# Traitements Numériques

# pour les Systèmes Embarqués

## 3) Optimisation de la précision des commandes

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# Introduction

Floating-point computations sensitive to how formulas are written

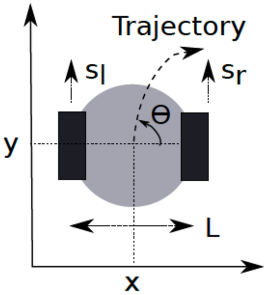
F.P. arithmetic not intuitive. Ex: in OCaml 4.01 (Ubuntu, Core I5):

```
# let f x = x ** 2.0 -. 2.0 *. x +. 1.0 ;;
# let g x = (x -. 1.0) *. (x -. 1.0) ;;
# f 0.99;;
- : float = 9.99999999999889866e-05
# g 0.99;;
- : float = 0.000100000000000000181
```

**Goal:** optimize accuracy almost like compilers do for exec. time

Main difference: ranges are given for the inputs of the programs

# Example: Odometry




$$\theta(t+1) = \theta(t) + \Delta\theta(t)$$

$$\Delta d(t) = \frac{\Delta d_r(t) + \Delta d_l(t)}{2}$$

$$\Delta\theta(t) = \frac{\Delta d_r(t) - \Delta d_l(t)}{L}$$

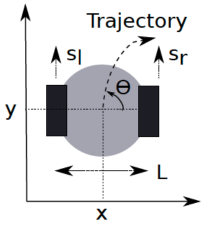
$$x(t+1) = x(t) + \Delta d(t+1) \times \cos\left(\theta(t) + \frac{\Delta\theta(t+1)}{2}\right)$$

$$\Delta d_l(t) = s_l(t) \times C \qquad \Delta d_r(t) = s_r(t) \times C$$

# Example: Odometry

## Source program:

```
sl = [0.52,0.53] ; sr = 0.785398163397 ;
theta = 0.0 ; t = 0.0 ; x = 0.0 ;
y = 0.0 ; inv_l = 0.1 ; c = 12.34 ;
while (t < 1.5) do {
  delta_dl = (c * sl) ;
  delta_dr = (c * sr) ;
  delta_d = ((delta_dl + delta_dr) * 0.5) ;
  delta_theta = ((delta_dr - delta_dl) * inv_l) ;
  arg = (theta + (delta_theta * 0.5)) ;
  cos = ((1.0 - ((arg * arg) * 0.5))
       + ((((arg * arg)* arg) * arg) / 24.0)) ;
  x = (x + (delta_d * cos)) ;
  theta = (theta + delta_theta) ;
  t = (t + 0.1)
}
```

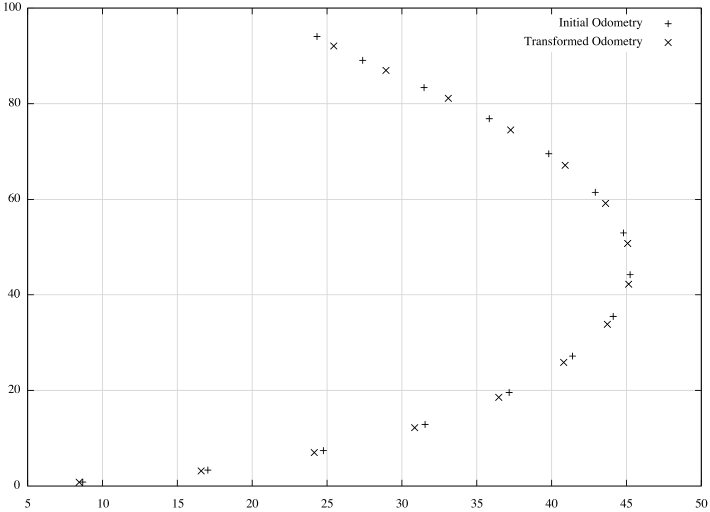


# Example: Odometry

## Optimized program:

```
sl = [0.52,0.53] ; sr = 0.785398163397 ;
theta = 0.0 ; y = 0.0 ; x = 0.0 ; t = 0.0 ;
while (t < 1.5) do {
   x = (x + ((0.5 * ((1.0 - ((theta + (((9.69181333632
     - (sl * 12.34)) * 0.1) * 0.5)) * (0.5 * ((0.5 * (0.1
     * (9.69181333632 - (sl * 12.34))))+ theta))))
     + (((((theta + (((9.69181333632- (sl * 12.34)) * 0.1)
     * 0.5)) * (theta + (((9.69181333632 - (sl * 12.34))
     * 0.1)* 0.5))) * (theta + (((9.69181333632 - (sl * 12.34))
     * 0.1) * 0.5)))* (theta + (((9.69181333632 - (sl * 12.34))
     * 0.1) * 0.5))) / 24.0)))*(9.69181333632 + (sl * 12.34))));
   theta = (theta + (0.1 * (9.69181333632 - (sl * 12.34)))) ;
   t = (t + 0.1)
}
```

