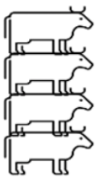
CSCI 570 - Summer 2021 - Homework 2
Due 2:59 PM, July 13th
Late submissions won't be graded.

I. Graded Problems

1) Suppose you are given two sets A and B, each containing n positive integers. You can choose to reorder each set however you like. After reordering, let a_i be the i-th element of set A, and let b_i be the i-th element of set B. You then receive a payoff on $\prod_{i=1}^{n} a_i^{b_i}$. Give an algorithm that will maximize your payoff. Prove that your algorithm maximizes the payoff, and state its running time.
2) When we have two sorted lists of numbers in non-descending order, and we need to merge them into one sorted list, we can simply compare the first two elements of the lists, extract the smaller one and attach it to the end of the new list, and repeat until one of the two original lists become empty, then we attach the remaining numbers to the end of the new list and it's done. This takes linear time. Now, try to give an algorithm using $\mathcal{O}(n \log k)$ time to merge k sorted lists (you can also assume that they contain numbers in non-descending order) into one sorted list, where n is the total number of elements in all the input lists. (Hint: Use a min-heap for k-way merging.)
3) Let G a connected graph with no edges having the same weight. Prove that the edge with the minimum weight must be on the minimum spanning tree.
4) Consider all spanning trees ordered by their weight. Show that the second best minimal spanning tree can be obtained from the minimal spanning tree by dropping only one edge and adding only one new edge instead.

5) Farmer John has N cows $(1, 2, \ldots, N)$ who are planning to escape to join the circus. His cows generally lack creativity. The only performance they came up with is the "cow tower". A "cow tower" is a type of stunt in which every cow (except for the bottom one) stands on another cow's back and supports all cows above in a column. The cows are trying to find their position in the tower. Cow i $(i = 1, 2, \ldots, N)$ has weight W_i and strength S_i. The "risk value" of cow i failing (R_i) is equal to the total weight of all cows on its back minus S_i.



Design an algorithm to help cows find their positions in the tower such that we minimize the maximum "risk value" of all cows.

II. PRACTICE PROBLEMS

1) Solve Kleinberg and Tardos, Chapter 3, Exercise 3.
2) Solve Kleinberg and Tardos, Chapter 3, Exercise 6.
3) Solve Kleinberg and Tardos, Chapter 4, Exercise 21.
4) Solve Kleinberg and Tardos, Chapter 4, Exercise 22
5) (a) Consider the problem of making change for n cents using the fewest number of coins. Describe a greedy algorithm to make change consisting of quarters (25 cents), dimes (10 cents), nickels (5 cents) and pennies (1 cents). Prove that your algorithm yields an optimal solution. (Hints: consider how many pennies, nickels and dimes and dime plus nickels are taken by an optimal solution at most.)

   (b) For the previous problem, give a set of coin denominations for which the greedy algorithm does not yield an optimal solution. Assume that each coin's value is an integer. Your set should include a penny so that there is a solution for every value of n.