Applying Spectral Methods To Mizer

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Feeding Kernels

Let $\{1, .., s\}$ denote the set of species.

The feeding Kernel Φ_i measures amount of preference that a predator has weight W_p has for a prey of weight w is

$$\Phi_i\left(\frac{w_p}{w}\right) = \exp\left[\frac{-\left(\ln\left(\frac{w}{w_p}\right) - \ln(\beta_i^*)\right)^2}{2\sigma_i^2}\right].$$

We shall normalize mass by dividing by the egg size w_0, and logs, to re-represent this information in the 'x-space' where $x = \frac{\log(w)}{log(w_0)}$, and $y = \frac{\log(w_p)}{log(w_0)}$. Before we consider the feeding kernel in these terms, let us note, that for any real number v we have:

$$\Phi_i\left(e^v\right) = \exp\left[\frac{-\left(\ln\left(e^{-v}\right) - \ln(\beta_i^*)\right)^2}{2\sigma_i^2}\right] = \exp\left[\frac{-\left(v + \beta_i\right)^2}{2\sigma_i^2}\right],$$

where $\beta_i = \ln(\beta_i^*)$, and $\Phi_i\left(e^v\right)$ is concentrated about $-\beta_i$.

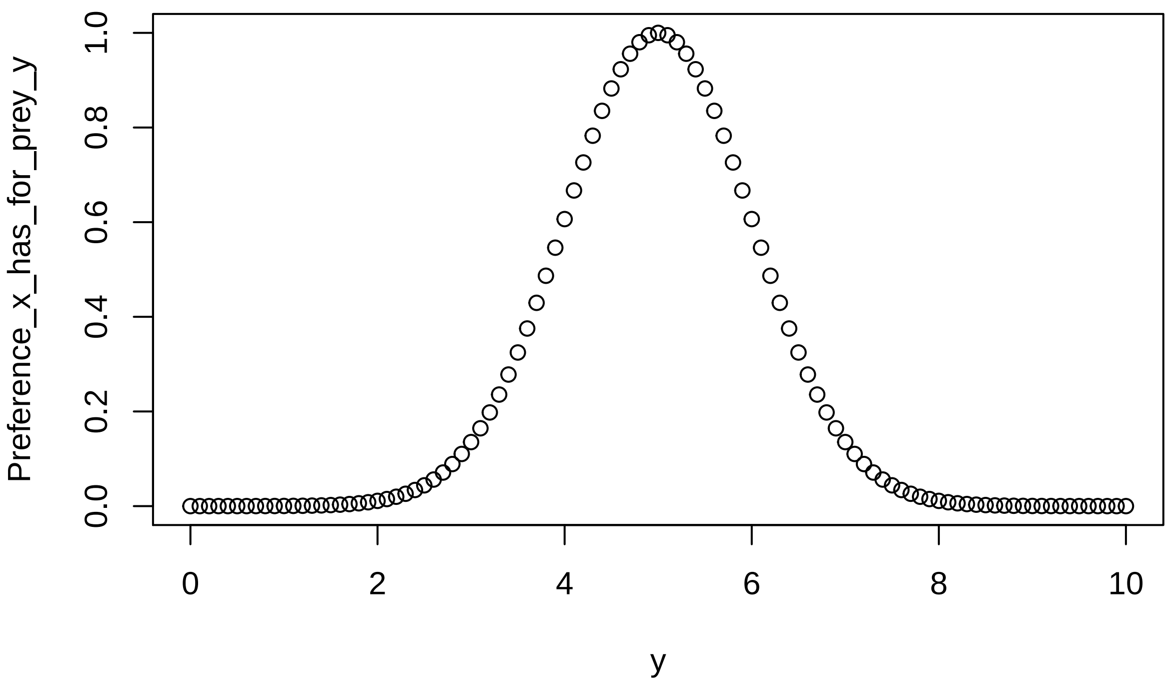
Let us return to consideration of $\Phi_i\left(\frac{w_p}{w}\right)$. By making our substitutions, we can rewrite Phi_i in terms of x and y as

$$\Phi_i\left(\frac{w_p}{w}\right) = \Phi_i\left(\frac{w_0 e^y}{w_0 e^x}\right) = \Phi_i(e^{y-x}) = \exp\left[\frac{-\left(y - x + \beta_i\right)^2}{2\sigma_i^2}\right]$$

To illustrate, let us consider a practical example where the width of the feeding distribution is $\sigma_i = 1$, the (log-space) predator size is $x = 8$, and the log of the preferred predator-prey mass ratio is $\beta_i = \ln(\beta_i^*) = 3$.