# Example: Odometry



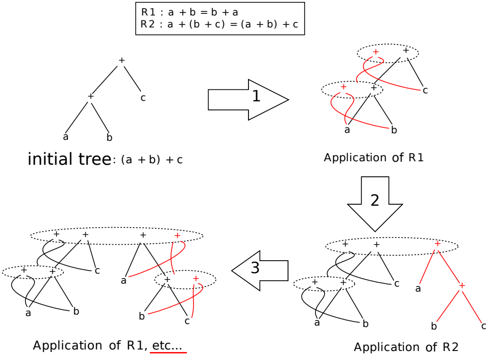
# Equivalent Program Expansion Graphs

Introduced for the *phase ordering problem*

Based on rewriting rules

Notion of equivalence class

[R. Tate, M. Stepp, Z. Tatlock and S. Lerner, *Equality Saturation: A New Approach to Optimization*, POPL'09]



# From EPEGs to APEGs

$(2n-1)!!$

EPEGs are exponential in size and possibly infinite (ex : $a = 1 \times a$)

APEG = Abstract Program Equivalence Graphs

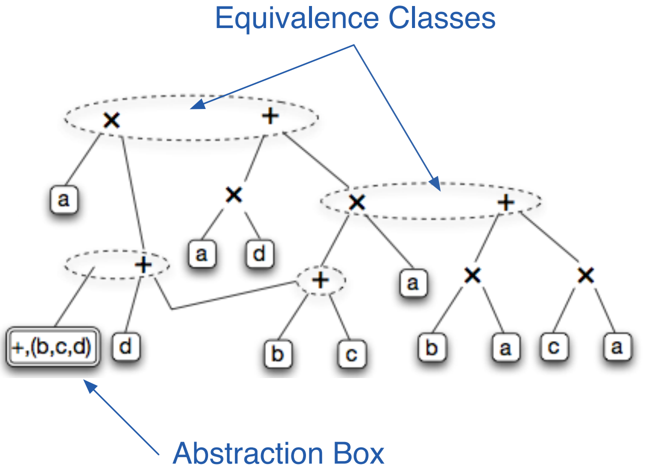
Represent many equivalent expressions in polynomial size

APEGs contain equivalence classes like EPEGs

APEGs contain abstraction boxes: $\boxed{*(e_1, \ldots, e_n)}$ ($*$ associative)

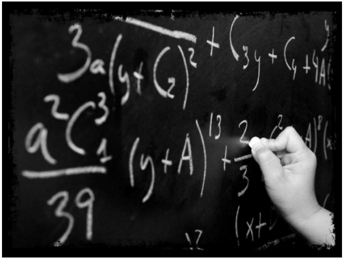
$\boxed{*(e_1, \ldots, e_n)}$ represents all the parsings of $e_1 * \ldots * e_n$

# Example of APEG



# Expression Transformation

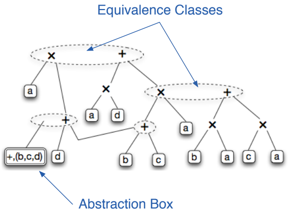
1 - **Construction of an APEG**



2 - Extraction of an expression optimizing the accuracy

# Insertion of New Nodes

In polynomial size :




Distribute products (largest factors)

Factorize every product (largest factors)

