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CSCI 570 – HW3

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* **[Question 1]**

1. , thus , and observed that which has *k* = 1. It is Case 2; thus, answer is:

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|  |

1. Observed that , and , thus which means that for any . It is Case 1; thus, the answer is:

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|  |

1. Observed that thus . , and , and = for any . Moreover:

, which is ,

which means

which means

which means

which means(and is roughly equal to).

Since there exists a value *k* such that , thus it is Case 3, therefore the answer is:

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| --- |
|  |

1. , thus . And for any . It is Case 3, thus answer is:

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|  |

1. Use the change of variables , to get . Then denote , so now we can transform:

and from the new algorithm we have , which leads to . And observe that which has *k = 0*. Thus, it is Case 2, and answer is:

and since we used the change of variables, and observed that , thus the real answer is:

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|  |

1. Cannot be solved by Master Theorem, since and the cannot be negative.
2. Cannot be solved by Master Theorem, since *a* is not constant.
3. , thus . And observed that = for any *0.01 > > 0*. Furthermore, if plugin *a* and *b* to , then we have:

, then

, then

we have , which means there exists a value *k* such that . Thus, it is Case 3, so answer is:

|  |
| --- |
|  |

1. Cannot be solved by Master Theorem, since *a* is less than 1.
2. , thus . Moreover, observed that for any *> 0*. Furthermore, if plugin the values of *a* and *b* into , then we have:

, then

, then

, then

, then

, then

, then (for ).

Therefore, we have seen that when , then there exists a value *k* such that . Thus, it is Case 3, so answer is:

|  |
| --- |
|  |

* **[Question 2]**

Sort

* **[Question 3]**

# I. Define Subproblem:

Sdfsdf

# II. Recurrence Relation:

Sdsdf

# III. Pseudo Code:

Sdasdasdasd

# IV. Runtime Complexity:

Sdasdasdasd

* **[Question 4]**

# I. Define Subproblem:

Asdasd

# II. Recurrence Relation:

Sdasdasdasd

# III. Pseudo Code:

Sdasdasdasd

# IV. Runtime Complexity:

Sdasdasdasd

* **[Question 5]**

# I. Define Subproblem:

Let 1-dimensional array *PointsAt* stores the maximum points that can be accumulated when starting from the index *i* (for ). And we need another 1-dimensional array *PointsAfter*, so that *PointsAfter*[*i*]stores the maximum points among *PointsAt* at or after index *i*. In other words: *PointsAfter*[*i*] *=* Max (*PointsAt*[*j*]*,* for).

# II. Recurrence Relation:

The base case is when starting from the position *n*, *PointsAt*[*n*] *= PointsAfter*[*n*]= *a*[*n*]. The Recursive formula is:

*PointsAt*[*i*] *= a*[*i*] + {*PointsAfter*[*i + a*[*i*]] if *i + a*[*i*] *n*, otherwise 0}

Skip position *i* unless starting from there can earn more points:

*PointsAfter*[*i*] = max (*PointsAt*[*i*], *PointsAfter*[*i*+1]).

And the final result is *PointsAt*[*1*].

# III. Pseudo Code:

PointsAt[n] = a[n];

PointsAfter[n] = a[n];

for (int i = n-1; i >0; --i) {

PointsAt[i] = a[i];

if(a[i] + i <= n) {

PointsAt[i] += PointsAfter[i];

}

PointsAfter[i] = Math.max(PointsAfter[i + 1], PointsAt[i]);

}

return PointsAt[1];

# IV. Runtime Complexity:

The runtime complexity is *O*(*n*). Because there are *n* subproblems and each costs constant time.

* **[Question 6]**

Prove by contradiction, so assume *a* is J-similar to *b* and they have different lengths. Since *a* and *b* have different lengths, so *a* cannot be equal to *b*, thus the case 1 fails, and we have to try case 2. In case 2, if *a* and *b* are both even-length, then we cut them in halves. However, since they have different lengths, their substrings will still have different lengths, thus we need to keep cutting them in halves until there is a substring has odd length and case 2 cannot be applied. At that point of time, we can see substrings of *a* and *b* cannot be equal to each other because they have different lengths, which further means *a* cannot be J-similar to *b*, but we assumed that *a* is J-similar to *b*. Contradiction!

* **[Question 7]**

Sort