of A to the top of C
  3     move ring 1 from the top of C to the top of B
  4   else
  5     SloTH(n-1, A, B, C); { so with ring n sitting on A, rings 1 through n - 1 have achieve all possible
  legal configurations on poles A, B and C with each configuration achieved just once }
  6     move ring n from the top of A to the top of C;
  7     SloTH(n-1, B, A, C); { so with ring n sitting on C, rings 1 through n - 1 have achieve all possible
  legal configurations on poles A, B and C with each configuration achieved just once }
  8     move ring n from the top of C to the top of B;
  9     SloTH(n-1, A, B, C); { so with ring n sitting on B, rings 1 through n - 1 have achieve all possible
  legal configurations on poles A, B and C with each configuration achieved just once }
  10  endif
  end_SloTH;

```