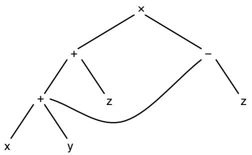
Propagate subtractions

Create a box for the left/right hand side of the operator

Create a box for the englobing homogeneous expression

Merges nested boxes

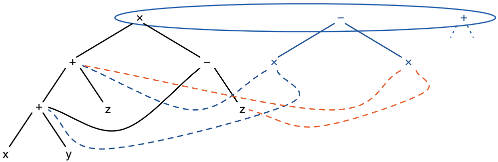
# Example (1/4)




$$((x + y) + z) \times ((x + y) - z)$$

# Example (2/4)

## Distribute:



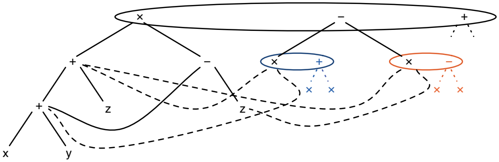

$$((x+y)+z)\times((x+y)-z)$$

$$((x+y)+z)\times(x+y)-((x+y)+z)\times z$$

$$(x+y)\times((x+y)-z)+z\times((x+y)-z)$$

# Example (3/4)

## Distribute:

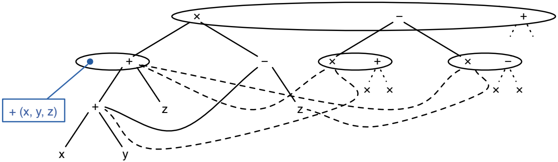



$$((x+y)+z) \times ((x+y)-z)$$
$$((x+y)+z) \times (x+y) - ((x+y)+z) \times z$$
$$(x+y) \times ((x+y)-z) + z \times ((x+y)-z)$$

$$(((x+y)+z) \times x + ((x+y)+z) \times y) - ((x+y)+z) \times z$$
$$(((x+y) \times (x+y))((x+y) \times z)) - ((x+y)+z) \times z$$
$$((x+y)+z) \times (x+y) - ((x+y) \times z + z \times z)$$
$$(((x+y)+z) \times x + ((x+y)+z) \times y)((x+y) \times z + z \times z)$$
$$(((x+y) \times (x+y))((x+y) \times z)) - ((x+y) \times z + z \times z)$$

# Example (4/4)

Box introduction: $\boxed{+(\text{x,y,z})} = \{(x+y)+z,\ x+(y+z),\ (x+z)+y\}$




$\boxed{+(\text{x,y,z})} \times ((x+y)-z)$

$\boxed{+(\text{x,y,z})} \times (x+y) - ((x+y)+z) \times z$

$(x+y) \times ((x+y)-z) + z \times ((x+y)-z)$

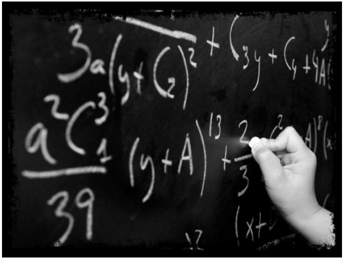
$(\boxed{+(\text{x,y,z})} \times x + \boxed{+(\text{x,y,z})} \times y) - \boxed{+(\text{x,y,z})} \times z$

$(((x+y)\times(x+y))((x+y)\times z)) - \boxed{+(\text{x,y,z})} \times z$

$\boxed{+(\text{x,y,z})} \times (x+y) - ((x+y) \times z + z \times z)$

$(\boxed{+(\text{x,y,z})} \times x + \boxed{+(\text{x,y,z})} \times y) - ((x+y) \times z + z \times z)$

# Expression Transformation



1 - Construction of an APEG

2 - **Extraction of an expression optimizing the accuracy**

# Formula Synthesis: The Case of Boxes

An abstraction box represents $(2n-1)!!$ expressions

Greedy heuristic - Complexity: $O(n^2)$

At each step, select terms $a$ and $b$ such that $\downarrow(a * b)$ is minimal

Example:

$$\boxed{+(\text{a,b,c,d,e})} \rightarrow \boxed{+(\text{a,c,e,}\boxed{+(\text{b,d})})} \rightarrow \boxed{+(\text{e,}\boxed{+(\text{a,c})},\boxed{+(\text{b,d})})}$$

$$\rightarrow \boxed{+(\boxed{+(\text{e,}\boxed{+(\text{a,c})})},\boxed{+(\text{b,d})})}$$

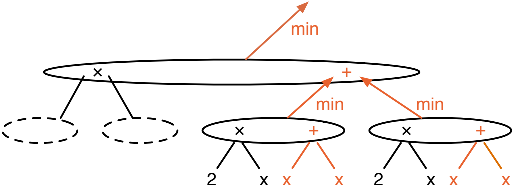
We synthetize $(e + (a + c)) + (b + d)$

# Formula Synthesis: Equivalence Classes

**Simplest approach:**

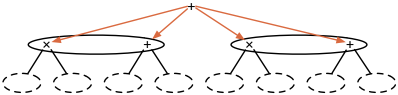
For each class, select the operation which yields the smallest error

