CSCI 570 - Summer 2021 - HW 4

due August 3, 2021

1 Graded Problems

1. The edge connectivity of an undirected graph is the minimum number of edges whose removal disconnects the graph. Describe an algorithm to compute the edge connectivity of an undirected graph with n vertices and m edges in $\mathcal{O}(m^2 n)$ time.

2. At a dinner party, there are n families $a_1, a_2, ..., a_n$ and m tables $b_1, b_2, ..., b_m$. The ith family a_i has g_i members and the jth table b_j has h_j seats. Everyone is interested in making new friends and the dinner party planner wants to seat people such that no two members of the same family are seated in the same table. Design an algorithm that decides if there exists a seating assignment such that everyone is seated and no two members of the same family are seated at the same table.

3. There is a precious diamond that is on display in a museum at m disjoint time intervals. There are n security guards who can be deployed to protect the precious diamond. Each guard has a list of intervals for which he/she is available to be deployed. Each guard can be deployed to at most A time slots and has to be deployed to at least B time slots. Design an algorithm that decides if there is a deployment of guards to intervals such that each interval has either exactly one or exactly two guards deployed.

4. Solve Kleinberg and Tardos, Chapter 7, Exercise 7.

5. Solve Kleinberg and Tardos, Chapter 7, Exercise 9.

2 Practice Problems

1. An edge of a flow network G is called critical if decreasing the capacity of this edge results in a decrease in the maximum flow. Is it true that with respect to a maximum flow of G, any edge whose flow is equal to its capacity is a critical edge? Give an efficient algorithm that finds a critical edge in a flow network.