Below we plot the resulting feeding kernel, the horizontal axis gives the (log-space) prey size, and the vertical axis measures the amount of preference that our size $x = 8$ predators have for this prey:

```
sigma <- 1
x <- 8
beta <- 3
dy <- 0.1
y <- seq(0, 10, by = dy)
Preference_x_has_for_prey_y <- exp((-((y-x+beta)^2))/(2*sigma^2))
plot(y,Preference_x_has_for_prey_y)
```



Because the size of the predator is $x = 8$, its most preferred prey size is

$$x - \beta_i = 8 - 3$$

, which is the y value at which the feeding preference function plotted above is concentrated at.

mass ratios,

Next: add x and y in, and make plot

and

The predation mortality for a species i, with a size w such that

$$x = \frac{\log(w)}{log(w_0)}$$

In the x space we want to evaluate the $\mu_{P,i}(x)$,$\forall x \in [x_0, X_i]$, where w_0 is the fish size, and X_i is the maximal size/ biomass of species $i \in 1, .., s$, and s is the number of species.

$$\mu_{P,i}(x) = \sum_{j=1}^{s} \int_{-\infty}^{\infty} \phi_j(y) q_{j,i}(x - y) dy$$

We can rewrite this expression as $\mu_{P,i}(x) = \sum_{j=1}^{s} \mathbb{I}_{j,i}(x)$,where $\mathbb{I}_{j,i}(x) = \int_{-\infty}^{\infty} \phi_j(y) q_{j,i}(x-y)$. Here $supp(q_{j,i}) = [x_0, X_j] = [0, X_j]$, and

Truncated gaussian feeding kernel from mizer

We define our truncated Gaussian feeding kernel $\phi_j(v)$ such that $\forall v \in \mathbb{R}$ we have

$$\Phi_j(e^v) = \phi_j(v) = \begin{cases} \exp\left(\frac{-(v+\beta_j)^2}{2\sigma_j^2}\right) & \text{if } v \in [-\beta_j - 3\sigma_j, 0] \\ 0 & \text{otherwise} \end{cases}$$

denotes the feeding kernel. Although in practice $\phi_j(v)$ is a truncated Gaussian, concentrated at $-\beta_j$. Naturally $\phi_j(v) \to 0$ as $v \to -\beta_j - 3\sigma_j$, but we also artificially the feeding kernel in mizer so that $v \geq \phi_j(v) \Rightarrow \phi_j(v) = 0$,to represent how predators will not eat prey larger than themselves.

Mortality Integral And Spectral Methods

In order to determine $\mu_{P,i}(x) = \sum_{j=1}^{s} \mathbb{I}_{j,i}(x)$, we wish to evaluate each

$$\mathbb{I}_{j,i}(x) = \int_{-\infty}^{\infty} \phi_j(y) q_{j,i}(x-y).dy = \int_{-\beta_j - 3\sigma_j}^{0} \phi_j(y) q_{j,i}(x-y).dy$$

$\forall x \in [x_0, X_i] = [0, X_j]$

where $supp(q_{j,i}) = [x_0, X_j] = [0, X_j]$

We can do this directly, using spectral methods.

In this integral, we gave that $q_{j,i}(v)$ is used $\forall v = x - y \in [0, X_i + \beta_j + 3\sigma_j]$. This means it makes treat $\mathbb{I}_{j,i}(x)$ is a convolution integral, with period

$$P_{i,j} = length\left([-\beta_j - 3\sigma_j, 0]\right) + length\left([0, X_i]\right) = X_i + \beta_j + 3\sigma_j = P_{j,i}$$

Suppose $\bar{\phi_j}(v)$ is the periodic extension of $\phi_j(v)$ that agrees with $\phi_j(v)$, for all $v \in [-\beta_j - 3\sigma_j, -\beta_j - 3\sigma_j + P_{j,i}]$.

Suppose $\overline{q_{j,i}(v)}$ is the periodic extension of $q_{j,i}(v)$ that agrees with $q_{j,i}(v)$, $\forall v \in [0, X_i + \beta_j + 3\sigma_j] = [0, P]$.

We can find the appropriate data to input into our spectral integration method by noting that $v \in [0, P_{j,i}] \Rightarrow \overline{q_{j,i}(v)} = q_{j,i}(v)$ and

Note that $\forall v \in [0.P] \Rightarrow \bar{\phi_j}(v) = \phi_j(v - P)$, and now we can evaluate

$$\mathbb{I}_{j,i}(x) = \int_{-\beta_j - 3\sigma_j}^{0} \phi_j(y) q_{j,i}(x-y).dy$$

$$\mathbb{I}_{j,i}(x) = \int_{-\beta_j - 3\sigma_j}^{-\beta_j - 3\sigma_j + P_{j,i}} \bar{\phi_j}(v) \overline{q_{j,i}(v)}.dy$$

$$\mathbb{I}_{j,i}(x) = \int_{-\beta_j - 3\sigma_j}^{-\beta_j - 3\sigma_j + P_{j,i}} \phi_j(v - P) q_{j,i}(x-y).dy$$

$$\mathbb{I}_{j,i}(x) = \int_{0}^{P_{j,i}} \phi_j(v - P) q_{j,i}(x-y).dy$$

Differences in notation

In the mizer vignette they write β_i. I write β_i^* to mean the same thing.

In the mizer vignette they write ϕ_i. I write Φ_i to mean the same thing.