Complexity: $O(n)$



# Formula Synthesis: Improvement

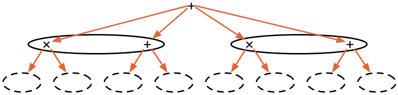
Minimize the error for one operator using the classes below




We compute the errors on:

$(a+a)+(b+b)$ $(a+a)+(2\times b)$ $(2\times a)+(b+b)$ $(2\times a)+(2\times b)$

**Generalization:** Consider all the classes up to $k$ levels



# Error Bound Computation

## Elementary operations: (current rounding mode $= \circ_\sim$)

$$
\begin{array}{cccc}
 & [x_1] & , & [\epsilon_1] \\
+ & [x_2] & , & [\epsilon_2] \\
\hline
= & [x_1] \oplus_\sim [x_2] & , & [\epsilon_1] \oplus_\leftrightarrow [\epsilon_2] \\
 & & & \oplus_\leftrightarrow \\
 & & & [\pm \frac{1}{2} ulp([x_1] +_\sim [x_2])]
\end{array}
$$

$$
\begin{array}{cccc}
 & [x_1] & , & [\epsilon_1] \\
\times & [x_2] & , & [\epsilon_2] \\
\hline
= & [x_1] \otimes_\sim [x_2] & , & [\epsilon_1] \otimes_\leftrightarrow [x_2] \\
 & & & \oplus_\leftrightarrow \\
 & & & [\epsilon_2] \otimes_\leftrightarrow [x_1] \\
 & & & \oplus_\leftrightarrow \\
 & & & [\epsilon_1] \otimes_\leftrightarrow [\epsilon_2] \\
 & & & \oplus_\leftrightarrow \\
 & & & [\pm \frac{1}{2} ulp([x_1] \otimes_\sim [x_2])]
\end{array}
$$

```
-] 0.1
      float64:  [9.999999999999999E-2,1.000000000000001E-1]
        error:  [-6.938893903907228E-18,6.938893903907229E-18]
-] 0.2
      float64:  [1.999999999999999E-1,2.000000000000001E-1]
        error:  [-1.387778780781445E-17,1.387778780781446E-17]
-] 0.1+0.2
      float64:  [2.999999999999999E-1, 3.000000000000001E-1]

        error:  [-4.857225732735059E-17, 4.857225732735060E-17]
```

# Overview

Simple imperative language

$$
\begin{aligned}
\mathsf{Cmd} \ni c \quad ::= \quad & \mathtt{x} \ := e \mid c \ ; \ c \mid \mathtt{END} \\
& \mid \mathtt{if}\ b\ \mathtt{then}\ c\ \mathtt{else}\ c \\
& \mid \mathtt{while}\ b\ \mathtt{do}\ c
\end{aligned}
$$

Programs terminate by `END`

Single static assignments

Same variables written in both branches of a conditionnal

# Transformation Rules

$$\mathsf{FEnv} \;:\; \mathsf{Id} \to \mathsf{Cmd} \qquad \mathsf{VEnv} \;:\; \mathsf{Id} \to \mathsf{Val}^\sharp$$

$$\begin{array}{ccccccccccc} \mathcal{T} & : & \mathsf{Cmd} & \times & \mathsf{FEnv} & \times & \wp(\mathsf{Id}) & \times & \mathsf{VEnv} & \to & \mathsf{Cmd} \\ & & (c & , & \varrho & , & \lambda & , & \varsigma) & \mapsto & c_{opt} \end{array}$$

$\lambda$ set of identifiers whose accuracy has to be optimized

## Transformation Rules: Assignments

$$\mathcal{T}\big(\mathtt{x} := e\ ;\ c, \varrho, \lambda, \varsigma\big) = \mathcal{T}\big(c, \varrho[\mathtt{x} \mapsto e], \lambda, \varsigma\big)$$

$$\mathcal{T}\big(\mathtt{END}, \varrho, \lambda, \varsigma\big) = \mathtt{REIFY}(\varrho, \lambda, \varsigma)$$

$$\begin{array}{rclcl} \mathtt{REIFY}(\varrho, \lambda) & : & \mathsf{FEnv} \times \wp(\mathsf{Id}) \times \mathsf{VEnv} & \to & \mathsf{Cmd} \\ & & (\varrho, \lambda, \varsigma) & \mapsto & c \end{array}$$

$$\mathtt{REIFY}(\mathtt{x}, \varrho, \varsigma) = \mathtt{x}\ := \mathcal{R}(\mathtt{INLINE}(e, \varrho), \varsigma)$$

$\mathcal{R}(e, \varsigma)$ optimizes the expression $e$ for the environment $\varsigma$

# Transformation Rules: Conditionals

$$\mathcal{T}\big(\texttt{if } b \texttt{ then } c_1 \texttt{ else } c_2 \texttt{ ; } c, \varrho, \lambda, \varsigma\big) =$$

$$\left|\begin{array}{l}
\texttt{REIFY}(\varrho, \texttt{VAR}(b), \varsigma) \texttt{ ;} \\
\texttt{if } b \texttt{ then } c_1' \texttt{ else } c_2' \texttt{ ;} \\
\mathcal{T}(c, \varrho, \lambda \setminus (\texttt{VAR}(b) \cup \lambda'), \varsigma) \\
\\
\textit{where}: \\
c_1' = \mathcal{T}(c_1 \texttt{ ; END}, \varrho, \lambda', \varsigma), \\
c_2' = \mathcal{T}(c_2 \texttt{ ; END}, \varrho, \lambda', \varsigma), \\
\lambda' = \texttt{WRITE}(c_1, c_2) \cap (\lambda \cup \texttt{READ}(c))
\end{array}\right.$$

# Transformation Rules: Loops

$$\mathcal{T}\big(\texttt{while}\ b\ \texttt{do}\ c_1\ ;\ c, \varrho, \lambda, \varsigma\big) =$$

$$\left|\begin{array}{l}
\texttt{REIFY}(\varrho, \texttt{VAR}(b), \varsigma)\ ; \\
\texttt{while}\ b\ \texttt{do}\ c_1'; \\
\mathcal{T}(c, \varrho, \lambda \setminus (\texttt{VAR}(b) \cup \lambda'), \varsigma) \\
\\
\textit{where}: \\
c_1' = \mathcal{T}(c_1\ ;\ \texttt{END}, \varrho, \lambda', \varsigma), \\
\lambda' = \texttt{WRITE}(c_1) \cap (\lambda \cup \texttt{READ}(c))
\end{array}\right.$$

# Transformation Rules: Functions

$$\mathcal{T}\Big(\texttt{x} \ := f(e_1, \ldots, e_n) \ ; \ c, \varrho, \lambda, \varsigma\Big) =$$

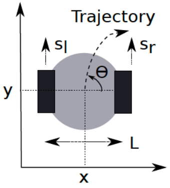
$$\left| \begin{array}{l} \texttt{REIFY}(\varrho, \texttt{VAR}(e_1) \cup \ldots \cup \texttt{VAR}(e_n), \varsigma) \ ; \\ \texttt{x} \ := f(\mathcal{R}(e_1, \varsigma), \ldots, \mathcal{R}(e_n, \varsigma)) \ ; \\ \mathcal{T}(c, \varrho, \lambda \setminus \{\texttt{x}_1, \ldots, \texttt{x}_\texttt{n}\}, \varsigma) \end{array} \right.$$

$$\mathcal{T}\Big(f(x_1, \ldots, x_n) \ \{c \ ; \ \texttt{return} \ x \ \}, \varrho, \lambda, \varsigma\Big) =$$

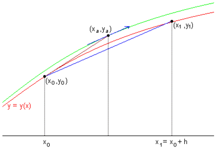
$$f(x_1, \ldots, x_n) \ \{\mathcal{T}(c, \bot, \{x\}, \varsigma); \ \texttt{return} \ x \ \}$$

# Case Studies

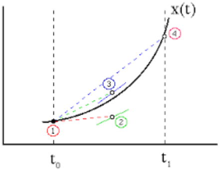
## Odometry



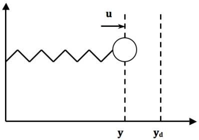

## Runge-Kutta 2



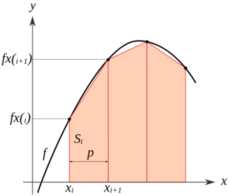

## Runge-Kutta 4



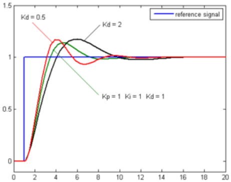

## Spring-Mass System



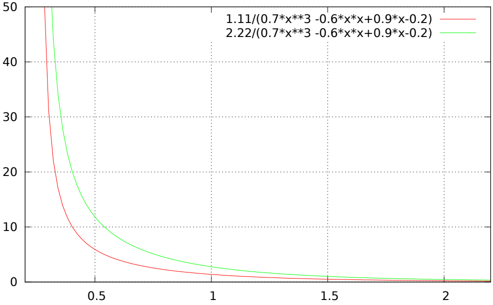

## Trapeze Method




## PID



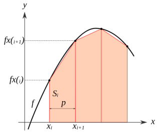
# Trapeze Method




$$I = \int_a^b f(x)dx$$

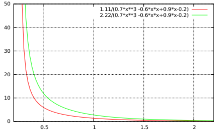
$$g(x) = \frac{u}{0.7x^3 - 0.6x^2 + 0.9x - 0.2} \qquad u \in [1.11, 2.22]$$

# Trapeze Method

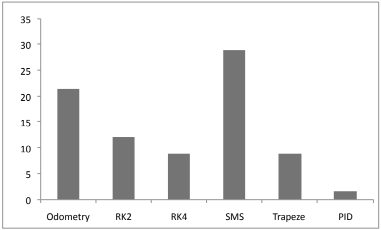



```
u = [1.11, 2.22]; a = 0.25;
b = 5000.0; n = 25.0; r = 0.0;
xa = 0.25; h = ((b - a) / n) ;
while (xa < 5000.0) do {
  xb = (xa + h) ;
  if (xb>5000.0) then xb=5000.0 ;
  gxa=(u/((((((0.7*xa)*xa)*xa)
    -((0.6*xa)*xa))+(0.9*xa))-0.2));
  gxb=(u/((((((0.7*xb)*xb)*xb)
    -((0.6*xb)*xb))+(0.9*xb))-0.2));
  r = (r+(((gxb+gxa)*0.5)*h)) ;
  xa = (xa + h)
}
```

```
u = [1.11, 2.22] xa = 0.25; r = 0.0;
while (xa < 5000.)  do {
  xb = (xa + 199.99) ;
  if (xb>5000.0) then xb=5000.0 ;
  r=(r+(99.995*((u/(((0.9*xa)
    -0.2)+(((xa*(xa*xa))*0.7)
    -((xa*xa)*0.6))))
    +(u/(((0.9*(xa+199.99))-0.2)
    +(((199.99+xa)*((199.99+xa)
    *((199.99+xa)*0.7)))-
    ((199.99+xa)*
    ((199.99+xa)*0.6))))))));
  xa = (xa + 199.99)
}
```



# Experimental Results





| Code | Initial Error | New Error |
|---|---|---|
| Odom | 0.111405189793203 e-10 | 0.881575893315158 e-11 |
| RK2 | 0.750448486755706 e-7 | 0.658915054553695 e-7 |
| RK4 | 0.201827996912328 e1 | 0.183745378465205 e1 |
| Spring | 0.395917695956325 e-13 | 0.281441385725130 e-13 |
| Trapeze | 0.536291684923368 e-9 | 0.488971110442931 e-9 |
| PID | 0.453945103062736 e-14 | 0.446837288725632 e-14 |

# Jacobi's Method

System of $n$ equations: $\mathbf{Ax} = \mathbf{b}$

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1, j\neq i}^{n} a_{ij} x_j^{(k)}}{a_{ii}} \qquad \textit{stop when} \qquad |x_i^{(k+1)} - x_i^k| < \epsilon$$

$$\begin{pmatrix} 0.62 & 0.1 & 0.2 & -0.3 \\ 0.3 & 0.602 & -0.1 & 0.2 \\ 0.2 & -0.3 & 0.6006 & 0.1 \\ -0.1 & 0.2 & 0.3 & 0.601 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1.0/2.0 \\ 1.0/3.0 \\ 1.0/4.0 \\ 1.0/5.0 \end{pmatrix}$$

*Sufficient condition for convergence* $|a_{ii}| > \sum_{j=1, j\neq i}^{j=4} |a_{ij}|$

# Jacobi's Method

## Original program

$e = 1.0;\ eps = 10^{e-16};\ a_{11} = 0.61;\ a_{22} = 0.602;\ a_{33} = 0.6006;\ a_{44} = 0.601;$

$b_1 = 0.5;\ b_2 = 1.0/3.0;\ b_3 = 0.25;\ b_4 = 1.0/5.0;$

$while\ (e > eps)\ \{$

$\quad x_{n1} = (b_1/a_{11}) - (0.1/a_{11}) * x_2 - (0.2/a_{11}) * x_3 + (0.3/a_{11}) * x_4;$

$\quad x_{n2} = (b_2/a_{22}) - (0.3/a_{22}) * x_1 + (0.1/a_{22}) * x_3 - (0.2/a_{22}) * x_4;$

$\quad x_{n3} = (b_3/a_{33}) - (0.2/a_{33}) * x_1 + (0.3/a_{33}) * x_2 - (0.1/a_{33}) * x_4;$

$\quad x_{n4} = (b_4/a_{44}) + (0.1/a_{44}) * x_1 - (0.2/a_{44}) * x_2 - (0.3/a_{44}) * x_3;$

$\quad e = x_{n1} - x_1;$

$\quad x_1 = x_{n1};$

$\quad x_2 = x_{n2};$

$\quad x_3 = x_{n3};$

$\quad x_4 = x_{n4};$

$\}$

# Jacobi's Method

Accuracy optimized up to 44.5%

## Transformed program

$e = 1.0;\ \ eps = 10^{e-16};$
$while\ (e > eps)\ \{$
$\quad TMP_1 = (0.553709856035437 - (x_1 * 0.498338870431894));$
$\quad TMP_2 = (0.166112956810631 * x_3);$
$\quad TMP_6 = (0.333000333000333 * x_1);$
$\quad x_{n1} = (((0.819672131147541 - (0.163934426229508 * ((TMP_1 + TMP_2) - (0.332225913621263 * x_4)))) - (0.327868852459016 * (((0.416250416250416 - TMP_6) + (0.4995004995005 * x_2)) - (0.166500166500167 * x_4)))) + (0.491803278688525 * (((0.332778702163062 + (0.166389351081531 * x_1)) - (0.332778702163062 * x_2)) - (0.499168053244592 * x_3))));$
$\quad x_{n2} = (((0.553709856035437 - (0.498338870431894 * x_{n1})) + (0.166112956810631 * (((0.416250416250416 - TMP_6) + (0.4995004995005 * x_2)) - (0.166500166500167 * x_4)))) - (0.332225913621263 * (((0.332778702163062 + (0.166389351081531 * x_1)) - (0.332778702163062 * x_2)) - (0.499168053244592 * x_3))));$
$\quad x_{n3} = (((0.416250416250416 - (0.333000333000333 * x_{n1})) + (0.4995004995005 * x_{n2})) - (0.166500166500167 * (((0.332778702163062 + (0.166389351081531 * x_1)) - (0.332778702163062 * x_2)) - (0.499168053244592 * x_3))));$
$\quad x_{n4} = (((0.332778702163062 + (0.166389351081531 * x_{n1})) - (0.332778702163062 * x_{n2})) - (0.499168053244592 * x_{n3}));$
$\quad e = (x_{n4} - x_4);$
$\quad x_1 = x_{n1};$

# Jacobi's Method

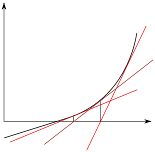
## Experimental Results

| $x_i$ | Initial it nbr | Optimized it nbr | Difference | Percentage |
|---|---|---|---|---|
| $x_1$ | 1891 | 1628 | 263 | 14.0 |
| $x_2$ | 2068 | 1702 | 366 | 17.3 |
| $x_3$ | 2019 | 1702 | 317 | 15.7 |
| $x_4$ | 1953 | 1628 | 325 | 16.7 |

Number of iterations of Jacobi's method needed before and after optimization to compute $x_i$, $1 \leq i \leq 4$

# Newton-Raphson's Method

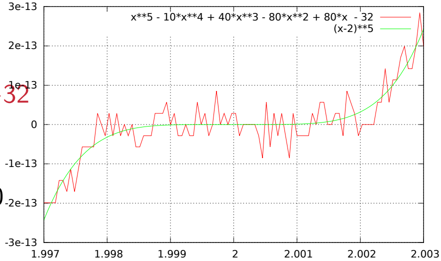
Newton's method converges quickly in $\mathbb{R}$
Problems due to floating-float errors



$$f(x) = (x-2)^5$$

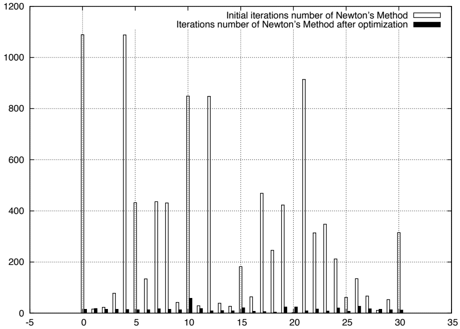
$$f(x) = x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$$

$$f'(x) = 5x^4 - 40x^3 + 120x^2 - 160x + 80$$



# Newton-Raphson's Method

## Experimental Results

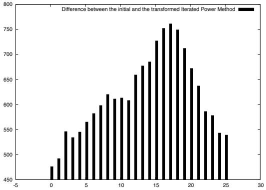



Number of iterations of the Newton-Raphson's Method before and after optimization for initial values ranging from 0 to 3 (30 runs with a step of 0.1)

## Iterated Power Method

Computes the largest eigenvalue

$$\mathbf{A} = \begin{pmatrix} \mathtt{d} & 0.01 & 0.01 & 0.01 \\ 0.01 & \mathtt{d} & 0.01 & 0.01 \\ 0.01 & 0.01 & \mathtt{d} & 0.01 \\ 0.01 & 0.01 & 0.01 & \mathtt{d} \end{pmatrix}$$

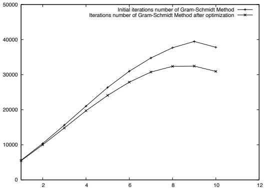



Difference between numbers of iterations of initial and optimized Iterated Power Method (tests done for $\mathtt{d} \in [175, 200]$ with a step of 1)

## Orthogonalization of a set of vectors

Numerical problems for small vectors

$$\left\{ \mathbf{q}_1 = \begin{pmatrix} 1/7\mathtt{n} \\ 0 \\ 0 \end{pmatrix}, \mathbf{q}_2 = \begin{pmatrix} 0 \\ 1/25\mathtt{n} \\ 0 \end{pmatrix}, \mathbf{q}_3 = \begin{pmatrix} 1/2592 \\ 1/2601 \\ 1/2583 \end{pmatrix} \right\}$$




Num. of it. of initial and optimized iterative GS for the family $(Q_n)_n$ of vectors, $1 \le n \le 10$.

# Performance Analysis

## Complementary results: speedups

| | Original Code Execution Time in $s$ | Optimized Code Execution Time in $s$ | Percentage Improvement | Mean on $n$ Runs |
|---|---|---|---|---|
| Jacobi | $1.49 \cdot 10^{-4}$ | $0.38 \cdot 10^{-4}$ | 74.5% | $10^4$ |
| Newton | $1.34 \cdot 10^{-3}$ | $0.02 \cdot 10^{-3}$ | 98.4% | $10^4$ |
| Eigenvalue | $4.50 \cdot 10^{-2}$ | $3.07 \cdot 10^{-2}$ | 31.6% | $10^3$ |
| Gram-Schmidt | $1.99 \cdot 10^{-1}$ | $1.70 \cdot 10^{-1}$ | 14.5% | $10^2$ |

Execution time measurements of programs

# Performance Analysis

| Method | $\sharp$ of $\pm$ per it Original Code | $\sharp$ of $\pm$ per it Optimized Code | Total $\sharp$ of $\pm$ Original Code | Total $\sharp$ of $\pm$ opt Optimized Code | Percentage of Improvement |
|---|---|---|---|---|---|
| Jacobi | 13 | 15 | 25389 | 24420 | 3.81 |
| Newton-Raphson | 11 | 11 | 3465 | 132 | 96.19 |
| Eigenvalue | 15 | 15 | 694080 | 685995 | 1.16 |
| Gram-Schmidt | 21 | 19 | 791364 | 715996 | 9.52 |
| Method | $\sharp$ of $\times$ per it Original Code | $\sharp$ of $\times$ per it Optimized Code | Total $\sharp$ of $\times$ Original Code | Total $\sharp$ of $\times$ opt Optimized Code | Percentage of Improvement |
| Jacobi | 28 | 14 | 54684 | 22792 | 58.32 |
| Newton-Raphson | 27 | 26 | 8505 | 312 | 96.33 |
| Eigenvalue | 19 | 19 | 879168 | 868927 | 1.16 |
| Gram-Schmidt | 22 | 20 | 712316 | 647560 | 9.09 |

Number of floating-point operations needed by the programs to converge

# Perspectives


Inter-procedural transformation

Lustre version

Generate correctness certificates

Improve static analysis

Estimate improvements on runs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